Laboratório 3
Filtros Ativos Passa-Banda

Roteiro Experimental

SEL393 – Laboratório de Instrumentação Eletrônica I
Escola de Engenharia de São Carlos - USP
Departamento de Engenharia Elétrica e de Computação
Laboratório 3 - Filtro Ativo Passa Banda

1. Implementação em Protoboard

- Implemente um filtro passa-banda de ordem 4 de Butterworh com topologia de realimentação múltipla (figura 1.1), frequência central de 10KHz, Q=10 e A_m=1. Utilize C=10nF.

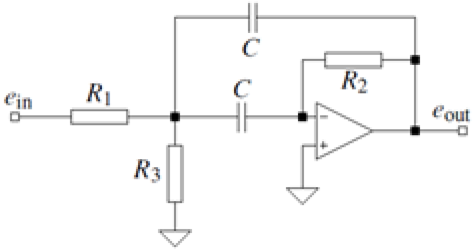


Fig. 1.1 – Topologia de realimentação múltipla de cada filtro de ordem 2 de um filtro passa-banda de ordem 4

a) Determine a frequência intermediária f_{mi}, o fator de qualidade Q_i e o ganho na frequência intermediária A_{mi} de cada filtro de ordem 2.

b) Determine a frequência intermediária f_m, o fator de qualidade **Q** e o ganho na frequência intermediária A_m do filtro de ordem 4.

2. Simulação no LTSpice

Simule o circuito da Fig. 1.1.

a) Determine a frequência intermediária $f_{mi,}$ o fator de qualidade Q_i e o ganho na frequência intermediária A_{mi} de cada filtro de ordem 2 e e compare com os valores teóricos, conforme equações abaixo.

Parâmetros do Filtro Passa Banda:

mid-frequency: $f_m = \frac{1}{2\pi C}\sqrt{\frac{R_1 + R_3}{R_1 R_2 R_3}}$ filter quality: $Q = \pi f_m R_2 C$

gain at f_m: $-A_m = \frac{R_2}{2R_1}$ bandwidth: $B = \frac{1}{\pi R_2 C}$

b) Determine a frequência intermediária f_m, o fator de qualidade Q e o ganho na frequência intermediária A_m do filtro de ordem 4.

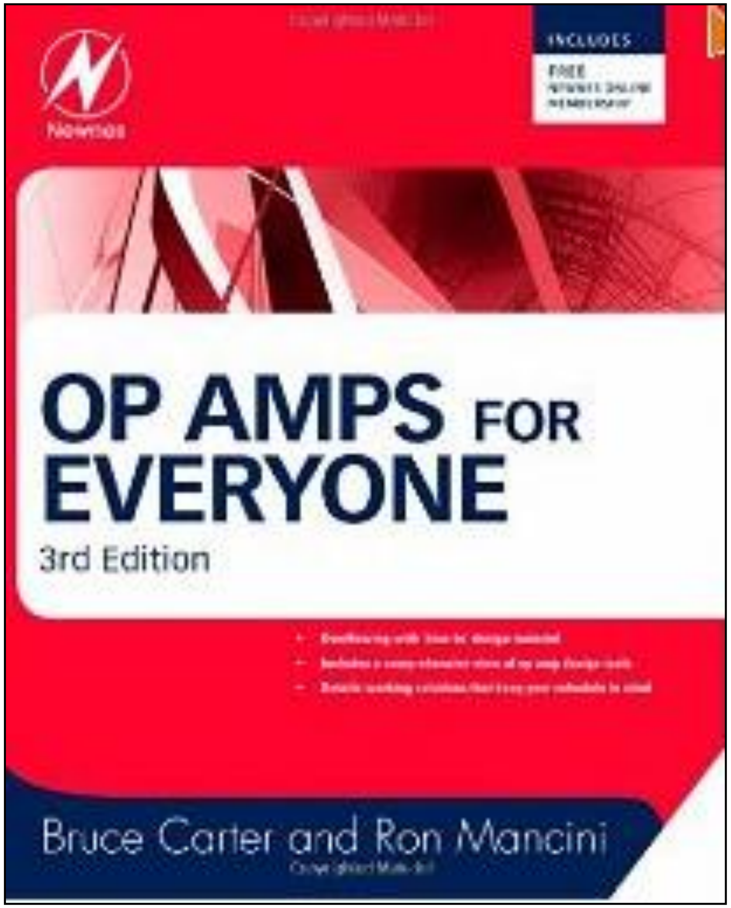
c) Verifique a influência de diferentes amplificadores operacionais na resposta do fittro utilizando na simulação o 741, o TL1022 e o LT081.

3. Referências Bibliográficas

Carter B, Mancini R. Op Amps for Everyone, In: Active Filter Design Techniques, Chapter 16, Newnes, 2009.

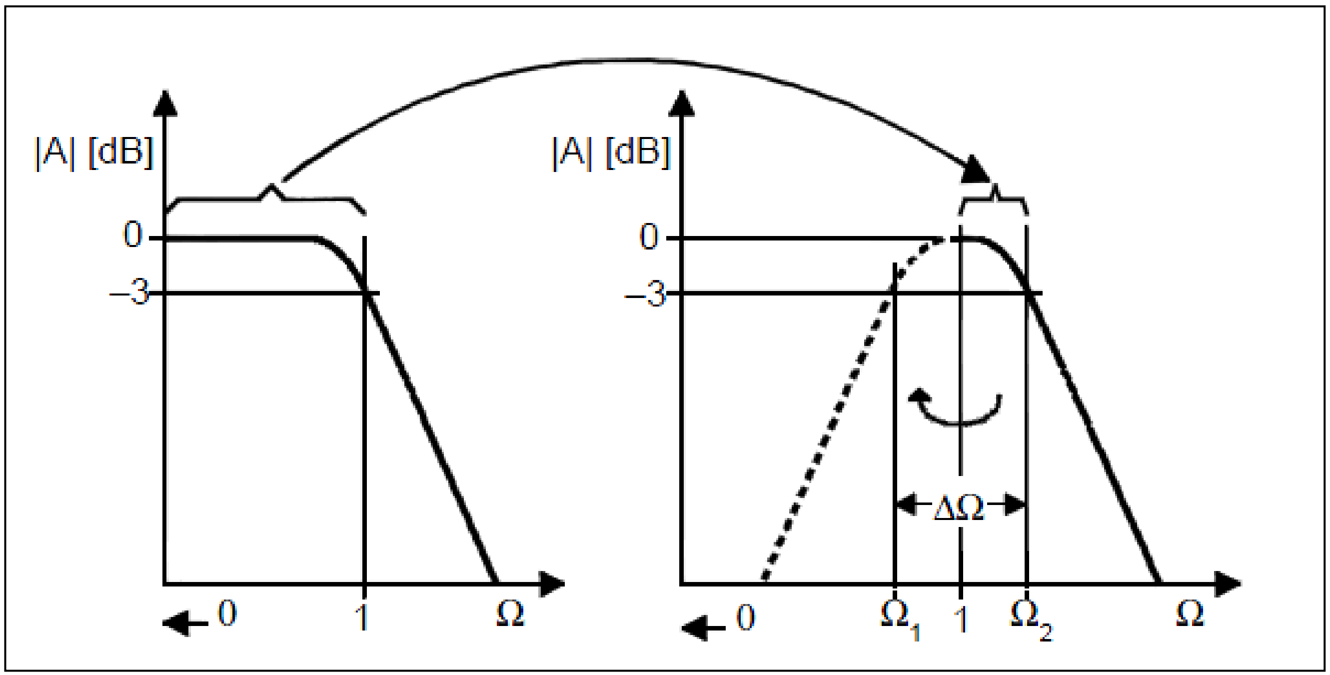
Referência Bibliográfica

OP AMPs for Everyone
Newnes, 2009



1

The passband characteristic of a low-pass filter is transformed into the upper passband half of a band-pass filter. The upper passband is then mirrored at the mid frequency, f_m (Ω=1), into the lower passband half



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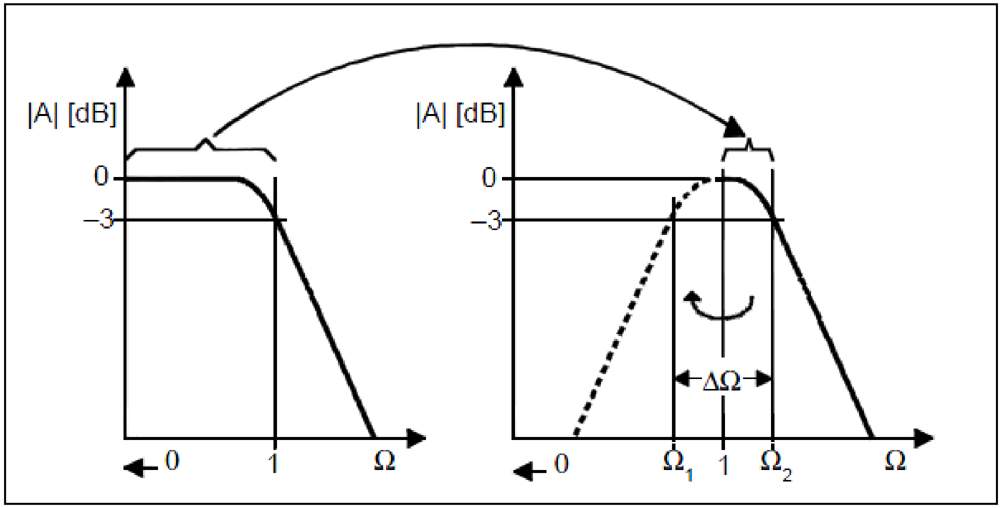

Normalized bandwith → $$\Delta\Omega = \Omega_2 - \Omega_1$$

Normalized mid bandwith → $$\Omega_m = 1 = \Omega_2 \cdot \Omega_1$$

Q factor → $$Q = \frac{f_m}{B} = \frac{f_m}{f_2 - f_1} = \frac{1}{\Omega_2 - \Omega_1} = \frac{1}{\Delta\Omega}$$

3 **The simplest design of a band-pass filter is the connection of a high-pass filter and a lowpass filter in series**, which is commonly done in wide-band filter applications.

4 A first order high-pass and a first-order low-pass provide a second-order band-pass.

5 A second-order high-pass and a second-order low-pass result in a fourth-order band-pass response.

6 Narrow-band filters of higher order consist of cascaded second-order band-pass filters that use the Sallen-Key or the Multiple Feedback (MFB) topology.

7

To develop the frequency response of a **second-order band-pass filter**, apply the

Transformation $s \rightarrow \frac{1}{\Delta\Omega}\left(s + \frac{1}{s}\right)$ to a **first-order low-pass transfer function**:

$$s \rightarrow \frac{1}{\Delta\Omega}\left(s + \frac{1}{s}\right)$$

$$\Downarrow$$

$$A(s) = \frac{A_0}{1 + s}$$

$$\Downarrow$$

$$A(s) = \frac{A_0 \cdot \Delta\Omega \cdot s}{1 + \Delta\Omega \cdot s + s^2}$$

8

When designing band-pass filters, **the parameters of interest are the gain at the mid frequency (A_m) and the quality factor (Q)**, which represents the selectivity of a band-pass filter.

$$Q = \frac{f_m}{B} = \frac{f_m}{f_2 - f_1} = \frac{1}{\Omega_2 - \Omega_1} = \frac{1}{\Delta\Omega}$$

Therefore, replace $A_0\Delta\Omega$ with A_m /Q and ΔΩ with 1/Q (Equation 16–7) to obtain:

$$A(s) = \frac{A_0 \cdot \Delta\Omega \cdot s}{1 + \Delta\Omega \cdot s + s^2}$$

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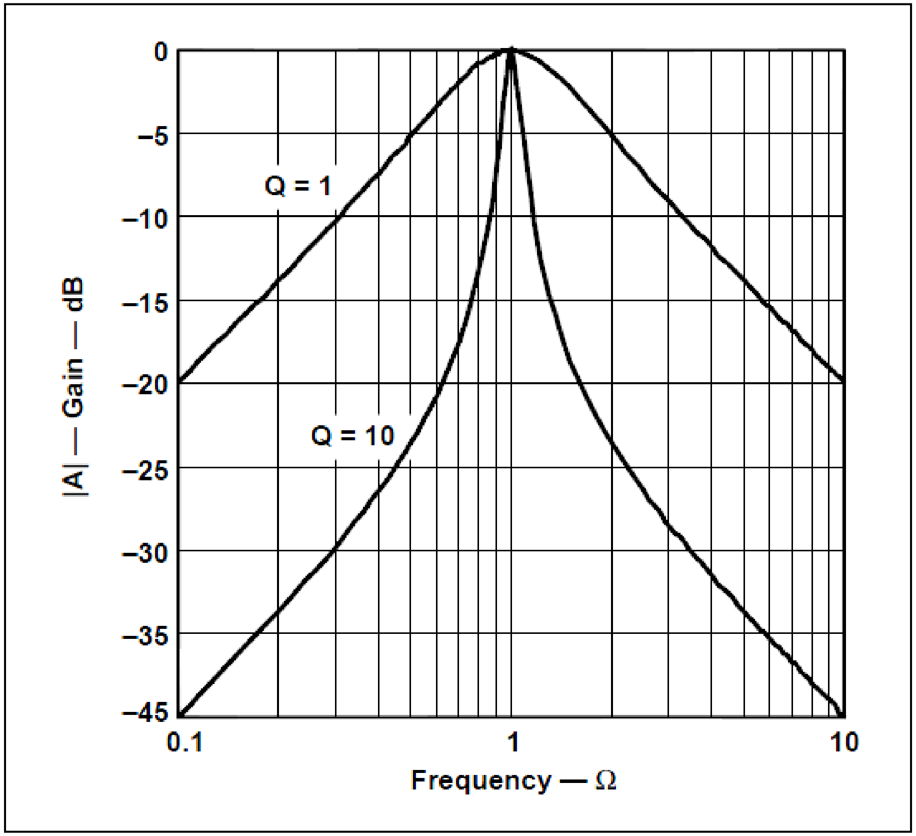
$$A(s) = \frac{\frac{A_m}{Q} \cdot s}{1 + \frac{1}{Q} \cdot s + s^2}$$

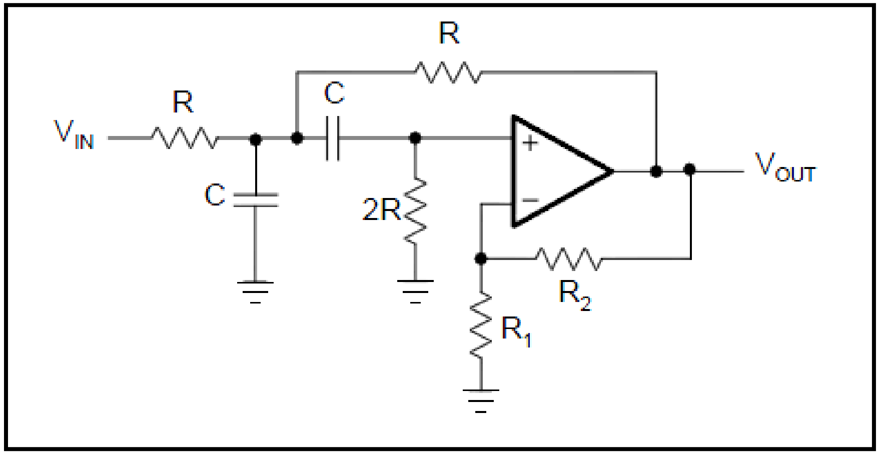


$$Q = \frac{f_m}{B} = \frac{f_m}{f_2 - f_1} = \frac{1}{\Omega_2 - \Omega_1} = \frac{1}{\Delta\Omega}$$

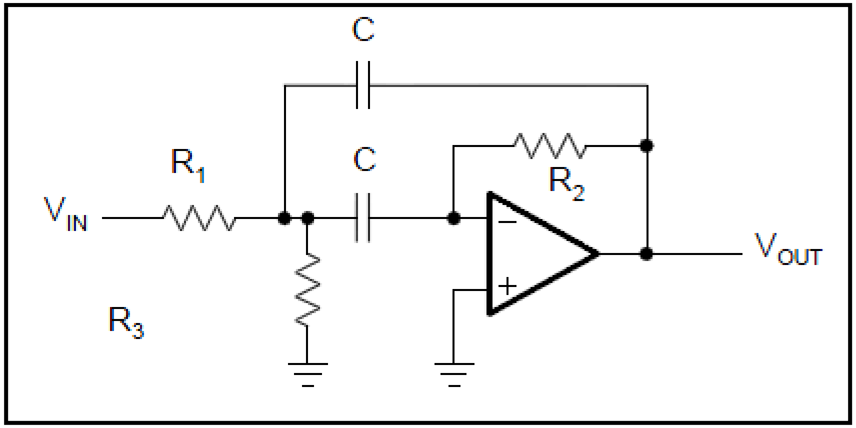
Normalized gain response of a second order bandpass filter

Band Pass Filters
Second Order Topology

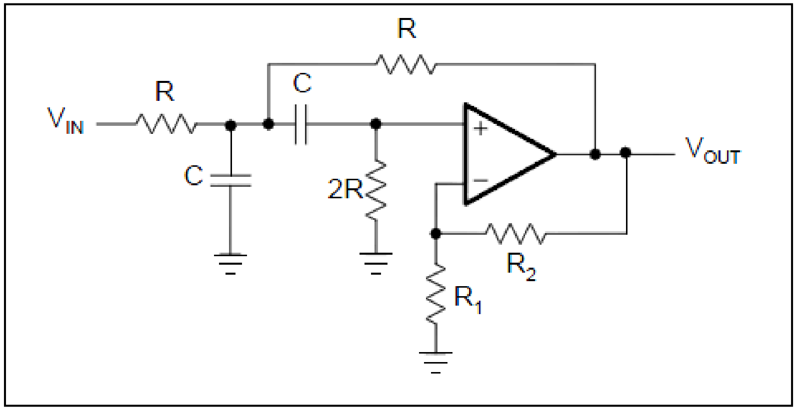
Sallen-Key Topology



Multiple Feedbak Topology



Sallen-Key Topology



$$A(s) = \frac{\frac{A_m}{Q}\cdot s}{1 + \frac{1}{Q}\cdot s + s^2}$$

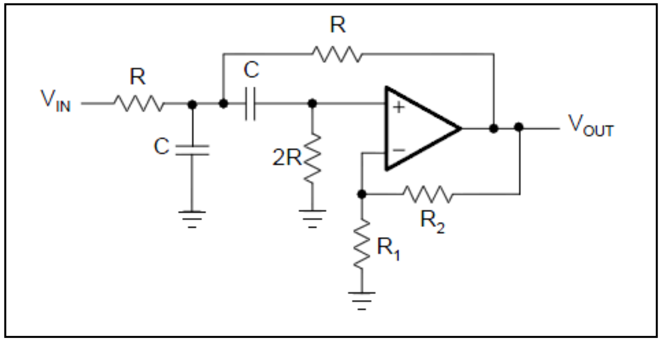
$$A(s) = \frac{G\cdot RC\omega_m\cdot s}{1 + RC\omega_m(3 - G)\cdot s + R^2C^2\omega_m{}^2\cdot s^2}$$

$G = 1 + R_2/R_1$

filter quality: $Q = \frac{1}{3 - G}$

mid-frequency: $f_m = \frac{1}{2\pi RC}$

gain at f_m: $A_m = \frac{G}{3 - G}$



1 The Sallen-Key circuit has the advantage that the quality factor (Q) can be varied via the inner gain (G) without modifying the mid frequency (f_m).

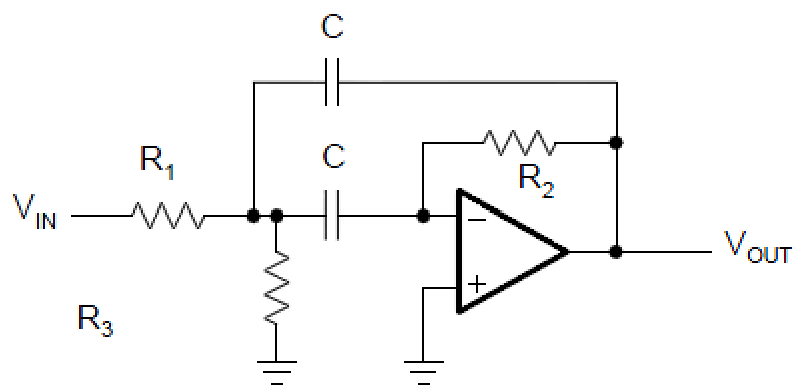
filter quality: $$Q = \frac{1}{3 - G}$$ mid-frequency: $$f_m = \frac{1}{2\pi RC}$$

2 A drawback is that Q and A_m cannot be adjusted independently.

filter quality: $$Q = \frac{1}{3 - G}$$ gain at f_m: $$A_m = \frac{G}{3 - G}$$

3 Care must be taken when G approaches the value of 3 because A_m becomes infinite.

Multiple Feedback Topology



$$A(s) = \frac{\frac{A_m}{Q}\cdot s}{1 + \frac{1}{Q}\cdot s + s^2}$$

$$A(s) = \frac{-\frac{R_2R_3}{R_1+R_3}C\omega_m\cdot s}{1 + \frac{2R_1R_3}{R_1+R_3}C\omega_m\cdot s + \frac{R_1R_2R_3}{R_1+R_3}C^2\cdot\omega_m{}^2\cdot s^2}$$

mid-frequency: $f_m = \frac{1}{2\pi C}\sqrt{\frac{R_1 + R_3}{R_1R_2R_3}}$

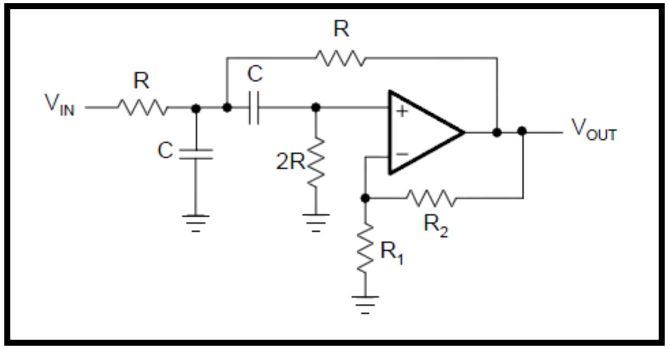
filter quality: $Q = \pi f_m R_2 C$

gain at f_m: $-A_m = \frac{R_2}{2R_1}$

bandwidth: $B = \frac{1}{\pi R_2 C}$

Designing a Band Pass Filters Second Order Topology

Sallen-Key Topology



1 Specify f_m and C

2 $$R = \frac{1}{2\pi f_m C}$$

3a Specify A_m, calculate G and R_2 / R_1

$$R_2 = \frac{2A_m - 1}{1 + A_m}$$

$$A_m = \frac{G}{3 - G}$$

$$G = 1 + R_2 / R_1$$

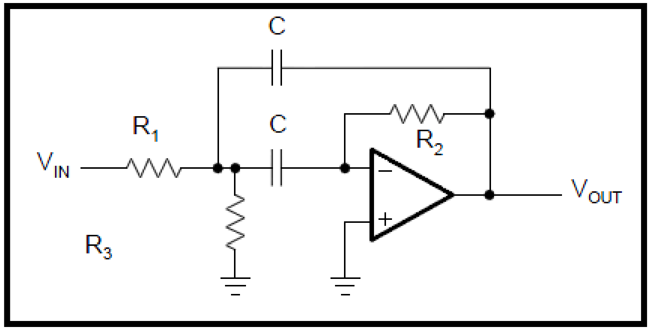
3b Specify Q, calculate G and R_2 / R_1

$$R_2 = \frac{2Q - 1}{Q}$$

$$Q = \frac{1}{3 - G}$$

$$G = 1 + R_2 / R_1$$

Multiple Feedback Topology



1 Specify f_m, Q, A_m and C

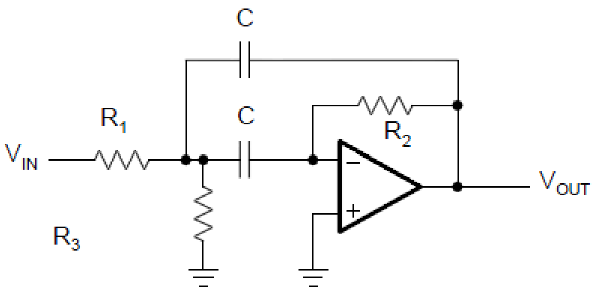
2 $$R_2 = \frac{Q}{\pi f_m C}$$

3 $$R_1 = \left|\frac{R_2}{-2A_m}\right|$$

4 $$R_3 = \left|\frac{-A_m R_1}{Q^2 + A_m}\right|$$

Exemple 1:

Design a second-order MFB band-pass filter with a mid frequency of f_m= 1 kHz, a quality factor of Q = 10, and a gain of A_m= –2. Assume a capacitor value of C = 100 nF.



$$R_2 = \frac{Q}{\pi f_m C} = \frac{10}{\pi \cdot 1\ \text{kHz} \cdot 100\ \text{nF}} = 31.8\ \text{k}\Omega$$

$$R_1 = \left| \frac{R_2}{-2A_m} \right| = \frac{31.8\ \text{k}\Omega}{4} = 7.96\ \text{k}\Omega$$

$$R_3 = \left| \frac{-A_m R_1}{2Q^2 + A_m} \right| = \frac{2 \cdot 7.96\ \text{k}\Omega}{200 - 2} = 80.4\ \Omega$$

Band Pass Filters
Higher Order Topology

Replacing the *s* term with the transformation $\frac{1}{\Delta\Omega}\left(s + \frac{1}{s}\right)$

in a **second order low pass transfer function** gives the general transfer function of a **fourth-order band-pass**:

$$\frac{1}{\Delta\Omega}\left(s + \frac{1}{s}\right) \rightarrow A(s) = \frac{A_0}{\prod_i \left(1 + a_i s + b_i s^2\right)}$$

$$A(s) = \frac{\frac{s^2 \cdot A_0 (\Delta\Omega)^2}{b_1}}{1 + \frac{a_1}{b_1}\Delta\Omega \cdot s + \left[2 + \frac{(\Delta\Omega)^2}{b_1}\right] \cdot s^2 + \frac{a_1}{b_1}\Delta\Omega \cdot s^3 + s^4}$$

$$A(s) = \frac{\frac{A_{mi}}{Q_i} \cdot \alpha s}{\left[1 + \frac{\alpha s}{Q_1} + (\alpha s)^2\right]} \cdot \frac{\frac{A_{mi}}{Q_i} \cdot \frac{s}{\alpha}}{\left[1 + \frac{1}{Q_i}\left(\frac{s}{\alpha}\right) + \left(\frac{s}{\alpha}\right)^2\right]}$$

$$A(s) = \frac{\frac{A_{mi}}{Q_i}\cdot \alpha s}{\left[1 + \frac{\alpha s}{Q_1} + (\alpha s)^2\right]} \cdot \frac{\frac{A_{mi}}{Q_i}\cdot \frac{s}{\alpha}}{\left[1 + \frac{1}{Q_i}\left(\frac{s}{\alpha}\right) + \left(\frac{s}{\alpha}\right)^2\right]}$$

This equation represents the connection in series of two second-order band-pass filters where:

1 A_{mi} is the gain at the mid frequency, f_{mi}, of each partial filter

$$A_{mi} = \frac{Q_i}{Q}\cdot\sqrt{\frac{A_m}{b_1}}$$

2 Q_i is the pole quality of each filter

$$Q_i = Q\cdot\frac{(1 + \alpha^2)b_1}{\alpha\cdot a_1}$$

$$A(s) = \frac{\frac{A_{mi}}{Q_i} \cdot \alpha s}{\left[1 + \frac{\alpha s}{Q_1} + (\alpha s)^2\right]} \cdot \frac{\frac{A_{mi}}{Q_i} \cdot \frac{s}{\alpha}}{\left[1 + \frac{1}{Q_i}\left(\frac{s}{\alpha}\right) + \left(\frac{s}{\alpha}\right)^2\right]}$$

3 α and $1/\alpha$ are the factors by which the mid frequencies of the individual filters, f_{m1} and f_{m2}, derive from the mid frequency, f_m, of the overall bandpass.

$$f_{m1} = \frac{f_m}{\alpha} \qquad f_{m2} = f_m \cdot \alpha$$

4 Factor α needs to be determined through successive approximation, using equation:

$$\alpha^2 + \left[\frac{\alpha \cdot \Delta\Omega \cdot a_1}{b_1(1 + \alpha^2)}\right]^2 + \frac{1}{\alpha^2} - 2 - \frac{(\Delta\Omega)^2}{b^1} = 0$$

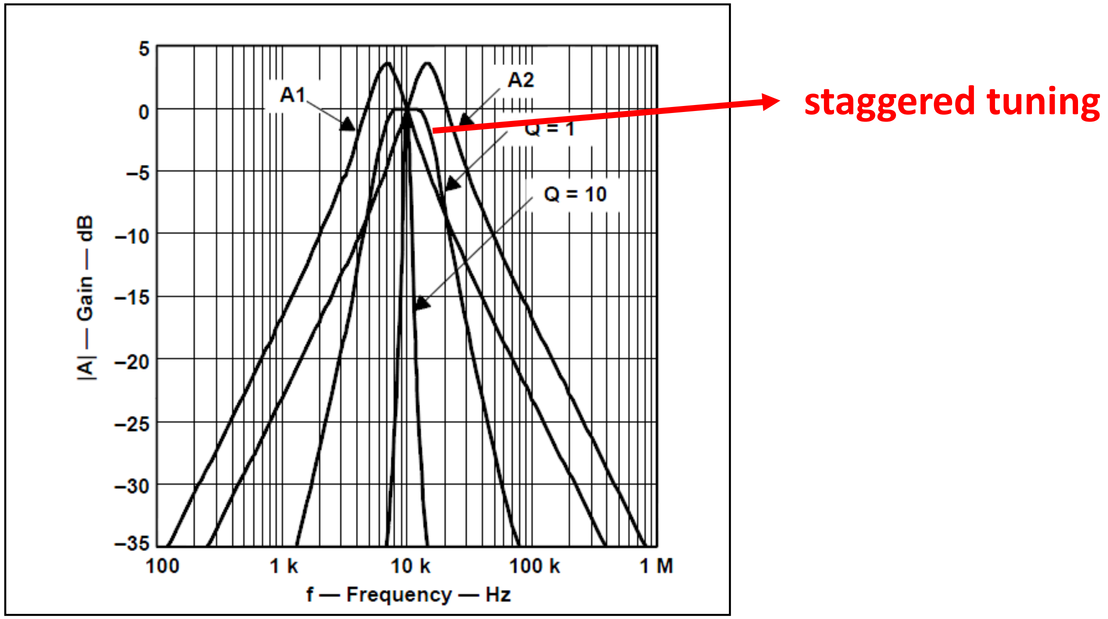
Values of α For Different Filter Types and Different valures of Q:

Bessel				Butterworth				Tschebyscheff			
a_1	1.3617			**a_1**	1.4142			**a_1**	1.0650		
b_1	0.6180			**b_1**	1.0000			**b_1**	1.9305		
Q	100	10	1	**Q**	100	10	1	**Q**	100	10	1
$\Delta\Omega$	**0.01**	**0.1**	**1**	$\Delta\Omega$	**0.01**	**0.1**	**1**	$\Delta\Omega$	**0.01**	**0.1**	**1**
α	1.0032	1.0324	1.438	α	1.0035	1.036	1.4426	α	1.0033	1.0338	1.39

$$A(s) = \frac{\frac{A_{mi}}{Q_i}\cdot \alpha s}{\left[1 + \frac{\alpha s}{Q_1} + (\alpha s)^2\right]} \cdot \frac{\frac{A_{mi}}{Q_i}\cdot \frac{s}{\alpha}}{\left[1 + \frac{1}{Q_i}\left(\frac{s}{\alpha}\right) + \left(\frac{s}{\alpha}\right)^2\right]}$$

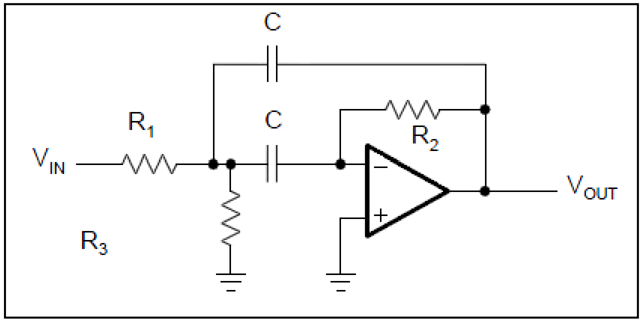
5 In a fourth-order band-pass filter with high Q, the mid frequencies of the two partial filters differ only slightly from the overall mid frequency. This method is called **staggered tuning. A flat gain response shows up as well as a sharp pass-band to stop-band transition.**

Example: Gain response of a fourth-order Butterworth band-pass filter with staggered tuning where with Q = 1. Its partial filters are shown as well as the gain of a non staggered tuning filter with Q = 10.



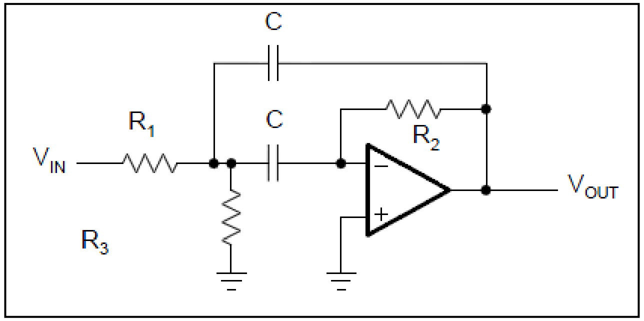
Designing a Band Pass Filters Higher Order Topology

Exemple:
Design a fourth-order Butterworth band-pass with f_m = 10KHz, Q = 10 and A_m = 1 using a second order multiple feedback topology.



1 From table: a_1 = 1.4142, b_1 = 1, α = 1,036

Bessel				Butterworth				Tschebyscheff			
a_1	1.3617			**a_1**	1.4142			**a_1**	1.0650		
b_1	0.6180			**b_1**	1.0000			**b_1**	1.9305		
Q	100	10	1	**Q**	100	10	1	**Q**	100	10	1
$\Delta\Omega$	**0.01**	**0.1**	**1**	$\Delta\Omega$	**0.01**	**0.1**	**1**	$\Delta\Omega$	**0.01**	**0.1**	**1**
α	1.0032	1.0324	1.438	α	1.0035	1.036	1.4426	α	1.0033	1.0338	1.39



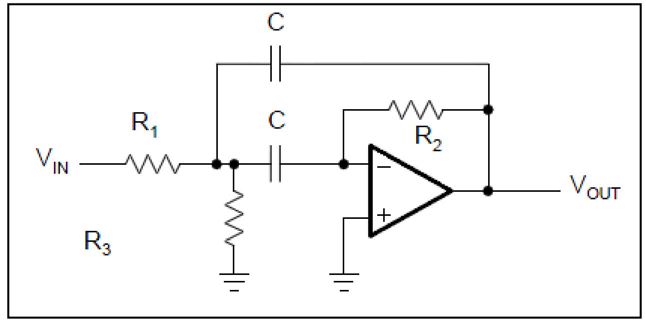
2 Calculate f_{m1} and f_{m2}

$$f_{m1} = \frac{f_m}{\alpha} \quad \Rightarrow \quad f_{mi} = \frac{10\ \text{kHz}}{1.036} = 9.653\ \text{kHz}$$

$$f_{m2} = f_m \cdot \alpha \quad \Rightarrow \quad f_{m2} = 10\ \text{kHz} \cdot 1.036 = 10.36\ \text{kHz}$$

3 Calculate Q_i

$$Q_i = Q \cdot \frac{(1 + \alpha^2) b_1}{\alpha \cdot a_1} \quad \Rightarrow \quad Q_i = 10 \cdot \frac{(1 + 1.036^2) \cdot 1}{1.036 \cdot 1.4142} = 14.15$$



4 Calculate A_{mi}

$$A_{mi} = \frac{Q_i}{Q} \cdot \sqrt{\frac{A_m}{b_1}} \quad \Rightarrow \quad A_{mi} = \frac{14.15}{10} \cdot \sqrt{\frac{1}{1}} = 1.415$$

5 Calculate the MF resistance components for filter 1 and filter 2 using C = 10nF

$$R_{2i} = \frac{Q_i}{\pi f_{mi} C} \qquad R_{1i} = \left|\frac{R_2}{-2A_{mi}}\right| \qquad R_{3i} = \left|\frac{-A_{mi}R_1}{2Q^2 + A_{mi}}\right|$$

Filter 1:

$$R_{21} = \frac{Q_i}{\pi f_{m1} C} = \frac{14.15}{\pi \cdot 9.653\ \text{kHz} \cdot 10\ \text{nF}} = 46.7\ \text{k}\Omega$$

$$R_{11} = \left|\frac{R_{21}}{-2A_m}\right| = \left|\frac{46.7\ \text{k}\Omega}{-2 \cdot + 1.415}\right| = 16.5\ \text{k}\Omega$$

$$R_{31} = \left|\frac{-A_{mi}R_{11}}{2Q_i^2 + A_m}\right| = \left|\frac{1.415 \cdot 16.5\ \text{k}\Omega}{2 \cdot 14.15^2 + 1.415}\right| = 58.1\ \Omega$$

Filter 2:

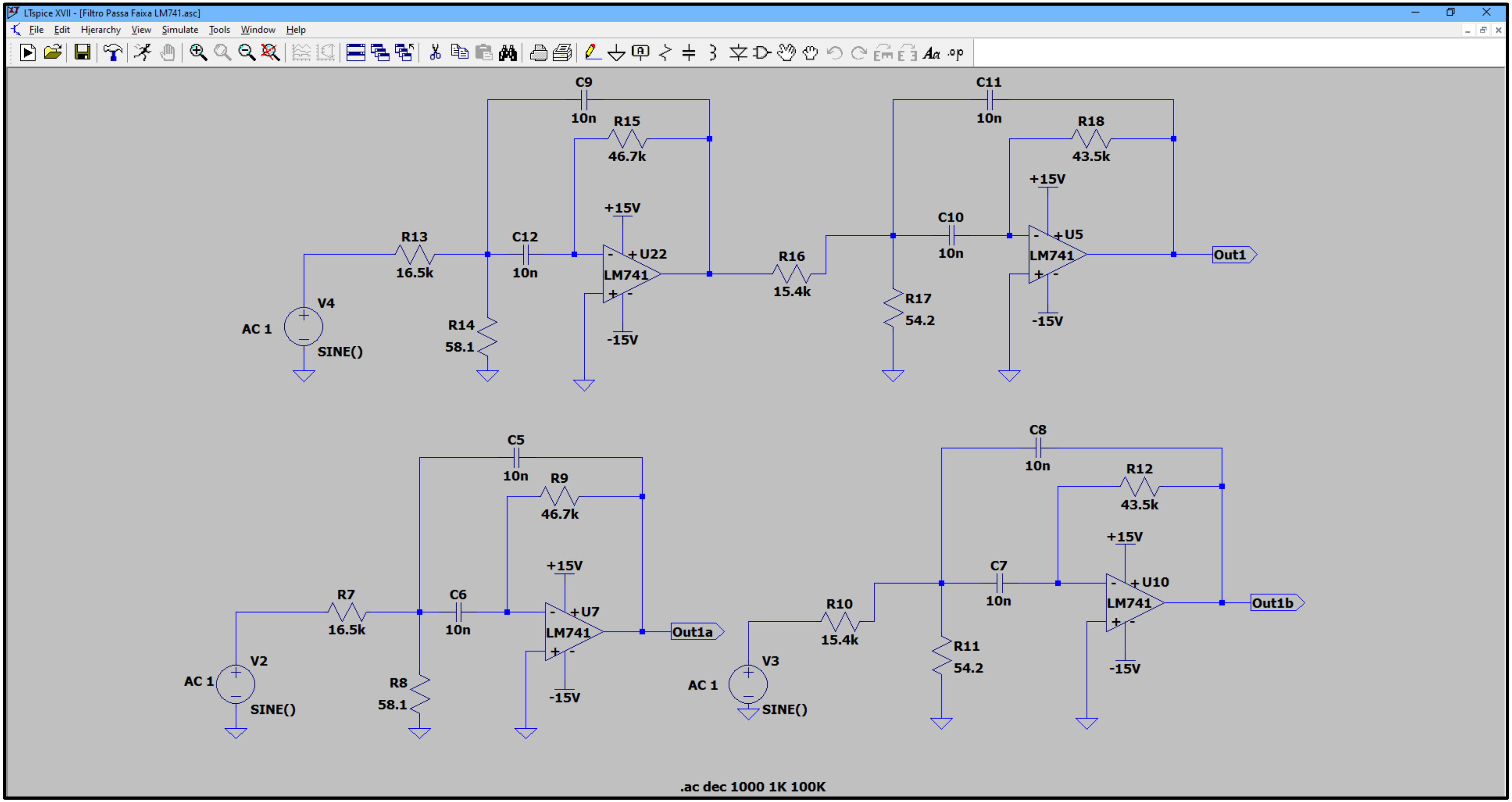
$$R_{22} = \frac{Q_i}{\pi f_{m2} C} = \frac{14.15}{\pi \cdot 10.36\ \text{kHz} \cdot 10\ \text{nF}} = 43.5\ \text{k}\Omega$$

$$R_{12} = \left|\frac{R_{22}}{-2A_{mi}}\right| = \left|\frac{43.5\ \text{k}\Omega}{-2 \cdot + 1.415}\right| = 15.4\ \text{k}\Omega$$

$$R_{32} = \left|\frac{-A_{mi}R_{12}}{2Q_i^2 + A_{mi}}\right| = \left|\frac{1.415 \cdot 15.4\ \text{k}\Omega}{2 \cdot 14.15^2 + 1.415}\right| = 54.2\ \Omega$$

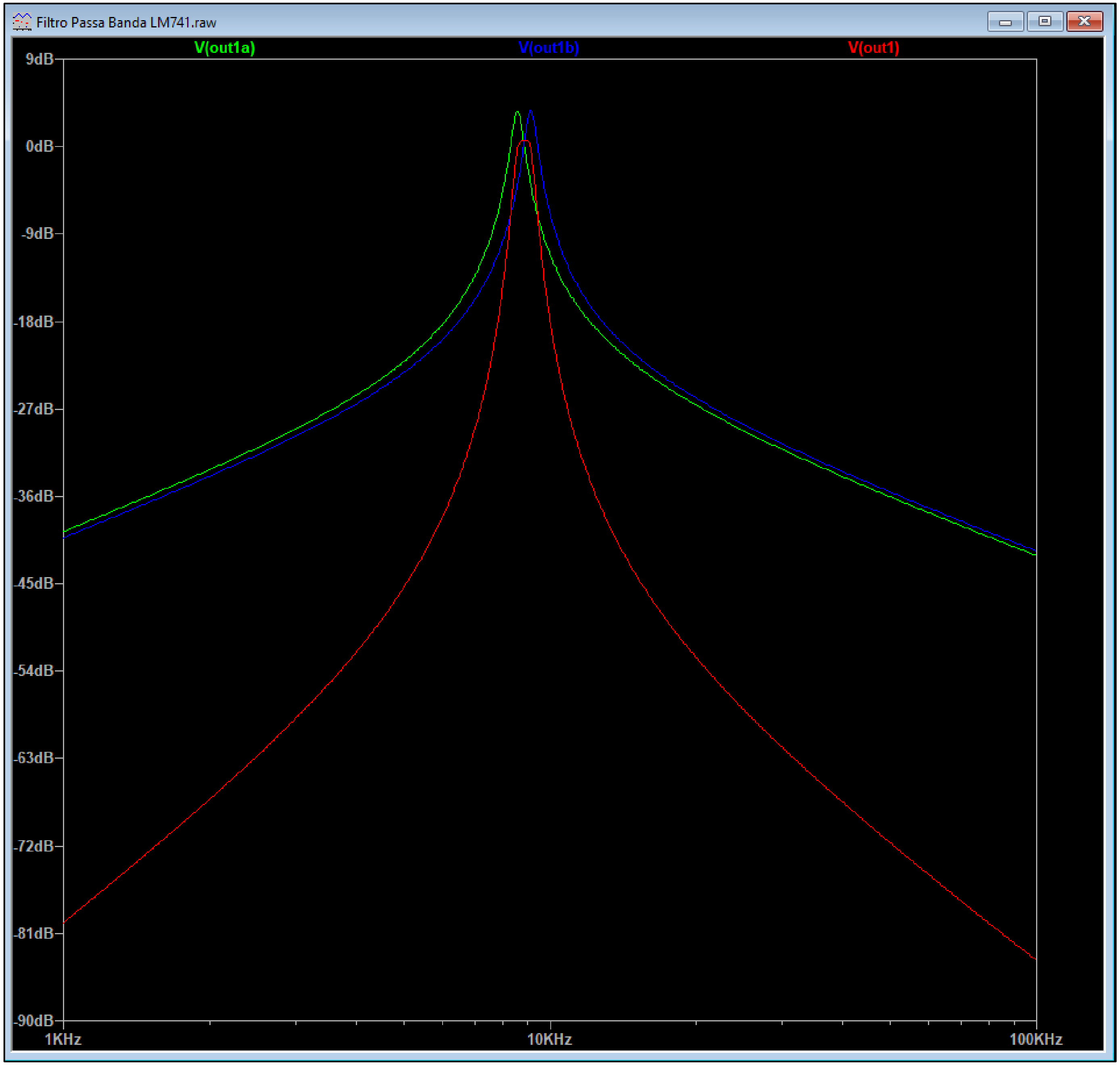
Resultados (Simulação LTPice)

Filtro Passa Banda (LM741)

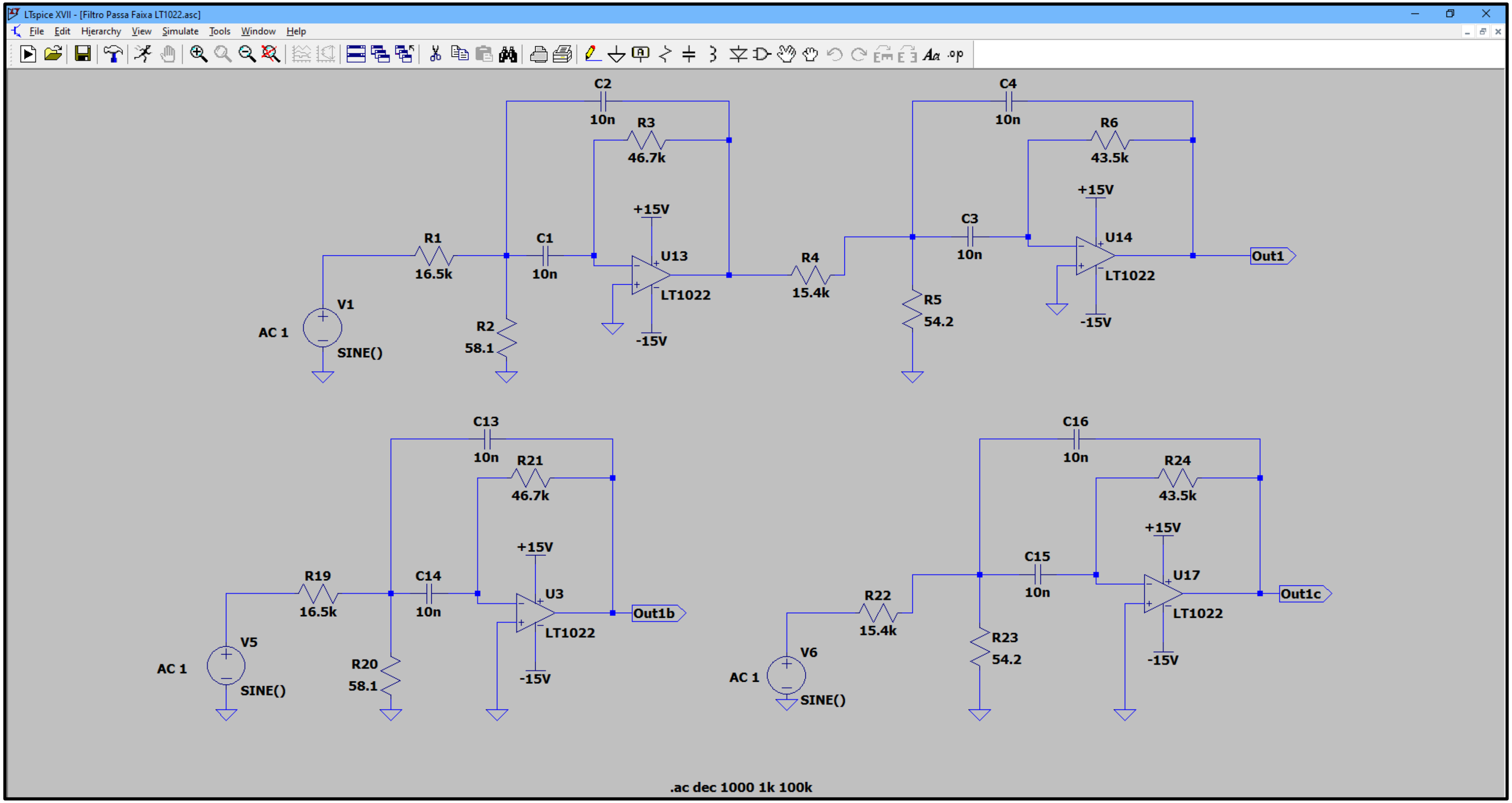


Os valores dos componentes foram medidos !

Filtro Passa Banda (LM741)

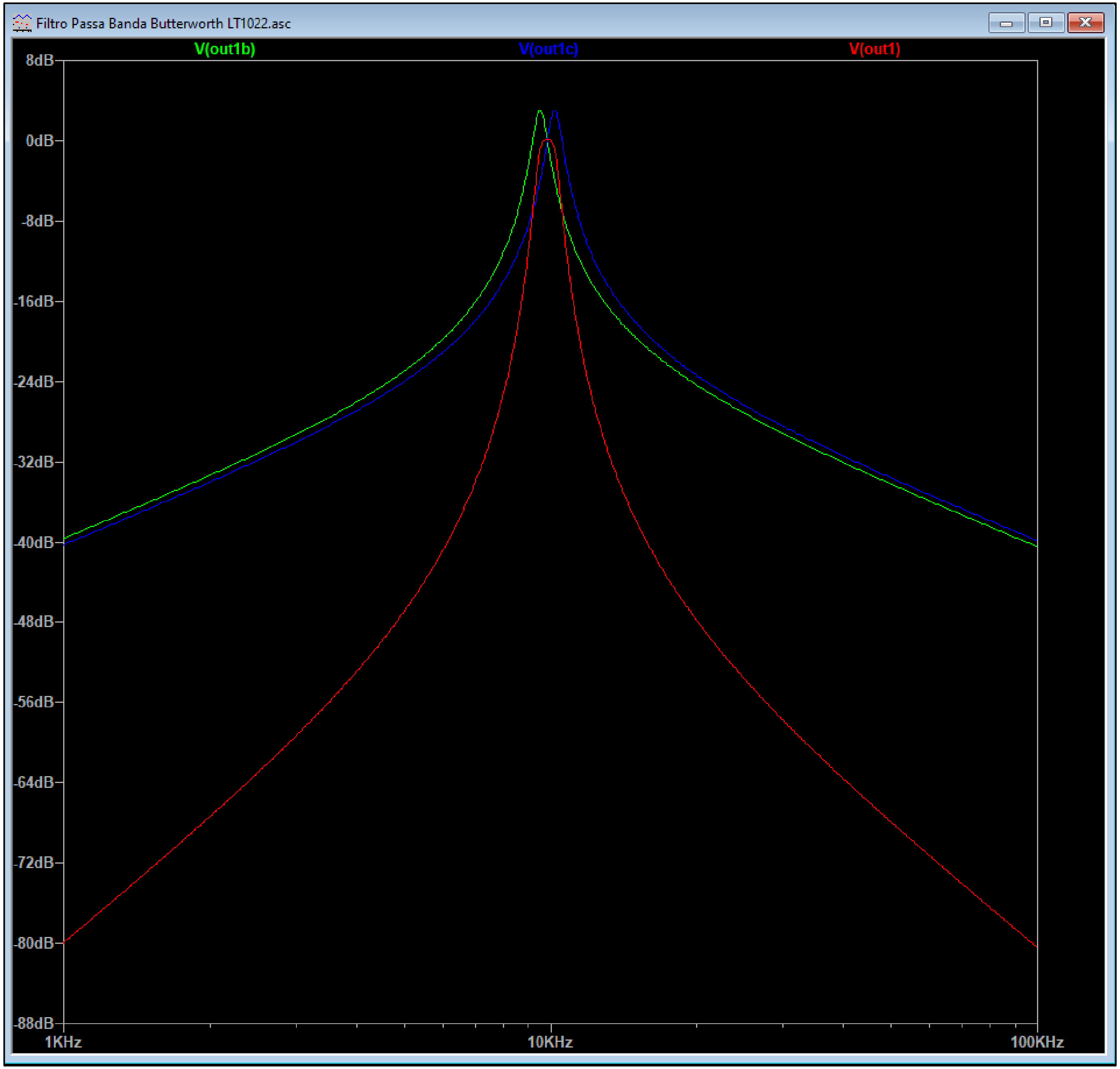


Filtro Passa Banda (LT1022)

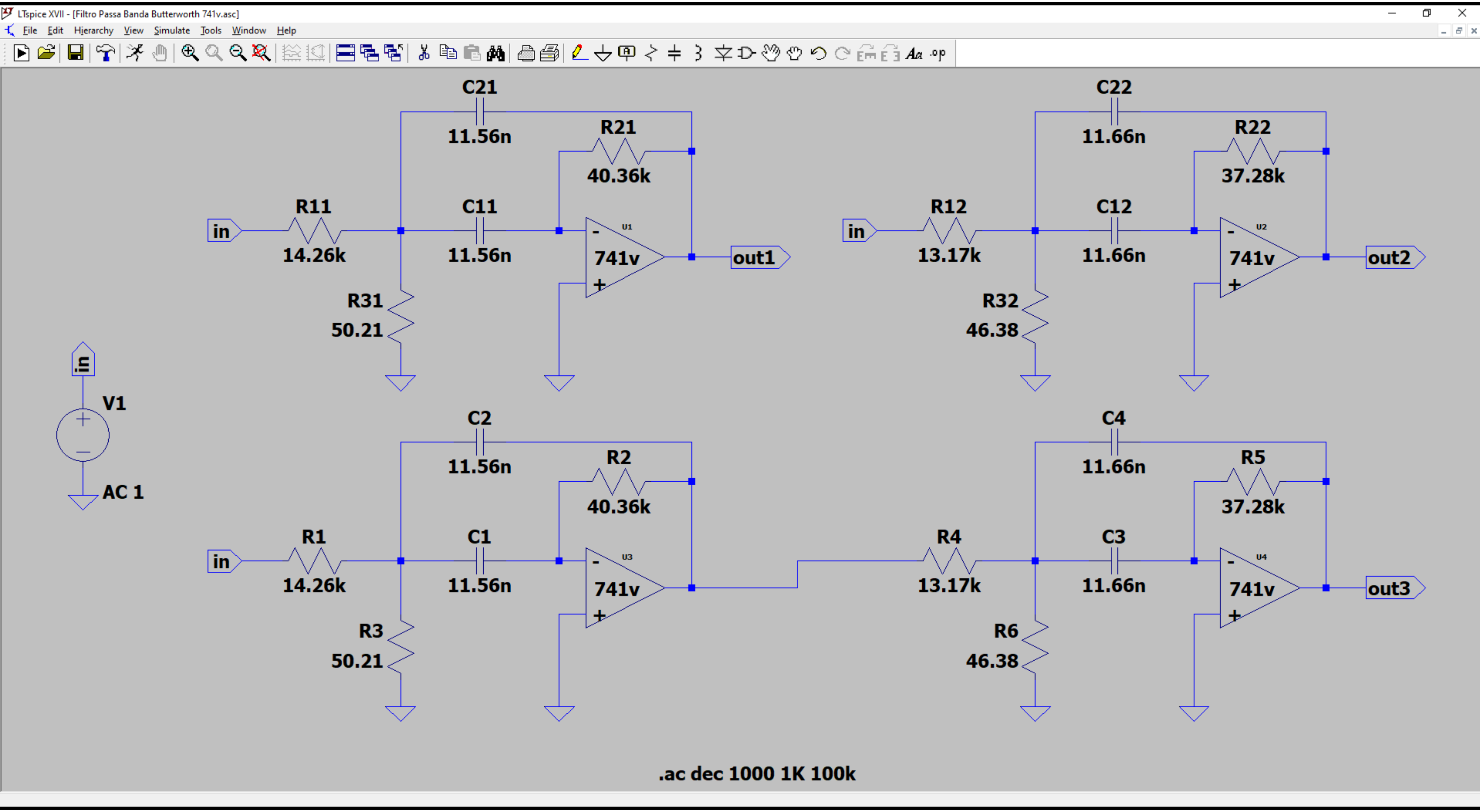


Os valores dos componentes foram medidos !

Filtro Passa Banda (LT1022)



Filtro Passa Banda (741v)
(op amp projetado na disciplina SEL0315)



Os valores dos componentes foram medidos !

Filtro Passa Banda (741v)