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The Power Grid as a complex network: A survey

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ABSTRACT

The statistical tools of Complex Network Analysis are of useful to understand salient properties of complex systems, may these be natural or pertaining human engineered infrastructures. One of these that is receiving growing attention for its societal relevance is that of electricity distribution. In this paper, we present a survey of the most relevant scientific studies investigating the properties of different Power Grids infrastructures using Complex Network Analysis techniques and methodologies. We categorize and explore the most relevant literature works considering general topological properties, physical properties, and differences between the various graph-related indicators and reliability aspects. We also trace the evolution in such field of the approach of study during the years to see the improvement achieved in the analysis.

1. Introduction

Complex Network Analysis (CNA) is a relatively young field of research. The first systematic studies appeared in the late 1990s [1–4] having the goal of studying the properties of large networks that behave as complex systems. The research owes a great deal of its foundations to the seminal work on Random Graphs of Erdős and Rényi [5,6] who studied asymptotic properties of stochastic graph processes. The research on the topic has embraced the spatial structure of networks [7], the dynamical aspects [8], and more and more applications in the natural and artificial world benefit from analyzing systems using the network approach [9]. Complex Network Analysis has been used in many different fields of knowledge, from biology [10] to chemistry [11], from linguistics to social sciences [12], from telephone call patterns [13] to computer networks [14] and web [15] to virus spreading [16] to logistics [17] and also inter-banking systems [18]. Men-made infrastructures are especially interesting to study under the Complex Network Analysis lenses. Especially, those characterized by large scale and growth following a decentralized and independent fashion, thus not the result of a global, but rather of many local autonomous designs. The Power Grid is a prominent example. But what do we mean by Power Grid in the context of the present treatment?

We focus on the electricity transmission and distribution Power Grid as it is essential for today's society as an enabling infrastructure. Power Grid efficiency and operations have major consequences, among other things, for the environment. Blackouts seem to have a special role in reminding us of the importance of the Grid and how much we give its availability for granted. From the technological point of view, the electrical system and Power Grid involve many scientific knowledge areas that contribute to the design, operations and analysis of power systems: Physics (electromagnetism, classical mechanics), Electrical engineering (AC circuits and phasors, 3-phase networks, electrical systems control theory) and Mathematics (linear algebra, differential equations). Traditional studies tend to focus on specific aspects of the Grid, e.g., defining how to design a transformer and predicting its functioning. Typically, studies tend to consider on the physical and electrical

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properties (e.g., Ref. [19]), or the characteristics of the Power Grid as a complex dynamical system [20], or again, the control theory aspects [21]. The move from a "local" to a "global" view of the Power Grid as a complex system is possible by resorting to Complex Network Analysis and statistical graph theory.

The goal of the present treatment is to provide a survey and compare the most relevant and well-known scientific studies conducted using Complex Network Analysis techniques concerning Power Grid systems. We consider the field mature enough to deserve a survey classifying the approaches, the geographies, the models used and the results of network reliability analysis. To the best of our knowledge, there are only two other surveys using Complex Network Analysisas its foundations and they are both older than five years [22,23]. The work of Sun [22] is a basic comparison of the then available works using Complex Network Analysis for the Power Grid more to show a new method than showing the differences and peculiarities followed by the studies. On the same tone is the work of Bai et al. [23]: very few works are surveyed (less than five) belonging to few geographies without stressing the commonalities or differences between the Grids and giving few space to the reliability aspects. Our survey is more comprehensive, with more than 30 works analyzed, considering several studies that have emerged recently that take into account not only pure topological analysis, but also physical parameters are considered in modeling of the network. In addition, our work examines several parameters (e.g., node degree, betweenness, geography, type of network, small-world, network attacks and improvement strategies) to assess the differences between the various studies under survey with particular stress on the resilience aspects of the analysis. Thus, our modern study allows us to have a broader view on the topic, emphasizing the differences between works that analyze different Power Grids and considering the latest results obtained in the recent years using enriched Complex Network Analysis models that use also physical properties of electrical lines. In addition, with the growing interest on the Smart Grid topic, we consider beneficial to have a broad view of the characteristics of the Grids in several geographies around the globe since the Smart Grid will also require changes and updates in the Power Grid infrastructures worldwide. The paper is organized as follows: we start by introducing the methods and metrics that are evaluated in this work (Section 2). Section 3 provides the main characteristics of all the studies surveyed. The actual comparison of these using CNA metrics are reported and discussed in Section 4, while Section 5 concludes the paper.

2. Background and survey methodology

Before going further analyzing the various studies in detail, we consider the reader familiar with basic notion of graph theory and Complex Network Analysis, otherwise we refer to established literature for a general introduction [24,25].

As described in Section 1, all the works that are examined in the present manuscript consider the Power Grid networks as graphs following the mathematical meaning of the term.

The main investigation that is usually performed when analyzing the Power Grid and that is almost always the motivation that drives Complex Network Analysis studies related to electrical infrastructures is the investigation of reliability. Usually, the investigation involves evaluating the disruption behavior of the graph when its nodes or edges are removed.

Other terms to compare the various Power Grid studies involve more general characteristics of the network under analysis. In particular, the geographical location of the analyzed Grid is responsible for topological properties due to the different morphological characteristics of different countries. Another relevant aspect deals with the layer of the Power Grid under investigation since differences can emerge from a topological perspective investigating the different ends in which the Grid is usually partitioned: High, Medium and Low Voltage. It is also important to have information if the type of Power Grid graph under analysis comes from a real network infrastructure or it is a synthetic sample extracted from blueprint models for the Power Grid such as the Bus models of IEEE.

The motivations to include the works in this survey are based on the quality of the research performed, the rigor in the application of Complex Network Analysis methodologies and the geography of the Power Grid analyzed in order to cover a broad spectrum of the infrastructure realized in the different countries and identify possible differences.

3. The Power Grid as a complex network

Complex Network Analysis studies are becoming more and more popular given the amount of natural and human complex systems. The Power Grid is clearly amenable to such studies and a number of these have been performed on the High Voltage Grid. Here we describe the most important aspects of each work under investigation. In particular, the works that are considered in this review are: Refs. [22,26–57]. These have been chosen based on the following factors: they are specifically about the Power Grid, they cover US, European, Chinese Grids or synthetic topologies of electrical engineering literature, they have samples of different sizes and, most importantly, these are the best-known and most representative works on the topic of CNA and Power Grid.

3.1. Basic Power Grid characteristics

The aspects considered in this first basic assessment of the studies take into account general and non-technical aspects so to give a global idea of the Grid considered, Table 1. Several aspects of comparison are considered: the number of nodes and

Table 1
Comparison between studies using CNA for the Power Grid.

Work	Number of nodes	Number of lines	Sample type	Network type	Geography
[26]	∼14,000	∼19,600	Real	HV	North America
[27]	∼300	∼500	Real	HV	Italy
[28]	∼314,000	NA	Real	HV	North America
[29]	∼4800	∼5500	Real	HV	Scandinavia
[30]	∼2700	∼3300	Real	HV	Europe
[31]	∼3000	∼3800	Real	HV	Europe
[32]	∼3000	∼3800	Real	HV	Europe
[33]	∼370	∼570	Real	HV	Italy, France and Spain
[34]	∼370	∼570	Real	HV	Italy, France and Spain
[35]	∼4900	∼6600	Real	HV	Western US
[58]	∼8500	∼13,900	Synthetic and real	HV	Western US and New York State Area
[36]	∼4850	∼5300	Real	MV/LV	Netherlands
[37][a]	∼210	∼320	Synthetic and real	HV	China
[38]	NA	NA	Real	HV	Europe
[39]	300	411	Synthetic	HV	
[40]	∼6400	∼8700	Synthetic and real	HV	North America, Scandinavia and Korea
[41]	300	411	Synthetic	HV	
[42]	∼8500	∼13,900	Synthetic and real	HV	Western US and New York State Area
[43]	∼30	∼13,900	Synthetic and real	HV	Western US and New York State Area
[44]	∼900	∼1150	Real	HV	China
[45]	∼3200	∼7000	Synthetic and real	HV	New York State Area
[46]	∼4900	∼6600	Real	HV	Western US
[47]	∼1700	∼1800	Real	HV	China
[48]	∼39	∼46	Syntethic	HV	
[49]	∼39	∼46	Syntethic	HV	
[50]	∼2500	∼2900	Real	HV	China
[22]	∼15,400	∼18,400	Real	HV	North America and China
[51]	∼550	∼800	Synthetic	HV	
[52]	∼14,000	∼19,600	Real	HV	North America
[53]	∼90	∼120	Synthetic	HV	
[54]	∼550	∼700	Synthetic and real	HV	Italy
[55]	∼29,500	∼50,000	Synthetic and real	HV	North America
[56]	∼400	∼700	Synthetic		
[57]	∼900	∼1300	Synthetic and real	HV	South-East US
[59]	∼60	∼110	Real	HV	India

[a] The values for nodes and lines in this table refer only to a snapshot of Shanghai Power Grid.

lines composing the Grid (second and third column)[1]; the type of sample considered either a real Grid or synthetic samples, for instance, coming from IEEE literature such as IEEE Bus systems (fourth column); the type of Grid analyzed (fifth column) in belonging either to the transmission part (High Voltage) or to the distribution part (Medium and Low Voltage); another essential information deals with the geography of the Grid (last column).

The data are in the most cases extracted from real samples, that is, they represent real electric infrastructures deployed. Other works in addition to real Power Grids consider synthetic models as shown in Fig. 1. Most of these studies investigating synthetic samples use IEEE blueprint networks such as IEEE Bus systems (a representation of the various IEEE Bus models used in the surveyed articles is shown in Fig. 2), while very few concentrate only on other synthetic networks (e.g., non-IEEE models, small-world models, random graphs). The number of synthetic models used is shown in Fig. 3 and almost 85% of them consider IEEE literature. Almost all samples belong to the High Voltage end of the Power Grid. High Voltage contains the lines used for long range transmission to which big power plants are attached too; the only exception is our study [36] that is focused on the distribution part of the Grid (i.e., Medium and Low Voltage network).

From a geographical perspective the samples are mainly localized in the United States or in Europe with some studies that consider Chinese High Voltage samples; a map of the countries whose Grids are analyzed is represented in Fig. 4, while the number of Grids analyzed for a given country is shown in Fig. 5. Another main commonality is to treat the Grid as an undirected graph where each substation or transformer represents a node and each line transporting electricity is an edge.

3.2. Statistical global graph properties

The main characteristics from a graph and Complex Network Analysis perspective of the Grids under analysis are summarized in Table 2. Several aspects of comparison are considered: the *order* (N) and *size* (M) of the graph (second and third column) corresponds to the number of nodes (*order*) and number of lines (*size*) actually in the Power Grids. The average

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[1] Notice that the numbers in the second and third column are not the exact numbers, but they are an approximation to give the idea of the importance of the sample.

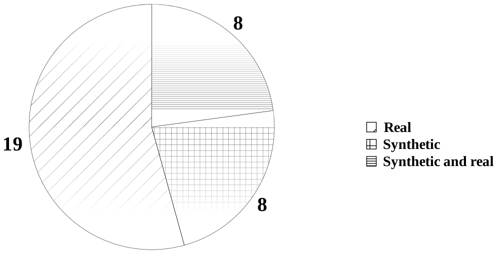

Fig. 1. Number of studies that consider real Power Grid samples or synthetic models.

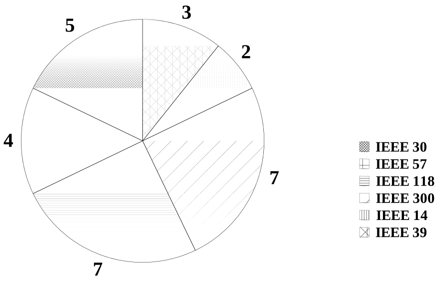

Fig. 2. IEEE literature bus model analyzed.

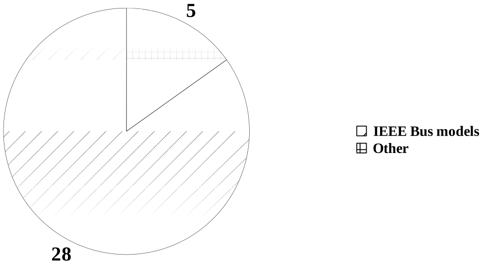

Fig. 3. Number of models used coming from the IEEE literature compared to other models.

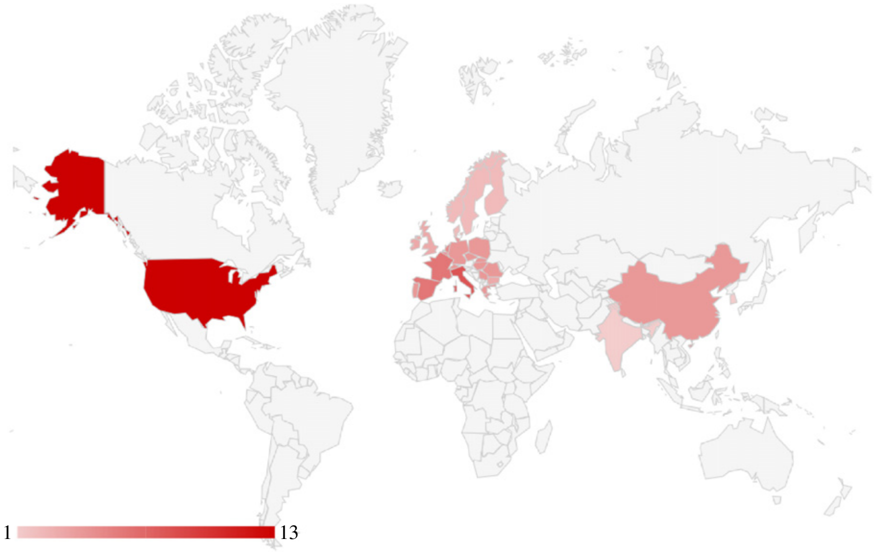

Fig. 4. Map of the Power Grid infrastructure studied using CNA approach.

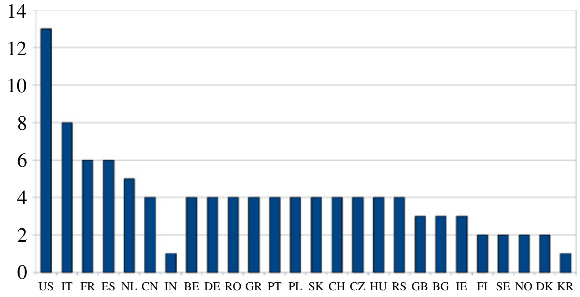

Fig. 5. Number of times the Power Grid of a given country is investigated.

degree, computed as $\langle k \rangle = \frac{2M}{N}$, gives a general idea of how many vertexes is an average vertex connected to (fourth column). Fifth, sixth and seventh column give information about the type of statistical analysis performed on the graph, in particular, the assessment of node degree distribution and betweenness distribution together with an evaluation of the path length are considered. Another term of comparison deals with the type of graph analyzed taking into account weights or simply use the unweighted definition of graph. Last two columns of the table consider the type of aim of the graph-based analysis either an investigation of the disruption behavior of the graph or the evaluation of the small-world properties.

Considering Table 2, a difference appears: the studies closer to a topological characterization uses unweighted representation of the edges of the Grid and consider always the node degree distribution in the analysis, since it is an important element to define the type of network under study (e.g., scale-free network). On the other hand, the studies that apply a weighted representation of the graph do not consider the node degree distribution statistics: neither considering the unweighted definition of node degree, nor using a definition that takes into account weights as proposed in Ref. [60]. This last aspect might worth to be considered since the degree distribution properties of the network might change the picture of the node degree distribution in comparison with unweighted studies.

Centrality measures are not often used, exceptions are Refs. [37,45,39,48] and the few others that compute betweenness distribution statistics, to identify the statistical distribution of critical nodes. More attention to centrality measures especially using weighted representation of the Power Grid graphs or models that provide the capacity or energy flows through the Grid might be beneficial in understanding the most critical nodes or lines in the power system.

Another recurring theme in the Complex Network Analysis involving the Power Grid is the reliability analysis, and actually it is the main motivation that drives these kind of studies. In fact many works were performed after major blackout occurred, such as the North American black-out of 2003[2] or the Italian one of 2003[3] (e.g., Refs. [26,61,27,28,52]) or anyway mention blackouts as the main motivation for the work. The fragility and resilience properties of the Power Grid has been the major reason of concern that has determined the focus of such Complex Network Analysis studies on the High Voltage network. That is why almost all studies consider the behavior of the Grid to various attacks to its nodes or edges.

3.3. The small-world property

The small-world property in network has received lots of attention starting with sociological studies [62,12], but more recently with application of this concept and model to many more classes of networks [1,35,2,63]. Among the studies analyzed small-world property investigation is performed by ten out of the thirty two considered in this survey. The various studies look for the satisfaction of the small-world property described by Watts [35] then together with Strogatz [1].

In general, the various studies tend not to have a common answer for the general question regarding the membership of Power Grid networks to the small-world group. It is indeed very specific to the samples analyzed and no conclusion can be drawn, this seems especially true for the High Voltage Grids, while the Medium and Low Voltage networks seem far from being a small-world network [36]. Figs. 6 and 7 provide a graphical representation of the similarity to random graphs (dashed lines) of the physical grid samples analyzed in the literature (dots in the figures). From the figures, one sees that the path lengths are similar to those of random graphs satisfying the condition ($L \approx L_{RG}$) proposed in Ref. [1], while analyzing the clustering coefficient (γ), one sees that only for few samples the condition of clustering coefficient much higher than a random graph with same *order* and *size* ($\gamma \gg \gamma_{RG}$) is satisfied. This justifies the statement made before stating that no general definitive answer can be given for the Power Grid network considering the membership to the small-world network category.

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[2] http://news.bbc.co.uk/2/hi/americas/3152451.stm

[3] http://news.bbc.co.uk/2/hi/3146136.stm

Table 2
Comparison of the main characteristics of the graphs related to Power Grids.

Work	Sample order	Sample size	Average degree	Node degree distribution statistics	Betweenness distribution statistics	Path length analysis	Weighted/unweighted analysis	Resilience analysis	Small-world investigation
[26]	~14,000	~19,600	~2.80	✓	✓		Unweighted	✓	
[27]	~300	~500	~3.33	✓	✓	Indirectly through efficiency metric	Weighted not based on physical properties	✓	
[28]	~314,000	NA	NA	✓			Unweighted	✓	
[29]	~4800	~5500	~2.29	✓		✓	Unweighted	✓	✓
[30]	~2700	~3300	~2.44	✓			Unweighted	✓	
[31]	~3000	~3800	~2.53	✓		✓	Unweighted	✓	✓
[32]	~3000	~3800	~2.53	✓			Unweighted	✓	
[33]	~370	~570	~3.08			Indirectly through efficiency metric	Unweighted	✓	
[34]	~370	~570	~3.08	✓		✓	Unweighted	✓	
[35]	~4900	~6600	~2.69			✓	Unweighted		✓
[58]	~8500	~13,900	~3.27	✓		✓	Unweighted and impedance analysis		✓
[36]	~4850	~5300	~2.18	✓	✓	✓	Both	✓	✓
[37][a]	~210	~320	~3.05			✓	Both	✓	✓
[38]	NA	NA						✓	
[39]	300	411	2.74				Both	✓	
[40]	~6400	~8700	2.72			✓	Unweighted	✓	✓
[41]	300	411	2.74	✓ (chart only)	✓		Both	✓	
[42]	~8500	~13,900	~3.27	✓			Unweighted	✓	
[44]	~900	~1150	~2.55	✓	✓	✓	Weighted	✓	✓
[45]	~3200	~7000	~4.375	✓ (chart only)	✓		Weighted	✓	
[46]	~4900	~6600	~2.69				Unweighted	✓	
[47]	~1700	~1800	~2.12			✓	Both		✓
[48]	~39	~46	~2.36				Weighted	✓	
[49]	~150	~46	~2.36				Weighted	✓	
[50]	~2556	~2892	~2.26				Weighted	✓	
[22]	~15,400	~18,368	~2.39	✓ (for one sample only)		✓	Unweighted	✓	✓
[51]	~550	~800	~2.91			✓	Unweighted	✓	
[52]	~14,000	~19,600	~2.80				Weighted not based on physical properties	✓	
[53]	~90	~120	~2.67				Weighted	✓	
[54]	~550	~700	~2.55				Weighted	✓	
[55]	~29,500	~50,000	~3.39				Weighted	✓	
[56]	~400	~700	~3.5			✓	Weighted	✓	✓
[57]	~900	~1300	~2.89				Unweighted	✓	
[59]	~60	~110	~3.67				Unweighted		

[a] The values for nodes and lines in this table refer only to a snapshot of Shanghai Power Grid.

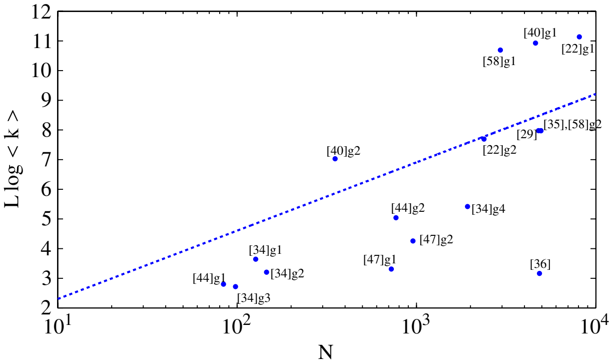

Fig. 6. Path length comparison between real Grids samples in the literature (dots) and random graphs (dashed line).

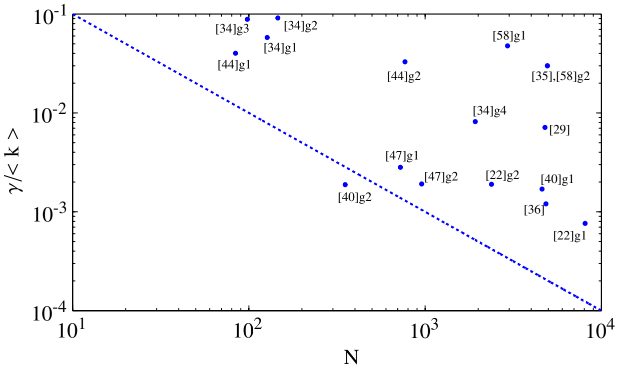

Fig. 7. Clustering coefficient comparison between real Gris samples in the literature (dots) and random graphs (dashed line).

Table 3
Comparison of the node degree cumulative distribution probability functions.

Work	Cumulative node degree distribution probability type	Fitted distribution
[26]	Exponential	$y(x) \sim e^{-0.5x}$
[27]	Exponential	$y(x) = 2.5e^{-0.55x}$
[28]	Power-law	$y_1(x) = 0.84x^{-3.04}$ $y_2(x) = 0.85x^{-3.09}$
[30]	Exponential	$y_1(x) \sim e^{-0.81x}$ $y_2(x) \sim e^{-0.54x}$
[31]	Exponential	$y(x) \sim e^{-0.56x}$
[32]	Exponential	$y(x) \sim e^{-0.61x}$
[34]	Exponential or sum of exponential terms	$y_1(x) = e^{-0.18x^2}$ $y_2(x) = e^{-0.21x^2} + 0.18e^{-0.25(x-4)^2}$ $y_3(x) = 0.96e^{-0.17x^2} + 0.25e^{-0.19(x-3.9)^2}$
[58,42]	Sum of truncated geometricaland irregular discrete terms	$y_1(x) \sim f_1(x)$ $y_2(x) \sim f_2(x)$
[36]	Power-law (unweighted) and sum of exponential terms (weighted)	$y_1(x) \sim x^{-1.49}$ $y_2(x) \sim 0.15e^{-21.47x} + 0.84e^{-0.49x}$
[44]	Exponential	$y_1(x) \sim e^{-0.65x}$ $y_2(x) \sim e^{-0.58x}$
[22]	Exponential	$y(x) \sim e^{-0.5x}$

3.4. *Node degree distribution*

The degree of a node is a property to understand how many other nodes it is connected to. However, this information is not particularly important for big graphs since keeping track of each node degree may not be manageable, instead it is better to have a general idea of the statistics of the node degree. In particular, its probability distribution gives us some insights of the general properties of the networks such as the likely or unlikely presence of nodes with very high degree (sometimes also referred as hubs). Table 3 shows the main information about the degree distribution in the works that perform this study. The second column gives a general idea of the type of cumulative node degree distribution that is investigated in the articles under review. What is interesting is to fit the distribution to a class of curves. This is shown in the third column.

As seen in the table, the results do not completely agree on the type of the distribution followed by the Power Grid networks, but generally they are close to an exponential decay. Figs. 8 and 9 represent the fitted node degree cumulative distribution reported in the third column of Table 3. For [58,42] presented in the table the functions $f_1(x)$ and $f_2(x)$ are not reported in the table for size reason, but in footnote.[4]

The plots in Figs. 8 and 9 give a general idea of the shape of the distribution. The charts have to be interpreted in a qualitative way since the details concerning the coefficients are not always available in the reviewed studies. In addition, for studies concerning multiple samples (i.e., [30,31,36]) averages between all samples, or particular significant samples have been chosen among the many available.

In general the various studies focusing on the High Voltage Grids agree on a statistical distribution for node degree that follows an exponential (or exponential based) distribution with characteristic parameters of the exponential curve that depend on the specific Grid analyzed. While High Voltage Grid have been quite extensively analyzed, the Medium and Low Voltage Grids have not found much attention so far and a deeper and wider investigation needs to be performed in different geographies since the only study (i.e., Ref. [36]) is representative of the Northern part of the Netherlands. In addition, the distribution Grid will be the section of the Power Grid mostly impacted by innovations in future Power Systems i.e., Smart Grid technology [64].

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[4] $f_1(x) = \sum_{x_i<x} 0.2269(0.7731)^{x_i} * \{0.4875\delta(1), 0.2700\delta(2), 0.2425\delta(3)\}$ $x_i = 1, 2, \ldots, 34$
$f_2(x) = \sum_{x_i<x} 0.4085(0.5916)^{x_i} * \{0.3545\delta(1), 0.4499\delta(2), 0.1956\delta(3)\}$ $x_i = 1, 2, \ldots, 16$
The $*$ symbol is here to be considered as the convolution operator and the δ is the Dirac delta function.

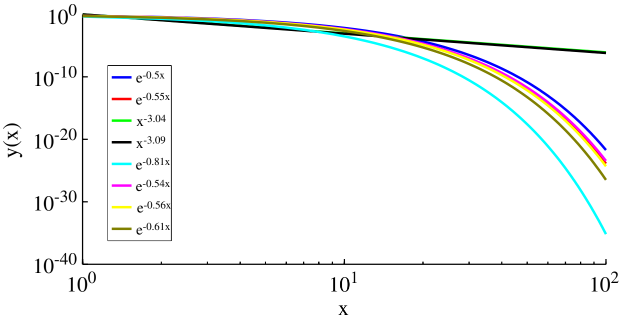

Fig. 8. Log–log plot of fitted node degree cumulative probability distribution corresponding to the first six rows of Table 3.

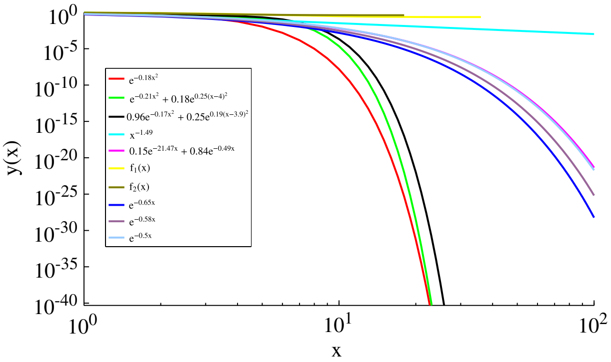

Fig. 9. Log–log plot of fitted node degree cumulative probability distribution corresponding to the last five rows of Table 3.

Table 4
Comparison of the betweenness cumulative distribution probability functions.

Work	Cumulative betweenness distribution probability type	Fitted distribution
[26]	Power-law	$y(x) \sim (2500 + x)^{-0.7}$
[27]	Power-law	$y(x) \sim 10000(785 + x)^{-1.44}$
[36]	Power-law and exponential	$y_1(x) \sim x^{-1.18}$
		$y_2(x) \sim 0.68e^{-6.8\cdot 10^{-4}x}$
[44]	Power-law	$y(x) \sim x_1^{-1.71} y(x) \sim x_2^{-1.48}$

3.5. Betweenness distribution

Betweenness is an important measure to assess how a node is central in a network. This metric in fact computes how many shortest paths traverse a node, therefore giving an information of the importance of the node in the path management. The main characteristics of the betweenness study are summarized in Table 4 where the second column shows the type of followed distribution, while the analytical function is represented in the third column. Unfortunately, this metric is computed by only five studies [26,27,36,44,45].

Although the studies that perform this type of analysis are only few, one can see that there is a tendency for the High Voltage network to have a betweenness distribution close to a Power-law. For the Medium and Low Voltage the situation is less clear: some samples analyzed in Ref. [36] follow an exponential decay, especially the Low Voltage ones, while other, usually the bigger belonging to the Medium Voltage network, follow a Power-law. In Fig. 10 the plot of the distributions is represented to show the difference between the trend of the Power-law and exponential decay: after a certain point the exponential distribution has a faster decay.

Power-law seems the dominating rule for betweenness probability distribution even if few studies consider this statistical property of graphs. To draw a general definitive conclusion regarding this property for the Power Grid more studies are required. Another aspect to be considered which has not received much attention so far is the study of betweenness statistics in weighted Grid models or when power flows are considered instead of the pure topological analysis.

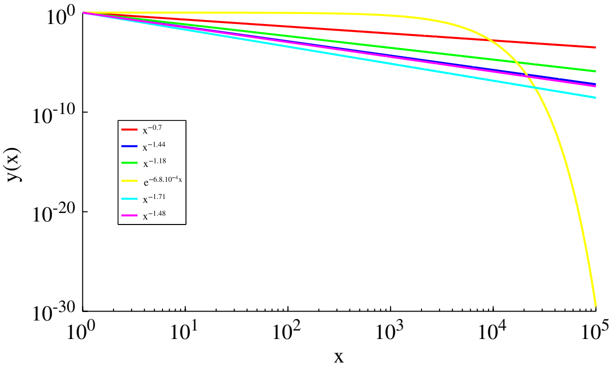

Fig. 10. Log–log plot of fitted betweenness cumulative probability distribution corresponding to Table 4.

3.6. Resilience analysis

The characterization of resilience is the main motivation for the studies involving Complex Network Analysis and Power Grid. In fact, the behavior in terms of connectivity of the network when nodes or edges are removed is the primary question in many works considering failures that happen in a random fashion or following an attack strategy. Table 5 describes the different types of resilience analysis that are performed by the various authors. In particular, the second column contains the metric that is used to assess the reliability of the network. The focus of the attack either it is related to nodes or edges is considered in third and fourth column, while the fifth column remarks the studies that in addition to the resilience analysis also propose a mitigation strategy for improving Grid reliability. The last column indicates which power engineering model is used, if any.

In general, the reliability is assessed by evaluating the connectivity or the ability to efficiently guarantee paths between nodes when nodes or edges in the network are removed. As a general result for failures related to nodes all the samples show a good resilience to random breakdowns. In fact, the network is always able to guarantee a certain connectivity until the number of nodes removed are the biggest part of it. On the other hand the Grids are extremely vulnerable to targeted attacks, that is failures that focus on key nodes for the entire network such as high degree nodes or nodes with high betweenness or nodes or lines that manage the highest amount of load or electricity flow. The thoroughness and space given to the vulnerability and blackout topics in Ref. [37] covering also aspects more related to the electrical domain than pure Complex Network Analysis, provide a view of the reliability of the power system that is more precise and complete. This last aspect (i.e., using also physical/electrical parameters in the analysis) is stressed also in other works [53–55,37,41] which provide motivations and results justifying the superiority of vulnerability analyses that have a more realistic view of the Power Grid network than a topological-only approach by applying power flow models [65] on top of the topological structure. Adding physical parameters to the network is beneficial for results concerning the way networks tend to disrupt and spread failures closer to reality. The works that take into consideration the physical parameters characterizing the power lines usually apply models and methods typical of the power engineering tradition to represent the flow of power that travels through the power lines (edges in the graph representation). Mainly, the works consider a simplified model to represent the power flow in the transmission lines that is the direct current (DC) power flow model. The DC flow approach lacks in the precision of the alternate current (AC) power flow model, but it has a much faster solution [65] by assuming that the Power Grid transfers only active power and that only linear equations describe the power lines and electric machinery. However, an aspect that is missing in all works is the cross-checking between nodes and lines identified as the most critical in topological analysis and the actual infrastructure failures experienced by the transmission operators. The comparison of the results of the vulnerability analysis using Complex Network Analysis and the critical lines failures considered by network operators would be beneficial to assess the one of the added values of the application of Complex Network Analysis in the power system realm. To the best of our knowledge, this has not been investigated so far. Only by proving the correspondence of problems and criticality of the Power Grid operators and the results of the Complex Network Analysis results, Complex Network Analysis tools and techniques can be applied in the real power system world.

4. Discussion

The survey of CNA studies and their comparison show how important properties of a real system such as the Power Grid can be studied using graph modeling tools and which conclusions about the reliability of the infrastructure can be drawn. Complex Network Analysis proofs to be an excellent set of tools that provide, although without dealing with the details and complexities of the electrical properties in the case of the Power Grid, a comprehensive and general understanding of the properties that characterize a network. We see an interesting trend in the various works analyzed, that is, the research towards more complex representation of the properties of the network than a simple graph. In fact, although Complex Network Analysis can help in understanding the foundational properties of the network of the

Table 5
Comparison of the resilience analysis and improvement studies.

Work	Resilience analysis type	Node attack	Edge attack	Grid improvement	Power engineering model
[26]	Connectivity loss	✓			None
[27]	Efficiency	✓			None
[28]	Loss of load probability	✓	✓		None
[29]	Influence on largest component size and path length	✓		✓	None
[30]	Robustness through mean degree, motifs and patch size analysis				None
[31]	Influence on largest component size	✓			None
[32]	Influence on largest component size e comparison with theoretical results	✓			None
[33]	Damages and improvements		✓	✓	None
[34]	Nodes disconnection and improvements		✓	✓	None
[36]	Influence on largest component size	✓	✓		None
[37]	Several criticality analysis and blackout models	✓	✓		DC and AC power flow
[38]	Reliability and disturbances				None
[39]	Unserved energy/load	✓	✓		Power injection/withdrawal at buses
[40]	Critical Path Length and clustering coeff.		✓		None
[41]	Sensitivity				Impedance of lines
[42]	Influence on largest spanning cluster size	✓			None
[44]	Loss of load and failure endurance	✓			Reactance of lines
[46]	Avalanche size	✓			None
[48]	Flow availability		✓		Admittance of lines
[49]	Efficiency		✓		Reactance of lines and active power
[50]	Largest power supply region	✓	✓		Not clearly specified
[51]	Influence in network connectivity and power degradation		✓		DC flow model
[52]	Efficiency	✓			None
[53]	Efficiency, net-ability, overload		✓		Impedance of lines and DC flow model
[54]	Efficiency, net-ability, overload		✓		Impedance of lines and DC flow model
[55]	Path length, connectivity loss	✓			Impedance and DC flow model
[56]	Line overload, cascade effects, network disruption		✓	✓	Impedance of lines and DC flow model
[57]	Overload, cascade effects, blackout size	✓		✓	None

Power Grid, it is always worth to remember that the Power Grid is subject to the law of physics and the principles of electrotechnics. From the initial studies (e.g., Refs. [35,26,28,29,22]) considering the Power Grid just as an indirected graph without any property (i.e., weight) on edges and with no characterization of the nodes, more recent studies take into account the electrical properties of the Power Grid system. Of course the aim of these later studies is always to provide, anyway, a simplification of the highly complex Power System, but they add those essential parameter to better simulate the Grid behavior: impedance parameters associated to the transmission lines, power limits supported by the substations (i.e., nodes) such as for instance in Refs. [37,54,55,50,45]. A more detailed description of the Power Grid under investigation (i.e., weighted graph representation) enables to better understand the dynamics guiding the Power Grid with a mixed approach: both preserving the idea of the Complex Network Analysis of having a general and statistical behavior of the overall Power Grid, and, on the other hand, to take into account the physical/electrical properties essential to characterize the Power Grid. Latest results in Refs. [55,54,53,66] show a better agreement to real Power Grid behavior of the models that take into account physical parameters, compared to the pure topological analysis, and the observed behaviors and critical points in real power systems thus justifying this *enhanced Complex Network Analysis* approach.

We emphasize the inappropriateness of purely topological measures, as indicated in Refs. [55,45], since they are not able to capture the essence of the power systems. In fact, not considering the peculiarity of the physical laws governing the power flows can bring to inaccurate results and considerations. We also see another gap in the current scientific work concerning the Power Grid and Complex Network Analysis techniques: the applicability in the real power systems of the results obtained by the network analysis, especially considering the vulnerability and cascading effects of the Power Grid, needs to be confirmed by the transmission and distribution operators. We remark here that basically all the works lack the cross-check of the theoretical results with the experience on the field. Another gap that limits the research, and consequently the applicability to real Power Grid systems, concerns the confidentiality that surrounds the topology and the properties of the power lines and power stations. As the grid is a critical infrastructure and amenable to powerful terroristic attacks, a strict level of confidentiality is kept on its structure and inner-workings. As a matter of fact, allowing scientists to investigate only realistic models (e.g., IEEE-bus models) that are much smaller and simpler than the real Power Grids and are characterized by artificial parameters, limits by definition the application to real power network problems, as noted by Casals in Ref. [38].

A noteworthy general aspect is the role that Complex Network Analysis has in the Power Grid infrastructure vulnerability analysis: CNA does not want to substitute the traditional approaches to Power Systems resilience and safety analysis since they have proved extremely successful in governing and managing the Electrical System with only occasional highly disruptive events. We stress once again that Complex Network Analysis techniques applied to the Power Grid world are high simplification of all the complexities governing the Power Systems, but such approach can anyway be helpful to give a general vision that can help in identifying quickly and in a simple way critical spots or aspects of the Power Grid which then may be further and deeply analyzed with traditional electrical engineering tools. One aspect that is only partially addressed so far in the approach is the reconciliation of the results of Complex Network Analysis with problems and failures identified by Grid operators. Such reconciliation is important on one hand to validate the CNA approach in the Power System world, and on the other hand to provide new insight and improve metrics and methodologies.

A recent emerging function that CNA is acquiring, deals with the design aspects of the Power Grid [67]. In fact, CNA can be used in generating Grid topological evolution, assessing the strategies to adapt actual topologies to improve them, especially in the panorama of the forthcoming Smart Grid. We consider the design and evolution of network to be one of the promising fields where complex networks can complement the traditional power system engineering techniques and provide a broader view. This is an interesting approach to create decision support systems for high level infrastructure planning.

5. Conclusion

The works surveyed almost entirely belong to the High Voltage network and are part either of the American Grid (or subsamples of it) or of the European one (or part of it) or Chinese Grid. Generally, the node degree distribution tends to follow an exponential distribution with some minor exceptions, allowing a general characterization of the properties of the network by the average degree parameter. Betweenness studies are also used to characterize the centrality (e.g., criticality) of certain nodes as essential for the ability of the network to guarantee its navigability. Unfortunately, the computation of betweenness is done only by a small part of the studies under assessment. However, the tendency is to have at least for High Voltage samples a behavior that is closer to a Power-law distribution for the probability characterizing this metric. It means having very few nodes with very high values that are responsible to allow the majority of the shortest paths across the network. We remark that almost none of literature studies has shown results of an accurate methods such as the one based on Kolmogorov–Smirnov method, shown by Clauset et al. [68]. As remarked in Ref. [68], many distributions that with a naive fitting analysis might be considered power-laws at first sight do not prove to belong to such a category when a more rigorous fit test is adopted. On the other hand the results from the different studies contrast regarding the small-world phenomenon in Power Grid networks. Indeed, some of the conditions imposed by Watts and Strogatz [1] are not met by Power Grid samples due to physical and economic reasons. This property must depend on the specific sample analyzed. The geography of the country whose sample is derived is sometimes important (e.g., Italy [34]) while for other studies it has lower impact [32]. A point of agreement between all the studies is about the reliability of the Power Grid networks when facing failures: a general good resilience to random breakdown, while extreme vulnerability is experienced by attacks that target the critical nodes (i.e., high node degree or high betweenness nodes).

An important result for the accuracy of the Complex Network Analysis studies is the similarity to the traditional electrical engineering results that this type of analyses provide [28,32,53,54]. This really shows how the theoretical study and the measured quantities in the real environment are very close. We emphasize that such a comparison with findings and issues faced by operators needs to be further explored and enhanced to make CNA results even more beneficial to the Power Systems world.

Of course to enable Complex Network Analysis to be a more useful tool in the Power Systems domain, especially in order to create models that take into account the physical/electrical parameters of the network, it is useful for CNA and Power Grid scientists to have precise and reliable information of the topology and physical characteristics of the electrical Grid. As remarked by Rosas-Casals [38] in order to better model and more precisely explain the behavior of the Power Grid, high quality and precise information are required about the Grids, their components, the actual flow of energy data the Grids deal with in their operations. Especially, properties of the power lines and for those part of the Grid which are held privately in the hands of Grid operators and distribution operators. Security and criticality of this data is a recognized aspect in a world that fears terrorist attacks, but without data feeding the models and research works it is not possible to discover actual vulnerability points and help to build more secure and reliable Power Grids.

Networks are an integral model of phenomena surrounding us, may these be biological networks (e.g., food-webs, protein interactions) or human generated ones (e.g., airline travel routes, computer chip wiring, telephone call graphs) [69]. Having methods and tools to better understand them and their dynamics is beneficial for knowledge advancement and better design of future systems. Complex Network Analysis is such a modeling technique that provides methods and metrics for an analytical comprehension of network behaviors. Public infrastructures are important for today's society, in particular the Power Grid, which is by nature a complex network, has a critical role for the economy in every country. Having an overall view of the Power Grid as a Complex System gives the ability to assess the potential issues the electrical system may face due to topological failures. In this paper, we have shown what are the main studies conducted on different Power Grid networks using metrics and techniques from the emerging CNA field of study. Although the basic methodology of study is the same, indeed the Complex Network Analysis techniques, the results show differences in some properties such as node degree distribution statistics, the presence or absence of the small-world property. On the other hand, the commonality is the behavior of the Power Grid networks when facing failures: a general good resilience to random breakdown, while they show extreme vulnerability when facing attacks that target the critical nodes (i.e., high degree or high betweenness nodes, nodes or lines that manage the highest electricity flow). The morphology of the country definitely influences the topology of the network and thus its properties and reliability, symbolic is the case of the Italian Grid [34].

The use of physical properties of the lines and the buses of the Power Grid allows a better and more precise representation closer to the real operation of the Power Grid, thus better suited in identifying failures.

To have a more complete idea of the Power Grid networks it is worth investigating other Grids from other countries, not only limiting oneself to networks in Europe, United States and China as the studies analyzed so far focus on. The investigation of Grids belonging to other geographies such as Asia and South America could lead to new topologies. On the other hand, it is important to study more samples belonging to the Medium and Low Voltage Grids as to the best of our knowledge the only study in this direction is our own [36]. This is interesting not only because it highlights some different properties from the High Voltage, but also because it can provide indications useful for the design of the future Smart Grid. In addition, Complex Network Analysis can be used not only as a tool for the analysis of the Grid, but also to consider how the electrical Grid might evolve according to design principles to be optimized at a topological level [58,67]. Another promising aspect to take into consideration is related to the influence of the network topology on electricity distribution costs for the future scenarios of Smart Grid solutions where local Grid where electricity is generated and distributed are likely to emerge [36].

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