

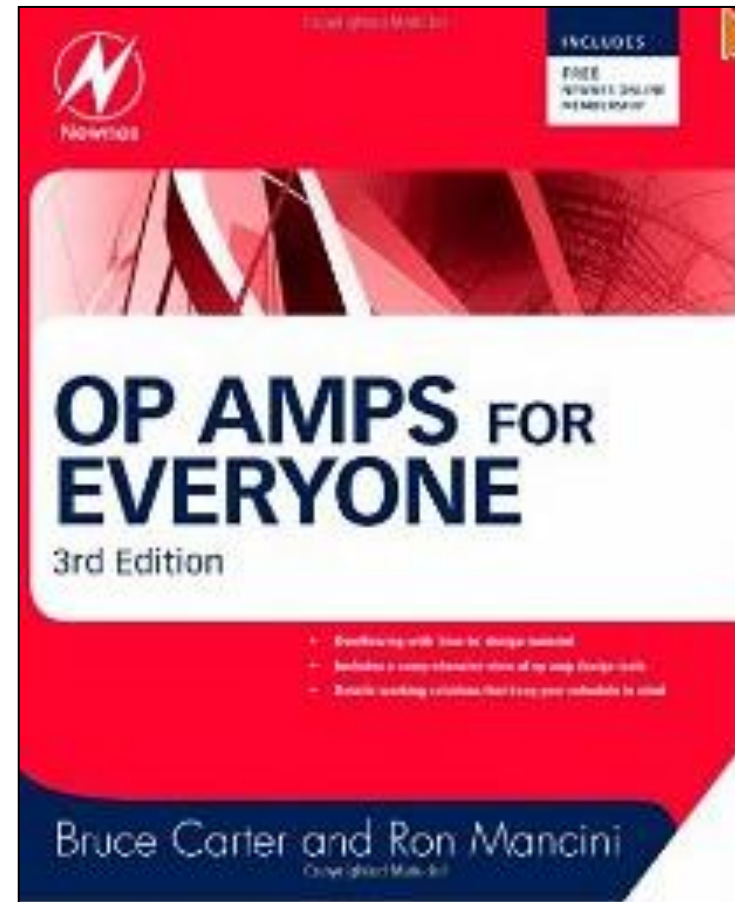
Laboratório 3

Filtros Ativos Rejeita Banda

Referência

[OP AMPS for Everyone](#)

Newnes, 2009



Roteiro Experimental

Laboratório 3 – Filtro Ativo Rejeita Banda

1. Implementação em Protoboard

Implemente em um protoboard um filtro rejeita banda de ordem 2 com a topologia Twin T, frequência central de 10KHz e $A_0=1,5$, conforme figura 1.1. Utilize $C=10nF$.

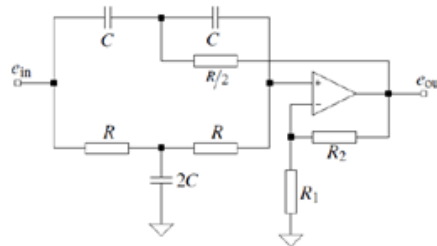


Fig. 1.1 – Circuito rejeita banda com topologia Twin T

mid-frequency: $f_m = \frac{1}{2\pi RC}$

passband gain: $A_0 = G$

inner gain: $G = 1 + \frac{R_2}{R_1}$

rejection quality: $Q = \frac{1}{2(2 - G)}$

Determine a frequência intermediária (f_m), o ganho na banda de passagem (A_0) e o fator de qualidade (Q)

2. Simulação no LTSpice

Simule o circuito da fig. 1.1.

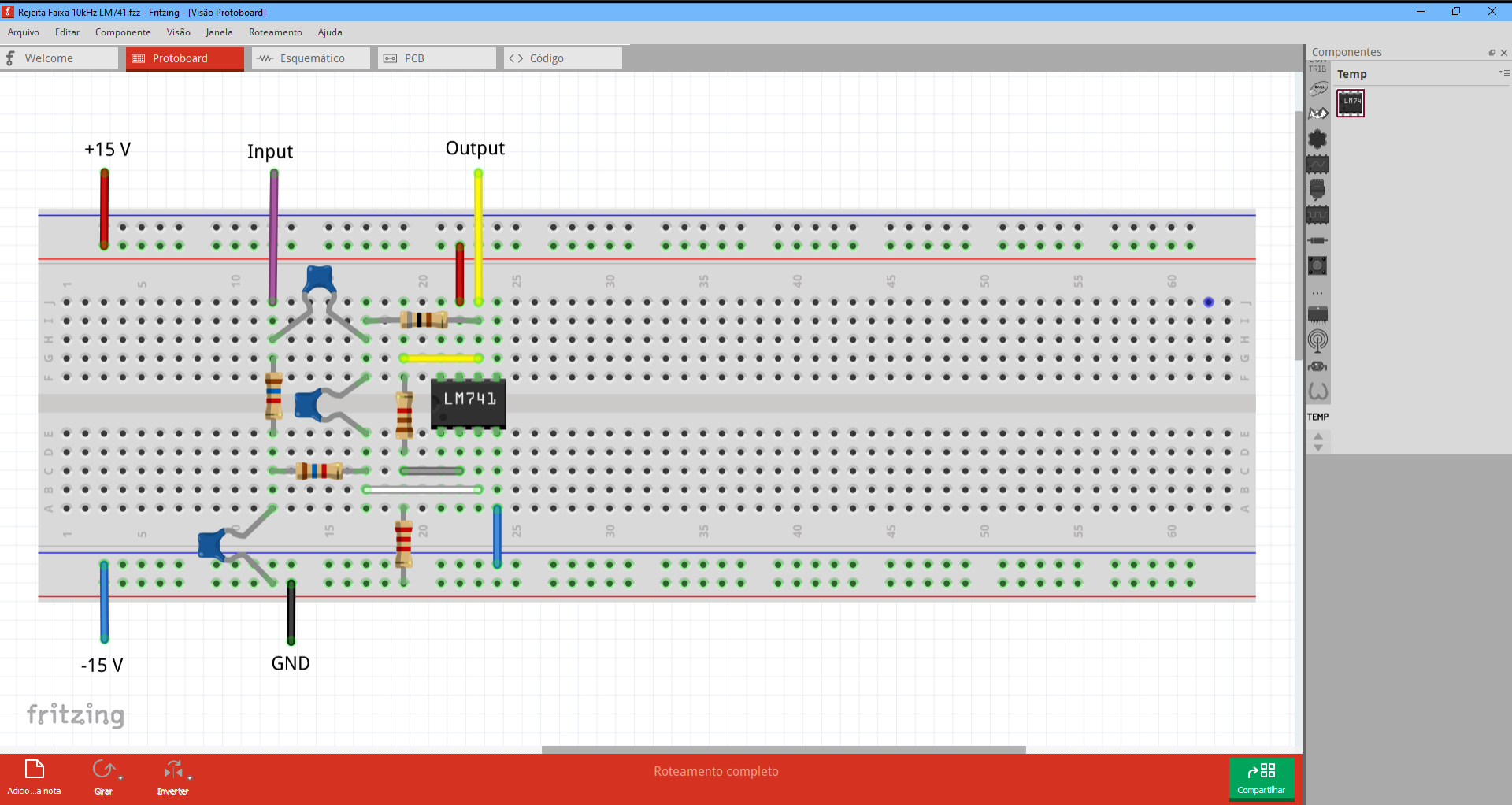
- a) Determine a frequência intermediária (f_m), o ganho na banda de passagem (A_0) e o fator de qualidade (Q) e compare com os valores teóricos.
- b) Verifique a influência de diferentes amplificadores operacionais na resposta do filtro utilizando na simulação o 741, o LT1022 e o TL081.

3. Referências Bibliográficas

Carter B, Mancini R. Op Amps for Everyone, In: Active Filter Design Techniques, Chapter 16, Newnes, 2009.

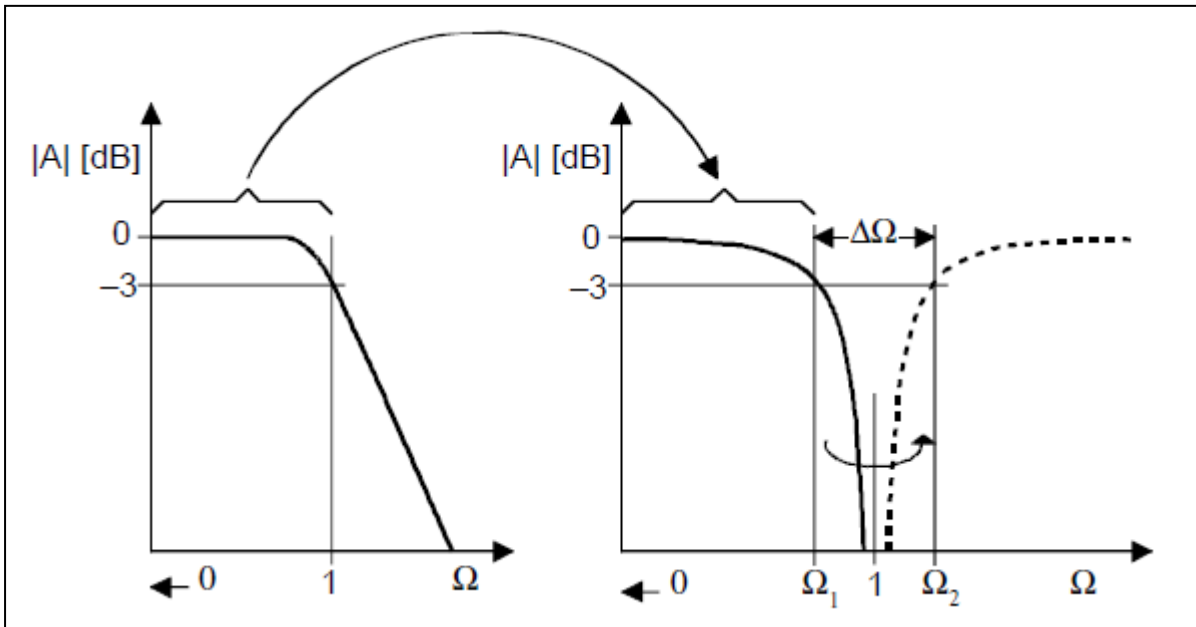
Band Reject Filter - Active Twin T

$f_m = 10\text{KHz}$, $A_o = 1.5$, $C=10\text{nF}$
(edited on Fritzing)



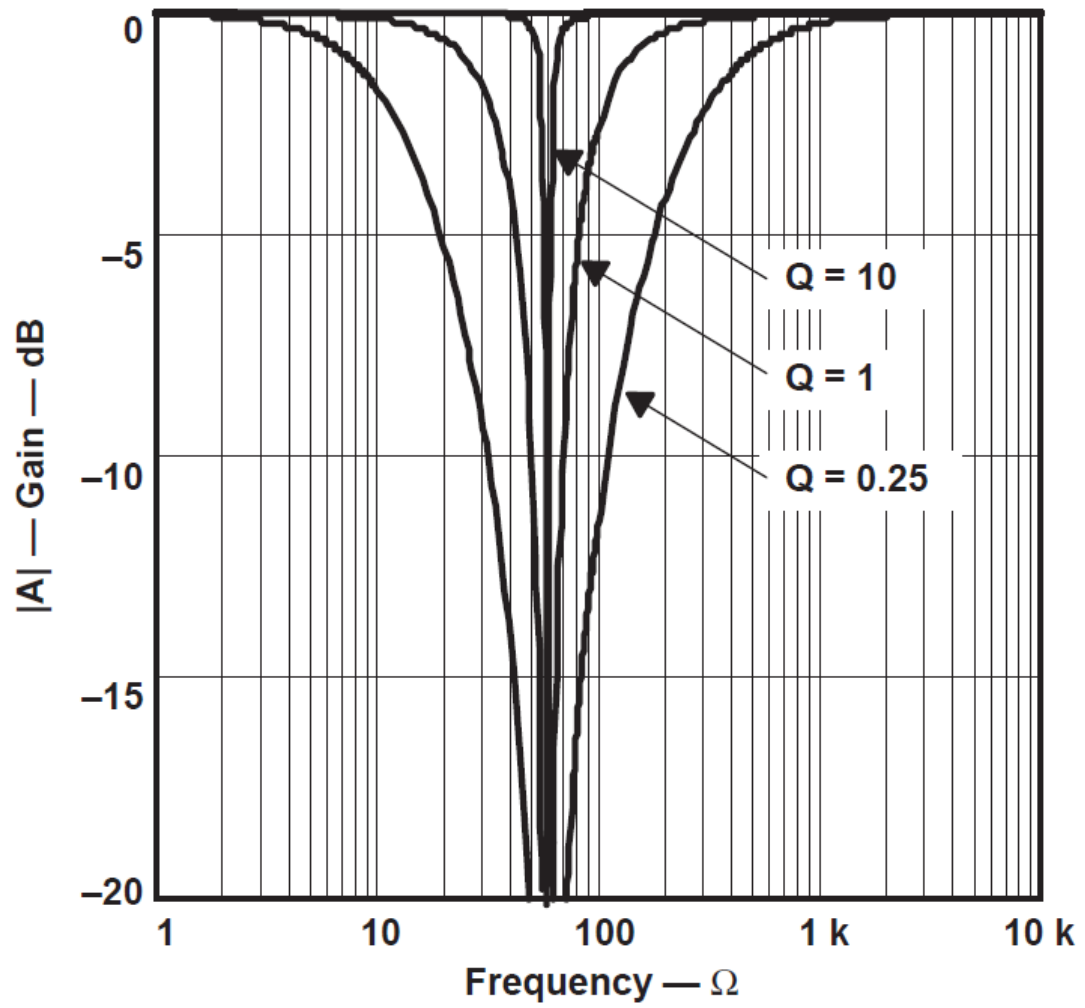
1 A band-rejection filter is used to suppress a certain frequency rather than a range of frequencies.

2 The passband characteristic of the low-pass filter is transformed into the lower passband of the band-rejection filter. The lower passband is then mirrored at the mid frequency, f_m ($\Omega=1$), into the upper passband half.



$$\Delta\Omega = \Omega_{\max} - \Omega_{\min}$$

$$Q = \frac{f_m}{B} = \frac{1}{\Delta\Omega}$$



- 3 To generate the transfer function of a **second-order band-rejection filter**, replace the s term of a **first-order low-pass** response with the transformation

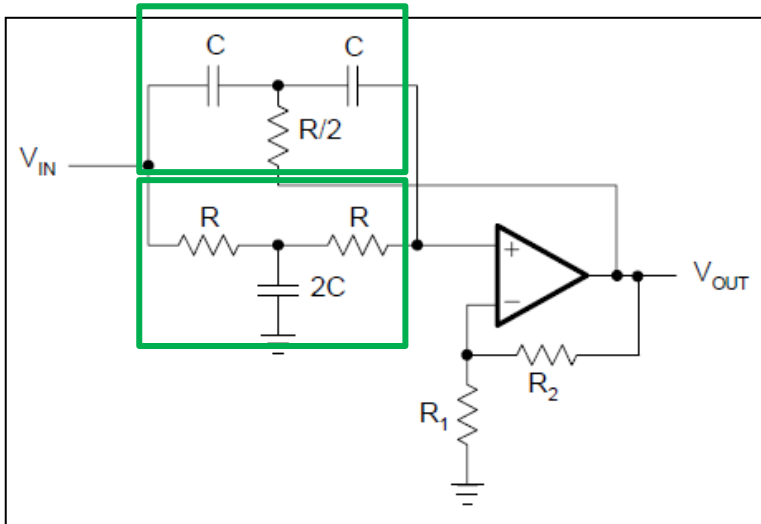
$$\frac{\Delta\Omega}{s + \frac{1}{s}} \quad \rightarrow \quad A(s) = \frac{A_0}{1 + s}$$

$$A(s) = \frac{A_0(1 + s^2)}{1 + \Delta\Omega \cdot s + s^2}$$

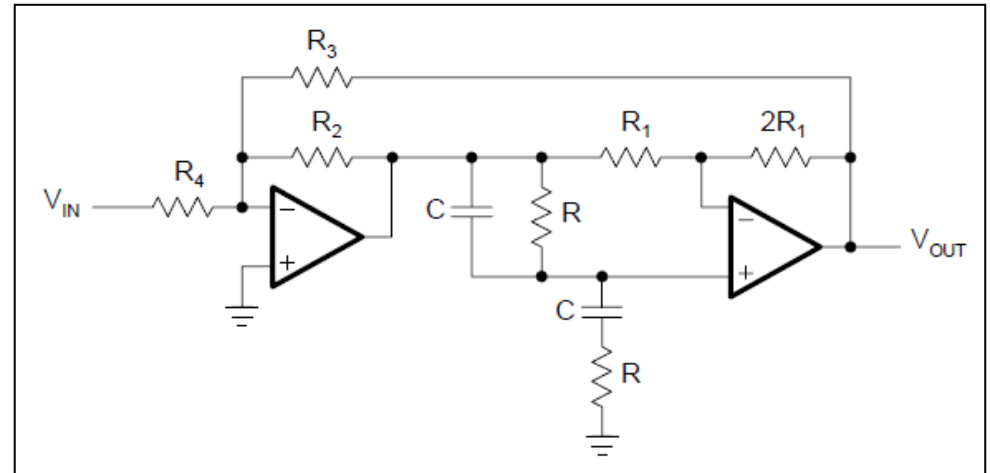
$$A(s) = \frac{A_0(1 + s^2)}{1 + \frac{1}{Q} \cdot s + s^2}$$

A_0 é o ganho na banda de passagem.

- 4 Two of the most popular band-rejection filters are the **active twin-T** and the **active Wien-Robinson** circuit, both of which are second-order filters.



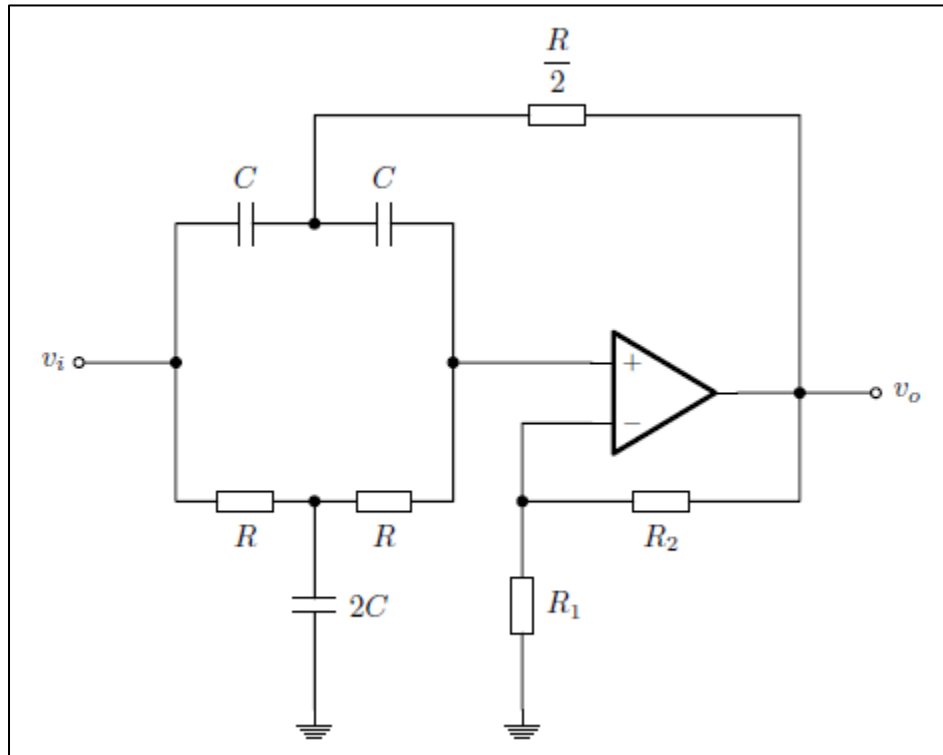
active twin-T



active Wien-Robinson

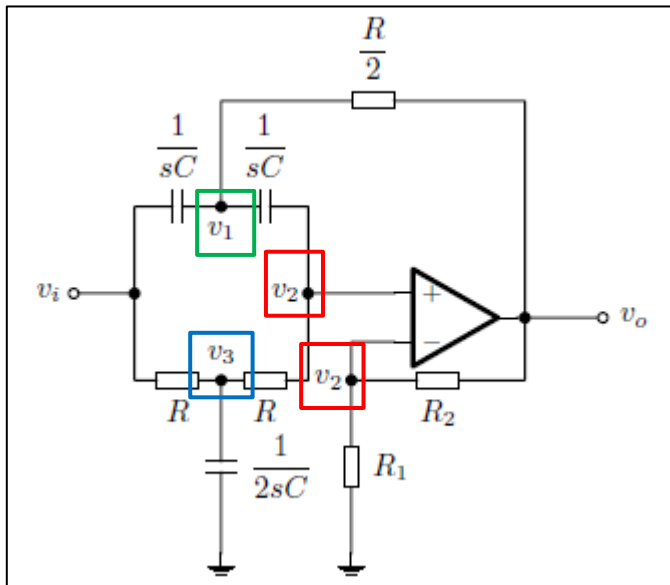
Band Rejection Filters

Função de Transferência



Filtro ativo rejeita banda com topologia *Twin T*

Função de Transferência H(s)



1

$$\text{nó 1:} \quad 0 = \frac{v_1 - v_i}{(sC)^{-1}} + \frac{2(v_1 - v_o)}{R} + \frac{v_1 - v_2}{(sC)^{-1}}$$

$$\text{nó 2(+):} \quad 0 = \frac{v_2 - v_1}{(sC)^{-1}} + \frac{v_2 - v_3}{R}$$

$$\text{nó 2(-):} \quad 0 = \frac{v_2}{R_1} + \frac{v_2 - v_0}{R_2}$$

$$\text{nó 3:} \quad 0 = \frac{v_3 - v_i}{R} + \frac{v_3}{(2sC)^{-1}} + \frac{v_3 - v_2}{R}$$

(1)

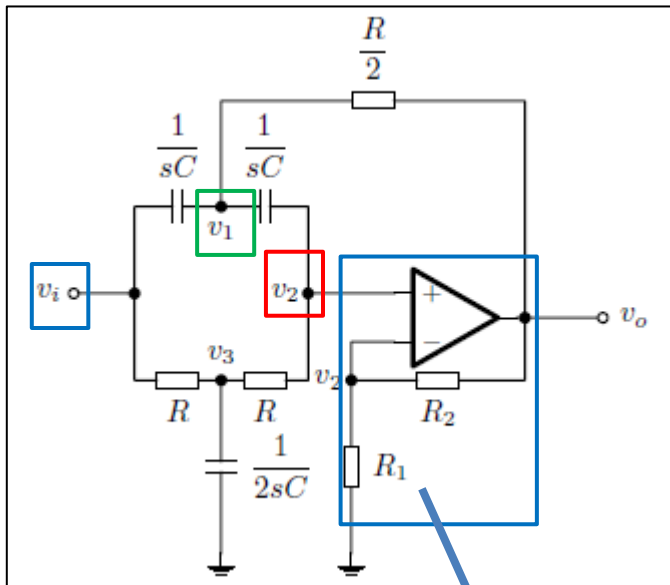
2

resolvendo o sistema (1)

$$H(s) = \frac{s^2(C^2R^2R_1 + C^2R^2R_2) + R_1 + R_2}{s^2(C^2R^2R_1) + s(2CRR_1 - 2CRR_2) + R_1} \quad (2)$$

Filtro de ordem 2 com dois zeros e dois pólos

3



Amplificador Não-Inversor

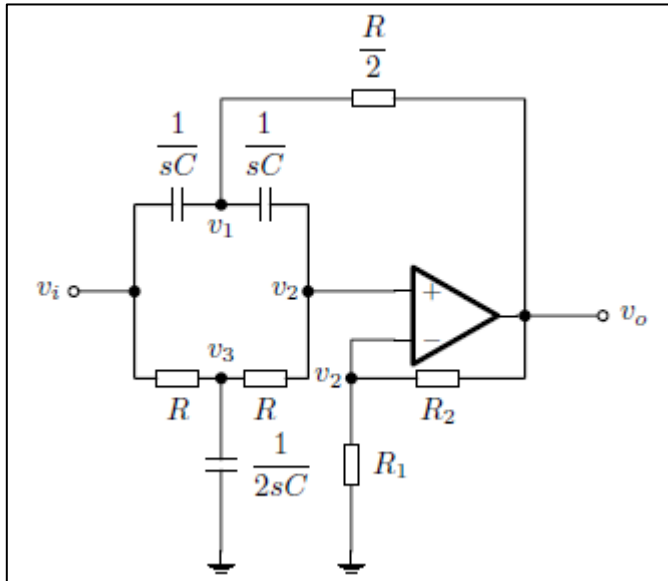
Ganho ($A_v=A_o$)

$$H(s) = \frac{s^2(C^2R^2R_1 + C^2R^2R_2) + R_1 + R_2}{s^2(C^2R^2R_1) + s(2CRR_1 - 2CRR_2) + R_1}$$

Para obtenção do ganho observa-se que o módulo de $H(s)$ quando $\omega \rightarrow 0$ é a igual a quando $\omega \rightarrow \infty$

Nota-se que quando $\omega \rightarrow \infty$ os capacitores se tornam curto circuitos e nessa situação $v_1=v_2=v_i$, tornando o circuito um amplificador não inversor, cujo ganho é:

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_2}{R_1}$$

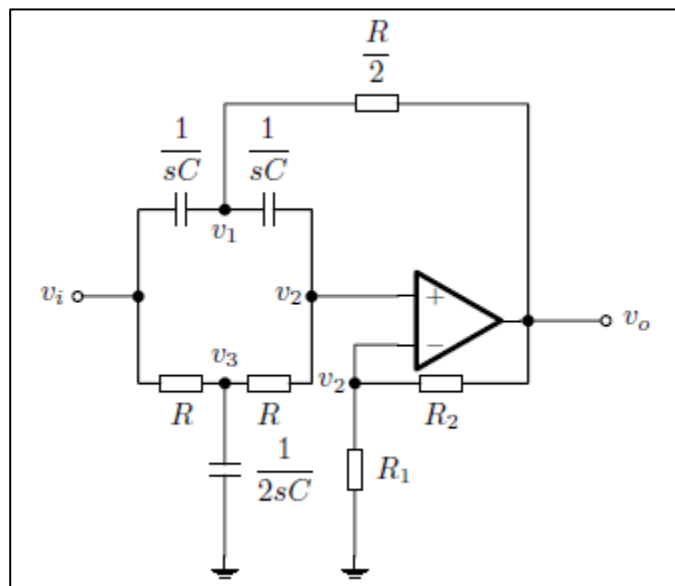


Como $s=j\omega$, então:

$$|H(j\omega)| = \left| \frac{\omega^2(-C^2R^2R_1 - C^2R^2R_2) + R_1 + R_2}{\omega^2(-C^2R^2R_1) + j\omega(2CRR_1 - 2CRR_2) + R_1} \right|$$

A frequência central é o valor ω_c que minimiza $|H(s)|$. Os valores de C , R , R_1 e R_2 são constantes positivas e o numerador de $|H(s)|$ é uma parábola com concavidade para baixo que cruza o eixo das ordenadas acima do zero, ou seja, essa parábola passa por zero. Sendo assim, para encontrar a frequência central basta encontrar ω_c tal que $|H(s)| = 0$, então:

$$\left| \omega^2(-C^2R^2R_1 - C^2R^2R_2) + R_1 + R_2 \right| = 0$$



$$|w^2(-C^2R^2R_1 - C^2R^2R_2) + R_1 + R_2| = 0$$

$$\Rightarrow \omega_c = \pm \frac{\sqrt{4(-C^2R^2R_1 - C^2R^2R_2)(R_1 + R_2)}}{2(-C^2R^2R_1 - C^2R^2R_2)}$$

$$\Rightarrow \omega_c = \pm \frac{1}{CR}$$

$$\xRightarrow{\omega_c > 0}$$



$$\omega_c = \frac{1}{RC}$$

Fator de Qualidade (Q)

5 Para filtros rejeita faixa o fator de qualidade é definido como

$$Q \triangleq \frac{\omega_c}{\omega_2 - \omega_1}$$

onde:

- ω_c é a frequência central, dada por (5)
- ω_2 é a frequência -3 dB superior, ou seja, $|H(j\omega_2)| = \frac{A_v}{\sqrt{2}}$ com $\omega_c < \omega_2$
- ω_1 é a frequência -3 dB inferior, ou seja, $|H(j\omega_1)| = \frac{A_v}{\sqrt{2}}$ com $\omega_c > \omega_1$

6 Substituindo A_v e ω_c em $|H(s)|$ resulta:

$$|H(j\omega)| = \left| \frac{\omega^2(-C^2R^2R_1 - C^2R^2R_2) + R_1 + R_2}{\omega^2(-C^2R^2R_1) + j\omega(2CRR_1 - 2CRR_2) + R_1} \right|$$

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_2}{R_1}$$

$$Q \triangleq \frac{\omega_c}{\omega_2 - \omega_1}$$

$$|H(j\omega)| = \left| \frac{A_v(\omega_c^2 - \omega^2)}{-\omega^2 + j\omega(2 - A_v)\omega_c + \omega_c^2} \right|$$

$$|H(j\omega)| = \left| \frac{A_v(\omega_c^2 - \omega^2)}{-\omega^2 + \omega 2j\omega_c(2 - A_v) + \omega_c^2} \right|$$

- 7 Para obter ω_2 (**frequência superior**) sabemos que $\omega_c < \omega_2$ e, portanto, o numerador será negativo :

$$\rightarrow \frac{-A_v(\omega_c - \omega_2^2)}{\sqrt{(-\omega_2^2 + \omega_c^2)^2 + (\omega_2 2\omega_c(2 - A_v))^2}} = \frac{A_v}{\sqrt{2}}$$

$$\omega_2 = \left(2 - A_v + \sqrt{A_v^2 - 4A_v + 5} \right) \omega_c$$

- 8 Para obter ω_1 (**frequência inferior**) sabemos que $\omega_c > \omega_1$ e, portanto, o numerador será positivo :

$$\rightarrow \frac{A_v(\omega_c - \omega_1^2)}{\sqrt{(-\omega_1^2 + \omega_c^2)^2 + (\omega_1 2\omega_c(2 - A_v))^2}} = \frac{A_v}{\sqrt{2}}$$

$$\omega_1 = \left(-2 + A_v + \sqrt{A_v^2 - 4A_v + 5} \right) \omega_c$$

9

$$Q \triangleq \frac{\omega_c}{\omega_2 - \omega_1}$$

$$\rightarrow \omega_2 - \omega_1 = \left(2 - A_v + \sqrt{A_v^2 - 4A_v + 5}\right) \omega_c - \left(-2 + A_v + \sqrt{A_v^2 - 4A_v + 5}\right) \omega_c$$

$$\omega_2 - \omega_1 = 2\omega_c(2 - A_v)$$

$$Q = \frac{\omega_c}{2\omega_c(2 - A_v)} = \frac{1}{2(2 - A_v)}$$

10 Das equações abaixo obtem-se a equação de H(s):

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_2}{R_1}$$

$$\omega_c = \frac{1}{RC}$$

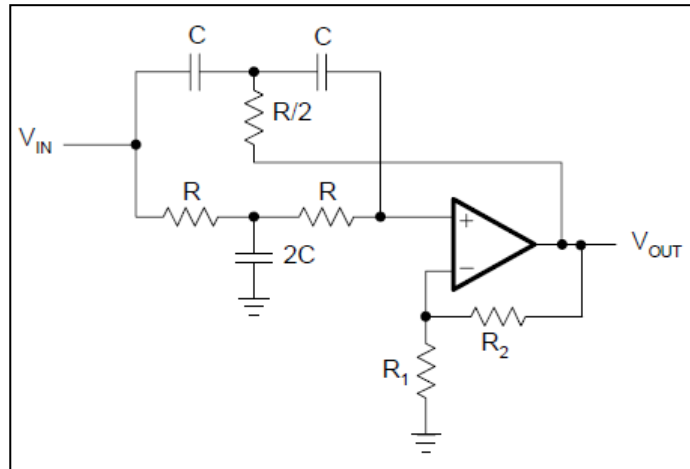
$$Q = \frac{\omega_c}{2\omega_c(2 - A_v)} = \frac{1}{2(2 - A_v)}$$

$$H(s) = \frac{s^2(C^2R^2R_1 + C^2R^2R_2) + R_1 + R_2}{s^2(C^2R^2R_1) + s(2CRR_1 - 2CRR_2) + R_1} \quad (2)$$

$$H(s) = \frac{A_v(s^2 + \omega_c^2)}{s^2 + s\frac{\omega_c}{Q} + \omega_c^2}$$

Designing Band Rejection Filters

Active Twin-T Filter



$$H_a(S) = \frac{|G(1 + S^2)|}{1 + 2(2 - G)S + S^2}$$



mid-frequency:

$$f_m = \frac{1}{2\pi RC}$$

inner gain:

$$G = 1 + \frac{R_2}{R_1}$$

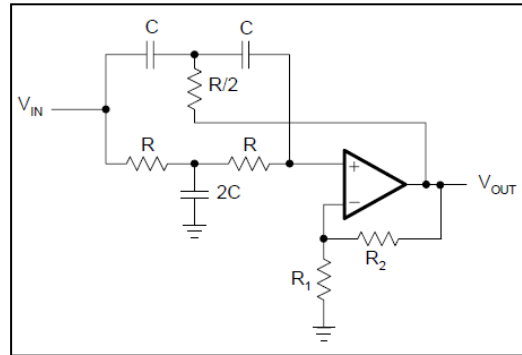
passband gain:

$$A_0 = G$$

rejection quality:

$$Q = \frac{1}{2(2 - G)}$$

Active Twin-T Filter



1 Specify f_m , $A_0 = G$ and C

2 Calculate R



$$R = \frac{1}{2\pi f_m C}$$

3 Calculate R_2 / R_1



$$R_2 = (A_0 - 1)R_1$$

4 Pick R_1 and calculate R_2

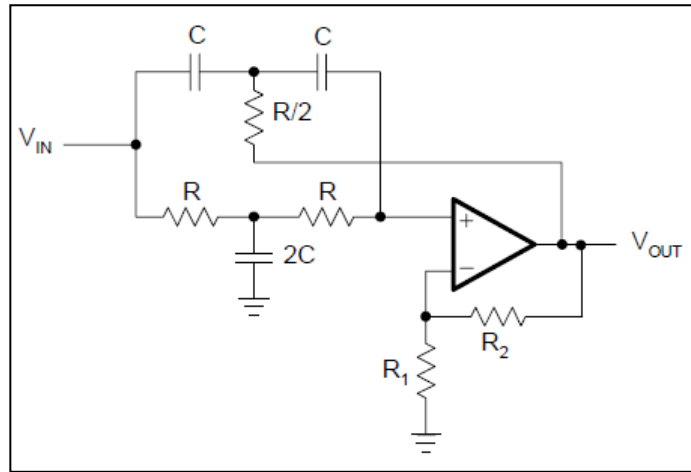
5 Cálculo de Q



$$Q = \frac{1}{2(2 - G)}$$

Example:

Design a rejection pass filter using the Twin-T topology with $f_m = 10\text{KHz}$, $A_o = 1,5$.
Pick $C=10\text{nF}$.

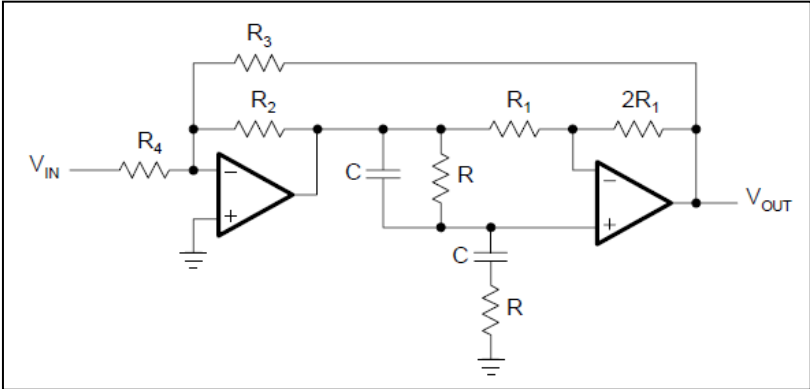


1 Calculate R $\rightarrow R = \frac{1}{2\pi f_m C} = 1,6\text{K}\Omega$

2 Calculate R_2 / R_1 $\rightarrow R_2 = (A_o - 1)R_1 \rightarrow R_2 / R_1 = 0,5$

3 Pick R_1 and calculate R_2 $\rightarrow R_1 = 2,2\text{K}\Omega$ and $R_2 = 1,1\text{K}\Omega$

Wien-Robinson Filter



$$H_b(S) = \frac{-\frac{\beta}{1+\alpha}(1+S^2)}{1 + \frac{3}{1+\alpha}S + S^2}$$



mid-frequency: $f_m = \frac{1}{2\pi RC}$

rejection quality: $Q = \frac{1 + \alpha}{3}$

$$\alpha = 3Q - 1$$

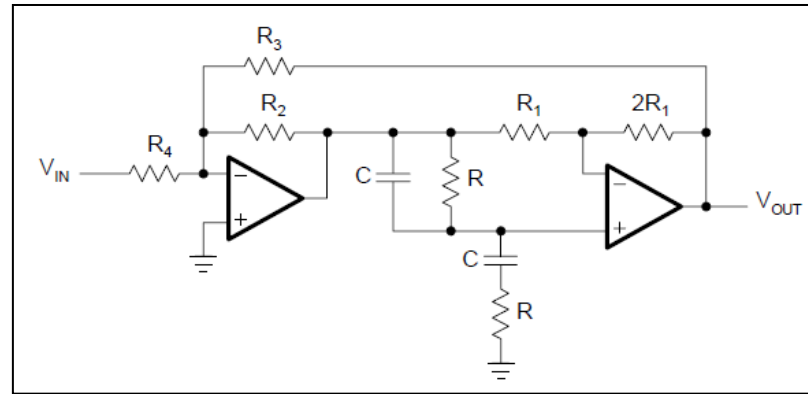
$$\alpha = \frac{R_2}{R_3}$$

$$\beta = -A_0 \cdot 3Q$$

$$\beta = \frac{R_2}{R_4}$$

passband gain: $A_0 = -\frac{\beta}{1 + \alpha}$

Wien-Robinson Filter



1 Define f_m and C , Q and A_0

2 Calculate R



$$R = \frac{1}{2\pi f_m C}$$

3 Determine α



$$\alpha = 3Q - 1$$

4 Determine β



$$\beta = -A_0 \cdot 3Q$$

5 Define R_2 and calculate R_3 and R_4



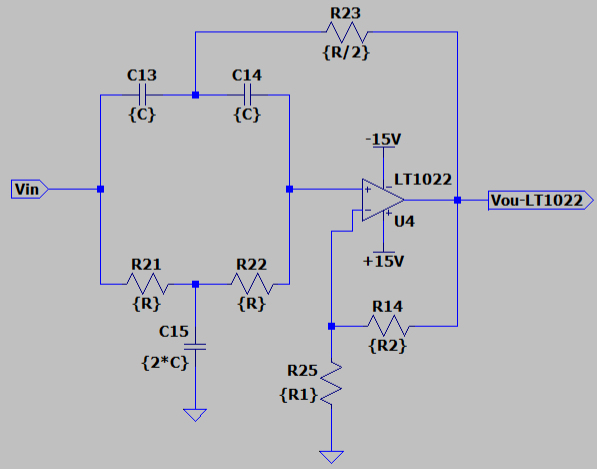
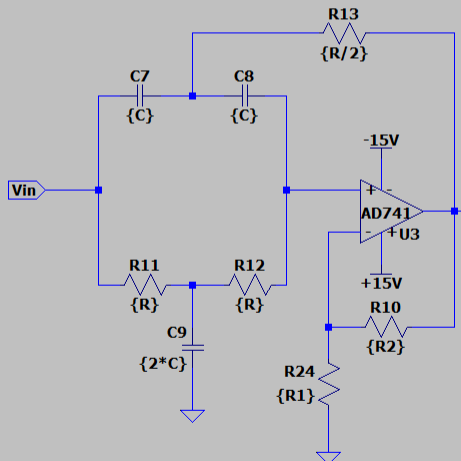
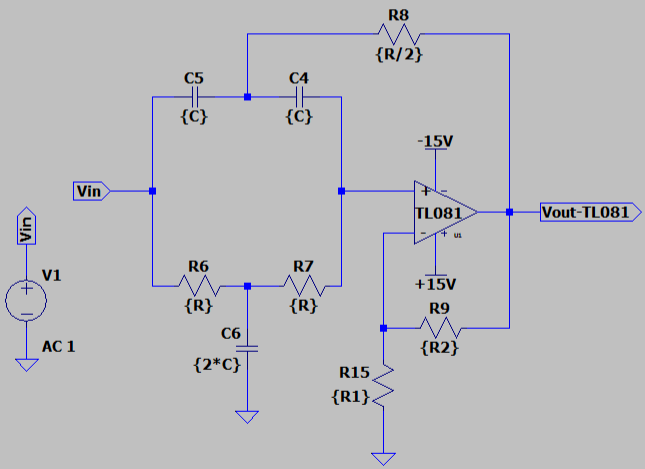
$$R_3 = \frac{R_2}{\alpha}$$

$$R_4 = \frac{R_2}{\beta}$$

Simulação LTSPice

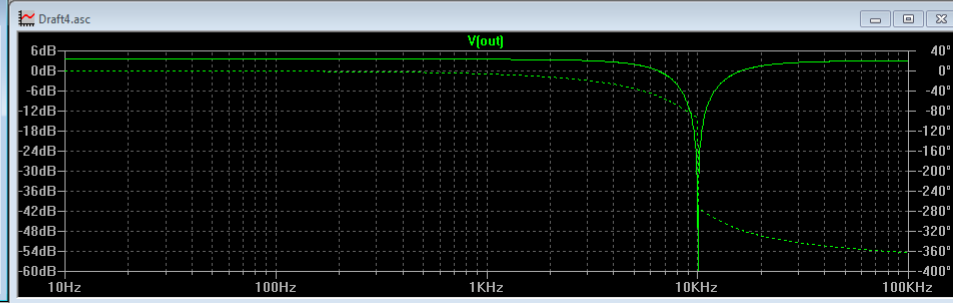
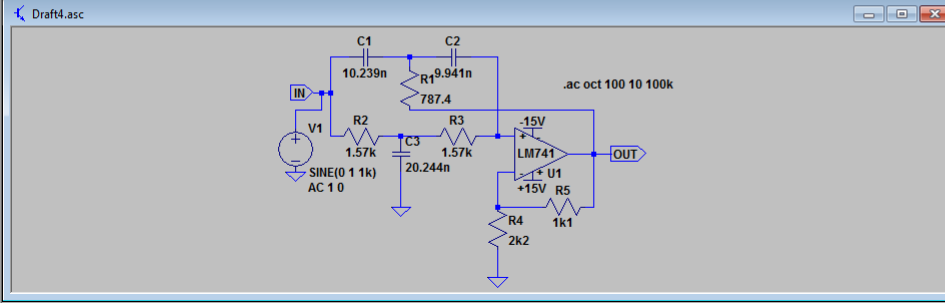
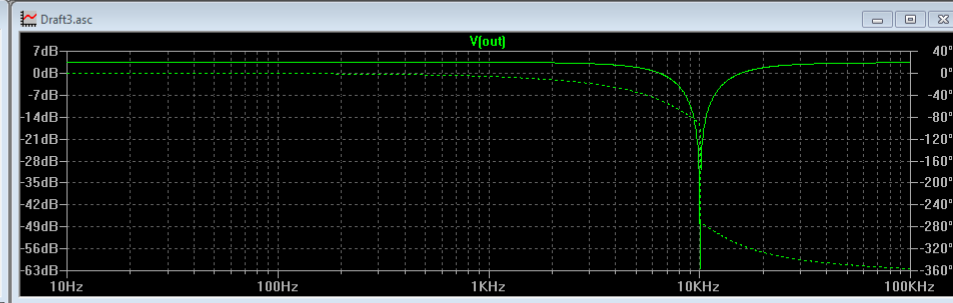
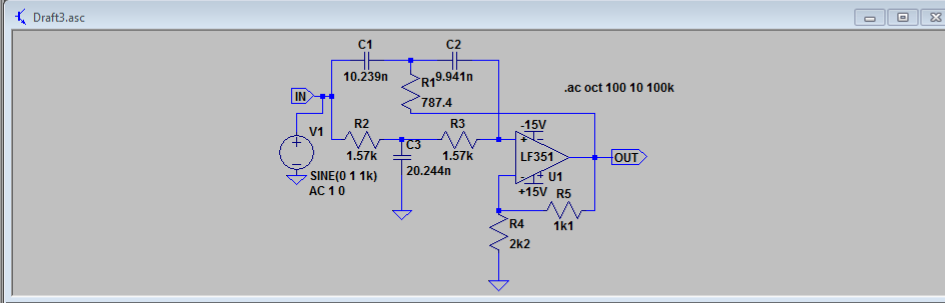
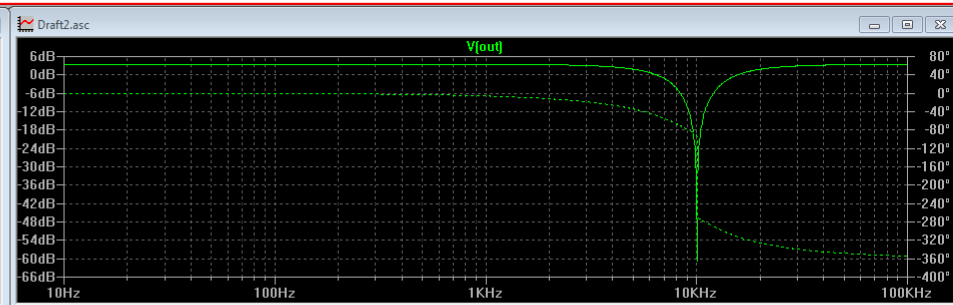
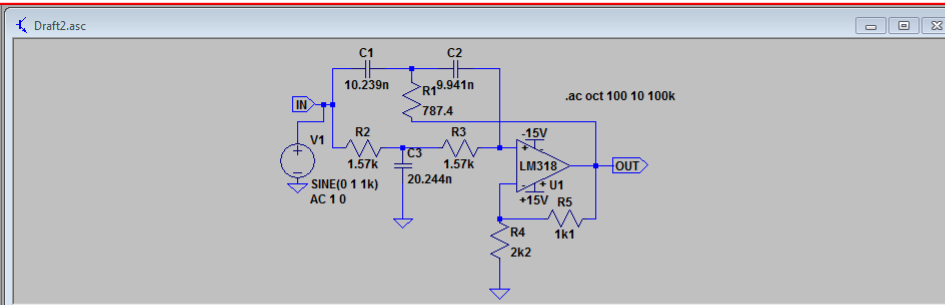
Resposta em Frequência

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.param R1={2k}  
.param C={10n}  
.param R={1591.55}
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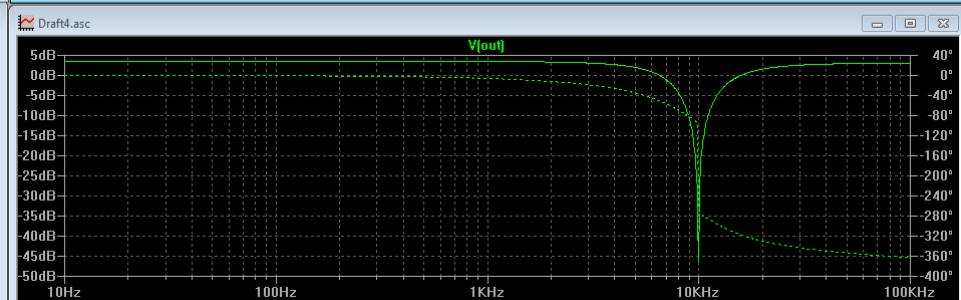
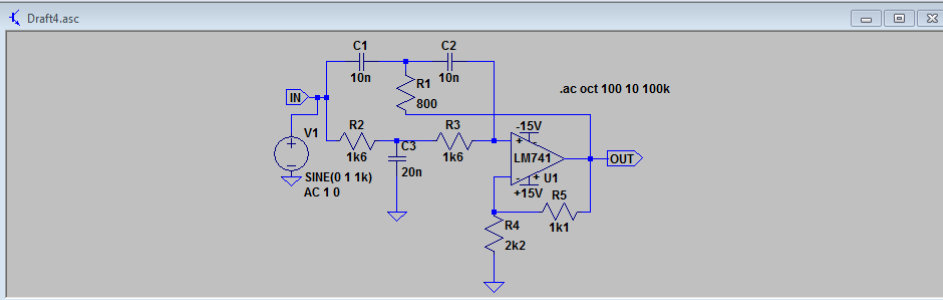
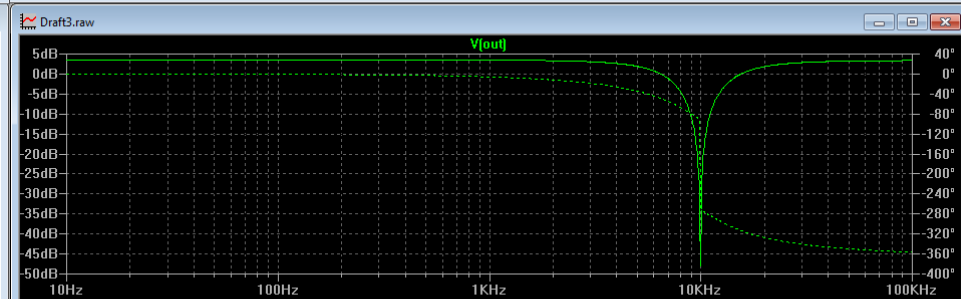
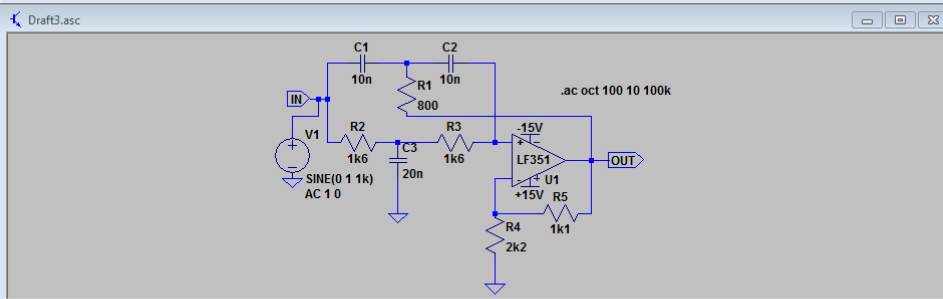
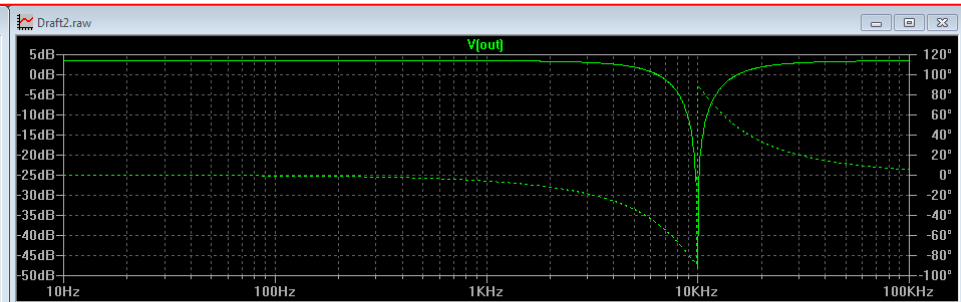
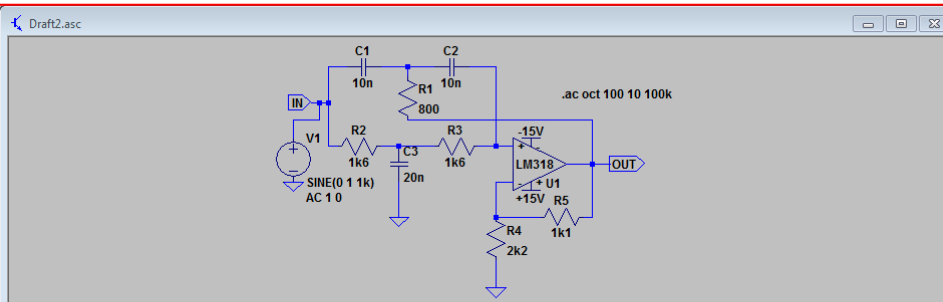


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.ac dec 1001 1 10Meg
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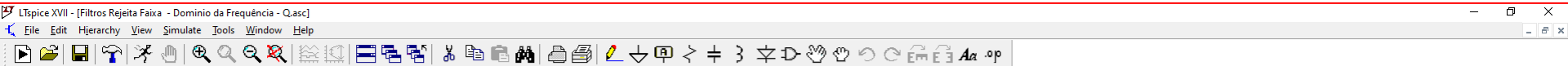
Filtro Rejeita Banda (componentes teóricos)



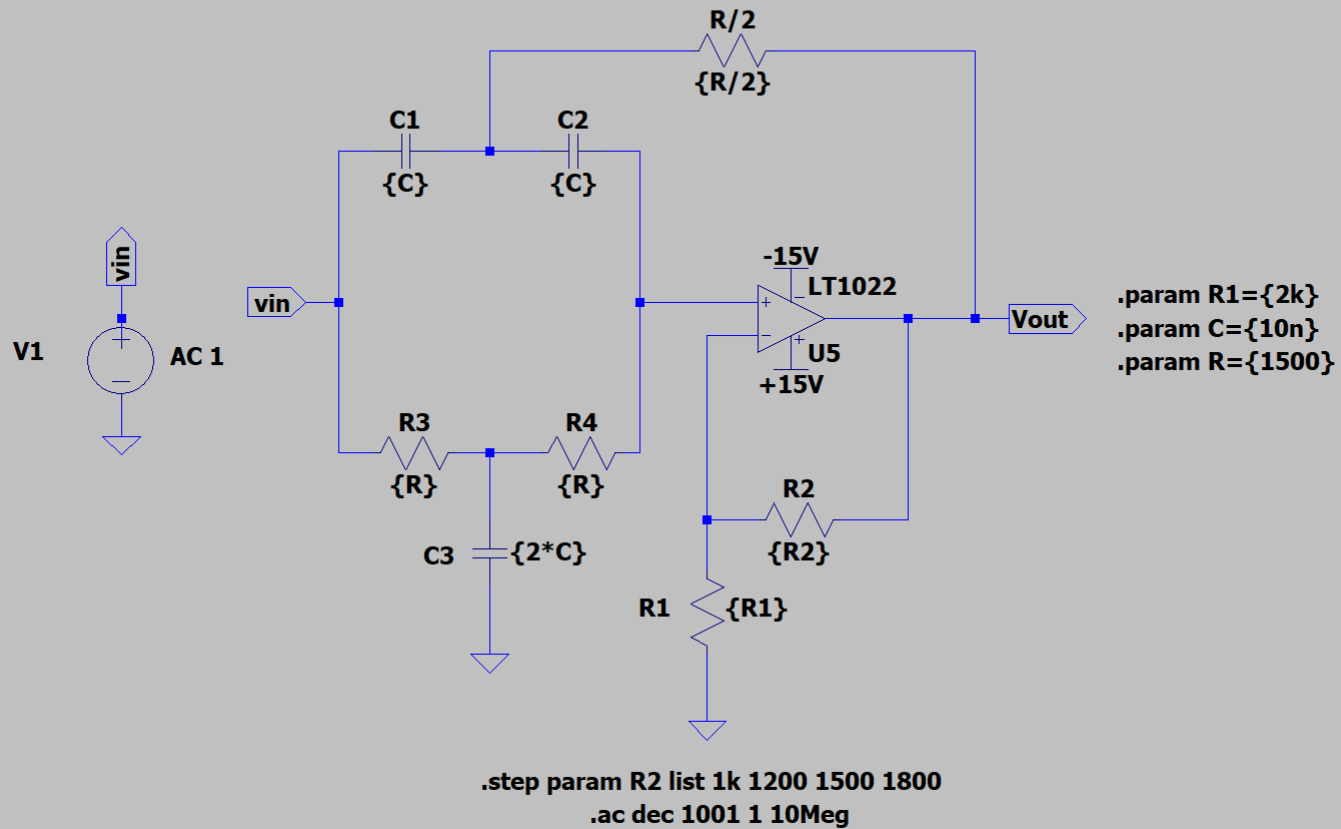
Filtro Rejeita Banda (componentes nominais)



Filtro Rejeita Banda (Variação de Q)



Variando o fator de qualidade Q



Varição do Fator de qualidade do filtro rejeita banda

