Lecture 4 – Crystal Oscillators

Classification of oscillators. Crystals. Ceramic resonators. Oscillation conditions. Oscillator configurations. The Pierce oscillator using digital IC inverters. Analysis of oscillators. Analysis of the Pierce oscillator.

Introduction

An electronic oscillator is an electronic circuit that produces a repetitive electronic signal, often a sine wave or a square wave. The output frequency is determined by the characteristics of the devices used in the circuit.

A *harmonic* oscillator produces quasi-sine-wave oscillations. In order to realize harmonic oscillations, the following are required:

Harmonic oscillator defined

1. An active element producing amplification.
2. Positive feedback.
3. A frequency selective network which mainly determines the oscillation frequency.
4. A nonlinearity, called "limiting", to maintain the oscillation amplitude in stable equilibrium.

The basic form of a harmonic oscillator is an electronic amplifier connected in a feedback loop with its output fed back into its input through a frequency selective electronic filter to provide positive feedback:

The basic form of a harmonic oscillator

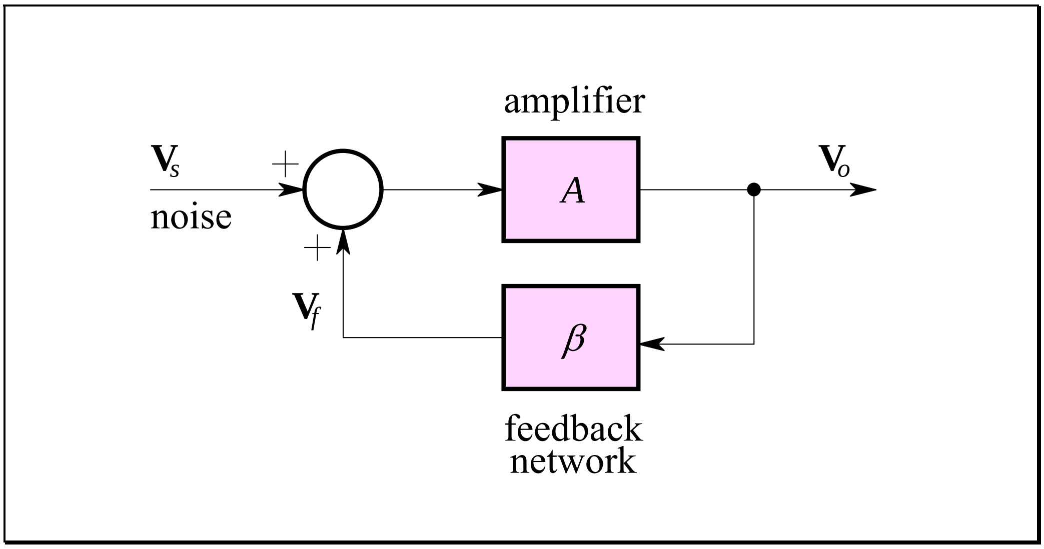


Figure 4.1 – The basic form of a harmonic oscillator

When the power supply to the amplifier is first switched on, the amplifier's output consists only of noise. The noise travels around the loop and is filtered and re-amplified until it increasingly resembles a sine wave at a single frequency.

Harmonic oscillator circuits can be classified according to the type of frequency selective filter they use in the feedback loop:

RC oscillators

- In an *RC oscillator* circuit, the filter is a network of resistors and capacitors. *RC* oscillators are mostly used to generate lower frequencies, for example in the audio range. Common types of *RC* oscillator circuits are the phase shift oscillator and the Wien bridge oscillator.

LC oscillators

- In an *LC oscillator* circuit, the filter is a tuned circuit (often called a tank circuit) consisting of an inductor and capacitor connected together. Charge flows back and forth between the capacitor's plates through the inductor, so the tuned circuit can store electrical energy oscillating at its resonance frequency. There are small losses in the tank circuit, but the amplifier compensates for those losses and supplies the power for the output signal. *LC* oscillators are often used at radio frequencies, when a tunable frequency source is necessary, such as in signal generators, tunable radio transmitters and the local oscillators in radio receivers. Typical *LC* oscillator circuits are the Hartley, Colpitts and Clapp circuits.

Crystal oscillators

- A *crystal oscillator* is a circuit that uses a piezoelectric crystal (commonly a quartz crystal) as a frequency selective element. The crystal mechanically vibrates as a resonator, and its frequency of vibration determines the oscillation frequency. Crystals have a very high *Q*-factor and also better temperature stability than tuned circuits, so crystal oscillators have much better frequency stability than *LC* or *RC* oscillators. They are used to stabilize the frequency of most radio transmitters, and to generate the clock signal in computers and quartz clocks. Crystal oscillators often use the same circuits as *LC* oscillators, with the crystal replacing the tuned circuit. Surface acoustic wave (SAW) devices are another kind of piezoelectric resonator used in crystal oscillators, which can achieve much higher frequencies. They are used in specialized applications which require a high frequency reference, for example, in mobile phones.

Relaxation oscillator defined

A *relaxation* oscillator produces a non-sinusoidal output, such as a square, sawtooth or triangle wave. It contains an energy-storing element (a capacitor or, more rarely, an inductor) and a nonlinear trigger circuit (a latch, Schmitt trigger, or negative resistance element) that periodically charges and discharges the energy stored in the storage element thus causing abrupt changes in the output waveform.

Triangular and rectangular waveform generator

An example of a relaxation oscillator that produces both a triangle and rectangular waveform is shown below:

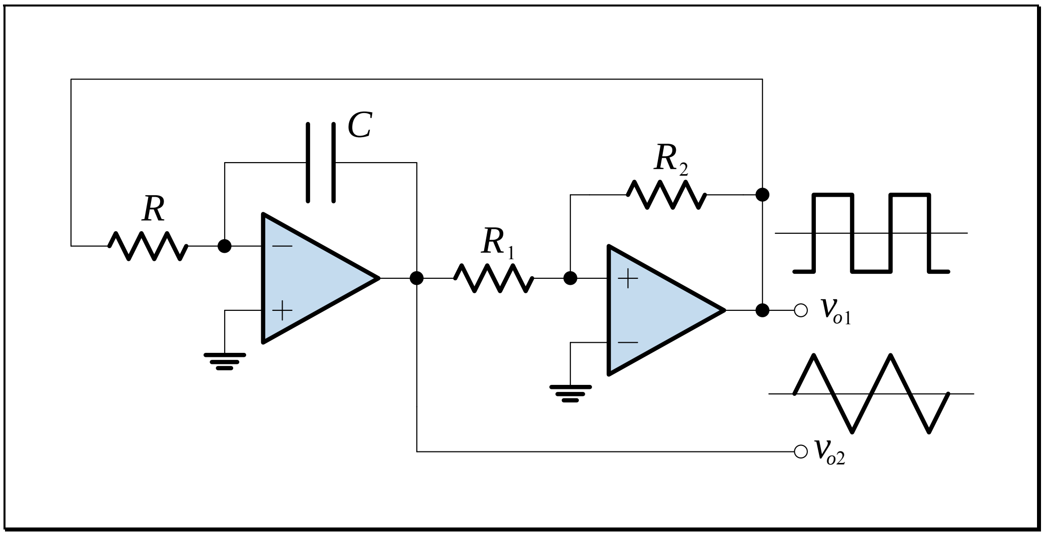


Figure 4.2 – A relaxation oscillator

4.4

Classification of Oscillators

There are many ways in which an oscillator can be classified:

1. Frequency range
2. Power output range
3. Function (e.g., the frequency can be readily modulated or shifted by an externally applied voltage)
4. Number of active devices (e.g. "single transistor" where the same transistor provides the amplification and the limiting)
5. Manner its frequency is stabilized for the changes in environment (e.g. oven controlled)
6. Manner of limiting (e.g. self-limiting, automatic level control)
7. Degree of frequency stability

To facilitate the classification process, a system of abbreviations has been gradually devised. Some of these abbreviations are:

Abbreviation	Meaning
O	oscillator
X	crystal
LC	inductor capacitor
VC	voltage controlled
OC	oven controlled
ALC	automatic level controlled

Table 4.1 – Oscillator abbreviations

These basic abbreviations can be combined to form a new abbreviation. For example, an OCXO would be an oven-controlled crystal oscillator.

Crystals

A “crystal” is a carefully oriented and dimensioned piece of quartz or other suitable piezoelectric material to which adherent electrodes have been applied. The crystals are held within sealed enclosures by mounting supports that also serve as connections between the electrodes and the external leads. Crystal enclosures are designated by HC- numbers (Holder, Crystal).

Crystal enclosures and mounting

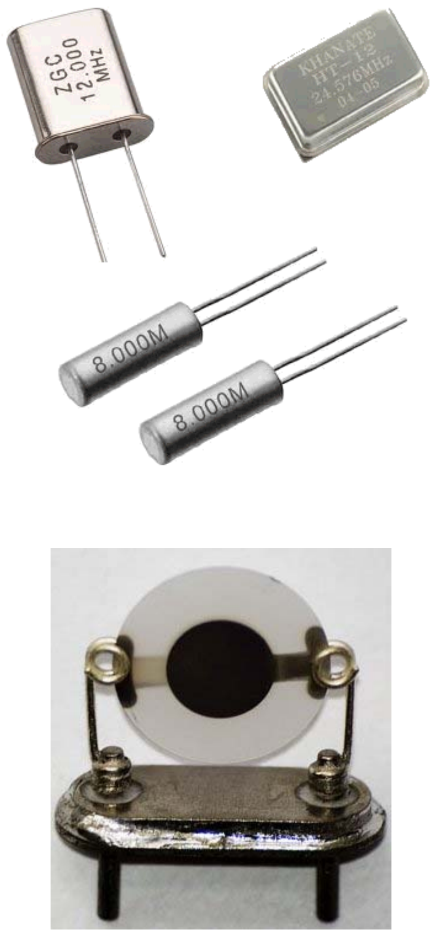


Figure 4.3 – Crystal enclosures and mounting

Crystals are unique not only because of the achievable combinations of circuit parameter values (i.e. high frequencies of resonance, small capacitance, etc.) but also because of other important features such as cost, size, and stability with time, temperature and other environmental changes.

Frequency Range

Crystal resonators are available to cover frequencies from below 1 kHz to over 200 MHz. At the low-frequency end wristwatch and real-time clock applications operate at 32.768 kHz and powers of two times this frequency. More conventional resonators span the range 80 kHz to 200 MHz; these utilise what is known as *bulk acoustic waves* (BAWs) that propagate within the crystal. Surface acoustic waves (SAWs) travel along the surface. Devices based on SAWs are available for the range above 50 MHz into the low GHz region.

Frequency Accuracy

The absolute frequency accuracy of crystal-stabilized commercial oscillators is between 10^{-6} and 10^{-7}. This figure includes variations over all environmental conditions, such as temperature, mechanical shock, and aging.

Frequency Stability

Precision quartz oscillators, held at constant temperature and protected from environmental disturbances, have fractional stabilities from 10^{-10} to 10^{-12}.

Aging Effects

Slow changes in frequency with time are referred to as aging. The principal causes of aging are contamination within the enclosure that is redistributed with time, slow leaks in the enclosure, mounting and electrode stresses that are relieved with time, and changes in atmospheric pressure. Changes in the quartz are usually negligible for most applications.

Environmental Effects

The frequency of a crystal resonator can vary due to a variety of environmental disturbances. Such disturbances include: thermal transients, mechanical accelerations in the forms of vibration, shock and turning the crystal over in a gravitational field (tip-over), magnetic fields, radiation, DC voltages, and variations in the *drive level* (the amount of power dissipated in the crystal).

Crystal Cuts

The resonator plate can be cut from the source crystal in many different ways. The orientation of the cut influences the crystal's aging characteristics, frequency stability, thermal characteristics, and other parameters. Some of the more popular cuts are:

The most common crystal cuts and their characteristics

Cut	Frequency Range	Description
AT	0.5 – 300 MHz	The most common cut, developed in 1934. The frequency-temperature curve is a sine-shaped curve with an inflection point at around 25–35°C. The *optimum* AT cut has a frequency variation with temperature of only ±12 ppm from -50°C to +100°C. Most (estimated over 90%) of all crystals are this variant. Sensitive to mechanical stresses, whether caused by external forces or by temperature gradients. The upper limit for the fundamental mode of vibration is around 30-40 MHz.
SC	0.5 – 200 MHz	A special cut (Stress Compensated), developed in 1974, is for oven-stabilized oscillators with low phase noise and good aging characteristics. Compared to the AT cut, it is less sensitive to mechanical stresses, has a faster warm-up speed, higher Q, better close-in phase noise, less sensitivity to spatial orientation against gravity, and less sensitivity to vibrations. The frequency-temperature curve has an inflection point at 96°C and the optimum SC cut has a much lower temperature sensitivity than the optimum AT cut. It is suitable for OCXOs, e.g. space and GPS systems. Aging characteristics are 2 to 3 times better than of the AT cuts.
BT	0.5 – 200 MHz	A special cut, similar to the AT cut. It has well known and repeatable characteristics. It has poorer temperature characteristics than the AT cut. It is used for oscillators vibrating at a fundamental mode which is higher than the AT cut, up to over 50 MHz.

Equivalent Circuit

Since a crystal is piezoelectric, the crystal changes shape when a signal is applied to the electrodes. If the applied signal frequency approaches a natural mechanical resonance, then these high-amplitude mechanical vibrations are very narrow in frequency and are ideally suited to oscillators (the crystal has a high "*Q*-factor"). The piezoelectric effect is responsible for converting the electrical signal to mechanical motion and it reconverts the vibratory motion of the crystal back into an electrical signal at its terminals. Therefore, looking at the crystal simply as a circuit component consisting of an enclosure and leads, we can formulate an equivalent circuit model that is valid around a particular region of mechanical vibration. The crystal circuit symbol, and a simplified equivalent circuit, are shown below:

Simplified equivalent circuit of a crystal

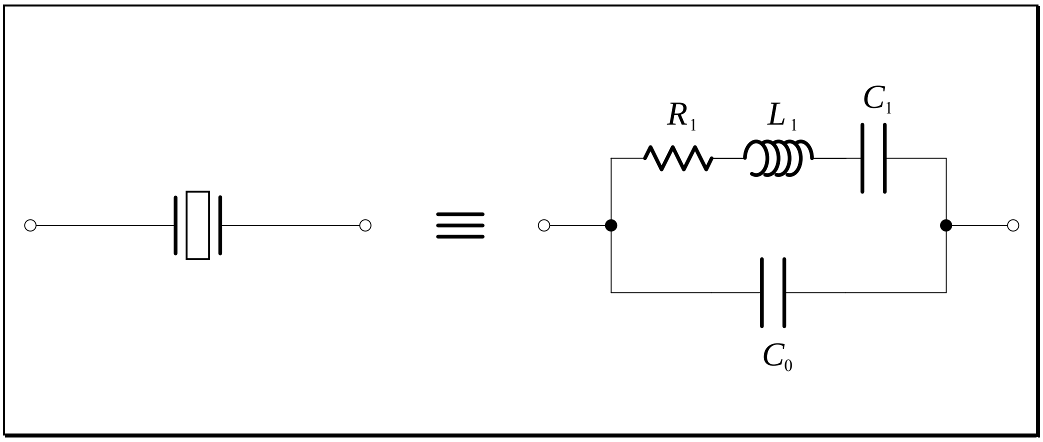


Figure 4.4 – Simplified equivalent circuit of a crystal

Motional arm defined

The series *RLC* portion is referred to as the *motional* arm of the circuit and arises from the mechanical crystal vibrations. It is a valid electrical model in the vicinity of a single mechanical resonance. The R_1 represents the heat losses due to mechanical friction in the crystal. The inductor L_1 is the electrical equivalent of the crystal mass, the capacitor C_1 represents the crystal elasticity.

Static capacitance defined

C_0 is called the *static* capacitance – it is the capacitance associated with the crystal and its adherent electrodes plus the stray capacitance internal to the crystal enclosure. It is a measured quantity that is specified on a datasheet. The value of C_0 does not include stray or wiring capacitance external to the enclosure.

Resonance

If we define:

$$\mathbf{Z}_0 = \frac{1}{j\omega C_0} \tag{4.1}$$

and:

$$\mathbf{Z}_1 = R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \tag{4.2}$$

then the impedance of the equivalent circuit in Figure 4.4 at any frequency is given by:

$$\begin{aligned}
\mathbf{Z}_e &= \frac{\mathbf{Z}_1\mathbf{Z}_0}{\mathbf{Z}_1 + \mathbf{Z}_0} \\
&= \frac{\left(R_1 + j\omega L_1 + \dfrac{1}{j\omega C_1}\right)\dfrac{1}{j\omega C_0}}{R_1 + j\omega L_1 + \dfrac{1}{j\omega C_1} + \dfrac{1}{j\omega C_0}} \\
&= \frac{\dfrac{\omega L_1 - 1/\omega C_1}{\omega C_0} - j\dfrac{R_1}{\omega C_0}}{R_1 + j\left(\omega L_1 - \dfrac{1}{\omega C_1} - \dfrac{1}{\omega C_0}\right)} \\
&= \frac{a + jb}{c + jd} \times \frac{c - jd}{c - jd} \\
&= \frac{ac + bd + j(bc - ad)}{c^2 + d^2} \\
&= R_e + jX_e
\end{aligned} \tag{4.3}$$

4.10

For a resonance to occur, $\mathbf{Z}_e$ must be resistive and, therefore:

$$X_e = \frac{bc - ad}{c^2 + d^2} = 0 \tag{4.4}$$

We then have:

$$\begin{aligned}
bc &= ad \\
\frac{-R_1^2}{\omega_r C_0} &= \left(\frac{\omega_r L_1 - 1/\omega_r C_1}{\omega_r C_0}\right)\left(\omega_r L_1 - \frac{1}{\omega_r C_1} - \frac{1}{\omega_r C_0}\right) \\
-R_1^2 &= \left(\omega_r L_1 - \frac{1}{\omega_r C_1}\right)\left(\omega_r L_1 - \frac{1}{\omega_r C_1} - \frac{1}{\omega_r C_0}\right)
\end{aligned} \tag{4.5}$$

$$\omega_r^2 L_1^2 - \frac{L_1}{C_1} - \frac{L_1}{C_0} - \frac{L_1}{C_1} + \frac{1}{\omega_r^2 C_1^2} + \frac{1}{\omega_r^2 C_1 C_0} + R_1^2 = 0 \tag{4.6}$$

$$\omega_r^4 + \left(\frac{R_1^2}{L_1^2} - \frac{2}{L_1 C_1} - \frac{1}{L_1 C_0}\right)\omega_r^2 + \frac{1}{L_1^2 C_1^2} + \frac{1}{L_1^2 C_1 C_0} = 0 \tag{4.7}$$

Using the quadratic formula and solving for ω_r^2 gives:

$$\begin{aligned}
\omega_r^2 = \frac{1}{2}\Bigg\{&\left(\frac{2}{L_1 C_1} + \frac{1}{L_1 C_0} - \frac{R_1^2}{L_1^2}\right) \\
&\pm\left[\left(\frac{2}{L_1 C_1} + \frac{1}{L_1 C_0} - \frac{R_1^2}{L_1^2}\right)^2 - 4\left(\frac{1}{L_1^2 C_1^2} + \frac{1}{L_1^2 C_1 C_0}\right)\right]^{1/2}\Bigg\}
\end{aligned} \tag{4.8}$$

Therefore:

$$\omega_r = \left\{ \left(\frac{1}{L_1 C_1} + \frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right) \pm \left[\left(\frac{1}{L_1 C_1} + \frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right)^2 - \left(\frac{1}{L_1^2 C_1^2} + \frac{1}{L_1^2 C_1 C_0} \right) \right]^{1/2} \right\}^{1/2} \tag{4.9}$$

We now consider the quantity in the inner square root:

$$\begin{aligned} &\left[\left(\frac{1}{L_1 C_1} + \frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right)^2 - \left(\frac{1}{L_1^2 C_1^2} + \frac{1}{L_1^2 C_1 C_0} \right) \right]^{1/2} \\ &= \left(\frac{1}{L_1^2 C_1^2} + \frac{1}{L_1^2 C_1 C_0} - \frac{R_1^2}{L_1^3 C_1} + \frac{1}{4L_1^2 C_0^2} - \frac{R_1^2}{2L_1^3 C_1 C_0} + \frac{R_1^4}{4L_1^4} - \frac{1}{L_1^2 C_1^2} - \frac{1}{L_1^2 C_1 C_0} \right)^{1/2} \\ &= \left[\left(\frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right)^2 - \frac{R_1^2}{L_1^3 C_1} \right]^{1/2} \end{aligned} \tag{4.10}$$

Therefore, the *exact* equation for the resonance condition is:

$$\omega_r = \left\{ \left(\frac{1}{L_1 C_1} + \frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right) \pm \left[\left(\frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right)^2 - \frac{R_1^2}{L_1^3 C_1} \right]^{1/2} \right\}^{1/2} \tag{4.11}$$

For any practical crystal, however, it is normally true that:

$$\left(\frac{1}{2L_1 C_0} - \frac{R_1^2}{2L_1^2} \right)^2 >> \frac{R_1^2}{L_1^3 C_1} \tag{4.12}$$

For example, typical values for an 8 MHz crystal are $L_1 = 14\,\text{mH}$, $C_1 = 27\,\text{fF}$, $R_1 = 8\,\Omega$ and $C_0 = 5.6\,\text{pF}$. Thus the inequality above is $7.972 \times 10^{21} >> 1.693 \times 10^{17}$, which is certainly true.

Then:

$$\left[\left(\frac{1}{2L_1C_0}-\frac{R_1^2}{2L_1^2}\right)^2-\frac{R_1^2}{L_1^3C_1}\right]^{1/2}\approx\left(\frac{1}{2L_1C_0}-\frac{R_1^2}{2L_1^2}\right) \tag{4.13}$$

We now have an *approximate* formula for the resonance frequencies:

$$\omega_r\approx\left\{\left(\frac{1}{L_1C_1}+\frac{1}{2L_1C_0}-\frac{R_1^2}{2L_1^2}\right)\pm\left(\frac{1}{2L_1C_0}-\frac{R_1^2}{2L_1^2}\right)\right\}^{1/2} \tag{4.14}$$

This equation gives two resonance frequencies; the first, obtained using the minus sign, is series resonance:

The approximate series resonance frequency of a crystal

$$\boxed{\omega_{rs}\approx\frac{1}{\sqrt{L_1C_1}}} \tag{4.15}$$

and the other is parallel resonance:

$$\omega_{rp}\approx\sqrt{\frac{1}{L_1C_1}+\frac{1}{L_1C_0}-\frac{R_1^2}{L_1^2}} \tag{4.16}$$

We make the further observation that a typical crystal has:

$$\frac{1}{L_1C_0}>>\frac{R_1^2}{L_1^2} \tag{4.17}$$

For example, for the crystal considered earlier, we have $1.276\times10^{13}>>3.265\times10^{5}$. Then:

The approximate parallel resonance frequency of a crystal

$$\boxed{\omega_{rp}\approx\sqrt{\frac{1}{L_1C_1}+\frac{1}{L_1C_0}}} \tag{4.18}$$

Furthermore, the parallel resonance frequency can be written as:

$$\omega_{rp} \approx \sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 C_0}} = \frac{1}{\sqrt{L_1 C_1}} \left(1 + \frac{C_1}{C_0}\right)^{1/2} \tag{4.19}$$

But $C_1/C_0 << 1$. Therefore, the binomial approximation:

$$(1+x)^n \approx 1 + nx \quad \text{if} \quad x << 1 \tag{4.20}$$

may be used, and:

$$\boxed{\omega_{rp} \approx \omega_{rs}\left(1 + \frac{C_1}{2C_0}\right)} \tag{4.21}$$

Hence, the frequency separation, Δf, between f_{rp} and f_{rs} is:

$$\boxed{\Delta f = f_{rp} - f_{rs} \approx f_{rs}\left(\frac{C_1}{2C_0}\right)} \tag{4.22}$$

The approximate frequency separation between the series and parallel resonance frequencies of a crystal

The frequency separation is very small, typically <0.3% of the series resonance frequency. The frequency range Δf is known as the *pulling range*.

Typical real and imaginary parts of the impedance plots versus frequency of the crystal impedance are shown below:

Typical real and imaginary plots of a crystal's impedance

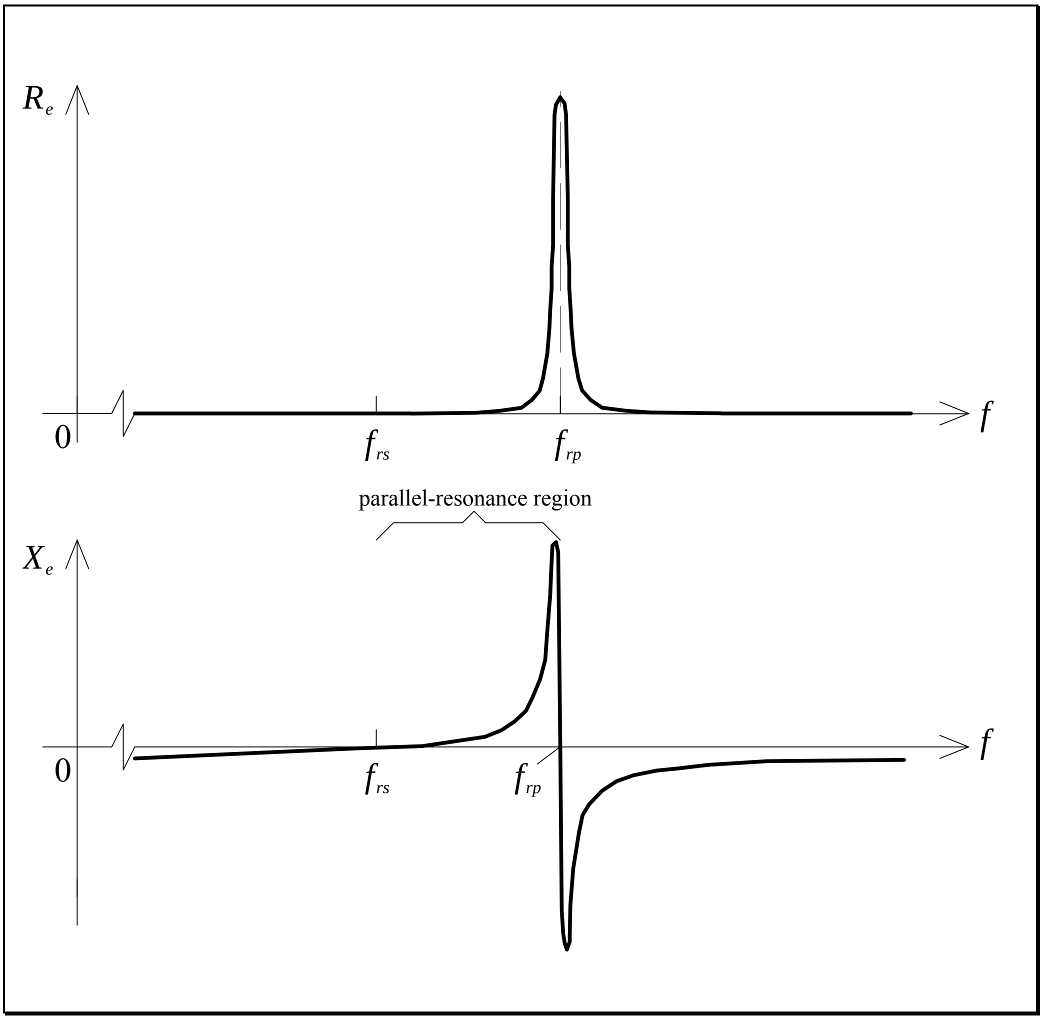


Figure 4.5 – Typical real and imaginary plots of a crystal's impedance

Note that the horizontal scale has been zoomed into the frequencies around resonance to show how the reactance is positive for only a very small range of frequencies between f_{rs} and f_{rp}.

Beware of using the approximate formula for parallel resonance – the reactance curve is nearly vertical in this region

Also note that the approximate formula for parallel resonance, given by Eq. (4.18), will make $d = 0$ in Eq. (4.3). Such a substitution in Eq. (4.3) will give $\mathbf{Z}_e = (a + jb)/c$, which has a non-zero imaginary component. However, the impedance is *highly* sensitive to frequency, and at the exact f_{rp}, the impedance $\mathbf{Z}_e$ will be purely resistive.

Typical magnitude (note the log scale) and phase plots of a crystal's impedance versus frequency are shown below:

Typical magnitude and phase plots of a crystal's impedance

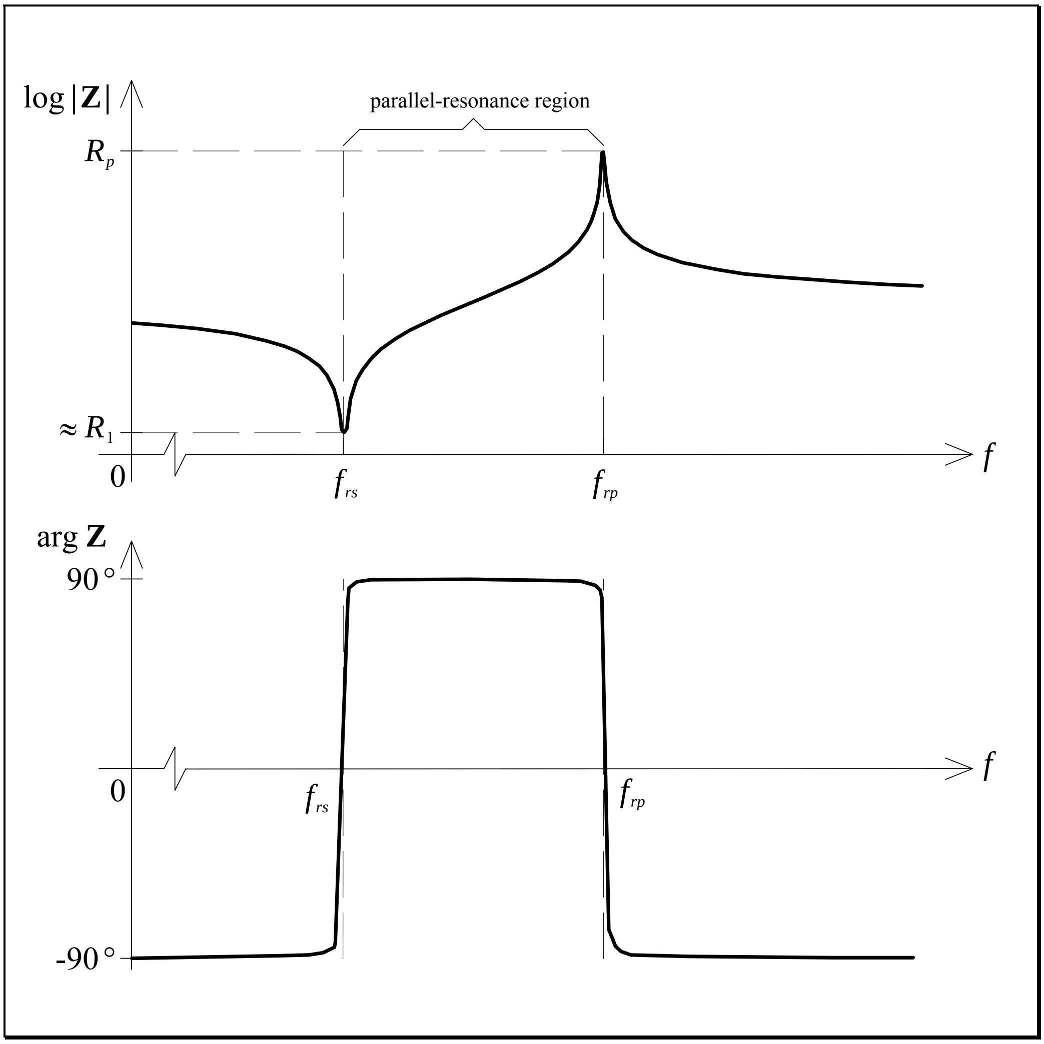


Figure 4.6 – Typical magnitude and phase plots of a crystal's impedance

This plot shows that the circuit has a series and a parallel resonance. At the series resonance frequency f_{rs}, $X_{L_1} = -X_{C_1}$ and the crystal branch impedance is simply R_1. The resistor R_1 is also called the equivalent series resistance (ESR) at series resonance. At series resonance, the resistor R_1 appears in parallel with the reactance X_{C_0}. However, $\left|X_{C_0}\right| >> R_1$ at this frequency, so the crystal essentially appears resistive. Between f_{rs} and f_{rp} the impedance is inductive with the phase being close to 90°. This is an important region of operation, which is called the *parallel-resonance region*.

ESR defined

The parallel-resonance region defined

Equivalent Circuit Near Resonance

The crystal is inductive in the parallel-resonance region between f_{rs} and f_{rp}, and so in this "region of operation" it seems appropriate to represent the crystal by its equivalent series resistance and inductance:

Equivalent model of a crystal in the parallel-resonance region

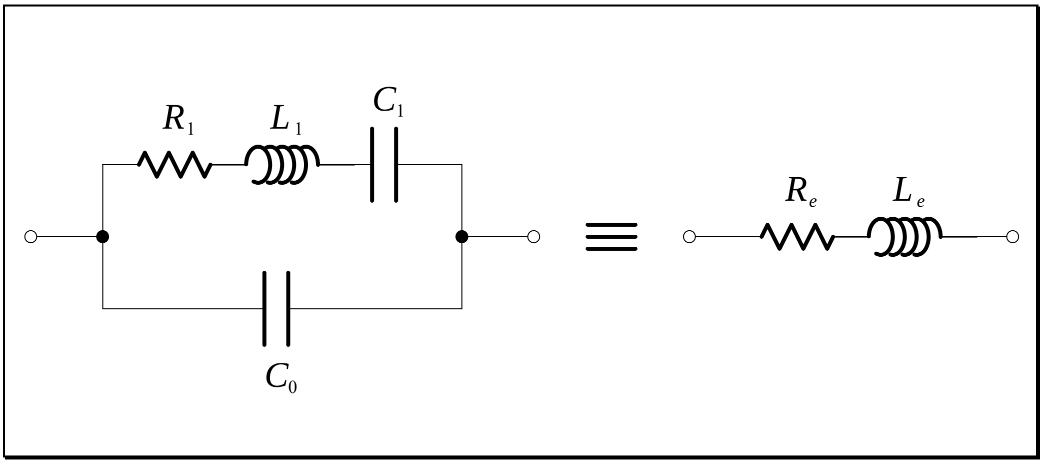


Figure 4.7 – Equivalent model of a crystal in the parallel-resonance region

If we let:

$$jX_1 = j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \tag{4.23}$$

then the impedance of the crystal is:

$$\begin{aligned}
\mathbf{Z}_e &= \frac{jX_{C_0}\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_{C_0}\right)} \\
&= \frac{R_1 X_{C_0}^2}{R_1^2 + \left(X_1 + X_{C_0}\right)^2} + j\frac{X_{C_0}\left[R_1^2 + X_1\left(X_1 + X_{C_0}\right)\right]}{R_1^2 + \left(X_1 + X_{C_0}\right)^2} \\
&= R_e + jX_e
\end{aligned} \tag{4.24}$$

This equivalent series impedance will be useful later when we analyse oscillators that use crystals.

Modes of Crystal Operation

Because of its resonant characteristics, a crystal is operated either as a series resonant circuit or in the parallel-resonance region.

The frequency listed on a crystal package is the desired frequency of operation - it can be for a series-mode oscillator or for a parallel-mode oscillator with a specific load capacitance

A series-mode oscillator uses a crystal in a series-resonant configuration where it appears as a pure resistor of value R_1 (C_0 is ignored) at the frequency f_{rs}.

A parallel-mode oscillator uses a crystal in the parallel-resonance region, where a "load capacitance" C_L is specified to be in parallel with the crystal, as shown below:

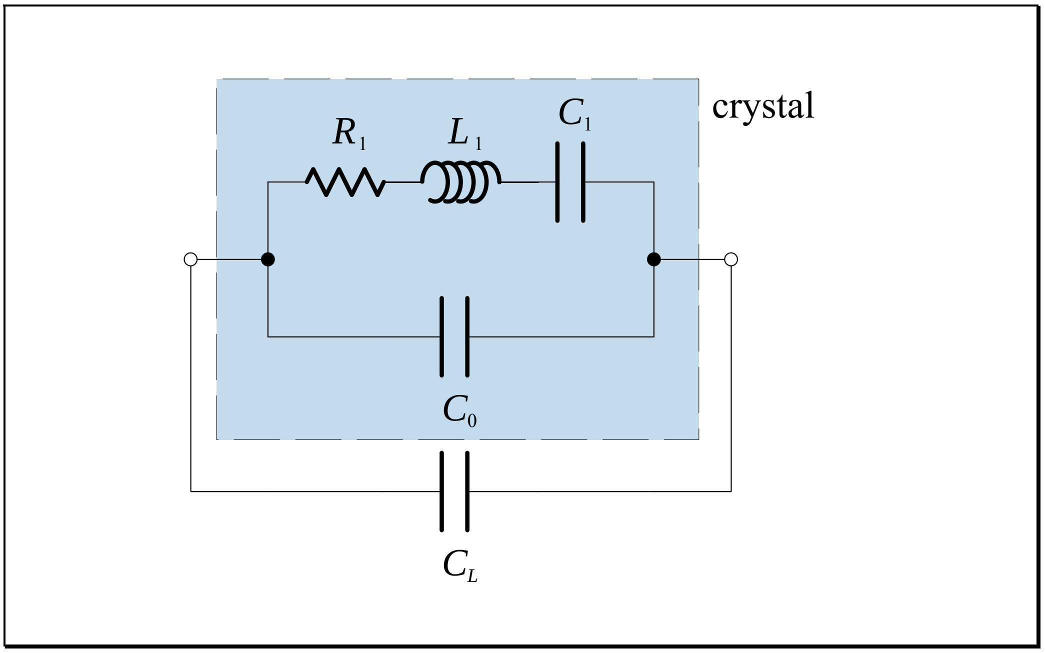


Figure 4.8 – Equivalent model of a crystal with a load capacitance

In the parallel mode of operation the crystal appears inductive. Its parallel resonance, denoted as the load-resonance frequency f_L, is given by:

A "loaded" crystal's resonance frequency

$$f_L \approx f_{rs}\left(1+\frac{C_1}{2(C_0+C_L)}\right) \tag{4.25}$$

This equation follows from Eq. (4.21) with C_0 replaced by $C_0 + C_L$.

Crystals in the parallel mode are specified by the manufacturer to resonate at the frequency f_L with a specific load capacitance C_L.

Ceramic Resonators

Ceramic resonators defined

A ceramic resonator (CR) is a polycrystalline ferroelectric material in the barium titanate and zirconate families of ceramic. A ceramic resonator is made by forming the ceramic mixture to the desired shape and then heating it above its ferroelectric transition point. A high voltage is applied, and the molecular electric dipoles align with the strong electric field. The ceramic is then slowly cooled, and the high voltage is disconnected. The preferential orientations of the electric dipoles of the polycrystalline aggregate yield a permanent electric moment which is equivalent to piezoelectricity.

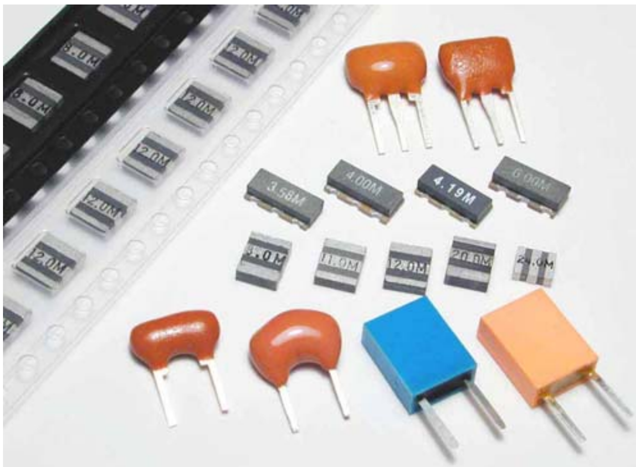


Figure 4.9 – Ceramic resonators

The circuit symbol of a ceramic resonator is the same as that of a crystal, and the equivalent electrical circuit is the same, except for the addition of a shunt resistance, R_0. It comes about from the presence of DC conduction paths around the polycrystalline material and is intrinsic to the material.

Ceramic resonators are similar to crystals, but have poorer performance

The performance of a CR oscillator is better than an *LC* tuned oscillator, and less than that of a crystal oscillator. CRs are smaller in size and less expensive than crystals, and are therefore used in lower-frequency applications where moderate stability is suitable and cost is an overriding consideration. One drawback is their relatively large temperature coefficient of frequency, being in the range -40 to -80 ppm / °C. Some ceramic resonators are pre-packaged with their “load capacitances” and therefore have three terminals.

Oscillation Conditions

A basic feedback oscillator is shown below:

The basic form of a harmonic oscillator

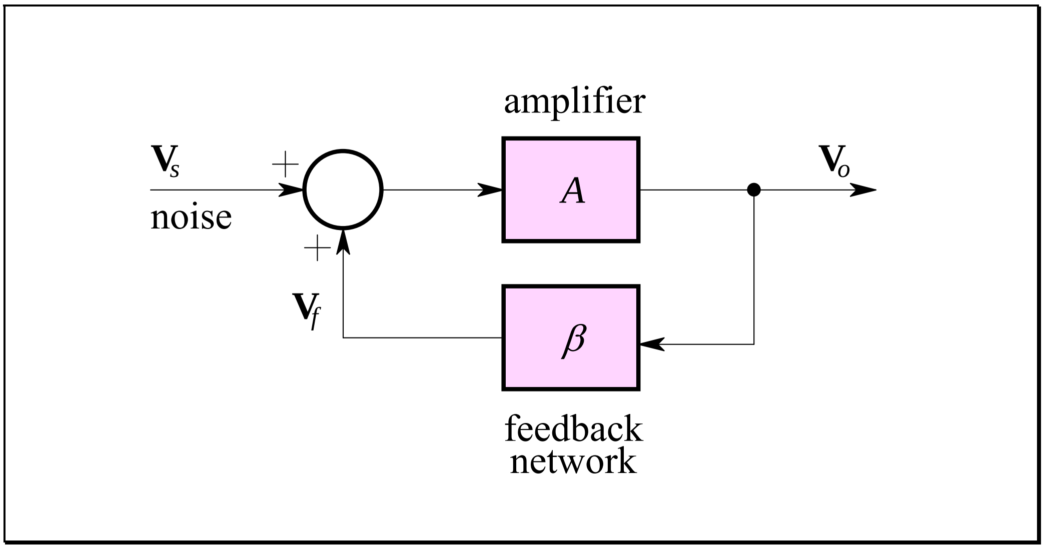


Figure 4.10 – The basic form of a harmonic oscillator

The amplifier's voltage gain is $A(j\omega)$, and the voltage feedback network is described by the frequency response $\beta(j\omega)$. In many oscillators, at the frequency of oscillation, the amplifier is operating in its midband region where $A(j\omega)$ is a real constant, denoted by A_o.

The summing junction in the figure suggests that the feedback is positive. However, the phase of $\mathbf{V}_f$ determines if $\mathbf{V}_f$ adds or subtracts from $\mathbf{V}_s$. From the figure we have:

$$\mathbf{V}_o = A(\mathbf{V}_s + \mathbf{V}_f) \tag{4.26a}$$

$$\mathbf{V}_f = \beta \mathbf{V}_o \tag{4.1b}$$

Thus, the closed-loop voltage gain is:

$$\boxed{\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{A}{1 - A\beta}} \tag{4.27}$$

Loop gain defined

The *loop gain* is defined as $A\beta$.

For oscillations to occur, an output signal must exist with no input signal applied. With $\mathbf{V}_s = 0$ it follows that a finite $\mathbf{V}_o$ is possible only when the denominator of Eq. (4.27) is zero. That is, when:

The Barkhausen criterion for oscillation in a harmonic oscillator

$$\boxed{A\beta = 1} \tag{4.28}$$

Thus, for oscillations to occur, the loop gain must be unity. This relation is known as the Barkhausen criterion. Since A and β are complex, it follows that:

$$\left|A\beta\right| = 1 \tag{4.29a}$$

$$\angle A\beta = \pm n360° \tag{4.4b}$$

where n is an integer.

If the amplifier is operating in the midband, and A_o is a negative number (i.e. the amplifier is an inverter), then the phase shift through the amplifier is $180°$. In this case the feedback network β needs to provide an additional phase shift of $180° \pm n360°$, so that the total phase shift associated with the closed-loop is $0°$ or a multiple of $360°$.

From circuit theory we know that oscillation occurs when a network has a pair of complex conjugate poles on the imaginary axis. However, in electronic oscillators the poles are not exactly on the imaginary axis because of the nonlinear nature of the loop gain. There are different nonlinear effects that control the pole location in an oscillator. One nonlinear mechanism is due to the saturation characteristics of the amplifier.

An intuitive explanation of the operation of a harmonic oscillator

A saturation-limited sinusoidal oscillator works as follows. To start the oscillation, the closed-loop gain in Eq. (4.27) must have a pair of complex-conjugate poles in the right-half plane. Then, due to the noise voltage generated by thermal vibrations in the network (which can be represented by a superposition of input noise signals v_s) or by the transient generated when the DC power supply is turned on, a growing sinusoidal output voltage appears.

The characteristics of the growing sinusoidal signal are determined by the complex conjugate poles in the right-half plane. As the amplitude of the induced oscillation increases, the amplitude-limiting capabilities of the amplifier (i.e., a reduction in gain) produce a change in the location of the poles. The changes are such that the complex-conjugate poles move towards the imaginary axis. Once the poles move to the left-half plane the amplitude of the oscillation begins to decrease, moving the poles toward the right-half plane. The process of the poles moving between the left-half plane and the right-half plane repeats, and some steady-state oscillation occurs with a fundamental frequency, as well as harmonics. This is a nonlinear process where the fundamental frequency of oscillation and the harmonics are determined by the location of the poles. Although the poles are not on the imaginary axis, the Barkhausen criterion in Eq. (4.28) predicts the fundamental frequency of oscillation fairly well. It can be considered as providing the fundamental frequency of the oscillator based on some sort of average location for the poles.

Oscillator Configurations

Oscillator circuits normally take on the name of their inventor. The Pierce, Colpitts and Clapp oscillators are the same circuit but with the ground point at a different location.

Pierce Oscillator

The Pierce oscillator configuration - the amplifier in a Pierce oscillator is an inverting amplifier

The most popular configuration is the Pierce oscillator shown below:

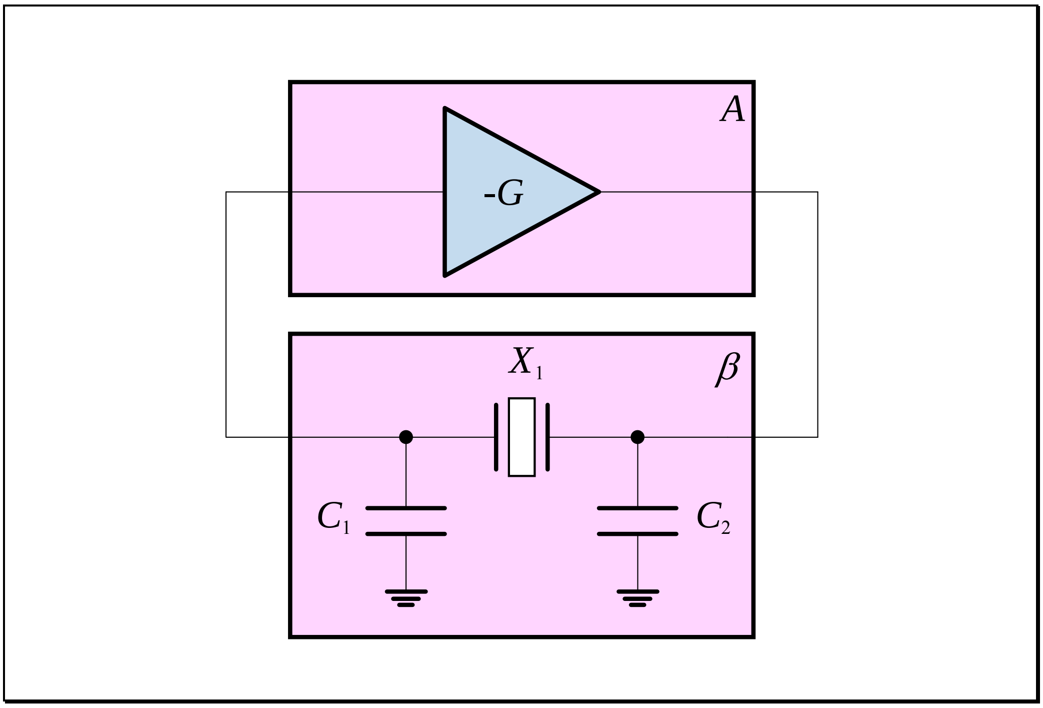


Figure 4.11 – The Pierce oscillator configuration

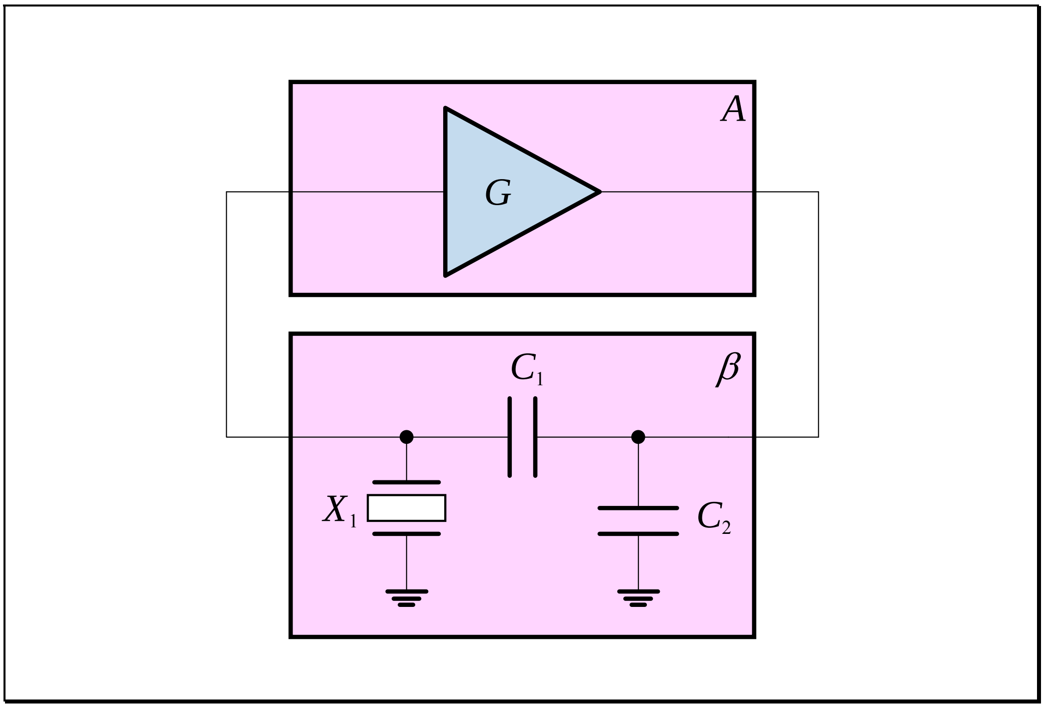
The Pierce oscillator has many desirable characteristics

It has many desirable characteristics. It will work at any frequency from the lowest to the highest – from 1 kHz to 200 MHz. It has very good short-term stability because the crystal's source and load impedances are mostly capacitive rather than resistive, which give it a high in-circuit Q. The circuit provides a large output signal and simultaneously drives the crystal at a low power level. Large shunt capacitances to ground on both sides of the crystal make the oscillation frequency relatively insensitive to stray capacitance, as well as giving the circuit a high immunity to noise.

The Pierce configuration does have one disadvantage – it needs a relatively high amplifier gain to compensate for relatively high gain losses in the circuitry surrounding the crystal.

Colpitts Oscillator

The configuration of the Colpitts oscillator is shown below:



The Colpitts oscillator configuration - the amplifier in a Colpitts oscillator is a noninverting amplifier

Figure 4.12 – The Colpitts oscillator configuration

The main advantage over the Pierce configuration is a very low current consumption, although the component selection is more critical. It also has the advantage of producing low amplitude sinusoidal oscillations, which reduces RF emissions.

Other Configurations

There are many other circuit configurations, such as the Clapp, Butler and Meacham oscillators. They are generally more complicated to design, and use a greater number of parts, than the Pierce and Colpitts circuits. They can be based on discrete transistor amplifiers or on digital IC gates. Some configurations are suitable only in particular applications.

The Pierce Oscillator using Digital IC Inverters

The popular Pierce oscillator, using digital IC inverters, is shown below:

The Pierces oscillator using digital IC inverters

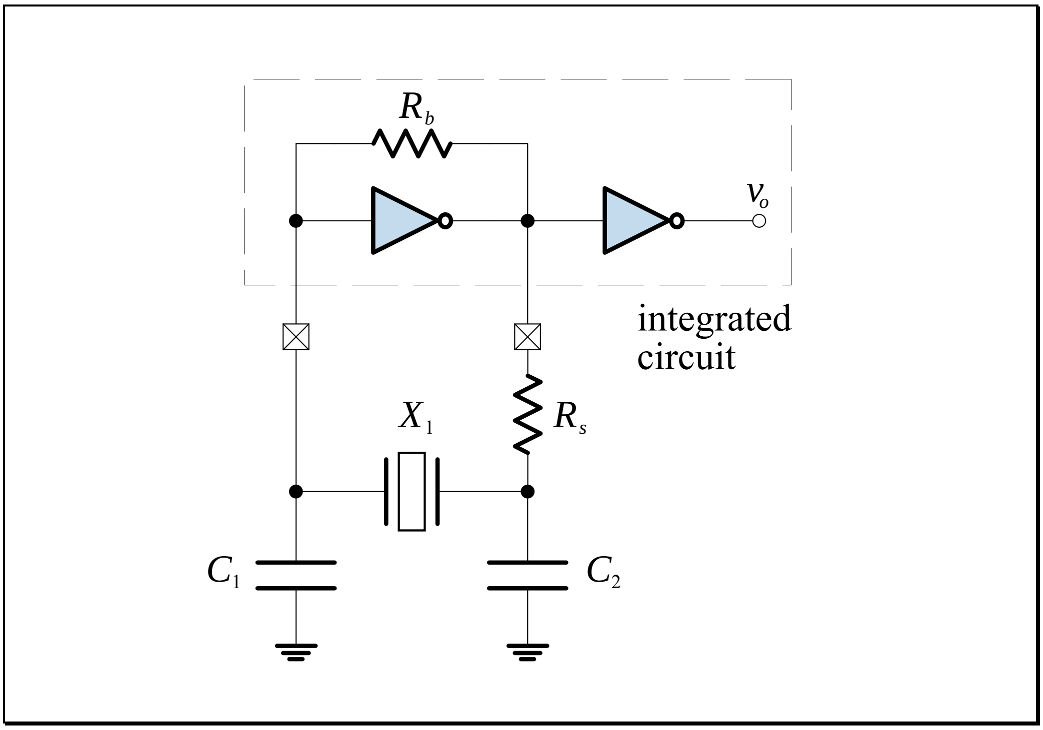


Figure 4.13 – The Pierce oscillator using digital IC inverters

The resistor R_b is necessary to bias the IC inverter so that it operates in the linear region of its input-output voltage characteristic. Thus, the inverter acts as a high gain inverting amplifier. This resistor is usually built into the dedicated clock module of a microcontroller. The resistor R_s is used to control the "crystal drive", i.e. the crystal current. The second inverter is used as a digital inverter and will produce a square wave signal instead of a sinusoid.

Virtually all digital IC clock oscillators are of the Pierce type, as the circuit can be implemented using a minimum of components: two digital inverters, two resistors, two capacitors, and the quartz crystal. The low manufacturing cost of this circuit, and the outstanding frequency stability of the quartz crystal, give it an advantage over other designs in many consumer electronics applications.

Analysis of Oscillators

The ideal structure of any oscillator configuration is shown below:

Ideal structure of any oscillator

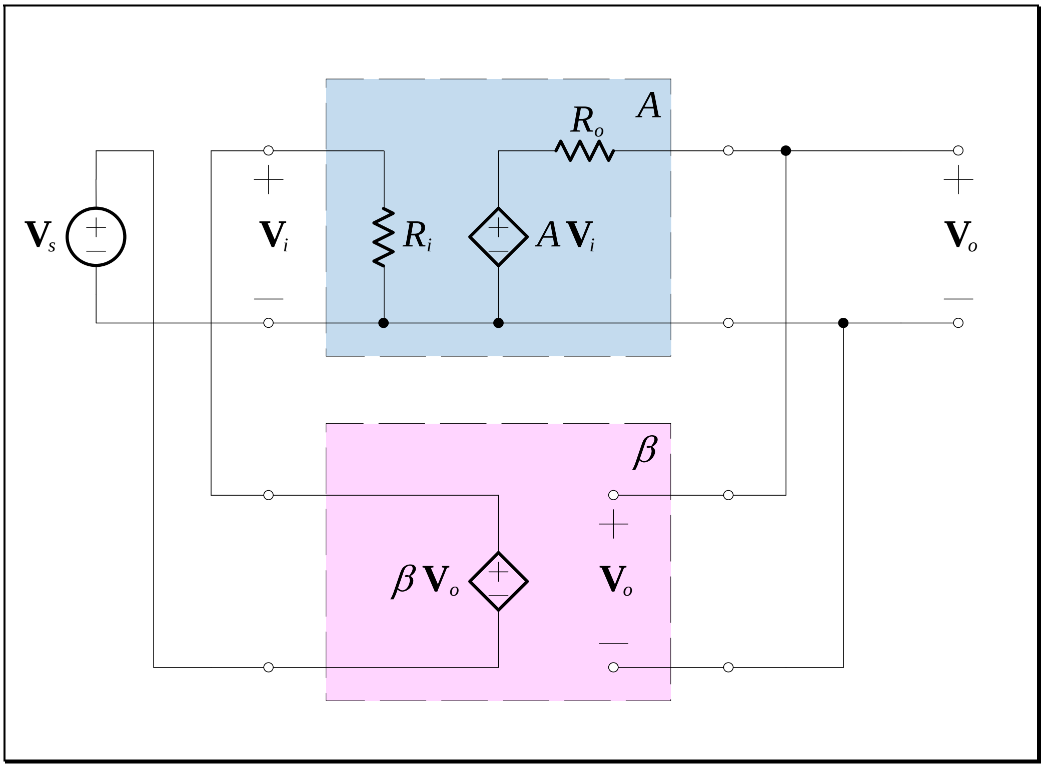


Figure 4.14 – Ideal structure of any oscillator

It consists of a *unilateral* open-loop amplifier (the A circuit) and an ideal voltage-sampling, voltage-mixing feedback network (the β circuit). The A circuit is represented by its Thévenin equivalent, and the β circuit does *not* load the A circuit.

The circuit of Figure 4.14 exactly follows the ideal feedback model of Figure 4.10. Therefore the closed-loop voltage gain is given by:

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{A}{1 - A\beta} \tag{4.30}$$

A practical oscillator is shown below:

Practical structure of any harmonic oscillator

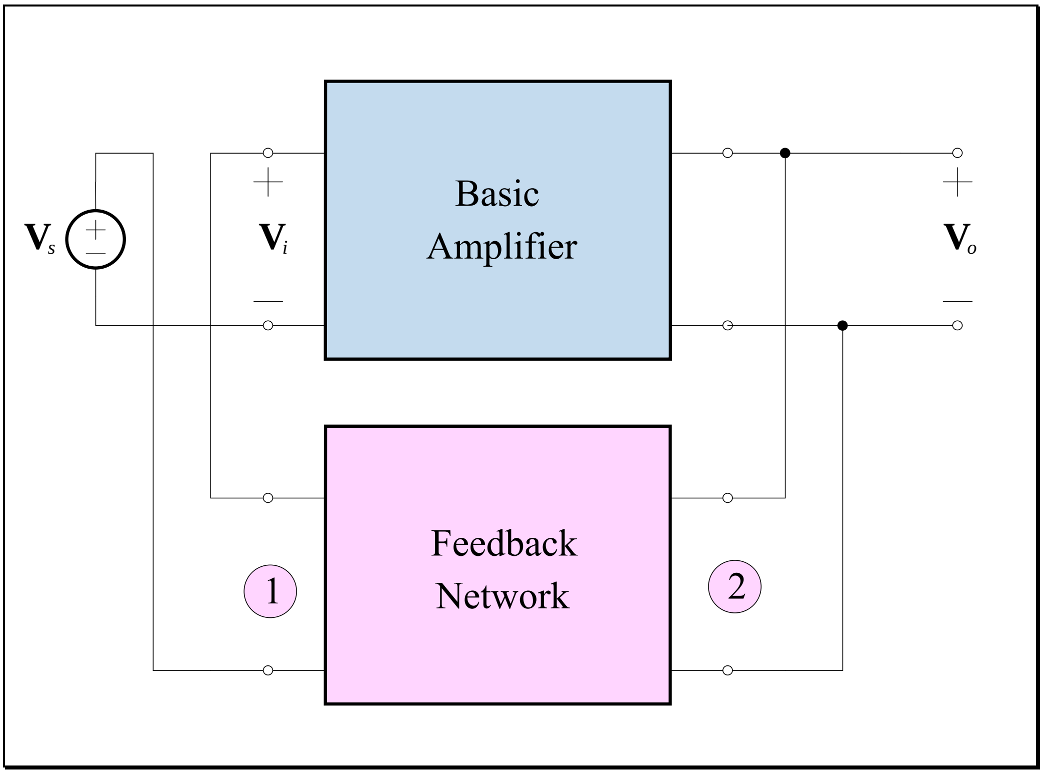


Figure 4.15 – Practical structure of any harmonic oscillator

In a practical oscillator, the feedback network will not be an ideal voltage-controlled voltage source, Rather, the feedback network is usually composed of reactive elements and hence will load the basic amplifier and thus affect the values of A, R_i and R_o.

We need to find the A circuit and the β circuit that corresponds to this practical case. The problem essentially involves representing the amplifier and feedback network of Figure 4.15 by the ideal structure of Figure 4.14.

As a first step, we can represent the two-port feedback network in terms of its *h* parameters (this choice is based on the fact that this is the only parameter set that represents the feedback network by a series network at port 1 and a parallel network at port 2):

Circuit showing the *h* parameters of the feedback circuit

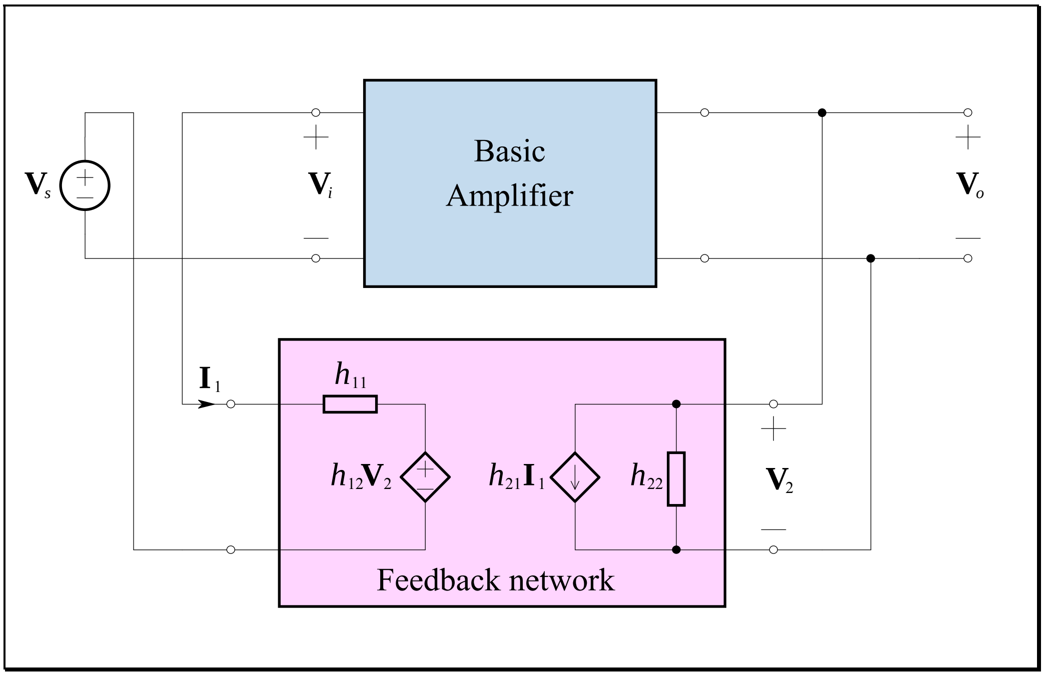


Figure 4.16 – Circuit showing the *h* parameters of the feedback circuit

The current source $h_{21}\mathbf{I}_1$ represents the forward transmission of the feedback network. Since the basic amplifier has a very large input resistance, the current $\mathbf{I}_1$ will be very small, and therefore the forward transmission $h_{21}\mathbf{I}_1$ of the feedback network can be neglected. We will thus omit the controlled source $h_{21}\mathbf{I}_1$ altogether.

If we now include h_{11} and h_{22} with the basic amplifier, we obtain the circuit shown below:

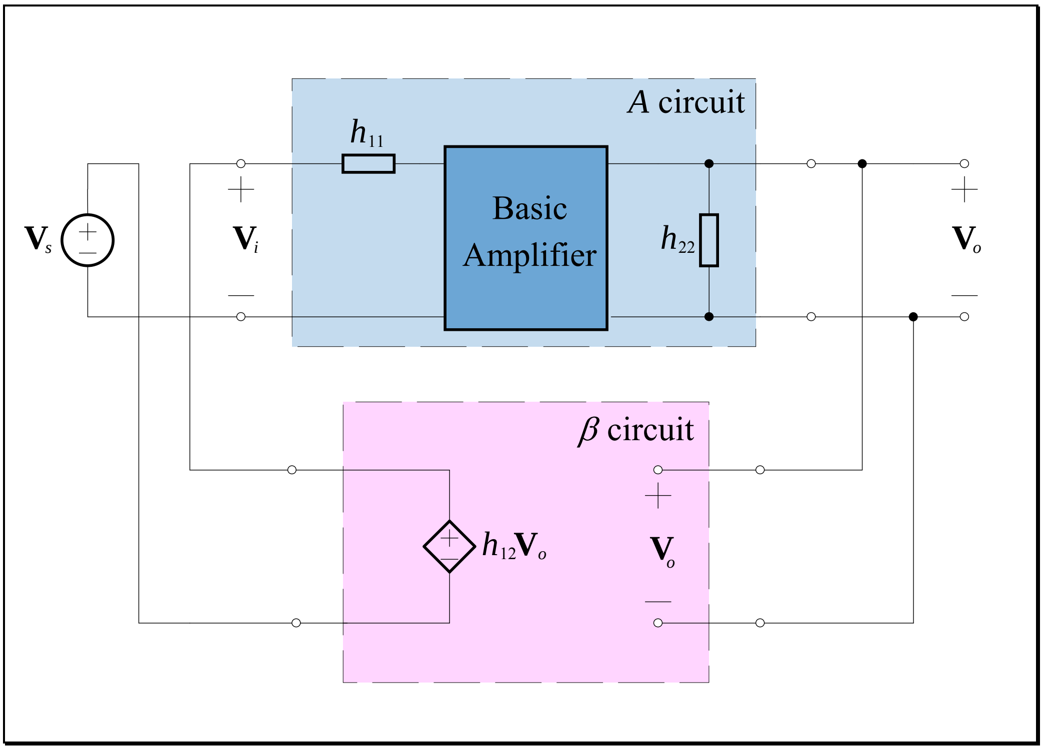


Figure 4.17 – The previous circuit with h_{21} neglected

If we now insert the Thévenin equivalent of the basic amplifier we get:

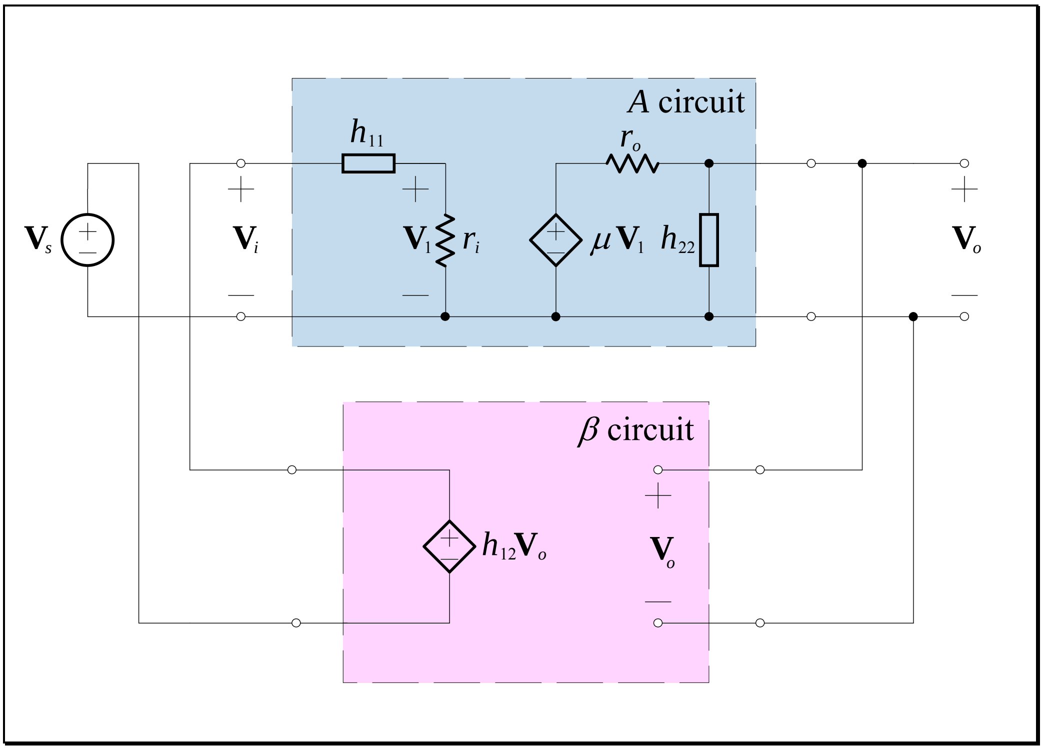


Figure 4.18 – The previous circuit with the basic amplifier shown

We can now determine the *A* circuit:

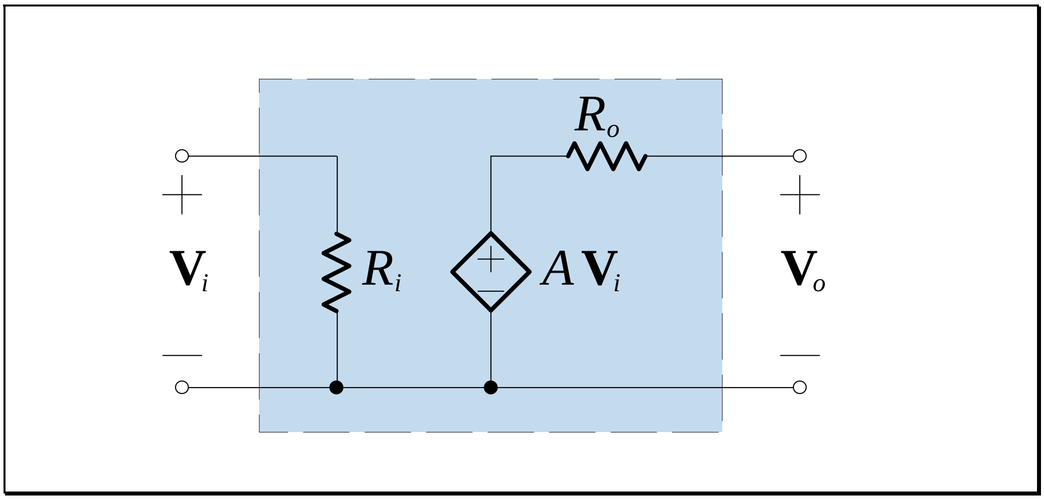


Figure 4.19 – The *A* circuit

where, from Figure 4.18, we have:

The *A* circuit's parameters

$$R_i = h_{11} + r_i$$
$$R_o = r_o \,\|\, h_{22}$$
$$A = \frac{h_{22}}{h_{22} + r_o} \mu \frac{r_i}{r_i + h_{11}} \tag{4.31}$$

We can also determine the β circuit:

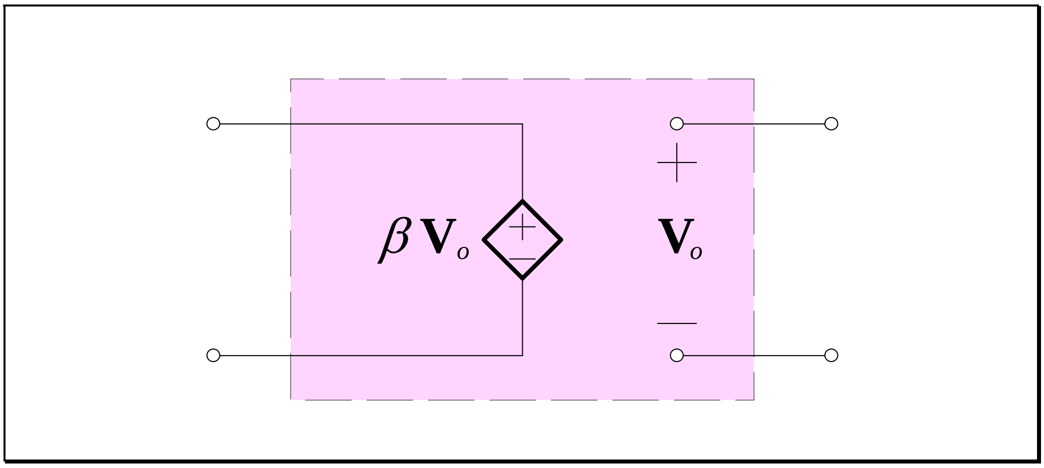


Figure 4.20 – The β circuit

where, from Figure 4.18, we have:

The β circuit's parameters

$$\beta = h_{12} \tag{4.32}$$

Analysis of the Pierce Oscillator

In many oscillators the feedback network is a frequency-selecting π-network:

The feedback network for many oscillators

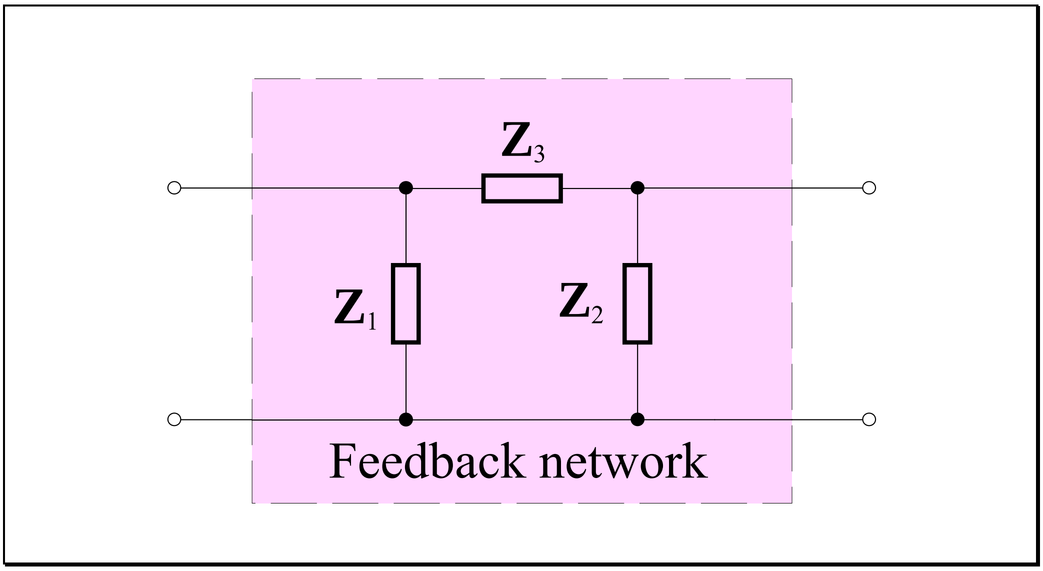


Figure 4.21 – The feedback network for many oscillators

From this network, we get:

The *h* parameters for the typical feedback network

$$
\begin{aligned}
h_{11} &= \mathbf{Z}_1 \parallel \mathbf{Z}_3 \\
h_{22} &= \mathbf{Z}_2 \parallel \left(\mathbf{Z}_1 + \mathbf{Z}_3\right) \\
h_{12} &= \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_3}
\end{aligned}
\tag{4.33}
$$

Therefore, we have:

$$
A = \frac{\mathbf{Z}_2 \parallel \left(\mathbf{Z}_1 + \mathbf{Z}_3\right)}{\mathbf{Z}_2 \parallel \left(\mathbf{Z}_1 + \mathbf{Z}_3\right) + r_o} \mu \frac{r_i}{r_i + \mathbf{Z}_1 \parallel \mathbf{Z}_3} \tag{4.34}
$$

and:

$$
\beta = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_3} \tag{4.35}
$$

We have determined the parameters that apply to the ideal model presented in Figure 4.14. The loop gain is:

$$A\beta = \left(\frac{\mathbf{Z}_2 \,\|\, (\mathbf{Z}_1 + \mathbf{Z}_3)}{\mathbf{Z}_2 \,\|\, (\mathbf{Z}_1 + \mathbf{Z}_3) + r_o} \mu \frac{r_i}{r_i + \mathbf{Z}_1 \,\|\, \mathbf{Z}_3} \right) \left(\frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_3} \right) \tag{4.36}$$

The loop gain equation for the typical configuration

In a Pierce oscillator utilising a gate inverter, the input resistance is very large, since the input is connected to the gate of a MOSFET amplifier. Therefore, if we let $r_i = \infty$, then the loop gain is:

$$\begin{aligned} A\beta &= \left(\frac{\mathbf{Z}_2 \,\|\, (\mathbf{Z}_1 + \mathbf{Z}_3)}{\mathbf{Z}_2 \,\|\, (\mathbf{Z}_1 + \mathbf{Z}_3) + r_o} \mu \right) \left(\frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_3} \right) \\ &= \frac{\mathbf{Z}_1 \mathbf{Z}_2}{\mathbf{Z}_2 (\mathbf{Z}_1 + \mathbf{Z}_3) + r_o (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3)} \mu \end{aligned} \tag{4.37}$$

If $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_3$ are purely reactive impedances given by $\mathbf{Z}_1 = jX_1$, $\mathbf{Z}_2 = jX_2$ and $\mathbf{Z}_3 = jX_3$, then the loop gain can be expressed in the form:

$$\begin{aligned} A\beta &= -\frac{X_1 X_2}{jX_2 (jX_1 + jX_3) + r_o (jX_1 + jX_2 + jX_3)} \mu \\ &= \frac{X_1 X_2}{X_2 (X_1 + X_3) - jr_o (X_1 + X_2 + X_3)} \mu \end{aligned} \tag{4.38}$$

Applying the Barkhausen criterion, we set the loop gain to one for stable oscillations. The phase shift of the loop gain is zero when the imaginary part of Eq. (4.38) is zero. That is, for stable oscillations we must have:

$$\boxed{X_1(\omega_0) + X_2(\omega_0) + X_3(\omega_0) = 0} \tag{4.39}$$

The first condition that must be achieved for stable oscillations to occur

This equation determines the frequency of oscillations, ω_0.

At this frequency the loop gain in Eq. (4.38) reduces to:

$$A(\omega_0)\beta(\omega_0) = \mu \frac{X_1(\omega_0)}{X_1(\omega_0) + X_3(\omega_0)} \tag{4.40}$$

and using Eq. (4.39) we obtain:

The second conditions that must be achieved for stable oscillations to occur

$$A(\omega_0)\beta(\omega_0) = -\mu \frac{X_1(\omega_0)}{X_2(\omega_0)} \tag{4.41}$$

For stable oscillations the loop gain in Eq. (4.41) must be unity. To start the oscillations, the loop gain must be greater than unity. Remembering that the amplifier is an inverter so that μ is a negative number, then $X_1(\omega_0)$ and $X_2(\omega_0)$ must have the same sign.

Thus, if $\mathbf{Z}_1(\omega_0)$ is capacitive (i.e. $X_1(\omega_0) = -1/\omega_0 C_1$), then $\mathbf{Z}_2(\omega_0)$ must also be capacitive (i.e. $X_2(\omega_0) = -1/\omega_0 C_2$). From Eq. (4.39) it also follows that $\mathbf{Z}_3(\omega_0)$ must be inductive since $X_3(\omega_0) = -X_1(\omega_0) - X_2(\omega_0)$ (i.e. $X_3(\omega_0) = \omega_0 L$).

This is precisely the configuration of the Pierce oscillator, where we set the feedback network to be:

The feedback network for a Pierce oscillator

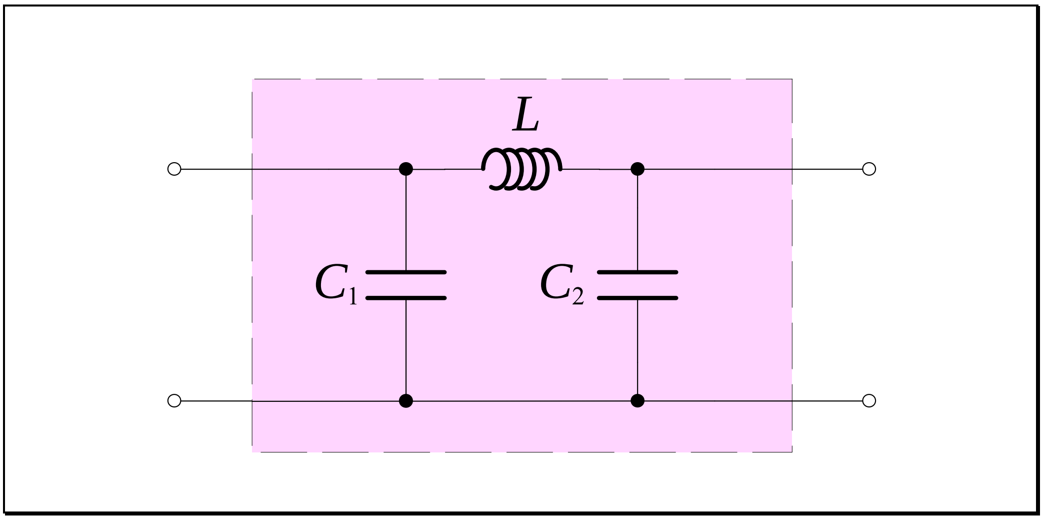


Figure 4.22 – The feedback network for a Pierce oscillator

For such a configuration, the frequency of oscillations, from Eq. (4.39), is given by:

$$-\frac{1}{\omega_0 C_1}-\frac{1}{\omega_0 C_2}+\omega_0 L=0 \tag{4.42}$$

Now we define the *load capacitance* as seen by the inductor as the series combination of C_1 and C_2:

$$\boxed{\frac{1}{C_L}=\frac{1}{C_1}+\frac{1}{C_2}} \tag{4.43}$$

Load capacitance defined

Then the frequency of oscillations is given by:

$$\boxed{\omega_0=\frac{1}{\sqrt{LC_L}}} \tag{4.44}$$

The frequency of oscillation for a Pierce oscillator

From Eq. (4.40), we can see that at ω_0 the gain of the amplifier is $A=\mu$ (which is a negative number) so the phase shift through the amplifier is 180°. From Eq. (4.41) we can see that:

$$\beta(\omega_0)=-\frac{X_1(\omega_0)}{X_2(\omega_0)}=-\frac{-1/\omega_0 C_1}{-1/\omega_0 C_2}=-\frac{C_2}{C_1} \tag{4.45}$$

The feedback network provides 180° of phase shift at the frequency of oscillation

Therefore, the phase shift through the feedback network is also 180° (as it must be for the loop gain to have a phase shift of $\pm n360°$). The gain condition follows from Eq. (4.41), namely:

$$\begin{aligned} A\beta &= -\mu\frac{X_1(\omega_0)}{X_2(\omega_0)}\geq 1 \\ \mu &\leq -\frac{X_2(\omega_0)}{X_1(\omega_0)}=-\frac{C_1}{C_2} \end{aligned} \tag{4.46}$$

The gain condition for oscillations to occur

Intuitive Analysis of the Pierce Oscillator

At ω_0, the load impedance seen by the amplifier:

$$h_{22} = \mathbf{Z}_2 \parallel (\mathbf{Z}_1 + \mathbf{Z}_3) = \frac{jX_2(X_1 + X_3)}{(X_1 + X_2 + X_3)} \tag{4.47}$$

is infinite. Therefore, at ω_0 the feedback network is:

The feedback network at resonance

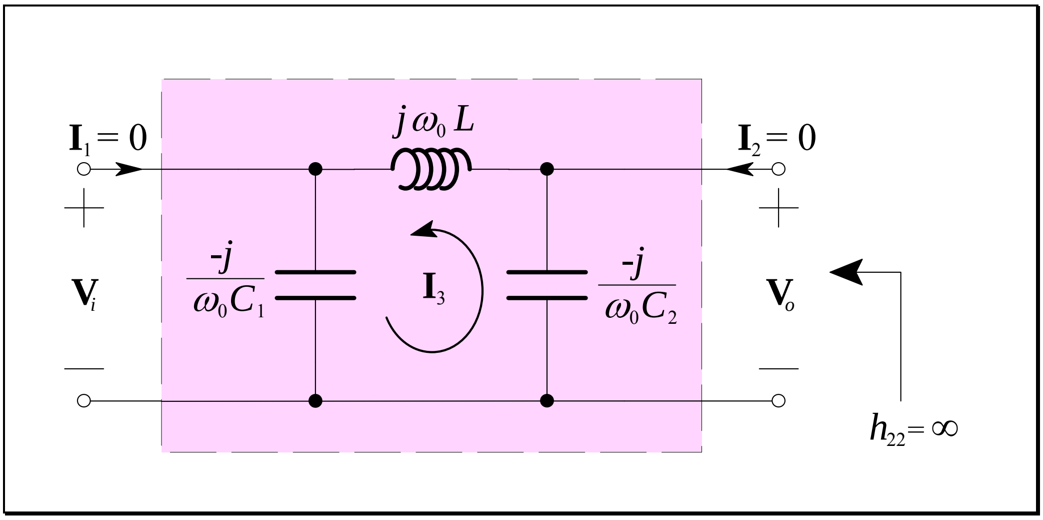


Figure 4.23 – The feedback network at ω_0

Since the feedback network appears as an open-circuit from port 2, then no current enters the network. Also, there is no current out of port 1 since the input impedance of the amplifier, R_i, is effectively an open-circuit. KVL around the interior mesh gives:

$$\left(\frac{-j}{\omega_0 C_1} + \frac{-j}{\omega_0 C_2} + j\omega_0 L \right) \mathbf{I}_3 = 0 \tag{4.48}$$

The current $\mathbf{I}_3$ can be found from Ohm's Law across C_2:

$$\mathbf{I}_3 = -j\omega_0 C_2 \mathbf{V}_o \tag{4.49}$$

which shows that $\mathbf{I}_3$ is non-zero.

Since $\mathbf{I}_3$ is non-zero, then Eq. (4.48) gives:

$$\frac{-j}{\omega_0 C_1} + \frac{-j}{\omega_0 C_2} + j\omega_0 L = 0 \tag{4.50}$$

which is the condition for oscillation again, and identical to Eq. (4.39).

We can also determine the voltage fed back to the amplifier:

$$\begin{aligned}\mathbf{V}_i &= \frac{1}{j\omega_0 C_1}\mathbf{I}_3 \\ &= \frac{1}{j\omega_0 C_1}\left(-j\omega_0 C_2 \mathbf{V}_o\right) \\ &= -\frac{C_2}{C_1}\mathbf{V}_o\end{aligned} \tag{4.51}$$

and thus the feedback factor is:

$$\beta = \frac{\mathbf{V}_i}{\mathbf{V}_o} = -\frac{C_2}{C_1} \tag{4.52}$$

which is identical to Eq. (4.45).

An intuitive description of how the Pierce oscillator works

In other words, since the feedback network as seen from port 2 appears open-circuit, the series combination of L and C_1 must appear inductive to produce with C_2 a parallel-resonant tuned circuit. Hence the current i_3 lags v_o by 90°, and v_i lags i_3 by 90°. Consequently v_i lags v_o by 180°. Since the amplifier provides a phase shift of 180°, the total phase shift around the loop is 360°, as required for oscillation.

Using a Crystal in the Feedback Network

One of the problems with using a real inductor in the Pierce oscillator feedback network is the fact that real inductors have a finite winding resistance, so that the inductor has a relatively low Q_0. As we saw before, a crystal oscillator has a very narrow region of frequencies where it appears inductive, and it has an extremely high Q_0 in this region, typically greater than 10, 000.

The feedback network using a crystal is:

The feedback network for a Pierce oscillator using a crystal

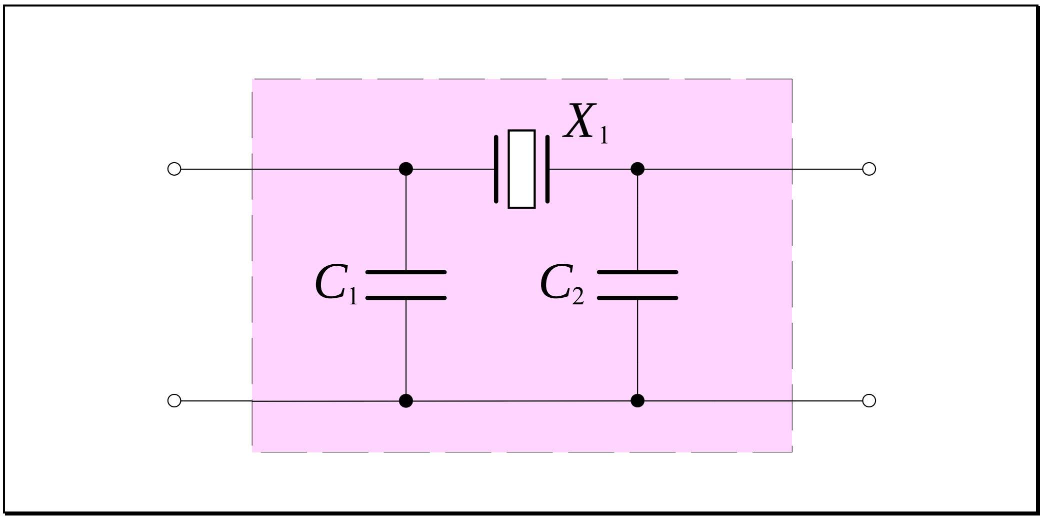


Figure 4.24 – The feedback network for a Pierce oscillator using a crystal

To derive the frequency of oscillation, we start with the loop gain equation, Eq. (4.37):

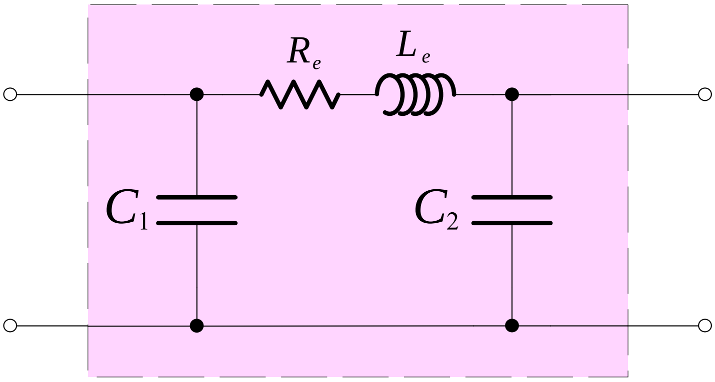
$$A\beta = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_2(\mathbf{Z}_1+\mathbf{Z}_3)+r_o(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3)}\mu \tag{4.53}$$

and let:

$$\mathbf{Z}_1 = jX_1, \quad \mathbf{Z}_2 = jX_2, \quad \mathbf{Z}_3 = R_e + jX_e \tag{4.54}$$

where the impedance $\mathbf{Z}_3$ is the equivalent impedance of the crystal, which needs to be evaluated at a certain frequency.

Thus, the equivalent feedback network is:



The equivalent feedback network for a Pierce oscillator using a crystal

Figure 4.25 – Equivalent feedback network using a crystal

The loop gain equation is then:

$$\begin{aligned} A\beta &= -\frac{X_1 X_2}{jX_2\left(jX_1 + R_e + jX_e\right) + r_o\left(jX_1 + jX_2 + R_e + jX_e\right)}\mu \\ &= -\frac{X_1 X_2 \mu}{-X_1 X_2 - X_2 X_e + r_o R_e + j\left(R_e X_2 + r_o\left(X_1 + X_2 + +X_e\right)\right)} \end{aligned} \tag{4.55}$$

According to the Barkhausen criterion, the imaginary part must be zero for oscillation. Hence, we must have:

$$\boxed{X_1 + X_2 + X_e = -\frac{R_e X_2}{r_o}} \tag{4.56}$$

The condition required for the loop gain to be real

The derivation of the frequency at which this criterion is met is quite complicated, and is detailed in Appendix A.

If we define the *load capacitance* on the crystal as the series combination of C_1 and C_2:

Crystal load capacitance defined

$$\boxed{\frac{1}{C_L} = \frac{1}{C_1} + \frac{1}{C_2}} \tag{4.57}$$

Then it turns out that:

The oscillation frequency of the Pierce oscillator – this is what manufacturers specify

$$\boxed{f_0 \approx f_{rs}\left(1 + \frac{C_1}{2(C_0 + C_L)}\right)} \tag{4.58}$$

As stated before, crystal manufacturers will cut the crystal to achieve resonance at a specified load capacitance, C_L. To design an oscillator, we simply have to choose a crystal which is "parallel" and choose C_1 and C_2 to match the manufacturer's specification for the load capacitance.

Returning to Eq. (4.55), if the imaginary part is zero then the gain requirement of $A\beta > 1$ to start oscillations requires that:

The conditions required to start oscillations

$$\boxed{-\mu X_1 X_2 > R_e r_o - X_2\left(X_1 + X_e\right)} \tag{4.59}$$

This puts a requirement on the open-circuit gain of the amplifier, μ.

Since the product $X_1 X_2$ for a given value of $\left(X_1 + X_2\right)$ is largest when $X_1 = X_2$, designers usually choose $C_1 = C_2$ and therefore:

The design equations for a Pierce oscillator

$$\boxed{C_1 = C_2 = 2C_L} \tag{4.60}$$

Drive Level

Drive level refers to the power dissipation in the crystal. This is important because the power dissipated in the crystal must be limited or the quartz crystal can actually fail due to excessive mechanical vibration. Also, the crystal characteristics change with drive level due to the nonlinear behaviour of the quartz.

Drive level defined

Manufacturers specify the “crystal maximum drive level” which is the maximum power dissipation allowable in the crystal. An overdriven crystal will deteriorate fast. The oscillator circuit determines the crystal drive level.

Using the equivalent feedback network:

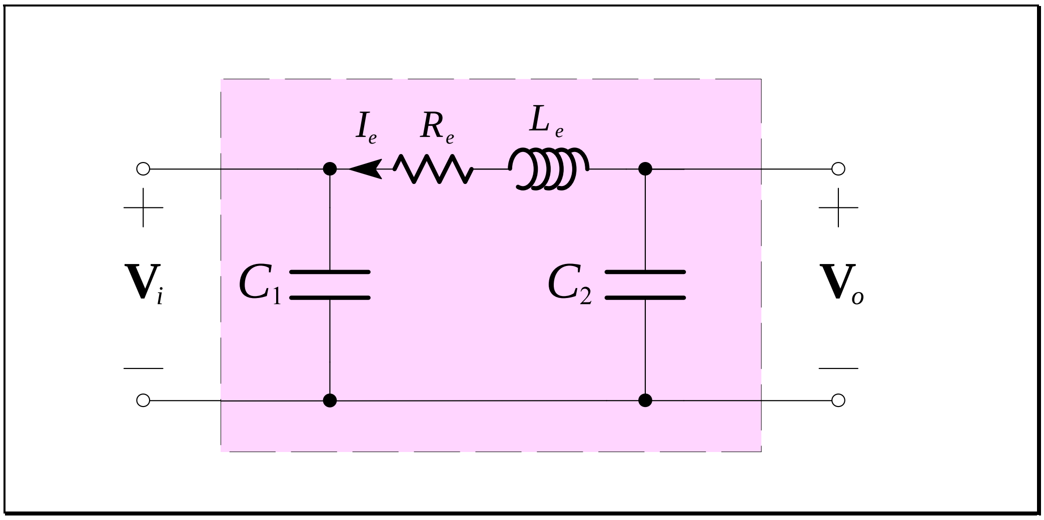


Figure 4.26 – Equivalent feedback network using a crystal

we can see that:

$$\mathbf{I}_e = \frac{\mathbf{V}_o}{R_e + j(X_e + X_1)} \tag{4.61}$$

At the oscillation frequency we have:

$$X_e = \omega_{01} L_e \approx \frac{1}{\omega_{01} C_L} = \frac{1}{\omega_{01} C_1} + \frac{1}{\omega_{01} C_2} \tag{4.62}$$

Therefore:

$$X_e + X_1 \approx \frac{1}{\omega_{01} C_2} = -X_2 \tag{4.63}$$

and:

$$\mathbf{I}_e \approx \frac{\mathbf{V}_o}{R_e - jX_2} \quad \text{and} \quad |\mathbf{I}_e| \approx \frac{|\mathbf{V}_o|}{X_2} \tag{4.64}$$

where we have made the reasonable assumption that $|X_2| >> R_e$ at the frequency of oscillation. The drive level can now be calculated as:

The power dissipated in the crystal

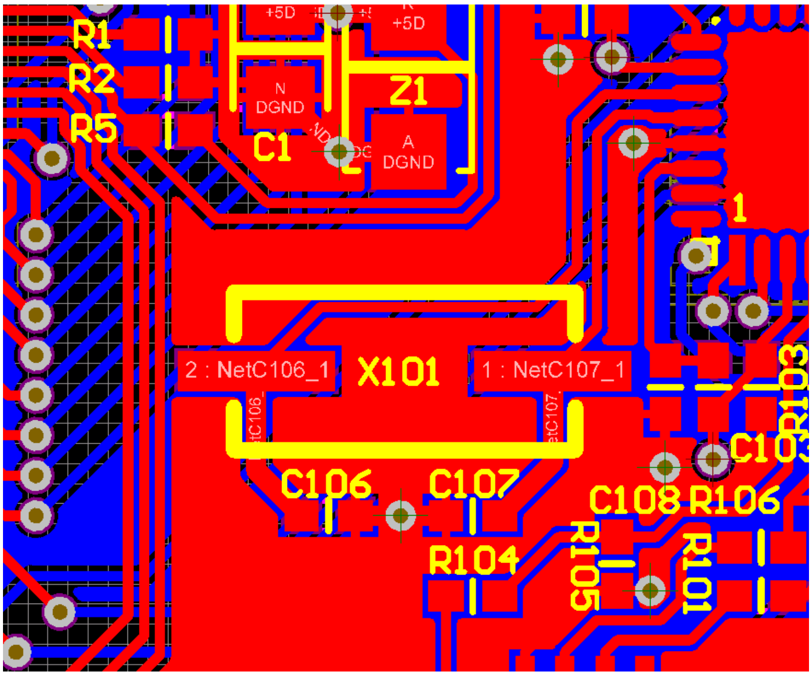
$$\boxed{P_{XTAL} = R_e |\mathbf{I}_e|^2 \approx R_e \frac{|\mathbf{V}_o|^2}{X_2^2} = R_e \left[\omega_{01} C_2 |\mathbf{V}_o|\right]^2} \tag{4.65}$$

where $\mathbf{V}_o$ is the RMS value of the fundamental component of the voltage appearing at the output of the amplifier, and R_e is the *effective resistance* of the crystal at the frequency of oscillation. In gate oscillators, the peak-to-peak variation of the amplifier's output voltage, v_o, is close to the supply voltage, V_{DD}, and so $\mathbf{V}_o = V_{DD} / \left(2\sqrt{2}\right)$ can be used as a maximum upper bound. In oscillators with amplitude limitation control, $\mathbf{V}_o$ is different and should be replaced with the actual RMS value of the output voltage.

To reduce the crystal drive level, it may be necessary to introduce a series damping resistor, R_s, as shown in Figure 4.13. It is important to have a good margin of safety between the power dissipated in the crystal and the maximum drive level specified by the crystal manufacturer, because loop gain can increase with colder temperatures and higher supply voltages, thus increasing the risk of overdriving the crystal. If the drive level is too low, the crystal may fail to oscillate or have degraded phase noise performance.

PCB Layout

The layout of the physical components should ensure that stray inductance in the loop containing the crystal and load capacitors is minimised. Therefore, careful grounding and routing of tracks around the crystal is usually required. An example of a PCB layout for a Pierce oscillator is shown below:



An example of a good PCB layout for a Pierce oscillator feedback network

Figure 4.27 – Example of a PCB layout for a Pierce oscillator

Notice that the load capacitors (C106 and C107) and the crystal (X101) form a small loop, since in the resonance condition there is a circulating current, and we wish to minimise parasitic inductance caused by connecting tracks. Also note that this is a 4-layer PCB, so the via between the load capacitors connects directly to a ground plane on an inner layer. The orientation of the crystal was unavoidable, due to surrounding components, and so the crystal tracks leading to the integrated circuit are surrounded by a copper ground plane to minimize mutual capacitive coupling from the high-frequency crystal tracks to surrounding signal tracks.

Example

For a particular microcontroller, we need to design a crystal oscillator for 8 MHz operation with an accuracy of 200 ppm over the crystal's operating temperature range. The lifetime of the product is 10 years.

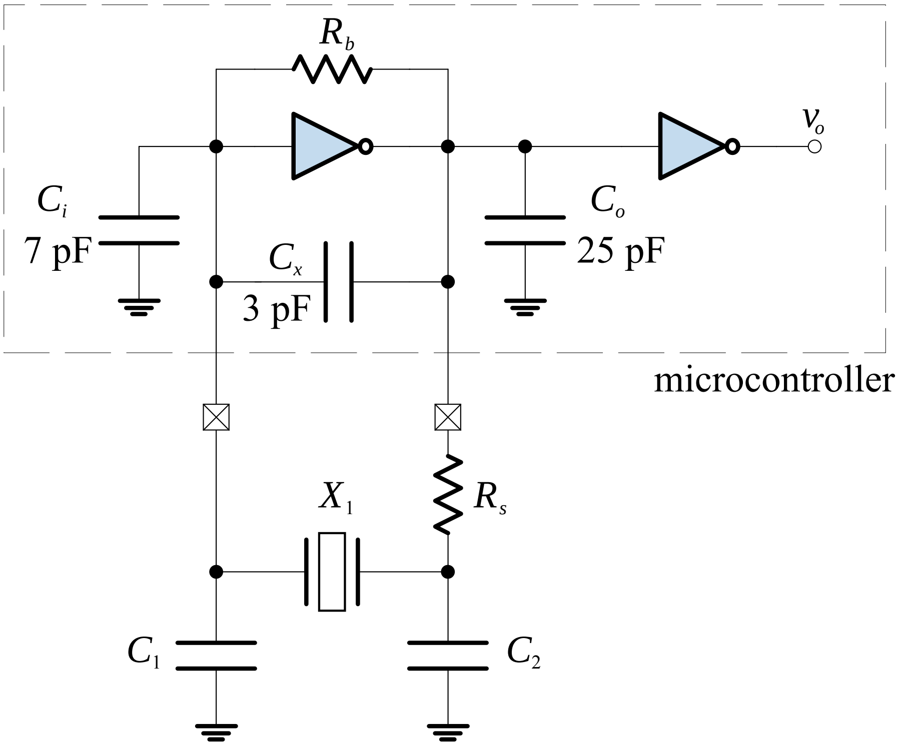
The following data is obtained from the datasheet:

Specification	Value
Nominal Frequency	8.000 MHz
Frequency tolerance at 25 °C	±30 ppm
Temperature stability	±50 ppm
Operating temperature range	-10 °C to 60 °C
Load capacitance	16 pF
Equivalent Series Resistance (ESR)	70 Ω max
Shunt capacitance, C_0	5 pF max
Drive level	100 μW max
Aging	±5 ppm per year

In addition, the microcontroller datasheet gives the following information on its oscillator pins:

Specification	Value
Input capacitance	7 pF typ
Xtal pin-to-pin capacitance	3 pF typ
Equivalent output capacitance (takes into account the internal propagation delay of the inverter)	25 pF typ
Output voltage level	0-5 V

The oscillator circuit diagram for the Pierce configuration is:



Notice that this is a realistic circuit diagram that shows the input and output capacitance, as well as the pin-to-pin capacitance, of the microcontroller's oscillator amplifier. The capacitance of the PCB tracks connecting the components to the microcontroller, if kept short, will have capacitances (to common) less than 1 pF, so they are not included in the model.

To achieve frequency accuracy, we have to present to the crystal the same load capacitance that it was adjusted for. The amplifier's input capacitance appears in parallel with C_1. The resistor R_s provides some isolation between the crystal feedback network and the amplifier's output capacitance C_o, as well as from the pin-to-pin capacitance, C_x. Let:

$$C_1' = C_1 + C_i$$

Thus, the load capacitance presented to the crystal is:

$$C_L = \frac{C_1' C_2}{C_1' + C_2}$$

4.44

Using a spreadsheet, set up with standard capacitor values, we can search for the series combination of C_1' and C_2 that is closest to C_L. Considerations of loop gain, start-up time and frequency stability over temperature suggest that C_1' and C_2 should be about equal. If we choose:

$$C_1 = 22\,\text{pF} \quad \text{and} \quad C_2 = 33\,\text{pF}$$

then:

$$C_1' = 22 + 7 = 29\,\text{pF} \quad \text{and} \quad C_2 = 33 = 33\,\text{pF}$$

which are approximately equal. Notice that the bulk external capacitors are larger than the microcontroller's internal capacitors, which will vary to some extent with temperature and the manufacturing process.

With these values, the load capacitance presented to the crystal is:

$$\begin{aligned} C_L &= \frac{C_1' C_2}{C_1' + C_2} \\ &= \frac{29 \cdot 33}{29 + 33} \\ &= 15.44\,\text{pF} \end{aligned}$$

We can determine the error in presenting a $15.44\,\text{pF}$ load capacitance to the crystal instead of a 16 pF load capacitance. We start with:

$$f_{01} \approx f_{rs}\left(1 + \frac{C_3}{2(C_0 + C_L)}\right)$$

then using the fact that the crystal has a shunt capacitance of 5 pF maximum, and was calibrated for $C_L = 16\,\text{pF}$:

$$\begin{aligned} 8{,}000{,}000 &\approx f_{rs}\left(1 + \frac{C_3}{2(5 + 16) \times 10^{-12}}\right) \\ &= f_{rs}\left(1 + \frac{C_3}{42 \times 10^{-12}}\right) \end{aligned}$$

The actual frequency of oscillation is:

$$
\begin{aligned}
f_{01} &\approx f_{rs}\left(1+\frac{C_3}{2(5+15.44)\times 10^{-12}}\right) \\
&= f_{rs}\left(1+\frac{C_3}{40.88\times 10^{-12}}\right)
\end{aligned}
$$

Dividing the actual and calibrated values gives:

$$
\frac{f_{01}}{8{,}000{,}000} \approx \frac{1+\dfrac{C_3}{40.88\times 10^{-12}}}{1+\dfrac{C_3}{42\times 10^{-12}}}
$$

To evaluate the error, we need an estimate of C_3, the motional arm capacitance, which is not generally specified on manufacturer's datasheets. Using $C_3 = 28\text{ fF}$ (which assumes the unloaded crystal $Q_0 = 10000$), we get:

$$
\frac{\Delta f}{f_0} = \frac{f_{01}-8{,}000{,}000}{8{,}000{,}000} \approx \frac{1+\dfrac{0.028}{40.88}}{1+\dfrac{0.028}{42}} - 1 = 18.25 \approx 20\text{ ppm}
$$

The type of capacitors used for C_1 and C_2 would be ceramic class 1, with a tolerance of 1% and a temperature coefficient of C0G / NPO. Even with a worst case tolerance of 1%, the frequency error is still within ≈ 25 ppm.

We can now tabulate an error budget:

Error Source	**Error Value**
Capacitors	25 ppm
Crystal frequency tolerance	30 ppm
Temperature stability	50 ppm
Aging (over 10 years)	50 ppm
TOTAL	**155 ppm**

Thus, the overall accuracy of the oscillator will be ±155 ppm over the operating temperature range and lifetime, which is better than the required accuracy of 200 ppm.

You can see from this example that the selection of capacitors C_1 and C_2 is not critical for the determination of the oscillation frequency, since most of the error is introduced by the characteristics of the crystal.

Next, we check the drive level. The crystal datasheet has already specified the maximum ESR at series resonance, which is equal to R_3. Therefore, at the oscillation frequency, the effective resistance is:

$$R_e \approx R_3\left(\frac{C_0 + C_L}{C_L}\right)^2 = 70\times\left(\frac{5+16}{16}\right)^2 = 120.6\,\Omega$$

The power dissipated in the crystal, assuming $R_s = 0$, is then:

$$\begin{aligned} P_{XTAL} &\approx R_e\left[\omega_{01}C_2|\mathbf{V}_o|\right]^2 \\ &= 120.6\times\left[2\pi\times 8\times 10^6\times 33\times 10^{-12}\times\frac{5}{2\sqrt{2}}\right]^2 \\ &= 1.037\text{ mW} \end{aligned}$$

Since this exceeds the maximum drive level specification of 100 μW by a factor of about 10 times, we need to add the series resistor R_s. This resistor is in series with the output resistance of the microcontroller's inverting amplifier, so it modifies the crystal oscillation equations. To take R_s, C_x and C_o into account, we can perform an analysis as in Appendix B.

To reduce the gain required by the amplifier, and therefore the crystal drive level, we will change the values of C_1 and C_2. Since $\beta \approx -C_2/C_1$, we can derive gain by reducing the value of C_1 and increasing the value of C_2. We will therefore set $C_1 = 5.6\text{ pF}$ and $C_2 = 100\text{ pF}$. We use a value for the output resistance of the gate inverter of around $r_o \approx 20\text{ k}\Omega$. We will also assume that the output voltage level of the gate inverter swings between 1 V and 4 V in the steady-state (since, from the inverter transfer characteristic, the gain drops off to zero beyond these values). The output is therefore assumed to be 3 V peak-to-peak.

The value of the series resistor is initially set to approximately match the impedance of C_2 at the oscillation frequency. Thus we set $R_s \approx 100\,\Omega$.

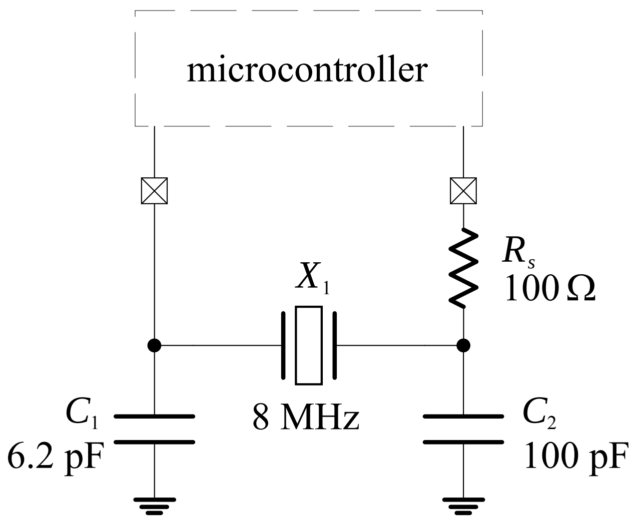
The MatLab code in Appendix B shows how to determine the frequency of oscillation, the voltages in the circuit, the crystal's equivalent impedance and the crystal power. The crystal parameters were derived from the ESR and nominal frequency given in the datasheet, and the assumption that the unloaded *Q* of the crystal was 10 000.

The results of the analysis are:

```
f0 = 8000221.39242875

error = 27.6740535943536 ppm

Ze = 123.662844964046 + 1308.12253917226i

mu = -18.4544198701407

Pxtal = 97.9151575444847 μW
```

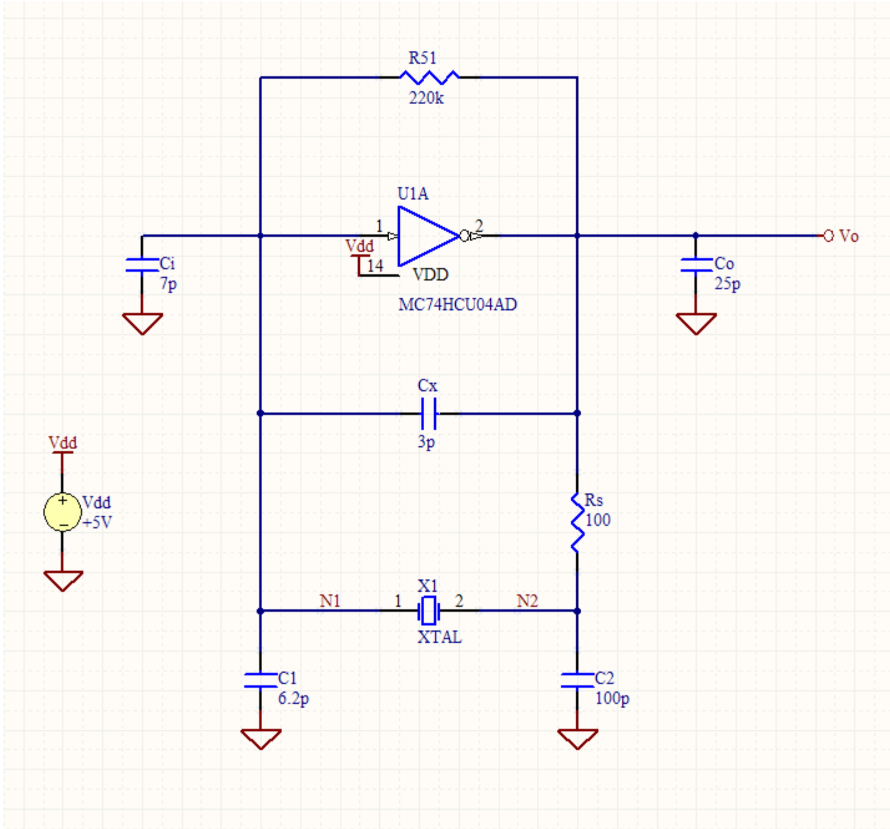
Notice that the frequency error is roughly the same as before (≈ 25 ppm). Also note that the minimum inverter gain required for oscillations is $\mu \approx -20$, which should be achievable in a single CMOS inverter. The power is also under the requirement of 100 μW.

The completed design is shown below:

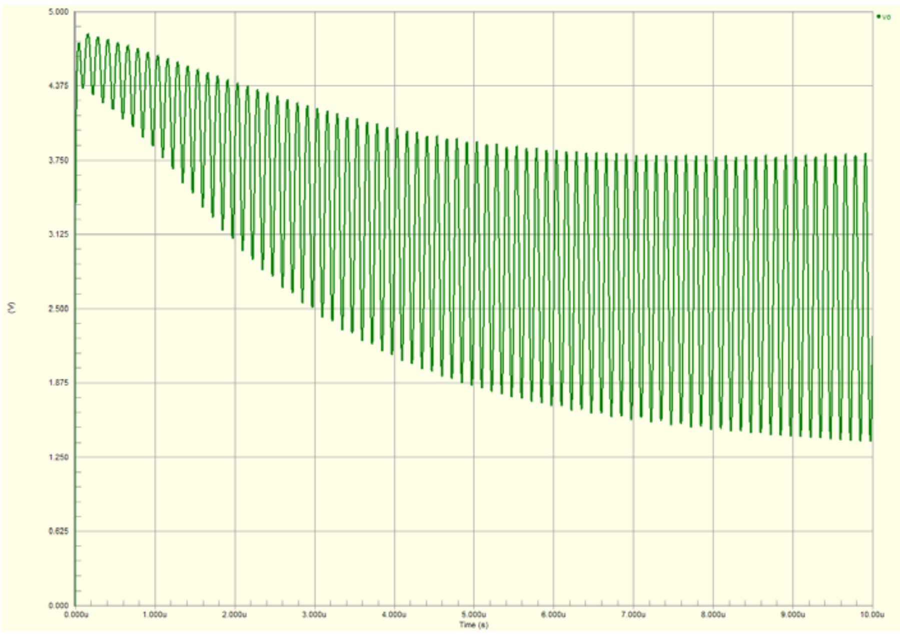


4.48

To verify the analysis, we can carry out a simulation in PSpice. The circuit below was entered into Altium Designer:



The output of the transient simulation is shown below:



The next stage would be to carefully lay out the components and the tracks on the PCB, taking care to keep tracks short and ensuring a good ground plane beneath and around the oscillator components and pads.

The last task would be to build and test the oscillator. Checking the design is virtually impossible without specialised test equipment. One check is to monitor the waveforms at the input and output of the inverting gate. This will require a high bandwidth oscilloscope and a specialised probe. The normal x10 oscilloscope probe will have an input impedance of ~10 MΩ in parallel with 15 pF. The 10 MΩ will form a DC potential divider to GND with the ~1 MΩ bias resistor R_b which will alter the inverters bias point. The 15 pF will appear directly across C_1 when measuring the inverter input waveform making $C_1 = 48\,\text{pF}$, not the designed 33pF. Any trace observed on the oscilloscope will be completely invalid (and most likely the probe will stop the oscillator from working anyway). A better choice of oscilloscope probe is an 'Active' or 'FET' probe which has a high input impedance buffer built into the probe tip. The input impedance of an 'Active' probe is typically >10 MΩ in parallel with <2 pF, but as before the effect of using this probe must be taken into account when probing the oscillator.

For this design the expected waveforms (assuming a suitable probe is used that will not alter the oscillator's working conditions) are a slightly distorted 8 MHz sine wave from 1 V to 4 V at the inverter's output and a clean 8 MHz sine wave of ≈ 0.5 V peak-to-peak at the inverter's input (both waves superimposed on the 2.5V bias point). It is important the input waveform peak-to-peak value is always less than the inverter's supply voltage to prevent the input from limiting due to the input protection diodes.

The actual crystal power dissipation is not easily measured. Assuming the actual crystal current could be measured (with a high bandwidth, ultra low inductance AC current probe for example) then we still need to know the actual crystal parameters, and not the maximum values as given on the datasheet. This requires measurements to be made on the crystal using a specialised crystal impedance meter before the crystal is used in the circuit.

References

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Appendix A

Oscillation Frequency for the Pierce Oscillator using a Crystal

Starting from the condition for oscillation:

$$X_1 + X_2 + X_e = -\frac{R_e X_2}{r_o} \tag{4A.1}$$

we substitute for R_e and X_e from Eq. (4.24), and using the subscript 3 for the motional arm of the crystal to avoid confusion with the external C_1, we have:

$$X_1 + X_2 + \frac{X_{C_0}\left[R_3^2 + X_3\left(X_3 + X_{C_0}\right)\right]}{R_3^2 + \left(X_3 + X_{C_0}\right)^2} = -\frac{X_2}{r_o}\frac{R_3 X_{C_0}^2}{R_3^2 + \left(X_3 + X_{C_0}\right)^2} \tag{4A.2}$$

Multiplying both sides by $R_3^2 + \left(X_3 + X_{C_0}\right)^2$, this simplifies to:

$$\left(X_1 + X_2\right)\left[R_3^2 + \left(X_3 + X_{C_0}\right)^2\right] + X_{C_0}\left[R_3^2 + X_3\left(X_3 + X_{C_0}\right)\right] = -\frac{X_2}{r_o} R_3 X_{C_0}^2 \tag{4A.3}$$

Factoring $R_3^2 + X_3\left(X_3 + X_{C_0}\right)$ gives:

$$\left(X_1 + X_2 + X_{C_0}\right)\left[R_3^2 + X_3\left(X_3 + X_{C_0}\right)\right] + \left(X_1 + X_2\right) X_{C_0}\left(X_3 + X_{C_0}\right) = -\frac{X_2}{r_o} R_3 X_{C_0}^2 \tag{4A.4}$$

Substituting for the reactances, we get:

$$\begin{aligned}&\left(\frac{-1}{\omega_0 C_1} + \frac{-1}{\omega_0 C_2} + \frac{-1}{\omega_0 C_0}\right)\left[R_3^2 + \left(\omega_0 L_3 - \frac{1}{\omega_0 C_3}\right)\left(\omega_0 L_3 - \frac{1}{\omega_0 C_3} - \frac{1}{\omega_0 C_0}\right)\right] \\ &+ \left(\frac{-1}{\omega_0 C_1} + \frac{-1}{\omega_0 C_2}\right)\left(\frac{-1}{\omega_0 C_0}\right)\left(\omega_0 L_3 - \frac{1}{\omega_0 C_3} - \frac{1}{\omega_0 C_0}\right) = \frac{R_3}{r_o}\frac{1}{\omega_0 C_2}\left(\frac{1}{\omega_0 C_0}\right)^2\end{aligned} \tag{4A.5}$$

In order to achieve a polynomial in ω_0 with positive powers, we multiply both sides by $\omega_0^3 C_0^2 C_1 C_2 C_3^2$:

$$\begin{aligned}&-\left(C_0C_1+C_0C_2+C_1C_2\right)\left[\omega_0^2R_3^2C_0C_3^2+\left(\omega_0^2L_3C_3-1\right)\left(\omega_0^2L_3C_0C_3-C_0-C_3\right)\right]\\&+\left(C_1+C_2\right)C_3\left(\omega_0^2L_3C_0C_3-C_0-C_3\right)=\frac{R_3C_1C_3^2}{r_o}\end{aligned} \tag{4A.6}$$

Expanding the inner product, we get:

$$\begin{aligned}&-\left(C_0C_1+C_0C_2+C_1C_2\right)\left[\omega_0^2R_3^2C_0C_3^2+\omega_0^4L_3^2C_0C_3^2-2\omega_0^2L_3C_0C_3-\omega_0^2L_3C_3^2+C_0+C_3\right]\\&+\left(C_1+C_2\right)\left(\omega_0^2L_3C_0C_3^2-C_0C_3-C_3^2\right)=\frac{R_3C_1C_3^2}{r_o}\end{aligned} \tag{4A.7}$$

Grouping like powers of ω_0, this can be written as:

$$\begin{aligned}&-L_3^2C_0C_3^2\left(C_0C_1+C_0C_2+C_1C_2\right)\omega_0^4\\&+\left\{\left[L_3C_3\left(2C_0+C_3\right)-R_3^2C_0C_3^2\right]\left(C_0C_1+C_0C_2+C_1C_2\right)+L_3C_0\left(C_1+C_2\right)C_3^2\right\}\omega_0^2\\&-\left(C_0+C_3\right)\left(C_0C_1+C_0C_2+C_1C_2+C_1C_3+C_2C_3\right)-\frac{R_3C_1C_3^2}{r_o}=0\end{aligned} \tag{4A.8}$$

Making the polynomial monic, we get:

$$\begin{aligned}&\omega_0^4\\&-\left(\frac{2C_0+C_3}{L_3C_0C_3}-\frac{R_3^2}{L_3^2}+\frac{1}{L_3}\frac{C_1+C_2}{C_0C_1+C_0C_2+C_1C_2}\right)\omega_0^2\\&+\frac{1}{L_3^2C_0C_3^2\left(C_0C_1+C_0C_2+C_1C_2\right)}\left[\left(C_0+C_3\right)\left(C_0C_1+C_0C_2+C_1C_2+C_1C_3+C_2C_3\right)+\frac{R_3C_1C_3^2}{r_o}\right]=0\end{aligned} \tag{4A.9}$$

Now we define the *load capacitance* on the crystal as the series combination of C_1 and C_2:

Load capacitance defined

$$\boxed{\frac{1}{C_L}=\frac{1}{C_1}+\frac{1}{C_2}} \tag{4A.10}$$

and define the total capacitance in parallel with the motional arm to be:

Total capacitance external to the motional arm defined

$$\boxed{C_T=C_0+C_L=C_0+\frac{C_1C_2}{C_1+C_2}} \tag{4A.11}$$

This simplifies the expression given by (4A.9) to:

$$\omega_0^4 - \left(\frac{2}{L_3 C_3} + \frac{1}{L_3 C_0} + \frac{1}{L_3 C_T} - \frac{R_3^2}{L_3^2} \right) \omega_0^2 + \frac{1}{L_3^2 C_3} \left(\frac{1}{C_3} + \frac{1}{C_0} \right) + \frac{1}{L_3^2 C_0 C_3} \frac{C_0 + C_3}{C_T} + \frac{1}{L_3^2 C_0 C_T \left(C_1 + C_2 \right)} \frac{R_3 C_1}{r_o} = 0 \tag{4A.12}$$

which can be further simplified into:

$$\boxed{\omega_0^4 - \left(\frac{2}{L_3 C_3} + \frac{1}{L_3 C_0} + \frac{1}{L_3 C_T} - \frac{R_3^2}{L_3^2} \right) \omega_0^2 + \left(\frac{1}{L_3 C_3} + \frac{1}{L_3 C_0} \right) \left(\frac{1}{L_3 C_3} + \frac{1}{L_3 C_T} \right) + \frac{1}{L_3^2 C_0 C_T \left(C_1 + C_2 \right)} \frac{R_3 C_1}{r_o} = 0} \tag{4A.13}$$

The exact equation that determines possible oscillation frequencies

This is an *exact* equation. In order to simplify it further, we call on the properties of the typical crystal. Firstly:

$$\frac{2}{L_3 C_3} + \frac{1}{L_3 C_0} + \frac{1}{L_3 C_T} >> \frac{R_3^2}{L_3^2} \tag{4A.14}$$

For example, typical values for an 8 MHz crystal are $L_3 = 14\,\mathrm{mH}$, $C_3 = 27\,\mathrm{fF}$, $R_3 = 8\,\Omega$ and $C_0 = 5.6\,\mathrm{pF}$. A typical load capacitance is $C_L = 32\,\mathrm{pF}$, giving $C_T = 37.6\,\mathrm{pF}$. Thus the inequality above is $5.306 \times 10^{15} >> 3.265 \times 10^{5}$, which is certainly true.

Secondly:

$$\left(\frac{1}{L_3 C_3} + \frac{1}{L_3 C_0} \right) \left(\frac{1}{L_3 C_3} + \frac{1}{L_3 C_T} \right) >> \frac{1}{L_3^2 C_0 C_T \left(C_1 + C_2 \right)} \frac{R_3 C_1}{r_o} \tag{4A.15}$$

For a typical output of a MOSFET inverter, $r_o = 5\,\mathrm{k}\Omega$, and the inequality is $7.037 \times 10^{30} >> 1.938 \times 10^{22}$, which again is certainly true.

With these inequalities, we can ignore the terms with R_3 in Eq. (4A.13), so that an *approximate* equation for the oscillation frequencies is:

$$\omega_0^4 - \left(\frac{2}{L_3C_3} + \frac{1}{L_3C_0} + \frac{1}{L_3C_T}\right)\omega_0^2 + \left(\frac{1}{L_3C_3} + \frac{1}{L_3C_0}\right)\left(\frac{1}{L_3C_3} + \frac{1}{L_3C_T}\right) \approx 0 \tag{4A.16}$$

which simplifies to:

The approximate equation that determines possible oscillation frequencies…

$$\boxed{\left[\omega_0^2 - \left(\frac{1}{L_3C_3} + \frac{1}{L_3C_0}\right)\right]\left[\omega_0^2 - \left(\frac{1}{L_3C_3} + \frac{1}{L_3C_T}\right)\right] \approx 0} \tag{4A.17}$$

Thus, the two *approximate* oscillation frequencies are:

…the first of which represents a series resonance with the capacitance external to the crystal's motional arm…

$$\boxed{\omega_{01} = \sqrt{\frac{1}{L_3C_3} + \frac{1}{L_3C_T}}} \tag{4A.18}$$

and:

…and the second which represents a parallel resonance with the crystal's static capacitance

$$\boxed{\omega_{02} = \sqrt{\frac{1}{L_3C_3} + \frac{1}{L_3C_0}}} \tag{4A.19}$$

Note that the first oscillation frequency, ω_{01}, *underestimates* the true frequency, typically by less than 1 ppm. The use of ω_{01} in determining the crystal's equivalent resistance and reactance will not lead to any significant error (less than 0.2%).

The second oscillation frequency, ω_{02}, *overestimates* the true frequency, typically by less than 1 ppm. However, due to the highly sensitive nature of the crystal reactance with frequencies near parallel resonance, the use of ω_{02} in determining the crystal's equivalent reactance will lead to large errors.

See Figure 4.5 for a graphical depiction of why the crystal reactance has different sensitivities to the two oscillation frequencies. Note that $\omega_{02} > \omega_{01}$.

Equivalent Circuit at the Oscillation Frequency

In the Pierce configuration, the crystal is inductive at each of its oscillation frequencies, and so it is appropriate to represent the crystal by its equivalent series resistance and inductance. If we let:

$$X_3 = \omega L_3 - \frac{1}{\omega C_3} \tag{4A.20}$$

then the impedance of the crystal is:

$$\begin{aligned} \mathbf{Z}_e &= \frac{jX_{C_0}\left(R_3 + jX_3\right)}{R_3 + j\left(X_3 + X_{C_0}\right)} \\ &= \frac{R_3 X_{C_0}^2}{R_3^2 + \left(X_3 + X_{C_0}\right)^2} + j\frac{X_{C_0}\left[R_3^2 + X_3\left(X_3 + X_{C_0}\right)\right]}{R_3^2 + \left(X_3 + X_{C_0}\right)^2} \\ &= R_e + jX_e \end{aligned} \tag{4A.21}$$

Thus:

$$\boxed{R_e = \frac{R_3 X_{C_0}^2}{R_3^2 + \left(X_3 + X_{C_0}\right)^2}} \tag{4A.22}$$

The crystal equivalent resistance…

and:

$$\boxed{X_e = \frac{X_{C_0}\left[R_3^2 + X_3\left(X_3 + X_{C_0}\right)\right]}{R_3^2 + \left(X_3 + X_{C_0}\right)^2}} \tag{4A.23}$$

…and reactance

At the frequency $\omega_{01} = \sqrt{\dfrac{1}{L_3 C_3} + \dfrac{1}{L_3 C_T}}$, we have:

$$\omega_{01} L_3 = \frac{1}{\omega_{01} C_3} + \frac{1}{\omega_{01} C_T} \tag{4A.24}$$

and so:

$$\begin{aligned} X_3 &= \omega_{01} L_3 - \frac{1}{\omega_{01} C_3} \\ &= \frac{1}{\omega_{01} C_T} \end{aligned} \tag{4A.25}$$

Substituting this into the formula for R_e gives:

$$\begin{aligned} R_e &= \frac{R_3 X_{C_0}^2}{R_3^2 + \left(\dfrac{1}{\omega_{01} C_T} - \dfrac{1}{\omega_{01} C_0} \right)^2} \\ &= \frac{R_3 X_{C_0}^2}{R_3^2 + \left(\dfrac{1}{\omega_{01} C_0} \right)^2 \left(\dfrac{C_0}{C_T} - 1 \right)^2} \\ &= \frac{R_3 X_{C_0}^2}{R_3^2 + X_{C_0}^2 \left(\dfrac{C_L}{C_0 + C_L} \right)^2} \end{aligned} \tag{4A.26}$$

If $\left| X_{C_0} \left(\dfrac{C_L}{C_0 + C_L} \right) \right| >> R_3$, as is usually the case, then:

The crystal equivalent resistance at the oscillation frequency ω_{01}

$$\boxed{R_e \approx R_3 \left(\frac{C_0 + C_L}{C_L} \right)^2} \tag{4A.27}$$

This formula has a typical error less than 0.05%.

The formula for X_e can be rearranged to give:

$$\begin{aligned} X_e &= \frac{X_{C_0}\left[R_3^2 + X_3\left(X_3 + X_{C_0}\right)\right]}{R_3^2 + \left(X_3 + X_{C_0}\right)^2} \\ &= X_{C_0}\left[1 - \frac{X_{C_0}\left(X_3 + X_{C_0}\right)}{R_3^2 + \left(X_3 + X_{C_0}\right)^2}\right] \end{aligned} \tag{4A.28}$$

Using Eq. (4A.25), we have:

$$X_3 + X_{C_0} = \frac{1}{\omega_{01} C_T} - \frac{1}{\omega_{01} C_0} = X_{C_0}\left(1 - \frac{C_0}{C_T}\right) = X_{C_0}\left(\frac{C_L}{C_0 + C_L}\right) \tag{4A.29}$$

and so:

$$\begin{aligned} X_e &= X_{C_0}\left[1 - \frac{X_{C_0}\left(X_3 + X_{C_0}\right)}{R_3^2 + \left(X_3 + X_{C_0}\right)^2}\right] \\ &= X_{C_0}\left[1 - \frac{X_{C_0}^2\left(\dfrac{C_L}{C_0 + C_L}\right)}{R_3^2 + X_{C_0}^2\left(\dfrac{C_L}{C_0 + C_L}\right)^2}\right] \end{aligned} \tag{4A.30}$$

Again, if $\left|X_{C_0}\left(\dfrac{C_L}{C_0 + C_L}\right)\right| >> R_3$, as is usually the case, then:

$$X_e \approx X_{C_0}\left[1 - \frac{C_0 + C_L}{C_L}\right] = \frac{-1}{\omega_{01} C_0}\left(-\frac{C_0}{C_L}\right) \tag{4A.31}$$

and therefore:

$$\boxed{X_e = \omega_{01} L_e \approx \frac{1}{\omega_{01} C_L}} \tag{4A.32}$$

The crystal equivalent reactance at the oscillation frequency ω_{01}

This formula has a typical error less than 0.2%.

The equivalent circuit of the crystal at the oscillation frequency ω_{01} is then:

The crystal equivalent circuit at the oscillation frequency ω_{01}

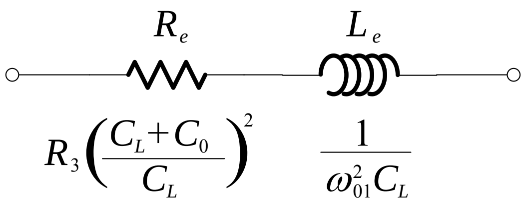


Figure 4.28 – Equivalent circuit of a crystal at the oscillation frequency ω_{01}

At the frequency $\omega_{02} = \sqrt{\frac{1}{L_3C_3} + \frac{1}{L_3C_0}}$, we have:

$$\omega_{02}L_3 = \frac{1}{\omega_{02}C_3} + \frac{1}{\omega_{02}C_0} \tag{4A.33}$$

and so:

$$\begin{aligned} X_3 &= \omega_{02}L_3 - \frac{1}{\omega_{02}C_3} \\ &= \frac{1}{\omega_{02}C_0} \end{aligned} \tag{4A.34}$$

Substituting this into the formula for R_e gives:

$$R_e = \frac{R_3 X_{C_0}^2}{R_3^2 + \left(\frac{1}{\omega_{02}C_0} - \frac{1}{\omega_{02}C_0}\right)^2} = \frac{X_{C_0}^2}{R_3} \tag{4A.35}$$

and so:

The crystal equivalent resistance at the oscillation frequency ω_{02}

$$\boxed{R_e = \frac{1}{\omega_{02}^2 C_0^2 R_3} = \frac{1}{\left(\frac{1}{L_3C_3} + \frac{1}{L_3C_0}\right)C_0^2 R_3} = \frac{L_3}{\left(1 + \frac{C_0}{C_3}\right)C_0 R_3}} \tag{4A.36}$$

This is accurate to within 0.5%, and its value is usually in the megohm range.

Dominant Oscillation Frequency

The two frequencies at which oscillation *could* occur according to the phase criterion are ω_{01} and ω_{02}. However, the gain condition to start oscillations, $A\beta > 1$, still has to be satisfied.

For ω_{01}, we have seen that the equivalent circuit for the crystal reduces to a resistance that is similar in value to R_3 in series with an inductor which has a reactance equal in magnitude to the reactance of the crystal's load capacitance. The crystal is effectively a high-*Q* inductor and we can resort to the equations derived for the pure reactance case. In particular:

$$\beta \approx -\frac{C_2}{C_1} \tag{4A.37}$$

The load on the amplifier, h_{22}, is not infinite and the calculation of the gain A is algebraically cumbersome. Typical values will result in a gain which is roughly half of the open-circuit gain, μ. Thus, the required amplifier gain for oscillation is not onerous, and is typically required to be $|\mu| > 2$ (remember that μ is a negative number).

For ω_{02}, we have seen that the equivalent circuit for the crystal reduces to a resistance that is very much greater than R_3 (with values in the megohms) in series with an inductor which also has a very large reactance. Thus, the amplifier's load is $h_{22} = \mathbf{Z}_2 \,||\, (\mathbf{Z}_1 + \mathbf{Z}_e) \approx \mathbf{Z}_2$ and the crystal and C_1 branch of the feedback network is effectively an open-circuit. Also, as a result of the large crystal impedance, the feedback factor typically has a magnitude $|\beta| < 10^{-4}$. Thus, to achieve $A\beta > 1$, the amplifier gain magnitude would need to be $|A| > 10^4$. This large gain is unachievable in a single transistor amplifier or logic gate inverter. Therefore, for ω_{02}, we have $A\beta < 1$, and oscillations at this frequency do not occur.

The crystal will only oscillate at ω_{01}

The Approximate Oscillation Frequency

We have now seen that ω_{01} will be the frequency of oscillation of the Pierce oscillator. We saw previously that for a crystal by itself, its series resonance frequency was given by Eq. (4.15):

$$\boxed{\omega_{rs} \approx \frac{1}{\sqrt{L_3 C_3}}} \tag{4A.38}$$

where the subscripts have been chosen to be 3 for consistency with the notation used in this appendix.

Thus, the approximate oscillation frequency ω_{01} can be written as:

$$\boxed{\omega_{01} = \omega_{rs}\sqrt{1+\frac{C_3}{C_T}}} \tag{4A.39}$$

But $C_3/C_T << 1$. Therefore, the binomial approximation:

$$(1+x)^n \approx 1+nx \quad \text{if} \quad x << 1 \tag{4A.40}$$

may be used, and:

$$\omega_{01} \approx \omega_{rs}\left(1+\frac{C_3}{2(C_0+C_L)}\right) \tag{4A.41}$$

Therefore:

The oscillation frequency that manufacturers tune the crystal to for a certain load capacitance

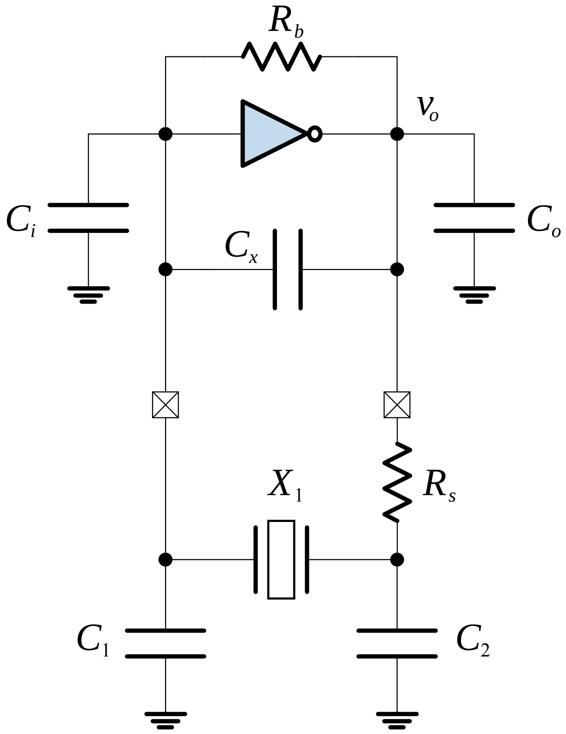
$$\boxed{f_{01} \approx f_{rs}\left(1+\frac{C_3}{2(C_0+C_L)}\right)} \tag{4A.42}$$

which is exactly the same as Eq. (4.25). The error in using this expression is typically less than 1 ppm.

Appendix B

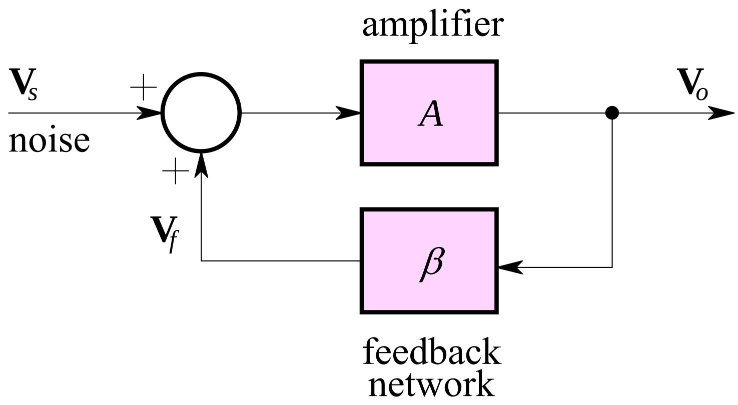
Analysis of a Realistic Pierce Oscillator using a Crystal

The realistic circuit diagram for the Pierce configuration, which includes the gate oscillator feedback resistor and the various pin-to-pin and pin-to-earth capacitances, is:

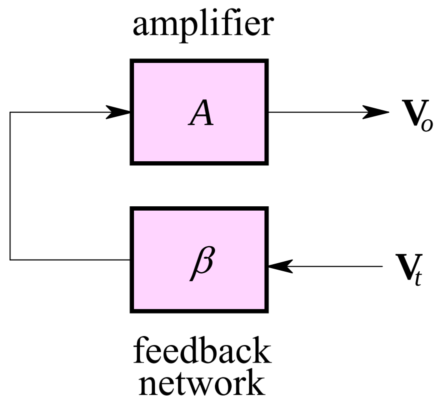


We are only interested in the loop gain of this circuit to establish the conditions necessary for oscillation. Therefore, we can “break” the feedback loop and analyse the quantity $A\beta$.

Consider the general feedback configuration:



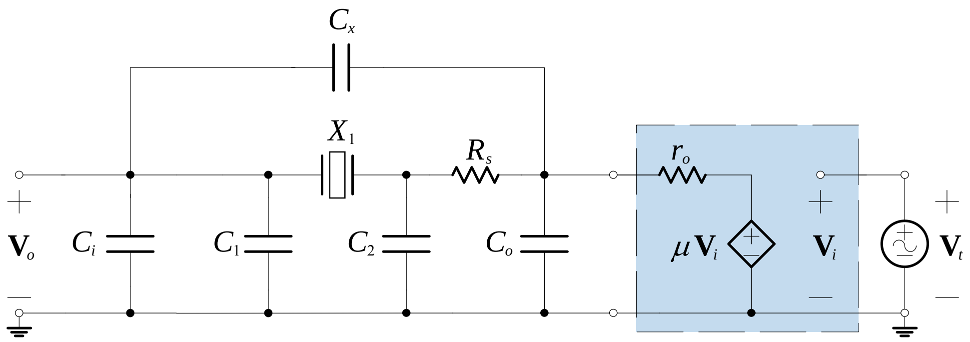
We set the external noise source to zero, and open the feedback loop by breaking the connection of $\mathbf{V}_o$ to the feedback network and apply a test signal $\mathbf{V}_t$:



It follows that the loop gain will be given by $A\beta = \mathbf{V}_o / \mathbf{V}_t$. It should also be obvious that this applies regardless of where the loop is broken. However, in breaking the feedback loop of a practical amplifier circuit, we must ensure that the conditions that existed prior to breaking the loop do not change. This is achieved by terminating the loop where it is opened with an impedance equal to that seen before the loop was broken.

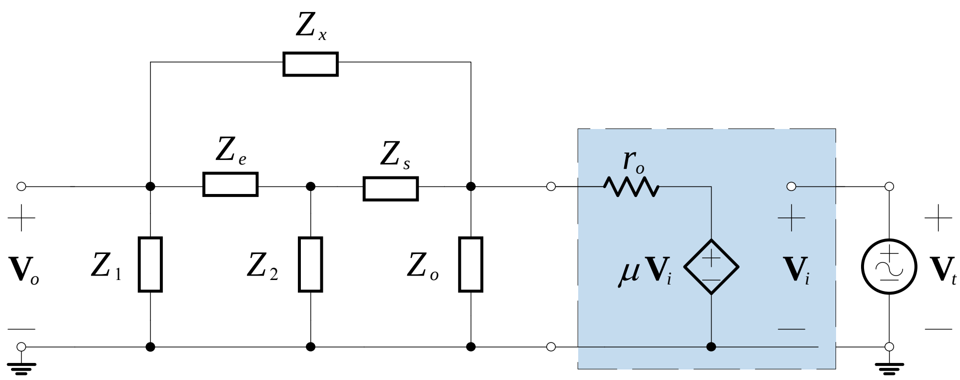
Returning to the Pierce oscillator circuit, we decide to break the loop at the input to the CMOS inverter, which is connected to the gate of an *n*-type and *p*-type MOSFET. The loop at this point effectively sees an open-circuit, so it is an easy place to break the loop.

We thus need to analyse the gain of the circuit represented by:



MOSFET inverter model

Using impedances, and lumping C_i and C_1 into $\mathbf{Z}_1$, this can be generalised to:



MOSFET inverter model

The crystal has been replaced with its equivalent circuit at resonance. Once the resonance frequency is found, analysis is straightforward using nodal analysis:

$$\begin{bmatrix} \mathbf{Y}_1 + \mathbf{Y}_e + \mathbf{Y}_x & -\mathbf{Y}_e & -\mathbf{Y}_x \\ -\mathbf{Y}_e & \mathbf{Y}_e + \mathbf{Y}_2 + \mathbf{Y}_s & \mathbf{Y}_s \\ -\mathbf{Y}_x & -\mathbf{Y}_s & \mathbf{Y}_x + \mathbf{Y}_s + \mathbf{Y}_o + g_o \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mu g_o \mathbf{V}_t \end{bmatrix} \tag{4B.1}$$

The algebraic solutions of the resonance frequency and loop gain are cumbersome and involve very large expressions for the coefficients. Symbolic analysis of this kind is best done on a computer, where the significance of terms appearing in the equations can be evaluated with the prospect of simplifying the expressions for human consumption. Alternatively, we can resort to numerical analysis and "play" with some of the more significant parameters of the circuit to achieve a desired outcome.

For example, if C_x is very small so that $\mathbf{Y}_x \approx 0$, then the expression for the oscillation frequency is:

$$\left(X_1 + X_2 + X_e\right)\left(1 + \frac{R_s}{r_o}\right) + \frac{R_e X_2}{r_o} - \frac{1}{X_o}\left[R_s R_r - X_2\left(X_1 + X_e\right)\right] = 0 \tag{4B.2}$$

where R_e and X_e are given by Eqs. (4A.22) and (4A.23). Again, this is an expression which is best solved numerically and a "try and see" approach is made to see how the series resistance R_s and output capacitance C_o affect the oscillation frequency and required amplifier gain.

The following function first finds the oscillation frequency, then solves for the required amplifier gain, circuit voltages and crystal power.

Xtal.m

```
% Xtal - calculates oscillator frequency,
% required amplifier gain and crystal power

clear all;
close all;
format long g;

% Xtal parameters
fnom=8e6;
R3=70;
L3=13.9354841820555e-3;
C3=28.4397636368481e-15;
C0=5.0e-12;
xtal=[R3 L3 C3 C0];

% amplifier parameters
ro=20e3;
Cin=7e-12;
Cout=25e-12;
Cx=3e-12;
Rs=0.1e3;
Cs=33e-12;
amp=[ro Cin Cout Cx Rs Cs];

% feedback parameters
C1=6.2e-12;
C2=100e-12;
fb=[C1 C2];
C1p=C1+Cin;

% oscillation frequency
f0=findosc(xtal,amp,fb,fnom*1.0001)
% error in ppm
error=(f0-fnom)/fnom*1e6
% nodal analysis
w0=2*pi*f0;
[V, Ze]=nodal(xtal,amp,fb,w0)

% RMS value of the output's fundamental component
Vdd=3;
Vs=Vdd/2/sqrt(2);

% evaluate required gain for AB=1 and resulting voltages
mu=1/real(V(1))
V=mu*V*Vs

% evaluate the "crystal drive"
% i.e. the power dissipated in the crystal
Vxtal=V(2)-V(1)
Ixtal=Vxtal/Ze;
Re=real(Ze);
Pxtal=Re*abs(Ixtal)^2*1e6          %in microwatts
myPxtal=Re*(w0*C2*abs(V(2)))^2*1e6 %in microwatts – approx.
```

The following function is used to find the frequency for which the imaginary part of the loop gain goes to zero. It uses the MatLab `fzero` function.

findosc.m

```
function y = findosc(xtal, amp, fb, f0)


% Normalise for numerical accuracy
wn=2*pi*f0;
rn=amp(1);

% Extract xtal parameters
R3=xtal(1)/rn;
L3=xtal(2)/rn*wn;
C3=xtal(3)*rn*wn;
C0=xtal(4)*rn*wn;

% Extract amplifier parameters
ro=amp(1)/rn;
Cin=amp(2)*rn*wn;
Cout=amp(3)*rn*wn;
Cx=amp(4)*rn*wn;
Rs=amp(5)/rn;
Cs=amp(6)*rn*wn;

% Extract feedback parameters
C1=fb(1)*rn*wn;
C2=fb(2)*rn*wn;

% Turn off Display
options = optimset('Display', 'off', 'TolX', 1e-15, 'TolFun', 1e-15);
y = fzero(@xtalfun, 1, options);

  function y = xtalfun(w) % Compute the polynomial
    V=nodal([R3 L3 C3 C0], [ro Cin Cout Cx Rs Cs], [C1 C2], w);
    AB=V(1);
    % the objective function value
    y=imag(AB)/abs(AB);
  end

  AB
  y=y*f0;
end
```

Note the use of frequency and magnitude scaling to normalise the coefficients in the admittance matrix. This avoids creating an “ill-conditioned” matrix and reduces round off errors.

The following function performs the nodal analysis and evaluates the crystal impedance:

nodal.m

```
function [V, Ze] = nodal(xtal, amp, fb, w)

  % Extract xtal parameters
  R3=xtal(1);
  L3=xtal(2);
  C3=xtal(3);
  C0=xtal(4);

  % Extract amplifier parameters
  ro=amp(1);
  Cin=amp(2);
  Cout=amp(3);
  Cx=amp(4);
  Rs=amp(5);
  Cs=amp(6);

  % Extract feedback parameters
  C1=fb(1);
  C2=fb(2);
  C1p=C1+Cin;

  % feedback network
  Y1=i*w*C1p;
  Y2=i*w*C2;

  % crystal equivalent impedance
  X3=w*L3-1./(w*C3);
  Z3=R3+i*X3;
  XC0=-1/(w*C0);
  den=R3^2+(X3+XC0)^2;
  Re=R3*XC0*XC0/den;
  Xe=XC0*(R3^2+X3*(X3+XC0))/den;
  Ze=Re+i*Xe;
  Ye=1/Ze;

  % amplifier and strays
  Yx=i*w*Cx;
  Ys=1/Rs;
  %Ys=i*w*Cs;
  Yo=i*w*Cout;
  go=1/ro;

  % admittance matrix
  Y=[Y1+Ye+Yx -Ye -Yx; -Ye Ye+Y2+Ys -Ys; -Yx -Ys Yx+Ys+Yo+go];
  % current source vector with mu=1
  I=[0 0 go]';
  % voltage solution
  V=Y\I;

end
```