PQI 3203: FENÔMENOS DE TRANSPORTE I

Tensão em fluidos – Navier Stokes

Exercício - 5

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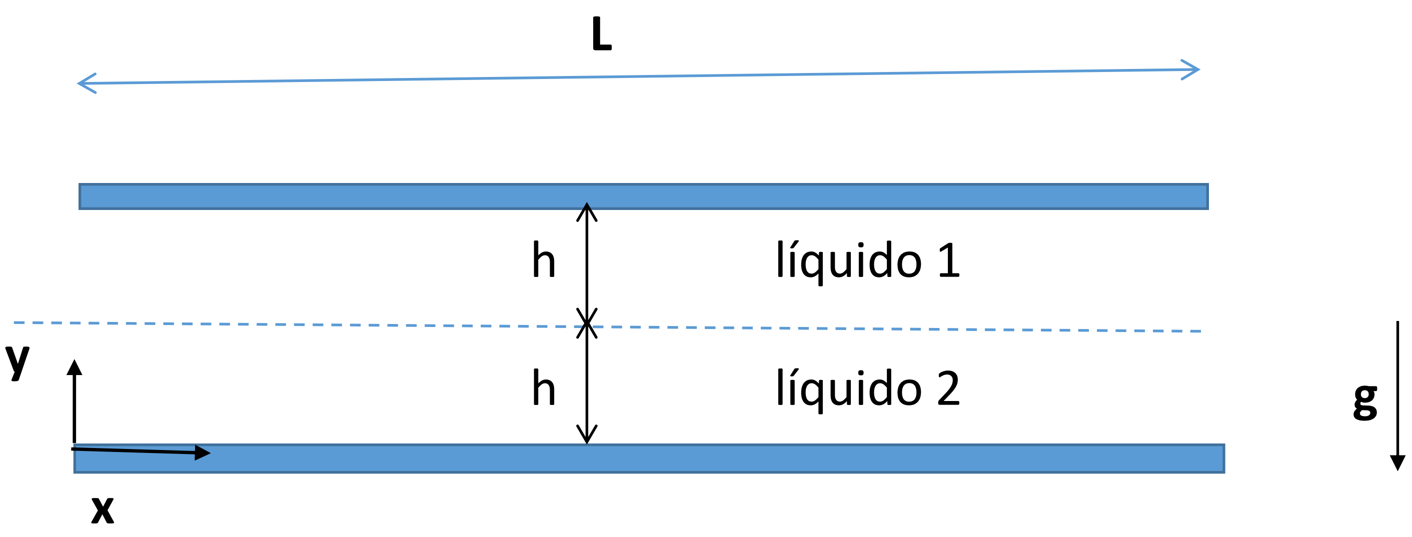
Dois líquidos imiscíveis, newtonianos e viscosos (densidades $\rho 1 < \rho 2$, viscosidades dinâmicas $\mu 1 > \mu 2$) escoam entre duas placas paralelas, horizontais e extensas. A placa inferior é mantida imóvel ea superior tem velocidade V. As espessuras de cada camada de líquido é h e a pressão não varia com x (figura). Deduzir as expressões: (a) do perfil de velocidades do escoamento, (b) da velocidade na interface, (c) do campo de pressões, (d) da tensão de cisalhamento no líquido e (e) da força tangencial do líquido sobre as paredes.



$$\begin{cases} 0 = \cancel{-\dfrac{\partial p_1}{\partial x}} + \mu_1 \dfrac{\partial^2 v_{x_1}}{\partial y^2} + \cancel{\rho_1 g_x} \\ 0 = \cancel{-\dfrac{\partial p_2}{\partial x}} + \mu_2 \dfrac{\partial^2 v_{x_2}}{\partial y^2} + \cancel{\rho_2 g_x} \end{cases}$$

$$\begin{cases} \mu_1 \dfrac{d^2 v_{x_1}}{dy^2} = 0 \Rightarrow \dfrac{dv_{x_1}}{dy} = C_1 \Rightarrow v_{x_1} = C_1 y + C_2 \\ \mu_2 \dfrac{d^2 v_{x_2}}{dx^2} = 0 \Rightarrow \dfrac{dv_{x_2}}{dy} = C_3 \Rightarrow v_{x_2} = C_3 y + C_4 \end{cases}$$

$$\begin{cases} \mu_1 \dfrac{d^2 v_{x1}}{dy^2} = 0 \Rightarrow \dfrac{dv_{x1}}{dy} = C_1 \Rightarrow v_{x1} = C_1 y + C_2 \\ \mu_2 \dfrac{d^2 v_{x2}}{dx^2} = 0 \Rightarrow \dfrac{dv_{x2}}{dy} = C_3 \Rightarrow v_{x2} = C_3 y + C_4 \end{cases}$$

C.C. $y = 0 \Rightarrow v_{x2} = 0 \Rightarrow C_4 = 0$

C.C. $y = 2h \Rightarrow v_{x1} = V \Rightarrow V = C_1 \times 2h + C_2$

C.C. $\begin{cases} x = h \Rightarrow v_{x1} = v_{x2} \Rightarrow C_1 \cdot h + C_2 = C_3 h + \cancel{C_4}^{\,0} \\ x = h \Rightarrow \mu_1 \dfrac{dv_{x1}}{dy} = \mu_2 \dfrac{dv_{x2}}{dy} \Rightarrow \mu_1 \cdot C_1 = \mu_2 C_3 \end{cases}$

$$C_1 h + C_2 = \frac{\mu_1}{\mu_2} C_1 \cdot h \Rightarrow C_2 = C_1 h\left(\frac{\mu_1}{\mu_2} - 1\right)$$

$$V - C_2 = C_1 \times 2h = V - C_1 h\left(\frac{\mu_1}{\mu_2} - 1\right) = C_1 \times 2h \Rightarrow V = C_1 h\left(1 + \frac{\mu_1}{\mu_2}\right)$$

$$\boxed{C_1 = \frac{V}{h\left(1 + \frac{\mu_1}{\mu_2}\right)}}$$

$$C_2 = C_1 h\left(\frac{\mu_1}{\mu_2} - 1\right) = \frac{V}{h\left(1 + \frac{\mu_1}{\mu_2}\right)} h \left(\frac{\mu_1}{\mu_2} - 1\right)$$

$$C_2 = \frac{V\left(\frac{\mu_1}{\mu_2} - 1\right)}{\left(1 + \frac{\mu_1}{\mu_2}\right)}$$

$$C_1 h + C_2 = \frac{\mu_1}{\mu_2} C_1 \cdot h \Rightarrow C_2 = C_1 h\left(\frac{\mu_1}{\mu_2} - 1\right)$$

$$V - C_2 = C_1 \times 2h = V - C_1 h\left(\frac{\mu_1}{\mu_2} - 1\right) = C_1 \times 2h \Rightarrow V = C_1 h\left(1 + \frac{\mu_1}{\mu_2}\right)$$

$$\boxed{C_1 = \frac{V}{h\left(1 + \frac{\mu_1}{\mu_2}\right)}}$$

$$C_2 = C_1 h\left(\frac{\mu_1}{\mu_2} - 1\right) = \frac{V}{h\left(1 + \frac{\mu_1}{\mu_2}\right)} h\left(\frac{\mu_1}{\mu_2} - 1\right)$$

$$C_2 = \frac{V\left(\frac{\mu_1}{\mu_2} - 1\right)}{\left(1 + \frac{\mu_1}{\mu_2}\right)}$$

$$V_{x1} = \frac{V}{h\left(1+\frac{\mu_1}{\mu_2}\right)} x + \frac{V\left(\frac{\mu_1}{\mu_2}-1\right)}{\left(1+\frac{\mu_1}{\mu_2}\right)} = \left(\frac{V}{1+\frac{\mu_1}{\mu_2}}\right)\left[\frac{x}{h} + \frac{\mu_1}{\mu_2} - 1\right]$$

$$c_3 = c_1 \frac{\mu_1}{\mu_2} = \frac{V}{h\left(1+\frac{\mu_1}{\mu_1}\right)} \frac{\mu_1}{\mu_2}$$

$$\boxed{V_{x2} = \frac{V}{h\left(1+\frac{\mu_1}{\mu_2}\right)} \frac{\mu_1}{\mu_2} x} = \boxed{\frac{V}{h\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)} \cdot \frac{1}{\mu_2} x}$$

$$\boxed{\tau\Big|_1 = \mu_1 \frac{dv_{x1}}{dy} = \mu_1 \frac{V}{h\left(1+\frac{\mu_1}{\mu_2}\right)}}$$

$$\boxed{\tau_2\Big| = \mu_2 \frac{dv_{x2}}{dy} = \mu_1 \frac{V}{h\left(1+\frac{\mu_1}{\mu_2}\right)}}$$

¡omepi!