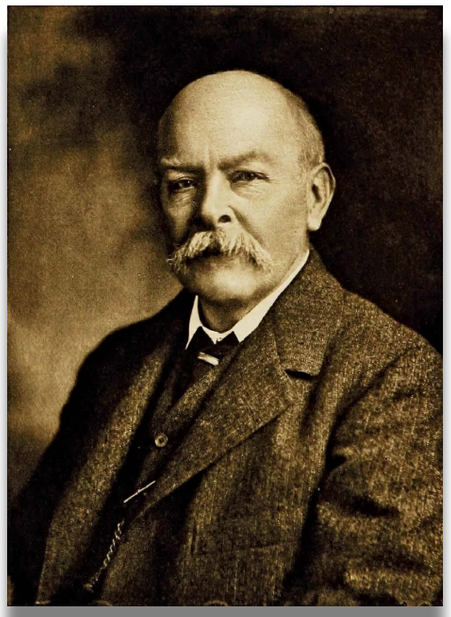
Eletromagnetismo Avançado

21 de agosto
Leis de conservação

Conservação do momento

$$\frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = \frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$



$$\frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = -\vec{\mathbf{B}} \times \frac{\partial \vec{\mathbf{E}}}{\partial t} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = -\vec{\mathbf{B}} \times \frac{\partial \vec{\mathbf{E}}}{\partial t} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} = \frac{1}{\mu_0}\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \frac{1}{\mu_0}\vec{\mathbf{J}}$$

$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = -\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}$$

$$\frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = -\vec{\mathbf{B}} \times \frac{\partial \vec{\mathbf{E}}}{\partial t} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

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$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = -\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}$$

$$\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = -\epsilon_0 \vec{\mathbf{B}} \times \frac{\partial \vec{\mathbf{E}}}{\partial t} + \epsilon_0 \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = -\epsilon_0 \vec{\mathbf{B}} \times \frac{\partial \vec{\mathbf{E}}}{\partial t} + \epsilon_0 \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} = \frac{1}{\mu_0} \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \vec{\mathbf{J}}$$

$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = -\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}$$

$$\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = -\vec{\mathbf{B}} \times \left(\frac{1}{\mu_0} \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \vec{\mathbf{J}}\right) - \epsilon_0 \vec{\mathbf{E}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right)$$

$$\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = -\epsilon_0 \vec{\mathbf{B}} \times \frac{\partial \vec{\mathbf{E}}}{\partial t} + \epsilon_0 \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} = \frac{1}{\mu_0} \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \vec{\mathbf{J}}$$

$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = -\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}$$

$$\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = -\vec{\mathbf{B}} \times \left(\frac{1}{\mu_0} \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \vec{\mathbf{J}}\right) - \epsilon_0 \vec{\mathbf{E}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0} \vec{\mathbf{B}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}\right) - \epsilon_0 \vec{\mathbf{E}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0}\vec{\mathbf{B}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}\right) - \epsilon_0 \vec{\mathbf{E}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0}\vec{\mathbf{B}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}\right) - \epsilon_0 \vec{\mathbf{E}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0}\vec{\mathbf{B}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}\right) - \epsilon_0 \vec{\mathbf{E}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0}\left(\frac{1}{2}\vec{\boldsymbol{\nabla}} B^2 - \vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}} \vec{\mathbf{B}}\right)$$

$$- \epsilon_0 \left(\frac{1}{2}\vec{\boldsymbol{\nabla}} E^2 - \vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}} \vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0}\vec{\mathbf{B}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}\right) - \epsilon_0 \vec{\mathbf{E}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0}\left(\frac{1}{2}\vec{\boldsymbol{\nabla}} B^2 - \vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}}\vec{\mathbf{B}}\right)$$
$$- \epsilon_0 \left(\frac{1}{2}\vec{\boldsymbol{\nabla}} E^2 - \vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}}\vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = \epsilon_0 (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} + \frac{1}{\mu_0}(\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}}\left(\frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2)\right)$$
$$- \epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0}\vec{\mathbf{B}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}\right) - \epsilon_0 \vec{\mathbf{E}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0}\left(\frac{1}{2}\vec{\boldsymbol{\nabla}} B^2 - \vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}}\vec{\mathbf{B}}\right)$$
$$- \epsilon_0 \left(\frac{1}{2}\vec{\boldsymbol{\nabla}} E^2 - \vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}}\vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = \epsilon_0 (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} + \frac{1}{\mu_0}(\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}}\left(\frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2)\right)$$
$$- \epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$

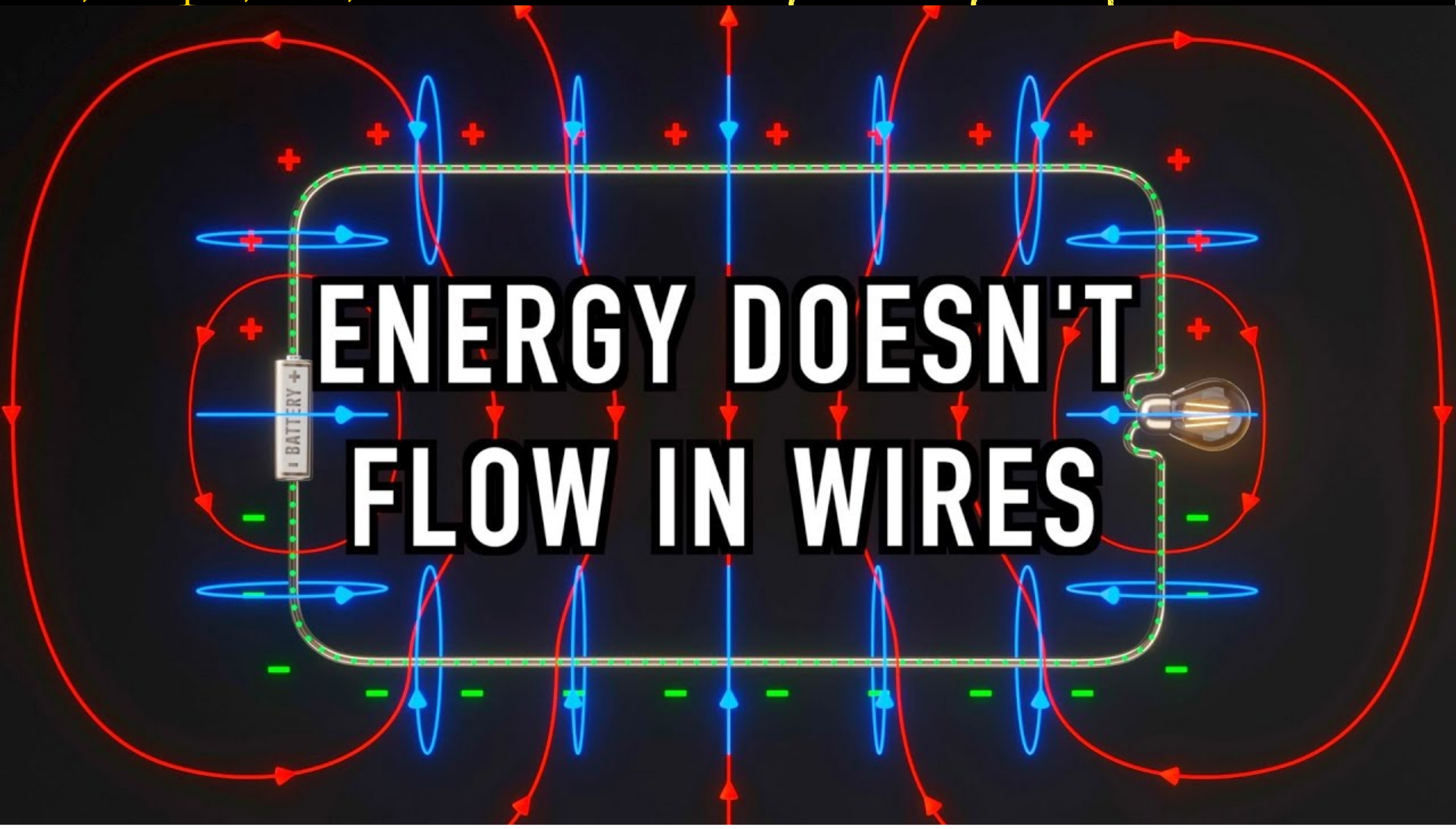
$$\boxed{\vec{\mathbf{f}} = \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}}$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0}\vec{\mathbf{B}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}\right) - \epsilon_0 \vec{\mathbf{E}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) - \frac{1}{\mu_0}\left(\frac{1}{2}\vec{\boldsymbol{\nabla}} B^2 - \vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}}\vec{\mathbf{B}}\right)$$
$$- \epsilon_0 \left(\frac{1}{2}\vec{\boldsymbol{\nabla}} E^2 - \vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}}\vec{\mathbf{E}}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = \epsilon_0 (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} + \frac{1}{\mu_0}(\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}}\left(\frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2)\right)$$
$$- \epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$

$$\boxed{\vec{\mathbf{f}} = \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}} \quad \Rightarrow \quad \vec{\mathbf{f}} = \epsilon_0 (\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}$$


ENERGY DOESN'T
FLOW IN WIRES
BATTERY