



AIRCRAFT FLIGHT CONTROL SYSTEMS II

Control Systems Applied to
Aviation

Aeronautical Engineering Department
São Carlos School of Engineering (EESC-USP)

Prof. Dr. João Paulo Eguea

Summary

[01](#) Stability augmentation

[02](#) Root locus

[03](#) Aircraft application

[04](#) Problem

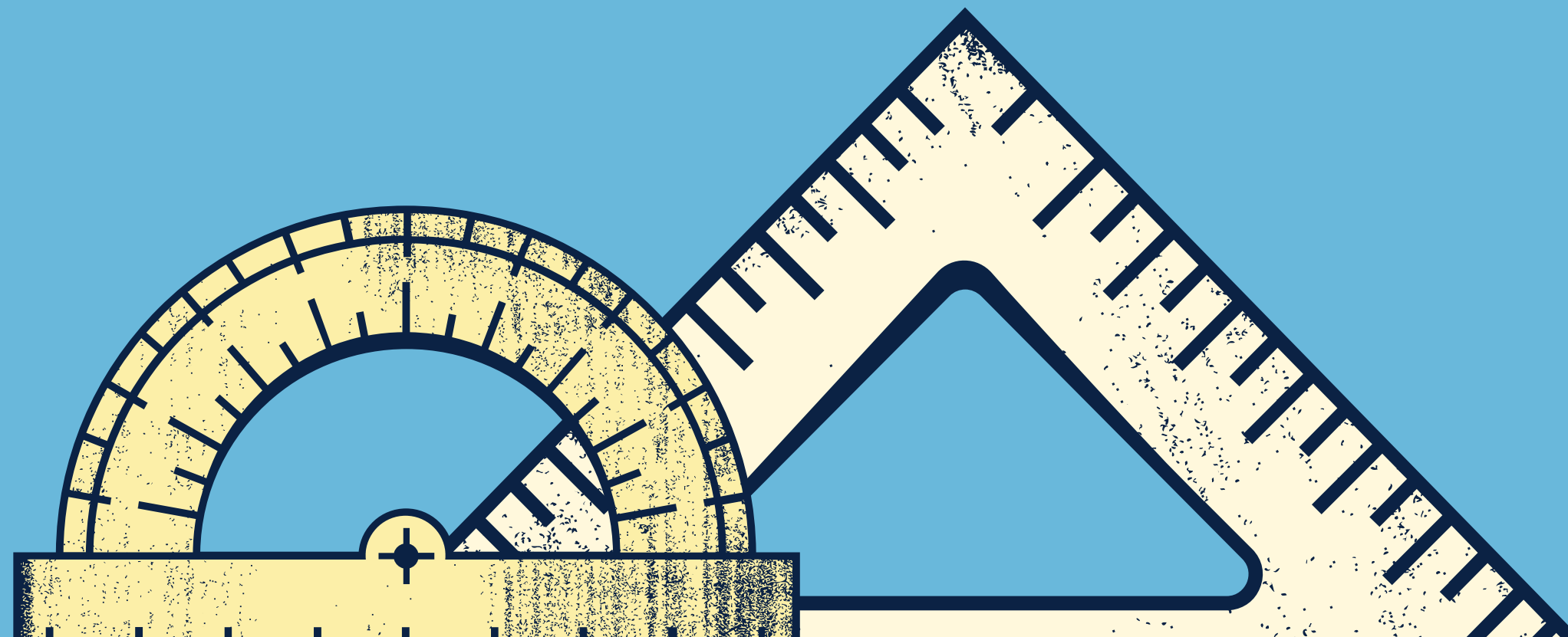
[05](#) Final remarks

Objectives

01 Describe the analysis methodology of a control system using root locus

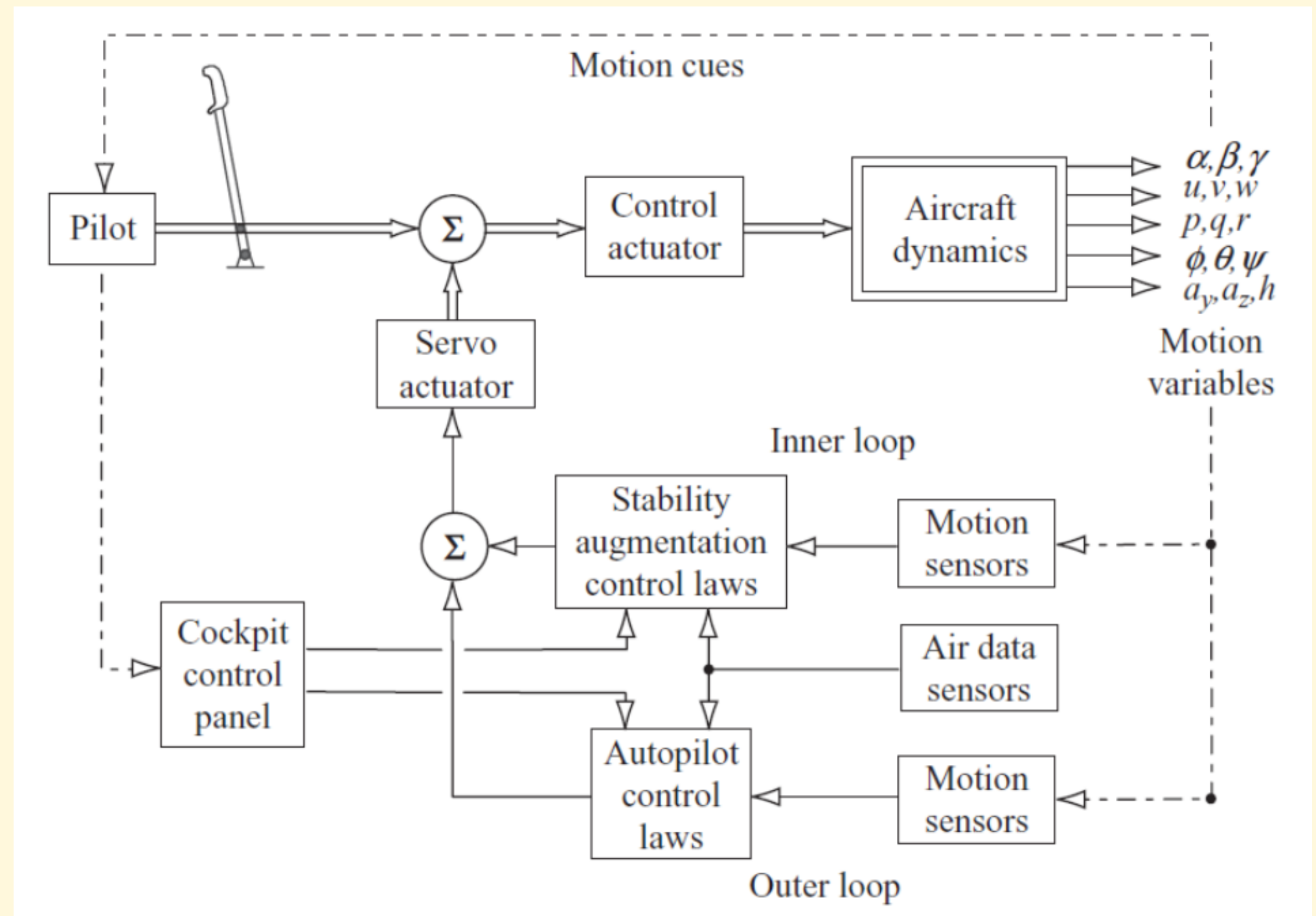
02 Identify the flight dynamics modes of an aircraft in the root locus diagram

03 Apply the gain adjustment method to a stability augmentation system



Stability augmentation

- Stability augmentation (inner loop)
 - May be deactivated
- Autopilot (outer loop)
 - Climb to level
 - Fixed velocity
 - Approach slope

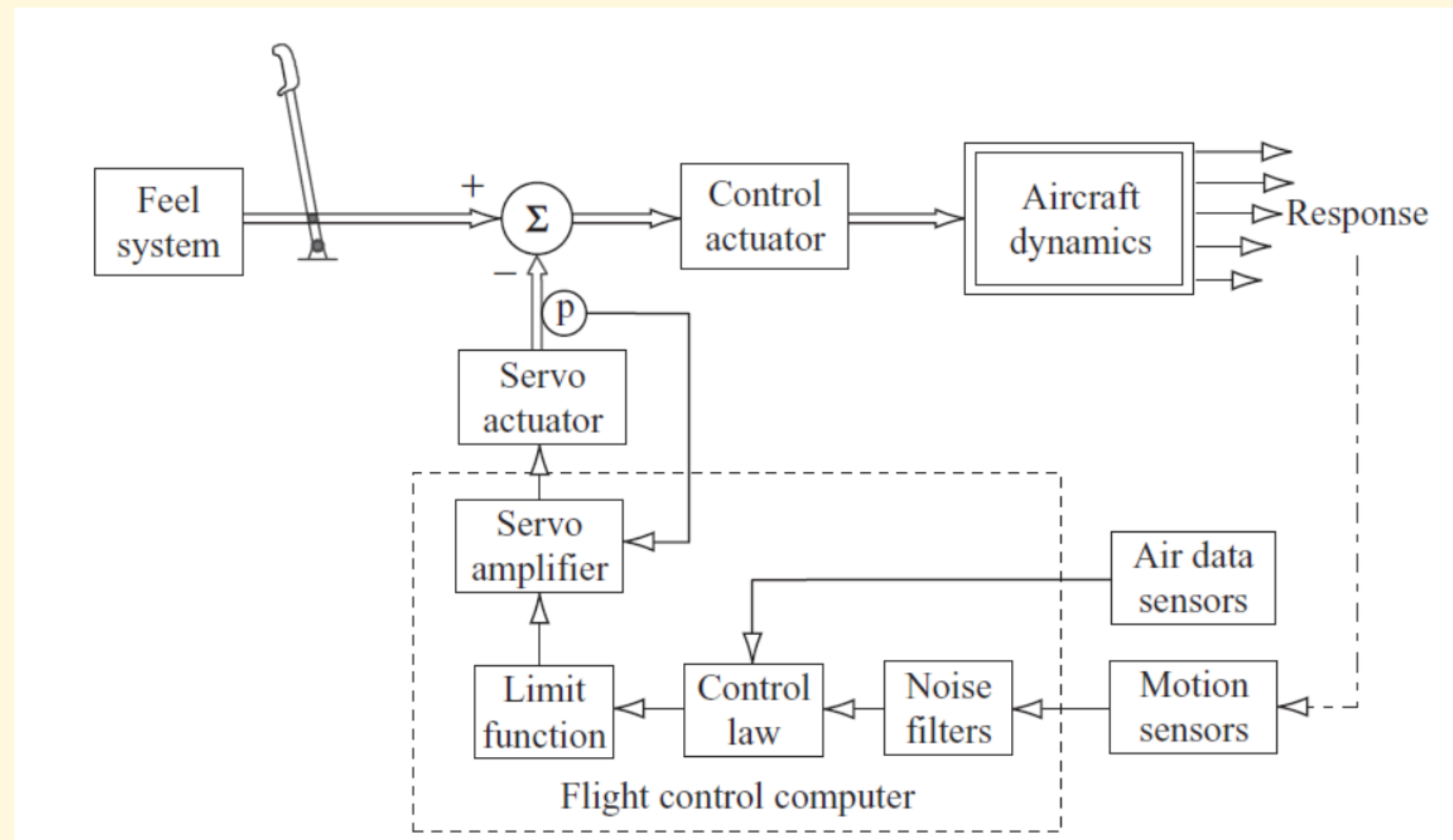


Source: Cook, M. V. (2007)

Stability augmentation

Control law - Yaw damper

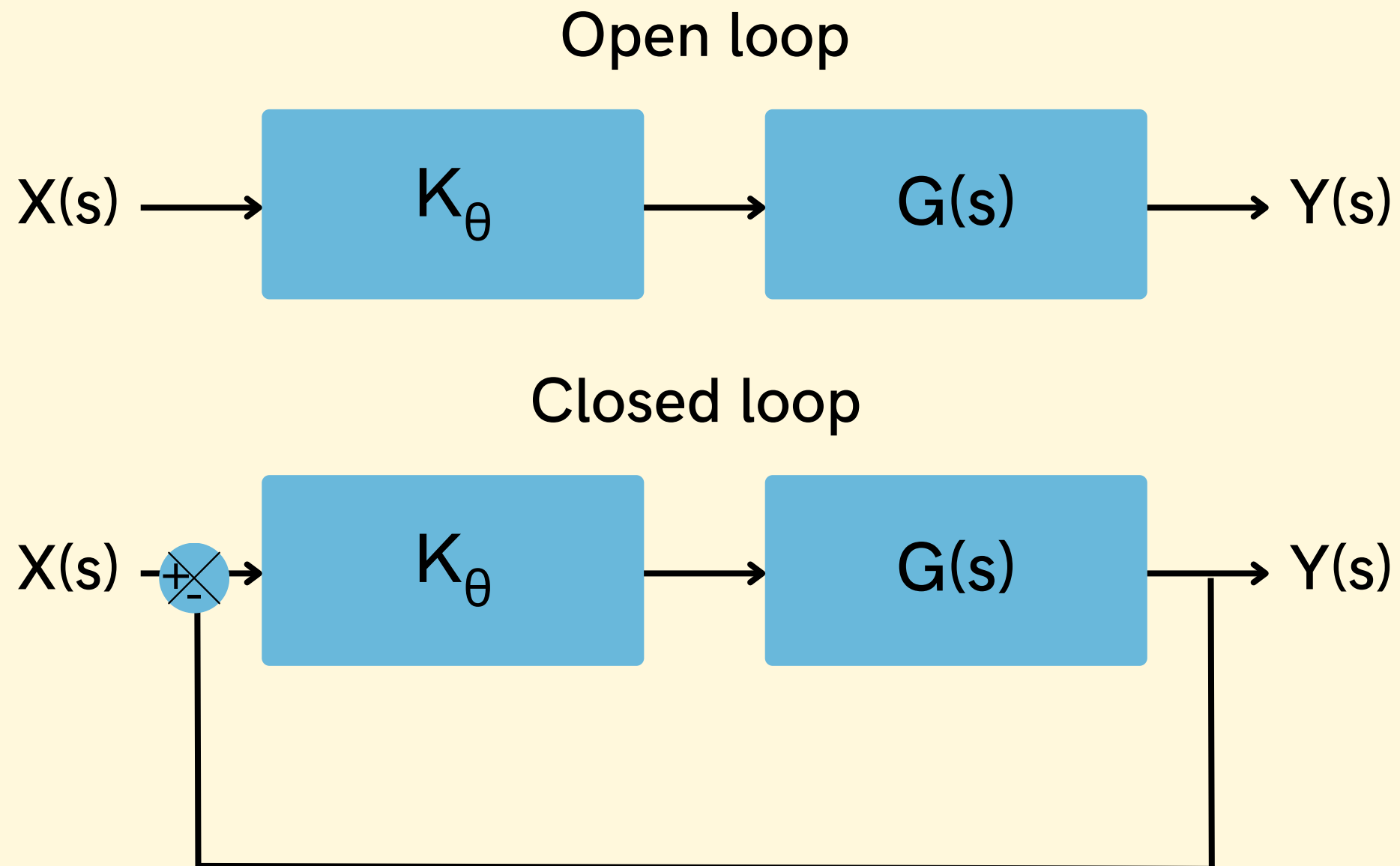
$$\zeta(s) = K_{\zeta} \delta_{\zeta}(s) - K_r \left(\frac{s}{1 + sT} \right) r(s)$$



Source: Cook, M. V. (2007)

Root locus

- Estimate characteristics of the closed-loop system from the open-loop system for a proportional driver



K_{θ} - Gain

$G(s)$ - Transfer function

$$G(s) = \frac{Y(s)}{X(s)} = \frac{\text{numerator}}{\text{denominator}}$$

Pole - Denominator root

Zero - Numerator root

$$\lambda_{1,2} = r_{1,2} \pm \omega_{1,2} \quad \omega_n = \sqrt{r^2 + \omega^2}$$
$$\lambda_{3,4} = r_{3,4} \pm \omega_{3,4} \quad \zeta = -\frac{r}{\omega_n}$$

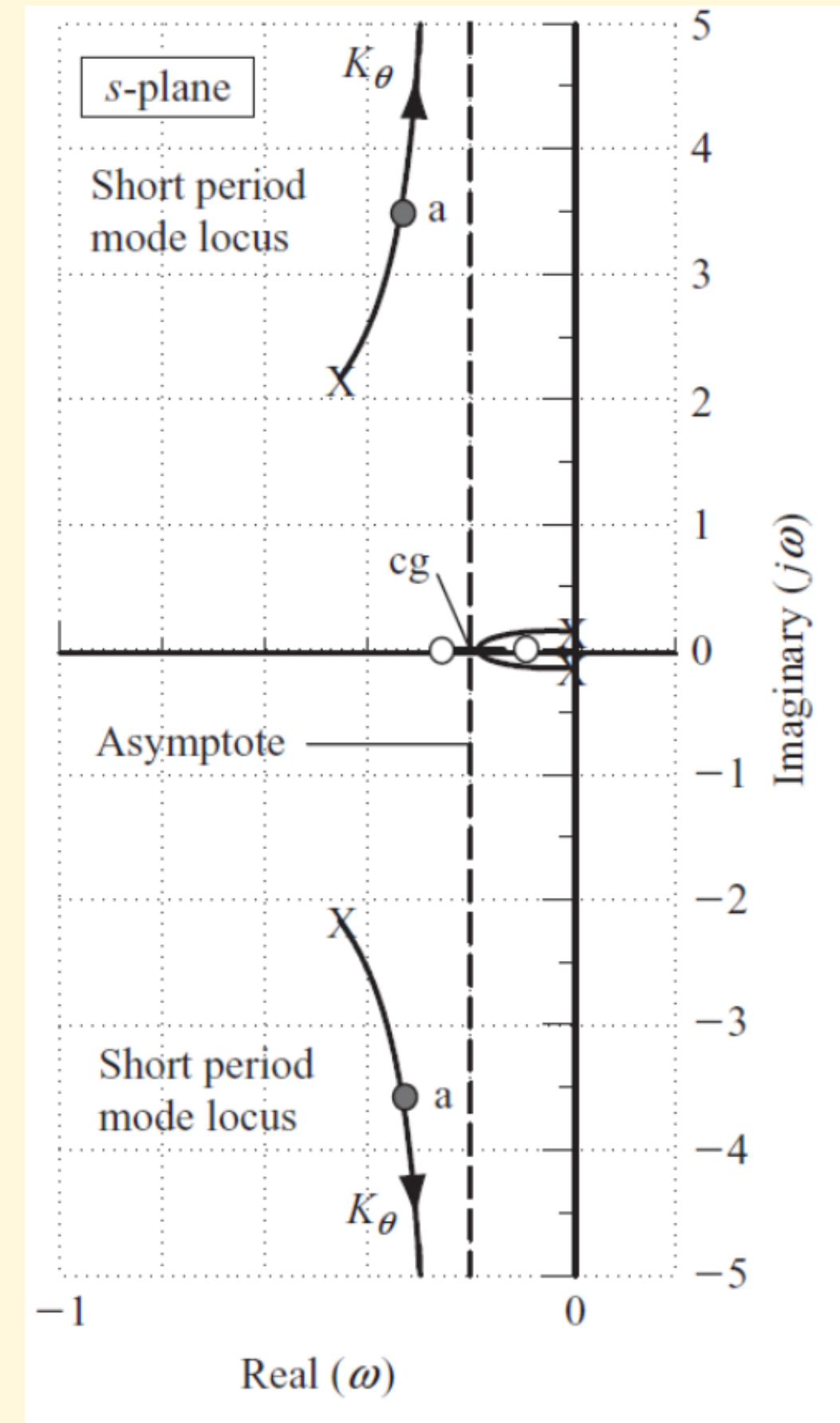
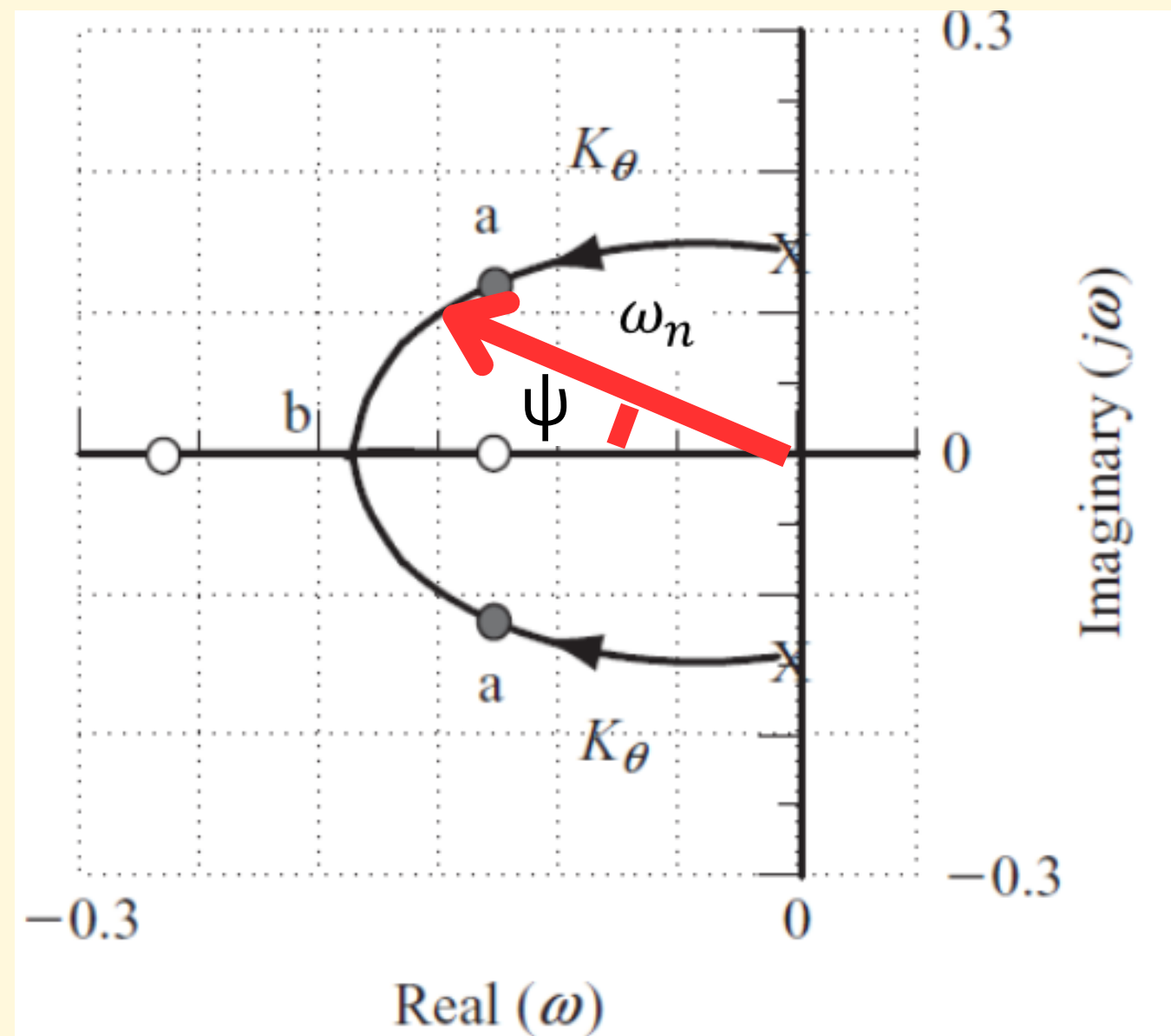
Root locus

$$\frac{\theta(s)}{\eta(s)} = \frac{-4.66K_{\theta}(s + 0.133)(s + 0.269)}{(s^2 + 0.015s + 0.021)(s^2 + 0.911s + 4.884)} \text{ rad/rad}$$

$$r = \omega n$$

$$\cos(\psi) = \zeta$$

- X – Open loop poles
- O – Open loop zeros
- – Gain test points



Aircraft application

McDonnell Douglas A-4D Skyhawk

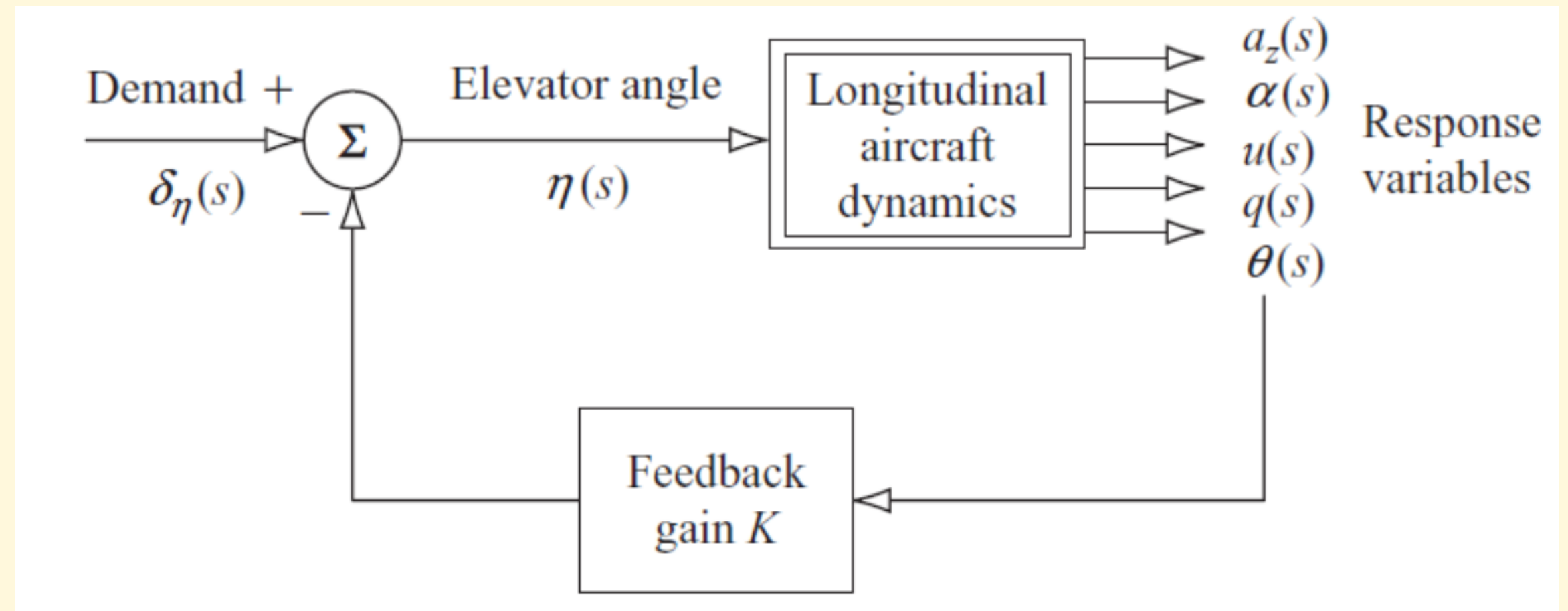


Source: Wikipedia

Aircraft application

Longitudinal movement

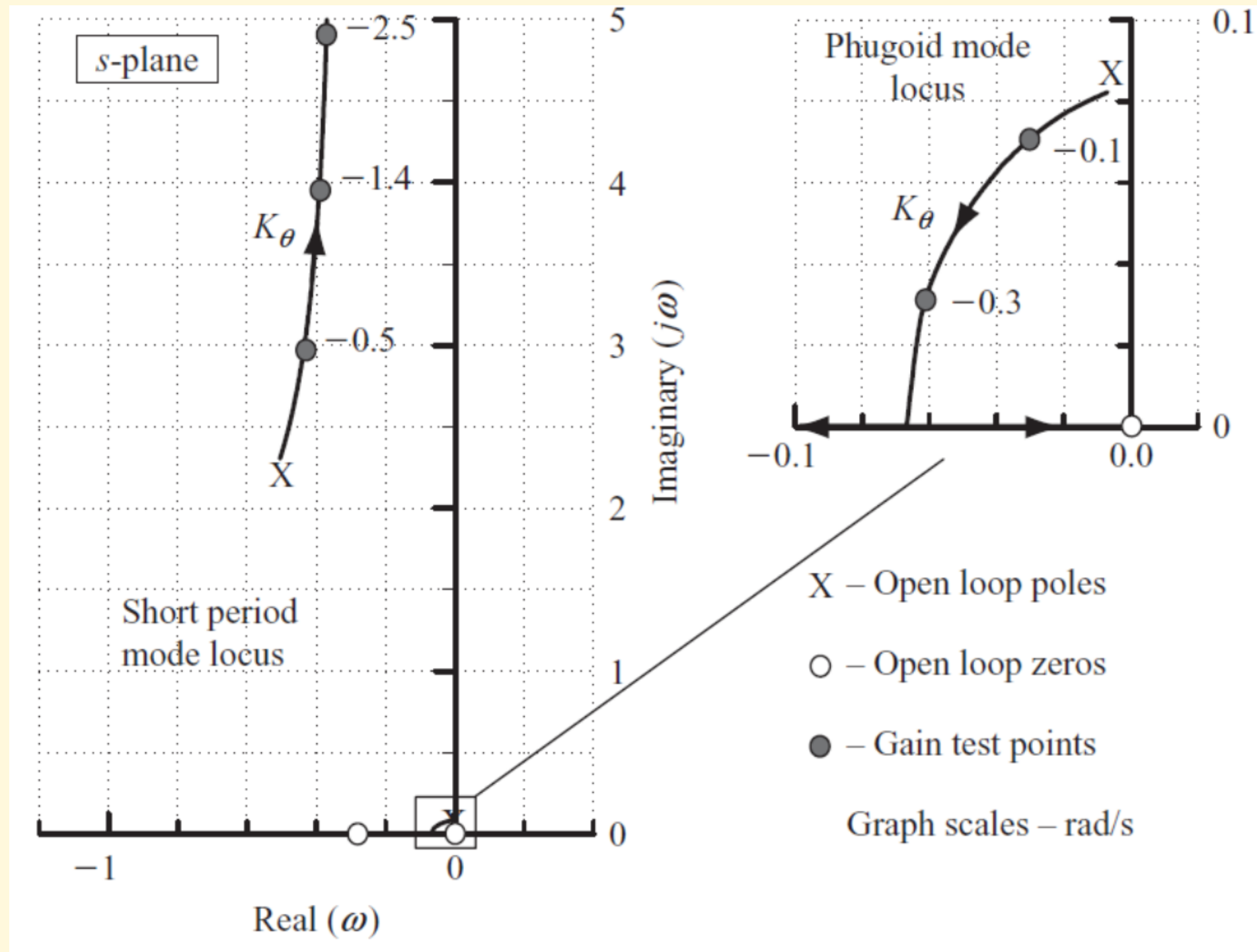
- Phugoid
 - Low frequency
 - Low damping
- Short-period
 - High frequency
 - High damping



Source: Cook, M. V. (2007)

Aircraft application

Longitudinal movement



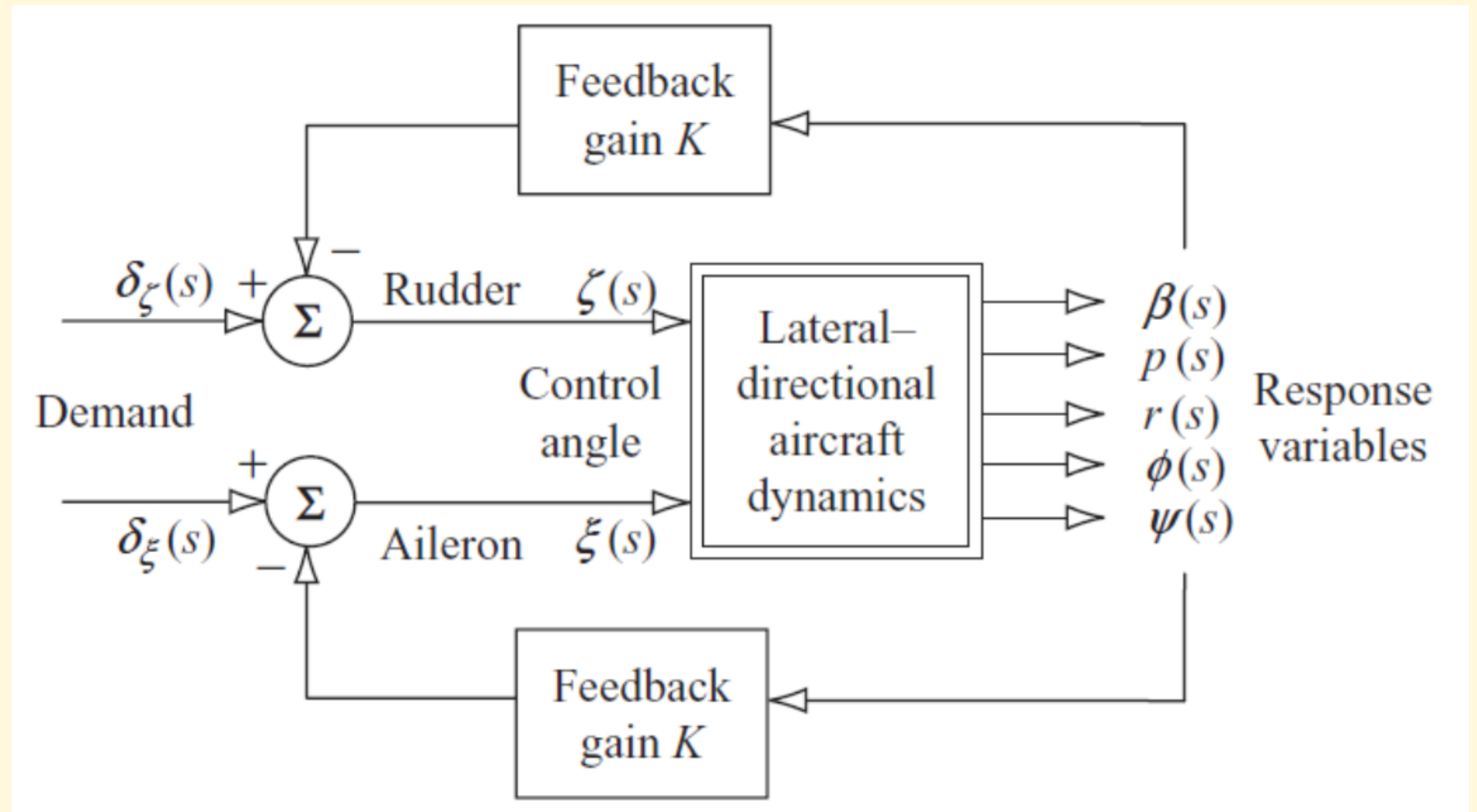
TF of the pitch due to elevator deflection

$$\frac{\theta(s)}{\eta(s)} \equiv \frac{-8.096(s - 0.0006)(s + 0.3591)}{(s^2 + 0.014s + 0.0068)(s^2 + 1.009s + 5.56)} \text{ rad/rad}$$

Aircraft application

Latero-directional movement

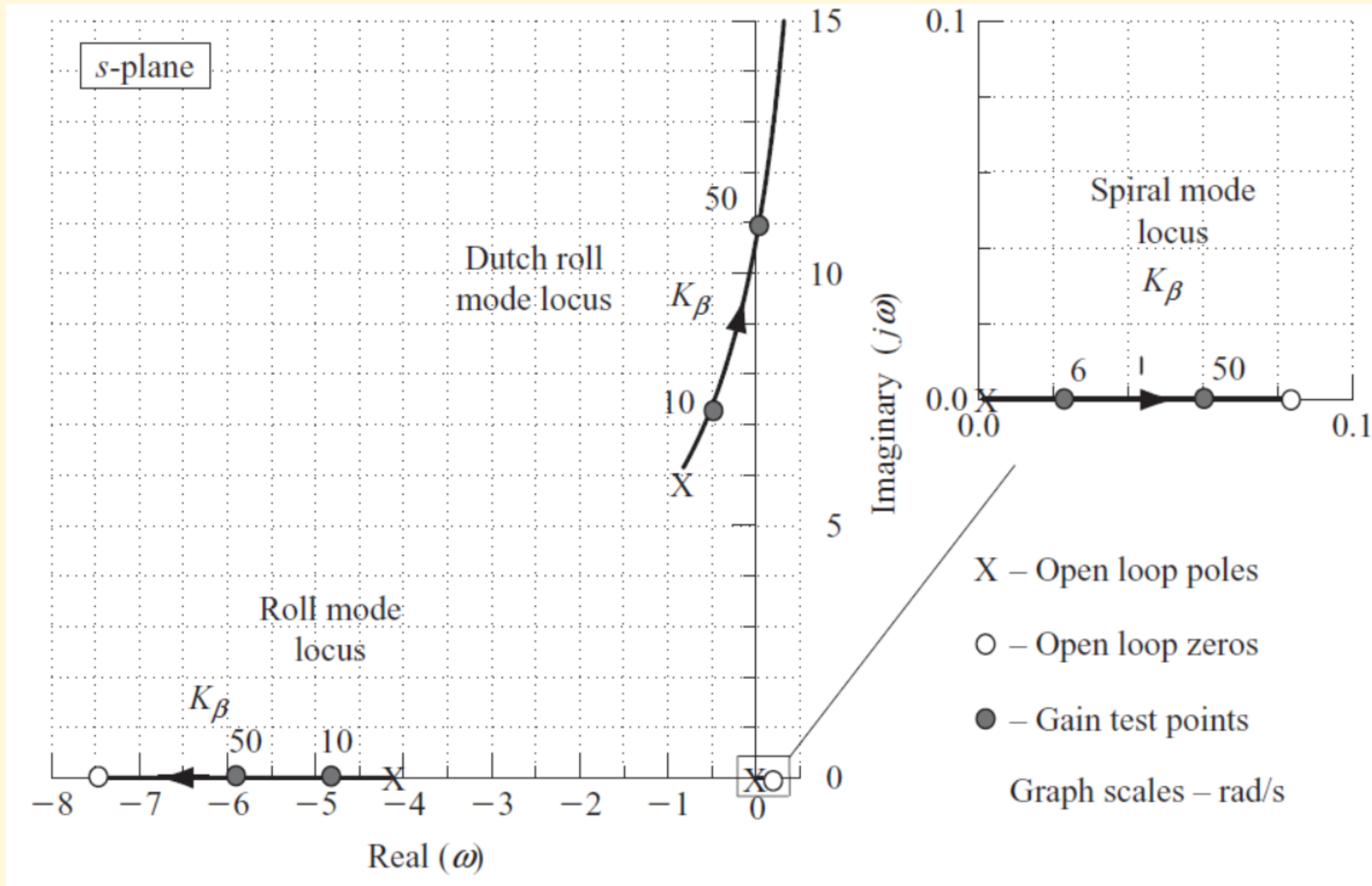
- Dutch roll
 - Oscillatory movement
- Spiral
 - High time constant
- Roll
 - Low time constant



Source: Cook, M. V. (2007)

Aircraft application

Latero-directional movement

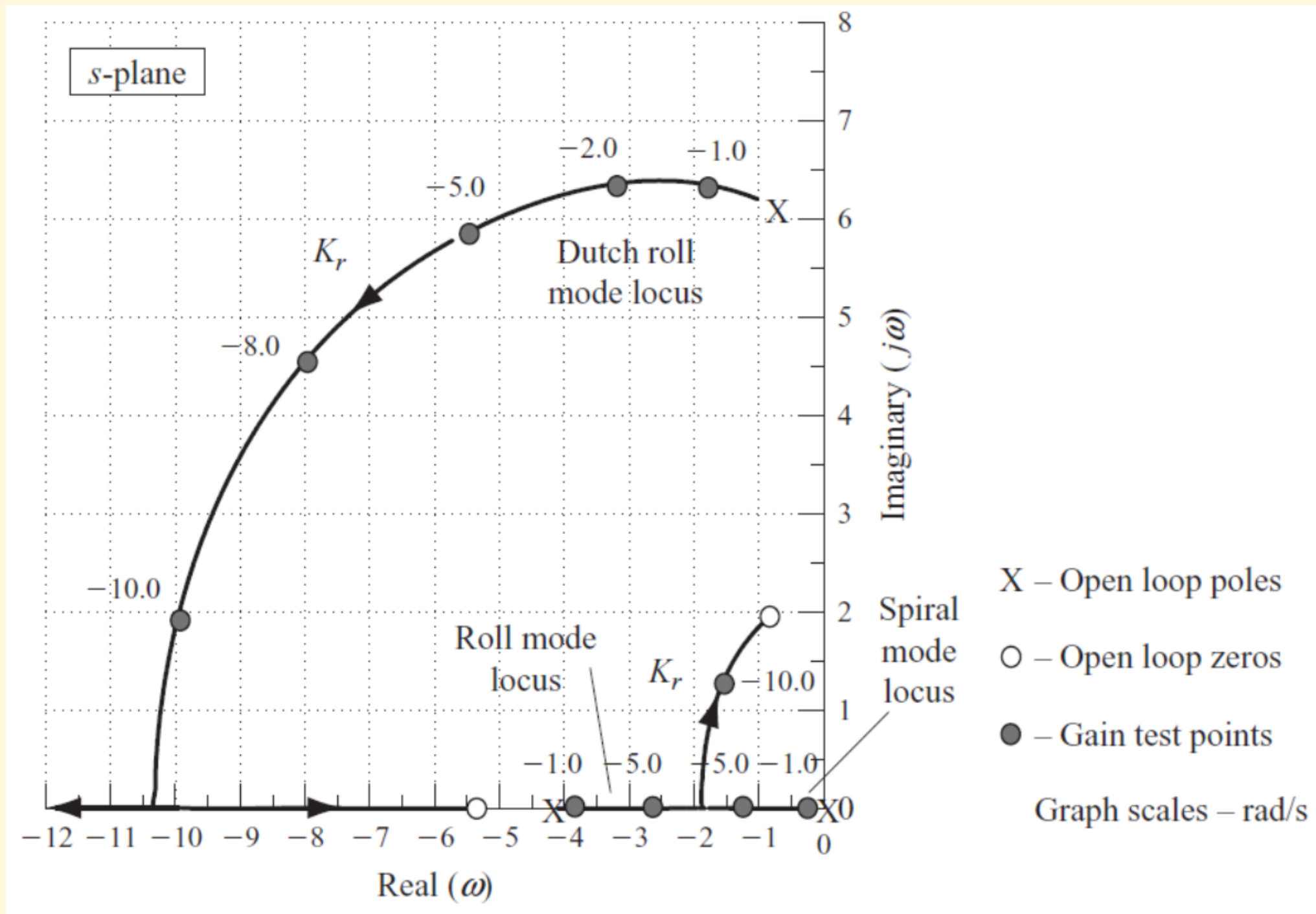


TF of skid due to aileron deflection

$$\frac{\beta(s)}{\xi(s)} \equiv \frac{1.3235(s - 0.0832)(s + 7.43)}{(s - 0.0014)(s + 4.145)(s^2 + 1.649s + 38.44)} \text{ rad/rad}$$

Aircraft application

Latero-directional movement



TF of yaw due to aileron deflection

$$\frac{r(s)}{\xi(s)} \equiv \frac{1.712(s + 5.405)(s^2 + 1.788s + 4.465)}{(s - 0.0014)(s + 4.145)(s^2 + 1.649s + 38.44)} \text{ rad/s/rad}$$

Problem

- Calculate
 - Natural frequencies
 - Damping ratio
- Identify the longitudinal movement modes
- Elaborate the root locus in Matlab
- Identify the longitudinal movement modes in the graph
- Adjust the gain to ensure:

Phugoid damping ratio $\zeta_p \geq 0.04$

Short period damping ratio $\zeta_s \geq 0.1$

Short period undamped natural frequency $0.8 \leq \omega_s \leq 3.0 \text{ rad/s}$

TF of the pitch due to elevator deflection

$$\frac{\theta(s)}{\eta(s)} = \frac{4.66(s + 0.133)(s + 0.269)}{(s^2 + 0.015s + 0.021)(s^2 + 0.911s + 4.884)}$$

Lockheed F-104 Starfighter



Source: Wikipedia

Final remarks

- Aircraft flight dynamics can be represented by transfer functions correlating the inputs on the control surfaces of the aircraft and its attitude, speeds, and angle of attack and slip
- The root locus method is a useful tool to determine the gains of a stability augmentation system using a proportional controller

References

BELO, E.M. **Sistemas de Controle de Aeronaves: Uma Introdução.**

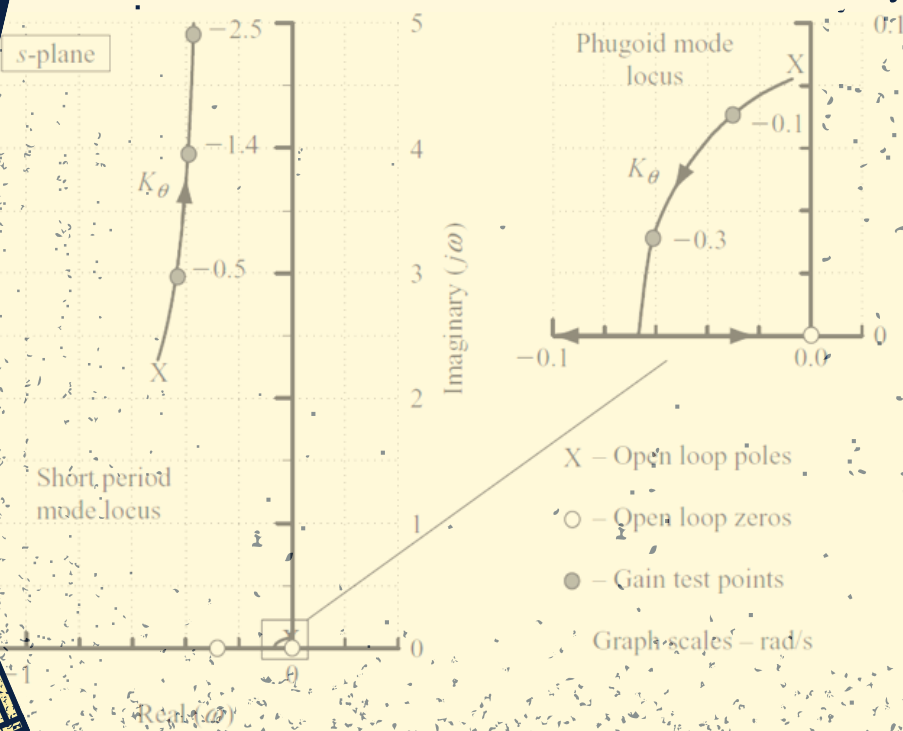
MCCLEAN, D. **Automatic flight control systems.** Old Tappan, NJ, USA: Prentice Hall, 1990.

MOIR, I.; SEABRIDGE, A.; JUKES, M. **Civil Avionics Systems.** 2. ed. Hoboken, NJ, USA: Wiley-Blackwell, 2013.

Cook, M. V. **Flight Dynamics Principles.** 2. ed. Elsevier, 2007



Wikipedia



© Cook, M. V. (2007)

Thank you!

Questions?