$$f(y;\theta,\phi) = \frac{\phi\, a(y,\phi)}{\pi(1+y^2)^{\frac{1}{2}}} \exp\left[\phi\left\{y\theta + (1-\theta^2)^{\frac{1}{2}}\right\}\right]$$

$0 < \theta < 1$, $y \in \mathbb{R}$ e $\phi > 0$

Segue que

$$f(y;\theta,\phi) = \exp\left[\phi\left\{y\theta + (1-\theta^2)^{\frac{1}{2}}\right\} + c(y;\phi)\right]$$

em que $b(\theta) = -\sqrt{1-\theta^2}$

e $c(y;\phi) = \log\left\{\dfrac{\phi\, a(y;\phi)}{\pi(1+y^2)^{\frac{1}{2}}}\right\}$

$$\Rightarrow b'(\theta) = \frac{-1}{2\sqrt{1-\theta^2}}(-2\theta) = \frac{\theta}{\sqrt{1-\theta^2}} = \mu$$

$$b''(\theta) = \frac{1}{1-\theta^2}\left\{\sqrt{1-\theta^2} - \frac{\theta}{2\sqrt{1-\theta^2}}(-2\theta)\right\}$$

$$= \frac{1}{1-\theta^2}\left\{\sqrt{1-\theta^2} + \frac{\theta^2}{\sqrt{1-\theta^2}}\right\}$$

$$= \frac{1}{(1-\theta^2)^{\frac{1}{2}}} \left[1 + \frac{\theta^2}{1-\theta^2} \right]$$

$$= \frac{1}{(1-\theta^2)^{\frac{1}{2}}} \cdot \frac{1}{1-\theta^2} = \frac{1}{(1-\theta^2)^{3/2}}$$

Como $\mu = \dfrac{\theta}{\sqrt{1-\theta^2}}$

$$\Rightarrow \mu^2 = \frac{\theta^2}{1-\theta^2}$$

$$\Rightarrow 1+\mu^2 = \frac{1}{1-\theta^2}$$

$$\Rightarrow (1+\mu^2)^{3/2} = \frac{1}{(1-\theta^2)^{3/2}}$$

$$\Rightarrow \boxed{V(\mu) = (1+\mu^2)^{3/2}}$$

$$\theta = \left(\frac{\mu^2}{1+\mu^2} \right)^{\frac{1}{2}}$$

$$b(\theta) = -\sqrt{\frac{1}{1+\mu^2}}$$

$$\tilde{\theta}_i = \left(\frac{y_i^2}{1+y_i^2} \right)^{\frac{1}{2}}$$

$$\hat{\theta}_i = \left(\frac{\hat{\mu}_i^2}{1+\hat{\mu}_i^2} \right)^{\frac{1}{2}}$$

$$b(\tilde{\theta}_i) = -\sqrt{\frac{1}{1+y_i^2}}$$

$$b(\hat{\theta}_i) = -\sqrt{\frac{1}{1+\hat{\mu}_i^2}}$$

$$D(y;\hat{\mu}) = 2\sum_{i=1}^{n}\left[y_i \left\{ \left(\frac{y_i^2}{1+y_i^2}\right)^{\frac{1}{2}} - \left(\frac{\hat{\mu}_i^2}{1+\hat{\mu}_i^2}\right)^{\frac{1}{2}} \right\} - \left(\frac{1}{1+\hat{\mu}_i^2}\right)^{\frac{1}{2}} + \left(\frac{1}{1+y_i^2}\right)^{\frac{1}{2}} \right]$$

$$D(y;\bar{y}) = 2\sum_{i=1}^{n}\left[y_i \left\{ \left(\frac{y_i^2}{1+y_i^2}\right)^{\frac{1}{2}} - \left(\frac{\bar{y}^2}{1+\bar{y}^2}\right)^{\frac{1}{2}} \right\} - \left(\frac{1}{1+\bar{y}^2}\right)^{\frac{1}{2}} + \left(\frac{1}{1+y_i^2}\right)^{\frac{1}{2}} \right]$$

$$R^2 = 1 - \frac{D(y;\hat{\mu})}{D(y;\bar{y})}$$

$$D^*(y;\hat{\mu}) = \phi D(y;\hat{\mu})$$