DYNAMICS OF A TURRET-FPSO SYSTEM AND HYDRODYNAMIC MODELS

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ABSTRACT

The dynamics of a Turret-FPSO system subjected to steady ocean current is addressed under standard bifurcation theory and under a heuristic hydrodynamic model (HM) that combines Finite Span Wing and Cross Flow models. Accounting for the equilibrium problem, it has been previously shown that if a classical third-order hydrodynamic derivatives model (HD) is used, the sign of the third derivative of the hydrodynamic moment with respect to the transversal velocity is responsible for a dramatic change in the pitchfork bifurcation scenario, from a super-critical to a sub-critical one (Pesce & Tannuri, 1997). On the other hand, under a heuristic approach, supported by captive model tests results, a bi-linear term on the heading angle has been shown to be dominant (Leite et al., 1997), destroying the previous conclusion. Additionally, such a quadratic term is responsible for a change in the super-critical pitchfork bifurcation pattern. This strong result has shown, quite clearly, the lack of robustness of the classical third-order HD model. On account of this fact, the static heuristic model has been extended dynamically, incorporating bi-linear terms involving the transversal relative velocity and yaw rate. The advantage are twofold: robustness and simplicity, as the *hydrodynamic coefficients* are written as simple functions depending solely on the ship principal dimensions and form coefficients. Time domain simulations are presented and used to exemplify the overall behavior discussed herein. The equilibrium bifurcation behavior is recovered and some dynamic scenarios are showed and discussed.

INTRODUCTION

The Floating Production Storage and Offloading System (FPSO's) is a modern concept for floating offshore oil exploration units, moored in deep water. A tanker is moored offshore and oil is stored before being transported by shuttle tankers that periodically are connected to the mother ship in a tandem formation. The vessels are subject to the environmental loads, due to the concomitant action of ocean currents, waves and wind. The 'Turret' type is a special kind of single point mooring system composed by a huge bearing system, fixed directly to the ship, the hawser cable being eliminated, and moored to the sea-bed, as shown in Fig 1.The primer motivation of such systems is to allow the ship to be aligned with the 'resultant' of the environmental forces, diminishing motions and structural loads on the mooring lines, hawsers and risers.

If wind and waves actions are not considered and if ocean current is taken as steady, a self-excited autonomous non-linear system of ordinary differential equations can be shown to govern the system dynamics. The highly non-linear dynamic nature of this kind of system gives rise to a rich behaviour scenario that may comprise from simple pitchfork point equilibrium bifurcations to Hopf bifurcations (limit cycles), or even chaotic regime. Bifurcation theory and modern system dynamics may form a proper theoretical basis for the analysis.

Under the bifurcation theory approach a large amount of research work has been done in this subject, primarily motivated by a close-related problem of a towed ship in either a straight course or maneuvering in a port site. Bernitsas & Kekridis, 1985, treat the towed-ship problem. Papoulias & Bernitsas, 1988, take the single point mooring problem into attention. Recently, Bernitsas & Garza-Rios, 1995, have studied the dynamics of some offshore spread but slacken mooring systems, that exhibit the same sort of dynamic behavior. Under such an approach the general equation of motion is derived on a "hydrodynamic derivative" model basis, in which the hydrodynamic forces due to the relative motion with respect to the water are represented through Taylor series expansions given in terms of relative velocity components.

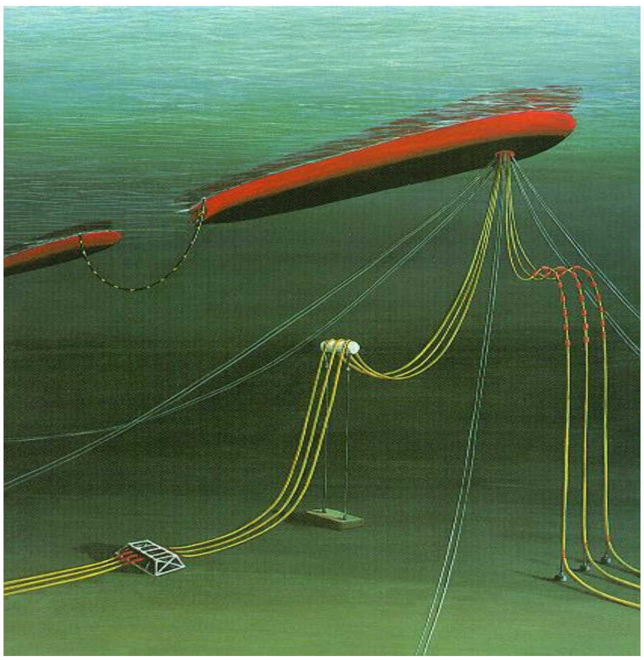

Figure 1 FPSO moored at a 'Turret' System with a shuttle tanker in tandem formation[1]

Pesce & Tannuri, 1997, under the same third-order model of the "hydrodynamic derivatives" type analyzed two excludent *pitchfork bifurcation* scenarios: the *sub*- and the *super-critical* ones. As well known, the position of mooring line attachment at the ship is the control parameter governing equilibria bifurcation. It has also been shown that this parameter can be responsible for a loss of structural stability of the system, switching bifurcation scenarios, whose type is controlled primarily by the 'hydrodynamic derivatives' coefficients. The sign of the third partial derivative of the yaw moment with respect to the lateral velocity, for instance, governs the type of bifurcation scenario that would appear. Nevertheless, this important coefficient is rather small, the experimental accuracy being usually poor, given rise even to changes in sign, depending on the analysis method that is used, as observed by Leite et al, 1997. The classical third-order hydrodynamic derivative model exhibits a square-root behavior in the vicinity of the bifurcation point.

[1] Cortesy: Orcina Cable Protection

More recently, Leite et al, 1997, developed a *heuristic hydrodynamic model* in order to assess captive model tests, aiming the equilibrium bifurcation analysis. Being β the angle of attack with respect to the flow, such a model asymptotically merges standard cross-flow expressions commonly used in ship hydrodynamics, (Faltinsen, 1990), which are valid when $|\sin(\beta)| \cong O(1)$, with low aspect ratio wing theory expressions, valid when $\beta << 1$, taking into account the linear maneuvering coefficients proposed by Clarke et al., 1983. Besides of a phenomenological nature this model has been experimentally verified for a class of tankers, and exhibits a special virtue: *the hydrodynamic coefficients are promptly evaluated from the principal geometrical characteristics of the hull*. More than that, Leite et al. have shown that the post-critical behavior of the super-critical pitchfork bifurcation scenario is dominated by a bi-linear term, being locally linear (in the vicinity of the bifurcation point), with respect to the bifurcation parameter, namely, the position of mooring line attachment. So, between these two hydrodynamic models, not only distinct qualitative behaviors but also quantitative discrepancies are expected.

The present paper extends the *heuristic hydrodynamic model* to the dynamic problem, recovering the above-mentioned results, concerning stability and bifurcation analysis, and presenting some examples of time domain simulations with both models, for a typical tanker.

GOVERNING EQUATIONS

We consider the horizontal plane motions of a FPSO system in a turret configuration in the presence of steady marine current as represented in Fig 2, below:

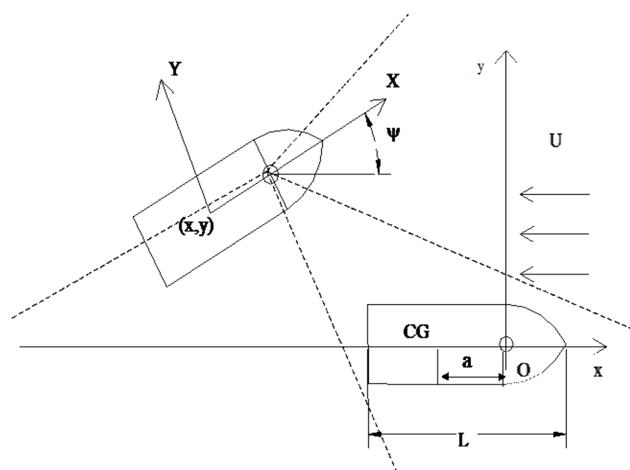

Figure 2 Reference frames and geometric definitions

The main variables that describe the dynamic problem are:

- *u*: velocity of the ship relative to the water, in the surge direction;
- *v*: velocity of the ship relative to the water, in the sway direction;

- (x,y): ship's center of mass coordinates, in the fixed Oxy reference frame;
- $(\dot{x},\dot{y})$: center of mass absolute velocity;
- ψ: yaw angle;
- r : yaw rate of the ship $(\dot{\psi})$;
- U: current velocity;
- β: instantaneous incidence angle of the current;

The parameter *a* measures the longitudinal distance between the turret and the ship's center of mass. In the present work this parameter is the only one that is allowed to vary and will then be referred to as the *bifurcation parameter*. The equations governing the system dynamics can be written as

$$\begin{aligned}
&-M_{11}\dot{u} + M_{22}vr + X_{moor} + X(u,v,r) = M(\dot{u} - vr)\\
&- M_{33}\dot{v} - M_{11}ur - M_{23}\dot{r} + Y_{moor} + Y(u,v,r) = M(\dot{v} + ur)\\
&- M_{33}\dot{r} - M_{32}(\dot{v} + ur) + N_{moor} + N(u,v,r) = I_Z\dot{r}
\end{aligned} \tag{1}$$

where

M: ship's mass;

I_z: ship's moment of inertia with respect to GZ axis;

$[M_{ij}]$: added mass matrix (symmetric by construction);

being

$(X_{moor};Y_{moor};N_{moor})$: the surge, sway and yaw generalized forces applied on the system by the mooring system;

$(X(u,v,r);Y(u,v,r);N(u,v,r))$: the hydrodynamic surge, sway and yaw generalized forces.

The hydrodynamic forces $(X(u,v,r);Y(u,v,r);N(u,v,r))$ depend on the particular hydrodynamic model that is adopted. The hydrodynamic model proposed herein is an extension of a heuristic model, developed in Leite et al.,1997, for the assessment of the captive model (static) forces. Dynamic forces, especially due to the yaw rotation of the ship, are then incorporated to address the dynamic problem.

THE HEURISTIC HYDRODYNAMIC MODEL

The static equilibrium version of this model has been proposed by Leite et al., 1997, under a captive model approach, in order to assess the static hydrodynamic forces acting on the hull. Being β the angle of attack with respect to the flow, such a model asymptotically merges standard cross-flow expressions commonly used in ship hydrodynamics, (Faltinsen, 1990), which are valid when $|\sin(\beta)| \cong O(1)$, with low aspect ratio wing theory expressions, valid when $\beta \ll 1$, taking into account the linear maneuvering coefficients proposed by Clarke et al., 1983.

Static Forces

The static hydrodynamic forces acting on the floating unit hull are written in a standard way, Leite et al., 1997, as

$$\begin{aligned}
X_s(\beta,V) &= \frac{1}{2}\rho TL.C_{1s}(\beta).|V|^2\\
Y_s(\beta,V) &= \frac{1}{2}\rho TL.C_{2s}(\beta).|V|^2\\
N_s(\beta,V) &= \frac{1}{2}\rho TL^2.C_{6s}(\beta).|V|^2
\end{aligned} \tag{2}$$

where V is the relative velocity vector and the hydrodynamic coefficients are given by:

$$\begin{aligned}
C_{1s}(\beta) &= \left[\frac{0.09375}{(\log(\mathrm{Re})-2)^2}\frac{S}{TL}\right]\cos(\beta) + \frac{1}{8}\frac{\pi T}{L}(\cos(3\beta) - \cos(\beta))\\
C_{2s}(\beta) &= \left[C_Y - \frac{\pi T}{2L}\right]\sin(\beta)|\sin(\beta)| + \frac{\pi T}{2L}\sin^3(\beta) + \frac{\pi T}{L}\left[1 + 0.4\frac{C_B B}{T}\right]\sin(\beta)|\cos(\beta)|\\
C_{6s}(\beta) &= \frac{-l_g}{L}\left[C_Y - \frac{\pi T}{2L}\right]\sin(\beta)|\sin(\beta)| - \frac{\pi T}{L}\sin(\beta)\cos(\beta) - \left[\frac{1+|\cos(\beta)|}{2}\right]^2\frac{\pi T}{L}\left[\frac{1}{2} - 2.4\frac{T}{L}\right]\sin(\beta)|\cos(\beta)|
\end{aligned} \tag{3}$$

being,

(L,B,T): ship's main dimensions (waterline length, breadth and draft);

S: wetted surface;

C_B: block coefficient;

C_Y : lateral force coefficient in transversally steady current.;

Re: Reynold's number (based on the length L);

The length l_g measures the longitudinal distance between the hull's center of mass and the midship section. The model assumes that the hull is also symmetric with respect to the midship section. The hydrodynamic force coefficients were compared to experimental data gathered from captive model tests, within a very good agreement (Leite et al., 1997), what provides a great confidence on the heuristic model. Notice that the computation is straightforward as only main particulars, the block coefficient and a cross flow coefficient are needed.

The angle β is defined so that the head current case corresponds to $\beta=\pi$. In the dynamic problem, the instantaneous value of this angle must be taken, being written in terms of the yaw angle ψ and the velocities components in the form,

$$\beta = (\pi - \psi) + \sin^{-1}\left(\frac{\dot{y}}{|V|}\right) \tag{4}$$

where

$$|V| = [u^2 + v^2]^{1/2} \tag{5}$$

Dynamic Forces

The hydrodynamic forces are here accounted for as being separable according to two distinct phenomena: those related to the crossflow along the ship length and those referring to vortices emission at bow and stern.

Forces and Moment due to Cross-flow:

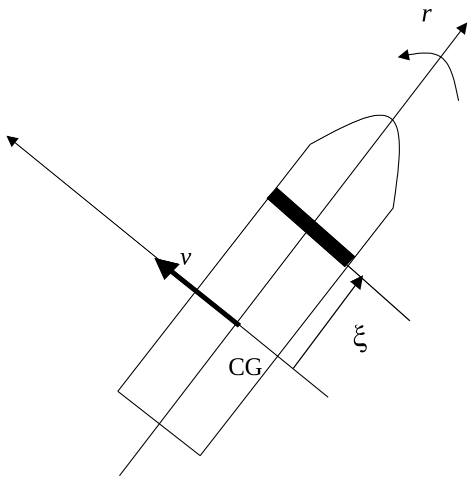

Figure 3 Definitions for the cross-flow dynamic effects

Considering the cross-flow effect, being r the yaw rotation rate, the force in the transversal direction and the yaw moment may be expressed as integrals along the ship length, in a standard strip-theory way, involving the transversal components of the relative velocities of each section with respect to the water, in the form

$$Y_{CF} = -\frac{1}{2}\rho.T_S.\int C_D(\xi).(v+\dot{\psi}.\xi).\left|v+\dot{\psi}.\xi\right|d\xi$$

$$N_{CF} = -\frac{1}{2}\rho.T_S.\int C_D(\xi).(v+\dot{\psi}.\xi).\left|v+\dot{\psi}.\xi\right|.\xi.d\xi \qquad (6)$$

where

$C_D(\xi)$: is the transversal drag coefficient for each station;

ξ : is the longitudinal coordinate of the considered station, with respect to the hull's center of mass

After some simple algebraic manipulations, Eq.(6) may be written as:

$$Y_{CF} = -\frac{1}{2}\rho.T_S.\left(v^2.I_0 + 2.v.\dot{\psi}.I_1 + \dot{\psi}^2.I_2\right)$$

$$N_{CF} = -\frac{1}{2}\rho.T_S.\left(v^2.I_1 + 2.v.\dot{\psi}.I_2 + \dot{\psi}^2.I_3\right) \qquad (7)$$

being:

$$I_0 = \int C_D(\xi).sign(v+\dot{\psi}.\xi)..d\xi$$

$$I_1 = \int C_D(\xi).sign(v+\dot{\psi}.\xi).\xi.d\xi$$

$$I_2 = \int C_D(\xi).sign(v+\dot{\psi}.\xi).\xi^2.d\xi$$

$$I_3 = \int C_D(\xi).sign(v+\dot{\psi}.\xi).\xi^3.d\xi \qquad (8)$$

As the terms involving v^2 in Eq. (7) have already being considered in the static model, Eq. (2), the only terms that must be added are the $v.\dot{\psi}$ and $\dot{\psi}^2$ terms, that is,

$$Y_{CF}^{D} = -\frac{1}{2}\rho.T_S.\left(2.v.\dot{\psi}.I_1 + \dot{\psi}^2.I_2\right)$$

$$N_{CF}^{D} = -\frac{1}{2}\rho.T_S.\left(2.v.\dot{\psi}.I_2 + \dot{\psi}^2.I_3\right) \qquad (9)$$

For consistency, the values of $C_D(\xi)$, (extracted from Faltinsen, 1990, in the present work), are normalized by the transversal coefficient C_Y as to make the v^2 component of the cross-flow lateral force equal to the static force Y_s (see Eq. (2)), for the incidence angle $\beta=90^o$, viz.

$$\int C_D(\xi)d\xi = C_Y L \qquad (10)$$

Moment due to vortices emission at bow and stern

An heuristic reasoning leads to a yaw moment component caused by vortices emission at bow and stern, as schematically shown in Fig 4, here referred to simply as a tip-vortex phenomenon,

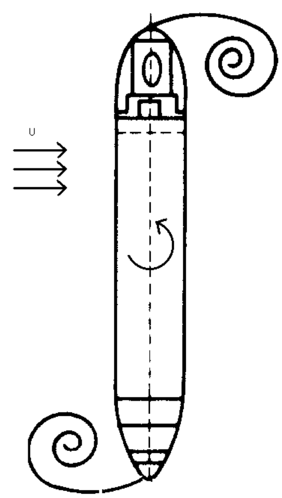

Figure 4 Tip-vortex phenomenon

As a first rough approximation, the hull can be considered as a flat plate of length L and infinite depth. Assuming these forces are applied at the very ends it can easily be shown that the corresponding yaw moment parcel can be written,

$$M_{\dot{\psi}}^{D} = -\frac{1}{16}.\rho.H.C_D^{plate}.L^4.\dot{\psi}.\left|\dot{\psi}\right| \qquad (11)$$

where the value of the coefficient C_D^{plate} can be taken, from Höerner, 1965, as 2.

A CLASSICAL THIRD-ORDER HYDRODYNAMIC DERIVATIVES MODEL

Usual ‘Hydrodynamic Derivatives’ (HD) models, are oftenly imported from ship maneuvering theory to the moored ship problem, under steady current. The classical one, as in Papoulias & Bernitsas, 1988, expands the generalized hydrodynamic forces into standard Taylor’s Series, in the relative velocity components. The coefficients in this expansion are the so-called ‘hydrodynamic derivatives’, being experimentally determined. Heuristically, a number of models, also present in the technical literature, add odd bi-linear terms of the form $v|v|$ to the Taylor’s Series; see, e.g., Takashina, 1986, Nishimoto et al., 1995.

In this particular section we shall address only the third-order, classical HD model, in order to discuss some crucial points, concerning stability and dynamics, that are originated when terms of the form $v|v|$ are disregarded. The hydrodynamic forces acting on the hull will then be given in terms of the hydrodynamic derivatives (in this case, up to the third order) in the form,

$$\begin{aligned} X(u,v,r) &= X_u u + \frac{1}{6} X_{uuu} u^3 \\ Y(u,v,r) &= Y_v v + \frac{1}{6} Y_{vvv} v^3 + Y_r r + \frac{1}{6} Y_{rrr} r^3 \\ N(u,v,r) &= N_v v + \frac{1}{6} N_{vvv} v^3 + N_r r + \frac{1}{6} N_{rrr} r^3 \end{aligned} \quad (12)$$

where *X(u,v,r)* and *Y(u,v,r)* are, respectively, the surge and sway forces and *N(u,v,r)* is the yaw moment acting on the floating unit.

It is assumed in this work that the surge force depends only on the longitudinal relative velocity (u). Also, as the forces above are odd functions with respect to the velocity components in their corresponding directions, the series expansions only present odd terms.

STABILITY ANALYSIS[2]

When the turret is located near the center of mass, the stable equilibrium position of the ship is usually different from $\psi = 0^o$. In other words, the ship is not aligned with the ocean current, but takes a new yaw angle of static equilibrium. The critical value of the distance a (between the turret and the center of mass) below which such a *bifurcation* takes place may be analytically calculated.

For the classical Hydrodynamic Derivatives model, it can be shown that (Papoulias & Bernitsas, 1988) this critical value is simply given by,

$$a_{CR} = \frac{N_v}{Y_v} \quad (13)$$

The new stable equilibrium position of the ship (when $a < a_{CR}$) is then,

$$\sin \overline{\psi} = \pm \frac{\sqrt{\lambda}}{U} \quad (14)$$

with,

$$\lambda = -6 \frac{N_v - aY_v}{N_{vvv} - aY_{vvv}} \quad (15)$$

a bifurcation control parameter. Under this HD model, as pointed out in Pesce & Tannuri, 1997, and easily seen from Eqs. (14-15), the qualitative behavior of the pitchfork bifurcation scenario can be dramatically changed, according to the sign taken by the third-order hydrodynamic derivative N_{vvv}, switching from a *super-* to a *sub-critical* pattern. In fact, if λ is negative there is no real stable bifurcated solutions. It should be noticed that the nondimensional absolute value of this ‘hydrodynamic derivative’ is rather small, compared to Y_{vvv}, and errors in experimentally evaluated high-order coefficients can be large. Notice, also, that $\lambda \le U^2$ must be satisfied, as well. In other words, at $\overline{\psi} = \pm\pi/2$ *structural stability* is broken. We shall return to this point later on.

On the other hand, as shown in Leite et al., 1997, the static heuristic hydrodynamic model leads to,

$$\frac{a_{CR}}{L} = \frac{1/2 + 2.4(T/L)}{1 + 0.4(C_B.B/T)} \quad (16)$$

and the angles of static equilibrium, for $a<a_{CR}$, are given by,

$$\begin{aligned} \overline{\psi} &= \pm\left\{ \frac{B(\hat{a})}{2A(\hat{a})} + \sqrt{\left(\frac{B(\hat{a})}{2A(\hat{a})}\right) + \frac{C(\hat{a})}{A(\hat{a})}} \right\}; A(\hat{a}) > 0 \\ \overline{\psi} &= \pm\left\{ \frac{B(\hat{a})}{2A(\hat{a})} + \sqrt{\left(\frac{B(\hat{a})}{2A(\hat{a})}\right) - \frac{C(\hat{a})}{A(\hat{a})}} \right\}; A(\hat{a}) < 0 \\ \hat{a} &= a/L \end{aligned} \quad (17)$$

where,

$$\begin{aligned} A(\hat{a}) &= \frac{2}{3}(\hat{a}_{CR} - \hat{a}) + \frac{\hat{a}/2 - 1/4 + 1.2T/L}{1 + 0.4(C_B.B/T)} \\ B(\hat{a}) &= \left(\frac{LC_Y}{\pi T} - \frac{1}{2}\right) \cdot \frac{\hat{a} + l_g/L}{1 + 0.4(C_B.B/T)} \\ C(\hat{a}) &= (\hat{a}_{CR} - \hat{a}) \end{aligned} \quad (18)$$

As pointed out by Leite et al., 1997, this latter model is much more robust, concerning bifurcation scenario predictability. It turns out that not only the bifurcation is of the supercritical type, but also the pitchfork is linear in $\hat{a}$, in the vicinity of a_{CR}.

Example on Stability Analysis

As an example on stability analysis and equilibrium bifurcations, the horizontal plane dynamics of a tanker in turret configuration will be studied, with hydrodynamic forces determined according to the two distinct models described above. The tanker’s main particulars, according to Papoulias & Bernitsas, 1988, are shown in Table 1. For simplicity, the mooring line system has been considered as linear, and no mooring line damping has been taken into account. Table 2 shows data used in the Heuristic Model (HM). Table 3 presents data referring to the Hydrodynamic Derivative Model (HD).

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[2] see also Leite et al, 1998, in this same Conference.

Table 1 Tanker's particulars

Length (Lbpp) (m)	325
Breadth (m)	52.9
Draft (m)	23.6
Displacement (m^3)	310669
M (t)	318436
M_{11} (t)	15834
M_{22} (t)	300840
$I_Z + M_{66}$ ($t.m^2$)	4.1254E9

Table 2 Data for the Heuristic Hydrodynamic Model

l_g	3.25 m
S	2.48E04m^2
C_B	0.75
C_Y	0.7

Table 3 Nondimensional data for the Hydrodynamic Derivatives Model.

Hydrodynamic Derivative	Factor	Nondimensional Hydrodynamic Derivative
X_u	$0.5\rho L^2 U$	-0.003
Y_v	$0.5\rho L^2 U$	-0.0261
Y_r	$0.5\rho L^3 U$	0.00365
N_r	$0.5\rho L^4 U$	-0.0048
N_v	$0.5\rho L^3 U$	-0.0105
Y_{vvv}	$0.5\rho L^2 / U$	-0.045
N_{vvv}	$0.5\rho L^3 / U$	0.00611
N_{rrr}	$0.5\rho L^6 / U$	0.00611

The Equilibrium Bifurcation

The angle of static equilibrium is plotted as a function of the bifurcation parameter *a*, according the two distinct hydrodynamic models. Both models lead to the same critical value for this parameter, $a_{CR}/L \cong 0.40$, (Eqs. (13) and (16)), below which super-critical pitchfork bifurcation occurs. Nevertheless, the post-critical behavior predicted by the distinct hydrodynamic models is significantly different, as may be seen in the figure below, shown only for positive values of equilibrium, due to symmetry.

The natures of the discrepancies in the static angles are twofold. First of all, a qualitative difference, which is inherent to the hydrodynamic models formulation. The heuristic model presents a component of yaw (static) moment that is proportional to $C_Y.\psi|\psi|$, which is not considered in the hydrodynamic derivatives model. In fact, one may demonstrate (see Leite et al. (1997)) that this parcel is dominant in the post-critical behavior. It turns out that, in the vicinity of the bifurcation point, the yaw angle predicted by the heuristic model varies linearly with the parameter *a/L*, while, according the hydrodynamic derivatives model, this variation obeys a square root law involving the bifurcation parameter; see Eq. (12).

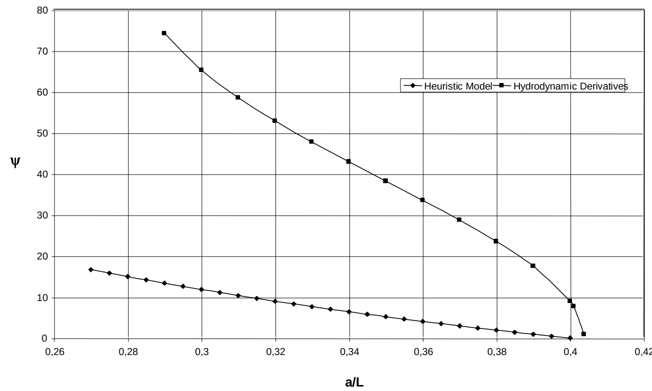

Figure 5 Pitchfork Bifurcation diagram.

Conversely, the observed quantitative discrepancy has a different reason. The dynamic behavior predicted by the hydrodynamic derivatives model depends strongly on the values of the third-order derivatives Y_{vvv} and N_{vvv}. It is well known, however, that accurate experimental determination of such derivatives is extremely difficult, especially for N_{vvv}, because the very small values usually obtained. Actually, very different results are observed for those derivatives (including changes in sign) depending on the source of experimental data; Leite et al, 1997. The relatively strong inaccuracy in such coefficients may even lead (see Pesce et Tannuri (1997)) to changes in the pitchfork bifurcation scenario predicted by the hydrodynamic derivatives model, switching from a super- to a sub-critical one. Furthermore, the bifurcated angles of static equilibrium, presented in the figure above, also strongly depend on the values of the third-order derivatives. As a matter of fact, according to Eq. (14), the angle of equilibrium depends on the parameter λ; see Eq. (15).

Assuming, for simplicity, that the error in the determination of the first-order derivatives is negligible and naming ($N_{vvv}-aY_{vvv}$) as Δ, one may easily verify, from Eq. (14) that, in the classical HD model, the error $\delta(\overline{\psi})$ in the angle of equilibrium is proportional to ($\delta\Delta/\Delta^{3/2}$). As Δ is usually a small quantity and the inaccuracy in its evaluation is significant, the error in the angle of equilibrium may also be expressive.

On the other hand, it must be emphasized that the post-critical behavior predicted by the heuristic model depends only on ship's main dimensions and, as mentioned above, is dominated by the parcel proportional to $C_Y.\psi|\psi|$. The experimental evaluation of the coefficient C_Y is relatively precise, due to the magnitude of the lateral cross-flow forces in beam current and this fact provides a desirable *robustness* to the heuristic hydrodynamic model which was also confirmed by experimental results; Leite et al., 1997.

Time-Domain Simulations

The dynamic system (1) was solved using a 5th-order Runge-Kutta integration scheme, with the following set of state variables: (u,v,r,x,y,ψ). Some time-domain simulations results, obtained according the two distinct hydrodynamic models, are presented in the figures below for different values of the bifurcation parameter *a*. *The main goal of this analysis is to exemplify the discrepancies that may arise* between the HD and the HM models. Initial conditions are taken the same for both models.

Figure 6 shows a stable equilibrium case around ψ=0. Notice that for the HM model this equilibrium is not asymptotically stable, eventhough the remaining oscillation is rather small. This can be easily explained. In fact as in equilibrium conditions; see, e.g., Papoulias & Bernitsas, 1988,

$$\begin{aligned} u &= \bar{u} = U\cos\bar{\psi} \\ v &= \bar{v} = -U\sin\bar{\psi} \end{aligned} \qquad (19),$$

the hydrodynamic damping is exactly null at ψ=0, in the present HM model, as can be seen from Eq. (9). It turns out that a linear term in *r* should then be incorporated, otherwise a small oscillation will last. Figs 7, 8 and 9 present some post-critical cases, concerning bifurcation of equilibrium. The discrepancies are really high. The heuristic hydrodynamic model (HM) leads always to asymptotically stable solutions, the time domain simulation recovering the equilibrium values analytically determined from Eq. (17).

For the HD model, however, the asymptotically stable regime is reached only in the situation shown in figure 7, when the condition $\lambda \le U^2$ is satisfied and Eq. (14) has real roots. Otherwise, as shown in Figs 8 and 9, the super-critical bifurcation scenario is broken, leading to a completely different one. A stable solution emerges (though not asymptotically), with an autonomous oscillation (limit cycle) taking place around $\psi=\pi$. In other words, a subcritical pitchfork bifurcation holds in these last two cases, $\psi=\pi$ being the new fixed point for the third-order hydrodynamic derivatives model. A pertinent question could be risen, however: *are the hydrodynamic derivatives*, experimentally evaluated, *valid for all range of yaw angles*? The answer is, probably, *no*.

Besides experimentally verified in a wide range values of the bifurcation parameter *a/L*, Leite et al, 1997, the HM sounds much more robust, since their coefficients depend solely upon geometrical parameters, with very accurate values, post-critical behavior being dominated by bi-linear terms.

Obviously, bi-linear terms could be added to the standard hydrodynamic derivatives model, as Takashina's for instance. But then, what's the point of doing that, since this adding would be totally heuristic as well, the coefficients still being determined experimentally?

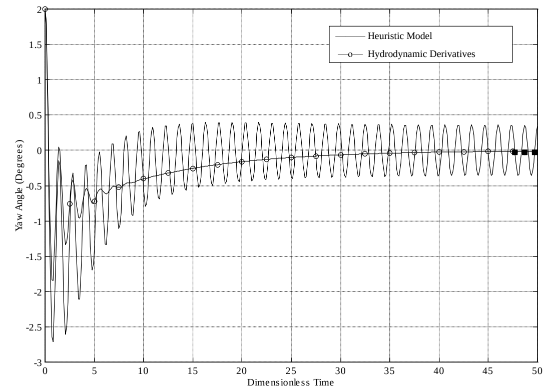

Figure 6 Yaw Angle for a/L=0.45. Initial Conditions: u=U; v,r,x,y=0; ψ =2º

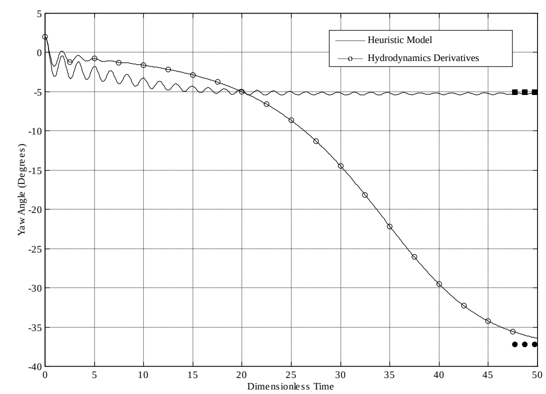

Figure 7 Yaw Angle for a/L=0.35. Initial Conditions: u=U; v,r,x,y=0; ψ =2º

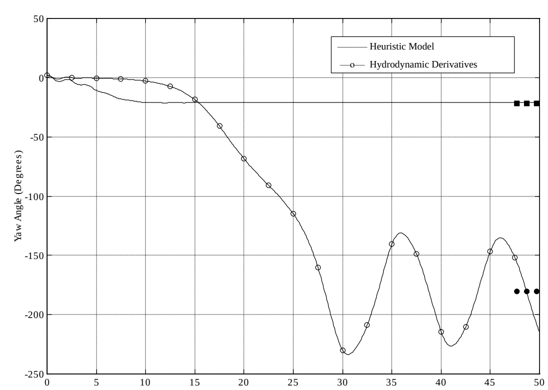

Figure 8 Yaw Angle for a/L=0.25. Initial Conditions: u=U; v,r,x,y=0; ψ =2º

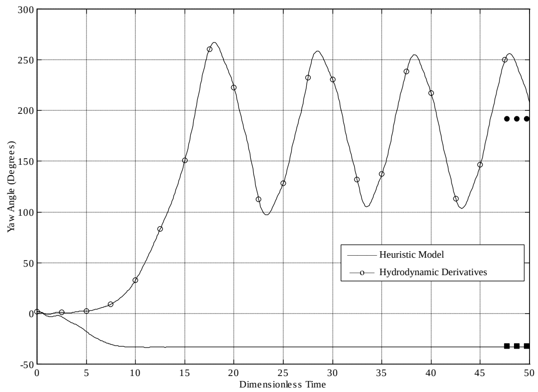

Figure 9 Yaw Angle for a/L=0.20. Initial Conditions: u=U; v,r,x,y=0; ψ =2°

(•••): analytical value of $\bar{\psi}$ for the third-order hydrodynamic derivatives model;

(■■■): analytical value of $\bar{\psi}$ for the heuristic model.

CONCLUSIONS

The present paper extended the static *heuristic hydrodynamic model* (HM), proposed by Leite et al, 1997, to the dynamic problem, recovering some important results concerning stability and bifurcation analysis, and presenting some examples of time domain simulations, for a typical tanker, with comparisons with a classical third-order hydrodynamic derivatives model. *The main goal of this analysis is to exemplify the discrepancies that may arise* between these two models.

If a classical third-order hydrodynamic derivative model is used, the bifurcation parameter, the position of the turret, can be responsible for a loss of structural stability of the system, switching bifurcation scenarios (Pesce & Tannuri, 1997), whose type is controlled primarily by the 'hydrodynamic derivatives' coefficients. The sign of the third partial derivative of the yaw moment with respect to the lateral velocity, for instance, governs the type of bifurcation scenario that would appear. Nevertheless, this important coefficient is rather small, the experimental accuracy being usually poor, given rise even to changes in sign, depending on the analysis method that is used, as observed by Leite et al, 1997. The classical third-order hydrodynamic derivative model exhibits a square-root behavior in the vicinity of the bifurcation point.

On the other hand, Leite et al, 1997, have shown that, in the *heuristic hydrodynamic model*, the post-critical behavior of the super-critical pitchfork bifurcation scenario is dominated by a bi-linear term in the relative velocities, being locally linear (in the vicinity of the bifurcation point), with respect to the bifurcation parameter, namely, the position of mooring line attachment. So, not only distinct qualitative behaviors but also quantitative discrepancies are expected, between these two hydrodynamic models.

Obviously, bi-linear terms could be added to the standard HD model, as Takashina's for instance. But, being experimentally verified for a class of tankers, in a wide range values of the bifurcation parameter *a/L* (Leite et al, 1997), the HM sounds much more simple and robust. In fact, besides being of a phenomenological nature the *heuristic hydrodynamic model* exhibits a special virtue*: the hydrodynamic coefficients are promptly evaluated from the main geometrical characteristics of the hull*, that are accurately known.

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