Estudo das deformações

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PEF 3302 – Mecânica das Estruturas I

Estudo das deformações

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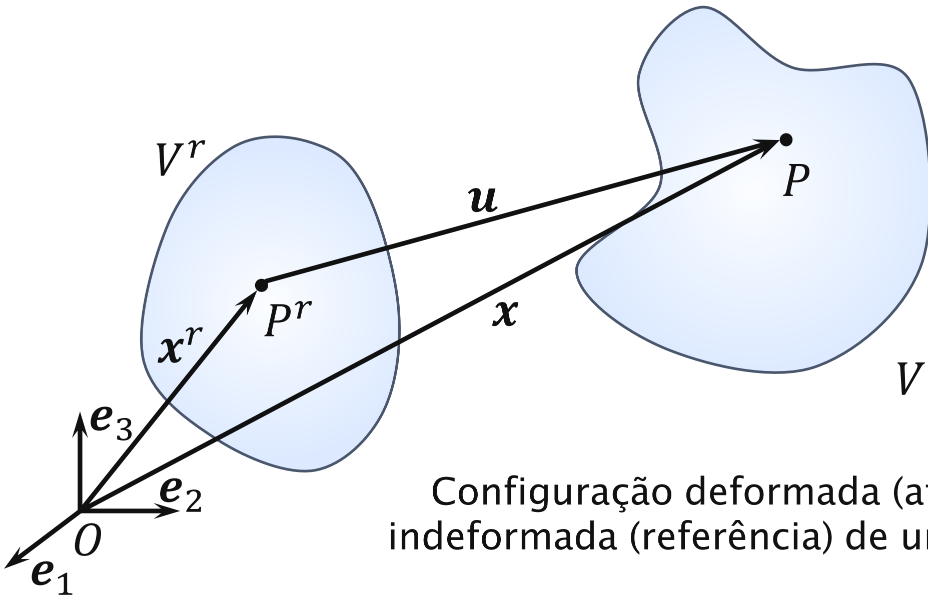

Configuração deformada (atual) e indeformada (referência) de um sólido

$$\boldsymbol{x}^r = \Sigma_{i=1}^{3} x_i^r \boldsymbol{e}_i = x_1^r \boldsymbol{e}_1 + x_2^r \boldsymbol{e}_2 + x_3^r \boldsymbol{e}_3 = x_i^r \boldsymbol{e}_i \quad (1)$$

$$\boldsymbol{x} = \Sigma_{i=1}^{3} x_i \boldsymbol{e}_i = x_1 \boldsymbol{e}_1 + x_2 \boldsymbol{e}_2 + x_3 \boldsymbol{e}_3 = x_i \boldsymbol{e}_i \quad (2)$$

Notação de Einstein

Campo de deslocamentos: $\boldsymbol{u} = \boldsymbol{x} - \boldsymbol{x}^r$ (3)

A deformação fica totalmente caracterizada conhecendo-se a função:

$$\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{x}^r) \qquad (4)$$

que fornece a posição de uma partícula material na configuração deformada a partir de sua posição na configuração indeformada.

Em componentes:

$$\begin{aligned} x_1 &= x_1(x_1^r, x_2^r, x_3^r) \\ x_2 &= x_2(x_1^r, x_2^r, x_3^r) \\ x_3 &= x_3(x_1^r, x_2^r, x_3^r) \end{aligned} \qquad (5)$$

e para o campo de deslocamentos: (6)

$$\begin{aligned} u_1 &= u_1(x_1^r, x_2^r, x_3^r) \\ u_2 &= u_2(x_1^r, x_2^r, x_3^r) \\ u_3 &= u_3(x_1^r, x_2^r, x_3^r) \end{aligned}$$

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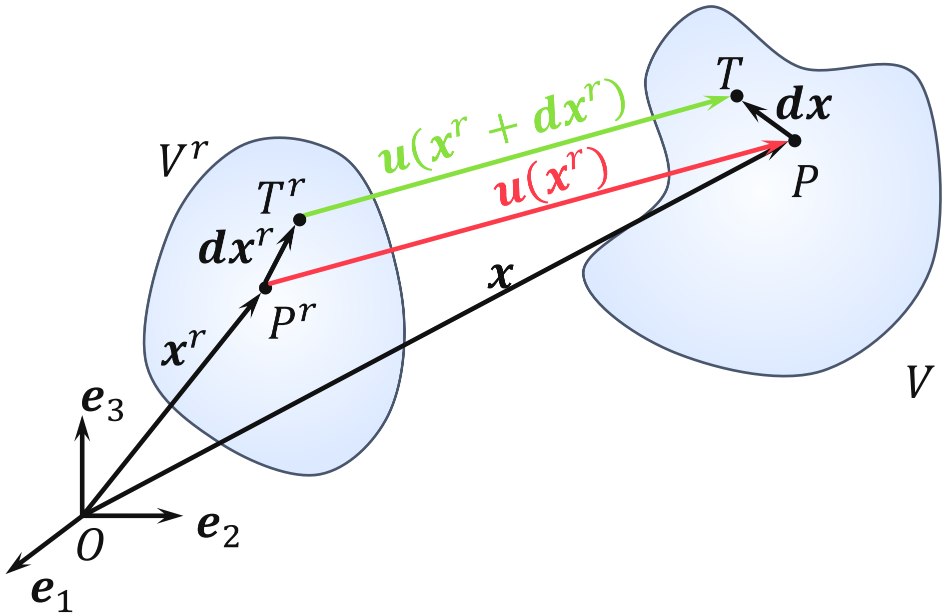

Fibras na configuração deformada e indeformada:

$$\boldsymbol{dx} = \boldsymbol{dx}^r + \boldsymbol{u}(\boldsymbol{x}^r + \boldsymbol{dx}^r) - \boldsymbol{u}(\boldsymbol{x}^r) \quad (7)$$

Em componentes:

$$dx_i = dx_i^r + u_i(x_1^r + dx_1^r, x_2^r + dx_2^r, x_3^r + dx_3^r) - u(x_1^r, x_2^r, x_3^r) \quad (8)$$

Lembrando do cálculo com múltiplas variáveis:

$$u_i(x_1^r + dx_1^r, x_2^r + dx_2^r, x_3^r + dx_3^r) - u(x_1^r, x_2^r, x_3^r) = \frac{\partial u_i}{\partial x_1^r} dx_1^r + \frac{\partial u_i}{\partial x_2^r} dx_2^r + \frac{\partial u_i}{\partial x_3^r} dx_3^r \quad (9)$$

Substituindo (9) em (8)

$$dx_i = dx_i^r + \frac{\partial u_i}{\partial x_1^r} dx_1^r + \frac{\partial u_i}{\partial x_2^r} dx_2^r + \frac{\partial u_i}{\partial x_3^r} dx_3^r \qquad (10)$$

que em forma matricial:

$$\begin{bmatrix} dx_1 \\ dx_2 \\ dx_2 \end{bmatrix} = \begin{bmatrix} dx_1^r \\ dx_2^r \\ dx_2^r \end{bmatrix} + \begin{bmatrix} \frac{\partial u_1}{\partial x_1^r} & \frac{\partial u_1}{\partial x_2^r} & \frac{\partial u_1}{\partial x_3^r} \\ \frac{\partial u_2}{\partial x_1^r} & \frac{\partial u_2}{\partial x_2^r} & \frac{\partial u_2}{\partial x_3^r} \\ \frac{\partial u_3}{\partial x_1^r} & \frac{\partial u_3}{\partial x_2^r} & \frac{\partial u_3}{\partial x_3^r} \end{bmatrix} \begin{bmatrix} dx_1^r \\ dx_2^r \\ dx_2^r \end{bmatrix} \qquad (11)$$

ou

$$\begin{bmatrix} dx_1 \\ dx_2 \\ dx_2 \end{bmatrix} = \begin{bmatrix} 1 + \frac{\partial u_1}{\partial x_1^r} & \frac{\partial u_1}{\partial x_2^r} & \frac{\partial u_1}{\partial x_3^r} \\ \frac{\partial u_2}{\partial x_1^r} & 1 + \frac{\partial u_2}{\partial x_2^r} & \frac{\partial u_2}{\partial x_3^r} \\ \frac{\partial u_3}{\partial x_1^r} & \frac{\partial u_3}{\partial x_2^r} & 1 + \frac{\partial u_3}{\partial x_3^r} \end{bmatrix} \begin{bmatrix} dx_1^r \\ dx_2^r \\ dx_2^r \end{bmatrix} \qquad (12)$$

São definidos:

Gradiente dos deslocamentos:

$$\nabla \boldsymbol{u} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1^r} & \frac{\partial u_1}{\partial x_2^r} & \frac{\partial u_1}{\partial x_3^r} \\ \frac{\partial u_2}{\partial x_1^r} & \frac{\partial u_2}{\partial x_2^r} & \frac{\partial u_2}{\partial x_3^r} \\ \frac{\partial u_3}{\partial x_1^r} & \frac{\partial u_3}{\partial x_2^r} & \frac{\partial u_3}{\partial x_3^r} \end{bmatrix} \tag{13}$$

Gradiente das deformações:

$$\boldsymbol{F} = \begin{bmatrix} 1 + \frac{\partial u_1}{\partial x_1^r} & \frac{\partial u_1}{\partial x_2^r} & \frac{\partial u_1}{\partial x_3^r} \\ \frac{\partial u_2}{\partial x_1^r} & 1 + \frac{\partial u_2}{\partial x_2^r} & \frac{\partial u_2}{\partial x_3^r} \\ \frac{\partial u_3}{\partial x_1^r} & \frac{\partial u_3}{\partial x_2^r} & 1 + \frac{\partial u_3}{\partial x_3^r} \end{bmatrix} = \boldsymbol{I} + \nabla \boldsymbol{u} \tag{14}$$

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Logo, pode-se escrever que:

$$\boldsymbol{dx} = \boldsymbol{dx}^r + \nabla \boldsymbol{u}\, \boldsymbol{dx}^r \qquad (15)$$

e

$$\boldsymbol{dx} = (\boldsymbol{I} + \nabla \boldsymbol{u})\, \boldsymbol{dx}^r = \boldsymbol{F}\, \boldsymbol{dx}^r \qquad (16)$$

Alternativamente, o gradiente das deformações pode ser definido como:

$$\boldsymbol{u} = \boldsymbol{x} - \boldsymbol{x}^r \Longrightarrow \boldsymbol{x} = \boldsymbol{x}^r + \boldsymbol{u}$$

Em componentes: $x_i = u_i + x_i^r$

$$\frac{\partial x_i}{\partial x_j^r} = \frac{\partial x_i^r}{\partial x_j^r} + \frac{\partial u_i}{\partial x_j^r}$$

delta de Kronecker

$$\boldsymbol{F} = \begin{bmatrix} \frac{\partial x_1}{\partial x_1^r} & \frac{\partial x_1}{\partial x_2^r} & \frac{\partial x_1}{\partial x_3^r} \\ \frac{\partial x_2}{\partial x_1^r} & \frac{\partial x_2}{\partial x_2^r} & \frac{\partial x_2}{\partial x_3^r} \\ \frac{\partial x_3}{\partial x_1^r} & \frac{\partial x_3}{\partial x_2^r} & \frac{\partial x_3}{\partial x_3^r} \end{bmatrix} \qquad (17)$$

Alongamentos lineares

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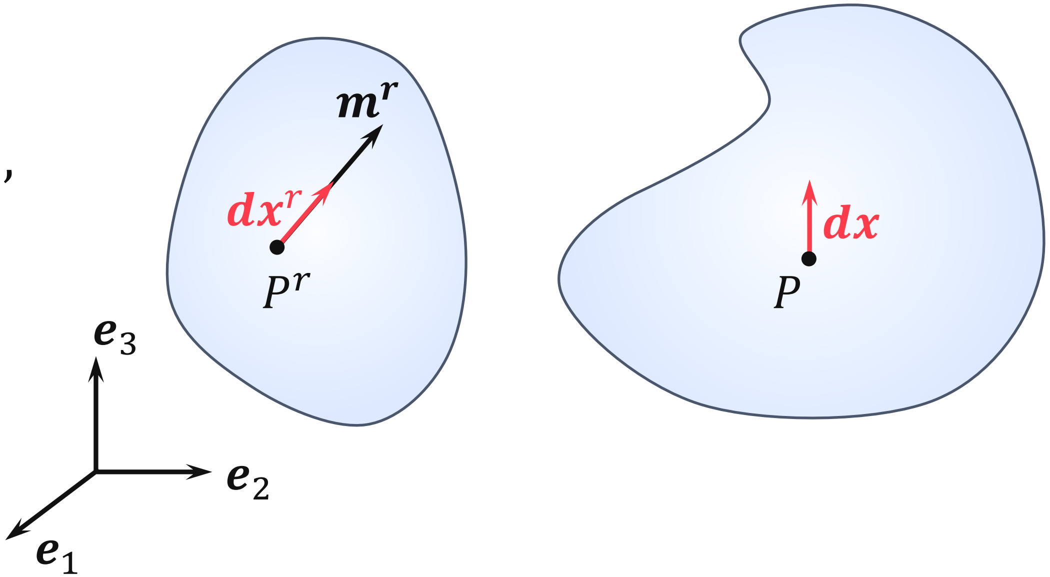
$$\boldsymbol{m}^r = \frac{\boldsymbol{dx}^r}{\|\boldsymbol{dx}^r\|},$$
$$\|\boldsymbol{m}^r\| = 1$$



Fibras infinitesimais nas configurações deformada e indeformada

Pode-se escrever:

$$ds^r = \|\boldsymbol{dx}^r\| = (\boldsymbol{dx}^r \cdot \boldsymbol{dx}^r)^{1/2} = \left((dx_1^r)^2 + (dx_2^r)^2 + (dx_3^r)^2\right)^{1/2} \quad (18)$$

$$ds = \|\boldsymbol{dx}\| = \left((dx_1)^2 + (dx_2)^2 + (dx_3)^2\right)^{1/2} \quad (19)$$

Alongamentos lineares

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Define-se alongamento linear por:

$$\varepsilon_l = \frac{ds - ds^r}{ds^r} = \frac{ds}{ds^r} - 1 \qquad (20)$$

e o estiramento por:

$$\lambda = \frac{ds}{ds^r} \qquad (21)$$

Portanto:

$$\varepsilon_l = \lambda - 1 \qquad (22)$$

Tem-se:

$$ds = \|\boldsymbol{dx}\| = \sqrt{\boldsymbol{dx} \cdot \boldsymbol{dx}} = \sqrt{\boldsymbol{F}\boldsymbol{dx}^r \cdot \boldsymbol{F}\boldsymbol{dx}^r} \qquad (23)$$

Define-se o transposto de um operador $\boldsymbol{A}$ como o operador $\boldsymbol{A}^T$ que verifica:

$$\boldsymbol{a} \cdot \boldsymbol{A}\,\boldsymbol{b} = \boldsymbol{b} \cdot \boldsymbol{A}^T\boldsymbol{a} \qquad (24)$$

Considerando a relação (24) em (23), temos:

$$ds = \sqrt{\underbrace{\boldsymbol{F}\boldsymbol{dx}^r}_{\boldsymbol{v}} \cdot \boldsymbol{F}\boldsymbol{dx}^r} = \sqrt{\boldsymbol{dx}^r \cdot \boldsymbol{F}^T\boldsymbol{v}} = \sqrt{\boldsymbol{dx}^r \cdot \boldsymbol{F}^T\boldsymbol{F}\boldsymbol{dx}^r} \qquad (25)$$

Resulta:

$$\varepsilon_l = \frac{ds}{ds^r} - 1 = \frac{\sqrt{\boldsymbol{dx}^r \cdot \boldsymbol{F}^T\boldsymbol{F}\boldsymbol{dx}^r}}{\sqrt{\boldsymbol{dx}^r \cdot \boldsymbol{dx}^r}} - 1 \qquad (26)$$

e como

$$\boldsymbol{dx}^r = ds^r\boldsymbol{m}^r \qquad (27)$$

$\|\boldsymbol{m}^r\| = 1$

Alongamentos lineares

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Resulta:

$$\varepsilon_l = \frac{ds}{ds^r} - 1 = \frac{ds^r\sqrt{\boldsymbol{m^r F}^T\boldsymbol{F}\,\boldsymbol{m}^r}}{ds^r\sqrt{\boldsymbol{m^r}\cdot\boldsymbol{m}^r}} - 1 \qquad (28)$$

ou

$$\varepsilon_l = \sqrt{\boldsymbol{m^r}\cdot\boldsymbol{F}^T\boldsymbol{F m}^r} - 1 = \sqrt{\boldsymbol{m^r}^T\boldsymbol{F}^T\boldsymbol{F m}^r} - 1 \qquad (29)$$

Alongamento linear para uma fibra infinitesimal que tem direção dada pelo versor $\boldsymbol{m}^r$ na configuração de referência.

Tem-se:

$$\boldsymbol{F}^T\boldsymbol{F} = (\boldsymbol{I}+\nabla\boldsymbol{u})^T(\boldsymbol{I}+\nabla\boldsymbol{u}) = \boldsymbol{I}+\nabla\boldsymbol{u}+\nabla\boldsymbol{u}^T+\nabla\boldsymbol{u}^T\nabla\boldsymbol{u} \qquad (30)$$

Quando se tem deslocamentos infinitesimais, pode-se desprezar $\nabla u_{ij}{}^T\nabla u_{ij}\,\frac{\partial u_i}{\partial x_j^r}\frac{\partial u_j}{\partial x_i^r}$ em relação a ∇u_{ij} e $\nabla u_{ij}{}^T$, onde:

$$\nabla u_{ij}{}^T\nabla u_{ij} = \frac{\partial u_k}{\partial x_i^r}\frac{\partial u_k}{\partial x_j^r} = \frac{\partial u_1}{\partial x_i^r}\frac{\partial u_1}{\partial x_j^r}+\frac{\partial u_2}{\partial x_i^r}\frac{\partial u_2}{\partial x_j^r}+\frac{\partial u_3}{\partial x_i^r}\frac{\partial u_3}{\partial x_j^r}$$

$$\nabla u_{ij} = \frac{\partial u_i}{\partial x_j^r}\ e\ \nabla u_{ij}{}^T = \frac{\partial u_j}{\partial x_i^r}$$

Portanto,

$$\boldsymbol{F}^T\boldsymbol{F} = \boldsymbol{I}+\nabla\boldsymbol{u}+\nabla\boldsymbol{u}^T \qquad (31)$$

Define-se então o tensor das deformações infinitesimais como:

$$\boldsymbol{E} = \frac{\boldsymbol{1}}{\boldsymbol{2}}(\nabla\boldsymbol{u}+\nabla\boldsymbol{u}^T) \qquad (32)$$

Em notação indicial:

$$E_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j^r}+\frac{\partial u_j}{\partial x_i^r}\right)$$

Em componentes:

$$\boldsymbol{E} = \begin{bmatrix} \dfrac{\partial u_1}{\partial x_1^r} & \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_2^r} + \dfrac{\partial u_2}{\partial x_1^r}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_3^r} + \dfrac{\partial u_3}{\partial x_1^r}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_2^r} + \dfrac{\partial u_2}{\partial x_1^r}\right) & \dfrac{\partial u_2}{\partial x_2^r} & \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial x_3^r} + \dfrac{\partial u_3}{\partial x_2^r}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_3^r} + \dfrac{\partial u_3}{\partial x_1^r}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial x_3^r} + \dfrac{\partial u_3}{\partial x_2^r}\right) & \dfrac{\partial u_3}{\partial x_3^r} \end{bmatrix} \quad (33)$$

$$\boldsymbol{E} = \boldsymbol{E}^T \quad (34)$$

De (31): $\boldsymbol{F}^T\boldsymbol{F} = \boldsymbol{I} + 2\boldsymbol{E}$

e a equação (29) pode ser reescrita como:

$$\varepsilon_l = \sqrt{\boldsymbol{m^r}^T(\boldsymbol{I} + 2\boldsymbol{E})\boldsymbol{m^r}} - 1 = \left(1 + 2\boldsymbol{m^r}^T\boldsymbol{E}\boldsymbol{m^r}\right)^{1/2} - 1 \quad (35)$$

$$\boldsymbol{m^r}^T\boldsymbol{E}\boldsymbol{m^r} = \Sigma_{i=1}^3\Sigma_{j=1}^3 \frac{\partial u_i}{\partial x_j^r} m_i^r m_j^r \quad (36)$$

Considere a identidade matemática:

$$(1+\delta)^s = 1 + s\delta + \frac{s(s-1)}{2}\delta^2 + \cdots \qquad (37)$$

Substituindo $\delta = 2\boldsymbol{m}^{\boldsymbol{r}^T}\boldsymbol{E}\boldsymbol{m}^{\boldsymbol{r}}$e $s = 1/2$, obtém-se:

$$\left(1 + 2\boldsymbol{m}^{\boldsymbol{r}^T}\boldsymbol{E}\boldsymbol{m}^{\boldsymbol{r}}\right)^{1/2} = 1 + \boldsymbol{m}^{\boldsymbol{r}^T}\boldsymbol{E}\boldsymbol{m}^{\boldsymbol{r}} + \text{termos de alta ordem em } \boldsymbol{m}^{\boldsymbol{r}^T}\boldsymbol{E}\boldsymbol{m}^{\boldsymbol{r}} \qquad (38)$$

Desprezando termos de alta ordem em $\frac{\partial u_i}{\partial x_j^r}$:

$$\left(1 + 2\boldsymbol{m}^{\boldsymbol{r}^T}\boldsymbol{E}\boldsymbol{m}^{\boldsymbol{r}}\right)^{1/2} = 1 + \boldsymbol{m}^{\boldsymbol{r}^T}\boldsymbol{E}\boldsymbol{m}^{\boldsymbol{r}} \qquad (39)$$

Deformações infinitesimais

De (35) resulta:

$$\varepsilon_l(\boldsymbol{m}) = \boldsymbol{m^r}^T \boldsymbol{E}\boldsymbol{m^r} = \boldsymbol{m^r} \cdot \boldsymbol{E}\boldsymbol{m^r} \qquad (40)$$

Alongamento linear para uma fibra infinitesimal $\boldsymbol{m^r}$ considerando deformações infinitesimais.

Mudança de ângulo entre fibras – Distorção

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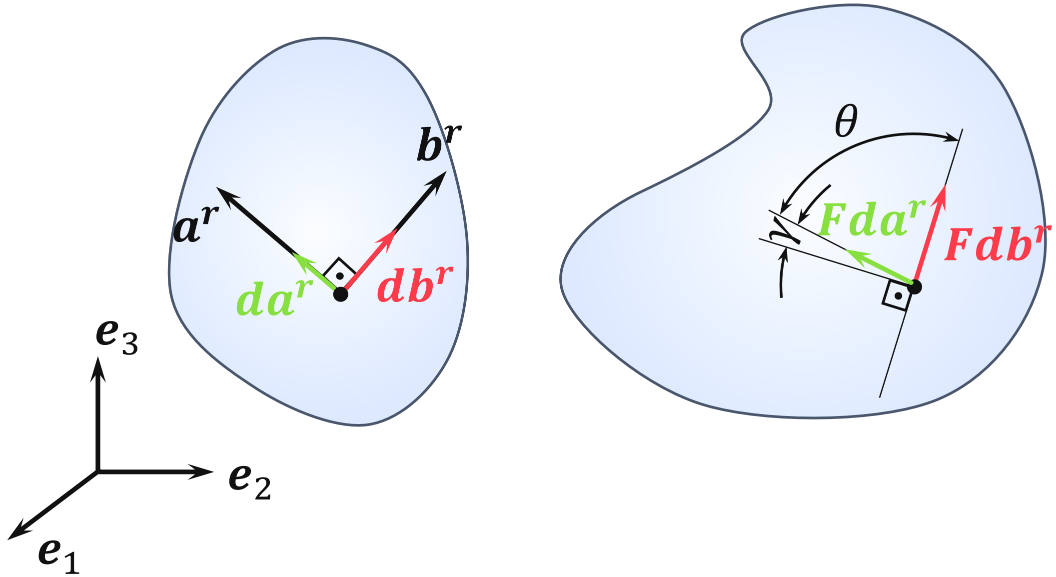

Fibras infinitesimais nas configurações deformada e indeformada.

Pode-se escrever, considerando a definição de produto escalar:

$$\boldsymbol{F}\,d\boldsymbol{a}^r \cdot \boldsymbol{F}\,d\boldsymbol{b}^r = \|\boldsymbol{F}d\boldsymbol{a}^r\|\|\boldsymbol{F}d\boldsymbol{b}^r\|\cos\theta \qquad (41)$$

e

$$\cos\theta = \operatorname{sen}\gamma = \frac{\boldsymbol{F}\,d\boldsymbol{a}^r \cdot \boldsymbol{F}\,d\boldsymbol{b}^r}{\|\boldsymbol{F}d\boldsymbol{a}^r\|\|\boldsymbol{F}d\boldsymbol{b}^r\|} = \frac{d\boldsymbol{a}^r \cdot \boldsymbol{F}^T\boldsymbol{F}\,d\boldsymbol{b}^r}{\|\boldsymbol{F}d\boldsymbol{a}^r\|\|\boldsymbol{F}d\boldsymbol{b}^r\|} \qquad (42)$$

Sabemos que:

$$\|\boldsymbol{F}\,\boldsymbol{da^r}\| = \sqrt{\boldsymbol{F}\,\boldsymbol{da^r}\cdot\boldsymbol{F}\,\boldsymbol{da^r}}\ \ e\ \ \|\boldsymbol{F}\boldsymbol{db^r}\| = \sqrt{\boldsymbol{F}\,\boldsymbol{db^r}\cdot\boldsymbol{F}\,\boldsymbol{db^r}}$$

$$\boldsymbol{da^r} = da^r\,\boldsymbol{a^r}\ e\ \boldsymbol{db^r} = db^r\,\boldsymbol{b^r} \qquad (43)$$

Substituindo a equação (43) na equação (42):

$$\sin\gamma = \frac{da^r\,\boldsymbol{a^r}\cdot\boldsymbol{F^T}\boldsymbol{F}\,db^r\,\boldsymbol{b^r}}{da^r\sqrt{\boldsymbol{F}\,\boldsymbol{a^r}\cdot\boldsymbol{F}\,\boldsymbol{a^r}}\,db^r\sqrt{\boldsymbol{F}\,\boldsymbol{b^r}\cdot\boldsymbol{F}\,\boldsymbol{b^r}}} = \frac{\boldsymbol{a^r}\cdot\boldsymbol{F^T}\boldsymbol{F}\,\boldsymbol{b^r}}{\sqrt{\boldsymbol{F}\,\boldsymbol{a^r}\cdot\boldsymbol{F}\,\boldsymbol{a^r}}\,\sqrt{\boldsymbol{F}\,\boldsymbol{b^r}\cdot\boldsymbol{F}\,\boldsymbol{b^r}}} \qquad (44)$$

que também pode ser escrita como:

$$\sin\gamma = \frac{\boldsymbol{a^r}^T\boldsymbol{F^T}\boldsymbol{F}\boldsymbol{b^r}}{\sqrt{\boldsymbol{a^r}^T\boldsymbol{F}^T\boldsymbol{F}\boldsymbol{a^r}}\,\sqrt{\boldsymbol{b^r}^T\boldsymbol{F}^T\boldsymbol{F}\boldsymbol{b^r}}}$$

Lembrando que:

$$1 + \varepsilon_l(\boldsymbol{a^r}) = \sqrt{\boldsymbol{a^r}^T \boldsymbol{F}^T \boldsymbol{F} \boldsymbol{a^r}} \;\; e \;\; 1 + \varepsilon_l(\boldsymbol{b^r}) = \sqrt{\boldsymbol{b^r}^T \boldsymbol{F}^T \boldsymbol{F} \boldsymbol{b^r}} \tag{45}$$

podemos então também expressar a equação (44) que calcula a distorção entre duas fibras infinitesimais ($\boldsymbol{da^r}$ e $\boldsymbol{db^r}$), inicialmente ortogonais, cujas direções são definidas pelos versores $\boldsymbol{a^r}$ e $\boldsymbol{b^r}$, da seguinte forma:

$$\sin\gamma\,(\boldsymbol{a^r}, \boldsymbol{b^r}) = \frac{\boldsymbol{a^r}^T \boldsymbol{F}^T \boldsymbol{F} \boldsymbol{b^r}}{\big(1 + \varepsilon_l(\boldsymbol{a^r})\big)\big(1 + \varepsilon_l(\boldsymbol{b^r})\big)} \tag{46}$$

Para deformações infinitesimais resulta:

$$\sin\gamma = \gamma = \boldsymbol{a}^{r^T}(\boldsymbol{I} + 2\boldsymbol{E})\boldsymbol{b}^r = \boldsymbol{2a}^{r^T}\boldsymbol{E}\boldsymbol{b}^r + \boldsymbol{a}^{r^T}\boldsymbol{b}^r$$

$$\gamma(\boldsymbol{a}^r, \boldsymbol{b}^r) = 2\boldsymbol{a}^{r^T}\boldsymbol{E}\boldsymbol{b}^r \qquad (47)$$

Distorção entre fibras infinitesimais ($\boldsymbol{da}^r$ e $\boldsymbol{db}^r$) considerando deformações infinitesimais.

Quadro resumido de alongamento linear e distorção

	Deformações finitas	Deformações infinitesimais
Alongamento linear para fibra na direção dada por $\boldsymbol{m}^r$	$\varepsilon_l(\boldsymbol{m}^r) = \sqrt{\boldsymbol{m}^{r^T}\boldsymbol{F}^T\boldsymbol{F}\boldsymbol{m}^r} - 1$	$\varepsilon_l(\boldsymbol{m}^r) = \boldsymbol{m}^{r^T}\boldsymbol{E}\boldsymbol{m}^r$
Distorção de um par de fibras ortogonais definidas nas direções $\boldsymbol{a}^r$ e $\boldsymbol{b}^r$	$\sin\gamma(\boldsymbol{a}^r, \boldsymbol{b}^r) = \dfrac{\boldsymbol{a}^{r^T}\boldsymbol{F}^T\boldsymbol{F}\boldsymbol{b}^r}{\big(1+\varepsilon_l(\boldsymbol{a}^r)\big)\big(1+\varepsilon_l(\boldsymbol{b}^r)\big)}$	$\gamma(\boldsymbol{a}^r, \boldsymbol{b}^r) = 2\boldsymbol{a}^{r^T}\boldsymbol{E}\boldsymbol{b}^r$

Componentes do tensor das deformações infinitesimais

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Considerando:

$$[\boldsymbol{E}] = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \quad (48)$$

Seja

$$\varepsilon_1 = \varepsilon_l(\boldsymbol{e}_1) = \boldsymbol{e}_1^T \boldsymbol{E}\, \boldsymbol{e}_1 \quad (49)$$

$$\varepsilon_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (50)$$

$$\boxed{\varepsilon_1 = E_{11}} \quad (51)$$

Analogamente:

$$\boxed{\begin{array}{l} \varepsilon_2 = E_{22} \\ \varepsilon_3 = E_{33} \end{array}} \quad (52)$$

Componentes do tensor das deformações infinitesimais

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Seja

$$\gamma_{12} = \gamma(\boldsymbol{e}_1, \boldsymbol{e}_2) = 2\boldsymbol{e}_1^T \boldsymbol{E}\, \boldsymbol{e}_2 \qquad (53)$$

$$\gamma_{12} = 2\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad (54)$$

$$\boxed{\gamma_{12} = 2E_{12}} \qquad (55)$$

Analogamente:

$$\boxed{\begin{aligned} \gamma_{13} = 2E_{13} = 2E_{31} = \gamma_{31} \\ \gamma_{23} = 2E_{23} = 2E_{32} = \gamma_{32} \end{aligned}} \qquad (56)$$

Referência bibliográfica

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