Slow Drift and Trapping of Waves on Submerged Bodies

J.A.P.ARANHA
C.P.PESCE
I.P.T.: S.Paulo, Brazil

SUMMARY

Low frequency nonlinear wave forces on ocean structures are usually divided into two distinct components. One, $F_1(t)$, associated with quadratic interaction of the first order (linear) potential; the other, $F_2(t)$, associated with the second order potential. The purpose of the present paper is to indicate that this second parcel must always be computed for a submerged body. On one hand, when the depth of submergence is large, $F_1(t)/F_2(t) \sim 0(1)$ and the effect of $F_2(t)$ cannot be neglected; on the other hand, when the depth of submergence is small, the ratio $F_1(t)/F_2(t)$ increases but the likelihood of trapped mode excitation also does. If this happens the second order problem is resonant and then $F_2(t)$ is, by far, the dominating parcel. The analysis in this work is restricted to two dimensional infinite water depth case and only diffraction problem is considered (body fixed in waves).

1. EXCITING FORCES IN BEAM SEA

Consider first a regular wave train and let $F_1 = \rho gAB\delta.Q_1(\omega)$ be the steady force in the mode being analysed, where:- ρ= density; g = acceleration of gravity; A = wave amplitude; B = beam; $\delta = A/B$ = small amplitude parameter; $Q_1(\omega)$ = steady force coefficient for a harmonic wave with frequency ω. If $K_o = \omega^2/g$ is the wave number, the wave steepness is given by $K_oA = K_oB.\ \delta << 1$ when $K_oB \lesssim 0(1)$. In sway mode a simple expression for $Q_1(\omega)$ has been obtained by Maruo (1960) and it is given by,

$$Q_1(\omega) = \frac{1}{2}|R|^2(1+0(\delta)),$$

where R = reflection coefficient. This regular wave train excites, at second order, the steady potential $\phi_{20}(y,z)$ where:

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$$\nabla^2\phi_{20} = 0,$$
$$\partial\phi_{20}/\partial z = L_{20}(y) \text{ on the free surface } z = 0,$$
$$\partial\phi_{20}/\partial n = 0 \qquad \text{on the body surface } \partial B, \qquad (1\text{-}2)$$
$$\nabla\phi_{20} \to 0 \quad \text{when } r = \sqrt{y^2+z^2} \to \infty.$$

In the above expression the exciting term $L_{20}(y)$ is given by,

$$L_{20}(y) = \frac{\omega}{4}\cdot\frac{\partial}{\partial y}\left(i\cdot\phi_L(y,0)\cdot\frac{\partial\phi_L^*}{\partial y}(y,0)\right) + (*), \qquad (1\text{-}3)$$

$\phi_L(y,z;\omega)$ being the linear diffraction potential at frequency ω. If U_K is the mode boundary condition ($U_K = n_y$ for sway, etc.) the following coefficient will be used later on:

$$Q_2(\omega) = -\omega\int_{\partial B}\phi_{20}\cdot U_K\cdot d\partial B. \qquad (1\text{-}4)$$

In reality the potential ϕ_{20} need not be determined to obtain $Q_2(\omega)$. In an actual computation the radiation potentials $\phi_K^{(o)}(y,z)$, at zero frequency, must be determined to compute the added mass. They satisfy the homogeneous boundary condition at the free surface and the mode boundary condition, $\partial\phi_K^{(o)}/\partial n = U_K$, at the body surface. If now,

$$G(\phi;\Psi) = \iint_A \nabla\phi.\nabla\Psi.dA; \quad A = \text{fluid region},$$
$$V_K(\Psi) = \int_{\partial B}\Psi.U_K.\,d\partial B, \qquad (1\text{-}5)$$
$$V_{20}(\Psi) = \int_{-\infty}^{\infty} L_{20}(y).\Psi(y,0).dy,$$

then,

$$G(\phi_K^{(o)};\Psi) = V_K(\Psi), \qquad (1\text{-}6)$$
$$G(\phi_{20};\Psi) = V_{20}(\Psi).$$

Since $Q_2(\omega) = -\omega.V_K(\phi_{20})$ from (1-6) it follows that,

$$Q_2(\omega) = -\omega.\,V_{20}(\phi_K^{(o)}). \qquad (1\text{-}7)$$

This latter expression is just Haskind's relation applied to the present problem and it shows that the knowledge of $\phi_K^{(o)}(y,z)$ is enough to compute $Q_{20}(\omega)$.

Consider now an irregular incident wavetrain consisting of a slow modulation of the harmonic wave at the frequency ω. The spatial and temporal modulation is described by a function $a(y,z,t) \sim O(1)$ with $(\partial/\partial_y;\ \partial/\partial_z;\ \partial/\partial_t)\ a \sim O(\Delta\omega) \ll 1$. The diffraction potential, correct to second order in the small amplitude parameter δ, is given by $\phi(y,z) = \delta.\phi_1(y,z) + \delta^2.\ \phi_2(y,z)$, where, obviously, $\phi_2(y,z)$ has energy in two distinct range of frequencies:- one around the frequency 2ω and another around the zero frequency. Only this latter parcel will be considered in the following. From Bernouilli's equation the low frequency exciting force is given by $F(t) = F_1(t) + F_2(t)$ where

$$F_1(t) = -0.5.\ \rho.\ \int_{\partial B} (\nabla\phi_1)^2.\ U_K.d\partial B,$$

$$F_2(t) = -\rho.\frac{d}{dt} \int_{\partial B} \phi_2.\ U_K\ .d\partial B. \qquad (1\text{-}8)$$

Due to the slow modulation, $\phi_1 \sim a.\phi_L(1+O(\Delta\omega))$, where ϕ_L is the diffraction potential at frequency ω. Placing this expression into (1-8) one obtains Newman's (1974) approximation,

$$F_1(t) = \rho g A B \delta\ .Q_1(\omega).\ |a(t)|^2\ .(1+O(\Delta\omega)). \qquad (1\text{-}9)$$

where $Q_1(\omega)$ is the steady force coefficient for harmonic waves. With the same relative error the free surface exciting term for ϕ_2 is given by $|a|^2.\ L_{20}(y)$, what suggests to write $\phi_2 = |a|^2\ .\phi_{20}.(1+O(\Delta\omega))$. After a detailed analysis of the far field radiation condition Aranha & Pesce (1986) have shown that this approximation is indeed consistent. Using it into (1-8) one obtains,

$$F_2(t) = \rho g A B \delta\ .Q_2(\omega).\ \frac{1}{\omega}\ \frac{d}{dt}(|a|^2).\ (1+(\Delta\omega)), \qquad (1\text{-}10)$$

with $Q_2(\omega)$ defined in (1-4). Since the derivative of $|a(t)|^2$ is of order $\Delta\omega$ the ratio between $F_1(t)$ and $F_2(t)$ can be gauged by,

$$r_1(\omega) = \frac{Q_1(\omega)}{\Delta\omega . Q_2(\omega)} = \frac{|R|^2}{2\Delta\omega . Q_2(\omega)}, \qquad (1\text{-}11)$$

where the expression on the right is valid only in sway, see (1-1). For a surface-piercing body in deep water, $|R|$ and $Q_2(\omega)$ are both of order 1 and $r_1(\omega) \sim 0(1/\Delta\omega) >> 1$ or in short:- the effect of $F_2(t)$ can be neglected in comparison to $F_1(t)$, a conclusion consistent with the one numerically obtained by Faltinsen & Løckens. (1980). For a circular submerged body $|R| \equiv 0$, see Ogilvie (1963), and in general $|R| << 1$ when the depth of submergence is relatively large. In this case $r_1(\omega)$ can be of order 1 and the effect of the second order potential cannot be neglected. When the body approaches the free surface, $|R|$ increases in general and so $r_1(\omega)$ also does. Simultaneously, however, increases the likelihood of a resonant phenomenon to be explained next.

2. TRAPPED MODE EXCITATION

A penetrating physical discussion about trapped waves over submerged bodies in shallow water can be found in Longuet-Higgins (1967), and Aranha (1987)a extended some of these results to the arbitrary water depth case. Mathematically a trapped wave is a function of the form,

$$\hat{T}(x,y,z,t) = T(y,z).\, e^{i(K_T x-\omega t)}$$

$$T(y,z) \sim A^{\pm} .\, e^{-\lambda_o|y|} .\, e^{K_o z} \quad ; K_o = \omega^2/g, \qquad (2\text{-}1)$$

$$\lambda_o = \sqrt{K_T^2 - K_o^2},$$

that satisfies the linear homogeneous water wave equation. The mode $\{K_T;\, T(y,z)\}$ can be determined from the following simple eigenvalue problem:- to find a non-trivial $T(y,z)$ for which the Lagrangian.

$$\mathcal{L}(T) = \frac{1}{2} \iint_A [(\nabla T)^2 + K_T^2 T^2]\, dA - \frac{\omega^2}{2} \int_{-\infty}^{\infty} T^2(y,0)\, dy, \qquad (2\text{-}2)$$

is stationary. Since $K_T > K_0$ these modes can be excited only by non-linear interaction of two incoming waves but, if this happens, the second order potential ϕ_2 will be of order $\delta^{2/3}$,see Aranha (1987)b, and so $Q_2(\omega) \sim O(\delta^{-4/3}) >> 1$. As it is usual in non-linear resonant phenomena a nonlinear cubic wave equation can be obtained by multiple scales, although the pertinent equation is non-dispersive in the present case, see Aranha (1987)b for details. If $\bar{\sigma}$ is the normalized detuning it can be shown that $\bar{\sigma} \sim O(\delta^{4/3}/(\lambda_0/K_0))$ or in short:- the more diffuse is the mode (the smaller is λ_0/K_0) the larger will be $\bar{\sigma}$ and the more unlikely the mode will be excited. Since λ_0/K_0 increases when the depth of submergence S decreases (see figure (5-1)) then the likelihood of excitation also increases when the body approaches the free-surface.

3. VARIATIONAL APPROXIMATION FOR $Q_1(\omega)$; $Q_2(\omega)$: λ_0/K_0

The present analysis depends on three coefficients:- $Q_1(\omega)$: $Q_2(\omega)$ and λ_0/K_0, where this latter one gauges the likelihood of trapped mode excitation. It is important to be observed that all of them can be expressed as stationary values of well defined functionals and so an order ε error in the potential leads to an order ε^2 error in these coefficients. In fact, λ_0/K_0 can be obtained by standard Rayleigh quotient, see (2-2), and Aranha & Pesce (1987) have shown how to express $Q_1(\omega)$, in its form (1-1), in a similar way. A short demonstration that this is also valid for $Q_2(\omega)$ will be given next. Let $\phi = \{\phi_{20};\phi_K^{(o)}\}$ be the solution of the weak equation $G(\phi,\Psi) = V(\Psi)$, see (1-6), and $\bar{\phi}$ its projection into the finite dimensional space $\bar{W}$. Then $\phi = \bar{\phi} + \Delta\phi$ where $G(\Delta\phi,\bar{\Psi}) = 0$ for all $\bar{\Psi} \in \bar{W}$. The related approximation for $Q_2(\omega)$ is given by (see (1-7), (1-6))

$$\bar{Q}_2(\omega) = -\omega\ V_{2o}(\bar{\phi}_K^{(o)}) = -\omega.G(\bar{\phi}_K^{(o)};\ \bar{\phi}_{2o}) = -\omega.G(\phi_K^{(o)};\phi_{2o}) - \omega.G(\Delta\phi_K^{(o)};\Delta\phi_{2o}) =$$

$$= -\omega.\ V_{2o}(\phi_K^{(o)}) - \omega.G(\Delta\phi_K^{(o)};\ \Delta\phi_{2o}) = Q_2(\omega) - \omega.G(\Delta\phi_K^{(o)};\ \Delta\phi_{2o}).$$

From this latter expression and Schwarz inequality it follows that the error in $Q_2(\omega)$ is smaller than $\varepsilon_{2o}\ .\varepsilon_K^{(o)}$ where $\varepsilon = \{\varepsilon_{2o};\ \varepsilon_K^{(o)}\} = (G(\Delta\phi;\Delta\phi))^{1/2}$ is the error in the potential.

4. NUMERICAL RESULTS

A variational approximation will be designated by the size (NxN) of the real and symmetric matrix that should be constructed and inverted in this formulation. Figure (5-1)b compares the phase of the transmission coefficient for a circular cylinder obtained from a (4x4) variational approximation with the ones obtained by Ogilvie (1963); Figure (5-1)d compares the values of $Q_1(\omega)$ in sway mode (rectangular section) obtained from a (4x4) variational approximation with the ones obtained by Inoue & Kyozuka (1985), where in this last work a (30x30) complex non-symmetric matrix was constructed and inverted. Both results agree quite well and this gives an indication of the power of the present variational approximation.

To compute $Q_2(\omega)$ a (1x1) variational approximation was used to represent $\phi_K^{(o)}$ in sway mode and the integral $V_{2o}\ (\phi_K^{(o)})$ has been computed only above the body. This coefficient is not supposed then to be determined with great precision although its order of magnitude - and this is enough in the present analysis - is thought to be correct. Since it can be shown that $T(y,z) \to e^{K_o z}$ when $S \to \infty$ this trial function has been used for a circle and compared with the values obtained by McIver & Evans (1985); see Figure (5-1)e to be noticed the drastic improvement when S changes from 1,1 to 1,5 and that, from minimum principle, the variational approximations are always lower bounds here. For the rectangle the same (1x1) approximation was used and compared with a (2x2) approximation, where the second trial function immitates shallow water trapped modes in the region above the rectangle. Again the discrepancy between both increases drastically when S decreases, see Figure (5-1)f.

As a whole two things are to be noticed:- $Q_2(\omega)$ is the dominating term when S is "large", see Figure (5-1)d where $\Delta\omega = 0.20$ has been used to compute $\tilde{Q}_2(\omega) = \Delta\omega . Q_2(\omega)$; λ_o/K_o increases when the body approaches the free-surface. So the effect of the second order potential should always be computed for a submerged body. Furthermore, variational approximation seems to be a powerfull tool to determine macroscopic quantities, like the exciting forces on a body. As it was shown here this approach can also be used to compute nonlinear exciting forces and this can affect significantly the required ammount of computing time. In the present case the computation of $Q_1(\omega)$, $Q_2(\omega)$ and $\lambda_o/K_o(\omega)$, in the range of frequencies shown in Figure (5-1), did not take more than 15 minutes in a IBM personal micro-computer.

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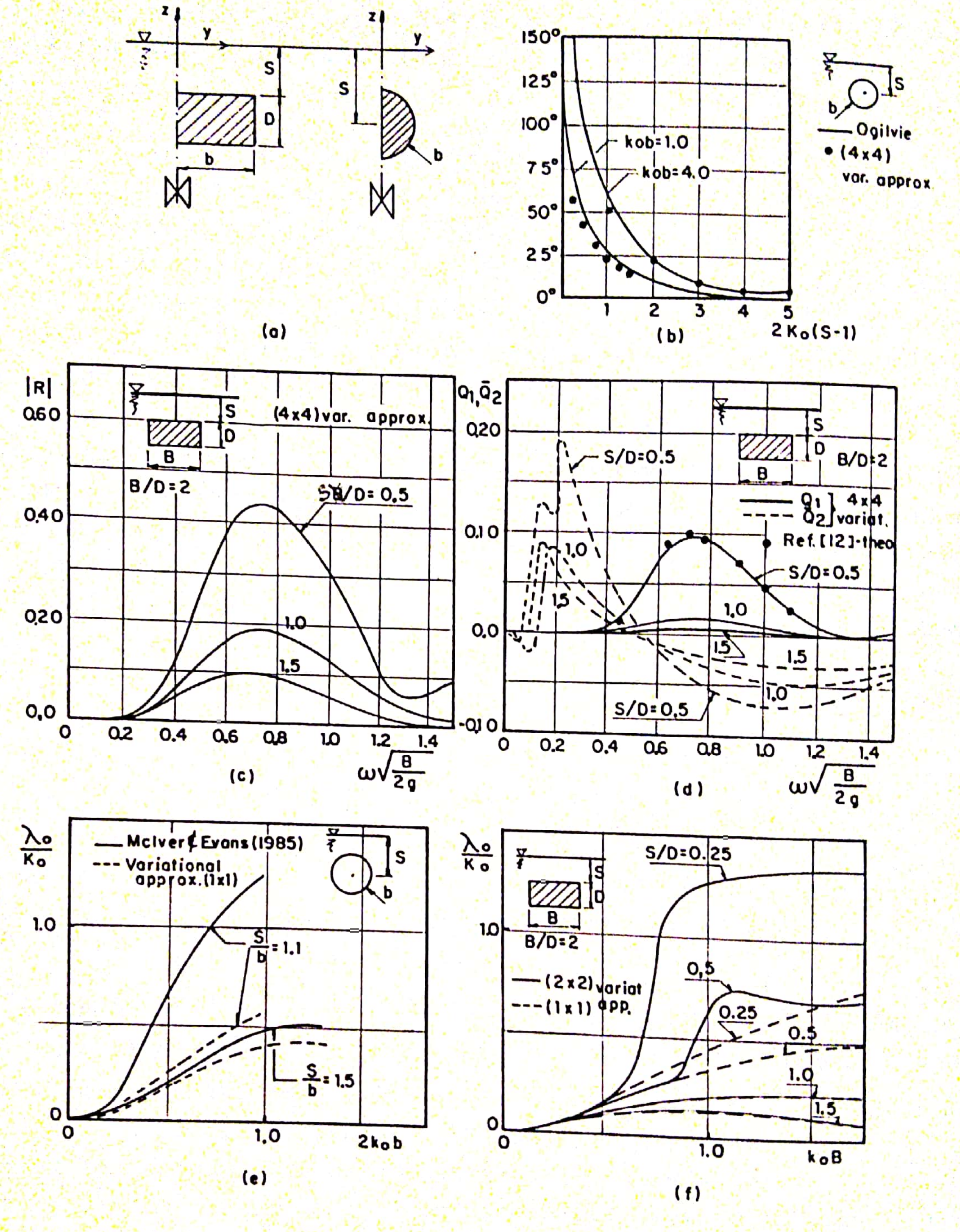

Fig. (5-1):- a) Analysed geometries; b) Phase of T for submerged circular cylinder compared with Ogilvie (1963); c) $|R(\omega)|$ for a rectangle with different submergence depth; d) Q_1 & Q_2 Drift Force Coefficients for a rectangle; e) λ_o/K_o for a circle compared with Mc Iver & Evans (1985); f) λ_o/K_o for a rectangle.