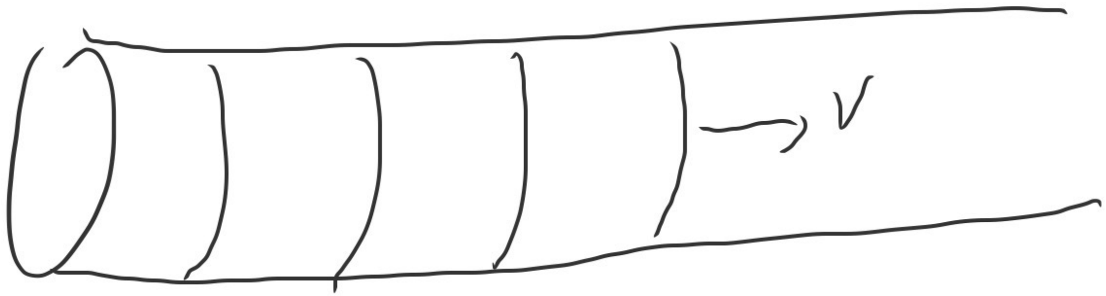
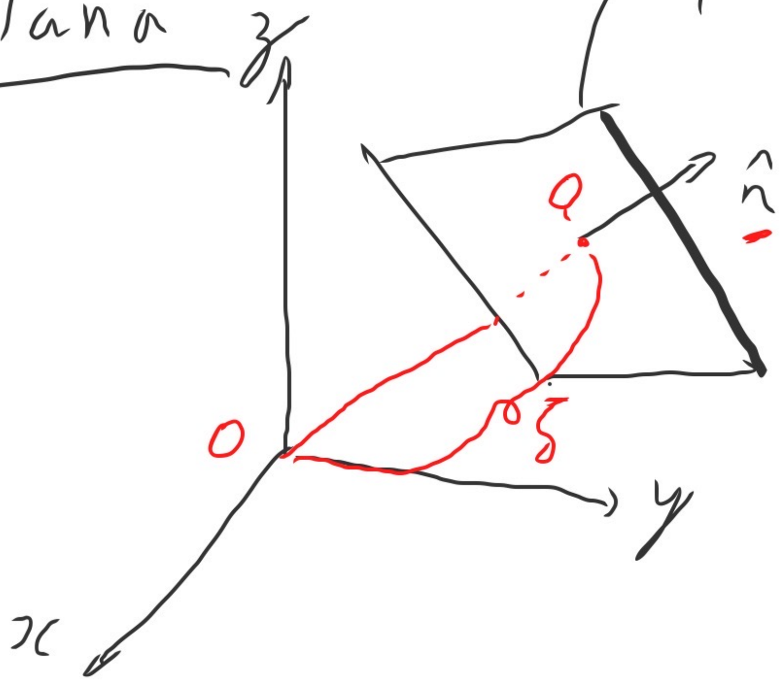
Ondas em 2D/3D



Onda Plana

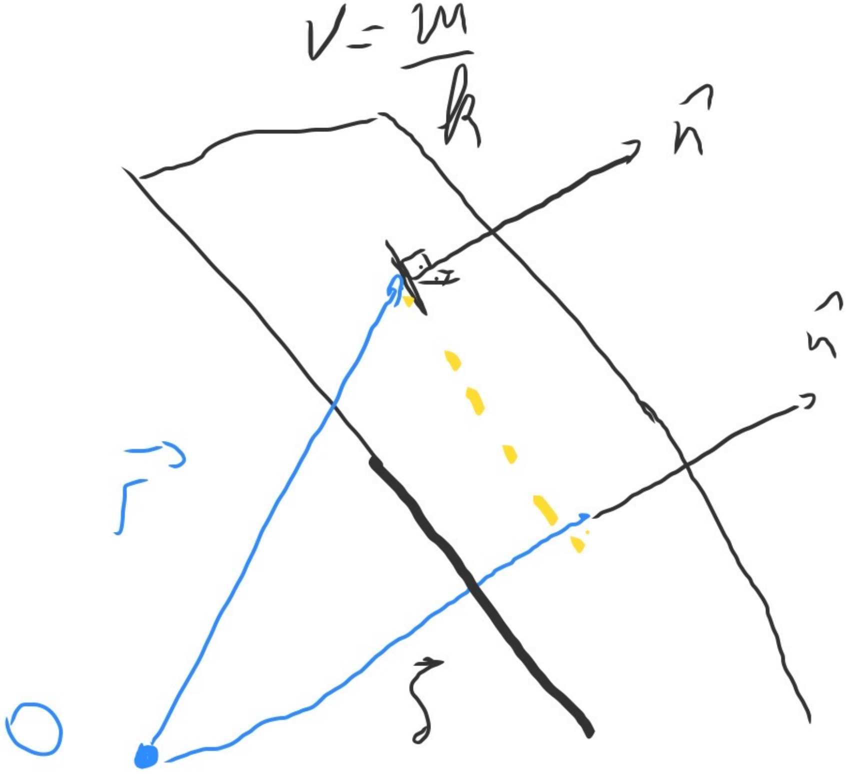
plano: definir a frente de onda

OQ



$$\varphi(\vec{r},t) = A\cos(k\cdot\zeta - \omega t + \delta)$$

$$\zeta = \hat{n}\cdot\vec{r} \qquad k = \frac{\omega}{v}$$



$\vec{k} = \hat{n}\cdot k \rightarrow$ vetor de onda

↳ número de onda

$$\varphi(\vec{r},t) = A\cos(\vec{k}\cdot\vec{r} - \omega t + \varphi)$$

$$= \mathrm{Re}\left[A e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right]$$

$$A = A e^{i\varphi}$$

Fase constante: $t \to \vec{k}\cdot\vec{r} = cte \to$ Frente de onda

$$\vec{k} = k_x\,\hat{x} + k_y\,\hat{y} + k_z\,\hat{z} \;;\; \vec{r} = x\,\hat{x} + y\,\hat{y} + z\cdot\hat{z}$$

$$|k|^2 = \underbrace{k_x^2 + k_y^2 + k_z^2}_{} = \frac{\omega^2}{v^2}$$

$$\varphi(\vec{r},t) = A\cos\left(k_x\cdot x + k_y\cdot y + k_z\cdot z - \omega t + \delta\right)$$

$$\frac{\partial^2}{\partial x^2}\varphi = -\underline{k_x}^2\cdot\varphi \;;\; \frac{\partial^2}{\partial y^2}\varphi = -\underline{k_y}^2\varphi \;;\; \frac{\partial^2}{\partial z^2}\varphi = -\underline{k_z}^2\varphi$$

Eq. de Onda 3D!

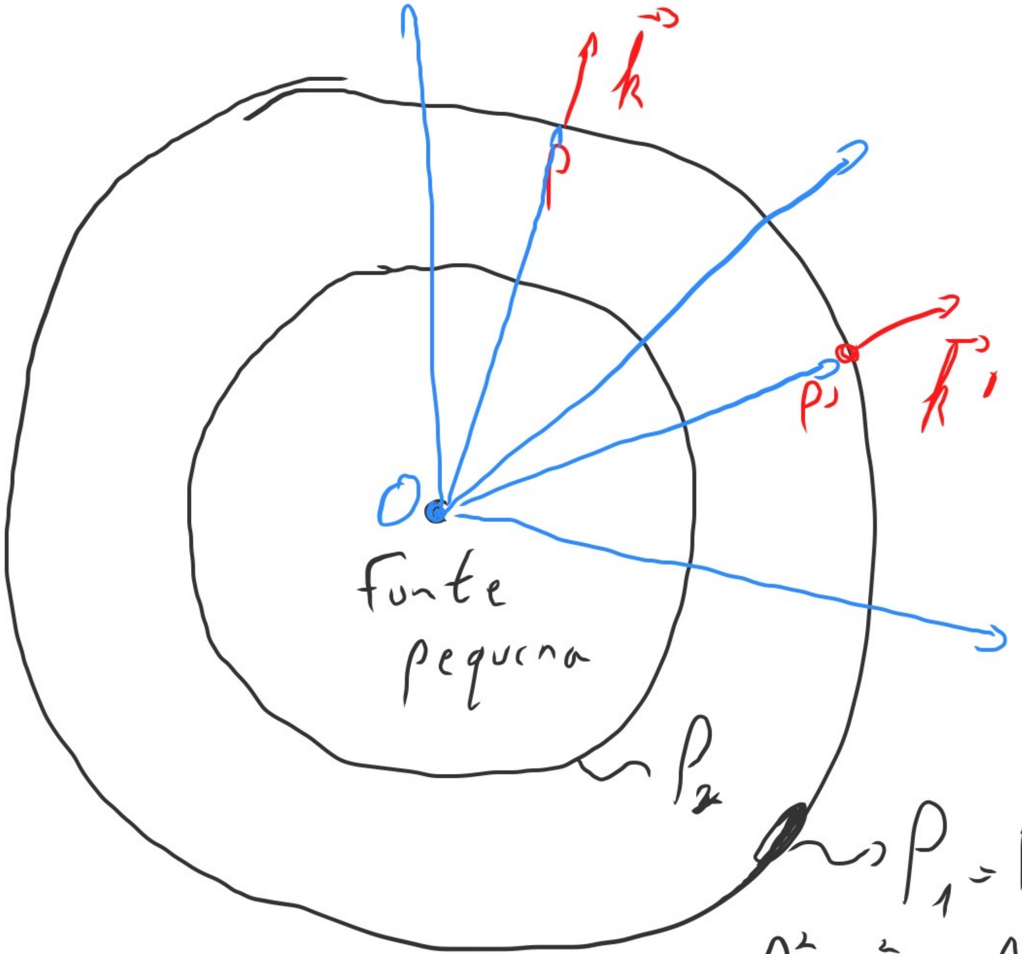
$$\frac{\partial^2}{\partial x^2}\varphi + \frac{\partial^2}{\partial y^2}\varphi + \frac{\partial^2}{\partial z^2}\varphi = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\varphi$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\varphi = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\varphi$$

Ondas $\Rightarrow$ conjunto de ondas planas!

Recorrente: ondas sonoras; luz; ondas de rádio, etc.

Ondas esféricas



$$\Theta(\vec{r},t) = \vec{k}\cdot\vec{r} - \omega t + \delta$$

$$= k\cdot r - \omega t + \delta$$

$$r = \sqrt{x^2+y^2+z^2}$$

$$\psi(\vec{r},t) = A\cos(kr - \omega t + \delta)$$

↓ ?

$A^2 \propto$ Intensidade da onda → $\dfrac{\text{Potência média}}{\text{Área}}$

$P_1 = P_2$

$\propto$ Intensidade $\quad A_1^2\, r_1^2 = A_2^2 \cdot r_2^2 \quad \rightarrow \propto$ Área

$$A(r) = \frac{a}{r} \; ; \; \varphi(\bar{r}, t) = \frac{a}{r} \cos(kr - \omega t + \delta)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\varphi = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\varphi$$

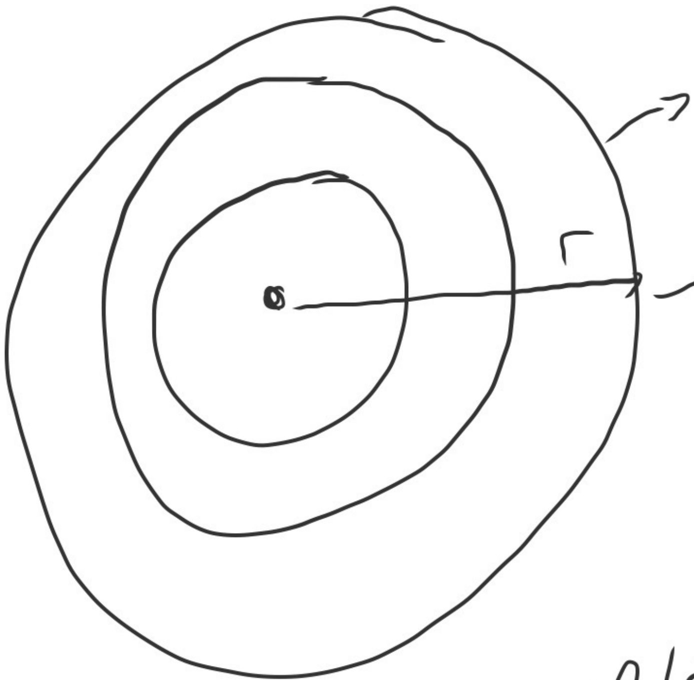
$\underline{r = 0} \rightarrow A(r) \rightarrow \infty$

Fonte <u>extensa</u> ; $\nexists$ Fonte pontual

Muito bom para sim. escalar $\left\{ \begin{array}{l} p \\ u \end{array} \right.$

Luz $\rightarrow$ aproximação $\rightarrow$ Feixe colimado $\rightarrow$ <u>transversa</u>

Ondas bidimensionais → ondas circulares



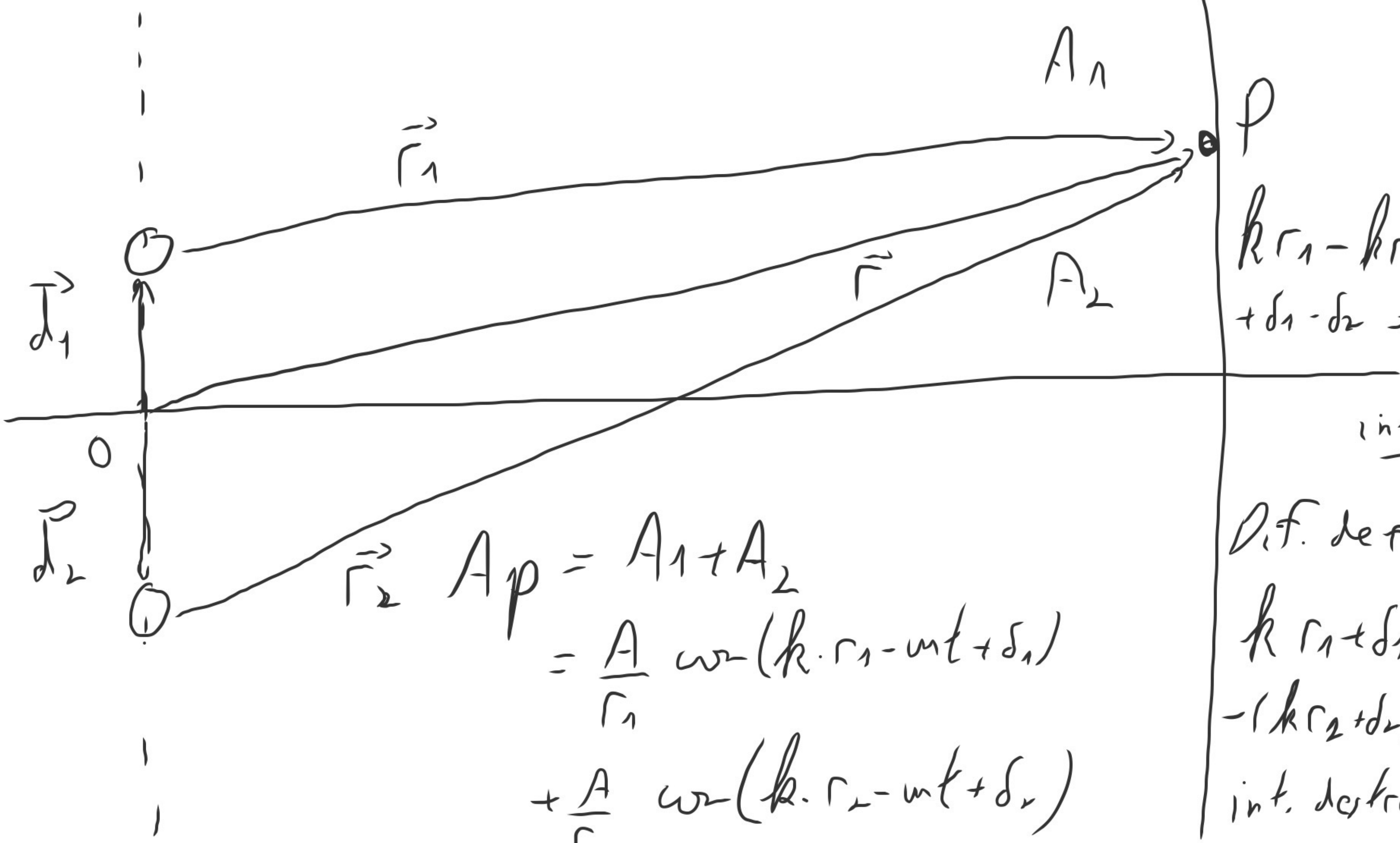
$\bar{P} = I \cdot 2\pi r \qquad I = \dfrac{\bar{P}}{2\pi r}$

$I \propto A^2 \Rightarrow \qquad A = \dfrac{a}{\sqrt{r}}$

$$\varphi(\vec{r}, t) = \frac{a}{\sqrt{r}} \cos(\vec{k} \cdot \vec{r} - \omega t + \delta)$$

Ondas confinadas em 2 Dimensões

Interferência



$$A_p = A_1 + A_2$$

$$= \frac{A}{r_1}\cos(k\cdot r_1 - \omega t + \delta_1)$$

$$+ \frac{A}{r_2}\cos(k\cdot r_2 - \omega t + \delta_2)$$

$$kr_1 - kr_2 + \delta_1 - \delta_2 = 2m\pi$$

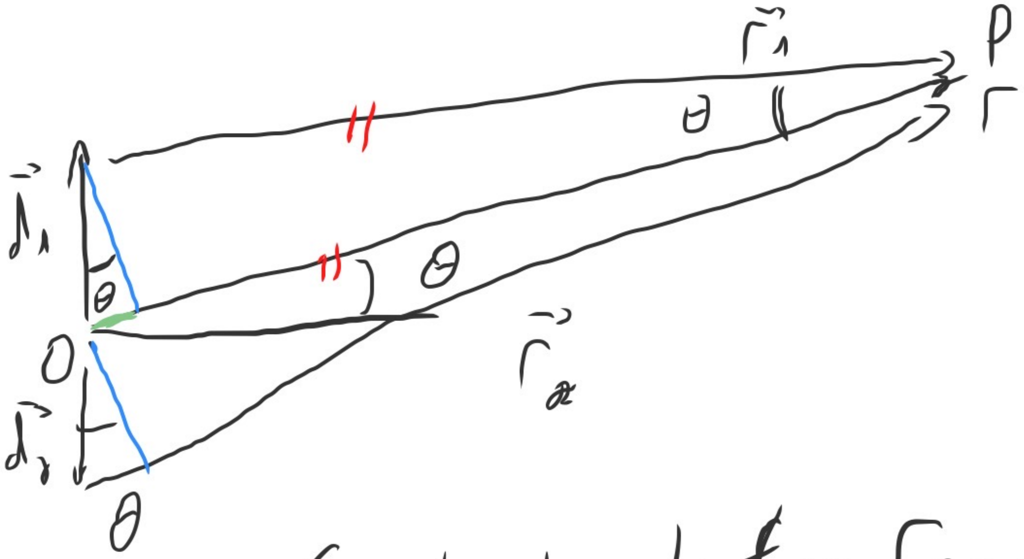
$\Downarrow$

<u>int. constr.</u>

Dif. de fase

$$kr_1 + \delta_1 - (kr_2 + \delta_2) = (2n+1)\pi$$

int. destrutiva



$$r_1 \sim r - \underline{d_1 \cdot \text{sen}\,\theta}$$

$$r_2 \sim r + d_2\,\text{sen}\,\theta$$

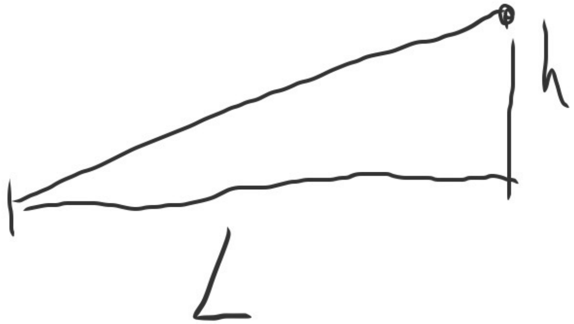
Cond. de Interferência

Construtivo, $\delta_1 = \delta_2$

Destrutivo, $\delta_1 = \delta_2 + \pi$

$$k(r_1 - r_2) = -k\cdot(d_1 + d_2)\,\text{sen}\,\theta =$$

$$= -k \cdot l \cdot \text{sen}\,\theta = 2m\pi$$

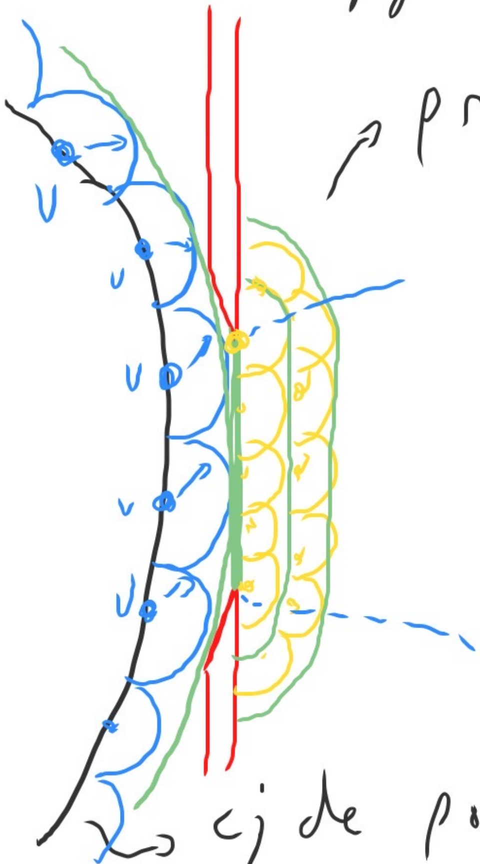


$$\text{sen}\,\theta_m = \frac{2\pi}{k\cdot l}\cdot m = \frac{\lambda}{l}\cdot m \qquad \boxed{\frac{h}{L} = \frac{\lambda}{l}\cdot m}$$

$$\text{sen}\,\theta_m \simeq \text{tg}\,\theta_m = h/L$$

Princípio de Propagação de Ondas

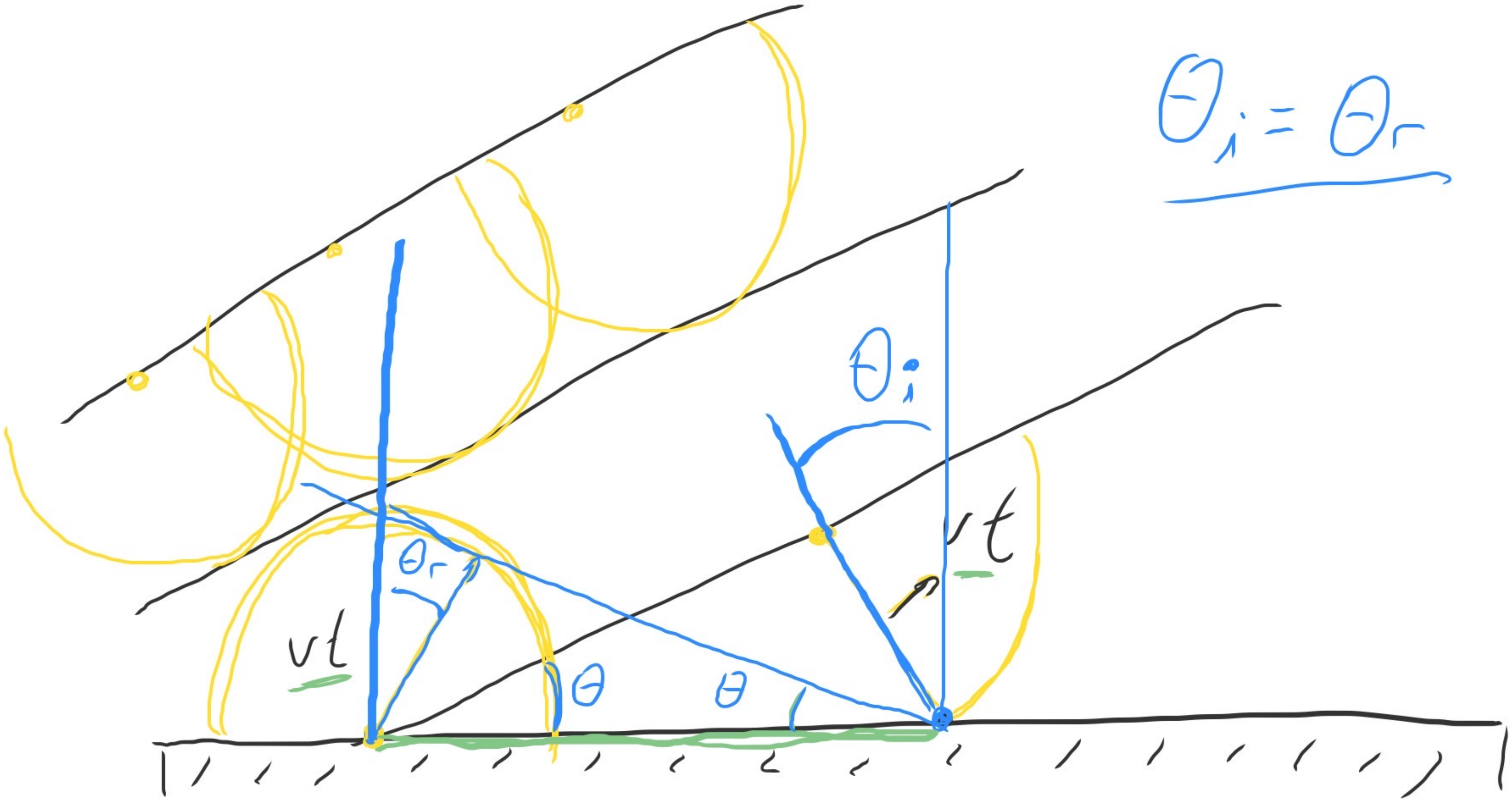
Huygens - Fresnel



↗ propagação na região de sombra?

↳ cj de pontos c/ mesma fase → frente de onda

Reflexão pelo Princípio de Huygens

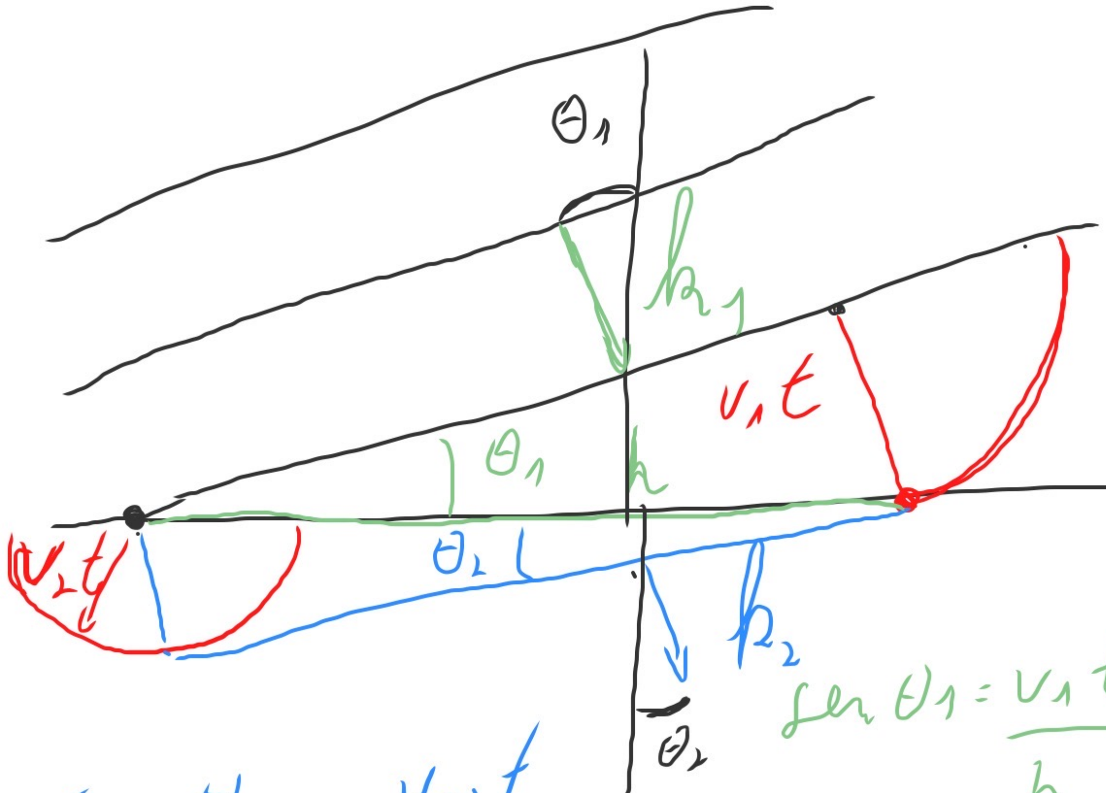


$$\theta_i = \theta_r$$

Refração, por Huygens:

$$\boxed{\frac{\operatorname{sen}\theta_1}{\operatorname{sen}\theta_2} = \frac{V_1}{V_2}}$$

Lei de Snell-Descartes (ótica)



V_1

V_2

$$\operatorname{sen}\theta_1 = \frac{v_1 t}{h}$$

$$\operatorname{sen}\theta_2 = \frac{v_2 \cdot t}{h}$$

$$\frac{\operatorname{sen}\theta_1}{v_1} = \frac{\operatorname{sen}\theta_2}{V_2}$$