Hydrodynamic Model Induced Differences in SPM Post Pitchfork Bifurcation Paths

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Several models of the hydrodynamic forces acting on a ship hull in maneuvering have been developed in the last 50 years. These models make possible analysis of ship maneuverability in high and low speeds. Following Bernitsas et al. [1], such hydrodynamic models may be classified into two major schools: the hydrodynamic derivatives (HD) models (first school) and "cross-flow" models (second school). The former is based on Taylor series expansion of the forces while the corresponding coefficients are determined experimentally and remain velocity independent for relatively low velocities. The second school heuristically combines short-wing theory (Jones) and cross-flow experimental data. The aim of this work is to establish and review a certain discrepancy observed in post pitchfork bifurcation paths depending on which school of modeling is adopted. This discrepancy exists in the practical problem of a Single-Point Mooring (SPM) system in a steady ocean current. This discrepancy appears immediately after the point of pitchfork bifurcation of the equilibrium yaw angle versus the longitudinal position of the line attachment point on the hull. According to HD models (e.g., Abkowitz [2]) such a bifurcation curve is a square-root post pitchfork path (e.g., Papoulias and Bernitsas [3]) while cross-flow models (e.g. Leite et al. [4]) predict a different shape of this path at the onset of the post bifurcation curve. Although the practical effect of such a discrepancy may be negligible for SPM systems, this is valuable in assessing an important difference in the distinct approaches followed by the hydrodynamic schools of modeling. Specifically, viscous forces are modeled by odd nonlinear terms in velocity, which are bilinear in the cross-flow models and cubic in the HD models. In this work, experimental results on the aforementioned post pitchfork bifurcation paths are presented and the origin and relevance of the observed discrepancy are discussed. Finally, results presented by Hooft [5] show that yaw angle dependence on bilinear velocity terms regarding cross-flow coefficients would be necessary for a more precise representation of bifurcation patterns near the pitchfork bifurcation. Such patterns may be strongly influenced by hull form.
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Keywords: Single-Point Mooring, Pitchfork Bifurcation, Hydrodynamic Derivatives, Cross-Flow Models

Introduction

Several hydrodynamic models have been proposed in the last 50 years aiming at describing the current forces acting on a ship hull. Those models were developed originally to simulate ship maneuvering. Extensive model and full-scale tests have been conducted in order to assess the accuracy of theoretical predictions.

Recently, the problem of determining current forces on a ship has been revisited due to the advent of Floating Production Storage and Offloading (FPSO) systems. For a moored ship the problem allows some simplifications, particularly free-surface effects can be neglected.

Following Bernitsas et al. [1], the existent hydrodynamic models may be classified into two distinct schools. Models of the first school approach the problem through a Taylor series expansion of the forces with respect to the relative velocities. The coefficients of such expansion (the so-called Hydrodynamic Derivatives, HD) must then be experimentally determined through model tests or field measurements. The models proposed by Abkowitz [2] and Takashina [6] are examples of first school models. Second school models, on the other hand, attempt to describe the current forces through a phenomenological approach. In general, these models heuristically blend Jones short-wing theory and cross-flow results for representing the current forces in any particular angle of incidence. The idea, already discussed by Newman [7], was followed by a series of models such as, for example, the ones proposed by Faltinsen et al. [8], Oltmann & Sharma [9] and, more recently, Simos et al. [10].

HD models are accurate but require extensive small-scale tests in order to determine all the slow motion derivatives. Also, they usually require third-order derivatives, which are relatively difficult to be measured. Discrepancies in higher-order derivatives for the same ship model measured by different testing facilities are not uncommon. The main advantage of cross-flow (CF) models is certainly the dependence on few hydrodynamic parameters, typically the drag coefficient for transverse flow and the hull friction coefficient. Such parameters can be determined easily by means of model tests and might even be approximated with acceptable precision based on data available in literature, as discussed in Simos et al. [10]. In order to achieve this desirable simplicity, however, CF models overlook some more subtle aspects of the flow and focus on the major ones.

When studying the static bifurcation that may occur in SPM systems and turret FPSO systems, a difference is observed at the onset of the post pitchfork bifurcation path, depending on the hydrodynamic model approach: HD models predict that the beginning of the secondary equilibrium path (which represents the equilibrium drift angle as a function of the longitudinal position of the

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mooring attachment on the hull) is a square-root curve. On the other hand, if the same analysis is performed based on a CF model, the beginning of the secondary equilibrium path is linear.

Although such a small discrepancy does not represent any significant practical problem, it shows explicitly a difference when modeling the forces according to the two distinct approaches, namely the inclusion or not of viscous forces terms that are bilinear in the transverse velocity (ν). HD models, as for example the Abkowitz [2] model, model viscous forces due to ν in cubic (odd) form in the Taylor series expansion. On the other hand, CF models use bilinear terms, which are also odd, but of second order in v. Recently, some models based on HD also proposed the inclusion of bilinear terms, e.g. Takashina [6].

In this work model tests were conducted at the University of Michigan Hydrodynamic Lab to determine experimentally the onset of the secondary equilibrium path for a container ship. Together with previous results obtained by Leite et al. [4] for tanker models and based also on experiments presented by Hooft [5] concerning the evaluation of drag coefficient for small angles of incidence, the origin and relevance of the aforementioned differences are analyzed.

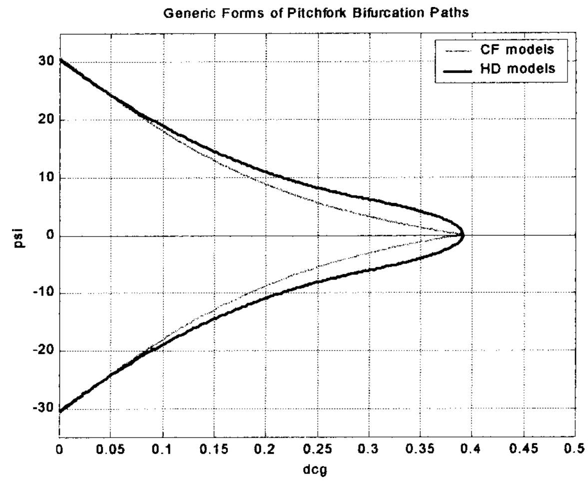

Fig. 2 Generic forms of static pitchfork bifurcation path according to HD and CF models

Pitchfork Bifurcation in SPM

SPM systems and turret FPSO's, which approximately behave as SPM's (Garza-Rios & Bernitsas [11]) typically experience static bifurcation under current action. If the mooring line attachment (or turret) is positioned near the bow, the trivial equilibrium position $\bar{\psi}=0$ may be stable,[1] where ψ is the drift angle as shown in Fig. 1 and the overbar indicates equilibrium values.

If the longitudinal position of attachment is placed closer to the center of gravity there is a bifurcation point beyond which the trivial equilibrium position ($\bar{\psi}=0$) becomes statically unstable and the ship typically finds another equilibrium point with $\bar{\psi}\neq 0$.

The equilibrium heading angle with respect to the current direction may then be plotted as a function of the parameter d_{CG}, which represents the location of the attachment (O) with respect to the center of gravity. The resulting curve is the secondary equilibrium path and its onset depends on the hydrodynamic model. HD models predict a typical square-root curve for the beginning of the path. CF models, on the other hand, produce a linear variation of the path in the vicinity of the bifurcation point. Figure 2 illustrates qualitatively the difference in the secondary equilibrium path at its onset for the two hydrodynamic modeling schools.

The origin of the discrepancy depicted in Fig. 2 was discussed in Simos et al. [12]. In summary, it is a direct consequence of modeling viscous terms due to ν by odd bilinear terms (CF models) or cubic terms (HD models). In CF models, the viscous cross-flow component is usually modeled in the form $C_Y v|v|$ where C_Y represents the hull total drag coefficient for transverse flow, e.g. Leite et al. [4]. Such coefficient must be measured in a captive model test and is assumed constant, i.e. independent of the incidence angle ψ.

Leite et al. [4] derived experimentally and plotted the post pitchfork bifurcation paths for two different tanker models in two distinct ballast conditions. The results were compared to those predicted by the CF model proposed in the same work. The agreement was very good. Unfortunately, their results did not present sufficient points in the immediate post bifurcation region. In this work, additional model tests were conducted and data collected to help determine the shape of the immediate post pitchfork bifurcation path.

[1]Dynamic loss of stability may occur as well in SPM systems, but this is not discussed in this work (Bernitsas et al., 1999).

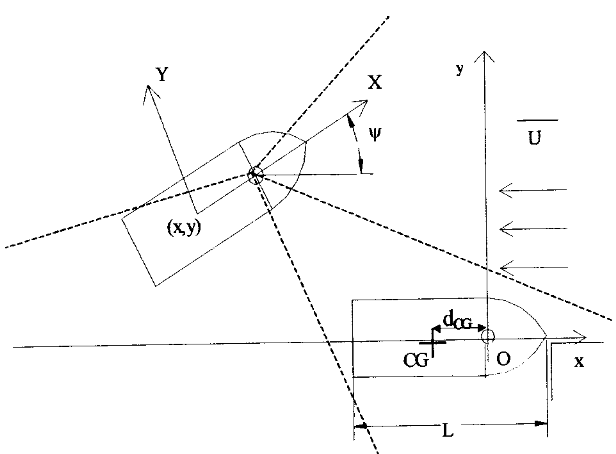

Fig. 1 Schematic representation of alternate static equilibrium

Model Tests and Results

Additional model tests were performed at the Marine Hydrodynamic Laboratory (MHL), Ann Arbor. Tests were conducted with a container ship model (model S175) so the influence of hull shape could also be inferred by comparison with results obtained for tankers by Leite et al. [4]. The model main dimensions are presented in Table 1.

Figure 3 presents a schematic representation of the experimental setup, which consisted of a graduated bar mounted along the ship centerline over which a potentiometer, connected to the tow carriage, could slide. The model was, then, free to rotate in yaw.

Table 1 Model S175 main dimensions

Model	S175
Lpp	3.50 m
B	0.51 m
T	0.19 m
C_B	0.57
scale	1:50

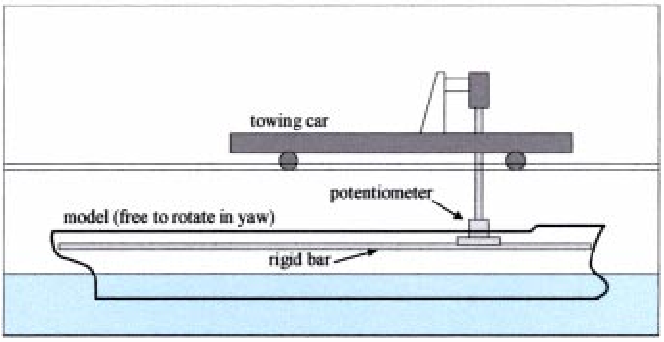

Fig. 3 Schematic representation of the experimental setup

Through the apparatus, also shown in Fig. 4, it was possible to change the attachment point distance d_{CG} in a continuous manner.

Tests were carried out with velocity $U=0.18$ m/s, corresponding to a Froude number $U/\sqrt{gB}=0.08$. For this value one may assume that free-surface effects are negligible (see [4]).

Figure 5 presents the experimental post bifurcation secondary equilibrium path obtained for the S175 container ship model.

Fig. 4 Experimental setup

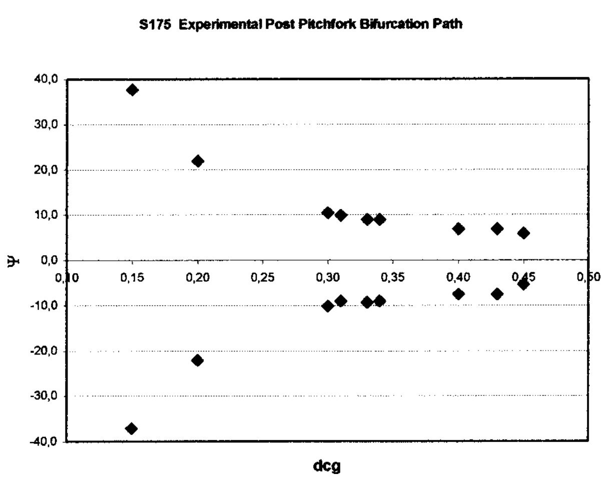

Fig. 5 Experimental post pitchfork bifurcation equilibrium path

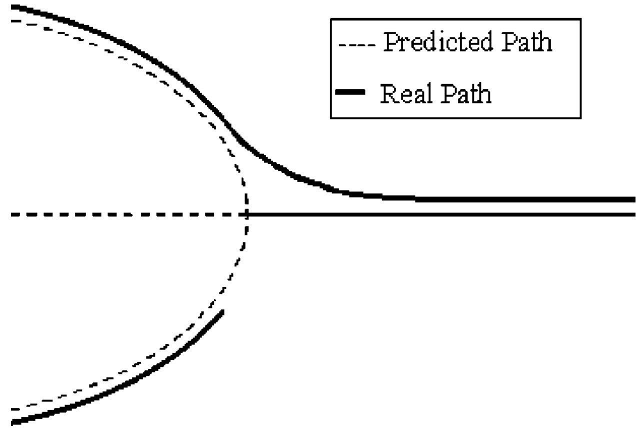

Fig. 6 Qualitative behavior of real and predicted bifurcation paths

Next, some important aspects concerning the results presented above are discussed:

(a) The experimental setup allowed moving the attachment point to a limit value of $d_{CG}=0.45$ but the bifurcation point for this model was located even forward of this point. This was expected since S175 is a slender container ship with $C_B=0.57$. Although there was a setup limitation, precise evaluation of the equilibrium angle $\bar{\psi}$ for positions closer to the bifurcation point would in fact be very difficult. Close to that point, the instability is weak and transient effects are so long that it becomes impossible to verify a stationary value of ψ within the length of the water tank (around 150 m).

(b) The results obtained allowed measurements of the secondary equilibrium path until a position very close to the bifurcation point. It can be seen from Fig. 5 that the curves have an inflection point around $d_{CG}=0.35$ and the angle remains almost constant from that point until $d_{CG}=0.45$. The shape of the curve is then similar to a classical square-root pitchfork and, therefore, in good agreement with the curve predicted by HD models. CF models do not allow any inflection point in the bifurcation curve and the variation of the angle must be linear with respect to parameter d_{CG} in the vicinity of the bifurcation point (see Fig. 2). Due to the limitations discussed above it cannot be inferred, however, that the angle of the experimental curve at the bifurcation point is 90 deg as predicted by the HD model).

(c) It is very difficult, if not impossible, to confirm that the onset of the secondary equilibrium path for pitchfork bifurcation occurs with an angle of 90 deg, as predicted by HD models. What we may expect at best is to confirm the existence of any inflection point. The reason is the following. A real model always presents some degree of imperfection in its shape and, even if it was indeed perfect, inaccuracies in adjusting skegs, weight distribution, ballast, etc. can easily introduce imperfections. As a result, the static bifurcation does not take the form of a pitchfork but that of a saddle node (see Bernitsas, Papoulias [13]). Then, near the saddle node one side of the pitchfork terminates abruptly while the other asymptotically follows the pre-pitchfork path on one side and the secondary path on the other side. The curve is then called the *real path* and it can exhibit an inflection point but not the theoretically predicted 90 deg angle of onset for perfect models. This behavior is shown qualitatively in Fig. 6.

(d) Post pitchfork bifurcation results obtained for the container ship model are quite different from those presented by Leite et al. [4] for two distinct tanker models.

This can be seen in Fig. 7, extracted from the aforementioned work, which presents the experimental values and the CF model prediction of the secondary equilibrium bifurcation paths for a VLCC tanker in loaded condition. Experimental results are pre-

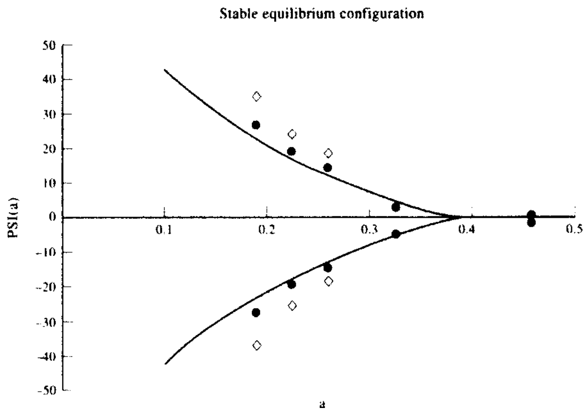

Fig. 7 Experimental values and CF model prediction of VLCC bifurcation paths (*a* represents d_{CG})-*extracted from Leite* et al. [4].

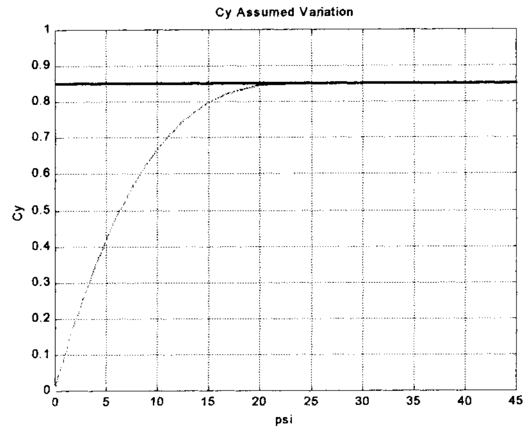

Fig. 8 Assumed variation of C_Y

sented for two different values of Froude number ($\bullet U/\sqrt{gB}=0.08$ and $\diamond\ U/\sqrt{gB}=0.24$). The agreement between theoretical and experimental results is indeed very good for the smallest value of Froude number, which is indeed representative of typical current velocities.

Although, as mentioned before, there are few experimental points in the vicinity of the bifurcation point, results for the tankers presented no evidence of inflection points and the diagram shape fits very well the one predicted by the CF model.

Comparison of experimental results obtained for the S175 model (Fig. 5) and the VLCC model (Fig. 7) clearly indicates that the shape of the post pitchfork bifurcation path is, as could be expected, strongly dependent on hull form.

Some Comments on CF Models

As mentioned earlier, in order to keep CF models simple, some assumptions are made which may overlook some more subtle aspects of the flow. This seems to be the case when assuming a constant value of the hull drag coefficient (C_Y, measured for transverse flow; $\psi=90$ deg) independently of the incidence angle. Variation of such coefficient might be expected for ψ close to 0 deg (which is the case in bifurcation tests close to the bifurcation point), since the flow pattern along the hull is quite different from the one observed for beam incidence.

Actually, there is experimental evidence of the variation of drag coefficient with ψ, as presented by Hooft [5]. His work analyzes the influence of the incidence angle on the cross-sectional drag distribution along the hull of a Todd 70 model. A drastic variation of drag distribution for values of ψ close to 0 deg is observed, especially a significant reduction of drag for bow sections. According to the results presented by Hooft [5], the value of the drag coefficient C_Y (obtained by integration of the drag distribution) reduces as ψ tends to 0 deg (bow incidence). It can also be inferred from Hooft's data that the drag coefficient for $\psi=8$ deg is almost 60% smaller than the value obtained for transverse flow.

To verify the influence that such drag reduction has on the shape of the post pitchfork bifurcation path predicted by CF models, the following qualitative comparison is performed: Post bifurcation paths were generated with the CF model proposed by Simos et al. [10] considering that C_Y varies with ψ as shown in Fig. 8. The results were then compared to those obtained for a constant value of $C_Y=0.85$ (also plotted in Fig. 8).

Figure 9 presents the secondary equilibrium path for the S175 container ship predicted by the CF model for a constant value of C_Y ($C_Y=0.85$) and for C_Y varying according to Fig. 8. Experimental points already presented in Fig. 5 are once again plotted for the sake of comparison.

It can be readily seen from Fig. 9 that the theoretical curves predicted by the CF model agree much better with the experimental points if the variable value of C_Y is adopted. Even the inflection points observed in the experimental bifurcation diagram are reproduced. This result clearly indicates that a more precise consideration of the drag coefficient variation for values of ψ close to 0 deg would be necessary for recovering the shape of the curve obtained experimentally for the S175 model near the bifurcation point.

It is important to observe that incorporating the drag coefficient dependence on ψ ($C_Y(\psi)$) is equivalent to assuming a cubic variation of the sway force since, for small angles, transversal velocity (ν) is linear in ψ (see Appendix). The proposed correction adds a cubic term to the CF model, which is significant for small values of ψ and, in this sense, approximates the CF model to HD models that incorporate bilinear terms such as the one presented by Takashina [6].

On the other hand, it must also be emphasized that although the assumed variation of C_Y for ψ smaller than 20 deg is very abrupt (see Fig. 8), the influence on the bifurcated angle is relatively weak, with a maximum increase in ψ around 3 deg. This happens because for small values of ψ the current forces predicted by CF models are dominated by components included in the short-wing model. Quadratic cross-flow components become dominant only for larger values of the incidence angle; for those values, the coefficient C_Y is estimated accurately. Therefore, as shown in Fig. 9, theoretical results assuming a constant value of C_Y agree very well with experiments for ψ around 10 deg and larger

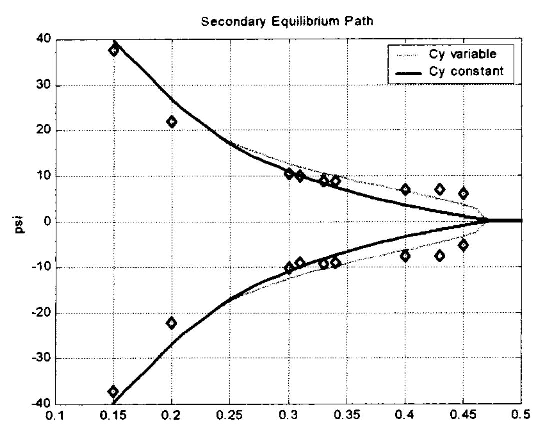

Fig. 9 Secondary equilibrium path for S175 according to CF model

Finally, reduction of drag coefficient with ψ certainly depends on hull form and, consequently, different ship hulls may exhibit very different static bifurcation paths. In fact, tanker models as the ones tested by Leite et al. [4] may present a much smaller drag reduction close to bow incidence compared to the container ship model. This may explain why CF models predict post pitchfork bifurcation paths for tankers better than for container ships.

Presently, available results for different kinds of ship hulls indicate that the assumption of constant drag coefficient is suitable for practical offshore purposes, since the effective variation of the equilibrium angle induced by C_Y reduction is small. Any attempt to describe C_Y variation for small angles would require several additional tests and, unless such variation proves to be important for practical applications, it would represent an unnecessary loss of the simplicity which constitutes the advantage of CF models.

Conclusions

The shape of the post pitchfork bifurcation path of SPM systems close to the bifurcation point strongly depends on hull form. Secondary equilibrium paths obtained for tankers (see Leite et al. [4]) differ significantly from the one obtained in this work for the S175 container ship model.

Hydrodynamic Derivative (HD) models, with or without inclusion of bilinear force components, are certainly able to recover the post pitchfork bifurcation secondary equilibrium path for any ship hull if the derivatives are measured with sufficient accuracy. This, however, requires extensive model-scale testing and implies that the slow motion derivatives have been measured experimentally.

Cross-Flow (CF) models consider, for the sake of simplicity, a value of the hull total drag coefficient that is independent of the incidence angle ψ, although it is only measured for transverse flow ($\psi = 90$ deg). Such assumption, however, may lead to a discrepancy in the shape of the post-pitchfork bifurcation path if the ship exhibits a significant reduction in the drag coefficient for small values of ψ. Experimental evidence that such reduction may in fact occur for some hull forms was presented by Hooft [5]. Dependence of drag variation on hull form may then explain, under the CF model approach, the difference observed between tankers and container ships at the onset of the secondary equilibrium path following a pitchfork bifurcation.

Finally, it must be observed that the effect of drag reduction on hydrodynamic forces predicted by CF models is quite small since, for small values of ψ, forces are dominated by short-wing components. Therefore, apart from the described discrepancy in the shape of the post-pitchfork bifurcation paths, the assumption of constant C_Y seems reasonable for studying most practical offshore applications of tankers in FPSO systems. Analysis involving other hull forms, such as towed container ships, may require a more profound investigation concerning the influence of drag variation as a function of ψ on CF models.

Appendix

Let $(O;x,y)$ be the fixed system of coordinates shown in Fig. 1, U the uniform current velocity, and (x,y) the coordinates of the center of gravity of the ship in the horizontal plane. Let (u,v) be the surge and sway components of the ship velocity with respect to the water, and r be the yaw angular velocity. Velocities are kinematically related in the form:

$$\begin{aligned} u &= (\dot{x}+U)\cos\psi + \dot{y}\sin\psi \\ v &= -(\dot{x}+U)\sin\psi + \dot{y}\cos\psi \end{aligned} \tag{1}$$

For small ψ, we derive the first-order approximation of the kinematic relations (1):

$$\begin{aligned} u &\cong (\dot{x}+U) + \dot{y}\psi \\ v &\cong -(\dot{x}+U)\psi + \dot{y} \end{aligned} \tag{2}$$

Further, for small ψ, $C_Y(\psi) \approx c\psi$ (c = constant). Then, transverse force term is approximately a third-order polynomial in ψ.

$$C_Y(\psi)v|v| \cong c\psi[-(\dot{x}+U)\psi + \dot{y}]|-(\dot{x}+U)\psi + \dot{y}|, \tag{3}$$

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