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Abstract	This paper presents a method for evaluating non-linear modes and the corresponding natural frequencies of hanging cables with small sag. The use of a Galerkin temporal scheme on the governing equations of motion associated with a fictitious normal force accounting for the effects of the resulting non-linear terms leads to a closed-form solution for the non-linear free vibration problem. The influence of amplitude on the modal shapes and frequencies are presented.	
Keywords	Non-linear normal modes - Extensible cable - Small sag - Closed form solution - Galerkin projection	

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# Chapter 23

## Non-linear Free Vibrations of a Hanging Cable with Small Sag



Guilherme Jorge Vernizzi, Guilherme Rosa Franzini and Celso Pupo Pesce

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 2 corresponding natural frequencies of hanging cables with small sag. The use of a  
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### 9 23.1 Introduction

10 Structural solutions based on tensioned cables with varying traction along the length  
 11 are commonly found in engineering applications. The study of the dynamic response  
 12 of those structures is of great importance in fatigue design and stability analysis.  
 13 Particularly, the study of the free-vibration problem is of interest, since it provides  
 14 intrinsic characteristics such as its natural frequencies and modes. Considering a  
 15 linear problem in free vibrations, the system oscillates with the form of a particular  
 16 normal mode if the initial conditions match this mode. This concept can be expanded  
 17 for non-linear systems by using the concept of the non-linear normal modes; see [1].

18 Reference [2] analytically investigates the linear free oscillations of a cate-  
 19 nary riser with negligible bending stiffness, providing a Wentzel–Kramers–Brillouin

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(WKB) (see for example [3]) closed-form solution for the problem. Following, the non-linear modes for a vertical beam with varying tension were addressed in [4], which presents a closed-form expression for the modal shape and the natural frequencies. In the latter paper, the authors employed a temporal Galerkin projection and a fictitious normal force similar to that previously proposed in [5].

The present paper aims at contributing with the planar non-linear dynamics of cables in free vibrations. Particularly, the major interest lies on determining the non-linear modes and frequencies of a cable hanging between two points at different heights, with a sag to span relation of order of 1:20 or smaller. The formulation herein presented extends the results of [4], allowing for use in cables that are in a configuration different from the vertical one. Furthermore, the formulation herein presented includes some non-linear effects neglected in [2].

## 23.2 Mathematical Model

Consider a cable made of an elastic-linear material, with axial stiffness  $EA$ , mass per unit length  $\mu$  and unstretched length  $l$ , as sketched in Fig. 23.1. Let  $u$  and  $v$  be, respectively, the displacements in the tangential and in the normal directions defined with respect to the static configuration. In addition to these quantities, we define  $T$  as the traction and  $\theta$  as the angle with the horizontal in the static configuration,  $\tau$  as the dynamic traction variation,  $\gamma$  as the dynamic variation of  $\theta$  and  $\varepsilon$  as the engineering strain component related to  $\tau$ . The definition  $\mathbb{T} = T + \tau$  is used in some mathematical steps. Throughout this paper, primes denote differentiation with respect to the arclength coordinate  $s$  in the static configuration and  $\dot{\phantom{x}}$  represents differentiation with respect to time. Since the sag is small and the tangential displacements are considered small compared to the transversal ones, the approximation  $\gamma \cong v'$  is used. This is possible due to the fact that the term  $u\theta'$ , although linear in the dynamical perturbations, becomes of second order when compared to  $v'$ . A detailed analysis on order of magnitude of terms arising from a dynamic perturbation approach around the equilibrium configuration may be found in [6].

The equations of motion herein analysed are based on the Clebsch–Love equations (see for example [7]). For the sake of a future generalization of this mathematical model, the static terms are not approximated using a parabolic static configuration as in [8]. Let  $b_u$  and  $b_v$  be the external forces per unit length in the tangential and transversal directions, respectively. Defining  $f_u$  and  $f_v$  as the corresponding elastic forces and neglecting rotatory inertial forces, the equations of motion are written as:

$$f_u + b_u = \mu \ddot{u} . \quad (23.1)$$

$$f_v + b_v = \mu \ddot{v} . \quad (23.2)$$

Considering a cable segment  $\delta s$ , the resulting elastic forces in the tangential and transversal directions are given by:

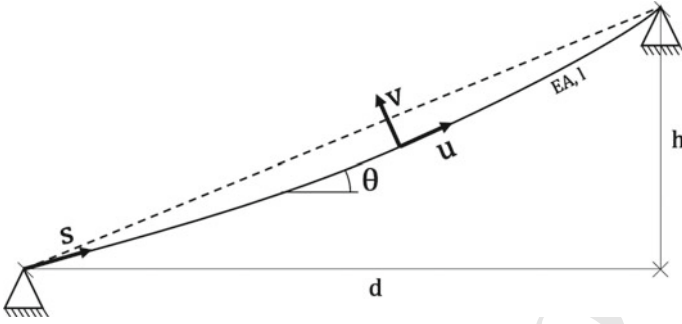


Fig. 23.1 Basic sketch and principal parameters

$$\delta F_u = \mathbb{T}(s + \delta s) \cos(\delta\theta + \gamma(s + \delta s)) - \mathbb{T}(s) \cos(\gamma(s)) . \quad (23.3)$$

$$\delta F_v = \mathbb{T}(s + \delta s) \sin(\delta\theta + \gamma(s + \delta s)) - \mathbb{T}(s) \sin(\gamma(s)) . \quad (23.4)$$

Taking the limit  $\delta s \rightarrow 0$  in Eqs. (23.3) and (23.4), considering that  $\gamma$  is small, the resulting terms are:

$$f_u = \lim_{\delta s \rightarrow 0} \delta F_u = \mathbb{T}' - \mathbb{T}(\theta' + \gamma') \gamma . \quad (23.5)$$

$$f_v = \lim_{\delta s \rightarrow 0} \delta F_v = (\mathbb{T}\gamma)' + \mathbb{T}\theta' . \quad (23.6)$$

The resulting equations of motion are then:

$$[T + b_u] + \tau' - T\gamma\theta' - T\gamma\gamma' - \tau\gamma\theta' - \tau\gamma\gamma' = \mu\ddot{u} . \quad (23.7)$$

$$[T\theta' + b_v] + \tau\theta' + (T\gamma)' + (\tau\gamma)' = \mu\ddot{v} . \quad (23.8)$$

Note that the expressions between brackets in Eqs. (23.7) and (23.8) are the equations of static equilibrium when the dynamical changes in the external forces can be disregarded or are, in fact, null. Now a static condensation procedure is applied. Following [2, 9], the inertial term in the tangential direction is disregarded. A price to be paid is missing the mutual inertial effect between tangent and transverse dynamics. The well known frequency cross-over phenomenon analysed in [8] is missing as well. However, the tangential component of the mode function may still be written as a function of the transversal one (see [2, 9]).

Also, a scaling analysis is used to simplify Eq. (23.7). The scaling is made considering  $v$  of unity order, which implies that  $v'$  is of order  $\eta$ , the **later** being a small parameter. The additional curvature  $v''$  is of order  $\eta^2$ , and the same order is considered for the small static curvature. This is in fact a strong hypothesis, limiting the dynamic amplitude to a fraction of the wave length of the modes that will be sought. Also, considering valid the scaling between tangential and transversal displacements obtained in [8],  $\tau$  is considered of order  $\eta$ . Keeping only terms of the smallest power of  $\eta$ , the condensed equation for the tangential displacements becomes:

$$EA\varepsilon' - Tv'\theta' - Tv'v'' = 0. \quad (23.9)$$

To ensure mathematical clearness, a dummy variable  $\xi$  is used when indefinite integrals of functions of  $s$  are required. Integration of Eq. (23.9) leads to:

$$EA\varepsilon = C_1 + \int_0^s Tv'\theta' d\xi + \int_0^s Tv'v'' d\xi. \quad (23.10)$$

Now, as made in [9], the constant  $C_1$  is obtained considering a spatial averaging of Eq. (23.10). Also, the strain measure is defined as  $\varepsilon = u' - v\theta' + (v')^2/2$ . The retained terms follow the smallest power of  $\eta$  that appears in the geometrically complete expression of  $\varepsilon$ . The constant  $C_1$  is given by:

$$C_1 = \frac{EA}{2l} \int_0^l (v')^2 ds - \frac{EA}{l} \int_0^l v\theta' ds - \frac{1}{l} \int_0^l \int_0^s Tv'\theta' d\xi ds - \frac{1}{l} \int_0^l \int_0^s Tv'v'' d\xi ds. \quad (23.11)$$

Equation (23.11) allows writing the equation of transversal motion in an isolated manner, i.e., decoupled from that associated with the tangential one. The resulting equation is given by Eq. (23.12).

$$\theta' \left( C_1 + \int_0^s Tv'\theta' d\xi + \int_0^s Tv'v'' d\xi \right) + (Tv')' + Tv'^2 (\theta' + v'') + v'' \left( C_1 + \int_0^s Tv'\theta' d\xi + \int_0^s Tv'v'' d\xi \right) = \mu\ddot{v}. \quad (23.12)$$

Supposing that the dynamics is governed by a single mode, the solution is sought in the form  $v = \psi(s) \sin(\omega t)$ . After a series of algebraic manipulations and the use of a Galerkin's temporal scheme (see [4]), the equation of the modal shape for the modes associated with the transversal direction becomes:

$$-\frac{EA\theta'}{l} \int_0^l \psi\theta' ds - \frac{\theta'}{l} \int_0^l \left( \int_0^s T\psi'\theta' d\xi \right) ds + \theta' \int_0^s T\psi'\theta' d\xi + T'\psi' + T\psi'' + \frac{3}{4}T\psi'^2\psi'' + \frac{3EA}{8l}\psi'' \int_0^l (\psi')^2 ds - \frac{3}{4l}\psi'' \int_0^l \int_0^s T\psi'\psi'' d\xi ds + \frac{3}{4}\psi'' \int_0^s T\psi'\psi'' d\xi + \mu\omega^2\psi = 0. \quad (23.13)$$

Following [4, 5], a fictitious or equivalent "normal force"  $N$  is proposed as:

$$\begin{aligned}
& -\frac{EA\theta'}{l} \int_0^l \psi \theta' ds - \frac{\theta'}{l} \int_0^l \left( \int_0^s T \psi' \theta' d\xi \right) ds \\
& + \theta' \int_0^s T \psi' \theta' d\xi + \frac{3}{4} T \psi'^2 \psi'' + \frac{3EA}{8l} \psi'' \int_0^l (\psi')^2 ds \\
& - \frac{3}{4l} \psi'' \int_0^l \int_0^s T \psi' \psi'' d\xi ds + \frac{3}{4} \psi'' \int_0^s T \psi' \psi'' d\xi = N \psi'' . \quad (23.14)
\end{aligned}$$

The numerical evaluation of this term is made using a spatial Galerkin projection considering a set of sinusoidal functions<sup>1</sup>  $\sin(n\pi s/l)$ , where  $n$  is the number of half-waves existing in the mode considered. The consideration of the number of half-waves is needed since, for inclined cables, the mode number is not necessarily the number of half-waves since mode hybridization can occur; see [10]. Using the wrong consideration regarding  $n$  leads to higher values of the fictitious normal force, specially for the lower modes.

For a catenary configuration, the approximation  $T \cong \bar{T} = \alpha + \beta s$  can be used as a simplification for the static traction with small errors (see [2]). The fictitious normal force is then associated with the number of half-waves  $n$  used in the Galerkin projection, and is indicated by  $N_n$ . The vibration modes will then be non-linear because some terms in Eq. (23.14) maintain a quadratic relation with the amplitude used in the projection functions when computing the fictitious normal force. Applying the approximation for the static traction and the evaluated fictitious normal force in Eq. (23.13), the modal shapes  $\psi_n$  must satisfy Eq. (23.15).

$$(\bar{T} + N_n) \psi_n'' + \bar{T}' \psi_n' + \mu \omega_n^2 \psi_n = 0 . \quad (23.15)$$

Notice that the averaging procedure represented by Eq. (23.14) transformed the nonlinear Eq. (23.13) into a linear one. Following [4], some new quantities are defined, being  $a = \beta/\mu\omega_n^2$ ,  $T_{bn} = \alpha + N_n$  and  $T_{tn} = \alpha + l\beta + N_n$ . Note that  $T_{bn}$  and  $T_{tn}$  are the modal tractions at the lower and upper ends of the cable respectively, while  $\omega_n$  is the natural frequency associated with the mode containing  $n$  half-waves. Defining now, as in [2, 4], a variable transformation, and the corresponding inverse transformation:

$$z = \frac{2\omega_n}{\beta} \sqrt{\mu(T_{bn} + \beta s)} , \quad (23.16)$$

$$s = \frac{az^2}{4} - \frac{T_{bn}}{\beta} . \quad (23.17)$$

<sup>1</sup>Sinusoidal functions are used for simplicity. Linear modes, given by the Bessel approximation or by the WKB closed form solution in [2] might be used instead.

134 Equation (23.15) turns out to a familiar Bessel form:

$$135 \quad \frac{d^2 \psi_n}{dz^2} + \frac{1}{z} \frac{d\psi_n}{dz} + \psi_n = 0 . \quad (23.18)$$

136 The solution of Eq. (23.18) can be written as a combination of zero-order Bessel  
 137 functions of first and second kinds ( $J_0(z)$  and  $Y_0(z)$ , respectively). The relations in  
 138 the combination and the natural frequencies are obtained by applying the essential  
 139 boundary conditions of the cable and using the solvability condition for non-trivial  
 140 solutions. Although the use of Bessel functions is already a solution, the high values  
 141 of  $z$  for a catenary cable with small sag allows the use of an asymptotic solution.  
 142 Following [4, 5], consider the following transformation:

$$143 \quad \psi_n = \frac{1}{\sqrt{z}} \Psi_n . \quad (23.19)$$

144 Equation (23.18) becomes then:

$$145 \quad \frac{d^2 \Psi_n}{dz^2} + \left(1 + \frac{1}{4z^2}\right) \Psi_n = 0 . \quad (23.20)$$

146 In the case of a catenary with small sag,  $1/4z^2 \ll 1$ . Such result allows substituting  
 147 this term in Eq. (23.20) by a small perturbation parameter, evaluated as the mean  
 148 value of  $1/4z^2$  along the cable. As shown in [4], the solution of Eq.(23.20) can then  
 149 be well approximated by:

$$150 \quad \Psi_n = A_n \sin(z) + B_n \cos(z) . \quad (23.21)$$

151 This leads finally to:

$$152 \quad \psi_n = \frac{1}{\sqrt{z}} (A_n \sin(z) + B_n \cos(z)) . \quad (23.22)$$

153 Notice that Eq. (23.22) resembles the WKB solution previously obtained in [2].  
 154 Now, since the transversal displacements must be zero at both ends of the cable, the  
 155 system of the boundary conditions reads:

$$156 \quad \begin{bmatrix} \frac{\sin z_0}{\sqrt{z_0}} & \frac{\cos z_0}{\sqrt{z_0}} \\ \frac{\sin z_l}{\sqrt{z_l}} & \frac{\cos z_l}{\sqrt{z_l}} \end{bmatrix} \begin{bmatrix} A_n \\ B_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \quad (23.23)$$

157 Being  $z_0$  and  $z_l$  the values of  $z$  at  $s = 0$  and  $s = l$ , respectively. Since is desired  
 158 to obtain non-trivial solutions of Eq. (23.23), the solvability condition leads to:



$$\frac{\sin(z_l - z_0)}{\sqrt{z_l z_0}} = 0 . \quad (23.24)$$

The solution of Eq. (23.24) is  $z_l - z_0 = n\pi$ , which, using Eq. (23.16) leads to:

$$\omega_n = \frac{n\pi}{2l\sqrt{\mu}} \left( \sqrt{T_{tn}} + \sqrt{T_{bn}} \right) . \quad (23.25)$$

The modal shapes can then be written as:

$$\psi_n = \sqrt{\frac{T_{bn}}{T_{bn} + \beta s}} \sin(z - z_0) . \quad (23.26)$$

Using Eq. (23.25) in Eq. (23.16), the coordinate  $z$  can be written in terms of the modal tensions and the number of half-waves as:

$$z = \frac{\sqrt{T_{bn} + \beta s}}{\sqrt{T_{tn}} - \sqrt{T_{bn}}} n\pi . \quad (23.27)$$

### 23.3 Numerical Example

To illustrate the effects of the non-linearities, preserved in the presented formulation, consider a cable with axial stiffness  $EA = 22970 \text{ kN}$ , diameter  $D = 1.57 \text{ cm}$  and  $\mu = 1.29 \text{ kg/m}$ . This cable is hanged such as  $h = 200 \text{ m}$  and  $d = 100 \text{ m}$ , and  $l = 223.73 \text{ m}$ . The length refers to the static equilibrium configuration length. In Fig. 23.2, the superposition of linear and non-linear modes is presented for the mode with  $n = 20$ , for a modal amplitude  $A_n = 3D$ . The modal shape functions are presented in dimensionless form, normalized by the maximum value of itself. As can be seen, there is no appreciable change in modal shape, since the modal amplitude is small.

Now, in Figs. 23.3 and 23.4, the superposition of linear and non-linear modes is presented for modes with  $n = 10$  and  $n = 20$  respectively, considering for the non-linear mode a modal amplitude of  $A_n = 20D$ . The change in modal shape now is visible, altering the position of nodal points and rate of change of the vibration amplitude along the cable. Those figures also show that higher modes are more affected by non-linearities compared to lower ones.

The effects of the non-linearities over the natural frequencies are shown in Table 23.1. The natural frequencies for some modal amplitude values and modes are shown. The modes are listed by the number of half-waves  $n$  in the modal shape. It is possible to conclude that the non-linearities have a hardening effect over the cable vibrations, and cause an increase in the natural frequencies. Such an increase is more significant for higher modes and for larger modal amplitude. Table 23.1 is graphically summarized in the backbone curves presented in Fig. 23.5. Those curves

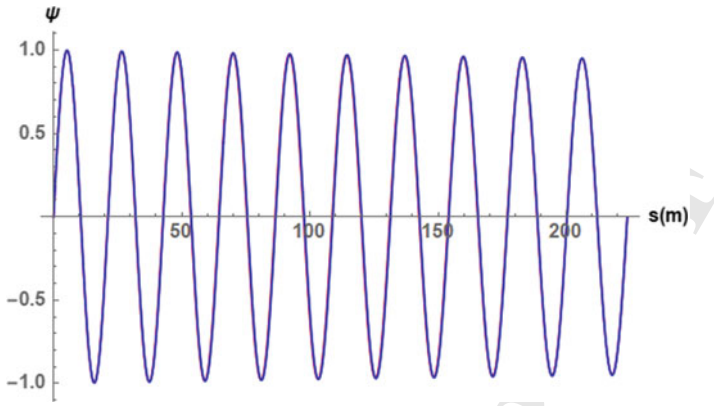


Fig. 23.2 Mode  $n = 20$ , linear solution in red and non-linear in blue with  $A_n = 3D$

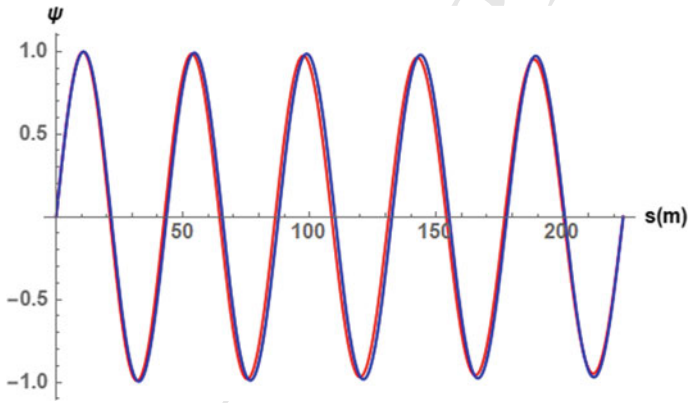


Fig. 23.3 Mode  $n = 10$ , linear solution in red and non-linear in blue with  $A_n = 20D$

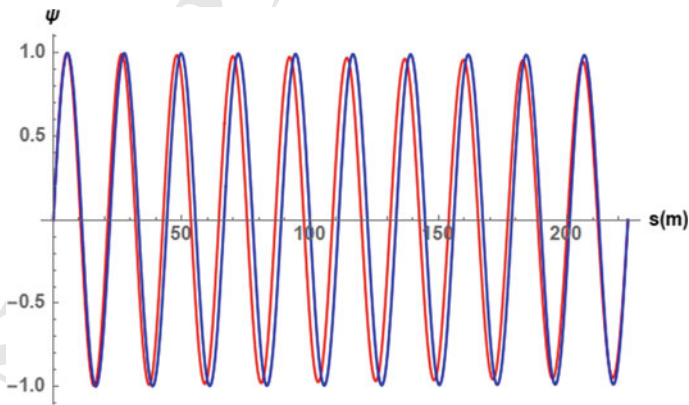
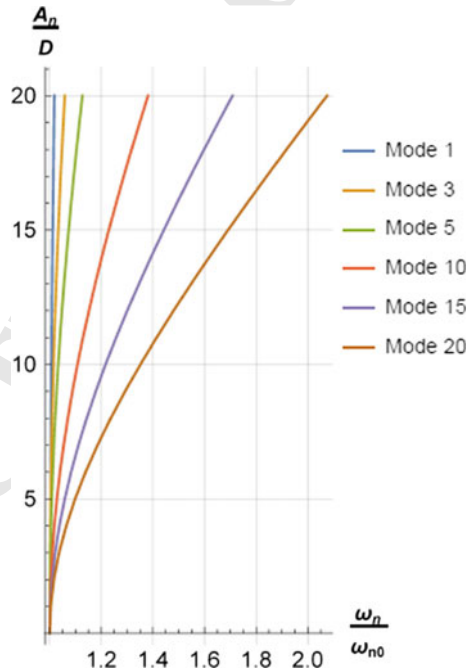


Fig. 23.4 Mode  $n = 20$ , linear solution in red and non-linear in blue with  $A_n = 20D$

**Table 23.1** Frequencies comparison (rad/s)

$n$	Linear	$A_n = 1D$	$A_n = 3D$	$A_n = 5D$	$A_n = 10D$	$A_n = 20D$
2	2.617	2.626	2.627	2.628	2.635	2.665
3	3.926	3.980	3.983	3.988	4.013	4.109
5	6.543	6.556	6.568	6.593	6.706	7.141
10	13.086	13.098	13.196	13.389	14.260	17.310
15	19.629	19.670	19.998	20.639	23.406	32.169
20	26.171	26.269	27.041	28.521	34.619	52.317
30	39.257	39.587	42.135	46.812	64.338	109.205

**Fig. 23.5** Backbone curves for the cable in study, being  $\omega_{n0}$  the natural frequency of the linear problem



189 were numerically obtained by applying the proposed model to some values of modal  
 190 amplitude in the range presented in the figure.

191 **23.4 Conclusions**

192 A closed-form solution for the non-linear modes and natural frequencies of a hang-  
 193 ing cable with small sag was obtained. The results showed the dependence of the  
 194 frequencies on the amplitude of motion and the change in the modal shape, resulting

195 in a shift of nodal points, changing the amplitude variation along the length. It is  
 196 important to highlight the increasing in natural frequencies due to the preserved non-  
 197 linearities, which may be significant for fatigue analysis for example. Finally, besides  
 198 giving intrinsic characteristics of the system, closed-form solutions for modal shapes  
 199 also allow for further direct implementations of projection methods in dynamic anal-  
 200 ysis, such as the Galerkin projection. Further work includes the search for non-linear  
 201 modes of hanging cables with arbitrary sag, and the application of non-linear modes  
 202 in Galerkin schemes to obtain reduced order models for problems of interest such  
 203 as cables subjected to vortex-induced vibrations or under the action of parametric  
 204 excitation.

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Transpose	└┐	└┐
Close up	linking ○ characters	⸸
Insert or substitute space between characters or words	/ through character or ⋈ where required	⸣
Reduce space between characters or words		⸣