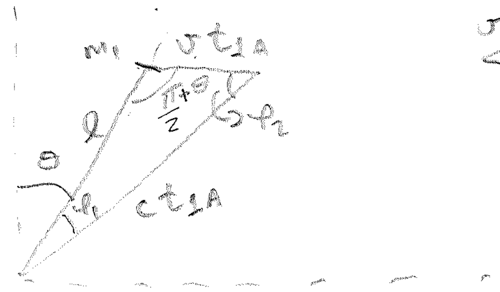
Exercício 2 - Lista 1

Utilizaremos a hipótese do éter, ou seja, a velocidade da luz é c em relação ao éter.

Para que a luz atinja o espelho M_1 ela tem que sair "para trás"



$v \leftarrow$ éter

onde t_{1A} é o tempo de ida da luz até M_1.

Pela lei dos senos temos:

$$\frac{\ell}{\sin\varphi_2} = \frac{v\, t_{1A}}{\sin\varphi_1} = \frac{c\, t_{1A}}{\operatorname{sen}\left(\frac{\pi}{2}+\theta\right)}$$

$$\varphi_1 + \varphi_2 + \frac{\pi}{2} + \theta = \pi \Rightarrow \varphi_1 + \varphi_2 = \frac{\pi}{2} - \theta$$

Logo

$$\operatorname{sen}\varphi_1 = \frac{v}{c}\operatorname{sen}\left(\frac{\pi}{2}+\theta\right) = \frac{v}{c}\cos\theta = \operatorname{sen}\left(\frac{\pi}{2}-\theta-\varphi_2\right) = \cos(\theta+\varphi_2)$$

$$= \cos\theta\cos\varphi_2 - \operatorname{sen}\theta \sin\varphi_2$$

ou

$$\cos^2\theta\cos^2\varphi_2 = \left(\frac{v}{c}\right)^2\cos^2\theta + \sin^2\theta\operatorname{sen}^2\varphi_2 + 2\frac{v}{c}\operatorname{sen}\theta\cos\theta\operatorname{sen}\varphi_2$$

$$\operatorname{sen}^2\varphi_2 + 2\frac{v}{c}\operatorname{sen}\theta\cos\theta\operatorname{sen}\varphi_2 - \left(1-\frac{v^2}{c^2}\right)\cos^2\theta = 0$$

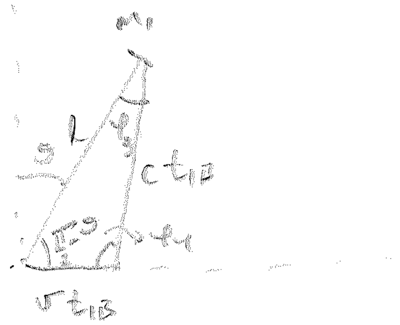
$$\operatorname{sen}\varphi_2 = \frac{-2\frac{v}{c}\operatorname{sen}\theta\cos\theta \pm \sqrt{4\frac{v^2}{c^2}\operatorname{sen}^2\theta\cos^2\theta + 4\left(1-\frac{v^2}{c^2}\right)\cos^2\theta}}{2}$$

$$\operatorname{sen}\varphi_2 = \cos\theta\left(-\frac{v}{c}\operatorname{sen}\theta + \sqrt{1-\frac{v^2}{c^2}\cos^2\theta}\right)$$

Logo

$$\boxed{t_{1A} = \frac{l}{c}\,\frac{1}{\sqrt{1-\frac{v^2}{c^2}\cos^2\theta}-\frac{v}{c}\operatorname{sen}\theta}}$$

Para o tempo de volta t_{1B} temos



Pela lei dos senos:

$$\frac{l}{\operatorname{sen}\varphi_4} = \frac{c\,t_{1B}}{\operatorname{sen}\left(\frac{\pi}{2}-\theta\right)} = \frac{v\,t_{1B}}{\operatorname{sen}\varphi_3}$$

e

$$\frac{\pi}{2}-\theta+\varphi_3+\varphi_4 = \pi \quad\rightarrow\quad \varphi_3+\varphi_4 = \frac{\pi}{2}+\theta$$

Logo

$$\operatorname{sen}\varphi_3 = \frac{v}{c}\cos\theta = \operatorname{sen}\left(\frac{\pi}{2}+\theta-\varphi_4\right) = \cos(\varphi_4-\theta)$$
$$= \cos\varphi_4\cos\theta + \operatorname{sen}\varphi_4\operatorname{sen}\theta$$

$$\cos^2\theta\cos^2\varphi_4 = \frac{v^2}{c^2}\cos^2\theta + \operatorname{sen}^2\theta\operatorname{sen}^2\varphi_4 - 2\frac{v}{c}\operatorname{sen}\theta\cos\theta\operatorname{sen}\varphi_4 \Rightarrow$$

$$\operatorname{sen}^2\varphi_4 - 2\frac{v}{c}\operatorname{sen}\theta\cos\theta\operatorname{sen}\varphi_4 - \left(1-\frac{v^2}{c^2}\right)\cos^2\theta = 0$$

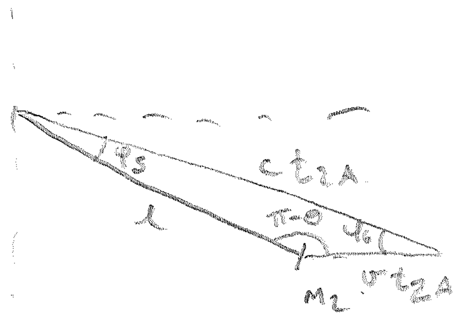
$$\operatorname{sen}\varphi_4 = \frac{2\frac{v}{c}\operatorname{sen}\theta\cos\theta \pm \sqrt{4\frac{v^2}{c^2}\operatorname{sen}^2\theta\cos^2\theta + 4\left(1-\frac{v^2}{c^2}\right)\cos^2\theta}}{2}$$

$$= \cos\theta\left(\frac{v}{c}\operatorname{sen}\theta \pm \sqrt{1-\frac{v^2}{c^2}\cos^2\theta}\right)$$

Logo

$$\boxed{t_{1B} = \frac{l}{c}\,\frac{1}{\sqrt{1-\frac{v^2}{c^2}\cos^2\theta} + \frac{v}{c}\operatorname{sen}\theta}}$$

Para o tempo de ida t_{2A} do espelho M_2 temos:



Pela lei dos senos

$$\frac{l}{\operatorname{sen}\varphi_6} = \frac{c\,t_{2A}}{\operatorname{sen}(\pi-\theta)} = \frac{v\,t_{2A}}{\operatorname{sen}\varphi_5} \qquad \begin{aligned}\varphi_5+\varphi_6+\pi-\theta &= \pi\\ \varphi_5+\varphi_6 &= \theta\end{aligned}$$

Logo

$$\operatorname{sen}\varphi_5 = \frac{v}{c}\operatorname{sen}\theta = \operatorname{sen}(\theta-\varphi_6)$$
$$= -\cos\theta\operatorname{sen}\varphi_6 + \operatorname{sen}\theta\cos\varphi_6$$

$$\operatorname{sen}^2\theta\cos^2\varphi_6 = \frac{v^2}{c^2}\operatorname{sen}^2\theta + \cos^2\theta\operatorname{sen}^2\varphi_6 + 2\frac{v}{c}\operatorname{sen}\theta\cos\theta\operatorname{sen}\varphi_6 = 0$$

$$\operatorname{sen}^2\varphi_6 + 2\frac{v}{c}\operatorname{sen}\theta\cos\theta\operatorname{sen}\varphi_6 - \left(1-\frac{v^2}{c^2}\right)\operatorname{sen}^2\theta = 0$$

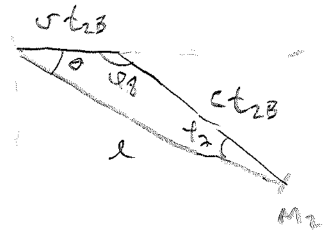
$$\operatorname{sen}\varphi_6 = \frac{-2\frac{v}{c}\operatorname{sen}\theta\cos\theta \pm \sqrt{4\frac{v^2}{c^2}\operatorname{sen}^2\theta\cos^2\theta + 4\left(1-\frac{v^2}{c^2}\right)\operatorname{sen}^2\theta}}{2}$$

$$= \operatorname{sen}\theta\left(-\frac{v}{c}\cos\theta \pm \sqrt{1-\frac{v^2}{c^2}\operatorname{sen}^2\theta}\right)$$

Logo

$$\boxed{t_{2A} = \frac{l}{c}\,\frac{1}{\sqrt{1-\frac{v^2}{c^2}\operatorname{sen}^2\theta} - \frac{v}{c}\cos\theta}}$$

Para o tempo da volta t_{2B} temos:



Pela lei dos senos:

$$\frac{l}{\operatorname{sen}\varphi_8} = \frac{ct_{2B}}{\operatorname{sen}\theta} = \frac{vt_{2B}}{\operatorname{sen}\varphi_7} \qquad \begin{aligned}\theta+\varphi_7+\varphi_8 &= \pi\\ \varphi_7+\varphi_8 &= \pi-\theta\end{aligned}$$

Logo $\operatorname{sen}\varphi_7 = \frac{v}{c}\operatorname{sen}\theta = \operatorname{sen}(\pi-\theta-\varphi_8) = \operatorname{sen}(\theta+\varphi_8)$

$$= \cos\theta\operatorname{sen}\varphi_8 + \operatorname{sen}\theta\cos\varphi_8$$

e daí

$$\operatorname{sen}^2\theta\cos^2\varphi_8 = \frac{v^2}{c^2}\operatorname{sen}^2\theta + \cos^2\theta\operatorname{sen}^2\varphi_8 - 2\frac{v}{c}\operatorname{sen}\theta\cos\theta\operatorname{sen}\varphi_8 = 0$$

$$\operatorname{sen}^2\varphi_8 - 2\frac{v}{c}\operatorname{sen}\theta\cos\theta\,\operatorname{sen}\varphi_8 - \left(1-\frac{v^2}{c^2}\right)\operatorname{sen}^2\theta = 0$$

$$\operatorname{sen}\varphi_8 = \frac{2\frac{v}{c}\operatorname{sen}\theta\cos\theta \pm \sqrt{4\frac{v^2}{c^2}\operatorname{sen}^2\theta\cos^2\theta + 4\left(1-\frac{v^2}{c^2}\right)\operatorname{sen}^2\theta}}{2}$$

$$= \operatorname{sen}\theta\left(\frac{v}{c}\cos\theta \pm \sqrt{1-\frac{v^2}{c^2}\operatorname{sen}^2\theta}\right)$$

Então

$$\boxed{t_{2B} = \frac{l}{c}\,\frac{1}{\sqrt{1-\frac{v^2}{c^2}\operatorname{sen}^2\theta}+\frac{v}{c}\cos\theta}}$$

Logo

$$\Delta t = t_{2A} + t_{2B} - t_{1A} - t_{1B}$$

$$= \frac{l}{c}\left[-\frac{1}{\sqrt{1-\frac{v^2}{c^2}\cos^2\theta}-\frac{v}{c}\operatorname{sen}\theta} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}\cos^2\theta}+\frac{v}{c}\operatorname{sen}\theta}\right.$$

$$\left. + \frac{1}{\sqrt{1-\frac{v^2}{c^2}\operatorname{sen}^2\theta}-\frac{v}{c}\cos\theta} + \frac{1}{\sqrt{1-\frac{v^2}{c^2}\operatorname{sen}^2\theta}+\frac{v}{c}\cos\theta}\right]$$

$$= \frac{l}{c}\left[-\frac{2\sqrt{1-\frac{v^2}{c^2}\cos^2\theta}}{1-\frac{v^2}{c^2}} + \frac{2\sqrt{1-\frac{v^2}{c^2}\operatorname{sen}^2\theta}}{1-\frac{v^2}{c^2}}\right]$$

Expandindo em potências de $\frac{v}{c}$

$$\Delta t = \frac{l}{c}\left[\cancel{-2} + \frac{v^2}{c^2}\cos^2\theta + \cancel{2} - \frac{v^2}{c^2}\operatorname{sen}^2\theta + O\left(\frac{v^3}{c^3}\right)\right] *$$

$$* \left[1 + \frac{v^2}{c^2} + O\left(\frac{v^3}{c^3}\right)\right]$$

Portanto

$$\Delta t = \frac{l}{c}\left[\frac{v^2}{c^2}\left(\cos^2\theta - \operatorname{sen}^2\theta\right) + O\left(\frac{v^3}{c^3}\right)\right]$$

ou se

$$\boxed{\Delta t \sim \frac{v^2}{c^3}\, l \cos 2\theta}$$