Charles University in Prague

Faculty of Mathematics and Physics

DOCTORAL THESIS

Tereza Bártlová

History and current state of recreational mathematics and its relation to serious mathematics

Department of Mathematical Analysis

Supervisor of the doctoral thesis:	prof. RNDr. Luboš Pick, CSc., DSc.
Study programme:	Mathematics
Study branch:	General Questions of Mathematics and Information Science

Prague 2016

I declare that I carried out this doctoral thesis independently, and only with the cited sources, literature and other professional sources.

I understand that my work relates to the rights and obligations under the Act No. 121/2000 Sb., the Copyright Act, as amended, in particular the fact that the Charles University in Prague has the right to conclude a license agreement on the use of this work as a school work pursuant to Section 60 subsection 1 of the Copyright Act.

In Prague on June 28, 2016 Tereza Bártlová

Title: History and current state of recreational mathematics and its relation to serious mathematics

Author: Tereza Bártlová

Department: Department of Mathematical Analysis

Supervisor: prof. RNDr. Luboš Pick, CSc., DSc., Department of Mathematical Analysis

Abstract: The present thesis is devoted to the study of recreational mathematics, with a particular emphasis on its history, its relation to serious mathematics and its educational benefits. The thesis consists of five papers. In the first one we investigate the history of recreational mathematics. We focus on the development of mathematical problems throughout history, and we try to point out the people who had an important influence on the progress of recreational mathematics. The second article is dedicated to Edwin Abbott Abbott and his book called Flatland. It is one of the first popularizing books on geometry. In the third article we review one of the prominent personalities of recreational mathematics, Martin Gardner. The fourth article is in some sense a sequel to the third one. It deals with treachery of mathematical intuition and mathematical April Fool's hoaxes. The last article is devoted to the implementation recreational mathematics to education of students.

Keywords: recreational mathematics, mathematical puzzles, mathematical games, science center

Název práce: Historie a současnost rekreační matematiky a její vztah k matematice odborné

Autor: Tereza Bártlová

Katedra: Katedra matematické analýzy

Vedoucí disertační práce: prof. RNDr. Luboš Pick, CSc., DSc., Katedra matematické analýzy

Abstrakt: Tato disertační práce je věnována studiu rekreační matematiky se zvláštním zřetelem na její historii, vztah k odborné matematice a didaktickému významu. Práce sestává z pěti článků a stručného úvodu. V prvním článku zkoumáme historii rekreační matematiky. Zaměřujeme se na vývoj matematických úloh v průběhu dějin a snažíme se jmenovat významné osobnosti, které měly vliv na vývoj rekreační matematiky. Druhý článek je věnován Edwinu Abbottovi Abbottovi a jeho knize Flatland. Jedná se o jednu z prvních popularizačních knih o geometrii. Ve třetím článku se věnujeme jedné z významných osobností rekreační matematiky, Martinu Gardnerovi. Na jeho práci volně navazuje čtvrtý článek, který se zabývá zrádností matematické a fyzikální intuice a ilustruje ji na matematických aprílových žertech. Poslední článek je věnovaný implementaci rekreační matematiky do vzdělávání studentů.

Klíčová slova: rekreační matematika, matematické hádanky, matematické hry, science centra

I would like to express my deep gratitude to Luboš Pick who was my PhD advisor. I am very grateful to him for introducing me into the field of recreational mathematics and his help during my study. I really enjoyed sharing the ideas with him. His endless optimism and great enthusiasm for everything he does helped to make my study at Charles University an unforgettable experience.

I would like to thank Vejtek Musil for his help with the TEX system.

Special thanks belong to my mother for her never-ceasing support of all kinds.

Contents

Which came first, the recreational mathematics or the serious mathematics?

Before we fully delve into the study of recreational mathematics, we should consider what this label exactly means. Perhaps the most concise definition of recreational mathematics is the one that has been provided by the leading figure of recreational mathematics of all times, namely Martin Gardner, who claimed that recreational mathematics is that part of mathematics that "includes anything that has a spirit of play about it".

However, of course, different people might have different opinions on which part of mathematics can be considered as fun. For some it may be a Sudoku in Sunday newspapers, for others it can be the Rubik's cube, and, not surprisingly, a completely different idea of what is "amusing mathematics" is shared by professional mathematicians. Not a single mathematician would say that he/she does not enjoy his/her job, even if he/she is studying functional analysis or other advanced mathematical topics. Many professional mathematicians regard their work as a form of play, in the same way professional golfers or basketball stars might. In general, math is considered recreational if it has a playful aspect that can be understood and appreciated by non-mathematicians. Consequently, such "definition" would be rather blurred as it would probably encompass almost all kinds mathematics and therefore it would be too general. So what is it that determines whether a problem or task is recreational or not?

There are four, somewhat overlapping, aspects which altogether cover most of the topics that could be possibly labelled by recreational mathematics.

1. **The popular-scientific aspect** - recreational mathematics is that part of mathematics which is fun and which is popular. That is, the corresponding problems should be understandable to an interested layman, though the solutions may be harder. By recreational mathematics we can understand the approach using which we can make serious mathematics understandable or, at least, more palatable.

2. **The amusement aspect** - recreational mathematics is a mathematics that is used as a diversion from serious mathematics to one's amusement. For example, one of the prominent contemporary recreational mathematicians Ian Stewart perceives the role of recreational mathematics precisely in this sense. He is trying to view mathematics as a source of inspiration and joy. He often writes in his books that the amusement mathematics is that part of it which is not taught at school. The same point of view was held by Martin Gardner who moreover believed that even in the school the mathematics that is taught there should be fun to a certain degree.

3. **The pedagogical aspect** - recreational mathematics can be used for the teaching purposes. It seen as a great pedagogical utility. Its parts have

been present in the oldest known mathematics and this situation continues to the present day.

4. **The historical aspect** – recreational mathematics has always played a very important part in the history of mathematics and it was responsible for the origin of whole important mathematical theories and concepts that would not exist without it.

All the four named aspects are interconnected and influence each other. They overlap considerably and there is no clear boundary between them and the "serious" mathematics. The recreational mathematics is somewhere on the border among all these four aspects and tries to find the balance between seriousness and frivolity. We shall now consider each of the above-mentioned aspects of recreational mathematics separately in detail.

0.1 The popular-scientific aspect

Mathematical riddles have been boggling the willing minds of interested intelligent people from the dawn of mankind. Moreover, important connections exist between problems originally meant to amuse and mathematical concepts. As a motivation force, then, the inclination to seek diversion and entertainment has resulted in the unintended revelation of mathematical truths, while, at the same time, it provides a good training of mathematical logic.

One of the areas in which recreational mathematics is interconnected with a serious one, include problems whose assignment is understandable to everyone, but not everyone knows how to solve them. These problems have usually caught the attention of both professional mathematicians and the public with its simple formulation and comprehensibility. In this way the recreational mathematics provides the utility of a way to communicate a mathematical ideal to the public at large. Sometimes we can even wait centuries before mathematicians find a solution to a particular problem because the solution requires a nontrivial and advanced mathematical knowledge. These problems include for example *Three classical geometrical problems* which have eventually been proved to be impossible to solve but these proofs took incredible twenty-four centuries. Similar fate we have seen in connection with further problems such as the *Archimedes's cattle problem*, the *Four-color theorem* or the *Fermat's last theorem*. All these problems and others are mentioned in more detail below in the article about the history of recreational mathematics [2].

Another area in which the recreational mathematics is intertwined with the serious one, includes problems that are both understandable and at the same time have quite an easy solution. On the one hand, for a mathematician, it is not difficult to solve such a task, on the other hand the problem might become more mathematically interesting when one tries to generalize it. Thanks to the wonderful universality of mathematics, ideas derived from a simple particular problem can turn out to illuminate a lot of others. Exactly these problems may shift the topic from the recreational toward the serious. Perhaps the clearest example is the problem of *Königsberg's bridges* from graph theory, where Euler made a simple model of the reality that he was studying, and then later workers found that model useful in other situations. Thus recreational mathematics helps as a major source of mathematical models, which are the raw material for mathematical research. As other example we can mention the origins of probability theory. The mathematics of games of chance began in the Middle Ages, but its development by Fermat and Pascal in the 1650s rapidly led to the development of the probability theory, and insurance companies based on this theory were founded in the middle of the eighteenth century. In the article [2] we can find that these and other popular problems have been a major (or the dominant) stimulus to the creation and evolution of the subject.

From the above example we can notice that the recreational problems are often the basis for some serious mathematics. Their major benefit is that they use a mixture of an abstract thought and the real world to motivate various mathematical ideas. The recreational mathematics includes elementary problems with elegant, and at times surprising, solutions. It also encompasses mind-bending paradoxes, ingenious games, bewildering magic tricks and topological curiosities such as Möbius bands and Klein bottles. In fact, almost every branch of mathematics simpler than calculus has areas that can be considered recreational. A lot of these topics we can find in the book devoted to recreational mathematics, for example in [4], [5], [6], [7], [8], [9], [10], [11] or in classic of the genre [13]. Consequently, the study of recreational topics is necessary to understanding the history of many, perhaps most, topics in mathematics.

Examples of problems of recreation mathematics in particular branches of mathematics:

0.1.1 Arithmetics

Elementary arithmetics is everywhere around us and it is used by almost everyone. It is essential to almost every profession and it is frequently used for solving tasks ranging from simple everyday counting to business calculations to scientific research. Riddles and entertaining problems whose solutions depend entirely on elementary arithmetical operations have been in evidence from ancient times to the present. The early puzzle problems were difficult at their time due to the lack of good symbolism and they lost their mystery when the algebraic relations had been developed. Modern arithmetic puzzles are often based on tricks or relations hidden under misleading statements. For example, at Fibonacci's time, algebra and its symbols have not been used often for solving problems as we would do it today. So Fibonacci was rather explaining in words the solution than solving explicit equations as the reader can find in detail in the section dedicated to Fibonacci in [2]. Modern arithmetic problems and riddles are often based on tricks or relations hidden under misleading statements. The example is the riddle "As I was going to St. Ives". For the exact formulation of the riddle see [2]. If we follow the changing of the formulation over time we notice that in these days the formulation of the riddle is presented as a trick question, in which it is not about calculating, but think about its purpose.

0.1.2 Number theory

Number theory is a vast and very attractive field of mathematics that studies the properties of whole numbers. Numbers have fascinated people from the dawn of civilization. It is full of many profound, subtle and beautiful theorems. The most of the problems have very simple formulations, but the proofs of corresponding theorems are often very difficult and lie in an exceeding obscurity. The beautiful example is the *Archimedes cattle problem*, more details in [1]. This problem can be easily understood by everyone, but its solution too difficult even for mathematicians.

Starting from the beginning of the computer era, when programmers have been testing their skills, the quality of the programs and the power of digital computers, plenty of problems from number theory have been solved and various curiosities in this field were discovered.

0.1.3 Geometry

Geometry, one of the oldest sciences, studies problems concerned with size, shape, and relative position of lines, circles, triangles and polygons (plane geometry) as well as spheres and polyhedrons (solid geometry) and with properties of space. Among the oldest recreational problems are those which belong to plane and solid geometry, for example the *Princess Dido's problem* or *Malfatti's problem*, more details in [2].

0.1.4 Combinatorics

Combinatorics is a branch of mathematics that studies arrangements of objects of finite sets, satisfying specified criteria. In particular, it is concerned with "counting" the objects in those sets and with deciding whether certain "optimal" objects exist. A great part of combinatorics has arisen from games and puzzles. This is caused mainly by the fact that combinatorial problems can be understood by a large audience as extensive prerequisites are not required. To solve most of the problems from elementary combinatorics, one often needs only patience, persistence, imagination and intuition. Once J. L. Synge said: "The mind is at its best when at play."

Combinatorics problems include a diverse set of famous entertaining problems such as, for instance, the ring puzzle, the Josephus problem, the Tower of Hanoi puzzle, or the river-crossing problems. Other problems we can find in [2].

0.1.5 Probability

Even if we could not say that for about any other branch of mathematics, we can claim with certainty that the development of probability started from recreational mathematics [2]. This theory which began with the consideration of games of chance should have become one of the most important objects of human knowledge. Probability deals with the estimation of a chance that some event will happen, or figuring out how often this event can occur under given conditions. Probability is full of surprising results and paradoxes, more than any other branch of mathematics. Indeed, thanks to our sense of risks and many aspects of chance, and direct life experience, we should have a good estimation of results.

0.1.6 Graph theory

The beginning of graph theory is associated with Euler's solution of the problem of Königsberg's seven bridges from 1736. His discoveries had a major impact on discrete mathematics and computer science. In the following years Königsberg's problem and Travelling Salesman Problem were important sources of graph theory and they constitute the basis of major fields of optimization, leading on to one of the major unsolved problem of the century: NP=P. These problems can be found for example in [12]. Nowadays, graph theory is extremely valuable not only in the field of mathematics but also in other scientific disciplines such as computer science, electrical engineering, network analysis, chemistry, physics, biology, operations research, social sciences, to name only some. Remember that the four-color problem, one of the most famous mathematical problems ever, was solved using graph coloring [2].

0.2 The amusement aspect

Particular problems of the most of recreational mathematics can be divided into three independent fields: games, mechanical puzzles and word-stated problems.

0.2.1 Mathematical games

A mathematical game is a game whose rules, strategies and outcomes are defined by clear mathematical parameters. Often, such games have simple rules and match procedures, such as Tic-tac-toe and Dots and Boxes. Generally, mathematical games need not be conceptually intricate to involve deeper computational underpinnings. For example, even though the rules of Mancala are relatively basic, the game can be rigorously analyzed through the lens of combinatorial game theory. Mathematical games differ sharply from mathematical puzzles in that mathematical puzzles require specific mathematical expertise to complete, whereas mathematical games do not require any deep knowledge of mathematics to play. Often, the arithmetic core of mathematical games is not readily apparent to players untrained to note the statistical or mathematical aspects.

When studying a game's core mathematics, arithmetic theory is generally of higher utility than actively playing or observing the game itself. To analyze a game numerically, it is particularly useful to study the rules of the game insofar as they can yield equations or relevant formulas. This is frequently done to determine winning strategies or to distinguish whether a given game has a solution. Games of chance and games of strategy also seem to be about as old as the human civilization itself. The mathematics of games of chance began in the Middle Ages and its development by Fermat and Pascal in the 1650 rapidly let to probability theory. Insurance companies based on this theory were founded in the middle of the eighteenth century.

0.2.2 Mechanical puzzles

There exist a huge number of mechanical puzzles that can be in some way or another connected to recreational mathematics. Some only require a certain amount of dexterity; other need ingenuity and logical thought; while yet other ones call for systematic application of mathematical ideas or patterns, such as Rubik's Cube, the Chinese rings, the Tower of Hanoi, Rubik's Clock. The creation of beauty often leads to the question of symmetry and geometry which are studied for their own sake, such as, for instance, the carved stone balls. The beauty of mathematical puzzles lies primarily in the fact that for dealing with them there is no age limit. For this fact, the popularity of mathematical puzzles and games is still bigger because they fulfil the need for diversion, the desire to achieve mastery over challenging subject matter or simply to test our intellectual capacities. Mathematical puzzles can be solved both by a child or an adult. It is of equal importance that mathematical amusements also offer an ample playing field to both the amateur and the professional mathematician. The fact that mathematicians from antiquity to the present day have always taken interest in solving puzzles and have been delighted by them might lead to a conclusion that creative stimulus and aesthetic considerations are closely interwoven. Moreover, it is very interesting that many a great scientist have stressed that they were just puzzlists in their early days, like Tyndall, Huxley, Humboldt, Darwin, Edison or Euler, so we can say that their early puzzle training gave the bent to their minds which in after years inclined them to grapple with problems of greater magnitude. Sam Loyd, an American chess player, a puzzle author and a great recreational mathematician said: "We see how the average boy, who abhors square root or algebra, will find delight in working out puzzles which involve identically the same principles. It makes one think that millions of earnest students who would really have loved to learn, have been abandoned as incorrigible blockheads, because those who had charge of their education did not know how to interest them in their studies. An aversion to figures and desire to forget all about mathematics as soon as one leaves school is almost universal, and yet, if the subject had been taught in more congenial way, the mathematics and inventive bumps might have developed in a way to astonish the family phrenologist."

0.2.3 Problems

Another branch of recreational mathematics deals with various specific problems such as for example "As I was going to St. Ives" or how to ferry a wolf, a goat and a cabbage across a river.

This outlines the convention scope of recreational mathematics, but there is some variation due to the personal taste. Mathematical puzzles and problems or games form a great branch of intellectual activity. They reflect its always youthful unspoiled and inquiring spirit. Whether we are dealing with a puzzle, solving a problem, or play a game, each time our patience and persistence is required, that is, the same qualities are needed that make for good careful mathematical research.

0.3 The pedagogical aspect

The mathematical learning process is not limited to memorization or drills. Memorizing formulas without understanding them leads to the dislike of mathematics. We know that abstract concepts which have not grown out of actual experience are likely to be misunderstood and treated almost like magical notions, and mathematics is more than arithmetic.

Instead of counting lots of meaningless equations that bore children, there is no greater learning experience than trying to solve a good problem. The goal is obtained by an increased understanding of just how broad the subject of mathematics is, of how much further it goes than anything that it has ever been taught at school, of its astonishingly wide range of applications, and of the surprising cross-connections that bind the entire subject together into a single, amazingly powerful package. All achieved by solving puzzles and playing games. Recreational mathematics provides many such problems and almost every problem can be extended or amended. Hence recreational mathematics is also a treasury of problems for student investigations.

Mathematics education is not just about the volume of knowledge, but a permanent mental mastery of skills and abilities to use them. We can consolidate this knowledge by using recreational mathematics.

Many students enjoy working on puzzles, and they can be easily motivated to adopt learning strategies that will improve their puzzle-solving skills. Grid number puzzles (for example *Magic square*) provide strong intrinsic motivation to solve for unknown numbers from a handful of clues. Since many math problems have a similar form, students who enjoy solving these puzzles can develop positive attitudes toward other forms of math in non-puzzle contexts as well.

Even without supervision, students can learn to be creative and persistent after working on many hard grid puzzles. For this purpose, in recent years were created several science centres. In these centres are created favourable conditions for education of children and they can learn while enjoying themselves in their spare time. More details about science centres and their importance we can find in [3].

0.4 The historical aspect of recreational mathematics

Recreational mathematics is very useful to the historian of mathematics. Recreational problems often are of great age and usually can be clearly recognised, they serve as useful historical markers, tracing the development and transmission of mathematics (and culture in general) in place and time. *Magic Square*, *The ass and mule problem* and others are excellent examples of this process. The number of topics which have their origins in China or India is quite surprising and it emphasizes our increasing realization that modern algebra and arithmetic derive more from Babylonia, China, India and from the Arabs than from Greece.

Maybe, for these reasons, it is difficult to determine which problem belongs to the recreational mathematics. In centuries past almost all mathematical problems (excluding, of course, real-life problems of measurement and counting) existed chiefly for intellectual pleasure and stimulation. Ultimately, however, deciding the recreational merits of a given problem involves imposing arbitrary distinctions and artificial boundaries. Over time, a significant number of recreational mathematics problems have become integral to the development of entirely new branches in the field. A lot of these examples we can find in [2].

Although several tasks may appear trivial to today's amateur mathematician, we must recall that several centuries ago, most of these problems were not easy to solve. While including such problems provides historical insight into mathematical studies, we must also remain alert to their historical context.

Recreational mathematics is a treasury of problems which make mathematics fun. In medieval arithmetic texts, recreational questions are interspersed with more straightforward problems to provide breaks in the hard slog of learning. These problems are often based on reality and they illustrate the idea that "Mathematics is all around you – you only have to look for it."

Another reason why recreational mathematics is so important for a historian is that we cannot simply personalize the history of mathematics. It is not enough to know the date, the name of mathematics and its discovery. The history of mathematics is very long and the recreational mathematics is an ideal vehicle for communicating historical and multicultural aspects of mathematics.

Bibliography

[1] Bártlová T.: *Archimedes' cattle problem,* (Czech) Archimédes. Několik pohledů do jeho života a díla, edice Dějiny matematiky, volume 54, Matfyzpress, Prague, 2012, pp. 99—108.

[2] Bártlová T.: *History of recreational mathematics,* Manuscript prepared for publication, 2016.

[3] Bártlová T.: *New way in teaching mathematics,* Manuscript submitted, 2016.

[4] Berlekamp E. R., Conway J. H., Guy R. K.: *Winning Ways for Your Mathematical Plays,* Vols. 1-2, Academic Press, London-New York, 1982.

[5] Dudeney H.: *Amusements in Mathematics,* London 1917 (reprinted by Dover Publications, New York, 1958).

[6] Gardner M: *The Scientific American Book of Mathematical Puzzles and Diversions,* Simon and Schuster, New York, 1959.

[7] Gardner M: *The Second Scientific American Book of Mathematical Puzzles and Diversions: A New Selection,* Simon and Schuster, New York, 1961.

[8] Gardner M: *Martin Gardner's New Mathematical Diversions from Scientific American,* George Allen and Unwin Ltd, London 1969.

[9] Gardner M: *Knotted Doughnuts and Other Mathematical Entertainment,* W. H. Freeman, New York, 1986.

[10] Gardner M: *The Colossal Book of Mathematics,* W. W. Norton and Company, New York-London, 2001.

[11] Gardner M: *The Colossal Book of Short Puzzles and Problems,* W. W. Norton and Company, New York-London, 2006.

[12] Petkovic M. S.: *Famous Puzzles of Great Mathematicians,* American Mathematical Society, Rhode Island, 2009.

[13] Rouse Ball W. W., Coxeter H. S. M.: Mathematical Recreations and Essays, Dover Publications, New York, 1987.

Paper I

T. Bártlová: *History of recreational mathematics,* Manuscript prepared for publication, 2016.

History of recreational mathematics

Mathematical puzzles and games have been in evidence ever since man first began posing mathematical problems. The history of mathematics is replete with examples of puzzles, games, and entertaining problems. The most of them were basically meant just for fun. Some problems attracted the minds of scientists so much that they found important connections existing between problems originally meant to amuse, and mathematical concepts critical to graph theory, geometry, optimization theory, combinatorics, and number theory.

In centuries past almost all mathematical problems (excluding real-life problem of measurement and counting) existed chiefly for intellectual pleasure and stimulation. Ultimately, however, deciding the recreational merits of a given problem involves imposing arbitrary distinctions and artificial boundaries. Over time, a significant number of recreational mathematical problems have become integral to the development of entirely new branches in the field. That means although several tasks may appear trivial to today's amateur mathematician, we must recall that several centuries ago, most of these problems were not easy to solve. While including such problems provides historical insight into mathematical studies, we must also remain alert to their historical context.

This text will be structured as a summary of important mathematical puzzles, games and entertaining problems over time. I tried to select objects that have a significant role in recreational mathematics and assign them to a data line, or monitor their progress over time. The text begins with *Carved Stone Balls* and ends with modern cartoon series *Futurama*. Despite the fact I have endeavored to describe the maximum of mathematical recreation, as I wrote in the previous paragraph, there is no clear definition of what does belong to recreational mathematics and what does not. I am aware of the fact that, for example, chess problems or games development would be worth a separate investigation. This text, however, does not attempt to enumerate exactly all the problems of recreational mathematics but to draw attention to the important ones which we still recall for their importance or which influenced the serious mathematics. At the end of this text I shall mention some of the famous recreational mathematicians, but for sure not of all. The number of mathematicians who are interested in recreational mathematics is increasing every day and hopefully it will remain so.

1 The Carved Stone Balls

Probably the oldest known problem which belongs to recreational mathematics is represented by the *Carved Stone Balls* from Scotland, forming an enigmatic class of objects. These are designed for manipulation and seem to date mainly to the Late Neolithic period (3000-2500 B.C.). They are made of various stones ranging from sandstone to granite. They all are of a relatively similar size and are decorated with carved evenly-spaced patterns of circular bosses or knobs around the surface of the sphere (figure 1[1]). The designs vary with the majority being based around a series of six bosses, but the number of bosses varies from 3 to 160. Some carved balls are more skilfully manufactured than others, and a rare few have additional decoration. All show an appreciation for symmetry in the design.

Figure 1: *Carved Stone Balls* from Scotland

Despite their huge numbers, very little is known about the *Carved Stone Balls* and their purpose is still unknown. Few of the balls are damaged or show any signs of use and they have not been found in contexts that would suggest a specific function. They are presumed to have been non-utilitarian objects with a symbolic

Mathematicians are interested in the *Carved Stone Balls* because of their aesthetic beauty. Their makers were generating spherical objects with maximal symmetry. *Carved Stone Balls* come in many shapes, but most have bosses equal to the number of faces on the Platonic solids. Some researchers have suggested that the *Carved Stone Balls* were attempts to realize the Platonic solids (figure 2[2]).

Figure 2: Five *Carved Stone Balls*

[1] *Carved Stone Balls* sketched according to the illustration from [8].
[2] Five *Carved Stone Balls* sketched according to the illustration from [9].

2 Magic square

Perhaps the very oldest known example of a computational problem is the magic square shown on the figure 3. Known as *lo-shu* to Chinese mathematicians around 2200 B.C., the magic square was supposedly constructed during the reign of the Emperor Yu. According to a Chinese myth, the Emperor Yu came across a sacred turtle in a tributary of the Yellow River, with strange markings on its shell (figure 3[3]). These markings are now known as the *lo-shu*, which means "Lo river writing".

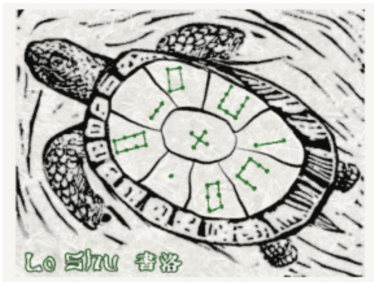
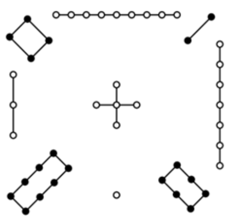


Figure 3: turtle *lo-shu*

The figure 4 on the left shows the *lo-shu* configuration where the numerals from 1 to 9 are composed of knots in strings with black knots for even and white knots for odd numbers, which is displayed on the figure 4 on the right.



4	9	2
3	5	7
8	1	6

Figure 4: square pattern with markings and numbers

In this configuration, the numbers in every row, every column and every diagonal add to the same sum (in this case it is 15). A number square with these properties is said to be magic, and the corresponding number is called its magic constant. Usually the square is made from successive whole numbers, 1, 2, 3, 4 and so on, but sometimes this condition is relaxed.

In these days the magic squares are very well known. The literature available nowadays on magic squares is vast, and it includes many variations such as magic squares of different sizes, magic squares whose entries are all distinct perfect squares, magic cubes, and many more, see for example [41].

[3]Mystic turtle, sketched according to the illustration from [46].

3 As I was going to St. Ives

The Rhind (of Ahmes) Papyrus, dating to around 1650 B.C., suggests that the early Egyptians often stated their mathematics problems in the form of a puzzle. As these problems had no application in everyday life, perhaps their main purpose was to provide intellectual pleasure. One of the earliest instances has the form of a nursery rhyme [21]:

Seven houses, in each are 7 cats,
each cat kills 7 mice,
each mouse would have eaten 7 ears of spelt,
each ear of spelt will produce 7 hekat.
What is the total of all of them?

The answer is given by the geometric series

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}$$

with $n = 5$ and $r = 7$. Therefore the total amounts to

$$\sum_{k=0}^{5} 7^k = \frac{1 - 7^6}{1 - 7} = 19\,607.$$

Nearly 3000 years later Fibonacci (who we will mention later in this section 13) in his *Liber Abaci* (1202) posed a very similar problem [37]:

Seven old women are traveling to Rome,
and each has seven mules.
On each mule there are seven sacks,
in each sack there are seven loaves of bread,
in each loaf there are seven knives,
and each knife has seven sheaths.
The question is to find the total of all of them.

It is tempting to suppose that these puzzles are related. If they are, could there be a historical connection across 5000 years also with a nursery rhyme states, named "As I was going to St. Ives", from the British 18th century *Mother Goose collection* [31]:

As I was going to St. Ives,
I met a man with seven wives.
Each wife had seven sacks,
Each sack had seven cats,
Each cat had seven kits.
Kits, cats, sacks and wives,
How many were going to St. Ives?

We can notice that in this formulation the riddle is presented as a trick question. In this variant, the readers are not supposed to calculate the total quantity of all the objects mentioned. Since the man and his wives, sacks, etc. were met by the narrator on the way to St. Ives, they were in fact leaving (not going to) St. Ives. The number going to St. Ives is therefore "at least one" (the narrator), but might be more since the problem does not mention if the narrator is alone.

4 Nine Men's Morris

Nine Men's Morris, often called *Merrills*, *Merels*, *Mills*, or any of thousands of other names, is a strategy board game for two players with a very long history. The pattern of the board has been found at a temple in Kurna, Egypt, dated to 1440 B.C. Other ancient boards appear in Troy, Athens, and Ireland, and a board from the 10th century was found on the Viking ship called Gokstad.

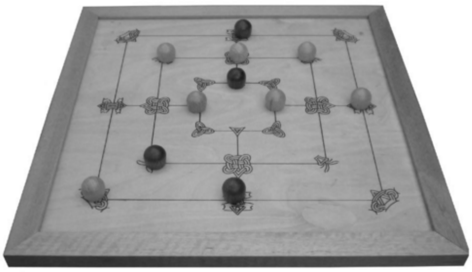


Figure 5: a Nine Men's Morris set

The game peaked in popularity in medieval England. Boards have been found carved into the cloister seats at the English cathedrals at Canterbury, Gloucester, Norwich, Salisbury and Westminster Abbey. These boards used holes, not lines, to represent the nine spaces on the board — hence the name "nine holes". Another board is carved into the base of a pillar in the Chester Cathedral. Giant outdoor boards were sometimes cut into village greens.

By Shakespeare's time the game was well known and very popular, and sometimes it was played as an outdoor pursuit on large boards cut in the turf: a passage in A Midsummer Night's Dream describes the *Nine Men's Morris* being filled with mud.

The board itself consists of three concentric squares, with the middle of each side of the inner square being joined to the other square by another line.

5 Three Classical Problems

Very interesting problems, which date back to the early days of Greek geometry, in the 5th century B.C., are three geometrical problems in particular, often referred to as the Three Classical Problems. We shall now describe The formulations of these well-known problems:

a) *the squaring* (*or quadrature*) *of the circle*: construct a square which has the same area as a given circle,

b) *the doubling* (*or duplicating*) *of the cube*: construct the edge of a cube that is double the volume of a given cube,

c) *the trisection of an angle* – divide an arbitrary angle into three equal angles.

Greeks tried to solve these problems using only a straight edge and a compass, which was later called Euclidean constructions. Though, naturally, they were never able to do so (as this is impossible, as is known now), the Greeks did find a series of many other related, remarkably clever, constructions. These intransigent problems were profoundly influential on future geometry and led to many fruitful discoveries, although their actual solutions (or rather, as it turned out, the proofs of the impossibility to solve them) had to wait until the 19th century. From today's perspective, the fact that the problem solving waited nearly a twenty-four centuries, makes these problems legendary.

6 The Cattle Problem

The ancient Greeks were deeply interested in numbers. Among other things, they were well aware of the necessity of being able to handle huge numbers in order to describe certain quantities. They created many various problems that illustrated the enormous power of numbers, both for practical applications and for amusement. One name outstandingly familiar to us is that of Archimedes of Syracuse, who lived around 250 B.C. He is known to be the author of the cattle problem which we shall formulate now [49] in detail:

If thou art diligent and wise, O stranger, compute the number of cattle of the Sun, who once upon a time grazed on the fields of the Thrinacian isle of Sicily, divided into four herds of different colours, one milk white, another glossy black, the third yellow and the last dappled. In each herd were bulls, mighty in number according to these proportions: Understand, stranger, that the white bulls were equal to a half and a third of the black together with the whole of the yellow, while the black were equal to the fourth part of the dappled and a fifth, together with, once more, the whole of the yellow. Observe further that the remaining bulls, the dappled, were equal to a sixth part of the white and a seventh, together with all the yellow. These were the proportions of the cows: The white were precisely equal to the third part and a fourth of the whole herd of the black; while the black were equal to the fourth part once more of the dappled and with it a fifth part, when all, including the *bulls, went to pasture together. Now the dappled in four parts were equal in number to a fifth part and a sixth of the yellow herd. Finally the yellow were in number equal to a sixth part and seventh of the white herd.*

If thou canst accurately tell, O stranger, the number of Cattle of the Sun, giving separately the number of well-fed bulls and again the number of females according to each colour, thou wouldst not be called unskilled or ignorant of numbers, but not yet shalt thou be numbered among the wise.

But come, understand also all these conditions regarding the cows of the Sun. When the white bulls mingled their number with the black, they stood firm, equal in depth and breadth, and the plains of Thrinacia, stretching far in all ways, were filled with their multitude. Again, when the yellow and the dappled bulls were gathered into one herd they stood in such a manner that their number, beginning from one, grew slowly greater till it completed a triangular figure, there being no bulls of other colours in their midst nor one of them lacking. If thou art able, O stranger, to find out all these things and gather them together in your mind, giving all the relations, thou shalt depart crowned with glory and knowing that thou hast been adjudged perfect in this species of wisdom.

In the first part of the problem we are supposed to count the number of cattle in each of the eight categories. These numbers are related by nine simple conditions which can be written in the form of linear equations:

$$\begin{aligned}
X &= \left(\frac{1}{2} + \frac{1}{3}\right) \cdot Y + T, \\
Y &= \left(\frac{1}{4} + \frac{1}{5}\right) \cdot Z + T, \\
Z &= \left(\frac{1}{6} + \frac{1}{7}\right) \cdot X + T, \\
x &= \left(\frac{1}{3} + \frac{1}{4}\right) \cdot (Y + y), \\
y &= \left(\frac{1}{4} + \frac{1}{5}\right) \cdot (Z + z), \\
z &= \left(\frac{1}{5} + \frac{1}{6}\right) \cdot (T + t), \\
t &= \left(\frac{1}{6} + \frac{1}{7}\right) \cdot (X + x).
\end{aligned}$$

Here the variables X, Y, Z, T denote the number of white, black, spotted and brown bulls, and, analogously, the variables x, y, z, t stand for the number of white, black, spotted and brown cows. The problem is to determine the size of the eight unknowns, and thus the size of the herd.

By solving the system of equations by the Gauss elimination, we get

$$x = \frac{6\,869\,229}{4\,657}\,k, \quad y = \frac{4\,893\,246}{4\,657}\,k, \quad z = \frac{3\,515\,820}{4\,657}\,k, \quad k \in \mathbb{N}.$$

However, the catch is, we are looking for an integer solution. The smallest possible solution therefore is

$$
\begin{array}{lllllllll}
X & = & 2\,226 \cdot 4\,657\, n & = & 10\,366\,482\, n & \quad & x & = & 7\,206\,360\, n \\
Y & = & 1\,602 \cdot 4\,657\, n & = & 7\,460\,514\, n & & y & = & 4\,893\,246\, n \\
Z & = & 1\,580 \cdot 4\,657\, n & = & 7\,358\,060\, n & & z & = & 3\,515\,820\, n \\
T & = & 891 \cdot 4\,657\, n & = & 4\,149\,387\, n & & t & = & 5\,439\,213\, n
\end{array}
$$

where n is an arbitrary integer.

The epigram of this problem is more complicated. In this part the problem it is required that $X + Y$ be a square number and $Z + T$ be a triangular number. At this time, we have to fully appreciate the comment by Archimedes in which he says that anyone who solves the problem should not be called *unskilled or ignorant of numbers, but not yet shalt thou be numbered among the wise.* Nothing could be more apt, since there was to elapse more then 2000 years before a computer finally found the solution.

In 1880, a German mathematician A. Amthor showed [27] that the total number of cattle in the Archimedes herd has to have 206 545 digits, beginning with 7 766. Not surprisingly, Amthor gave up at that point. Over the next 85 years, a further 40 digits were worked out. But it was not until 1965 when the mathematicians at the University of Waterloo in Canada finally found the complete solution. It took over seven and a half hours of computation on an IBM 7040 computer.

Let us note that some historians disputed the authorship of Archimedes. Some of them have commented that the solution is senseless, giving a remarkably huge number. On the other hand, the outstanding Danish researcher of Archimedes' work, Johan L. Heiberg, as well as some other mathematicians, was convinced that the above problem should indeed be attributed to Archimedes. Clearly Archimedes had a mischievous streak in addition to his principles, and maybe he was trying to pull a fast one on his Alexandrian rival Eratosthenes. At any rate, the problem will always fascinate us with its simple formulation, which is in a deep contrast to its extraordinarily difficult solution. Further details about the *Cattle Problem* can be found in [1].

7 The Princess Dido's problem

The classical Roman poet Virgil (70 B.C. – 19 B.C.) described in the *Aeneid* the legend of the Phoenician princess Dido:
After escaping tyranny in her home country, the princess Dido arrived on the coast of North Africa where she asked the Berber king Iarbas for a small bit of land for a temporary refuge until she could continue her journeying, only as much land as could be encompassed by a bull's hide. They agreed. The clever Dido then cut the bull's hide into the thinnest

possible strips so that she had enough to encircle an entire nearby hill, which was therefore afterward named Byrsa (which means "hide").

Although the assignment of this problem comes from Greek mythology, a solution was described by a Greek mathematician Zenodorus already in the 2nd century B.C. who described that the closed curve which has the maximum area for a given perimeter is circle. Today the problem of enclosing the maximum area within a fixed boundary is recognized as a classical isoperimetric problem. It is regarded as the first problem in a new mathematical discipline, established 17 centuries later, as calculus of variations. Detailed solutions to the problem, we can find in [3].

8 The ass and mule problem

Some problems, which now safely belong to the category of recreational mathematics, were created in other parts of the world. One such problem can be found in a Chinese mathematical text *Nine chapters*, dated 179 A.D. [26]:

Now there are 2 persons, A [*and*] *B. Each has an unknown amount of coins. A gets 1/2 of B's, then* [*has*] *50 coins. B gets 1/3 of A's, then* [*has*] *50 coins also. Tell: what is the amount of coins A* [*and*] *B has each?*

If x is the amount of coins which A has and y is the amount of coins which B has, then we have the identities:

$$x + \frac{1}{2}\,y = 50$$
$$\frac{1}{3}\,x + y = 50$$

which form a system of two linear equations. The solution is $x = 37,5;\ y = 25$.

Versions of this problem also appear in the work of other authors, for example of Zhang Quijian in the 5th century. Actually the earliest known version of this problem is attributed to Euclid, from about 300 B.C. Heiberg's edition gives the problem in Greek and Latin verses.

Alcuin of York formulated in his *Problems to Sharpen the Young* a similar problem [24]: *Two men were leading oxen along a road, and one said to the other: 'Give me two oxen and I'll have as many as you have.' Then the other said: 'Now you give me two oxen and I'll have double the number you have.' How many oxen were there, and how many did each one have?*

This version leads to the system of equations:

$$\begin{aligned} x+2 &= y-2 \\ y+2 &= 2\,(x-2), \end{aligned}$$

where x is the count of the oxen belongs to the first man and y is the count of the oxen belongs to the second man.

Alcuin of York gives an unusual variant of the problem because he assumes the second person starts from the situation after the transfer mentioned by the first person has taken place.

By about the 9th century, the problem was considered standard, not only in Europe but also in India and the Arabian world. From the 12th century onwards, the problem appears in many more Western sources. It became well known when Euler included it in his *Elements of Algebra* (1770). It is now known as "The ass and mule problem" [16]:

A mule and an ass were carrying burdens amounting to several hundred weight. The ass complained of his, and said to the mule, I need only one hundred weight of your load, to make mine twice as heavy as yours; to which the mule answered. But if you give me a hundred weight of yours, I shall be loaded three times as much as you will be. How many hundred weights did each carry?

The question has remained a standard problem ever since. A generalization of this problem has been studied by David Singmaster. His modified version states [39]:

'If I had ***a*** *from you, I'd have* ***b*** *times you,' and the second responds: 'And if I had* ***c*** *from you, I'd have* ***d*** *times you.'*

This leads to the system of equations:

$$x + a = b\,(y - a)$$
$$y + c = d\,(x - c),$$

where x means the count of something belongs to the first person and y means the count of something of the second person and a, b, c and d are parameters. In Alcuin version these parameters are $a = c = d = 2$ and $b = 1$. In Euler's problems the parameters are $a = c = 100$, $b = 2$ and $d = 3$.

9 The Josephus problem

Another of the problems from antiquity is known as the Josephus problem [51]:

In the Romano-Jewish conflict of 67 A. D., the Romans took the town Jotapata which Josephus was commanding. He and 40 companions escaped and were trapped in a cave. Fearing capture they decided to kill themselves. Josephus and a friend did not agree with that proposal but were afraid to be open in their opposition. However, in this extreme distress, he was not destitute of his usual sagacity; but trusting himself to the providence of God and he suggested following: "And now," said he, "since it is resolved among you that you will die, come on, let us commit our mutual deaths to determination by lot. He whom the lot falls to first, let him be killed by him that hath the second lot, and thus fortune shall *make its progress through us all; nor shall any of us perish by his own right hand, for it would be unfair if, when the rest are gone, somebody should repent and save himself.”*

Mathematically speaking: *A group of 41 men stand in a circle so that every third man is killed. Where should Josephus and his friend stand to survive?*

Denote the men in the circle by number from 1 to 41. During the first round, the men who have been assigned the numbers 1, 5, 10, 14, 19, 23, 18, 32, 37, and 41, are subsequently killed. In the second round, the numbers 7, 13, 20, 26, 34, and 40 are eliminated. During the third round, the numbers 8, 17, 29, and 38 disappear. And within the final round, the numbers 11, 25, 2, 22, 4, and 35 are gone. At the end, only the men standing in the positions having numbers 16 and 36 are left alive. Incredibly, Josephus and his friend placed themselves exactly to these positions, allowing them to ensure survival.

For the first time, the puzzle appeared in the work of an unknown author, possibly Ambrose of Milan, who wrote in 370, under the nom de plume of Hegesippus, a work entitled *De bello iudaico*. Later on, the problem found its way not just into European manuscripts, but also into Arabian and Japanese books (figure 6[4]). Depending on the time and location where the particular version of the Josephus problem was raised, the survivors and victims were sailors and smugglers, Christians and Turks, sluggards and scholars, good guys and bad guys, and so on. This puzzle attracted attention of many outstanding scientists, including Gerolamo Cardano, Leonhard Euler, Peter Tait, Herbert Wilf, Ronald Graham and Donald Knuth.

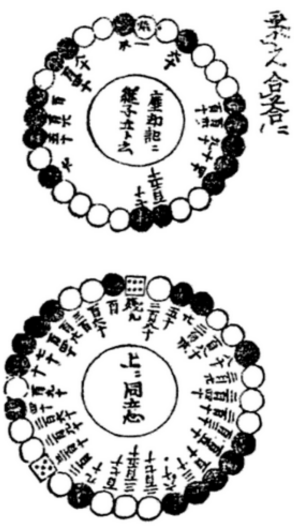


Figure 6: the Josephus problem in Japan

[4] Josephus problem, sketched according to the illustration from [40].

We can consider the Josephus problem in a fairly general form involving any number of people:

There are n people standing in a circle so that every third person is eliminated. Gradually every other person will be executed until there are no survivors except one, who will be pardoned. Where should he stand to survive?

A group of n people are standing in a circle, numbered consecutively clockwise from 1 to n. Let $j(n)$ denote the last person remaining. Obviously, $j(1) = j(2) = 1$. For $j(n)$, where $n \geq 3$, we shall deduce a recursive equation. If n is even, that is, $n = 2k$, then after the k-th elimination there will remain k people enumerated $1, 3, \ldots, 2k-1$. If the person with the number $j(2k)$ should survive, then his number is $2j(k) - 1$. We get the identity

$$j(2k) = 2\,j(k) - 1.$$

If n is odd, that is, $n = 2k+1$ for some k, then after the $(k+1)$-th elimination there will remain k people with the numbers $3, \ldots, 2k-1, 2k+1$. If the person with the number $j(2k+1)$ should survive, then his number is $2j(k)+1$. We thus have

$$j(2k+1) = 2j(k) + 1.$$

With these recursively equations we can get $j(n)$ for any integer n.

Moreover, if we make the table of values for small n, then we can see that for $n = 2^k$ one has $j(n) = 1$ and each time n is increased by 1, $j(n)$ increases by 2 until we reach the next power of two. Thus if $n = 2^k + l$, where $0 \leq l \leq 2^k$, then $j(n) = 2l + 1$, as can be proved by mathematical induction.

Our problem, however, did not concern only saving Josephus, but also saving his friend. Thus for $n \geq 2$ we find $i(n)$, the number of the person who will be the last one eliminated and also we assume that the person $j(n)$ is the only one who gets a pardon. We can proceed in solving an analogous set of equations and get

$$\begin{aligned} i(2n) &= 2\,i(n) - 1 \\ i(2n+1) &= 2\,i(n) + 1. \end{aligned}$$

The initial conditions are different. Instead of $j(1) = 1$, we get $i(2) = 2$ and $i(3) = 1$.

If we make again the table of values for small n, we can see that for $n = 3 \cdot 2^k + l$, where $l < 3 \cdot 2^k$, one has $i(n) = 2l + 1$, which can be once again also proved by mathematical induction.

R. L. Graham, D. E. Knuth and O. Patashnik describe the Josephus problem in their book [23].

Ernest Dudeney's book *Amusements in Mathematics*, published in 1917, presents to the reader a puzzle similar to the Josephus problem but involving cats and mice. It does not really advance our quest to discover the truth about Flavius's calculatory prowess, but does provide an entertaining variation on the theme [15]:

"*Play fair!*" *said the mice.* "*You know the rules of the game.*"

"*Yes, I know the rules,*" *said the cat.* "*I've got to go round and round the circle, in the direction that you are looking, and eat every thirteenth mouse, but I must keep the white mouse for a tit-bit at the finish. Thirteen is an unlucky number, but I will do my best to oblige you.*"

"*Hurry up, then!*" *shouted the mice.*

"*Give a fellow time to think,*" *said the cat.* "*I don't know which of you to start at. I must figure it out.*"

While the cat was working out the puzzle he fell asleep, and, the spell being thus broken, the mice returned home in safety. At which mouse should the cat have started the count in order that the white mouse should be the last eaten?

When the reader has solved that little puzzle, here is a second one for him.

What is the smallest number that the cat can count round and round the circle, if he must start at the white mouse (*calling that "one" in the count*) *and still eat the white mouse last of all?*

And as a third puzzle try to discover what is the smallest number that the cat can count round and round if she must start at the white mouse (*calling that "one"*) *and make the white mouse the third eaten.*

10 The Chinese rings

The figure below shows a very familiar toy known as the *Chinese rings*. According to the legend, Chinese general Hung Ming (181-234 A.D.) made this toy puzzle to amuse his wife while he was away at the wars [12]. But despite its name, no one has ever proven its Chinese origin.

The French call this puzzle *La Baguenodier* and the English call it *Tiring Irons*. According to Steinhaus, this device was originally used by French peasants to lock chests. The word "baguenaudier" means "time-waster" in French.

Gerolamo Cardano was apparently the first one to describe this puzzle in 1550 in his *De Subtilitate*.

The *Chinese rings* puzzle consists of a number of rings hung upon a long wire loop. Each ring is connected loosely by a post to a platform below the loop. Each of the connecting posts is linked to the corresponding ring to prevent removal of a ring from the loop (figure 7[5]). The ring can slide along the loop to its end (denoted by A) and can be taken off or put on the loop in such a way that any other ring can be taken off or put on only when the one next to it towards A is on, and all the rest towards A are off the loop. The order of the rings cannot be changed. The aim is to remove all the rings in the minimal number of moves.

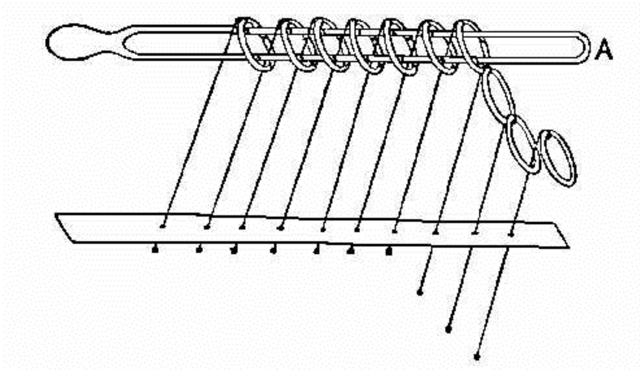


Figure 7: *Chinese rings* puzzle

The solution to the *Chinese rings* puzzle is similar to that of the *Tower of Hanoi* (section 27), in that they both require a reversal of the procedure, in other words, putting the rings back on the loop.

W. W. Rouse Ball and H. S. M. Coxeter in [34] describe a procedure to find the total number of steps necessary to remove all of the rings. The minimal number of moves is either $\frac{1}{3}(2^{n+1}-1)$ if n id odd, or $\frac{1}{3}(2^{n+1}-2)$ if n is even. These numbers can be expressed by the recurrence relation

$$A_0 = 1,\ A_2 = 2,\ A_n = A_{n-1} + 2A_{n-2} + 1.$$

Although very old, this toy is sold even today in toy shops all over the world.

11 A hundred birds

An already mentioned Chinese mathematician Zhang Qiujian wrote his mathematical text *Zhang Qiujian suanjing* (*Zhang Qiujian's Mathematical Manual*) some time between 468 A.D. and 486 A.D. In one chapter he introduces perhaps the most famous problem in the entire Zhang's treatise, called *A hundred fowls* [6]:

If a cock is worth 5 cash, a hen 3 cash, and 3 chickens together 1 cash; how many cocks, hens and chickens, in all 100, may be bought for 100 cash?

[5] *Chinese rings* puzzle, sketched according to the illustration from[10].

In Western Europe this problem was described by Alcuin of York (735-804). In his version we can find a camel which costs 5 solidi, an ass that costs 1 solidus, and 20 sheep that together cost 1 solidus.

A similar problem can be found in the *Book of mathematical curiosities* by an Indian mathematician Abu Kamil (850-930). His treatise introduces a collection of ten solved problems motivated by selling various birds who apparently serve as food (chickens, ducks, pigeons, wood pigeons, sparrows and larks are being sold in this version). All the mentioned problems ultimately lead to the necessity of solving a system of diophantine equations from three to six unknowns.

Euler (1770) used the problem in his *Elements of Algebra* as an introductory example before plunging into a theoretical approach [16]:

A certain person buys hogs, goats and sheep, to the number of 100, for 100 crowns; the hogs cost him $3\frac{1}{2}$ crowns apiece; the goats $1\frac{1}{3}$ crown; and the sheep, $\frac{1}{2}$ a crown. How many had he of each?

Euler translates his problem into modern algebraic form with the system of equations:

$$p + q + r = 100$$
$$3\frac{1}{2}p + 1\frac{1}{3}q + \frac{1}{2}r = 100$$

Elimination of r leads to $q = 60 - \frac{18p}{5}$ and since q is an integer, p is divisible by 5. The solutions are (5,42,53), (10,24,66), (15,6,79).

12 River-crossing problems

As Europe emerged from the Dark Ages, interest in arts and sciences reawakened. In 8th century England, the mathematician and theologian Alcuin of York (735-804) wrote a book *Problems to sharpen the young* in which he included a problem that involved a man wishing to ferry a wolf, a goat and a cabbage across a river [24]:

A man had to take a wolf, a goat and a bunch of cabbages across a river. The only boat he could find could only take two of them at a time. But he had been ordered to transfer all of these to the other side in good condition. How could this be done?

River-crossing problems under specific conditions and constraints were very popular in medieval Europe. Alcuin, Tartaglia (section 17), Bachet, Trenchant and Leurechon studied puzzles of this type. A variant involves how three couples should cross the river in a boat that cannot carry more than two people at a time. The problem is complicated by the jealousy of the husbands, each husband is too jealous to leave his wife in the company of either of the other men.

Essentially the same puzzle is also found in Africa, in Ethiopia, in the Cape Verde Islands, in Cameroon, and among the Kpelle of Liberia and elsewhere. Since these African versions are often logically distinct from the Western ones, they may well be entirely independent: the difficulty of transporting uncongenial items across a river used to be universal.

This kind of problem can be solved using a graph. It is a truly wonderful example of how one can apply elements of graph theory to solving Alcuin's problem. Let M, W, G, and C stand for the man, the wolf, the goat, and the cabbage. According to the puzzle's conditions, the following set of symbols denote the permissible states on the starting bank: $MWGC$, MWG, $MWC\, MGC$, MG, CW, W, G, $C\, O$. The symbol 0 refers to the state once the river crossing has been accomplished. The figure 8 shows the graph of all possible transits among the accepted states and now we can simply reduce the solution of the puzzle to the determination of the shortest path between the vertex $MWGC$ and the vertex 0.

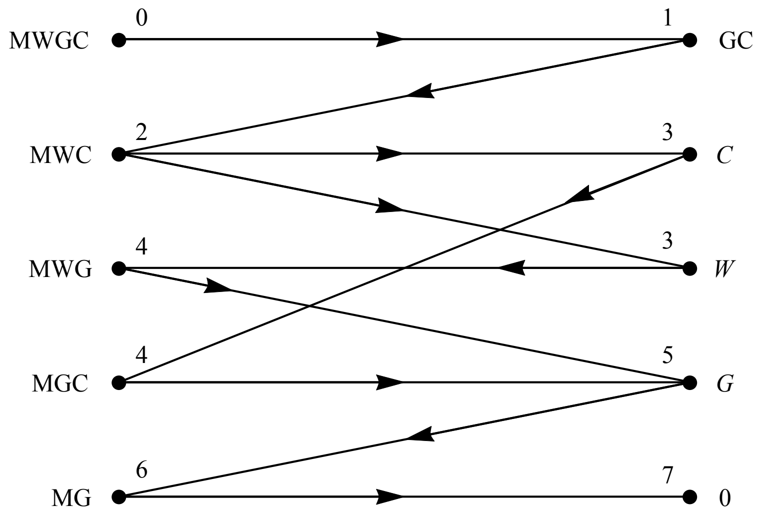


Figure 8: *Crossing the river* - a graph

There are two minimal solutions which appear due to the possibilities of taking a fourth and a fifth step. Each of these solutions require seven transits:

a) $MWGC$, CW, MWC, W, MWG, G, $MG\,0$,

b) $MWGC$, CW, MWC, C, MGC, G, $MG, 0$.

Now we can easily record, as a result, lists of passengers in the boat:

a) $MG - M - MC - MG - MW - M - MG$,

b) $MG - M - MW - MG - MC - M - MG$.

This problem allows a solution not only using a graph, but also using a geometric approach. The picture 9 displays the three-dimensional space, where x-axis, y-axis and z-axis state wolf, cabbage and goat position. This space consists of triples (x, y, z) where each symbol either 0 (on this side of the river) or 1 (on the far side). So the problem is to get $(0, 0, 0)$ to $(1, 1, 1)$ without anything being eaten. When we use this description, we do not need to specify the position of the man, since he always travels in the boat.

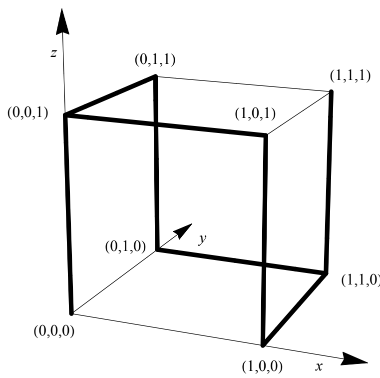


Figure 9: *Crossing the river* - 3-dimensional space

There are eight possible triples, and they can be thought of as the vertices of a cube. Because only one item can be on the boat beside the man, the permissible moves are the edges of the cube. The permitting edges are shown in black. The remaining edges (black) do not cause mayhem. So the puzzle reduces to a geometric one: find a route along the black edges, from $(0, 0, 0)$ to $(1, 1, 1)$. The two solutions are immediately evident.

The newest version of the problem appeared in a television show The Simpsons in an episode "Gone Maggie Gone" [22]. Homer finds himself trapped on one side of a river with his baby Maggie, his dog Santa's Little Helper, and a large bottle of poison capsules. Homer is desperate to cross the river. The boat can only carry Homer and one other item at a time – baby, dog, and thing. In this case, the dog Santa's Little Helper is essentially equivalent to wolf, Maggie has the same role as the goat and the poison is in place of the cabbage. I shall mention more details about this television show in section 37.

13 Leonardo Fibonacci

Another scholar who has studied many interesting problems and created many mathematical puzzles that nowadays belong to the field of recreational mathematics, was Leonardo Fibonacci. His most famous work is the book called *Liber Abaci*, published in 1202. His approach to solving problems is very unique because in many cases, Fibonacci gave more than one version of the problem, and he demonstrated an astonishing versatility in the choice of several methods of solution. In addition, he was not often using algebra in solving the problem, as we would do today. Instead, he was explaining in words the solution rather than solving explicit equations. A nice example of one of the problems that appear in *Liber Abaci* is the following one [37]:

A man whose end was approaching summoned his sons and said: "Divide my money as I shall prescribe." To his eldest son, he said, "You are to have 1 bezant [a gold coin first struck at Byzantium] and a seventh of what is left." To his second son he said, "Take 2 bezants and a seventh of what remains." To the third son, "You are to take 3 bezants and a seventh of what is left." Thus he gave each son 1 bezant more than the previous son and a seventh of what remained, and to the last son all that was left. After following their father's instruction with care, the sons found that they had shared their inheritance equally. How many sons were there, and how large was the estate?

First, we can solve the problem using modern algebra. Let us denote the entire estate by E and the share (in bezants) of each son by x. The first son received:

$$x = 1 + \frac{1}{7}(E-1),$$

the second son received:

$$x = 2 + \frac{1}{7}(E-2-x).$$

Because they all shared the inheritance equally, equating the two shares we get

$$1 + \frac{1}{7}(E-1) = 2 + \frac{1}{7}(E-2-x),$$

and solving the equation we get

$$x = 6.$$

Therefore, each son received 6 bezants. We can easily calculate that the total estate was 36 bezants.

For comparison, let us see the original Fibonacci solution. The total inheritance has to be a number such that when I times 6 is added to it, the result will be divisible by 1 plus 6, or 7; when 2 times 6 is added to it, the resulting number is divisible by 2 plus 6, or 8; when 3 times 6 is added, then the result is divisible by 3 plus 6, or 9, and so forth. The number is 36. 1/7 of 36 minus 1/7 is 35/7; plus 1 is 42/7 is 6; and this is the amount each son received; the total inheritance divided by the share of each son equals the number of sons, or 36/6 equals 6.

Another of the tasks that Fibonacci solved was in his time considered quite difficult. This problem we can find for example in the prologue of the book *Liber quadratorum* [28]:

Find such a rational number [a whole number or a fraction] that when 5 is either added to or subtracted from its square, the result [in either case] is also the square of a rational number.

Today we have to be impressed by the fact that without relying on computers or calculators of any sort, simply through his command of number theory, Fibonacci was able to find out that the solution to the problem is 41/12.

Besides mathematical puzzles Fibonacci also dealt with the Golden Ratio, which he described in a short book on geometry entitled *Practica Geometriae*. His role in the history of the Golden Ratio is truly fantastic. On one hand, in problems in which he consciously used the Golden Ratio, he is responsible for a significant but not spectacular progress. On the other hand, by simply formulating a problem that on the face of it has no relation whatsoever to the Golden Ratio, he expanded the scope of the Golden Ratio and its application dramatically.

Finally, in connection with Fibonacci we must mention his famous sequence. In *Liber Abaci* he introduced the following problem [37]:

A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year of it is supposed that every month each pair begets a new pair which from the second month on becomes productive?

I think that in these days it is clear how can the numbers of the offspring of rabbits have significant mathematical consequences. The sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ... in which each term (starting with the third) is equal to the sum of the two preceding terms, was appropriately dubbed the Fibonacci sequence in the 19th century, be the French mathematician Edouard Lucas (who I shall mention in section 27). Number sequences in which the relation between successive terms can be expressed by a mathematical expression are known as recursive. The Fibonacci sequence was the first such recursive sequence known in Europe. The general property that each term in the sequence is equal to the sum of the two preceding ones is expressed mathematically as $F_{n+2} = F_{n+1} + F_n$, where F_n represents the nth number if the sequence.

The reason that Fibonacci's name is so famous today lies with the fact that the appearance of the Fibonacci sequence is far from being confined to the breeding of rabbits. We encounter the Fibonacci sequence in an incredible variety of seemingly unrelated phenomena. Fibonacci numbers appear unexpectedly often in mathematics, so much so that there is an entire journal dedicated to their study, the *Fibonacci Quarterly*, [43]. Applications of Fibonacci numbers include computer algorithms such as the Fibonacci search technique and the Fibonacci heap data structure, and graphs called Fibonacci cubes used for interconnecting parallel and distributed systems. They also appear in biological settings, such as branching in trees, phyllotaxis (the arrangement of leaves on a stem), the fruit sprouts of a pineapple, the flowering of an artichoke, an uncurling fern and the arrangement of a pine cone's bracts.

14 ibn Khallikan

Yet another medieval mathematician, an Islamic scholar ibn Khallikan, who lived in the 13th century, formulated a brain teaser requiring the calculation of the total number of wheat grains placed on a standard 8×8 chessboard. This problem is formulated as a story which takes place in India [29]:

I have met a great number of persons who believed that as-Suli was the inventor of chess, but this is an erroneous opinion, that game having been imagined by Sissah Ibn Dahir the Indian, for the amusement of king Shihram.

[*Although*] *the sages of the time declared it superior to the game of nerd, and that for reasons too long to be explained...*

It is said that, when Sissah invented the game of chess and presented it to Shihram, the latter was struck with admiration and filled with joy; he ordered chess-board to be placed in the temples, and considered that game as the best thing that could be learned, in as much as it served (as an introduction) to (the art of) was, as an honour to religion and the world, and as the foundation of all justice. He manifested also his gratitude and satisfaction for the favour which heaven had granted him in illustrating his reign by such an invention, and he said to Sissah. "Ask me for whatever you desire." – "I then demand," replied Sissah, "that a grain of wheat be placed in the first square of the chess-board, two in the second, and that the number of grains be progressively doubled till the last square is attained: whatever this quantity may be, I ask you to bestow it on me." The king, who meant to make him a present of something considerable, exclaimed that such a recompense would be too little, and reproached Sissah for asking so inadequate a reward. Sissah declared that he desired nothing but what he had mentioned, and, heedless of the king's remonstrances, he persisted in his demand. The king, at length, consented, and ordered that quantity of wheat to be given him. When the chiefs of the government office received orders to that effect, they calculated the amount, and answered that they did not possess near so much wheat as was required. These words were reported to the king, and he, being unable to credit them, ordered the chiefs to be brought before him. Having questioned them on the subject, they replied that all the wheat in the world would be insufficient to make up the quantity. He ordered them to prove what they said, and, by a series of multiplications and reckonings, they demonstrated to him that such was the fact. On this, the king said to Sissah: "Your ingenuity in imagining such a request is yet more admirable than your talent in inventing the game of chess."

If we want to place one grain on the first square, two grains on the second square, four grains on the third one, eight grains on the fourth and so on, doubling the number for each successive square, then the solving of this problem leads to the geometric progression. The resulting number of grains is then $2^{64} - 1$.

Speaking in broad terms, we can consider the ibn Khallikan's problem as one of the earliest chess problems. His problem of the number of grains is a standard illustration of geometric progressions, copied later by many including Fibonacci, Pacioli, Clavius and Tartaglia.

15 Johannes Buteo

Arithmetic progressions were also used in other entertaining problems. At the turn of the 14th and the 15th century, one of the most challenging problems appeared in book *Logistical* from French mathematician Johannes Buteo (also known as Jean Borrel) [33]:

A mouse is at the top of a poplar tree 60 braccia[6] *high, and a cat is on the ground at its foor. The mouse descends* $\frac{1}{2}$ *of a braccia a day and at night it turns back* $\frac{1}{6}$ *of a braccia. The cat climbs one braccia a day and goes back* $\frac{1}{4}$ *of a braccia each night. The tree grows* $\frac{1}{4}$ *of a braccia between the cat and the mouse each day and it shrinks* $\frac{1}{8}$ *of a braccia every night. In how many days will the cat reach the mouse and how much has the tree grown in the meantime, and how far does the cat climbs?*

Buteo's *Logistica* covers a lot of other problems of mathematics and algebra.

16 Guarino Guarini

In 1512 G. Guarini devised a chessboard problem in which the goal is to effect the exchange of two black and two white knights, with each pair placed at the corners of a 3×3 chessboard, in the minimum number of moves [50]:

The white knights and the black knights wish to exchange places. Their situation is shown in figure 10[7]*. A knight can move on a chessboard by going two squares in any horizontal or vertical direction, and then turning either left or right one more square. Find the minimum number of moves required for these knights to exchange places.*

Figure 10: Guarini's problem: switch the knights

This puzzle belongs to the class of problems that can be solved in an elegant manner using the theory of planar graphs. The squares of the chessboard represent nodes of a graph, and the possible moves of the pieces between the corresponding squares (the nodes of the graph) are interpreted as the connecting lines of the graph. The corresponding graph for the board and the initial positions of the knights are shown in figure 11[8].

[6] "Braccia" is an old Italian unit of length, usually about 26 or 27 inches (66 or 68 cm), but varying between 18 and 28 inches (46 and 71 cm).

[7] Guarini's problem, sketched according to the illustration from [50].

[8] Graph to Guarini's problem, sketched according to the illustration from [50].

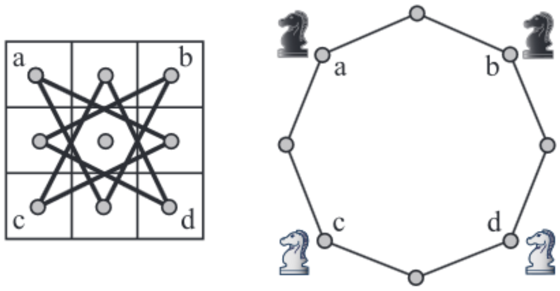


Figure 11: Graph to Guarini's problem

The initial positions of the knights are indicated and all possible moves of the knights between the squares (the nodes of the graph) are marked by lines. We can use Dudeney's famous method of "unraveling a graph", which is described in [17]. Starting from any node, the graph in figure 11 can be replaced by an equivalent graph in figure 12[9], which is much clearer and more convenient for the analysis.

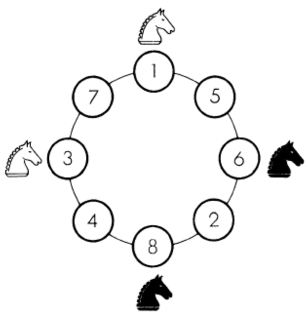


Figure 12: Dudeney's method

Obviously, the topological structure and the connectedness are preserved. To find the solution it is necessary to write down the moves (and reproduce them on the 3×3 board according to some correspondence), moving the knights along the circumference of the graph until they exchange places. The minimum number of moves is 16 although the solution is not unique (because the movement of the knights along the graph is not unique). Here is one solution:

$$1-5 \quad 6-2 \quad 3-7 \quad 8-4 \quad 5-6 \quad 2-8 \quad 7-1 \quad 4-3$$

$$1-5 \quad 8-4 \quad 6-2 \quad 3-7 \quad 5-6 \quad 7-1 \quad 2-8 \quad 4-3.$$

Note that people's interest in chess problems and the challenge they provide has lasted from the Middle Ages, through the Renaissance and to the present day.

[9]The sketch of Dudeney's method, according to the illustration from [17].

17 Niccolo Tartaglia

While the Italian mathematicians Niccolo Tartaglia (1499-1557) and Gerolamo Cardano (1501-1576) labored jointly to discover the explicit formula for the solution of a cubic algebraic equation, they also found time for recreational problems and games in their mathematical endeavors. More precisely, in Tartaglia's work *General Trattato* (1556) described several interesting tasks that were considered as challenging and serious in Tartaglia's time but which today have acquired a recreational character. These problems include for example the weighing problem, the division of 17 horses, the wine and water problem, and so on, which we can find in [33]. Tartaglia also proposed a more elaborate edition of the river-crossing. This version of the problem featured three beautiful brides and their young, handsome, and intensely jealous husbands, who come to a river [13]:

Three beautiful brides with their jealous husbands come to a river. The small boat which is to take them across holds only two people. To avoid any compromising situations, the crossings are to be so arranged that no woman shall be left with a man unless her husband is present. How can this be done, if any man or woman can be the rower?

In this case, nine crossings are required.

The first version of the river-crossing problem was mentioned in section 12 in work of Alcuin of York. Other more complicated versions of this problem involving more people and animals, can easily be constructed. However, not all are solvable. For instance, as the well-known puzzlists Sam Loyd and Henry E. Dudeney discovered, it is impossible to arrive at a solution under the conditions stipulated by Tartaglia's puzzle for four couples. In such a case, a solution is possible only if there is an island in midstream for use as a "transit stop". The Loyd-Dudeney puzzle is assigned as an exploration exercise, further on.

18 Gerolamo Cardano

Gerolamo Cardano was one the most famous scientists of his time and an inventor in many fields. In an earlier book, *De Subtilitate* (1550), Cardano was the first one from Europe who presented a game, often called the Chinese ring puzzle, that made use of a bar with several rings on it that remains popular even now, (section 10). The puzzle's solution is closely related to Gray's error-correcting binary codes introduced in the 1930s by the engineer Frank Gray.

19 Gaspar Bachet

Many mathematicians consider the book *Problems Plaisans et Delectables*, by Claude Gaspar Bachet (1581-1638), to be the first book on mathematical puzzles and tricks. Most of the famous puzzles and curious problems invented before the 17th century may be found in Bachet's delightful book. The book begins with problems of the following type:

I am thinking of a number that. . . .

Bachet presents many such problems, some with unusual variants, such as this one [42]:

If a given number is multiplied by another, the product [then] divided again by another [number], there will be a like proportion of the given number to the quotient of the division, [as] there exists a factor to the multiplier.

Or this [42]:

If four numbers are proportional, then the number produced from the first and fourth equals the number produced from the second and third; and, if the number produced from the first and fourth equals that produced from the second and third, then the four numbers are proportional.

In addition to Bachet's original "delectable" problems, the book contains puzzles by Alcuin of York, Pacioli, Tartaglia and Cardano, and other puzzles of Asian origin.

Bachet's book, first published in 1612 and followed by the second edition published in 1624, probably served as the inspiration for subsequent works devoted to mathematical recreation. The book became very popular, going through five editions over three centuries. In style and content, it became the forerunner of modern books of mathematical recreations.

20 Jean Leurechon

Other important writers on the subject include the Jesuit scholar Jean Leurechon (1591-1670), who published under the name of Hendrik van Etten, and Jacques Ozanam (1640-1717). Etten's work, *Mathematical Recreations, or a Collection of Sundry Excellent Problems Out of Ancient and Modern Philosophers Both Useful and Recreative*, first published in French in 1624 with an English translation appearing in 1633, is a compilation of mathematical problems interspersed with mechanical puzzles and experiments in hydrostatics and optics that most likely borrowed heavily from Bachet's work.

21 Fermat's Last Theorem

Pierre de Fermat was a very important person in mathematics of the 17th century. Today we think of Fermat as a number theorist, in fact as perhaps the most famous number theorist who ever lived. It is therefore surprising to find that Fermat was in fact a lawyer and only an amateur mathematician. Pierre de Fermat had a great liking for mathematics and dealt with it in his spare time. He created the *Last Theorem* while studying *Arithmetica*, an ancient Greek text written in about 250 A.D. by Diophantus of Alexandria. This was a manual on number theory, the purest form of mathematics, concerned with the study of whole numbers, the relationships between them, and the patterns they form.

The page of *Arithmetica* which inspired Fermat to create the *Last Theorem* discussed various aspects of *Pythagoras' Theorem*, which states that in a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. In other words:

$$x^2 + y^2 = z^2 ,$$

where z is the length of the hypotenuse, the longest side, and x and y are the lengths of the other two sides.

Fermat tried to find solutions to *Pythagoras' equation*, such that x, y, and z could be any whole number, except zero. For example, 3, 4, 5 ($9+16 = 25$), or 5, 12, 13 ($25+144 = 169$). Fermat must have been bored with such a tried and tested equation, and as a result he considered a slightly mutated version of the equation:

$$x^3 + y^3 = z^3.$$

Suddenly it seemed that this new equation has no integer solutions. Fermat was very impressed with the idea that it is possible that such a small change in the *Pythagorean equation*, which has infinitely many solutions, caused the new equation will have no solution. But Fermat went even further, believing that if the power of the equation were increased further, then these equations would likewise have no solutions:

$$\begin{aligned} x^3 + y^3 &= z^3, \\ x^4 + y^4 &= z^4, \\ x^5 + y^5 &= z^5, \\ &\vdots \\ x^n + y^n &= z^n, \end{aligned}$$

where n is any number greater than 2.

According to Fermat, none of these equations could be solved and he noted this in the margin of the *Arithmetica* in his comment around 1637 [32]: "It is impossible for a cube to be the sum of two cubes, a fourth power to be the sum of two fourth powers, or in general for any number that is a power greater than the second to be the sum of two like powers. I have discovered a truly marvelous demonstration of this proposition that this margin is too narrow to contain." Fermat probably believed he could prove his theorem, but he never committed his proof to paper. After his death, mathematicians across Europe tried to rediscover the proof of what became known as *Fermat's Last Theorem*.

Throughout the eighteenth and nineteenth centuries no mathematician could find a counter-example, a set of numbers that fitted Fermat's equation. Hence, it seemed that the Last Theorem was true, but without a proof nobody could be as sure as Fermat seemed to be. Some of the greatest mathematicians were able to devise specific proofs for individual equations (e.g. $n = 3$ and $n = 5$), but nobody was able to match Fermat's general proof for all equations. The longer that the Last Theorem remained unproven, the more important it became, and the more effort was put into finding a proof.

It is worth noting that finding proof was unlikely to yield any useful application, but the simple joy of solving an innocent riddle was enough to spur on generations of number theorists. Although all their attempts ended in failure, a great deal of new mathematics was inspired along the way, and it can be argued that the progress of number theory has been largely inspired by the desire to prove Fermat's Last Theorem. Not only professional mathematicians but also the general public were impressed by this theorem. The beauty of this problem lies in its stunningly simple task, we can say that the problem statement will understand every schoolboy, while the proof of this problem seemed to be very difficult. As far as the enthusiasm for this problem by the general public is concerned, let us recall that toward the end of the 19th century Paul Wolfskehl, a German industrialist and amateur mathematician, bequeathed in his will 100,000 Marks (worth \$2 million in today's money) to whoever may succeed in proving Fermat's Last Theorem. Soon after his death in 1906, the Wolfskehl Prize was announced, generating an enormous amount of publicity and introducing the problem to the general public. Within the first year, 621 proofs were sent in, most of them from amateur problem-solvers, all of them flawed.

Soon after the World War II computers helped to prove the theorem for all values of n up to five hundred, then one thousand, and then ten thousand. This may seem to be a significant contribution toward finding a complete proof, but the standards of mathematical proofs demand absolute confidence that no numbers fit the equations for all values of n. One of the reasons why *Fermat's Last Theorem* is so difficult to prove is that it applies to an infinite number of equations: $x^n + y^n = z^n$, where n is any number greater than 2. Even the advent of computers was of no help, because, although they could be employed to help perform sophisticated calculations, they always could at best deal with only a finite number of equations.

After three centuries of failure mathematicians were beginning to lose hope that a proof for Fermat's Last Theorem would ever be found. To prove that something is true for an infinite number of cases required to pull together some of the most recent breakthroughs in number theory, and in addition invent new techniques of his own. This heroic feat was accompanied by a British mathematician Andrew Wiles until 1993. On June 23rd he announced his seven-year calculation to a stunned audience at a mathematical conference in Cambridge.

The *Fermat's Last Theorem* shows that despite its simple task, its proof weaves the whole history of mathematics as a red thread and affects all fields of number theory. The technique of its proof belongs to the difficult part, what the current mathematics knows. It is clear that Fermat did not have the mathematical knowledge and could not know the technique in his time, which Wiles used in his proof at the end of the 20th century. The question still remains if Fermat did not have Wiles' proof, then what did he have? Some sceptics believe that Fermat did not in fact know the proof. Although Fermat wrote:

"I have discovered a truly marvelous proof", he had probably only found a flawed proof. Other mathematicians, believe that Fermat may have had a genuine proof. Whatever this proof might have been, it would have been based on 17th century techniques and would have involved an argument so cunning that it has eluded everybody else. Indeed there are plenty of mathematicians who still believe that they can still achieve fame and glory by discovering Fermat's original proof.

The popular science writer Simon Singh wrote a nice book [36] about *Fermat's Last Theorem*. He tells the story of the search for a proof of Fermat's last theorem, first conjectured by Pierre de Fermat in 1637.

In 1995 in the "Homer ^{3}" episode [25] of the television show *The Simpsons*, the equation

$$1\,782^{12} + 1\,841^{12} = 1\,922^{12}$$

appears at one point in the background. Seemingly, a solution of the Fermat's Last Theorem which is described here, is a so-called near-miss solution of Fermat's equation, which means that if we match the sum of the first two squares to the sum of the third square, then the results are accurate for the first nine digits. Although the discrepancy in equation is very small, it is a false solution.

Moreover there is a quick way to spot that $1\,782^{12}+1\,841^{12} = 1\,922^{12}$ is a false solution, without having to do any lengthy calculations. The trick is to notice that we have an even number 1 782 raised to the twelfth power added to an odd number 1 841 raised to the twelfth power supposedly equaling an even number 1 922 raised to the twelfth power. But an odd number raised to any power will always give an odd result, whereas an even number raised to any power will always give an even result. Since an odd number added to an even number always gives an odd result, the left side of the equation is doomed to be odd, whereas the right side of the equation must be even. Therefore, it is clear that the solution is false.

Yet another false solution to Fermat's Last Theorem is the episode "The Wizard of Evergreen Terrace" [47] which was broadcast in 1998. There is an equation

$$3987^{12} + 4365^{12} = 4472^{12}.$$

This equation is very clever, because if we have a calculator with ten digits onto its display, we can think that the solution is true.

Both equations which are in the episodes of *The Simpsons*, are a mathematical prank on those viewers who were quick enough to spot the equations and not smart enough to recognize its link with Fermat's Last Theorem. By the time these episodes aired, Wiles's proof was published, so academics were well aware that Fermat's Last Theorem had been conquered. The writer of these episodes David S. Cohen probably wanted to pay homage to Pierre de Fermat and Andrew Wiles by creating a solution that was so close to being correct that it would apparently pass the test if checked with only a simple calculator.

I shall mention more details about this television show in section 37.

22 The origins of probability theory

First steps to probability theory were made by Gerolamo Cardano, in the 16th century. In 1525, Cardano described the method, how to assign possible results numerical values when throwing dice and everything summarized in his book *Liber de Ludo Alea* (*Book of games of chance*)

The probability theory was being developed primarily from the 17th century. The basis of probability theory is the correspondence between the French mathematician Blaise Pascal and Pierre de Fermat. During their correspondence they together discovered the first laws of probability theory, a discipline whose main characteristic is uncertainty.

Pascal's attention to this theory was attracted by a French salon theorist Antoine Gombaud, who was not a nobleman but nevertheless adopted the title Chevalier de Méré. Gombaud was interested in gambling, and in particular in one of the variants of craps called points. During the game, one is adding up the points which are scored by the dice thrown, and the one who reaches the agreed number of points wins and gets the money wagered. At one occasion Gombaud and his teammate were forced to stop the game before its natural end. Therefore, a problem arose, what to do with the money wagered. The simplest solution would be to give money to anyone who had scored by that very moment the most points, but Gombaud asked Pascal whether there is a fairer distribution of money. How should the gamblers divide the banks fairly when playing dice at 5 and the game is prematurely ended at 2:1?

Pascal was tasked to calculate the probability of winning each of the opponents, if the game continued or if each of your opponents has the same chance to earn additional points. Bet money would be split in the ration thus calculated.

Until the 17th century the laws of probability were recorded only in intuition and experience of gamblers. One of the goals of correspondence that Pascal exchanged with Fermat was to reveal the mathematical rules that describe "laws governing coincidence". Pascal and Fermat dismantled Gombaud's question and soon came to learn that the answer is relatively simple, if we analyze carefully all the possible outcomes of games and if we assign them to their respective probabilities. Both Pascal and Fermat were able to solve the problem alone, but their co-discovery solutions accelerate and they led to a deeper examination of other, more subtle and complex issues associated with probability.

23 Leonhard Euler

Although in the 17th century, some of the newer mathematical disciplines are beginning to slowly shape, people are not interested that mathematics is able to deal with abstract concepts. They wanted it mainly used to solve practical problem and often competed among themselves, who hires a better thinker and a better known for this purpose.

Euler began his career at the headquarters of the Russian tsar, and later became the invitation of the Prussian ruler Frederick the Great, a member of the Berlin Academy. Later, he returned to Russia and his remaining years were spent as a subject of Catherine the Great. During his career he has dealt with a wide range of issues from different areas of the ship navigating in the financial sector, the acoustics after irrigation.

World of practical mathematics course does not weaken his abstract mathematical abilities. Any problem, to which he embarked, inspired him so much that on the basis of often brilliantly improve current mathematical knowledge. Moreover his deep and exacting investigations led to the foundation and development of new mathematical disciplines, often studied mathematical puzzles and games. Euler's results from the seven bridges of Königsberg problem presage the beginnings of graph theory. The thirty-six officers problem and orthogonal Latin squares (or Eulerian squares), discussed by Euler and later mathematicians, have led to important work in combinatorics.

Euler's conjecture on the construction of mutually orthogonal squares found resolution nearly two hundred years after Euler himself initially posed the problem. Another problem, which Euler occupied with, was the chessboard knight's re-entrant tour problem. A knight's re-entrant path consists of moving a knight so that it moves successively to each square once and only once and finishes its tour on the starting square. This famous problem has a long history and dates back to the 6th century in India.

23.1 The problem of Königsberg's bridges

The problem, which employed Euler mind one day, concerned the Prussian town Königsberg, but now known as Kaliningrad and part of Russia, which is situated on both banks of the river Pregel.

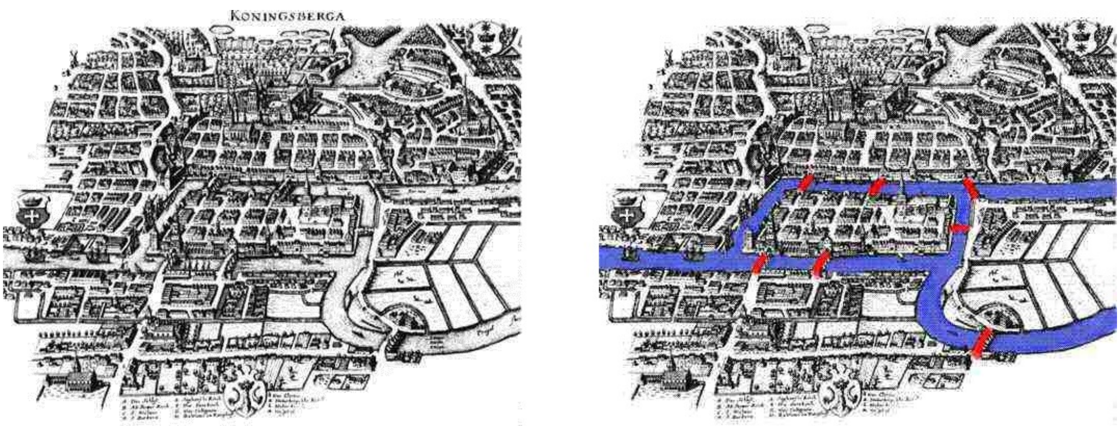


Figure 13: Königsberg bridges in Euler's day

The river flowed through the city and it divided the city into four distinct regions which were connected by the seven bridges (figure 13[10]). According to lore, the citizens of Königsberg used to spend Sunday afternoons walking around their beautiful city. While walking, the people of the city decided to create a game for themselves, their goal being to devise a way in which they could walk around the city, crossing each of the seven bridges only once and return to the same place from which they came out. None of the citizens of Königsberg could invent a route that would allow them to cross each of the bridges only once. So they ask for help a mathematician Euler.

Neither Euler failed to find the coveted trip, but managed to explain why this is impossible. First, Euler took the town plan, under which made a simple sketch. On this draw a dry area was represented by points and bridges by as lines connecting the points (figure 14).

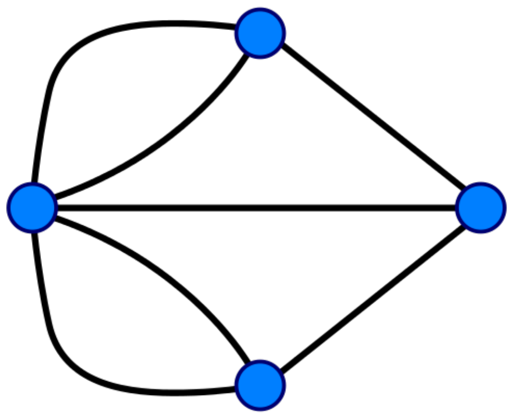


Figure 14: Königsberg bridges in graph

Euler arrived at the view that if is possible to carry out the walk (i.e. go each bridge exactly once), then from each of the points must outings even number lines – bridges. If a traveler is on some parts of the mainland, he had come to it along some bridge and will again have to leave it along other bridge. There are only two exceptions – the beginning and end of the route. But if the journey begins and ends in the same part of the mainland, then this part have, like all others, be connected with the world even number of bridges.

Euler then explains that it is obvious that if there are two landmasses with an odd number of bridges then the journey will always be possible if the journey starts in one of the regions with an odd number of bridges. This is because if the even numbers are halved, and each of the odd ones are increased by one and halved, the sum of these halves will equal one more then the total number of bridges. However, if there are four or more landmasses with an odd number of bridges, then it is impossible for there to be a path. Generally

[10]The sketch of Königsberg bridges, according to the illustration from [35].

speaking, any assembly bridges can pass on the condition that we go through each bridge exactly once, only when each part of the land based on an even number of bridges, or just two parts of mainland is based on an odd number of bridges. In Königsberg, from each of the four parts of land come out an odd number of bridges – three bridges come out of three parts and five bridges come out of one part. Thus, Euler was able not only to explain why it is not possible to pass each Königsberg's bridge just once, moreover formulated a general law applicable to all bridges of all cities in the world.

This idea is also beautiful in its simplicity and represents perhaps one of those problems that Euler solved before dinner. Problem of Königsberg's bridges is one of the problems of applied mathematics, in particular the role of graph theory. This problem inspired Euler to think about much more abstract graphs. He gradually discovered a fundamental relationship is valid for all graphs, today called Euler's formula, which he has been able to proof in just a few logical steps.

23.2 Eulerian squares

Euler was interested in the topic of magic squares. In 1783 Euler wrote the article *Recherches sur une nouvelle espece de quarres magiques*, in which he described the kind of magic squares, which today known as Latin squares and Graeco-Latin squares (or the Eulerian squares). A *Latin square* of order n consists of n distinct symbols, arranged in the form of a square scheme in such a way that each symbol occurs once in every row and once in every column. In other words, every row and every column is a permutation of n symbols. Table 1 (left) shows a fourth order *Latin square* where four Latin letters a, b, c, d are arranged in the described manner.

b	a	d	c		β	α	δ	γ		$b\,\beta$	$a\,\alpha$	$d\,\delta$	$c\,\gamma$
d	c	b	a		δ	γ	β	α		$d\,\delta$	$c\,\gamma$	$b\,\beta$	$a\,\alpha$
c	d	a	b		γ	δ	α	β		$c\,\gamma$	$d\,\delta$	$a\,\alpha$	$b\,\beta$
a	b	c	d		α	β	γ	δ		$a\,\alpha$	$b\,\beta$	$c\,\gamma$	$d\,\delta$

Table 1: Eulerian squares

Table 1 (center) shows *Graeco-latin square*, which differs from a *Latin square* that its cells labeled with the four corresponding Greek letters. If we superpose these two squares, as shown in the table 1 right, we find that each Latin letter combines once and only once with each Greek letter. When two or more *Latin squares* can be combined in this way, they are said to be *orthogonal squares*. The combined square is known as a *Graeco-Latin square*.

In Euler's day it was easy to prove that no *Graeco-Latin* square of order 2 is possible. Squares of orders 3, 4 and 5 were known, but Euler wondered about the order 6. In 1782, Euler formulated a problem familiarly known as *Euler's officers problem* [18]:

Each of six different regiments has six officers, one belonging to each of six different ranks. Can these 36 officers be arranged in a square formation so that each row and file contains one officer of each rank and one of each regiment?

Euler shows that the problem of n^2 officers, which is the same as the problem of constructing a *Graeco-Latin square* of order n, can always be solved if n is odd, or if n is a number divisible by 4. Moreover, he stated that *Graeco-Latin square* of the order 6, 10, 14 and in general to all even numbers which not divisible by 4 cannot be constructed. This became famous as Euler's conjecture. In other words: There does not exist a pair of orthogonal *Latin squares* of order $n = 4k + 2$ for any positive integer k.

In 1901, more than one century later, the French mathematician Gaston Tarry proved Euler's conjecture for the particular case $n = 6$. After Tarry, several matheamticians even published proofs that the conjecture was true, but later the proofs were found to contain flaws.

Being restricted to pencil-and-paper mathods, it was exhaustive to solve next unknown case, the order 10. This case in particular had to wait until the computer era. In 1959, 177 years later, Euler's conjecture was disproved by mathematicians from the University of North Carolina.

23.3 Knight's re-entrant route

The knight's re-entrant route is doubtless the most interesting and familiar to many reader among all the chessboard puzzles and problems. This remarkable and very popular problem was formulated in the 6th century in India, but this task has delighted people for centuries and continues to do so to this day [34]:

Find a re-entrant route on a standard 8×8 *chessboard which consists of moving a knight so that it moves successively to each square once and only once and finishes its tour on the starting square.*

Closed knight's tours are often called "knight's circles". The knight's circle also interested such great mathematicians as Leonhard Euler, Alexandre-Théophile Vandermonde, Adrien-Marie Legendre, Abraham de Moivre, Pierre Rémond de Montmort, and others. De Montmort and de Moivre provided some of the earliest solutions at the beginning of the 18th century. Their method is applied to the standard 8×8 chessboard divided into an inner square consisting of 16 cells surrounded by an outer ring of cells two deep. If the knight starts from a cell in outer ring, it always moves along this ring filling it up and continuing into an inner ring cell only when absolutely necessary.

But Euler was who made the first serious mathematical analysis of this subject. In his letter to the mathematician Christian Goldbach, Euler gave a solution to the knight's re-entrant path shown in figure 15.

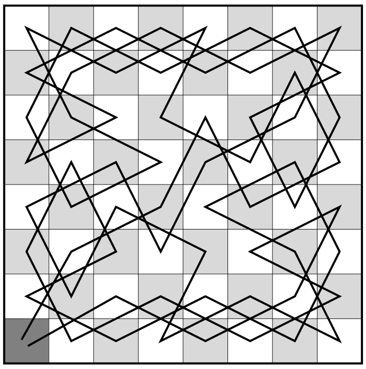


Figure 15: Euler's knight's circle solution

Euler's method consists of a knight's random movement over the board as long as it is possible, taking care that this route leaves the least possible number of untraversed cells. The nest step is to interpolate these untraversed cells into various parts of the circuit to make the re-entrant route. Details on this method may be found in the book [34].

There is an extensive literature on the knight's re-entrant tour. In 1823 H. C. Warnsdorff provided one of its most elegant solution. His method is very efficient, not only for the standard chessboard but also for a general $n \times n$ board as well. In 2000 Ingo Wegener calculated that there are 13 267 364 410 532 closed knight's tours. The same number was previously claimed by Brendan McKay in 1997. One of the ways to find a knight's tour is the application of backtracking algorithms, but this kind of search is very slow so that even very powerful computers need considerable time. Another algorithm developed by A. Conrad and his college is much faster and finds the knight's re-entrant tours on the $n \times n$ board for $n \geq 5$. This solution we can find in their book [11]. An extensive study of tthe possibility of the knight's re-entrant routes on general $m \times n$ chessboard can be found in [50].

24 Malfatti's problem

Giovanni Francesco Malfatti (1731-1807) was a brilliant Italian mathematician who devoted himself to the promotion of many new ideas and wrote many papers in different fields of mathematics including algebra, calculus, geometry, and probability theory. Malfatti appears in the mathematical literature of the last two centuries mostly in connection with a geometrical problem he raised and discussed in a paper in 1803 and today the problem is known as Malfatti's problem [14]:

To draw within a given triangle three circles each of which is tangent to the other two and to two sides of the triangle.

This problem can also be found earlier in Japanese temple geometry, where it is attributed to Chokuyen Naonobu Ajima (1732–1798).

Malfatti's approach was clearly algebraic. He computed the coordinates of the centers of the circles involved, and noticed that the values of the expressions can be constructed using ruler and compass. He assumed that the solution consisted of three mutually tangent circles, each also tangent to two edges of the triangle 16.

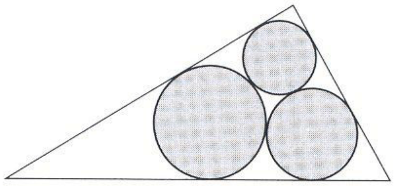


Figure 16: Malfatti's solution

In 1826 Steiner published an elegant solution of Malfatti's construction problem. He also considered several variations including analogous problems where the sides of the triangle are replaced by circular arcs, or when these arcs are placed on a sphere.

This problem has not been solved only by algebraical methods but also by methods of mathematical analysis. We want to maximize the area of three circles within a triangle. In 1811 Gergonne asked about the existence of a similar extremal arrangement in three-dimensional space, using a tetrahedron and four spheres instead of a triangle and three circles. The extremal arrangement of spheres was constructed by Sansone in 1968. In the 19th century many mathematicians, including Cayley, Schellbach, and Clebsch, worked on various generalizations.

It was very surprising when in 1930 Lob and Richmond observed that in an equilateral triangle the triangle's inscribed circle together with two smaller circles, each inscribed in one of the three components left uncovered by the first circle, produces greater total area than Malfatti's arrangement. The figure 17 shows Malfatti's solutions and new arrangement.

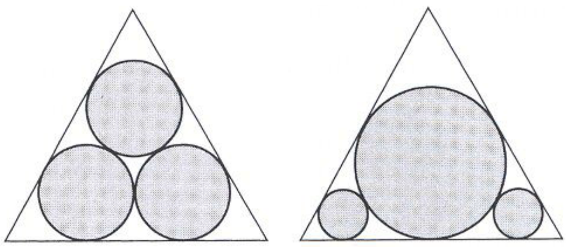


Figure 17: Malfatti's solution Lob and Richmond's solution

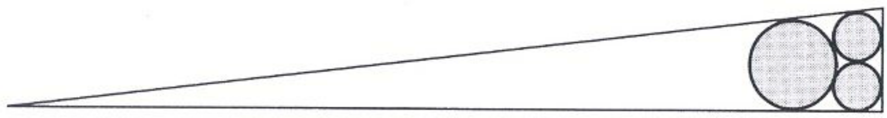
The calculation shows how much their solution better than Malfatti's assumption. Malfatti's construction takes

$$\frac{\pi\sqrt{3}}{(1+\sqrt{3})^2} \approx 0,729,$$

but new construction takes

$$\frac{11\,\pi}{27\,\sqrt{3}} \approx 0,739.$$

In 1965 Howard Eves pointed out that in a very tall triangle placing three circles on top of each other also produces greater total area (figure 18). We say that n circles in a given region form a greedy arrangement, if they are the result of the n-step process, where at each step one chooses the largest circle which does not overlap the previously selected circles and is contained by the given region.



Malfatti's solution

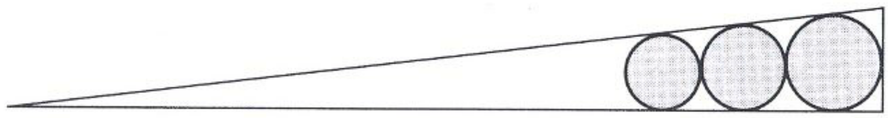


Figure 18: Eves's solution

In 1967 Michael Goldberg outlined a numerical argument that the greedy arrangement is always better than Malfatti's (figure 19).

Although Malfatti's solution seemed at the first view very natural and intuitive, 164 years later it shows that in this case we are cheated by our intuition.

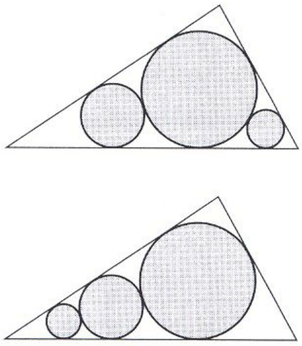


Figure 19: The greedy arrangement

25 Kirkman's schoolgirl problem

In 1847, an English vicar and expert in group theory and combinatorics, Thomas Kirkman set the following problem [48]:

Fifteen young ladies in a school walk out three abreast for seven days in succession: it is required to arrange them daily so that no two shall walk twice abreast.

This problem belongs to the area of combinatorics called block-design theory. These types of problems were studied intensively in the 19th century, but at the beginning they were seen as problems of recreational mathematics. Over time it was discovered that such problems are closely related to topics as diverse as statistics, error-correcting codes, higher-dimensional geometry and projective geometry.

The first solution gave Kirkman in 1847. He solved a problem on "triad systems" which would be generalized by Steiner several years later and today is known as a Steiner triple systems. With this system we find that Kikrman problem the number of days required for the walks is $\frac{1}{2}(n-1)$. The above numbers will be integers only if n is an odd multiple of 3, that is, of the form $3(2k+1) = 6m+3$. Thus, the sequence of possible values is 3, 9, 15, 21 and so on.

Since 1922, Kirkman's original problem is known have 80 solutions for the case $n = 15$, but only seven of which are the basic solutions. There are many methods for solving Kirkman's problem. For example geometric solution was described by Martin Gardner in [20].

Although the form $6m + 3$ for n is necessary for the solution of the general form of Kirkman's problem, it is not sufficient. In the second half of the 19th century many papers of n. A general solution for all n (of the form $6m + 3$) was given in 1970 when two mathematicians D. K. Ray-Chaudhuri and Richard M. Wilson of Ohio State University proved that the answer is yes. However, the number of solutions remains unknown, and it was found only for small values of n. The number of the Steiner triple systems S_n increases very rapidly; for example, there are more than 2×10^{15} non-isomorphic solutions for $n = 31$.

26 The Four-Colour Theorem

Some problems in history of mathematics that are easy to state can sometimes be very hard to answer. The four-colour theorem is a notorious example. This story began in 1852 at the time when Francis Guthrie, a graduate student at University College, London, wrote a letter to his younger brother Frederick, containing what he thought would be a simple little puzzle. He had been trying to colour a map of the English counties, and had discovered that he could do it using four colours, so that no two adjacent counties were the same colour. He wondered if *the regions of any simple planar map can be colored with only four colors, in such a way that any two adjacent regions have different colors,* or this fact was special property of the map of England.

His brother Frederick Guthrie could not answer this question, so he asked for help the mathematician Augustus De Morgan. But neither De Morgan could. It is easy to prove that at least four colours are necessary for some maps, because there are maps with four regions, each adjacent to all the others. The figure 20 shows four counties in the map of England form such an arrangement, which proves that at least four colours are necessary in this case.

Figure 20: map of England

De Morgan finally managed to prove that it is not possible to find an analogous map with five regions, each adjacent to all four of the others. However, this does not prove the four-colour theorem, because we do not know whether there might be a very complicated map with, for example, a hundred regions, which can't be coloured using only four colours because of the way long chains of regions connect to their neighbours.

The first printed reference to the problem dates from 1878, when Arthur Cayley wrote a letter to the Proceedings of the London Mathematical Society to ask whether anyone had solved the problem yet. They had not, but in the following year a barrister Arthur Kempe published a proof. However, his proof was wrong. The error in the proof was appeared by Percy Heawood, in 1890. A lof of mathematicians tried to fix this mistake or adapt Kempe's method, but all of them were not successful. The gap was tantalising, and quickly became a disgrace.

In 1922, Philip Franklin made a partial progress when he proved that all maps with 26 of fewer regions can be four-coloured. In early 1974, the mathematician Jean Mayer managed to prove that theorem is true for a planar map that contains a maximum of 95 states. Everybody expected that the problem would be soon resolved. Various symposiums on graph theories were held and the problem of the four colours was discussed everywhere.

Note, that this advantage of the atmosphere was took by Martin Gardner in 1975 and in the April issue of the Scientific American magazine [19], he published the article about the discovery of the counterexample of the four-colour-map theorem. He informed his readers that in autumn 1974, the American mathematician William McGregor managed to construct an example of a planar map with 110 countries, where minimum of five colours is needed for colouration. More details we can find in [5]. This article was clearly april hoax but most of people believed in.

For many, then it was a bigger surprise when in 1976 Kenneth Appel and Wolfgang Haken proved the four colour theorem by the use of a computer. The computation took about 1 000 hours in those days. Today is the computation faster thanks to work of another mathematicians, but still an unaided human cannot verify the proof. Nevertheless, we now know the answer to the question which Francis Guthrie asked 124 years ago. Yes, the four-colour theorem is true.

27 The Tower of Hanoi

The familiar *Tower of Hanoi* was sold as a toy in 1883. It originally bore the name of "Prof. Claus" of the College of "Li-Sou-Stian", but these were soon discovered to be anagrams for its inventor Edouard Lucas, it means "Prof. Lucas" of the College of "Saint Louis".

The figure 21 depicts the toy as it is usually made. The tower puzzle consists of three vertical pegs set into a board, and a number of disks graded in size, eight disks in the case of Lucas' toy. These disks are initially stacked on one of the pegs so that the largest rests at the bottom of the stack, the next largest in size atop it, and so on, ending with the smallest disk placed at the top. The problem is to transfer the tower of eight disks to either of the two vacant pegs in the fewest possible moves using a few basic rules:

a) a player can shift the disks from one peg to another one at a time,

b) no disk may rest upon a disk smaller than itself.

How we can transfer the tower of disks from the peg upon which the disks initially rest to one of the other pegs in the minimum number of moves?

It is not hard to prove that there is a solution regardless of how many disks are in the tower. Denote the required minimal number of moves with H_n. It is evident that $H_1 = 1$ and $H_2 = 3$. The figure 21 displays particular moves of the Tower of Hanoi with five disks.

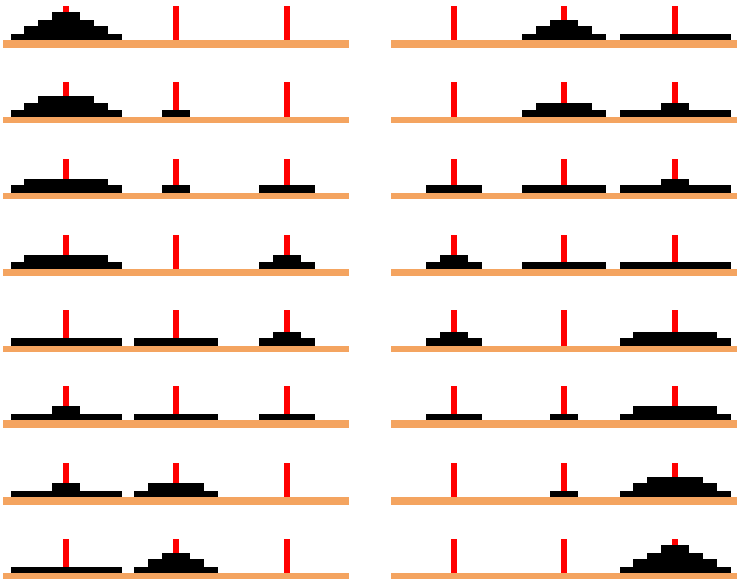


Figure 21: The Tower of Hanoi

The figure also shows, in order to transfer the largest disk from the first peg to the second peg , we must first construct a tower composed of the remaining $n-1$ disks on the third peg, using in this process the second peg. The minimal number of moves necessary for this transfer is H_{n-1}. After that, one move is needed for the transfer of the largest disk to peg B and at least H_{n-1} moves to transfer $n-1$ disks from the third peg to the second peg by using the first peg. Therefore, the required number is given by the recurrence relation

$$H_n = 2\,H_{n-1} + 1,\ \ n \geq 2,\ \ H_1 = 1.$$

By adjusting this relation we obtain that the minimum number of moves required can be expressed by the formula

$$H_n = 2^n - 1,$$

where n being the number of disks.

In the case of Lucas' toy with $n = 8$ disks, this number is $2^8 - 1 = 255$.

The original description of the toy called it a simplified version of a mythical *Tower of Brahma* in a temple in the Indian city of Benares. We can find retells this story in the book [34]:

In the great temple at Benares, says he, beneath the dome which marks the centre of the world, rests a brass-plate in which are fixed three diamond needles, each a cubit high and as thick as the body of a bee. On one of these needles, at the creation, God placed sixty-four discs of pure gold, the largest disc resting on the brass plate, and the others getting smaller and smaller up to the top one. This is the Tower of Bramah. Day and night unceasingly the priests transfer the discs from one diamond needle to another according to the fixed and immutable laws of Bramah, which require that the priest must not move more than one disc at a time and that he must place this disc on a needle so that there is no smaller disc below it. When the sixty-four discs shall have been thus transferred from the needle on which at the creation God placed them to one of the other needles, tower, temple, and Brahmins alike will crumble into dust, and with a thunder-clap the world will vanish.

According to the derived general formula, the number of separate transfers of golden disks of the Tower of Brahma in Benares is $2^{64} - 1 = 18\,446\,744\,073\,709\,551\,615$. Assuming that the priests can transfer one disk per second, the end of the world will occur in about 585 billion years.

Note, that the numbers having the form of $2^n - 1$ are called *Mersenne numbers*. If n is composite number then $2^n - 1$ is also composite number. If n is prime then also $2^n - 1$ may be prime or not, for example $2^3 - 1 = 7$. We call these numbers *Mersenne primes*. Lucas himself was the first man to verify that $2^{12} - 1$ was a prime. This huge 39-digit number was the largest known prime until 1952, when a large electronic computer was used to find five higher *Mersenne primes*. We don't known all *Mersenne primes* yet. We do not even know if their quantity is finite [30]. The newest *Mersenne prime* was discovered in January 2016 and it has 22 338 618 digits.

28 Lewis Carroll

Charles Lutwidge Dodgson (1832–1898) is better known by his pen name Lewis Carroll. This pseudonym was a play on his real name: Lewis was the anglicised form of Ludovicus, which was the Latin for Lutwidge, and Carroll an Irish surname similar to the Latin name Carolus, from which comes the name Charles. The transition went as follows: "Charles Lutwidge" translated into Latin as "Carolus Ludovicus". This was then translated back into English as "Carroll Lewis" and then reversed to make "Lewis Carroll".

Charles Dogson was an English writer, mathematician, logician, Anglican deacon, and photographer. Dodgson grew up in a family which was predominantly northern English, with Irish connections, conservative and High Church Anglican. Charles' father was an active and highly conservative cleric of the Church of England who later became the Archdeacon of Richmond. During his early youth, Dodgson was educated at home. He suffered from a stammer that often influenced his social life throughout his years. Despite this drawback, however, there has never been any doubt about his high intellect. In 1852, he obtained first-class honours in Mathematics Moderations, and was shortly thereafter nominated to a Studentship by his father's old friend Canon Edward Pusey. In 1854, he obtained first-class honours in the Final Honours School of Mathematics, standing first on the list, graduating as Bachelor of Arts. He remained at Christ Church studying and teaching, but the next year he failed an important scholarship through his self-confessed inability to apply himself to study. Even so, his talent as a mathematician won him the Christ Church Mathematical Lectureship in 1855, which he continued to hold for the next twenty-six years.

Dodgson also expressed interest in other fields. He was an early member of the Society for Psychical Research and wrote some studies of various philosophical arguments. In 1895, he developed a philosophical regressus-argument on deductive reasoning in his article *What the Tortoise Said to Achilles*. From a young age, Dodgson wrote poetry and short stories. Among his famous poems belong *The Hunting of the Snark*, all examples of the genre of literary nonsense. In 1856, he published his first piece of work under the name that would make him famous. A romantic poem called *Solitude* appeared in The Train under the authorship of "Lewis Carroll".

But definitely his the unrivaled books are *Alice's Adventures in Wonderland* (commonly shortened to *Alice in Wonderland*), published in 1865 and the sequel from 1872, *Through the Looking-Glass*. If we wanted to do a detailed analysis of *Alice*, we would probably be busy for several years. *Alice* is an extraordinary book from many views. The book rapidly became a bestseller, it has never been out of print since it first appeared, and it has been translated into well over 100 languages.

Some people who read *Alice in Wonderland* find it a whimsical adventure into a world of fun little paradoxes. Other people consider it a creepy march through a world of characters who seem to be set on making life as frustrating as possible as manically as they can. The both sides might possibly have some bearing on their view of the world. For sure, *Alice* isn't just fun and games.

Dodgson added a lot of paradoxes and puzzles as he was poring over the new math that was springing up in the middle of the 1800s. These days were a turbulent period for mathematicians, with the subject rapidly becoming more abstract. The discoveries of non-Euclidean geometries, the development of abstract algebra that was not tied to arithmetics or geometry, and the growing acceptance – or at least use – of imaginary numbers were just some of the developments that shook the discipline to its core. Dogson liked good old-fashioned no nonsense algebra and Euclidean geometry — areas of study that could prove things about the natural world. Suddenly students of mathematics, and even teachers, were using different mathematical methods to prove things like one and one not equaling two.

It seemed to Dogson that they were just being difficult on purpose, so he skewered them in prose in which he satirised these radical new ideas.

His criticism, however, was hidden in the story of Alice and so cleverly that mathematicians get to know his wicked satire on Those new developments, but readers who are not interested in mathematics disturb anything from reading a fairy tale for adults. In this context, we must mention the great book of Martin Gardner, *The Annotated Alice* published in 1960. Gardner who first decoded many of the mathematical riddles and wordplay that lay ingeniously embedded in Dogson's both stories.

Dodgson outgunned the specialist press with *Alice's Adventures in Wonderland*, because he was the first one who took his mathematics to his fiction.

29 Samuel Loyd

Samuel Loyd (1841-1911) is perhaps a greatest ever puzzle maker, who invented and refined thousands of puzzles in his lifetime. He was also a great chess player and a chess composer. As a chess composer, he authored a number of chess problems, often with interesting themes. He enriched the field of recreational mathematics many times, his best-known contribution being the ingenious toy called *15 Puzzle* (known also as the *Boss Puzzle*, or *Jeu de Taquin*) which is popular even today. The *15 Puzzle* (figure 22[11]) consists of a square divided into 16 small squares and holds 15 square blocks numbered from 1 to 15. The task is to start from the given initial arrangement and set these numbered blocks into the required positions (say, from 1 to 15), using the vacant square for moving blocks.

Figure 22: Loyd's *15 Puzzle*

Its author has even pledged to pay $ 1,000 reward to anyone who can throw a series of movements of blocks 14 and 15 and gets them back to place. Although many people claimed that they desired task succeeded, no one ever appeared to collect the reward. No wonder. Loyd did not output the reward due to a surplus of money. He simply knew that one cannot just swap the two blocks without undermining the overall structure of the puzzle again elsewhere. He knew that the problem is impossible to solve.

[11]Loyd's *15 Puzzle*, according to the illustration from [52].

30 Henry Ernest Dudeney

Henry Ernest Dudeney (1857–1930) was England's greatest maker of puzzles. With respect to mathematical puzzles, especially problems of more than trivial mathematical interest, the quantity and quality of his output surpassed that of any other puzzlist before or since, in or out of England. He came from a family which had a mathematical tradition and also a tradition of school teaching. Henry himself was a self-taught mathematician who never went to college. He was understandably proud to be a grandson of this famous shepherd-mathematician.

Henry learnt to play chess at a young age and soon became interested in chess problems. From the age of nine he was composing problems and puzzles which he published in a local paper. Although he only had a basic education and has never attended any college, he had a particular interest in mathematics and studied mathematics and its history in his spare time.

Dudeney wrote articles for magazines and joined a group of authors which included Arthur Conan Doyle. At this stage his earliest work, published under the pseudonym of "Sphinx". He started cooperating with the American puzzlist Sam Loyd. Loyd had been sending his puzzles to England since 1893, and a correspondence started between him and Dudeney. The two were the main creators of mathematical puzzles and recreations of their day and it was natural that they should exchange ideas. Readers familiar with the work of Sam Loyd will notice that many of the same puzzles appear, in different story forms, in the books of Loyd and Dudeney. This is hardly a surprise, because they were in frequent correspondence and they had been sharing ideas all the time.

Dudeney contributed to the *Strand Magazine* for over 30 years, starting at the time when his collaboration with Loyd ended, and from around the same time he began publishing in *Blighty, Cassell's, The Queen, Tit-Bits*, and the *Weekly Dispatch*. Dudeney's very popular collections of mathematical puzzles *The Canterbury Puzzles* (1907), *Amusements in Mathematics* (1917), and *Modern Puzzles* published in 1926, contain a wealth of fascinating examples which would provide any teacher of mathematics with a treasure trove of material.

31 Flatland: A Romance of Many Dimensions

In 1884, a thin book titled *Flatland: A Romance of Many Dimensions* appeared. The author of the book was the English minister, headmaster, and biblical and Shakespearean scholar Edwin Abbott Abbott. The book covered both an introduction to the notion of higher dimensions and a satire of Victorian society and norms. At that time, there was a substantial interest in the idea of higher dimensions, both within the scientific community and also in the more general population. Abbott's work provided a simple story that allowed lay audiences to grasp the idea of dimensions beyond the familiar three.

Flatland is a unique book because there exist only very few books about mathematics by amateur mathematicians that would last so long in popularity or that would have such a dramatic impact. From a scientific perspective, we can even say that Flatland helped to set the stage for many of the scientific advances to come. Generations of students have gained their first true appreciation of higher dimensions by reading this thin book written by a schoolmaster more than a century and a quarter ago. Of the more than 50 books that Abbott wrote, this is the one for which he is remembered.

Flatland is the story of a two-dimensional creature, A Square, who is actually a square and is one of the inhabitants of Flatland, a world consisting of a single plane. Other residents of Flatland include polygons, which means isosceles triangles, equilateral triangles, squares and other regular polygons. The social status depends on the numbers of edges. The rule says that the larger one's number of edges, the larger one's angles, and larger angles provide greater intelligence and higher social status. More details information about *Flatland* and Edwin Abbott Abbott we can find in article [2].

For a modern-day audience, Abbott's description of the society might be painful to read, but his portrayal of women in the book was intended as parody of Victorian customs that he himself deplored.

Later, many writers followed up on Abbott's *Flatland*. Among the most important include *Sphereland* written by the Dutch physicist Dionys Burger, *The Planiverse* written by Alexander Keewatin Dewdney or *Flatterland* by Ian Stewart.

32 Martin Gardner

Martin Gardner (1914-2010) was an American popular amateur mathematician and a popular science writer. The area of his interest included encompassing micromagic, scientific skepticism, philosophy, religion, and literature, especially the work of Lewis Carroll, L. Frank Baum, and G. K. Chesterton. Gardner is the best known for creating and sustaining interest in recreational mathematics which had been appearing in his regularly column in *Scientific American* magazine for twenty-five years.

His *Mathematical Games* column first appeared in the 1956 issue of *Scientific American* with an article on hexaflexagons. These curious structures, created by folding an ordinary strip of paper into a hexagon and then gluing the ends together, could be turned inside out repeatedly, revealing one or more hidden faces. The structure was invented in 1939 by a group of Princeton University graduate students. Hexaflexagons are not only fun to play with, but they also constitute a very important link between recreational puzzles and "serious" mathematics. This claim is supported by the fact that one of their inventors was Richard Feynman, one of the most famous theoretical physicists of the century. At the time when Gardner started writing his column, he drew inspiration from books on recreational mathematics available at that time. These included classic of the genre – *Mathematical*

Recreations and Essays, written by an English mathematician W. W. Rouse Ball in 1892, or *Mathematical Recreations*, written by a Belgian number theorist Maurice Kraitchik. But he also collected some puzzles and read the books of many distinguished mathematicians, for example Ian Stewart, John H. Conway, Richard K. Guy, Elwyn R. Berlekamp. Articles on recreational mathematics also appeared in mathematical periodical, for example the quarterly *Journal of Recreational Mathematics*, published between 1968 and 2014.

During his life he was a prolific and versatile author, who published more than 100 books on the subjects of mathematics, skepticism, literature, magic, and religion. More detail information about his life, his career and his contribution to recreational mathematics and science, can be found in article [4].

33 Richard Kenneth Guy

Richard Kenneth Guy (1916-) is a British mathematician and a Professor Emeritus in the Department of Mathematics at the University of Calgary.

He is best known for the co-authorship (jointly with John Conway and Elwyn Berlekamp) of *Winning Ways for your Mathematical Plays* and the authorship of *Unsolved Problems in Number Theory*, but he has also published over 100 papers and books covering combinatorial game theory, number theory and graph theory.

His research interests include combinatorial games, enumerative combinatorics, combinatorial geometry, number theory and graph theory. Around 1959, Guy discovered a unistable polyhedron having only 19 faces. No such structure with fewer faces has ever been found until 2012. Guy also discovered the glider in Conway's *Game of Life*. He is said to have developed the partially tongue-in-cheek "Strong Law of Small Numbers" which says there are not enough small integers available for the many tasks assigned to them — thus explaining many coincidences and patterns found among numerous cultures.

34 Raymond Smullyan

Raymond Smullyan (1919-) is a unique person, because he has had a remarkably diverse sequence of careers. Raymond Smullyan's live could be described as a set of four lives: as a mathematical logician, a musician, a magician, and a writer.

Raymond early showed a musical talent, winning a gold medal in a piano competition when he was aged 12. His first teaching position was at Roosevelt College in Chicago, where he taught piano. Unfortunately, or maybe fortunately, at about that time he developed tendonitis in his right arm forcing him to abandon piano performances as his primary career. As a result he turned his attention to mathematics which he equally loved. He had learned most of it on his own, with very little formal education at the time. He then took a few advance courses at the University of Chicago, and supported himself at the time as a professional magician. Curiously enough, before he had a college degree, or even a high school diploma, he received an appointment as a mathematics instructor at Dartmouth College on the basis of some brilliant papers he had written on mathematical logic.

After teaching at Dartmouth College for two years, the University of Chicago awarded him a Bachelor of Arts degree, based partly on courses he had never taken, but had successfully taught. He then went to Princeton University for his Ph.D. in mathematics in 1959. While a Ph.D. student, Smullyan published a paper in the 1957 Journal of Symbolic Logic showing that Gödelian incompleteness held for formal systems considerably more elementary than that of Gödel's 1931 landmark paper. Later, he also generalized some theorems of the great logicians Gödel and Tarski, and discussed logic in general.

Smullyan is world-wide well known as a writer, having authored over twenty books, many of which have been translated into seventeen languages. His writings cover an amazing variety of subjects. He is the author of many books on recreational mathematics and recreational logic. His recreational logic puzzles are designed to introduce the general reader to deep results in mathematics; retrograde chess problems encapsulated into Sherlock Holmes and Arabian Nights stories; stereo photography. Most notably, one is titled *What Is the Name of This Book?* or *The Lady or the Tiger?* He published six books about mathematical logic and over forty research papers in the field. Other fields of his interest are Chinese Taoism; the psychology of religious and mystical consciousness; philosophical fantasies; and essays on various aspects of life. His latest book, *Some Interesting Memories* (Published by Thinkers? Press, Davenport, Iowa) contains a delightfully charming account of some of his more memorable adventures and is replete with jokes, anecdotes, puzzles, and paradoxes.

35 John Horton Conway

John Horton Conway (1937-2016) was the outstanding British Professor of mathematics at Cambridge and later at Princeton, and a member of the Royal Society of London. He is widely known among mathematicians and amateur mathematicians for his remarkable contributions to combinatorial game theory and many branches of recreational mathematics. His book *Winning Ways for Your Mathematical Plays*, written with E. R. Berlekamp and R. K. Guy, has attracted the attention of a wide audience for many years.

J. H. Conway became instantly famous when he launched in 1970 the *Game of Life*, a kind of artificial simulation of life. Researching certain games, Conway came to a new system of numbers, named surreal numbers by Donald Knuth. John Conway proposed also many mathematical puzzles.

His celebrated career and eccentricities were profiled in the 2015 biography *Genius At Play: The Curious Mind of John Horton Conway*, by Siobhan Roberts.

36 Ian Stewart

Ian Stewart (1945-) is an Emeritus Professor of Mathematics at the University of Warwick, England, and an Emeritus Professor of Gresham College, London. He has held visiting positions in Germany, New Zealand, and the USA. He has five honorary doctorates (Open University, Westminster, Louvain, Kingston, and Brighton) and is an honorary wizard of Unseen University on Discworld.

He is best known for his popular science writing—mainly on mathematical themes and for his science-fiction writing. He earned a lot of various awards, for example the Royal Society's Faraday Medal (1995), the IMA Gold Medal (2000), the AAAS Public Understanding of Science and Technology Award (2001), the LMS/IMA Zeeman Medal (2008), and the Lewis Thomas Prize (2015), joint with Steven Strogatz.

He has published more than 80 books. Two from these books, *Nature's Numbers* and *Why Beauty is Truth* were shortlisted for the 1996 Rhone-Poulenc Prize for Science Books. Among his famous book belong also *Does God Play Dice?* or *Letters to a Young Mathematician*. Two wonderful books covering a lot of problems of recreational mathematics: *Professor Stewart's Cabinet of Mathematical Curiosities* and *Professor Stewart's Hoard of Mathematical Treasures*.

He wrote the US bestseller *Flatterland*, the sequel of the novel *Flatland* which was mentioned in section 31.

He is also a critically acclaimed science-fiction author. His interest in science fiction is reflected in the collaboration with Terry Pratchett and Jack Cohen on the series *The Science of Discworld I, II, III*, and *IV*, all of which reached number 1 or 2 in the Sunday Times bestseller list. The first book in the series was nominated for a Hugo award at the World Science Fiction Convention in 2000.

He has contributed to a wide range of newspapers and magazines in the UK, Europe, and the USA, including *New Scientist* and *Scientific American* magazine. Moreover, he followed the long time work of Martin Gardner, Doughlas Hofstadter and Kee Dewdney and for twelve years he wrote the monthly "Mathematical Recreations" column in *Scientific American*, see [4].

37 The Simpsons

The Simpsons is an American animated sitcom created by Matt Groening for the Fox Broadcasting Company. The series is a satirical depiction of a working class lifestyle epitomized by the Simpson family, which consists of Homer, Marge, Bart, Lisa, and Maggie. The show is set in the fictional town of Springfield and illustrates in a funny way many areas of American life.

The Simpsons is probably the most successful television show in history. Its enduring popularity attracted the attention of not only children, teenagers, adults but also many academics. From a certain perspective one can claim that *The Simpsons* essentially provides its viewers with a weekly lecture in mathematics, physics, computer science, history, psychology and philosophy. We can identify some links between various episodes and the issues raised by history's great thinkers, including Aristotle, Sartre, Kant and others. We can also focus on the spiritual significance of *The Simpsons* and wonder why Homer consistently resist pressure to attend church each Sunday. For sure we can say that no viewer gets bored while watching this series. But we are mainly interested in the show's writers and in the mathematics behind *The Simpsons*.

The core team behind the first season of *The Simpsons* consisted of several Los Angeles's smartest comedy writers. They created scripts that included references to sophisticated concepts from all areas of human knowledge, and calculus was particularly high on the agenda because two of the writers were devotees of mathematics. One of them was Mike Reiss. Reiss was not a professional mathematician, he studied actually English at Harvard. Ever since at a young age Reiss loved the books of Martin Gardner, he was the regular reader of Gardner's columns and attended several mathematics competitions. The other mathematician on the team was Al Jean, who shared Reiss's love of Martin Gardner's puzzles and was also a mathlete. Thanks his interest in mathematics he was accepted to study mathematics at Harvard when he was only sixteen years old. Both Reiss and Jean were always interested in comedy writing and when they joined the writing team for the first season of *The Simpsons*, they felt that this was the ideal opportunity to connect their love of mathematics and writing.

The first episode of *The Simpsons* was broadcast on December 17, 1989. Immediately it was a massive success, both in terms of audience figures and with the critics. And for the second episode called "Bart the Genius" contained a serious dose of mathematics. In many ways, this episode set the tone for what was to follow over the next two decades, namely a relentless series of numerical references and nods to geometry.

In the mid-1990s David S. Cohen joined to mathematically writers of *The Simpsons*. Cohen earned his bachelor's degree in Physics at Harvard University and master's degree in Computer Science at the University of California in Berkeley. In addition to his formal education, like Jean and Reiss, Cohen had exhibited a genuine talent for mathematics at a young age, and he too loved Gardner's monthly column. Due to his education, Cohen could include also physics (apart from mathematics) into the episodes of *The Simpsons*.

The team of writers further included mathematicians J. Stewart Burns and Ken Keeler and a physicist Jeff Westbrook.

In the series one can find many mathematical allusions. Some of these are intended just for mathematicians, for example, the *Fermat's last theorem*, which was mentioned in section 21. But some jokes incorporate mathematical concepts that will be familiar to many viewers. A classic example is *Crossing the river* which was introduced in section 12 or also the number π, which has made several guest appearances in the series. The jokes about the number π rely on the fact that "pie" and "π" are homophones. Thanks this property of π the mathematics was no longer just about long multiplication and some fractions, but was also about something interesting, elegant and universal. Because the number π appears in connection with every circle in the world, from Ferris wheels to Frisbees, from chapattis to the Earth's equator.

37.1 About the number π

For example in the episode "Simple Simpson" [38] is a pun-based joke referring to π. In this episode, Homer disguises himself as a superhero named Simple Simon, Your Friendly Neighborhood Pie Man, and punishes evildoers by flinging pies in their faces.The Pie Man's first act of superheroism is to deliver retribution to someone who bullies Lisa. This is witnessed by a character named Drederick Tatum, Springfield's famous ex-boxer, who proclaims: "We all know 'πr^2', but today 'pie are justice'. I welcome it."

Another reference to number π appears in the episode "Bye, Bye, Nerdie" [7]. This time the authors were inspired by history. In the article [april] about April fool's hoaxes we can read the section about an incident that took place in Indiana in 1897, when politicians attempted to legislate an official (and wildly incorrect) value for π as 3,2. This nonsense bill was known also Al Jean who used it for the scenario. In this episode, Lisa is supposed to speak in front of an audience of a scientific conference. But no one can hear Lisa, because the atmosphere is intense and the audience excitable thus the moderator struggles to bring the conference to order. So he exclaims: "Pi is exactly three!" Suddenly, the noise stops. This absurd declaration is a neat reminder of the postponed law from Indiana.

37.2 The pancake sorting problem

The other episode worth mentioning is "The Twisted World of Marge Simpson" [45]. In this episode the famous problem of recreational mathematics called the *pancake sorting problem* appears. A waiter in a restaurant prepares for Homer a stack of n pancakes of a random size. On the way to the table the waiter rearranges the pancakes (so that the smallest winds up on top, and so on, down to the largest at the bottom) by grabbing several from the top and flipping them over, repeating this (varying the number I flip) as many times as necessary. The question is how many flips will be required to put them into the correct size? This number of flips is called the pancake number P_n.

The question is mathematically interesting, because the problem is related to the solving of certain computer science problems. The idea behind this is based on the fact that the rearranging of pancakes has parallels with the rearranging of data.

Let us first find the solution of the problem for the simple cases. If we have just one pancake, then the pancake number is obviously zero: $P_1 = 0$, because the pancake cannot arrive in the wrong order. For two pancakes, the solution is also easy. Either the pancakes arrive in the correct order, or in the reverse one. The worst case is thus easy to identify, and it requires only one flip to overturn both pancakes at once to transform them into the correct arrangement, hence $P_2 = 1$. The calculation of the pancake number for three items is trickier, because there are six possible starting arrangements. Depending on the starting arrangement, the number of flips required to reach the correct arrangement varies from zero to a worst case of three: $P_3 = 3$. The table below displays that as the pile of pancakes grows, the problem becomes increasingly difficult due to the fast growing number of possible pancake permutations and flipping strategies.The difficulty of the calculation is also well illustrated by the fact that even very powerful computers have not yet been able to calculate the twentieth (or higher) pancake number.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
P_n	0	1	3	4	5	7	8	9	10	11	13	14	15	16	17	18	19	20	22	?

After more than three decades, nobody has been able to find a clever equation that predicts pancake numbers without using computers. So far, the only breakthroughs have been in finding equations that set limits on the pancake number. In 1979, the upper limit for the pancake number was shown to be less than $(5n+5) \times 3$ flips. This result was published in a paper that was co-authored by William H. Gates and Christos H. Papadimitriou. William H. Gates is better known as Bill Gates, the co-founder of Microsoft, and this is thought to be the only research paper that he has ever published. In this paper, further a devious variation of the problem called the *burnt pancake problem* is mentioned. The burnt pancakes are burnt on one side, so the challenge is to flip them into the right orientation (burnt side down), as well as flipping them into the correct size order. This is the problem that was addressed by David S. Cohen. He proved that the lower and the upper bounds for burnt pancake flipping lie between $3n/2$ and $2n$–2. Now it is clear that the upper limit for the twentieth pancake number is 35 and in the case of burnt pancakes the number is set between the number 30 and 38.

In *The Simpsons* we can find a lot more mathematical curiosities and problems. A very nice description of some of them, as well as more detailed information about the mathematics of *The Simpsons* can be found in the book [36].

38 The Futurama

After the huge success of *The Simpsons* in 1996, Groening teamed up with David S. Cohen to create a sister series titled *Futurama*, which is set one thousand years in the future. Moreover, the science fiction scenario permits the writers to explore mathematical themes in even greater depth. Thanks to this every episode of *Futurama* contains some links to various mathematical theorems, conjectures, and equations.

The series follows the adventures of a late-20th-century New York City pizza delivery boy, Philip J. Fry, who, after being unwittingly cryogenically frozen for one thousand years, finds employment at Planet Express, an interplanetary delivery company in the retro-futuristic 31st century. Apart from the pizza delivery boy, other members of the Planet Express team contain a one-eyed mutant Leela, a robot named Bender, the 160-year-old founder of Planet Express Inc. Professor Hubert J. Farnsworth, the company's lobster-like alien doctor John A. Zoidberg, an ex-Olympic limbo champion and the company's accountant Hermes Conrad and intern Amy Wong. Although the crew composition suggests that this science fiction series will contain plenty of science facts, the more erudite viewers are at every episode impressed by the sheer quantity and quality of facts from mathematics, physics and computer science. More detailed information about the mathematics of *The Simpsons* can be found in the book [36].

38.1 The Futurama theorem

From a recreational mathematics point of view, the most interesting episode is "The Prisoner of Benda" [44]. In it the Professor invents the Mind-switcher, which is a machine that takes two sentient beings and swaps their minds, allowing them to inhabit each other's bodies. There is no need to justify the capabilities of the machine which will take the whole team of Planet Express. In total there are seven switches among various members mentioned in the episode. The mathematics is not in the mind-switching perse, but rather it is required to help unravel the mess caused by such mental juggling. The machine has only one fault. Once two bodies have swapped minds, the Mind-switcher cannot perform a reverse swap between this pair of bodies. Hence, it is not at all clear that the various minds can return to their own bodies. This Mind-switcher glitch was introduced by the writers to make the plot more interesting. However, someone then had to find a way to overcome this barrier to reach a happy ending, and the responsibility fell to Ken Keeler, the leading writer for this episode. Keeler assumed that one way to break the deadlock would be to introduce fresh people into the scenario, characters who could provide indirect paths by which the minds of the Professor and everyone else could return to their correct bodies. The main question that Keeler put on was how many fresh people would be needed to be introduced into a group of any size to unravel any conceivable mind-switching muddle?

Finally, Keeler concluded that introducing just two fresh people would be enough to untangle the mind-switching chaos of any magnitude, as long as those two people were deployed in the correct manner. Keeler's proof essentially describes a clever unmuddling strategy, which begins by a realization that individuals with switched minds can be placed into well-defined sets. This proof which is somewhat technical has come to be known as the *Futurama theorem* or *Keeler's theorem.*

James Grime, a mathematician based in Cambridge, England, examined the proof in detail. He explored a trick which results in a reduction of the necessary switches. This trick is based on the using of an existing set to provide the two fresh people required to unmuddle another set. Hence, some people refer to this trick as *Grime's corollary*, a mathematical statement that emerges from the Futurama theorem.

Clearly, this episode is based on some genuinely interesting and innovative mathematics and it is the pinnacle of all the mathematical references that have ever appeared in both *The Simpsons* and *Futurama*. The fact Ken Keeler created an entirely new theorem in order to help the Planet Express crew. Indeed, Keeler can claim the honor of being the first writer in the history of television to have created a new mathematical theorem purely for the benefit of a sitcom, what we can for sure consider as recreational mathematics.

References

[1] Bártlová T.: *A romance in many dimensions (A story of Edwin Abbott Abbott),* In Proceedings of the Fourth Recreational Mathematics Colloquium, Lisboa: Associacao Ludus, 2016, pp. 167–177.

[2] Bártlová T.: *Archimedes' cattle problem,* (Czech) Archimédes. Několik pohledů do jeho života a díla, edice Dějiny matematiky, volume 54, Matfyzpress, Prague, 2012, pp. 99—108.

[3] Bártlová T.: *Izoperimetrické nerovnosti,* (Czech) Prague, 2012, Master's thesis, Charles University in Prague, Faculty of Mathematics and Physics.

[4] Bártlová T.: *Martin Gardner's Mathemagical Life,* Recreational Mathematics Magazine, Issue 2, 2014, pp. 21—40.

[5] Bártlová T.: *Where are (pseudo)science fool's hoax articles in April from?,* Manuscript submitted, 2016.

[6] Burton D. M.: *The history of mathematics,* An introduction, McGraw-Hill 2006, pp. 230.

[7] Bye, Bye, Nerdie, In: *The Simpsons*, Series 12, episode 16, Fox Television, 11 March 2001.

[8] *Carved Stone Balls* [online], last modified on 31 March 2016 [cit. 2016-06-29]. Available from World Wide Web: `https://en.wikipedia.org/wiki/Carved_Stone_Balls`

[9] *Carved Stone Balls from Scotland,* [online]. [cit. 2016-06-29]. Available from World Wide Web:
`http://www.ashmolean.org/ash/britarch/highlights/stone-balls.html`

[10] *Chinese Ring Puzzle Applet,* [cit. 2016-06-20]. Available from World Wide Web:https://`www.cs.oberlin.edu/ jwalker/puzzle/`

[11] Conrad A., Hindrichs T., Morsy H., Wegener I.: Solution of the knight's Hamiltonian path problem on chessboards, Discrete Applied Mathematics 50 (1994).

[12] Culin S.: *Games of the Orient: Korea, China, Japan,* Rutland: Charles E. Tutlle, 1965.

[13] Danesi M.: *The Liar Paradox and the Towers of Hanoi: The Ten Greatest Math Puzzles of All Time,* John Wiley & Sons, 2011.

[14] Dörrie H.: *Malfatti's Problem,* §30 in 100 Great Problems of Elementary Mathematics: Their History and Solutions. New York: Dover, 1965, pp. 147–151.

[15] Dudeney H.: *Amusements in Mathematics,* London 1917 (reprinted by Dover Publications, New York, 1958).

[16] Euler L.: *Elements of Algebra,* London: Printed for Longman, Rees, Orme, and co. (translated from the French), 1828. [online], [cit. 2016-06-20]. Available from World Wide Web: `https://archive.org/details/elementsofalgebr00eule`

[17] Gardner M.: *Mathematical Puzzles and Diversions,* New York: Penguin, 1965.

[18] Gardner M.: *New Mathematical Diversions,* New York: The Mathematical Association of America, 1995, pp. 162–172.

[19] Gardner M.: *Six sensational descoveries that somehow or another have escaped public attention,* Scientific American *April* (1975), pp. 126–133.

[20] Gardner M.: *What unifies dinner guests, strolling schoolgirls and handcuffed prisoners?* Scientific American *May* (1980).

[21] Gillings R. J.: *Mathematics in the Time of the Pharaohs,* New York: Dover Publications; Revised ed. edition (June 1, 1982).

[22] Gone Maggie Gone, In: *The Simpsons*, Series 20, episode 13, Fox Television, 15 March 2009.

[23] Graham R. L., Knuth D. E., Patashnik O.: *Concrete Mathematics,* Massachusetts: Addison-Wesley, 1994.

[24] Hadley J., Singmaster D.: *Problems to Sharpen the Young*, The Mathematical Gazette, Vol. 76, No. 475, The Use of the History of Mathematics in the Teaching of Mathematics (Mar., 1992), pp. 102–126.

[25] 'Homer^3', Treehouse of Horror VI, In: *The Simpsons*, Series 7, episode 6, Fox Television, 29 October 1995.

[26] Kangshen S., Crossley J. N., Lun A. W.-C.: *The Nine Chapters on the Mathematical Art: companion and commentary,* New York: Oxford University Press, 1999, pp. 411.

[27] Krumbiegel B., Amthor A.: *Das Problema bovinum des Archimedes.* Zeitschrift für Mathematik und Physik (Historisch-literarische Abtheilung) 25(1880), pp. 121–136, pp. 153–171.

[28] Livio M.: *The Golden Ratio: The Story of PHI, the World's Most Astonishing Number,* New York: Brodway Books, 2002.

[29] MacGuckin de Slane W.: *Ibn Khallikan's Biographical Dictionary,* (translated from the Arabic), volume III, Paris: Oriental translation fund of Great Britain and Ireland, 1868, pp. 69–70.

[30] *Mersenne prime,* [online], last modified on 14 June 2016 [cit. 2016-06-29]. Available from World Wide Web: `https://en.wikipedia.org/wiki/Mersenne_prime`

[31] *Mother Goose's Nursery Rhymes*, London: George Routledge and Sons, 1877, pp. 318.

[32] Nagell T.: *Fermat's Last Theorem,* §68 in Introduction to Number Theory. New York: Wiley, 1951, pp. 252.

[33] Petkovic M. S.: *Famous Puzzles of Great Mathematicians,* Rhode Island: American Mathematical Society, 2009.

[34] Rouse Ball W. W., Coxeter H. S. M.: Mathematical Recreations and Essays, Dover Publications, New York, 1987.

[35] *Seven Bridges of Königsberg* [online], last modified on 4 June 2016 [cit. 2016-06-29]. Available from World Wide Web:
`https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg`

[36] Singh S.: Fermat's Last Theorem, London: Harper Perennial, 2005.

[37] Sigler L. E.: *Fibonacci's Liber Abaci: Leonardo Pisano's Book of Calculation,* New York: Springer-Verlag, 2002.

[38] Simple Simpson, In: *The Simpsons*, Series 15, episode 19, Fox Television, 2 May 2004.

[39] Singmaster D.: *A Variation of the Ass and Mule Problem,* Crux Math. 28, 2002, pp. 236-238.

[40] Smith D. E.; Mikami Y.: *A History of Japanese Mathematics,* New York: Cosimoclassics, 2007.

[41] Stewart I.: *Professor Stewart's Cabinet of Mathematical Curiosities,* Basic Books, New York, 2008.

[42] Swetz F. J.: *Mathematical Treasure: Bachet's Mathematical Recreations,* Convergence (August 2014).

[43] *The Fibonacci Quarterly*, [cit. 2016-06-20]. Available from World Wide Web: http://www.fq.math.ca/

[44] The Prisoner of Benda, In: *The Futurama*, Series 6, episode 10, Fox Television, 19 August 2010.

[45] The Twisted World of Marge Simpson, In: *The Simpsons*, Series 8, episode 11, Fox Television, 19 January 1997.

[46] *The 3x3 Magic Square, More Magical than You Thought!* [cit. 2016-06-20]. Available from World Wide Web: http://pballew.blogspot.cz/2013/02/the-3x3-magic-square-more-magical-than.html

[47] The Wizard of Evergreen Terrace, In: *The Simpsons*, Series 10, episode 2, Fox Television, 20 September 1998.

[48] Toal R.: *Solving the Kirkman Schoolgirl Problem,* [online]. [cit. 2016-06-29]. Available from World Wide Web: http://cs.lmu.edu/ ray/notes/kirkman/

[49] Vardi I.: *Archimedes' Cattle Problem,* Amer. Math. Monthly 105, 1998, pp. 305–319.

[50] Watkins J. J.: *Across the Board: The Mathematics of Chessboard Problems,* New Jersey: Princeton University Press, Princeton and Oxford, 2004. Available from World Wide Web: http://press.princeton.edu/chapters/s7714.pdf

[51] Whiston W.: *The Wars Of The Jews Or History Of The Destruction Of Jerusalem: By Flavius Josephus,* [cit. 2016-06-20]. Available from World Wide Web: http://www.gutenberg.org/files/2850/2850-h/2850-h.htm

[52] *15 puzzle,* [online], last modified on 7 April 2016 [cit. 2016-06-29]. Available from World Wide Web: https://en.wikipedia.org/wiki/15_puzzle

Paper II

T. Bártlová: *A romance in many dimensions (A story of Edwin Abbott Abbott),* In Proceedings of the Fourth Recreational Mathematics Colloquium, Lisboa: Associacao Ludus, 2016, pp. 167-177.

The preface to the article *A romance in many dimensions (A story of Edwin Abbott Abbott)*

Among great examples of recreational mathematics one often meets problems from geometry. Geometry is regarded the aspect of clarity and thus so many problems invite the general public to solve it. But this aspect we can apply only if we talk about Euclidean geometry. The geometry which consist in assuming five Euclid's axioms. But if we cross the boundaries of Euclidean geometry, we get into a very difficult and abstract part of mathematics.

The non-Euclidean geometry and the introduction of a fourth of higher dimensional space, makes problems also to the mathematicians themselves. Consider, for example, the fact for how long the mathematicians tried to prove the fifth Eulid's postulate. From the 3rd century B.C. to 1826 a lot of mathematicians worried away at its proof. And even in the 19th century when mathematical knowledge was so advanced that Nikolaj I. Lobacevkij reported the idea of non-Euclidean geometry, it means new geometry in which the fifth postulate does not apply, a lot of mathematicians did not want to believe. Many of them considered this idea not scientific, but devilish.

Recall, for example, Lewis Carroll who internally struggled with the notions of abstract mathematics. His dismissive attitude toward abstract mathematics is reflected satirically in his book *Alice's Adventures in Wonderland.* Conversely mathematicians who had no problem with the abstract ideas and non-Euclidean geometry was absolutely permissible for them, often failed to communicate these ideas to the general public. No wonder, for the people who live in the three dimensional world, that the idea of the world of four and more dimensions quite elusive.

How to pass the information about the discovery of non-Eulidean geometry to the general public in the way they would adopt it? Or rather, should people ever know about this discovery? Edwin Abbott Abbott believed that the answer is yes. It is necessary to make the people aware of the newest mathematical discoveries. Mathematics should not be shrouded in mysticism to which only a select few can glimpse. Edwin Abbott Abbott coped with this task thanks to his thin novel *Flatland: A Romance of Many Dimensions.* In his book, he used the analogy between the two dimensions of world, Flatland, and the three dimensions of space. He described the difficulties why a two-dimensional being who is exposed to the three-dimensional experience, to help people in a three-dimensional space to accustom themselves to the radical but enormously popular idea of a fourth dimension.

Abbott's *Flatland* is from a certain view very similar to Carroll's *Alice*, but it contains quite distinctive elements. But as in the case of Alice, the reader could be anyone. For some readers it is the scientific popularization about the fourth dimension, for someone it is a satire, someone can find that it is mathematical fantasy.

A romance in many dimensions (the story of Edwin Abbott Abbott)

Tereza Bártlová (Charles University in Prague)

Edwin Abbott was born in London on 20th December, 1838. His parents were Edwin Abbott, the headmaster of the Philological School in Marylebone, and Jane Abbott. His unusual double-barreled name originates from the fact his parents were first cousins – both bearing the name of Abbott. This explains Edwin having "Abbott" as both a surname and a middle name.

Abbott was educated at the City of London School, which was remarkable at that time in several respects. It had earned its considerable reputation under the headmaster Doctor George Ferris Whidborne Mortimer. He did not discriminate against students on the grounds of their religion (at the time when most public schools had an Anglican emphasis). Mortimer's religious tolerance led him to open the school for boys from non-conformist and Jewish families. In this school Abbott met another student, Howard Candler, who became his lifelong friend.

Abbott was a very talented student. In 1857, he won a scholarship at St. John's College at the University of Cambridge. At this university, Abbott took the highest honours in classics, mathematics and theology. After finishing an outstanding academic career as an undergraduate in 1861, he was awarded the Senior Classics medal. In the following year he was elected to a fellowship at his college. In the same year he was ordained a deacon and in 1863, he became a priest. But at this time college fellows were not allowed to marry, so when Abbott wished to marry Mary Elizabeth Rangeley, he had to resign the fellowship.

1 Abbott and education

Among Abbott's lifelong passions belonged not only theology but also education. He started his teaching career in 1862, when he became an assistant master at King Edward's School in Birmingham and in 1864 he moved to Clifton College. Then he decided to apply for the headship at his old school. He had a difficult task ahead of him because he had to preside over a substantial number of much older teachers who taught him when he was a student himself, some of whom had also applied for the headship. Despite this, Abbott was very successful (see [3]): "*He won their respect and became the school's best-know* *headmaster.*" As a result, Abbot has held the position of a headmaster since 1965 to 1889.

Abbott spent most of his professional life as a teacher. Not only he had extraordinary intellectual abilities but also he was a great educator. He liked Shakespeare and literature in general and managed to pass his own enthusiasm to the students at school. For example, once in a term, students performed Shakespeare's plays (or dialogues) (see [3]): "*The boys had to study one Greek and one Shakespearean play every term. On 'Beaufoy Day,' when the prizes were awarded, it became traditional to have recitations from Shakespeare; under Abbott these developed into dialogues and eventually the performances of entire scenes.*" He also won his fame thanks to his deep engagement in public matters. Thereby, he inspired many students who later became known in a wide range of subjects and professions. This included, for instance, the prime-minister-to-be, Herbert Henry Asquith.

Edwin Abbott had been the headmaster for long 24 years. And he was not just an outstanding teacher, but also a very effective administrator. Thanks to him, the school became one of the top ones in the country. Rather unusually for that period, he put emphasize on the importance of science, namely chemistry, mathematics or comparative philology, and he made many innovations to the curriculum taught at the school. Branchoff writes (see [**?**]): "*His greatness as an educator derived partly from his organization of new methods of instruction, partly from his initiation of many innovations in the school curriculum, and partly from what can only be called his genius for teaching. Having a reverence for physical science not often found among the classical scholars of his day, he made an elementary knowledge of chemistry compulsory throughout the upper school.*" He managed to reduce the size of classes, which, when he first arrived, had often reached seventy students per teacher. He focused on hiring high-quality teachers.

Abbott was relatively young when he retired, being only 50 years old. There were several reasons why he decided to retire so early. By late 1870s, he saved his school from closing. The school was declared too small and outdated and closing was ordered. In December 1874, Abbott wrote to his friend Candler (see [3]): "*Next year I shall agitate for the removal of the School.*" In 1875, a lengthy and debilitating struggle was started, which ultimately cost Abbott a tremendous lot of energy and effort. It took several years till the new school was opened in the late 1882. During this tiresome period, Abbott devoted his attention to writing. It was in 1889 when he finally retired and decided to spend his newly-gained free time on his literary efforts.

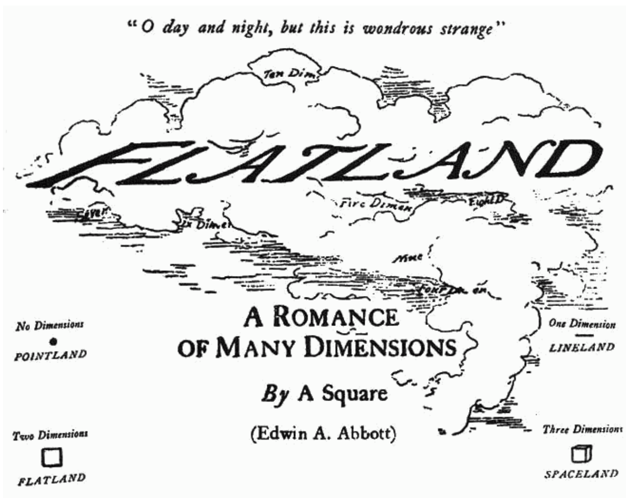
Abbott started writing books in 1871. In total he published over fifty books. Using his vast knowledge, he wrote excellent works on a wide variety of topics. He published *Shakespearean Grammar* (1870), which is a permanent contribution to English philology. Next came *English Lessons for English People* (1871) and *How to Write Clearly* (1872). He was a leading expert on Francis Bacon. He published *Bacon and Essex* (1877) and wrote an introduction to *Bacon's Essays* (1886). Some of his works on textual criticism contain excellent statistical analyses, for example *Johannine Vocabulary* (1905) and *Johannine*

Grammar (1906). Some of his outputs were used at schools as textbooks. His theological writings include three anonymously published religious romances - *Philochristus* (1878), in which he tried to raise interest in Gospels reading, *Onesimus: Memoirs of a Disciple of Paul* (1882), and *Silanus the Christian* (1906), or the theological discussion *The Kernel and the Husk* (1886.)

Undoubtedly, one of his best-known books of all times, which worthy of being remembered for its individual, literary merits, is an engaging fable called *Flatland: A Romance of Many Dimension.*

2 Flatland: A Romance of Many Dimensions

Flatland: A Romance of Many Dimensions shortly *Flatland* is unique in many directions, and it stands out among all of Abbott's writings as it falls into the category of mathematical fantasy.



Flatland: A Romance of Many Dimensions

Flatland was probably written during the summer of 1884. In October 1884, Abbott distributed a draft of the book to a selected group of friends, and in November of the same year the first edition was published.

Flatland was not published under Abbott's name. He used the pseudonym "A. Square". Not surprisingly, several Abbott's works had been before initially published also under different pseudonyms. This concerns for example *Philochristus*, *Onesimus*, or *The Kernel* *and the Husk*. There exists a number of theories which try to explain the origin of the pen name "A. Square". Perhaps the most credible one finds the reason in the fact that Abbott's middle name was the same as his surname. In other words, Edwin Abbott Abbott might have been nicknamed as "Abbott Squared". However, A. Square is also the name of the main hero and narrator in the book. So Abbott probably felt a considerable degree of identification with the character.

The setting for the tale is a two-dimensional world, an infinite Euclidean plane: Flatland. Flatland is inhabited by intelligent creatures shaped like polygons. First of all, the narrator A. Square is indeed a person who has the shape of a square.

In the first part of the book, A. Square describes Flatland's customs, society, and history. This part is no doubt aimed at Victorian social prejudices of the time. The second part is mainly devoted to an attempt of an introduction of the concept of the fourth dimension. At first, A. Square has a vision of the one-dimensional world Lineland. In his dream, he tries to explain to the inhabitants of the Lineland that there exists a two-dimensional world, known well to him, but fails repeatedly and badly, because of the lack of imagination of such a world on their side. Even worse situation, of course, can be found in Pointland. Next, Flatland is visited by an unthinkable three-dimensional creature called Sphere. The Sphere opens Square's eyes to the possibility of existence of a third dimension. As a result, the Square suggests to the Sphere that there might exists also a world with four or more dimensions. The Sphere rejects the idea immediately and savagely as absurd.

This idea bears a central scientific message of the book – imagine a two-dimensional creature trying to comprehend our three-dimensional world; then beef everything up by one more dimension.

Abbott's story does not end happily. Just as a Linelander cannot be convinced by the Square that there is a second dimension, the Sphere refuses to believe that there might be a fourth or higher dimension. So, in the end, the Sphere laughs at this suggestion. When A. Square tells his fellow Flatlanders about the third dimension, nobody believes him, and he is finally sent to a mental institution.

3 Scientific popularization

Abbott's impulse for writing *Flatland* was, in the first place, his reader's innate prejudice for three-dimensional ways of thinking. So he used the analogy between the two dimensions of Flatland and the three dimensions of space. Flatland is a world inhabited by beings whose experience of space is limited to two dimensions. The character A. Square depicts an everyday life and learning issues within the world of two dimensions. For example, how members of that world recognize one another if recognizing planar shapes commonly requires an elevated point, which assumes the third dimension? The narrator wanders into a one-dimensional world and its mysterious mirrors reflect the reader's experience of two-dimensional reality. Finally, the narrator is transferred into a three-dimensional world in awe described experiences that the figures of this world are taken for granted (as well as the reader). But when he starts thinking about the world of four-dimensional, a three-dimensional being tells him that it is no such thing.

The difficulties of comprehending the third dimension by a two-dimensional being are used to help Victorians living in a three-dimensional space to accustom themselves to the radical, but enormously popular at that time idea of a fourth dimension. This is an important moment of the novel, because it shows the limited sight and life of the people not developing further in their way of life.

Abbott was not a mathematician. And with the exception of a short passage to geometry in his book for home teaching *Hints on Home Teaching*, he wrote nothing more about mathematics besides *Flatland*. Despite the fact that Abbott was a very clever man, it could not be easy for him to write over one hundred pages of anything geometrical without falling into all sorts of errors. Abbott managed it, many years prior to Einstein's four dimensional world of relativity. How did he develop this dimensional analogy without falling into error? It might have been his friend Howard Candler who helped him. It had been already mentioned that Candler and Abbott met when they were both schoolboys at the City of London School. They went to Cambridge together and stayed in touch for the rest of their lives. With Candler being at Uppingham, they exchanged letters every week for long 25 years. (Abbott's letters to Candler were still available as late as 1939, but when they were supposed to be used as one of the sources for the history of the City of London School, which was in preparation at that time, they suddenly got lost.) Abbott consistently consulted Candler about his theological writings, so it is almost certain that he would have written to him about the geometrical part of *Hints on Home Teaching*. And Abbott certainly shared his thoughts on *Flatland with Candler*. It is likely that their mutual correspondence led Abbott to Charles Howard Hinton, a new science master, whose ideas he then started to follow.

Apart from the fact that Abbott was not the first one who used the role of the fundamental analogy – to appreciate the difficulties of understanding such higher geometry, it is useful to imagine the situation of a creature from a lower dimension attempting to come to terms with the third one. This thought exercise captured the imagination of many writers. And, for sure, he was not the first person to use a two-dimensional universe inhabited by flat beings, but he was the first to explore what it would mean for such individuals to interact with phenomena from a dimension higher than their own.

4 Educational aspects

Perhaps the very fact that Abbott was not a mathematician but a literature teacher makes *Flatland* so popular, since it can be read by anybody. It is nearly a flawless introduction to the method of analogy in the treatment of different dimensions.

Flatland is inspired by school geometry texts, which at that time were universally derived from a classical text: the *Elements* of the ancient Greek geometer Euclid. This is Abbott the schoolmaster, so he knew that a sheet of paper is the easiest way how we can imagine the two-dimensional space (see [1]): "*I call our world Flatland, not because we call it so, but to make its nature clearer to you, . . . imagine a vast sheet of paper. . .*"

The influence of the Euclid's *Elements* also appears elsewhere. Abbott gave Flatland's inhabitants the shape of polygons. At the beginning of the story he talks for example about Triangles, Squares, Pentagons, Hexagons. It means that he cautiously described only those polygons which are easy to construct with Euclidean constructions. It means creating them with nothing more than a pair of compasses and an unmarked ruler. This is probably the reason why he deliberately avoids heptagons or nonagons.

He also described very aptly the Archimedean method of exhaustion when talking about Circle-process (see [1]): "*. . . from thence rising in the number of their sides till they receive the honourable title of Polygonal, or many-sides. Finally when the number of the sides becomes so numberous, and the sides themselves so small, that the figure cannot be distinguished from a circle.*"

5 Flatland as a satire

Flatland is not only the dimensional analogy but also a reflection of Abbott´s times.

5.1 Victorian Era

The period in which Abbott lived is called the Victorian era. Queen Victoria was the longest British reigning monarch of all times. It was the era of social and economic changes which were caused by industrialization and progress in natural science.

The social structure at this time could be divided up into a class system beginning with the working class, the lower middle class, the middle class and ending with the aristocracy. On one hand, this era was progress-oriented because of industrialization. It meant new jobs, technological development and social mobility. On the other hand, it caused urbanization through growing towns and cities to which the rural population was moving. New jobs and employees were needed for production of goods and building railroads. As a result, the middle class rose and grew in population and diversity. Its members worked as businessmen, shopkeepers or teachers, and so on. The lower class which included the working class, worked in towns or factories. But, the problem was that they found it difficult to adapt to the new circumstances and learn other jobs than those known to them before. The life of the working class was unsecure because there was no unemployment insurance or any medical or health plans. Since all depended on financial status in Victorian society, the poor working class was not able to rise in the social stratum. In comparison to the working class, the upper classes and the aristocracy had a luxurious and wealthy life summed up by the term "Victorian prosperity".

5.2 Flatland's social structure

The novel *Flatland* does not try to show the good sides of the Victorian era, but it lets the reader see what else is hidden behind social changes during this time and which changes it had for the population and the social structure. It is a biting satire on Victorian values.

The society in *Flatland* is built up in a social class system, whereas the class someone belongs to becomes visible through one's physical form. It means that the rights of every person as well as his or her status in society depends on the shape of the body (see [1]): "*Our Soldiers and Workmen are Triangles with two equal sides...*"

The position in the class gets better with the growing number of sides of a person (see [1]): "*...from thence rising in the number of their sides till they receive the honourable title of Polygonal, or many-sides. Finally when the number of the sides becomes so numberous, and the sides themselves so small, that the figure cannot be distinguished from a circle... and this is the highest class of all.*" It is impossible for someone of the lower class to become in the highest class because his physical form will never allow him to do so. Abbott probably appeals to Victorian society's class distinction.

This situation is not entirely hopeless. Abbott also described an evolutionary process because he said that a male child shall always have one more side than his father (see [1]): "*...each generation shall (as a rule) rise up one step in the scale of development and mobility.*" It is a Victorian idea in which the main point of progress in evolution is related to progressing in a higher status. Stewart claims in his book [3] that this belief could probably come from misinterpretation of Darwin's theory of evolution which he elaborated during Victorian time.

But this "Law of Nature" does not apply to everyone. The lower classes, including soldiers or women, are at the bottom of the hierarchy and are not able to rise in society (see [1]): "*...the son of Isosceles...remains Isosceles still.*" Stewart states [3] that it is a common feature of hierarchical society in Victorian era that the lower classes "know their place" and do not strive to improve their position. In effect they help to maintain the hierarchy, possibly because they know that any attempts to overthrow it will lead to a violent repression, but mostly because life is much simpler if you know your place.

Abbott strongly criticized the Victorian class system and the differences between the classes, especially the lower classes who do not have a chance on rising up in the social scale and depend on decisions of the reigning classes. Thanks to progress through industrialization, the class system was much strengthened and the situation of the lower classes became worse than before.

5.3 Flatland's women

During the Victorian era, the role of a woman was also an important aspect. A woman was regarded as being inferior and she was not allowed to vote. In Victorian era, the domestic family life was the core of the society in which the man, as the patriarch, had to be supported by his wife. A woman with no rights depended entirely on her husband and therefore she had to obey him. The only woman's tasks were to take care of the household and to up bring the children.

Moreover the women did not have the same educational opportunities as men. They were not allowed to receive any education other than that connected to taking care of the household. The Victorian society cherished a very clear idea of what was suitable for women, and their intellectual activities. In the 1800s, the Cambridge University refused to let women qualify for a degree. However, women were permitted to take the examinations on an informal basis. Their answers would have been graded by the examiners, but only to indicate what level of understanding they had reached: they would not be given any formal credit. Plenty of time elapsed until, finally, in 1970s, individual colleges admitted students of both sexes (see [3]).

In *Flatland*, Abbott intends to point out the discriminatory role of women. He underlines the big oppression and assignment to domesticity alluding women's tasks during the Victorian era. Likewise, neither in Flatland the women are allowed to vote. And, of course, they have to obey their husbands entirely. Moreover, in Flatland, women have a special status because they are at the bottom of the class system.

Flatland's women are one-dimensional, hence they belong to the lowest class of all (see [1]): "*Our Women are Straight Lines.*" Flatland's females are one-dimensional and are even lower in the social hierarchy than males of the most degraded type. Males of Flatland, even the most unintelligent ones, are always two-dimensional figures. Moreover, women, as well as isosceles triangles, have no chance to rise up in the social scale of development (see [1]): "*Once a Woman, always a Woman is a decree of Nature.*"

Since the Flatland's women belong to the lowest class, there is nobody who cares about them or appreciates them. They do not have any intelligence and thus nobody wants to listen to them (see [1]): "*... the wife has absolutely nothing to say, and absolutely no constraint of sense, or conscience to prevent her from saying it, not a few cynics have been found to aver that they prefer the danger of the death-dealing but inaudible sting to the safe sonorousness of a Woman's other end.*" In Flatland, the women have no reason and do not understand rational thinking, thus, in particular, they are not educated in mathematics.

Abbott believed that the conventional roles of women in the Victorian society were needlessly limiting. He was in regular contact with several female intellectuals, for example the novelist Mary Ann Evans (writing under the male pseudonym George Eliot) or the educator Dorothea Beale. Abbott himself was an active reformer who argued strongly for improvements in the education of women. Therefore, he described the role of women in *Flatland* with exaggeration and a great deal of satire (stewart xviii): "*By making Flatland men treat their women with undisguised contempt, he was pointing out how common this attitude was in Victorian society.*"

Rather unfortunately, a lot of people did not understand that *Flatland* was meant to be a parody to the Victorian class structure. They thought that the opinions expressed in *Flatland* were identical to that of Abbott himself. We can deduce this from the preface to the second edition (see [3]): "*where Abbott includes an explicit (and slightly pained) reaction to criticisms of the book's treatment of women.*" This account can prove some excerpts from the first to review, for example The Architect magazine wrote (see [4]): "*. . . we might be tempted to say that Mr. Square is not so impartial as he professes to be and his statement that the female sex consists of straight lines, dangerously acute at each end, is probably evidence that Mr. Square is rather advanced in years, and has been jilted more than once.*"

5.4 Only regular figures

In the previous paragraph we mentioned that the rights and education of the individuals belonging to their classes depend on "one's social pedigree" which predominates all other things "-talent, intelligence, ability - just as it did for the Victorians" (see [3]).

But in Flatland, not only the social status but also intelligence are closely associated and both depend on a physical form. Intelligence of Flatlanders can be measured directly by the size of their angles. It means the bigger the angle, the bigger the brain. The bigger the brain, the greater the intelligence. This view stems from Victorian belief that a bigger brain implies a superior intelligence (see [3]).

And, on top of that, in the Victorian England, very little tolerance for irregularity has been exercised. Often, it was associated with criminal tendencies. This is reflected in Flatland by locking up irregular figures.

This is a very interesting moment of the book, since Abbott seems to have discovered such facts many years ahead of other, more famous writers on the opposition to totalitarian establishments, in particular 65 years prior to Orwell and 40 years prior to Zamjatin.

6 Hinton, Abbott, Wells

For sure Abbott was not the first one who wrote about the fourth dimensional world. In the late 1800s, a lot of scientists and mathematicians were very interested in it, and their excitement spread out to the general public. An important role in this regard was played by a mathematician Charles Howard Hinton, who published an article about fourth dimension called *What Is the Fourth Dimension?* in the Dublin University Magazine in 1880. Banchoff claims (see [5]): "*... we can almost certainly discover the primary association that led Abbott to take up this idea as the basis of his combination of social satire and philosophical exercise, namely his encounter with the work of Charles Howard Hinton.*" It is also possible that Abbott met Hinton in person after Hinton had become a science master at the Uppingham School, where, by coincidence Abbott's friend and mathematics master Howard Candler at the time.

It is clear that Hinton's article had inspired Abbott. We can find several essential plot elements of *Flatland* that indicate it. In both stories the plane world, creatures shaped like circles, polygons and finally also the dimensional analogy can be found. Unlike Abbott, Hinton often related the fourth dimension to pseudoscientific topics that ranged from ghosts to the afterlife.

And the inspiration probably had to be mutual because the appearance of Abbott's *Flatland* did not prevent Hinton from using the same name the very next year in his *An Episode of Flatland*.

Despite the fact that both authors wrote works on the same topic, Banchoff claimed that the situation was not viewed as hostile by neither of them. Each referred to the other in later works which indicated that they saw their efforts as complementary rather than competitive. Banchoff took the reference for key from Abbott's side in his book *The Kernel and the Husk*. In discussing spirits Abbott wrote (see [2]): "*You know - or might know if you would read a little book recently published called Flatland, and still better, if you would study a very able and original work by Mr C. H. Hinton - that a being of Four Dimensions, if such there were could come into our closed rooms without opening door or window, nay could even penetrate into, and inhabit our bodiesEven if we could conceive of Space of Four Dimensions - which we cannot do although we can perhaps describe what some of its phenomena would be if it existed - we should not be a whit better morally or spiritually. It seems to me rather a moral than an intellectual process, to approximate to the conception of a spirit: and toward this no knowledge of Quadridimensional space can guide us.*" In 1884 and 1886 Hinton published his collected speculative essays about his ideas on the fourth dimension called *Scientific Romances*. In the introduction to the chapter on a plane world, Hinton replied (see [6]): "*And I should have wished to be able to refer the reader altogether to that ingenious work, Flatland. But on turning over its pages again, I find that the author has used his rare talent for a purpose foreign to the intent of our work. For evidently the physical conditions of life on the plane have not been his main object. He has* *used them as a setting wherein to place his satire and his lessons. But we wish, in the first place, to know the physical facts.*"

Hinton's collected essays probably captured Herbert George Wells´s attention. In 1885 the *Nature Magazine* reviewed Hinton's first series of *Scientific Romances* and summarized several of his ideas on the fourth dimension. Shortly afterwards, a reader, signed S, responded to the article and proposed considering time as the fourth dimension. We do not know who this mysterious S was. James E. Beichler surmised that it might have been a mathematician James Joseph Sylvester. Anyway, this proposal probably impressed Wells, a regular reader of Nature, because in 1895 he published his famous story *The Time Machine*, using this idea.

Both *The Time Machine* and *Flatland* are widely admired in science fiction circles and belong firmly to the prehistory of science fiction. And we can only credit Sylvester and Hinton with these inspiring books.

7 Abbott, Carroll and their adventures world

Besides *Flatland* belonging among science fiction, it is also one of the earliest works of what might be termed as mathematics fiction; that is, a speculative fiction with a mathematical theme. In this respect it was preceded by *Alice's Adventures in Wonderland* (1865) and *Through the Looking-Glass and What Alice Found There* (1871) by more conventional mathematician Lewis Carroll.

These books have much in common. At the first sight, both books can be considered as fairy tales for children or adults. In deeper reflection on the content and adaptation, however, we also appreciate the mathematical essence of the story. Both stories are interwoven with a number of serious mathematical ideas and propositions. In both cases, it is fantasy that takes place in the mysterious world in which certain beings live which cannot be encountered on the street. But *Flatland* is somewhat different from *Alice's Adventures*.

The first difference we can find is the narrative style of *Flatland*. The narrative style may emphasize the importance of our consciousness when experience the physicality of space, for example, that the body of the character can grow or shrink to human size. The novel *Alice in Wonderland* affects our relationship to space by pointing out how the movement, object manipulation, management and interpersonal relationships are influenced by scaling. In *Alice's Adventures*, the story is told by Alice who visits another wonderland - it means in this book, the story is told not by the visitor but the person visiting. In contrast, in *Flatland*, the story is told by A. Square who is visited by the Sphere. It is as if the adventures of Alice were told by the White Rabbit. The Flatland's reader has to stay with the story all the way through to appreciate the change that takes place in the storyteller. There is also important educational aspect in the book, because the narrator of *Flatland*, namely A. Square, is seeking answers to questions that, at the same time, baffle the reader. Abbott was a very good teacher and his legacy is widely used in all sorts of teaching practice.

And the second difference lies in the fact that *Flatland* recounts the daily life and exploring questions of the world. Actually, this novel fits into the context of Abbott's life. *Flatland* gives him a chance to examine more closely the limitations of Victorian society, its preoccupations with class consciousness, social Darwinism, resistance to the rights of women or minorities or misfits, and a growing two-cultures mentality separating the rational from the intuitive and the theoretical from the practical order. So unlike Carroll's Alice, Abbott tells us that we do not enter into a strange world ourselves. A strange world surrounds us, and we have to deal with it.

8 Waiting for success

At the time of Flatland's publication it did absolutely not attract as much attention as it deserved. Only one month after Abbott published the first edition of *Flatland*, he had to release the second edition, because a lot of readers did not understand his irony. Abbott wrote the new preface to the second edition of *Flatland* makes it clear that Abbott must have found himself on the receiving end of criticism about the regard in which *Flatland* held its women.

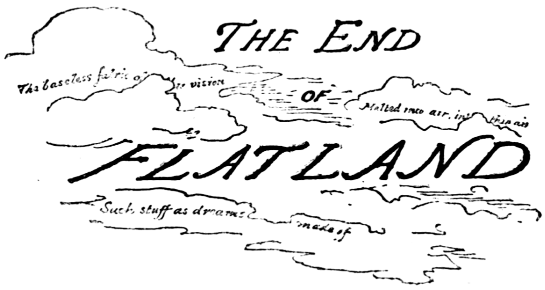
Maybe this is the reason why Abbott never wrote anything similar, so *Flatland* stayed unique among all of his writings.

Abbott believed that his most significant contributions to human culture would be some more serious writings. It is true that the broad issues, Abbott tackled in his literary and theological works, remain important, but the details have become as obsolete as the Victorian society, in which he lived, itself.

In these days it can be easily understood that Flatland has outlasted his other books because it is timeless. A twenty-first-century reader can identify himself with the poor A. Square, and with his lonely battle against mindless orthodoxy and social hypocrisy, as easily as a nineteenth-century one.

Of course, *Flatland* is not perfect. It includes some discrepancies. Hinton's story is more exactly and mathematically sophisticated. Wells's books probably have more engaging tale. Abbott was not a mathematician, so he did not have pretensions to a logically consistent scientific treatise. And unlike Wells, Abbott, as a teacher, probably did not intend to write something containing logical mistakes. His lifelong passion was education. Maybe, he felt some responsibility for his writing and for his ideas.

Flatland is a mathematical fantasy with many dimensions and every reader can find his. For the non-mathematical reader, there is the scientific popularization about the fourth dimension, for a mathematician, there is a satire on society.



The end

References

[1] Abbott, Edwin A. *Flatland: A Romance of Many Dimensions.* Second Edition, 1992. New York: Dover Publications.

[2] Abbott, Edwin A. *The Kernel and the Husk: Letters on Spiritual Christianity.* 1887, Boston : Roberts Bros.

[3] Abbott, Edwin A.; Stewart, Ian. *The Annotated Flatland: A Romance of Many Dimensions.* First Edition, 2002. Cambridge: Perseus Publishing.

[4] *The Architect*, November 15, 1884, pp. 326–327.

[5] Banchoff, Thomas F. From Flatland to Hypergraphics: Interacting with Higher Dimensions.*Interdisciplinary Science Reviews*, 01 December 1990, vol. 15, no. 4, pp. 364–372.

[6] Hinton, Charles H. *Scientific romances.* 1886, London : S. Sonnenschein & co., lim.

Paper III

T. Bártlová: *Martin Gardner's Mathemagical Life,* Recreational Mathematics Magazine, Issue 2, 2014, 21—40.

The preface to the article *Martin Gardner's Mathemagical Life*

In the article about history of recreational mathematics I am shortly describing the evolution of recreational mathematics. From the description two aspect of progress should be clear. First, the recreational mathematics has been a part of a mathematics since long time. And second, the line between entertaining mathematics and serious mathematics is a blurry one.

Martin Gardner collected mathematical problems whose formulation was understood to everyone and whose solution did not require knowledge of higher mathematics, and, especially, these problems were fun. This way he accomplished that many people found motivation in them, an interest in them, and a desire to solve them.

Many skeptics would say that it is fine, but it is nothing new. Gardner was not a mathematician and certainly did not aim to think up something new in mathematics. Additionally, he openly said that his ideas were not original. At the time when he started writing the column of recreational mathematics, some books on recreational mathematics were already in print. We can mention, for example, the classic of this genre – Mathematical Recreations and Essays, written by an English mathematician W. W. Rouse Ball in 1892 and others.

So why Gardner is so much extolled? Maybe because he chose the right approach and used a good psychology. The development of recreational mathematics is in fact dependent not so much on mathematicians, but on the general public. Non-mathematicians determine the direction and the pace at which the recreational mathematics will take. On the one hand it sounds a little paradoxical that non-mathematicians should determine mathematics. But on the other hand, we have to realize to which extent people enjoy specific fields of the recreational mathematics by seeing their response. And Gardner for sure listened to his readers.

The first of Gardner's strengths can be called amateurism in mathematics. He was not a professional mathematician. He was trying to understand himself first the problem which he solved and then he tried to retell it to his readers in the way which they could understand. He was absolutely right when he claimed that if he could not understand what he was writing about, then his readers would not either.

Another of his specialities was the form which he chose, namely that of the regular short columns. It is a growing problem to convince people to read a thick book, moreover if the mathematics is in the title. Gardner found a good compromise – every month he published just a short column. A number of people finally dared to read a short article. If the readers gained the impression that the problem intrigued after the reading, Gardner won. And so over time he learnt his readers, or rather trained on a regular dose of mathematics.

As a third aspect of his success I would classify interactivity. By publishing articles in magazines, the readers had always the possibility to write to the editor their opinions or requests for advice or help. Additionally, Gardner answered them all. Over the time Gardner also served as a mathematical counsel. The people knew that if they happened to come upon a problem, they could turn to him and he would help them.

All these aspects (and there certainly could be found others which I did not mention) resulted in a huge motivation of people to solve mathematical problems in Gardner's columns and thus contribute to the development of recreational mathematics and serious mathematics. For this fact, Gardner is rightfully considered to be an important person of the recreational mathematics.

ISSN 2182-1976

Recreational Mathematics Magazine

Martin Gardner's Mathemagical Life

Tereza Bártlová
Charles University
bartlova@karlin.mff.cuni.cz

Number 2
September, 2014

Ludus CIUHCT

Articles

Martin Gardner's Mathemagical Life

Tereza Bártlová
Charles University
bartlova@karlin.mff.cuni.cz

ON THE OCCASION OF THE 100TH ANNIVERSARY OF BIRTH

Abstract: *Martin Gardner has turned dozens of innocent youngsters into math professors and thousands of math professors into innocent youngsters.*

Persi Diaconis [32]

Key-words: Martin Gardner, 100th anniversary.

1 Introduction

There are a lot of people who devote a great deal of their lives to mathematics, but it is difficult to find those who have done more for the promotion of mathematics than Martin Gardner. Despite the fact that Gardner had not a formal mathematical education, his position in the world of mathematics is unique.

Being the author of the "Mathematical Games" column that ran for twenty-five years in Scientific American magazine, he opened the eyes of the general public to the beautiful and fascination of mathematics and inspired many to go on to make the subject their life's work. His column was the place where several important mathematical notions, such as Conway's Game of Life and Penrose tiles, first became widely known. It was also a place where the sheer fun of mathematical games and puzzles was celebrated and savored.

Allyn Jackson [26]

Martin Gardner had many lifelong passion and mathematics was one of them. In spite of, or perhaps because of, lacking proper mathematical education, Gardner's articles influenced generations of mathematicians. Thanks to his boundless enthusiasm and careful choice of topics, his articles got the general public interested in mathematics. Apart from mathematics he was an amateur magician, a well-known skeptic and also a leading figure in refuting pseudoscientific theories

Recreational Mathematics Magazine, Number 2, pp. 21–40

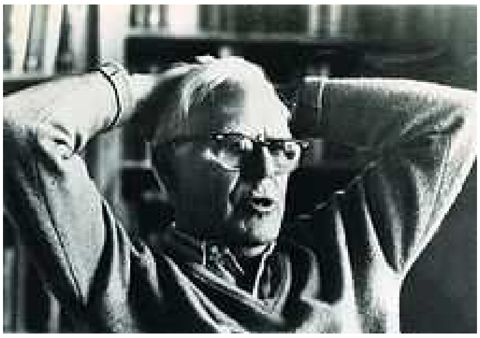


Figure 1: Martin Gardner.

ranging from modern diets to flying saucers. He showed great interest in religion, was a writer of fiction and poetry. He wrote more than 70 books concerning magic, philosophy, mathematics or commented on other authors' books.

2 Personal life and education

Martin Gardner was born October 21, 1914, in Tulsa, Oklahoma. His father was a geologist who owned a small oil company. His mother once worked as a kindergarten teacher, but after the delivery of her third child she decided to stay at home and became a housewife. From an early age Martin was fascinated by various puzzles, mathematical games, resolving paradoxes or mysterious stories. "My mother read the *Wizard of Oz* to me when I was a little boy, and I looked over her shoulder as she read it," he remembered from his childhood [2]. With his childhood friend John Bennett Shaw they collected different kinds of brain teasers. Later on, J. B. Shaw's extensive collection consisting of various brain teasers and Sherlock Holmes' mementos was awarded a prize[1].

Figure 2: Martin's brother Jim, Martin's father Dr. Gardner, and Martin.

[1] More detailed information about Gardner parents and his childhood is described in the article [2].

Martin Gardner loved physics in high school. He admired his physics teacher and his hopes were to become a physicist too. He applied for his dream job to Cal Tech[2]and found out that he was supposed to spend two years at university before he might be accepted. He wasn't discouraged and decided to study at the University of Chicago. During his studies Gardner got absolutely enthused by philosophy, especially philosophy of science. He chose to change his field of study and in 1936 obtained a bachelors degree in philosophy.

In his free time Gardner kept in touch with mathematics, unfortunately he couldn't take any maths courses at university because of his study plan. The main idea was to introduce the widest possible range of scientific disciplines to students in the first two years of their study. Optional subjects (maths in Gardner's case) could be chosen in the third year.

Gardner was always very keen on philosophy, but he already knew he would not make living as a philosopher [13]: "If you're a professional philosopher, there's no way to make any money except teach. It has no use anywhere." Gardner was sure he did not want to teach - partly this was due to his shyness but also because by then he knew really liked writing. He started occasionally to write some articles for various magazines but he didn't get paid. Then he worked shortly for the *Tulsa Tribune* as an assistant oil editor.

Before World War II he returned to Chicago to work in the public relation office of the *University of Chicago Office of Press Relation*, mainly writing science. During World War II he served for four years in the U. S. Navy. He spent about a year at Madison, Wisconsin, which bad a radio training school there [8]: "I handled public relations for the school, and edited a school newspaper," he described his work. The following three years he served as a yeoman on DE-134[3], a destroyed escort, in the Atlantic.

Figure 3: Martin Gardner in the Navy.

[2]Caltech or California Institute of Technology is private research university located in Pasadena, California, United States.

[3]USS Pope (DE-134) was an Edsall-class destroyer escort built for the U.S. Navy during World War II. She served in the Atlantic Ocean and provided destroyer escort protection against submarine and air attack for Navy wessels and convoys.

After World War II he went back to the University of Chicago, where he took some graduate courses and started to sell short stories to *Esquire Magazine*. His first story for magazine called "The Horse on the Escalator". It was a humorous and crazy story about a man who collected shaggy dog jokes[4] about horses. Shortly after his first story was published he wrote a second one named No-Sided Professor, in which Gardner explains the basis for topology with the help of the Möbius Strip. He made his living by writing for *Esquire Magazine* for about a year or two. Most stories are collected in the book called *The No-Sided Professor and Other Stories*[5].

In the early 1950s, Gardner moved to New York City and started to work for a children's magazine *Humpty Dumpty*. Not only did he write short stories and various columns, he was also in charge of inventing various paper-folding toys and cutouts for children. He was inspired by magazine called *John Martin's Book* where he found lots of interesting sources (activity features, where you cut things out of the page and fold them into different things, pictures that turn upsidedown, or you hold them up to the light and see through) [4]: "I grew up on this magazine." And these puzzles and folding paper toys had influence on his writing style for the magazine *Scientific American*, where he was employed from 1956 until 1981.

In 1979, Gardner moved with his wife Charlotte to Hendersonville in North Carolina. After her death in 2002, he decided to move to Norman, Oklahoma, where his son lived. In May 22, 2002 he died here.

Figure 4: Martin and with his wife.

[4]Shaggy dog joke is an extremely long-winded anecdote characterized by extensive narration of typically irrelevant incidents and terminated by an anticlimax or a pointless punchline.

[5]More detailed description we can find in the article [3].

3 Recreational mathematics and Scientific American

I dare say so, what Martin Gardner has done is of far greater originality than work that has won many people Nobel Prizes.

Douglas Hofstadter [22]

3.1 Recreational mathematics

Gardner always tried his best to give the general public insight into mathematics and mathematical research. He though that one of the reasons for unpopularity of mathematics lies in its isolation from the outside world [8]: "Well, that's a tough one, because almost all the really exciting research going on in mathematics is not the sort of thing that the public can understand. It takes considerable knowledge of mathematics to know what the breakthroughs are. And the really big breakthroughs that take place are just-it seems almost impossible to put it in terms that the general public can understand, whereas big breakthroughs in biology and so on are popularized, I think, fairly easily." In this connection he appreciated Sherman Stein's[6] job of popular articles on the subject, and books that the layman can read and understand. Gardner wished more similar mathematicians had written some popular articles and had introduced mathematical research to the public.

The method that Gardner chose to hook the interest in mathematics, was based on the attractiveness of the so-called *recreational mathematics* [8]: "...if they don't make mathematics to a certain degree fun to those first coming to it..." He believed that if mathematics teachers would use some problems from recreational mathematics in their lessons, the interest of their students could be attracted: "...the students are so bored that they get turned off by the topic, especially if the teachers are dull teachers." Gardner defined the term *recreational mathematics* in the very broad sense: "include anything that has a spirit of play about it".

Gardner was not only a "popularizer" of mathematics. American writer and cognitive scientists Douglas Hofstadter[7] considers Gardner's approach and his ways of combining ideas are truly unique and truly creative [22]: "In each column Martin managed to point out some little known but profound issue, and to present it in such a clear (and often humorous) fashion..."

[6]Sherman Stein (*1953) is a professor emeritus of mathematics at the University of California at Davis. He is the author of books *How the Other Half Thinks: Adventures in Mathematical Reasoning* (McGraw-Hill, 2002), *Strength in Numbers: Discovering the Joy and Power of Mathematics in Everyday Life* (Wiley, 1999) and *Mathematics: The Man-Made Universe* (Dover Publications, 1998) [49].

[7]Douglas Richard Hofstadter (*1945) is an American professor of cognitive scientists of wide interests. In his early career he focused on the logic, mathematics, computer science and other cognitive sciences. Later he got interested in interdisciplinary themes. He produced program in computer modeling of mental processes (he called "artificial intelligence research"). He was appointed adjunct professor of history and philosophy of science, philosophy, comparative literature, and psychology. He is best known for his book *Gödel, Escher, Bach: an Eternal Golden Braid*, first published in 1979. It won both the Pulitzer Prize for general non-fiction, in 1980 [16].

He put special emphasis also on the applications. He held the view that it is necessary to combine mathematics with its applications [8]: "...If the math can be applied somehow thats useful in the childs experience and things can be introduced so theyre challenging and have a play aspect..." With rapidly evolving technology and computers he saw success of popularization of mathematics in conjunction recreational maths with computer programming.

Martin Gardner didn't consider himself a "true mathematician". At the same time he believed that in this lies his advantage [8]: "If I can't understand what I'm writing about, why, my readers can't either." Maybe this is the reason why more people have probably learned more from him. He devoted a lifetime to work with mathematics and we could say that he kept himself busy with recreational mathematics in the U.S. for most of the twentieth century. However, he became really famous for his column entitled Mathematical Games in magazine *Scientific American*.

3.2 Scientific American and other recreational literature with mathematical topics

Everything began in 1952 when Gardner sent off an article on history of logic machines[8] to *Scientific American*. Editors of the magazine were so pleased with the article and they showed interest in an other contribution. So, in December 1956, Gardner published his article on hexaflexagons. A hexaflexagon is a paper model with hexapolygon shape, folded from straight strip of paper. One of the ways to construct such hexaflexagon is shown in the image below. Hexaflexagons have the fascinating property of changing their faces when they are "flexed". When we pinch two adjacent triangles together and push the opposite corner of the hexaflexagon toward to center, the model would open out again, a "budding flower" according to Gardner, and show a completely new face[9].

The article had overwhelmingly positive response from readers. A lot of people were folding hexaflexagon models, drew some motives on them and sent them back to the magazine or some of them were used in advertising. The magazine publisher Gerard Piel did not hesitate a minute to ask for a monthly column but also wanted to know if Martin thought there was enough material to warrant a monthly column. Although Gardner did not own any books on recreational mathematics, he knew that there are a big field out there [8]. His first column called *A new kind of magic square with remarkable properties*[10] appeared in the January 1957 issue and it was called Mathematical Games[11].

Since then the column appeared regularly every month exactly for a quarter of a century. It is remarkable that all this time Gardner was taking care of the articles on his own, as well as inventing new topics, gathering correspondence from his readers and answering to it. Gardner never wanted any assistants. He claimed that he had learned to type fast when being a yeoman in the Navy and

[8] Logic Machines 186, 68-73, Mar 1952

[9] A detailed description of the construction of hexaflexagon and its other curious property we can find, for example in the book [18].

[10] A new kind of magic square with remarkable properties Jan 1975 (169,1,-)

[11] Note that the initial letters of words in the title of the article are also Gardner's initials.

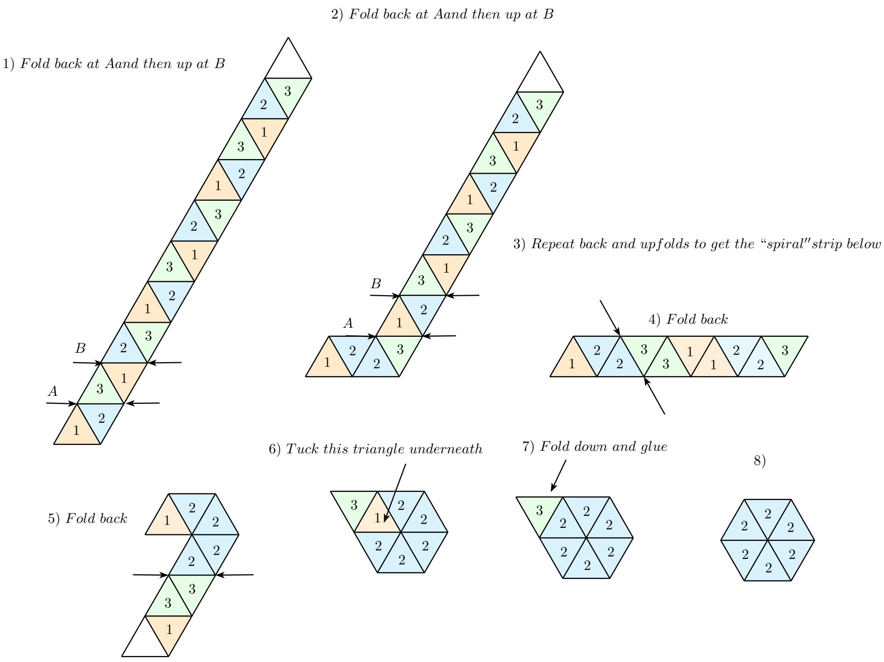


Figure 5: The construction of hexaflexagon

so that it was faster for him to type himself rather than to dictate anything. Only his wife Charlotte was allowed to help him. She proofread for him, checked the text for grammatical errors and spelling.

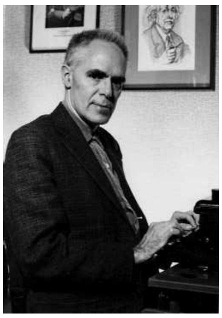


Figure 6: Martin Gardner during the writing

Gardner chose the topics of his column with great care [8]:"I try to pick a topic that is as different as possible from the last few topics; that's one of my criteria in choosing topics, so that I get a maximum variety from month to month," said Gardner. He kept gathering and piling possible future topics over the years. He drew inspiration from some books that came out in the recreational maths field, periodicals (he subscribed to about ten journals), and, of course, from a big correspondence with his readers who sent him ideas [8]: "Once the column became popular and the people interested in recreational math started reading it, why, they started writing to me. And then if I replied on my own stationery, why then they could write to me directly, and not have to go through Scientific American. So about half the correspondence I get comes through the magazine and about half I get directly." Among his column correspondents were several distinguished mathematicians and scientists as John Horton Conway[12], Persi Diaconis[13], Ron Graham[14], Douglas Hofstadter, Richard Guy[15], Donald Knuth[16], Sol Golomb[17] and Roger Penrose[18].

Gardner particularly enjoyed writing columns where philosophical things were interfering into mathematical issues and the other way round, for example a marvelous paradox called *Newcomb's paradox.* It is a thought experiment (a game) between two players, when one of them claims that he is able to predict the future [19]:

Two closed boxes, B1 and B2, are on a table. B1 contains $1 000. B2 contains either nothing or $1 million. You do not know which. You have an irrevocable choice between two actions:

1. Take what is in both boxes.

2. Take only what is in B2.

At some time before the test a superior Being has made a prediction about what

[12]John Horton Conway (*1937) is a British mathematician active in the theory of finite groups, knot theory, number theory, combinatorial game theory and coding theory. He has also contributed to many branches of recreational mathematics, notably the invention of the cellular automaton called the *Game of Life* [27].

[13]Persi Diaconis (*1945) is an American mathematician and former professional magician. He is the Mary V. Sunseri Professor of Statistics and Mathematics at Stanford University. He is particularly known for tackling mathematical problems involving randomness and randomization, such as coin flipping and shuffling playing cards [36].

[14]Ronald Graham (*1935) is an American mathematician credited by the *American Mathematical Society* as being "one of the principal architects of the rapid development worldwide of discrete mathematics in recent years". He has done important work in scheduling theory, computational geometry and Ramsey theory [41].

[15]Richard Guy (*1916) is a British mathematician, Professor Emeritus in the Department of Mathematics at the University of Calgary. He is best known for co-authorship of *Winning Ways for your Mathematical Plays* and authorship of *Unsolved Problems in Number Theory*, but he has also published over 100 papers and books covering combinatorial game theory, number theory and graph theory [1].

[16]Donald Knuth (*1938) is an American computer scientist, mathematician, and Professor Emeritus at Stanford University. He is the author of the multi-volume work *The Art of Computer Programming*. Knuth has been called the "father" of the analysis of algorithms [15].

[17]Solomon Golomb (*1932) is an American mathematician, engineer and a professor of electrical engineering at the University of Southern California. He is best known for his works on mathematical games. Most notably he invented *Cheskers* or *Polyominoes* and *Pentominoes*, which were the inspiration for the computer game *Tetris*. He has specialized in problems of combinatorial analysis, number theory, coding theory and communications [47].

[18]Roger Penrose (*1931) is an English mathematical physicist, recreational mathematician and philosopher. He is the Emeritus Rouse Ball Professor of Mathematics at the Mathematical Institute of the University of Oxford, as well as an Emeritus Fellow of Wadham College. He is best known for his scientific work in mathematical physics, in particular for his contributions to general relativity and cosmology [40].

you will decide. If the Being expects you to choose both boxes, he has left B2 empty. If he expects you to take only B2, he has put $1 million in it. If he expects you to randomize your choice by, say, flipping a coin, he has left B2 empty. In all cases B1 contains $1 000. What should you do?

Due to the popularity of Gardner's columns many of these articles have been collected in a book *The Scientific American Book of Mathematical Puzzles and Diversions*, published in 1959. Over the next forty years, he published another fourteen books[19].

In 1981, Gardner handed his coulmn over to Douglas Hofstadter. Gardner had so many other writing interests in these days that he felt he could no longer maintain the column. Hofstadter was looking up to Gardner. On one hand he feared being put in Martin Gardner's shoes, but on the other hand he understood if he hadn't taken the chance he would have regretted it later. He didn't want the readers to expect him to copy Gardner's style so he decided to rename the column. So Metamagical Themas (anagram of the earlier title) came to existence.

Hofstadter managed to publish his column regularly for almost three years. But in 1983 he was swamped with work to such an extent, it became clear that he would be unable to continue producing columns at a monthly pace. And so a Canadian mathematician Kee Dewdney[20] took over Hofstadter. The change of the author meant the change of title again. So this time Computer Recreations first saw the light.

In September 1987, a Scottish mathematician Ian Stewart[21] got the opportunity to contribute to this column. Stewart also gratefully accepted the chance of writing articles for *Scientific American*. Although he had never met Martin Gardner, Stewart admitted that was a regular and faithful reader of Gardner's column, since he was sixteen years old [48]: "Every column contained something new to attract my attention, and it was mathematical, and it was fun. There was plenty of room for new ideas and creative thinking. It is probably fair to say that Martin Gardners column was one of the reasons I ended up becoming a mathematician."

In December 1990, there was another change of the column's title. It was renamed the Mathematical Recreations and a few months later Ian Stewart became officially its author. Stewart had problem with choice of the topics. He thought that Martin Gardner had already used loads of interesting themes. He identified himself with Gardner's point of view [43]: "The way to explain math to nonspecialists is to understand it thoroughly yourself, to strip away needless technicalities, and to focus on the central story." And exactly this principle he tried to follow.

[19]Most Martin Gardner bibliography we can find on the Web side [31].

[20]Alexander Keewatin Dewdney (*1941) is a Canadian mathematician, computer scientist and author who has written a number of books on mathematics, computing, and bad science [6].

[21]Ian Nicholas Stewart (*1945) is a professor of mathematics at the University of Warwick, England, and a widely known popular-science and science-fiction writer [24].

The last one to finish the famous columns was an American mathematician Dennis Shasha[22]. He started to work on the column, which changed its name for the last time in early 2001. The Puzzling Adventures columns were published in print until May 2004, and since the following month the columns were accessible only on the website of the magazine. The very last article was published in June 2009, and then column came to an end.

From 1977 until 1986 Gardner also was contributive to the magazine *Asimov's Science Fiction*. His column was focused primarily on "puzzle tale".

Retirement did not stop Gardner from working. He only focused more on writing scientific literature and updating his older books such as *Origami, Eleuis and the Soma Cube*.

3.3 Gathering for Gardner

Despite the fact that Gardner was very popular among people, he was known for his shy personality. He refused to receive several awards just because he would have to take part in the public ceremony [17]: "I hate going to parties or giving speeches. I love monotony. Nothing pleases me more than to be alone in a room, reading a book or hitting typewriter keys." Once he told Colm Mulcahy [30]: "...I have never given any lecture in my life and most probably I wouldn't know how to do it."

However, in 1993, Atlanta puzzle collector Tom Rodgers persuaded Gardner to attend a special evening occasion devoted to Gardner's puzzle-solving efforts. The event met with roaring success and was repeated in 1996, again with Gardner's presence. No wonder that Rodgers and his friends decided to organize the gathering on regular basis. Since then it has been held every other year (even-numbered) in Atlanta, and the programme consists of any topic which is concerning Gardner and his writing career in any way. The event is named *Gathering for Gardner*, in short G4Gn, when n stands for the number of the event (the 2010 event thus was G4G9) [23]. Gardner attended the 1993 and 1996 events.

4 Pseudoscience

Even when a pseudoscientific theory is completely worthless there is a certain educational value in refuting it.

Martin Gardner [17]

Despite his introverted nature Gardner was considered to be one of the leading polemics against pseudoscientific and fringescientific theories, astounding

[22]Dennis Shasha is a professor of computer science at the Courant Institute of Mathematical Sciences, a division of New York University. He does research in biological computing (including experimental design), pattern recognition and querying in trees and graphs, pattern discovery in time series, cryptographic file systems, database tuning, and wireless [14].

[23]Detailed event program we can find, for example in the article [12].

discoveries, the paranormal and everything what became later known as *pseudoscience.*

In his articles, he tried to put all these misleading and confusing information appearing in the media straight. He was irritated by boundless human gullibility. He warned scientists that at the time they do not write any popular articles attacking pseudoscience and do not acquaint the general public with scientific discoveries, there is space for pseudoscientists to popularize their dubious discoveries and inventions and it may easily happen that the general public would consider pseudoscience a real science. He believed that if he explained everything rationally, he would be able to influence people's opinions and also mitigate the damage caused by pseudoscientists. "Bad science contributes to the steady dumbing down of our nation", declared Gardner [17].

For many years he tirelessly researched and studied different pseudoinventions and pseudofacts from a scientific view point, and wrote various articles concerning these topics. The first article which had the scent of distrustful spirit and reacted negatively to the results of pseudoscience was called The Hermit Scientist and was published in 1950 in the journal *Antioch Review.* This article wasn't definitely the last one and only two years later he published his first book dealing with these issues entitled *In the Name of Science.* It was a skeptical book by its nature - it explored myriads of dubious outlooks and projects including modern diet, fletcherism[24], creationism[25], Charles Fort[26], Rudolf Steiner[27], scientology[28], dianetics[29], UFOs, dowsing[30], extra-sensory perception[31] and psychokinesis[32]. Not only this book but many others, for example *Science: Good,*

[24]Fletcherism is a kind of special diets named after Horace Fletcher (1849–1919). The basic tenets of diet were these: one should eat only when genuinely hungry and never when anxious, depressed, or otherwise preoccupied; one may eat any food that appeals to the appetite; one should chew each mouthful of food 32 times or, ideally, until the food liquefies (see [23]).

[25]the belief that the universe and living organisms originate from specific acts of divine creation, as in the biblical account, rather than by natural processes such as evolution [28].

[26]Charles Fort (1874–1932) was an American writer and researcher into anomalous phenomena [10].

[27]Rudolf Joseph Lorenz Steiner (1861–1925) was an Austrian philosopher, social reformer, architect, and esotericist. Steiner gained initial recognition as a literary critic and cultural philosopher. At the beginning of the twentieth century, he founded a spiritual movement, anthroposophy (see [42]). Anthroposophy is a human oriented spiritual philosophy that reflects and speaks to the basic deep spiritual questions of humanity, to our basic artistic needs, to the need to relate to the world out of a scientific attitude of mind, and to the need to develop a relation to the world in complete freedom and based on completely individual judgments and decisions [7].

[28]Scientology a religious system based on the seeking of self-knowledge and spiritual fulfilment through graded courses of study and training. It was founded by American science fiction writer L. Ron Hubbard (191186) in 1955 (see [44]).

[29]Dianetics is a system developed by the founder of the Church of Scientology, L. Ron Hubbard, which aims to relieve psychosomatic disorder by cleansing the mind of harmful mental images. [45].

[30]Dowsing a technique for searching for underground water, minerals, ley lines, or anything invisible, by observing the motion of a pointer (traditionally a forked stick, now often paired bent wires) or the changes in direction of a pendulum, supposedly in response to unseen influences [38].

[31]Extrasensory perception (ESP) involves reception of information not gained through the recognized physical senses but sensed with the mind [34].

[32]Psychokinesis or telekinesis is an alleged psychic ability allowing a person to influence a physical system without physical interaction [39].

Bad and Bogus, (1981); *Order and Surprise*, (1983), *Gardner's Whys & Wherefores*, (1989), caused that number of fierce opponents and critics arose in the fields of fringe science and New Age philosophy, with many of them he kept up in touch (both publicly and privately) for decades.

Another reason for Gardner's uncompromising attitude towards pseudoscience it was its impact on the real science. It often happened that pseudoscientists used some serious scientific discovery, which they interpreted erroneously and applied it as the basis for their pseudoresearch. In the worst case, it became the very opposite - scientists did not recognize a pseudodiscovery was not based on the real facts and they took it seriously. In many cases, they made fools out of themselves.

Gardner wanted to prevent these situations, and so in 1976, he was a founding member of the *Committee for the Scientific Investigation of Claims of the Paranormal*, in short CSICOP[33]. It is to serve as a sort of neutral observer that examines various psychic phenomena from a scientific point of view. From 1983 until 2002 Gardner wrote a column called Notes of a Fringe Watcher (originally Notes of a Psi-Watcher) in magazine *Skeptical Inquirer* (originally *Zeletic*). All the articles were later collectively published in several books. Especially in his old age, Gardner was an excellent sceptic about paranormal phenomena. In August, 2010, Gardner´s contributions in the skeptical field earned him, in memoriam, an award rom the *Independent Investigations Group* on its 10th Gala Anniversary.

5 Religion

During his life Gardner found lifelong fascination for religion. As a youngster he was influenced by a Sunday school teacher and the Seventh-day Adventist Church. Young Gardner became convinced that the Second Coming of Jesus was close [52]:

"I grew up believing that the Bible was a revelation straight from God," he recounted. He had lived in this belief before he began studying at university and met some other points of view on Christianity and religion. University life and some ideas of authors whose books Gardner read slowly weakened his fundamental beliefs. Among these authors belonged e.g. Platon[34], Immanuel Kant[35],

[33]The mission of the Committee for Skeptical Inquiry is to promote scientific inquiry, critical investigation, and the use of reason in examining controversial and extraordinary claims. More detailed information we can find on the Web site www.csicop.org.

[34]Platon (427 BC – 347 BC) was a philosopher in Classical Greece. He is one of the most important founding figures in Western philosophy. Plato's sophistication as a writer is evident in his Socratic dialogues, his dialogues have been used to teach a range of subjects, including philosophy, logic, ethics, rhetoric, religion and mathematics [37].

[35]Immanuel Kant (1724–1804) was a German philosopher who is widely considered to be a central figure of modern philosophy. He argued that human concepts and categories structure our view of the world and its laws, and that reason is the source of morality [25].

Gilbert Keith Chesterton[36], William James[37], Charles Sanders Pierce[38], Rudolf Carnap[39] and Herbert George Wells[40]. Gardner tried to catch pearls of wisdom from every single one of them [52]: "From Chesterton I got a sense of mystery in the universe...", he explained. "From Wells I took his tremendous interest in and respect for science. That's why I do not accept the virgin birth of Christ or a blood atonement for the sin of Adam and Eve." Gardner was also inspired by the theology of a Spanish philosopher Miguel de Unamuno[41] According to Unamuno belief in God and the desire for immortality were as important as any scientific and rational view of the world. He claimed that one feels the need for faith in God and at the same time he yearns for recognition of his personality as an individual [33]: "the most tragic problem of philosophy is to combine the intellectual with the emotional needs, and also with free will."

With highest respect to all religious convictions Gardner described his own belief as *philosophical theism* [9]: "I am a philosophical theist. I believe in a personal god, and I believe in an afterlife, and I believe in prayer, but I dont believe in any established religion."

Gardner professed his faith in God as creator, but criticized and rejected everything which was beyond human understanding: God's revelation, prophecy, miracles, the authority of the Church. Despite all the criticism of the Church he believed in God and asserted that this belief cannot be confirmed or denied by science. At the same time, he was sceptical about claims that God has communicated with human beings through spoken or telepathic revelation or through miracles in the natural world [30]: "There is nothing supernatural, and nothing in human reason or visible in the world to compel people to believe in any gods. The mystery of existence is enchanting, but a belief in *The Old One* comes from faith without evidence. However, with faith and prayer people can find greater happiness than without."

Gardner often compared parapsychology with religion in his comments, and

[36] Gilbert Keith Chesterton (1874–1936) was an English writer,lay theologian, poet, dramatist, journalist, orator, literary and art critic, biographer, and Christian apologist [20].

[37] William James (1842–1910) was an American philosopher and psychologist who was also trained as a physician. James was one of the leading thinkers of the late nineteenth century and is believed by many to be one of the most influential philosophers the United States has ever produced, while others have labelled him the "Father of American psychology". He is considered to be one of the greatest figures associated with the philosophical school known as pragmatism [51].

[38] Charles Sanders Peirce (1839–1914) was an American philosopher, logician, mathematician, and scientist, sometimes known as "the father of pragmatism". He is appreciated largely for his contributions to logic, mathematics, philosophy, scientific methodology, and semiotics, and for his founding of pragmatism [11].

[39] Rudolf Carnap (1891–1970) was a German philosopher, mathematician and logician. He made significant contributions to philosophy of science, philosophy of language, the theory of probability, inductive logic and modal logic [35].

[40] Herbert George Wells (1866–1946) was an English writer, now best known for his work in the science fiction genre (see [21]).

[41] Miguel de Unamuno (1864–1936) is a Spanish writer, philosopher and one of the main leaders of the Group Generation 98. The central theme of his essays and poetry is faith. He touched topics such as finding personal spirituality, mental anguish, time, death, pain caused by confidentiality God and others. Second quote is from his the most famous book *Del sentimiento trágico de la vida*, published in 1913. Other information we can find, for example on the Web side [33].

claimed that he considered parapsychology and other researches on paranormal phenomena completely the same as "God temptation" and "looking for signs and wonders".

His attitude towards religion is best explained and described in his novel with autobilgraphical features *The Flight of Peter Fromm*[42] of 1973. This novel is not purely autobiographical, because Gardner does not identify himself with Peter, the main character. Nevertheless, the main character goes through the same changes of his own faith as Gardner which makes the book partly autobiographical.

6 Magic

Apart from brain teasers and puzzles Gardner expressed his interest in magic from his early age. Magic was hobby of his father who showed him some magic tricks (see [2]): "I learned my first tricks from him, in particular one with knife and little pieces of paper on it..." Gardner never made a living by magic. He only got paid once for doing magic at the occasion of presenting Gilbert´s magic set at the Marshall Field department store[43].

It is no surprise that Gardner prefered tricks with a touch of maths, particularly those that are breaking topological laws [2]: "The most important thing is to startle people, and have them wonder how it's done." Close-up magic is very different from the stage illusion that David Copperfield does. In close-up magic or micromagic hold true that "hands must be always quicker than eye".

It is not surprising that Gardner prefered tricks with a mathematical flavor and especially those that they are violating topological laws [2]: "In recent years magicians have gotten interested in rubben band tricks that are all topologically based... I did a book for Dover Publications on mathematical tricks that has a chapter on topological tricks."

Gardner wrote two voluminous books for magicians: *The Encyclopedia of Impromptu Magic* and *Martin Gardner Presents*. Both books have about five hundred pages where original tricks with cards, matches, dices, coins or mental magic tricks can be found.

[42]Character of novel is better explained in the article [8] and [3].

[43]Alfred Carlton Gilbert (1884–1961) was an American scientist, inventor, illusionist, athlete and scholar. Gilbert turned to magic by his studies at Yale. He began performing illusions on street corners and in shop windows. At these performances, Gilbert sold tricks and magic kits to his audience. He developed a local following and the Mysto Manufacturing Company became interested in publishing his magic toys. In 1990, Gilbert collaborated with the New Haven-based company to produce fis first toy, the Mysto Magic Set [46].

7 Literature

Gardner was considered a leading expert on Lewise Carroll[44]. They both shared love for mathematics, puzzles, formal logic and conjuring. Carroll was delighted to do simple magic tricks for his little audience and often took children to magic performances or wrote books for children. Among his best-known books belong *Alice's Adventures in Wonderland* and *Through the Looking Glass*. Although it might seem that it is just a fairy tale, in fact, both Alice books are full of logical and mathematical tricks and wordplays. Gardner admitted that he appreciated the depth of Carroll's stories when he was a grown-up [5]: "I did not discover the richness of this kind of humor in the Alice books until I was in my twenties, but since then I have felt a close kinship with Carroll."

In 1960, Martin Gardner published his annotated version of both Alice stories. Garnder revealed and explained all the mathematical riddles, wordplays, and literary references hidden in the Alice books. Later Gardner published a follow-up book with new annotations called *More Annotated Alice* and in 1999, the last edition combining the notes from earlier editions and new pieces of knowledge *The Annotated Alice: The Definitive Edition* was released.

Over the years Gardner's annotated Alice book has become a best seller [8]: "I was lucky there in that I really didn't have anything new to say much in The Annotated Alice, because I just looked over the literature and pulled together everything in the form of footnotes in the book. But it was a lucky idea because thats been the best seller of all my books." In following years more editions of the book appeared and it was translated into many languages.

Figure 7: Martin Gardner with Alice in Central Park

[44]Charles Lutwidge Dodgson (1832–1898), better known by his pen name, Lewis Carroll, was an English writer, mathematician, logician, Anglican deacon and photographer. His most famous writing are *Alice's Adventures in Wonderland* and its sequel *Through the Looking Glass and What Alice Found There*), as well as the poems "The Hunting of the Snark", all examples of the genre of literary nonsense [29].

From childhood Gardner fell in love with the books by Lyman Frank Baum about *The Wizard of Oz.* He wrote several forewords in additional issues of Baum's books and in 1998 Gardner published his own book called *Visitors From Oz.* Although Gardner's Visitors from Oz is an imitation of Baum's Wizzard of Oz book, again, he added some mathematics into it. Gardner made use of Klein's bottle, which appears throughout the story, as a magical feature for transition between parallel worlds[45].

Apart from Alice books Gardner published annotated edition of the books by Gilbert Keith Chesterton *The Innocence Of Father Brown* and *The Man Who Was Thursday.* He also commented on famous poems e.g. *The Rime of the Ancient Mariner, Casey at the Bat*, *The Night Before Christmas* and *The Hunting of the Snark* too.

Over the years he was expressing his concerns with many present-day problems. In 1993 he described his philosophical opinions and attitudes in his book *The Whys of a Philosophical Scrivener* [5]: "It is my favorite because it is a detailed account of everything I believe... Well, the book is controversial because almost everybody who believes in a personal god is into an established religion." Later Gardner harshly panned his own book in a review written under the pseudonym George Groth for *New York Review of Book* [52]: "I heard that people read the review and didn't buy the book on my recommendation."

After Martin Gardner's death his autobiographical book under title *Undiluted Hocus-Pocus* was released. The book was meant as a present for his fans. There are no dramatic revelations to be found, it only summarizes the story of his life, ideas and beliefs.

8 Closing

Martin Gardner was a man of wide interests. He was passionate about all types of paradox and revealing secrets. His columns and writings are unique considering constant novelty of human thoughts. He managed to get freed from expected patterns of thoughts, broke seemingy-solid laws, and discovered unexpected connections and revelations.

He died at the age of 95 and there is no doubt that he attracted many people of all ages to recreational mathematics during his lifetime. His infectious enthusiasm and brilliant choice of topics are unrivalled. There were plenty of those who tried to emulate him, but nobody has succeeded. American mathematics Douglas Hofstadter paid Martin Gardner a compliment saying [13]: "... is one of the great intellects produced in this country in the 20th century".

References

[1] *About: Richard K. Guy* [online], [cit. 2013-12-11]. Available from World Wide Web: `http://dbpedia.org/page/Richard_K._Guy`

[45] The story is described in the article [3].

[2] Albers, Don. *The Martin Gardner Interview* [online]. c2008 [cit. 2013-8-10]. Available from World Wide Web: http://www.cambridgeblog.org/2008/09/the-martin-gardner-interview/

[3] Albers, Don. *The Martin Gardner Interview Part 2* [online]. c2008 [cit. 2013-8-10]. Available from World Wide Web: http://www.cambridgeblog.org/2008/09/the-martin-gardner-interview-part-2/

[4] Albers, Don. *The Martin Gardner Interview Part 2* [online]. c2008 [cit. 2013-8-10]. Available from World Wide Web: http://www.cambridgeblog.org/2008/09/the-martin-gardner-interview-part-3/

[5] Albers, Don. *The Martin Gardner Interview - Part 5* [online]. c2008 [cit. 2013-8-10]. Available from World Wide Web: http://www.cambridgeblog.org/2008/10/the-martin-gardner-interview-part-5/

[6] *Alexander Dewdney* [online], last modified on 20. October 2013 [cit. 2013-12-15]. Available from World Wide Web: http://cs.wikipedia.org/wiki/Alexander_Dewdney

[7] *What is Anthroposophy?* [online], [cit. 2013-12-26]. Available from World Wide Web: http://www.waldorfanswers.com/Anthroposophy.htm

[8] Barcellos, Anthony; Gardner, Martin. "A Conversation with Martin Gardner", *The Two-Year College Mathematics Journal*, vol. 10, no. 4, p. 233-244, September 1979.

[9] Carpenter, Alexander. *Martin Gardner on Philosophical Theism, Adventists and Price* [online], c2008 [cit. 2013-12-25]. Available from World Wide Web: http://spectrummagazine.org/node/1091

[10] *Charles Fort* [online], last modified on 29. October 2013 [cit. 2013-10-30]. Available from World Wide Web: http://en.wikipedia.org/wiki/Charles_Fort

[11] *Charles Sanders Peirce* [online], last modified on 27. September 2013 [cit. 2013-12-15]. Available from World Wide Web: http://cs.wikipedia.org/wiki/Charles_Sanders_Peirce

[12] Crease, Robert P. *Gathering for Gardner* [online]. c2010 [cit. 2013-06-08]. Available from World Wide Web: http://online.wsj.com/news/articles/SB10001424052702304370304575151970094262604

[13] *Deceased–Martin Gardner.* Philosophy of Science Portal: A Venue for Discussions of Science, Philosophy and the Arts [online]. c2010 [cit. 2013-10-29]. Available from World Wide Web: http://philosophyofscienceportal.blogspot.cz/2010/05/deceased-martin-gardner.html

[14] Shasha, Dennis. *Dennis Shasha* [online], [cit. 2013-12-16]. Available from World Wide Web: http://cs.nyu.edu/shasha/

[15] *Donald Knuth* [online], last modified on 7. December 2013 [cit. 2013-12-10]. Available from World Wide Web: http://en.wikipedia.org/wiki/Donald_Knuth

[16] *Douglas Hofstadter* [online], last modified on 20. December 2013 [cit. 2013-12-24]. Available from World Wide Web: http://en.wikipedia.org/wiki/Douglas_Hofstadter

[17] Frazier, Kendrick. *A Martin Gardner Sampler* [online]. c2010 [cit. 2013-2-10]. Available from World Wide Web: http://www.csicop.org/si/show/a_martin_gardner_sampler

[18] Gardner, Martin. *Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi*, First published, 2008. New York: Cambridge University Press, p. 1-15. ISBN 978-0-521-75615-0, 2008.

[19] Gardner, Martin. *The Colosall Book of Mathematics: Classic Puzzles, Paradoxes, and Problems: Number Theory, Algebra, Geometry, Probability, Topology, Game Theory, Infinity, and Other Topics of Recreational Mathematics*, First Edition, 2001, New York: W. W. Norton & Company Ltd., p. 580-591. ISBN 0-393-02023-1, 2001.

[20] *Gilbert Keith Chesterton* [online], last modified on 24. December 2013 [cit. 2013-12-15]. Available from World Wide Web: http://en.wikipedia.org/wiki/G._K._Chesterton

[21] *Herbert George Wells* [online], last modified on 13. December 2013 [cit. 2013-12-25]. Available from World Wide Web: http://en.wikipedia.org/wiki/H._G._Wells

[22] Hofstadter, Douglas. *Martin Gardner: A Major Shaping Force in My Life* [online]. c2010 [cit. 2013-12-06]. Available from World Wide Web: http://http://www.scientificamerican.com/article.cfm?id=martin-gardner-hofstadter

[23] *Fletcherism* [online], c2013 [cit. 2013-12-26]. Available from World Wide Web: http://www.merriam-webster.com/medical/fletcherism

[24] *Ian Stewart* [online], last modified on 16. December 2013 [cit. 2013-12-16]. Available from World Wide Web: http://en.wikipedia.org/wiki/Ian_Stewart_(mathematician)

[25] *Immanuel Kant* [online], last modified on 24. December 2013 [cit. 2013-12-25]. Available from World Wide Web: http://en.wikipedia.org/wiki/Immanuel_Kant

[26] Jackson, Allyn. Interview with Martin Gardner, *Notices of the American Mathematical society*, vol. 52, no. 6, p. 602-611, June/July 2005.

[27] *John Horton Conway* [online], last modified on 6. December 2013 [cit. 2013-12-11]. Available from World Wide Web: http://en.wikipedia.org/wiki/John_Horton_Conway

[28] *creationism: definition of creationism in Oxfort dictionary* [online], c2013 [cit. 2013-12-16]. Available from World Wide Web: http://www.oxforddictionaries.com/definition/english/creationism?q=creationism

[29] *Lewis Carroll* [online], last modified on 15. December 2013 [cit. 2013-12-25]. Available from World Wide Web: http://en.wikipedia.org/wiki/Lewis_Carroll

[30] *Martin Gardner* [online], last modified on 19. December 2013 [cit. 2013-12-26]. Available from World Wide Web: http://en.wikipedia.org/wiki/Martin_Gardner

[31] *Martin Gardner bibliography* [online], last modified on 30 July 2013 [cit. 2013-12-10]. Available from World Wide Web: http://en.wikipedia.org/wiki/Martin_Gardner_bibliography

[32] Memories of Martin Gardner. *Notices of the American Mathematical society*, vol. 58, no. 3, p. 418-422, March 2011.

[33] *Miguel de Unamuno y Jugo: ensayista, dramaturgo, novelista, poeta, pensador, 1864-1936* [online], [cit. 2013-12-11]. Available from World Wide Web: http://www.swarthmore.edu/Humanities/mguardi1/espanol_11/unamuno.htm

[34] *Extrasensory perception* [online], last modified on 17. December 2013 [cit. 2013-12-26]. Available from World Wide Web: http://en.wikipedia.org/wiki/Extrasensory_perception

[35] Murzi, Mauro. *Rudolf Carnap* [online], [cit. 2013-12-25]. Available from World Wide Web: http://www.iep.utm.edu/carnap/

[36] *Persi Diaconis* [online], last modified on 15. September 2013 [cit. 2013-12-11]. Available from World Wide Web: http://en.wikipedia.org/wiki/Persi_Diaconis

[37] *Platon* [online], last modified on 24. December 2013 [cit. 2013-12-25]. Available from World Wide Web: http://en.wikipedia.org/wiki/Plato

[38] *dowsing: definition of dowsing in Oxford dictionary* [online], c2013 [cit. 2013-12-25]. Available from World Wide Web: http://www.oxforddictionaries.com/definition/english/dowsing?q=dowsing

[39] *Psychokinesis* [online], last modified on 13. December 2013 [cit. 2013-12-26]. Available from World Wide Web: http://en.wikipedia.org/wiki/Psychokinesis

[40] *Roger Penrose* [online], last modified on 11. December 2013 [cit. 2013-12-12]. Available from World Wide Web: http://en.wikipedia.org/wiki/Roger_Penrose

[41] *Ronald Graham* [online], last modified on 20. December 2013 [cit. 2013-12-24]. Available from World Wide Web: http://en.wikipedia.org/wiki/Ronald_Graham

[42] *Rudolf Steiner* [online], last modified on 29. October 2013 [cit. 2013-09-31]. Available from World Wide Web: http://en.wikipedia.org/wiki/Rudolf_Steiner

[43] *Scholars and Others Pay Tribute to "Mathematical Games" Columnist Martin Gardner* [online], last modified on 24. May 2010 [cit. 2013-12-16]. Available from World Wide Web: http://www.scientificamerican.com/article.cfm?id=scholars-and-others-pay-t

[44] *Scientology: definition of Scientology in Oxford dictionary* [online], c2013 [cit. 2013-12-26]. Available from World Wide Web: http://www.oxforddictionaries.com/definition/english/Scientology?q=Scientology

[45] *Dianetics: definition of Dianetics in Oxford dictionary* [online], c2013 [cit. 2013-12-26]. Available from World Wide Web: http://www.oxforddictionaries.com/definition/english/Dianetics

[46] Scott, Sharon M, *Toys and American Culture: An Encyclopedia*, ABC-CLIO, c2010, p. 130. ISBN 0-31-335111-2, 2010.

[47] *Solomon W. Golomb* [online], last modified on 2. December 2013 [cit. 2013-12-09]. Available from World Wide Web: http://en.wikipedia.org/wiki/Solomon_W._Golomb

[48] Stewart, Ian. *Math Hysteria: Fun and games with mathematics*, Oxford University Press, p. vii-xi. ISBN 0-19-164745-4, 2004.

[49] *Survival Guide for Outsiders: About the Author* [online], [cit. 2013-12-22]. Available from World Wide Web: http://www.shermanstein.com/about-the-author/

[50] *UFO: definition of UFO in Oxford dictionary* [online], c2013 [cit. 2013-12-26]. Available from World Wide Web: http://www.oxforddictionaries.com/definition/english/UFO?q=ufo

[51] *William James* [online], last modified on 20. December 2013 [cit. 2013-12-24]. Available from World Wide Web: http://en.wikipedia.org/wiki/William_James

[52] Yam, Philip. *Profile: Martin Gardner, the Mathematical Gamester (1914-2010).* Scientific American [online]. c2010 [cit. 2013-10-30]. Available from World Wide Web: http://www.scientificamerican.com/article.cfm?id=profile-of-martin-gardner

Paper IV

T. Bártlová: *Where are (pseudo)science fool's hoax articles in April from?*, Manuscript submitted, 2016.

The preface to the article *Where are (pseudo)science fool's hoax articles in April from?*

In the article we can find some April Fool's hoaxes, which amused or sometimes frightened many of people. In today's perspective this article also reflects the enormous influence of the media on our views even when it comes to unquestioned mathematics.

The aim of the article is to show that sometimes we have to take authors of the articles with a little humor and mainly rely on our own wits. We can forget that what was true in mathematics today, will be true also tomorrow.

Where are (pseudo)science fool's hoax articles in April from?

Tereza Bártlová, Prague *

1 Introduction

April Fool's Day, sometimes called All Foll's Day, is one of the most light-hearted days of the year. April 1 has long been celebrated as a day to celebrate, well, foolishness to be exact. More specifically, April Fools' Day is about making other people look stupid with practical jokes.

The origin of April Fools' Day is uncertain. Somebody considers it as a celebration related to the turn of the seasons. Ancient cultures, including those of the Romans and Hindus, celebrated New Year's Day on or around April 1. It closely follows the vernal equinox (March 20 or March 21). In medieval times, much of Europe celebrated March 25, the Feast of Annunciation, as the beginning of the new year.

Others believe it stems from the adoption of a new calendar. In 1582, Pope Gregory XIII ordered a new calendar (the Gregorian Calendar) to replace the old Julian Calendar. The new calendar called for New Year's Day to be celebrated January 1. That year, France adopted the reformed calendar and shifted New Year's day to January 1. According to a popular explanation, many people either refused to accept the new date, or did not learn about it, and continued to celebrate New Year's Day on April 1. Other people began to make fun of these traditionalists, sending them on "fool's errands"or trying to trick them into believing something false. Eventually, the practice spread throughout Europe. But we have no direct historical evidence for this explanation, only conjecture.

It is worth noting that many different cultures have had days of foolishness around the start of April, give or take a couple of weeks. The Romans had a festival named Hilaria on March 25, rejoicing in the resurrection of Attis. The Hindu calendar has Holi, and the Jewish calendar has Purim. Perhaps there's something about the time of year, with its turn from winter to spring, that lends itself to lighthearted celebrations. To these days, April Fools' Day is observed throughout the Western world. Practices include sending someone

*Mgr. Tereza Bártlová, Department of Mathematical analysis MFF UK v Praze, Sokolovská 83, 186 75 Prague 8, Czech Republic e-mail: bartlova@karlin.mff.cuni.cz

on a ”fool’s errand,“ looking for things that don’t exist; playing pranks; and trying to get people to believe ridiculous things.

Although it might seem that April has experienced a slight decline of Fame in recent years, April 1 is still a welcome opportunity for usually serious media to banter with impunity. With the April tradition media can go beyond the seriousness and let some sensation to the world. There is nothing more exciting then catching a million people out at a time. If we investigated the influence of the media, we could not forget the date of April 1. In this day, the number of TV and radio stations and newspapers are testing their readers, listeners and viewers when catching their attention on the grotesque reports. The newspaper sensations in the form of serious-looking reports are on a daily basis.

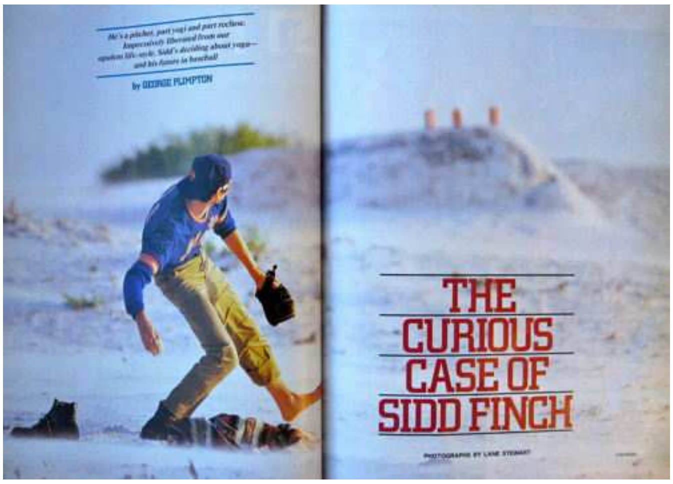
Even the smallest local newspaper tries to cheat their readers by informing them about the blast of the chimney in the local factory, saving the collapsing bridge or the unexpected arrival of a celebrity in the local discos. People attracted by these sensational titbits are gathering in front of the screens to find out soon that they were just taken in by the bunch of journalists who are more than happy to report how many people fell for their bait in the following issue.

We must also be vigilant following days. Although the first in a joking occupy mainly daily periodicals, including Internet, some journals do not want to be left behind and publish the April issue. The readers have to be alert only April 1 but basically the throughout month.

And they grip on not only but also in the choice of hoaxes. The most success jokes will go down the history of April hoaxes. Their international rankings can be found on the website of The Museum of Hoaxes. In the first place there is a report of the British BBC television from 1975. Contribution in the transmission informed viewers that Swiss farmers grew spaghetti on trees due to warm weather and had a record harvest this year. Although it may seem highly improbable, many viewers believed it.

The Swiss Spaghetti Harvest.

For the best-rated newspaper sensation is considered an article about incredible rookie baseball Mets's player which was published in Sports Illustrated magazine, in 1985. The boy named Sidd Finch could reportedly pitch a baseball about 65 mph faster than previously recorded speed for a pitch. But this was not all, Finch had "learned the art of the pitch" in Tibet where he learned the teachings of the "great poet-sain Lama Milaraspa". Mets's fans were amazed by this rumor and clamored for more information. However, to their great disappointment, they read in the next issue of the magazine, they were victims of April fools hoax.



New Mets's player.

Basically, we can divide the journalistic April fool's hoax into several categories: seriously conceived message, which you can hardly see through; report with a clear overstatement, in which you can just having fun with the adaptation or absolutely humorous messages that can be true, just they do not have any place in the current edition. There are no reliable "rules of April" how we can recognize a hoax report. Everything depends on the sophistication of journalists and presence of mind of the reader. Sometimes we can use some tools such as print text upside down or using apparently fiction names and titles, but the author of the hoax article throw such a lifebuoy exceptionally. Every day, we are witness of events that we do not dare to believe for many reasons, even if they are sometimes true. So how we can know what is true and what is not? The chapter itself is, when scientific journals make fun of us; for example, reports of breakthroughs and inventions.

In a time, in which we live, the science become more progressive every day. Media constantly spewing on us messages about new scientific results. But how we can distinguish that discovery is crucial or extremely stupid? And do we have the ability to do it?

There are plentiful of pranks or hoaxes. In the following chapters, we will gradually trade the April fool's hoaxes from mathematicians of physicists. We will show some examples of such scientific April hoax articles. You can also read where the authors draw inspiration and what happens when readers take such a discovery seriously.

2 The great Moon hoax

After reading the introductory chapter, we have idea how an April fool's hoax article might look like. In this chapter, we introduce the first ever article which passed into history and triggered an avalanche of newspaper sensation.

Historically, the first false news sensation, which went down in history, is associated with the New York daily press *The Sun*.[1] In August 1835 *The Sun* published a series of briefings on astronomical observation of Sir John Frederick William Herschel that took the newspaper and scientific worlds by the ears.

The first angle of the plot appeared on *The Sun* on August 21, 1835. The report was allegedly taken from the English magazine *Edinburgh Courant* [27]: "*We have just learnt from an eminent publisher in this city that Sir John Herschel, at the Cape of Good Hope, has made some astronomical discoveries of the most wonderful description, by means of an immense telescope of an entirely new principle.*"

This announcement only serve as an initial report to the much more complicated series of articles that began appearing in the newspaper four days later. Everyday news brought readers news of astronomical observations. All articles cited the results published in a supplement to the English scientific magazine.


THE SUN.

NEW YORK, TUESDAY MORNING, AUGUST 25, 1835

GREAT ASTRONOMICAL DISCOVERIES

Front page of daily New York *The Sun*: August 25, 1835

Articles describing the landscape of the Moon, which seemed to telescope observers in the same similarities, as a viewer on Earth offering distance less than a hundred meters. The continuation of the story focused not only on the lunar landscape, but also for

[1] *The Sun* was a New York newspaper that was published from 1833 until 1950. The founder and first editor of the newspaper was Benjamin Day. He came up with the idea of daily newspapers with an extremely low price, so he decided to create and fund his press based on the results from the sale of individual issues. Profit from each issue had to cover production costs. To newspapers sold well, spreadsheet content to offer simple and fun to read. Day emphasis on sensational news reports about crimes, sex and so on [21].

animals and creatures that lived on the Moon. It talked about different life forms on the moon, including such fantastic animals as unicorns, two-legged beavers and furry, winged humanoids resembling bats. The articles also offered vivid description of the moon's geography, complete with massive craters, enormous amethyst crystals, rushing rivers and lush vegetation. Then, suddenly, it followed a short statement that further discoveries failure prevented the telescope. This story ended.

Figure showing beings from the Moon.

Exotic landscapes, flora and fauna of the Moon has issued the first article became a sensation overshadowing reports in other newspapers. People all over New York to discuss whether the story is true. Readers were completely taken in by the story, however, and failed to recognize it as satire. A number of New York newspapers reprinted article, or at least its excerpts in order to they did not lose their readers (see [27]):

The Daily Advertiser wrote that: "No article has appeared for years that will command so general a perusal and publication. Sir John has added a stock of knowledge to the present age that will immortalize his name and place it high on the page of science."

The Times said that everything in the *The Sun* story was probable and plausible, and had an "air of intense verisimilitude".

The New York Sunday New advised the incredulous to be patient: "Our doubts and incredulity may be a wrong to the learned astronomer, and the circumstances of this wonderful discovery may be correct".

The craze over Herschel's supposed discoveries even fooled a committee of Yale University scientists, who traveled to New York in search of the *Edinburgh Journal* articles.

The articles were an elaborate hoax. It was a perfect mystification, which succumbed not only readers, but newspapers and some scientists. The only thing on the whole stunt was real, was the figure of John Herschel. Sir John Frederick William Herschel was the greatest astronomer of his time, moreover he was the son of the celebrated astronomer Sir William Herschel. In truth, in January 1834, he went to South Africa and established an observatory near Cape Town, with the intention of completing his survey of the sidereal heavens by examining the southern skies as he had swept the northern, thus to make the first telescopic survey of the whole surface of the visible heavens. Nevertheless Herschel had not really observed life on the moon, nor and he accomplished any of the other astronomical breakthroughs credited to him in the article. In fact, Herschel was not even aware what it was happening in New York. That the such discoveries had been attributed to him, he found out until much later. At first he was rather amused, and only complained that his own observations, sadly, never will nor half as entertaining as they are described in the paper. Later, however, he was much annoyed because he had to constantly face questions from people who thought that the articles are true [35]: "*I have been pestered from all quarters with that ridiculous hoax about the Moon – In English, French, Italian and German!!*“

The author of this newspaper stunt was reportedly reporter Richard A. Locke,[2] who in August of that year began working for the newspaper *The Sun*. There are persistent rumors that Locke confessed to his friend Finn authorship of articles in a weak moment. Reveal the secrets of a friend who works in a competing newspaper, was proved a poor choice. The next day, the newspaper *Journal of Commerce*published the news that a series of monthly articles about discoveries is a hoax, and Locke identified the author as a fiddle. There are also speculations that by writing articles involving more people. In connection with the mentioned articles is most often named yet one man: Jean-Nicolas Nicollet,[3] French astronomer passengers at the time in America. However, there is no specific evidence about who was the real author of those sensational articles, and even Locke never publicly admit authorship.

Regarding the intention of the entire newspaper fiddle, assumptions are somewhat nebulous. The first option is entirely pragmatic: Locke's aim could be to create a sensational story that would increase the sales of *The Sun*. Another reason could be targeted ridicule from some rather extravagant astronomical theories, which were published at that time. It comes into consideration also the option that Locke was inspired by a story by Edgar Allan Poe[4] about similar inhabitants of the moon called *The Unparalleled Adventure of One Hans Pfaall*.

Whatever the intentions of any of Locke, his hoax sparked sharp criticism from the other New York newspapers. For the "robust“ newspapers of that time, which came from

[2]Richard Adam Locke (1800–1871) was an English journalist, writer and later editor of *Somerset paper*. In 1835, he first met with the editor Benjamin Day and began working for *The Sun* [35].

[3]Jean-Nicolas Nicollet (1786–1843), also known as Joseph Nicolas Nicollet, the French geographer, astronomer and mathematician who is most notable thanks to the mapping of the upper stream Mississippi in 1830 [18].

[4]Edgar Allan Poe (1809–1849) was an American Romantic poet, novelist, literary theorist and essayist. He was the author usually fantastic and mystical stories and founder of the detective genre [4].

the Enlightenment concept of the press as a medium of education for the "common people", it was similar to the stigma content "relegation" task newspaper. Despite the fact that the New York newspaper condemned all competition from fraud, *The Sun* for a long time he was not ready to disappoint their readers and confess. Until September 16, 1835, more than two weeks after the conclusion of the story, was imprinted long article on the topic of authenticity discoveries. At the end of this article *The Sun* the whole affair has concluded that although the articles about discoveries on the moon initially written as a satire to entertain readers, later unexpectedly encountered new circumstances, which may confirm the authenticity of some breakthroughs, and therefore it is necessary explore everything properly again. This statement apparently after previous experience nobody believed, but because readers and most newspapers fell for a ruse, it is better not to pursue the case further too and take the whole affair rather humorously. "That the public were misled, even fop an instant"Poe declared in his critical essay on Locke's writings [27], "merely proves the gross ignorance which, ten or twelve years ago, was so prevalent on astronomical topics."

However, it is clear that gambling confidently readers did not erode sales of the newspaper *The Sun*, quite the contrary. Thanks to a sensational articles, the number of his prints has increased dramatically and even after the discovery of fraud had not fallen. *The Sun* every day began to publish more or less fanciful reports about various attractions, stories and tidbits from the world of crime, sex and so on. Over the next year cost of *The Sun* several times higher than the average cost of a conventional printed in the USA. No wonder then that this convincing success prompted several attempts to infiltrate into the same area newspaper market.

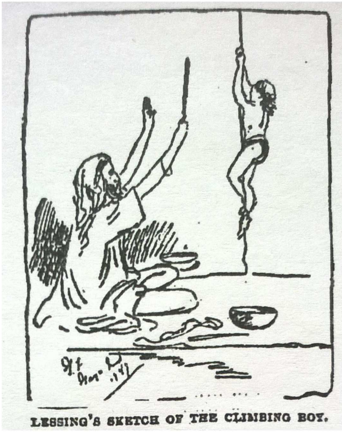
3 The Indian rope trick

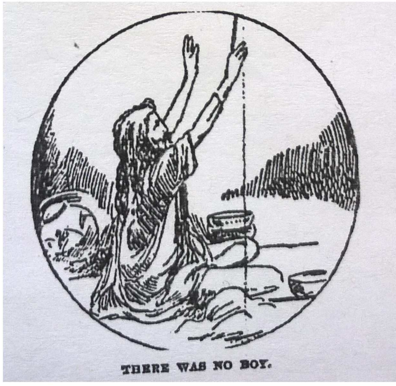
Another in a series of successful newspaper sensation came from *Chicago Daily Tribune.* [5] On August 9, 1890 the newspaper published a report describing the breathtaking illusionist performances of the Indian fakir. [?]

The story is narrated by a young amateur photographer Frederick S. Ellmore, who traveled to India with his friend George Lessing. During his stay in Gaya he attended a performance of a local snake charmer. The performance was stunning. The young travelers were most attracted by a rope trick. The fakir took a ball of gray twine in a hand. Taking the loose and between his teeth he with a quick upward motion, tossed the ball into the air. Instead of coming back to him it kept on going up and up until out of sight and there remained only the long swaying end. At the same moment a young boy appeared beside the fakir. He began climbing it and vanished in clouds. A moment later the twine disappeared.

The story was also added to the pictures. While the fakir was going through his performances Lessing was to make a rapid pencil sketch of what he saw while Ellmore at the same moment would take the photographs with kodak. Strangeness was that the scene outlined in Lessing's sketches did not agree with the Ellmore's photographs. Despite the fact that the sketch showed the boy climbing the twine, the camera said there was no boy and no twine. Therefore Ellmore concluded that the fakir had hypnotized the crowd into believing the trick had been performed(see [5]): "...his eyes were remarkable both for their brilliancy and their intense depth, if I may so term it. They seemed to be almost jet black and were set unusually deep in his head. When we stepped into the little circle about him those eyes took us in from sole to crown...I'm compelled believed that my theory is absolutely correct - that Mr. Fakir had simply hypnotized the entire crowd, but couldn't hypnotize the camera."In the conclusion of the article he promised to sent the copies of the pictures to the London Society for Psychical Research to closer investigation.

[5] *Chicago Daily Tribune* is a major daily newspaper based in Chicago, Illinois, United States. It was founded in 1847 and formerly self-styled as the "World's Greatest Newspaper". In the year 1854–1901 Joseph Medill was a managing editor of the newspaper.





Picture from *Chicago Daily Tribune* with Lessing's sketch and Ellmore's photograph

Both readers and other newspapers believed the article. The story quickly spread and gained credibility, first in the United States, then in Great Britain and very soon it was translated into many European languages. Over the next four months this article provoked so much debate that the editor *Daily Tribune* decided to clarify the whole affair and admit that the story was completely fabricated. On December 6, 1890 the short notice was published at the bottom of a *Queries and Answers* column (see [37]): "The article on hypnotism referred to in the above query was written for the purpose of presenting a theory in an entertaining form... The principal character was Mr. F. S. Ellmore (sellmore), and the writer considered that the name would suggest to a careful reader that ir was a 'sell'."How many reader devoted attention to the confession, it is hard to estimate. Although press transfers the messages better than other media, simultaneously it allows people the disclaimer to just skip. Whatever the readers may read the disclaimer or not, the story about the Indian rope trick definitely did not fit.

Over the ensuing decades, this trick has inspired much controversy in magic and psychical research circles. Other eyewitness accounts to the trick were presented, but finally, they collapsed under investigation. Despite all the statements of the witnesses were false, with every repetition the story became more real and realistic not only rumor. [6] Magicians and illusionists bet among themselves who will perform the air-open show as the first. In 1930's, Lt Col Elliot of the London Magic Circle, when offering a substantial reward for an outdoor performance, found it necessary to define the trick. He demanded that (see wiki):"the rope must be thrown into the air and defy the force of gravity, while someone climbs it and disappears."Challenges to perform the trick in the open air, not on a stage, were taken up but never met by magicians.

[6]Rumor is a tall tale of explanations of events circulating from person to person and pertaining to an object, event, or issue in public concern. It contains nothing from which we could assess the veracity. (see [20])

We must admit that this newspaper sensation was really successful. Many people (including scientists) did not want to believed that the story was not inspired by a reality and so they traveling to India inquired about the trick and helped to spread the legend there; soon Indians were defending their trick against claims it had originated centuries prior in China. In 1996 Peter Lamont [7] a historian and performer of magic dealt with the origin of the trick and concluded that there were no known references to the trick predating 1890, and later stage magic performances of the trick were inspired by article. By now the rumor about the Indian rope trick lived its own life and with time diverse accounts of the trick begun to appear in print which differ in the degree of theatricality displayed by the magician and his helper. [8] According to the version of the story, the tricks they used to be attributed to various Indian or Moroccan fakirs.

The question still is, how did we discover the identity of the story's anonymous author? In 1891 Andrew Stewart, the editor of a popular British weekly *People's Friend*, had read various copies of the original story and sent a letter to the *Daily Tribune* seeking more information of the rope trick. One John E. Wilkie[9] responded to his letter (see [24]): "I am led to believe... that the little story attracted more attention than I dreamed it could, and that many accepted it as perfectly true. I am sorry that any one should have been deluded." And the letter was signed, with no obvious sense of irony, "sincerely yours, John E. Wilkie". Richard Hodgson of the American Society for Psychical Research has on the question of authorship a different opinion. R. Hodgson did not credit Wilkie either, but instead and noted that (see [24]): "[t]he story of the boy climbing the rope and disappearing is, in one form or another, very old". This would be another reason why Wilkie would not be remembered as the man who launched the legend. For while he was responsible for making the story famous, the story itself was not entirely original. There had been, for centuries, many stories of ropes, cords, chains and the like rising into the air, of humans and animals climbing to the top and disappearing. Such stories, in fact, can be found in the mythologies and folklore of several cultures, not only in India but in Europe and China, in North America and Australia.

[7]Peter Karl Lamont is a research fellow at Edinburgh University. He is also historian and performer of magic (see [29]) In 1996, he pulls of a neat trick himself in making 264 pages appear on so slim a topic. Moreover, his book contains the names the alleged hoaxer and follows the tricky caroms the legend took over the years [24].

[8]One of them threw a rope into the air which hitched itself up to apparently nothing in the sky above; one could see the rope going straight up as far as one could see anything... A small boy then swarmed up this rope, becoming smaller and smaller, till he likewise vanished from sight, and a few minutes later bits of his (apparently mangled) remains fell from the sky, first an arm, then a leg, and so on till all his component parts had descended; these the juggler covered with a cloth, mumbled something or other, made a pass or two and behold! there was the boy smiling and whole before us (see [24]).

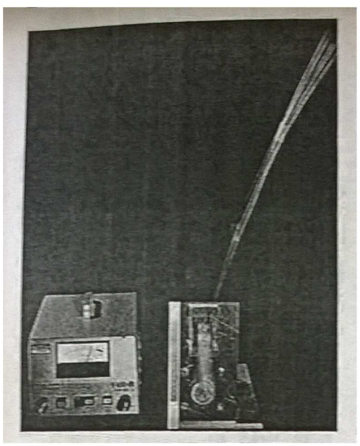
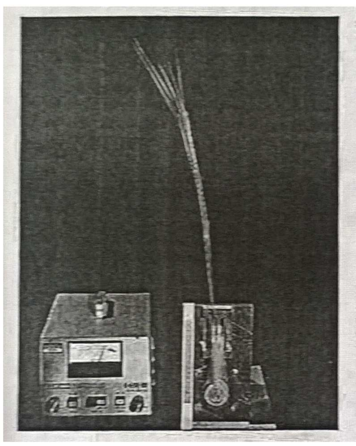
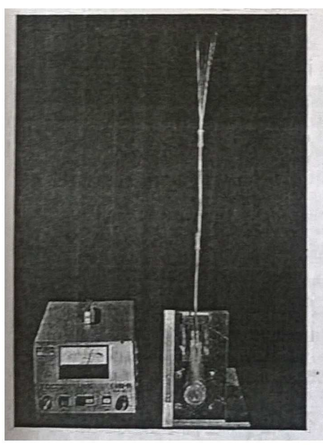
[9]John Elbert Wilkie (1860–1934) was an American journalist and Director of the United States Secret Service from 1898 to 1911. His father, Franc B. Wilkie,was an editorial writer at the newspaper; the two traveled to Europe and served as the *Chicago Times*' European correspondents. In 1911 John E. Wilkie joined the staff of the *Chicago Tribune*, where he initially served as financial editor and later city editor (viz [17]).

And even if we knew the real author of this hoax, today we can only guess, what was the main impetus for the writing this hoax. For example, Lamont believes Wilkie's ruse was inspired be debates at the time about conjurors and psychic phenomena.

Very curiously is that although Wilkie have dreamed trick which the magician have not succeeded in doing it till now, but it does not mean that it is totally unrealistic. When Wilkie unleash his imagination, surely he did not know that one hundred years later, something similar will realize through scientific and technological methods.

Very similar analogy is described by the Swiss mathematician Daniel Bernoulli. In 1738 he published the article on pendulum motion. Bernoulli was interested in multiple pendulums that means N pendulums suspended from one another. He discovered that this system can oscillate at any one of N different natural frequencies $f_1, \ldots, f_N$, where f_1 denotes the smallest and f_N the largest. In the lowest-frequency mode the pendulums swing to and more of less together, much as if they formed one long, single pendulum. The other way around, in the highest-frequency mode, adjacent pendulums swing in opposite directions at any given moment.

In 1992 a mathematician David Acheson and a physics Tom Mullin began to study more closely the Indian rope trick. They turned everything upside-down and discovered that it is possible to take N linked pendulums, turn then upside-down, so that they are all precariously balanced on top of one another, and then stabilize them in this position by vibrating the pivot up and down. They conclusion said that (viz [2]): "the trick can always be done, so long as the pivot is vibrated up and down by a small enough amount and at a high enough frequency." By the experiment they even discovered, that with a 50 cm inverted triple pendulum, for example, the pivot was vibrating up and down through 2 cm or so at a frequency of about cycles per second. Moreover the inverted state is quite stable because when they push them over by as much as 40 degrees or so and they would still gradually wobble back to the upward vertical.

In 1993 they published their results in *Nature* magazine and two years later they made a brief appearance, with the experiment, on the British BBC television on the TV programme *Tomorrow's World*. Sometimes, however, fictions and fantasies of journalists seem closer to reality than advances of science. While the *Daily Tribune* was enjoyed greater sales after publishing hoax article, Acheson and Mullin did not get popularity. After the TV program, the BBC received a telephone call of from outraged viewers who claimed that (viz [2]): "[the] balancing act was obviously impossible, and contrary to the laws of physics" and he seemed "genuinely upset with *Tomorrow's World* for 'lowering their usually high standards' and 'falling prey to two tricksters from Oxford'.

4 Gravity Nullified

Let's return back to the sensational newspaper articles. Fascination with the power of newspapers keep also subsequent periods. Another newspapers the idea of news sensationalism further developed and promoted it with a far more aggressive. Over time heals completely withdrawing from publishing completely fictional reports, as undermining the credibility of the sheet. Newspaper stunts were replaced by sensationalist headlines and graphic illustrations that complement the only real event.

All this has led to the fact that at the beginning of the 20th century, people began to look at the newspaper with a certain disdain due to the lack of objective information and pandering to readers. Efforts to regulate the work of journalists and provide relevant information to readers culminated in the release of the ethical code of journalists [10] The ethical code serves primarily as ensuring moral support for journalists and their readers and sets a limit on what is good, moral and what is not. Although none of the ethical code has legal force, but its compliance is mandatory and violations can be punished. Penalties for violations are different in every country. Generally, a person in violation of the Code may be sued, and the result is a matter of court proceedings - from moral sanctions such as a reprimand, despite the temporary suspension, to the exclusion of professional associations.

This regulation of the press is the reason why today's press, especially if we focus on professional scientific journals to his readers a little more forgiving and canards deleted only once a year, on April. However, in this day readers really need to be alert. A chance to catch their readers from time to time will not miss even reputable scientific journals.

Moreover it is becoming harder and harder to recognize when it is a hoax, and when it is a real discovery. Additionally, if we are not experts in the field and do not understand the issues, we have little chance to detect fraud and not be fooled. On April, Other times, serious scientific journals, often use (or abuse) the reader's ignorance about technical topics.

[10] The first ethical code of journalists was named the *French Charter of the duties of journalists* and it was publish in France in 1918. In 1926 followed the Ethics Code in USA. Furthermore, in 1947, again in USA, it was publish the Hutchins Commission on press freedom and in 1950's *International Federation of Journalists* was founded by journalistic organizations of the USA and Western Europe.

On April 1927 a German journal *Radio Umschau* [11] publish a report called Überwindung der Schwerkraft? Ein neuer Erfolg der Quarzkristallforschung about discovery of antigravity device.

The discovery was made in a newly established central laboratory of the Neuuartadline-Werke in Darreskein, Poland, by Krowsky and Frost. While experimenting with piezoelectric properties of quartz crystals they discovered that the constants of very short waves, carried on by means of quartz resonators, a piece of quartz which was used, showed a clearly altered appearance. It means quartz crystal changed the entire structure. Moreover it lost its weight, had become practically negative, and its was able to levitate.


Fig. 1. Im Quarzkristall-Laboratorium.

Ueberwindung der Schwerkraft?

Ein neuer Erfolg der Quarzkristallforschung.

the article from German journal *Radio-Umschau*

It is possible that the Polish scientists were actually at the birth of methods to overcome gravity? It is true that science goes forward every day and we must admit that theoretically everything is possible. With the development of technology we do not know the day or the hour, when it will be found for the way gravity to break and perhaps a similar manner

[11] *Radio Umschau*

to that described in the article. However, it did not happen, and although the Frost's and Krowsky's experiment would surely bring a revolutionary discovery for science and humanity, the facts in the article are not true.

One of the readers who was very amused interested in the article, was Hugo Gernsback. [12] Gernsback is still highly praised for his contribution to the science fiction genre. Due his enthusiasm for science fiction, or maybe just because, he was a great opponent of pseudoscience such as astrology spiritualism and especially the attack on alternative medicine. In 1913 he founded the magazine *Science and Invention* which was given up until 1929. The magazine was focused on science and technology, especially for amateur science experiments, construction of radio and notable inventions. There were often published speculative articles about upcoming technologies and even science fiction stories (see [26]). So when April fools article about the discovery of antigravity device appeared in the German magazine, Gernsback was so impressed article, he could no resist the joke and did not share this with your readers as well.

And because history repeat itself Gernsback used the same schema such as the *New York Sun* – he published the report called Gravity Nullified (Quartz Crystal Charged by High Frequency Currents Lose Their Weight) with the remark that (see [12]): "this report appears in a reliable German journal, Radio Umschau."

Článek Gravity Nullified z časopisu Science and Invention

[12]Hugo Gernsback (1884-–1967), born Hugo Gernsbacher, was a Luxembourgian American inventor, writer, editor, and magazine publisher, best known for publications including the first science fiction magazine. His contributions to the genre as publisher were so significant that, along with the novelists H. G. Wells and Jules Verne, he is one person sometimes called "The Father of Science Fiction". In his honor, annual awards presented at the World Science Fiction Convention are named the "Hugos". (see /citehugo)

In this case, the editor propably aware that many readers take the article seriously, and so he tried to throw a lifeline in the form of final notes (see [12]): "Don't Fail to See Our Next Issue Regarding This Marvelous Invention." Above sentence should tell readers that this is only joking article. Also, if the readers better view individual images, they found that their labels do not completely correspond to what is shown on each image. Unfortunately, not all readers were so receptive and farsighted, and so a large proportion of them would fall.

In the following October issue of tfhe magazine *Science and Invention* the article appeared which was called Gravity Nullified - A Hoax, in which the editors tried to put everything into the correct perspective and explain to readers that the article was only hoax and they cannot take this seriously (see [12]): "As a matter of fact, most of the statements are true, with the exception, of course, of those statements referring to the expanded crystal and to the loss of weight caused by the supposed high frequency currents." One of the disadvantages of periodicals is that are not daily published and disclaimer of the article may come up next month, so readers lived a lie for quite a long time. Moreover the Achilles' heel of this disclaimer certainly is that it is disprovable. It means that the original report claim that anything goes and disclaimer suddenly claims that it is not. Generally, we can say that every statement about something that does not exist, has in terms of verifiability terrible handicap. In addition, for the article, where we are not able to self-assess his truthfulness on the based on our knowledge and skills, we come to the knowledge of the basic paradox: the belief in denial follows the same logic as the belief in the original statement. In both cases, the point is to believe one's word. Questions of the readers: "What to believe?", depends largely on what "Where it says?" And if readers evaluated as a credible source of information, and the authors themselves acknowledge the fact that in the future may be all different, we can not be surprised that some readers were so excited about an experiment that did not want to believe that it is a hoax.

And therefore the deception did not left entirely without consequences. In 1981, in the February issue of the journal *Planetary Association for Clean Energy* John G. Gallimore published an article entitled Anti-Gravity Properties of Crystalline Lattices. Gallimore informed the readers that in the summer of 1927, two Polish scientists Kowsky and Frost described the specific anti-gravitational properties of crystals. The report about their discover appears in magazines *Science and Invention* and *Radio Umschau* shortly after their experiment, some photographs of the tests was published as well.

Also David Hatcher Childress and W. P. Donavan are convinced of the truth of this discovery. In the book *The Anti-Gravity Handbook*, they polemise whether the magazine misunderstood when the article declared false.

Perfection, however, that the joke was delivered by authors who publish on the Internet. For example, the above mentioned W. P. Donavan (also acting under the name Bill Donavan) wrote in his work *Glimpses of Epiphany* that he is not only deep convinced that the results of Frost's and Krowsky's experiment are true, but also that "it seems that something got out, that wasn't supposed to get out" and now it is an effort to conceal everything.

Of course, we can speculate what the editor followed through publishing this article. Someone accuses the magazine that the article was published with the intention to profit from higher sales.

The article eventually became one of the more popular jokes publications and occupies pride of place on the website of The Museum of Hoaxes. Needless to say, the editor's note in the explanatory article Gravity Nullified - A Hoax did not speak clearly to the contrary (see [12]): "Scientific hoaxes are no novelty. One of the most famous, which was no exposed as quickly as this one, appeared in no less than the *New York Sun*. At that time, in August, 1835, a certain professor was supposed to have submitted his report on a fantastic moon people to the *Edinburgh Journal of Science*, to which manuscript the *New York Sun* obtained the first rights, and the article ran consecutively over a period of the time." On the other hand, he also notes (see [12]): "The moral is that we should not believe everything that we see, but do a little original thinking ourselves, because we may never know, otherwise, what are facts and what are not." Therefore, it is also possible that the article should have more educational aspect.

5 Six sensational discoveries that somehow have escaped public attention

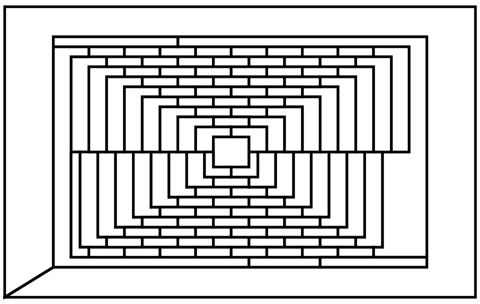
In 1975, in the April issue of the Scientific American magazine was published the article about six sensational discoveries of recent years [8]. The readers were informed about a huge discovery in the number theory, about the finding of the counterexample four-colour-map theorem, about local flaw in the special theory of relativity, about revolutionary chess-playing programme, about discovery of page from Leonardo da Vinci's notebook, and about the invention of psychic-energy-working motor.

In fact any of these revolutionary discoveries were not of great importance to the world. Although they were based on true facts, lots of information has been thought through. And in many cases the names of scientists, who have studied the given problem, were altered. All conclusions in the article were completely fabricated.

The author of this article was Martin Gardner [13] who published column called Mathematical Games for a long time. Mathematics became his lifelong passion, but he was also an expert magician, a well-known sceptic or a leading figure in refusing pseudoscientific theories ranging from modern diets to flying saucers. All of these hobbies are reflected in his April hoax article.

5.1 Four-color-map theorem

Discovery of the counterexample of the four-colour-map theorem created a considerable stir among the readers. In autumn 1974, the American mathematician William McGreor managed to construct an example of a planar map with 110 countries (picture below), where minimum of five colours is needed for colouration.

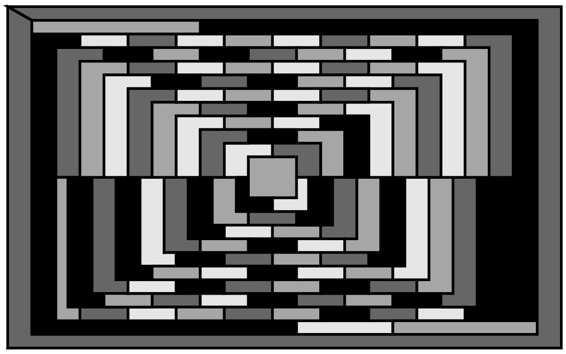


The planar map constructed by William McGregor

[13] Martin Gardner (1914—2010) wrote columns in *Scientific American* magazine for long twenty five years and published more than 70 books. In spite of, or perhaps because of, lacking proper mathematical education, Gardner's articles and books influenced generation of people. Thanks to his boundless enthusiasm and careful choice of topics, his articles got the general public interested in math. He also had other hobbies apart from mathematics. He wrote a lot of books concerning magic, philosophy, or commented on other authors' books.

Since 1740s all mathematicians tried to prove the fact that four colours are enough to colour any political map in the way that any of two neighbouring states would have the same colour. For decades they were unable to come up with the longed-for evidence of the theorem, so they tried to construct at least a counterexample. When examining small maps it was shown that four colours were enough for colouration. It seemed, however, that it would be about much more complicated maps. In early 1974, the mathematician Jean Mayer managed to prove that theorem is true for a planar map that contains a maximum of 95 states. Everybody expected that the problem would be soon resolved. Various symposiums on graph theories were held and the problem of the four colours was discussed everywhere. Gardner took the advantage of the atmosphere which longed for resolving the problem, and offered readers a simple solution.

The map, which is showed in the picture, was designed by a correspondent William McGregor (his real name). If you try to colour the map, you will discover soon, that four colours are enough.



William McGregor's map coloured by foour colours.

In fact, the four colour theorem was proved in 1976 by Kenneth Appel and Wolfgang Haken. It was the first major theorem to be proved by the use of a computer (see [1]).

Gardner received thousands of letters from his readers who sent him copies of the coloured maps where only four colours were used. Some of them claimed they spent days before working it out. A large number of readers, including mathematicians, fell for the Gardner's joke as many articles at that time mentioned.

When Norman Kent Roth published the article called *Map colouring* in December 1975, he was snowed under with letters from readers who kept informing him that *Scientific American* had already published a map disproving the four colour theorem whose author was Martin Gardner.

In December, 1976, a British mathematician George Spencer-Brown announced that he had found the proof of four colour theorem without using a computer. Although the experts finally agreed that the evidence contained errors, his announcement did not pass unnoticed. In January 17th, 1977, Canadian journal *Vancouver Sun* published a letter from a woman in British Columbia, in which she protested against Brown's proof because of Gardner's article.

In 1978 *Artificial Intelligence* magazine published the article (see [?]) whose author stated that he managed to colour McGregor's map using a computer programme. The author obviously did not realize it was a hoax.

Not only Gardner, but also the staff of *Scientific American* magazine, was little bit taken aback by some reactions of the readers. The following letter, signed by a mathematician Ivan Guffvanoff III. at the University of Wisconsin, was a bit frightening for the staff of Scientific American magazine, unless they realized that it, too, was a joke (see [9]): „This is to inform you that my lawyer will soon be contacting you for a damage case of $25 million. In the mathematics section of your April 1975 issue, Martin Gardner wrote that the four-colour problem had been solved. I have been working on this problem for 25 years. I had prepared a paper to be submitted to American Mathematical Monthly. The paper was over 300 pages in length. In it I had proved that the answer to the four-colour problem was no and that it would take five colours instead of four. Upon reading Gardner's article, that someone else would publish the solution before I could, I destroyed my paper. Last week I read in Time magazine that Gardner's article was a farce. I did not read Gardner's entire article, only the part on the four-colour problem, so I was not aware of the farce. Now that I have destroyed my article, it will not be possible to reproduce all 300 pages, since the work has extended over such a long time. I therefore believe that damages are due me. I believe that Gardner's article was the most unprofessional article I have ever seen in yours or any other journal. This kind of activity is below the dignity of what I thought your magazine stood for. I am not only suing you but I am cancelling my membership, and I will ask all my friends to cancel theirs".

A reference of Gardner's article can also be found in the Italian magazine *Rendiconti*. In 1975, mathematician Serge Benjamino published series of articles in which he showed that McGregor's map can be coloured using four colours.

5.2 Ramanujan's constant

Another piece of news Martin Gardner came up with was a surprising discovery in number theory claiming that the number $e^{\pi\sqrt{163}}$ is integer.

This exciting result was discovered thanks to American mathematician John Brillo in 1974. He was supposed to find an ingenious way of applying Euler's constant to prove that

$$e^{\pi\sqrt{163}} = 262\,537\,412\,640\,768\,744\,.$$

Mathematicians of the 18th century were already interested in number $e^{\pi\sqrt{163}}$. This number was discovered by Indian mathematician Srinivasa Ramanujan. [14] In any case, it is not an integer. S. Ramanujan occupied himself with several similar powers of Euler's number in the article *Modular equations and approximations to* π. But it was clear to him that all numbers were transcendental numbers (viz [30]): „[from equations] we can find

[14]Srinivasa Ramanujan (1887–1920) was an Indian mathematician with a wide range of interests such as heuristic aspects in number theory, mathematical analysis, infinite series (see [32]).

whether $e^{\pi\sqrt{n}}$ is very nearly an integer for given values of n, and ascertain also the number of 9's or 0's in the decimal par… "

John Brillo, to whom this hoax is attributed, is a play on the name of the distinguished number theorist John Brillhart.

Gardner's idea, that the prime number 163 manages to convert the expression to an integer, implies from the fact that the number 163 is, in many respects, interesting. For example the number 163 is one of the Heegner numbers. To understand better the importance of Heegner numbers we recall some qualities of complex numbers.

There are many possibilities how to express complex numbers. The most famous method is by using the Gaussian integers. By *Gaussian integer* is meant a complex number $z = a+bi$ an integer when a, b are integers:

$$\mathbb{Z}[\sqrt{-1}] = \mathbb{Z}[\mathrm{i}] = \{\mathrm{z} = \mathrm{a} + \mathrm{bi} : \ \mathrm{a}, \mathrm{b} \in \mathbb{Z}\}\,.$$

Gaussian numbers form a square lattice in the complex plane. The mappings between complex numbers and Gaussian numbers are of one-to-one correspondence. It means that every complex number is paired with just one Gaussian number and the other way round. Gauss also discovered that every Gaussian number can be uniquely factored into Gaussian primes. [15] Gaussian primes are of shape

$$\begin{cases} a + b\mathrm{i}, & \text{when } a^2 + b^2 = p \text{ is prime, or} \\ up, & \text{when } u = \{\pm 1,\ \pm \mathrm{i}\} \text{ a } p \text{ is prime of shape } 4k+3. \end{cases}$$

For example::

$$\begin{aligned} &2 = (1+i)(1-i) = 1^2 + 1^2, \\ &3 \text{ is prime,} \\ &5 = (2+i)(2-i) = 2^2 + 1^2, \\ &7 \text{ and } 11 \text{ is prime,} \\ &13 = (3+2i)(3-2i) = 3^2 + 2^2 \text{and so on.} \end{aligned}$$

An alternative system how to define the complex „whole numbers" is through Eisenstein integers. Like Gaussian numbers form a square lattice in complex plane, Eisenstein integers form a triangular lattice. Every number is of shape $z = a + \omega b$ when $a, b \in \mathbb{Z}$ and $\omega = e^{2\pi i/3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}$ we call *Eisenstein integer*:

$$\mathbb{Z}[\sqrt{-3}] = \mathbb{Z}[\omega] = \{z = a + \omega b : \ a, b \in \mathbb{Z},\ \omega = e^{2\pi i/3}\}\,.$$

So the Eisenstein integers also have unique factorization and every nonzero Eisenstein integer is uniquely the product of Eisenstein primes. [16] Eisenstein primes are of the shape

[15] Gaussian number z

[16] Eisenstein number z

$$\begin{cases} a+\omega b, & \text{when } a^2-ab+b^2=p \text{ is 3 or prime of shape } 3k+1, \text{ or} \\ up, & \text{when } u=\{\pm 1,\ \pm\omega,\ \pm\omega^2\} \text{ and } p \text{ is prime of shape } 3k+2. \end{cases}$$

It was not entirely clear whether complex integers can be always uniquely factored into prime numbers. We know that this is true for numbers containing $\sqrt{-1}$ nebo $\sqrt{-3}$. How we can factorize the numbers of shape $a+b\sqrt{-5}$? This is not unique factorization in this system of numbers. For example 6 factorizes in two different ways:

$$6 = 2\cdot 3 = (1+\sqrt{-5})(1-\sqrt{-5})\,.$$

Neither of the numbers $2, 3, 1+\sqrt{-5}$ or $1-\sqrt{-5}$ cannot be further factorized. We have two different ways of factorization. We can ask the question: Which negative numbers in a number system $\mathbb{Z}[\sqrt{-d}]$ can be uniquely factorized? The answer is *Heegner numbers*:

$$-1, -2, -3, -7, -11, -19, -43, -67, -163.$$

For a long time mathematicians were aware of these nine numbers but the question was whether there were more numbers meeting the requirements. At the beginning of the 20th century they came to the conclusion that if other number did exist, it would be only one. It gave rise to “tenth discriminant problem”. In 1936 Hans Arnold Heilbronn and Edward Linfoot showed that if other discriminant existed, it would be bigger than 10^9. In 1952 mathematician Kurt Heegner [17] produced evidence that such a tenth discriminant didn't exist and the list of nine was complete. Unfortunately, the experts didn't accept his proof because they expressed doubts about its validity. In 1966–67, two young mathematicians, Harold Stark and Alan Baker, both gave independent proofs. H. Stark also focused on Heeger's proof and two years later he confirmed its validity.

Heegner numbers have a lot of interesting qualities. It is given the formula

$$n^2-n+k,$$

when $k>1$. This formula represents primes for the consecutive numbers $n=1,2,\ldots,k-1$ as long as $1-4k$ is one of the Heegner numbers. Heegner number we have in case $k = 2, 3, 5, 11, 17$ a 41.

n^2-n+2	$n=1$	2
n^2-n+3	$n=1,2$	3, 5
n^2-n+5	$n=1,2,3,4$	5, 7, 11, 17
n^2-n+11	$n=1,2,\ldots,10$	11, 13, 17, 23, 31, 41, 53, 67, 81, 101
n^2-n+17	$n=1,2,\ldots,16$	17, 19, 23, 29, 37, 47, 59, 73, 89, 107, 127, 149, 173, 199, 227, 257
n^2-n+41	$n=1,2,\ldots,40$	41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151, 173, 197, 223, 251, 281, 313, 347, 383, 421, 461, 503, 547, 593, 641, 691, 743, 797, 853, 911, 971, 1033, 1097, 1163, 1231, 1301, 1373, 1447, 1523, 1601

[17]Kurt Heegner (1893 – 1965) was a German mathematician who was famous for his discoveries in number theory (see [23]).

Another remarkable fact of Heegner numbers is that the numbers $\mathrm{e}^{\pi\sqrt{\mathrm{d}}}$ are getting closer to integers, the bigger Heegner number d is:

$$\begin{aligned}
\mathrm{e}^{\pi\sqrt{43}} &= 884\,736\,743,999\,777\ldots \\
\mathrm{e}^{\pi\sqrt{67}} &= 147\,197\,952\,743,999\,998\,66\ldots \\
\mathrm{e}^{\pi\sqrt{163}} &= 262\,537\,412\,640\,768\,743,999\,999\,999\,999\,250\,07
\end{aligned}$$

5.3 Chess-playing programme

Readers who like playing chess must have found a big chess discovery interesting. In 1973 the Artificial Intelligence Laboratory in the Massachusetts Institute of Technology designed a special-purpose chess-playing computer.

The programme known as MacHic, was made by Richard Pinkleaf with the help of ex-world-chess-champion Mikhail Botvinnik of the U.S.S.R. Unlike most chess-playing programmes, MacHic used methods of artificial intelligence – a special learning machine that profited from mistakes by keeping records of all games in its memory and thus was steadily improving. In 1974, after many games of chess, the programme arrived at a surprising result: "It had established, with a high degree of probability, that pawn to king's rook 4 is a win for White." This was quite unexpected because such an opening move used to be regarded as poor. The machine MacHic constructed a "game tree" and analysed which position were about to win.

The chess-playing programme, described by Martin Gardner, was built by the Artificial Intelligence Laboratory in the Massachusetts Institute of Technology. In fact, it was designed between 1966-67; and its creators were Richard D. Greenlatt and Donald E. Eastlake III. and it was known as Mac Hack or also the Greenblat chess programme. The truth is that it was a revolutionary chess programme at the time. This programme was the first one, which could simulate human conditions while playing chess. It was also the first programme, which was able to compile and analyze the game and thanks to these qualities it was able to win the game against a man. Hhowever, it never counted the probability of winning in different positions.

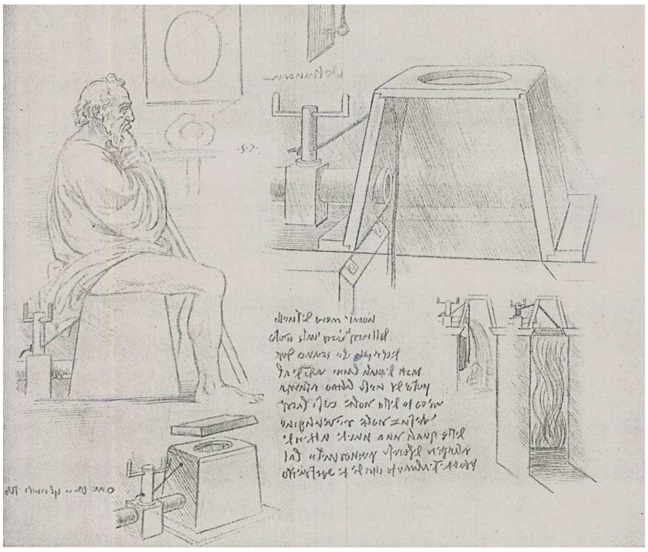
Unfortunately, people did not know much about computers at the time Gardner's article came out. And that´s why high number of readers took this article seriously.

5.4 Discovery of the missing page from Leonardo da Vinci's notebook

For lovers of art and various inventions Gardner published a groundbreaking discovery of Leonardo da Vinci.

In the 1960s the famous manuscript of Leonardo da Vinci, known as *Codex Madrid I*, was found in the National Library in Madrid. In the manuscript there were some sketches and treatises on theoretical and applied mechanics. Later, it was discovered that one page was missing. For years, the nature of the missing page was speculated about. Augusto

Macaroni of the Catholic University of Milan thought that the page might have dealt with some type of flushing mechanism, because the sketch was in a section on hydraulic devices. In December 1974 the missing page was finally found (see figure below). It turned out that A. Macaroni was right. Ramón Paz y Bicuspid came across the missing page when he browsed the 15th-century treatise on the Renaissance art of perfume making. The sketch became a great discovery, because the drawing established Leonardo as the first inventor of the valve flush toilet.



The missing page from Leonardo da Vinci's notebook.

The drawing, which is shown in the illustration, inspired Gardner to write his "discovery". In fact, Leonardo da Vinci's sketch was drawn by Anthony Ravielli, a graphic artist well known for his superb illustrations in books on sports, science, and mathematics. Gardner claims that: "Many years ago a friend of Ravielli's had jokingly made a bet with a writer that Leonardo had invented the first valve flush toilet. The friend persuaded Ravielli to do a Leonardo drawing in brown ink on faded paper. It was smuggled into the New York Public Library, stamped with a catalogue file number, and placed in an official library envelope. Confronted with this evidence, the writer paid off the bet".

The writer obviously was not the last one who was taken in by this joke. Gardner's hoax also gained entrance to Wikipedia. We can find this information under the reference to the European toilet paper holders (see [38]): "An important gap in the history of toilet paper receptacles was filled in 1974, with the discovery of a missing page from the Codex Madrid I, a notebook of Leonardo da Vinci found in Madrid's National Library in the 1960s. Ramon Paz y Bicuspid found the missing page, which verified a long-held belief that Leonardo had invented the first valve flush toilet. As you can see from the sketch (at right), the valves involved clearly double as toilet paper holders, and one of them is conveniently within an arm's-length of the seat."

The fact is that Leonardo da Vinci was one of the broad-based inventors and dealt with sewer system. One of the major advances in urban hygiene, which Leonardo wanted to implement, was an underground collection and disposal of household and street waste. This was the main cause of serious health hazards all over Europe and of several pestilence epidemics. When in 1484–86 pestilence epidemics hit Milan, Leonardo tried to create a plan for the ideal city for the French King Francouis I., where people would live better and healthier than in existing cities. The project, created by Leonardo da Vinci in 1516, was called Romorantin. A crucial role in this project played an underground collection and disposal of household and street waste. Romorantin, on the bank of the river Sauldre, is the capital of the Sologne region in France. The project also included a palace for the king with the series of flush toilets, including run-off channels in the walls inside and a ventilation system going through the roof. Unfortunately, as well as his plans for flying machines and military tanks, this project was destroyed and declared nonsense.

The name Augusto Macaroni is a wordplay on Augusto Marinoni, a da Vinci specialist at the Catholic University of Milan. Ramón Paz y Bicuspid is a play with words on Ramón Paz y Remolar, the man who actually found the two missing da Vinci notebooks.

5.5 Logical flaw in the special theory of relativity

Stunning is also the discovery of the mistake in the theory of relativity.

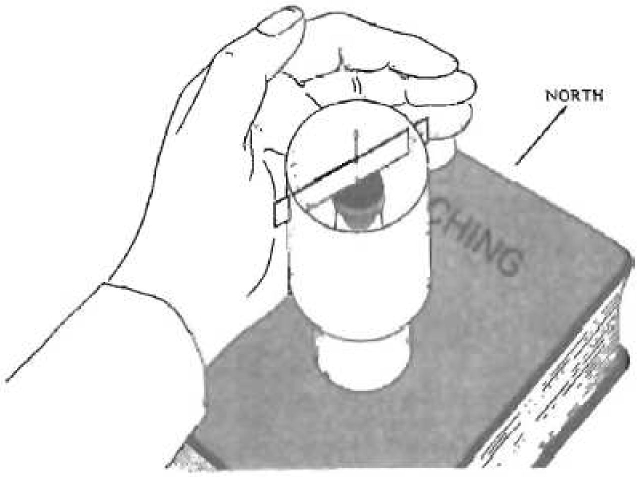
The crucial "thought experiment" is described in the paragraph about the mistake in the theory of relativity. It proves that a meter stick travels at a high speed along horizontal plate with a circular hole with diameter 1m centered. The plate is parallel with the stick's path and moves perpendicularly to it. In terms of system associated with a moving stick the situation appears that the front end of the stick exceeds a hole long before the rear section of the stick enters the hole, so the stick would not fall into the hole. These two situations are equivalent, however, and therefore the basic assumption of the theory of relativity is broken.

This relativistic paradox is well known. It is often referred to as the Meterstick and the hole paradox or Fast walking man paradox. The solution of this paradox lies in admitting the fact that rigid stick is relativistically unacceptable. The rigid sticks do not exist in relativity. The front part of the stick slightly bents at the entrance of the hole and bent stick can pass through the opening.

5.6 The psychic motor

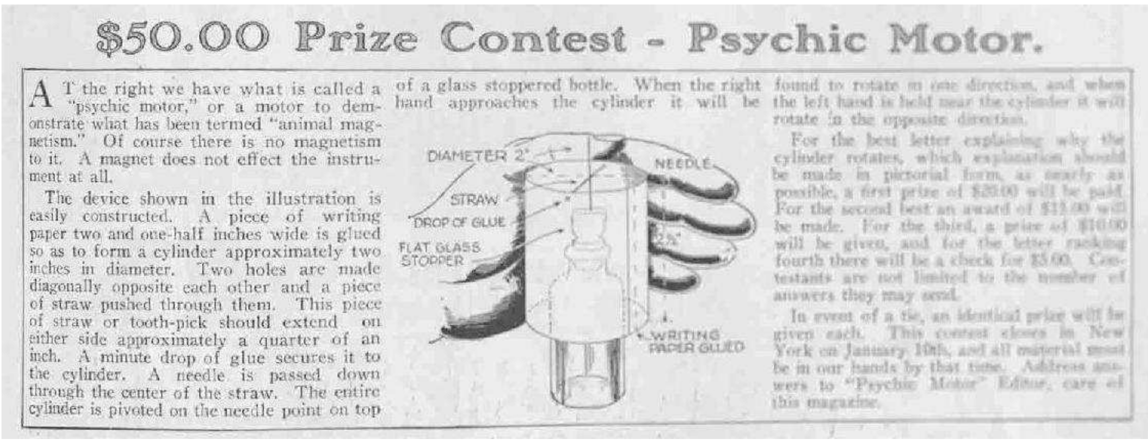
At the end of the article there was another blockbuster waiting for the hungry readers and it was a great discovery in parapsychology in the form of a simple motor that runs on psychic energy.

The motor was constructed by Robert Ripoff, the noted Prague parapsychologist and founder of the *International Institute for the Investigation of Mammalian Auras*, in 1973. Construction of the motor is not difficult – we need paper, needle, glass bottle, and the *Bible* or the *I Ching*. And then we have no choice but to lend a hand on it, make our mind blank and focus our mental energy on the motor until the psychic energy coming from our aura takes effect and the motor starts to rotate slowly.



The psychic motor from *Scientific American* magazine.

"Psychic motor" that is shown in the picture above, is a modification of Ripoff rotor, which was described by Hugo Gernsback in the magazine *Science and Invention* (see [11]). Readers of the magazine were asked to send the explanation what makes the cylinder turn. The best response should be rewarded with $20.



$50.00 Prize Contest - Psychic Motor.

AT the right we have what is called a "psychic motor," or a motor to demonstrate what has been termed "animal magnetism." Of course there is no magnetism to it. A magnet does not effect the instrument at all.

The device shown in the illustration is easily constructed. A piece of writing paper two and one-half inches wide is glued so as to form a cylinder approximately two inches in diameter. Two holes are made diagonally opposite each other and a piece of straw pushed through them. This piece of straw or tooth-pick should extend on either side approximately a quarter of an inch. A minute drop of glue secures it to the cylinder. A needle is passed down through the center of the straw. The entire cylinder is pivoted on the needle point on top of a glass stoppered bottle. When the right hand approaches the cylinder it will be found to rotate in one direction, and when the left hand is held near the cylinder it will rotate in the opposite direction.

For the best letter explaining why the cylinder rotates, which explanation should be made in pictorial form, as nearly as possible, a first prize of $20.00 will be paid. For the second best an award of $15.00 will be made. For the third, a prize of $10.00 will be given, and for the letter ranking fourth there will be a check for $5.00. Contestants are not limited to the number of answers they may send.

In event of a tie, an identical prize will be given each. This contest closes in New York on January 10th, and all material must be in our hands by that time. Address answers to "Psychic Motor" Editor, care of this magazine.

The article about the psychic motor from *Science and Invention* magazine.

In fact, the motion can be caused by any of three forces: slight air currents in the room, convection currents produced by heat of the hand, and currents cause by breathing. The three forces could be combined in many unpredictable ways, so we cannot say nor influence, in which side the cylinder would rotate.

Martin Gardner was considered to be one of the leading polemics against pseudoscientific and fringescientific theories, astounding discoveries, the paranormal and everything what became later known as pseudoscience. He was a founding member of the *Committee for the Scientific Investigation of Claims of the Paranormal*, in short CSICOP in 1976. The mission of the committee was to promote scientific inquiry, critical investigation, and the use of reason in examining controversial and extraordinary claims. From 1983 until 2002 Gardner wrote a column in magazine *Skeptical Inquirer*. Gardner was able to influence people's opinions and also mitigate the damage caused by pseudoscientists. "Bad science contributes to the steady dumbing down of our nation", declared Gardner (see [7]). In his articles, he always tried to put all these misleading and confusing information appearing in the media straight. Although his critics considered him as a very serious man, Gardner had a playful mind. He was often rather amused than outraged of many "amazing discoveries". He and H. L. Mencken said that (see [7]): "one horselaugh is worth ten thousand syllogisms." And he hoped that his readers would understand his article.

Several psychic motor appeared on sale after Gardner's hoax. Most readers, however, did not take this discovery seriously.

When Gardner published his article, he did not have the faintest ideas of the great acclaim he was going to get. Gardner claimed that the main purpose of the article was to entertain his readers. To his surprise, both the public and also professionals did not understand his joke and took the article seriously. He got thousands of letters from mathematicians and physicists. Many readers pointed out that he made a mistake in his article — but only in the field they specialized in, but the rest they considered unquestionable. Speaking up for the cheated readers, Gardner's article was a truly brilliant hoax in full details. When readers read only one paragraph, about the topic they were interested in, it was not clear at first sight that it was a hoax.

Gardner was one of the few authors who signed the article and proudly claimed responsibility for it. Benevolent readers forgave him soon. Everyone loved Gardner's columns for their originality, playfulness and witty spirit.

6 Legislating the value of π

Human imagination, however, will never be wild enough for some politicians' decisions. In 1998 *New Mexicans for Science and Reason*[18] magazine published the report that the Alabama state legislature passed a law redefining a mathematical constant π.

The law was passed on March 30, 1998, redefining the value of π to exactly integer three. The law took the state's scientific community by surprise ([25]): "It would have been nice if they had consulted with someone who actually uses π." Mathematicians form University of Alabama tried to explain to state legislature that π is a universal constant, and cannot be arbitrarily changed by lawmakers. In addition, we can never express it exactly because π is an irrational number, which means that it has an infinite number of digits after the decimal point. However, the mover did not listen to any arguments (see [25]): "I think that it is the mathematicians that are being irrational, and it is time for them to admit it. The Bible very clearly says in I Kings 7:23 that the altar font of Solomon's Temple was ten cubits across and thirty cubits in diameter, and that it was round in compass." On the contrary, he called into question the usefulness of any number that cannot be calculated exactly, and suggested that not knowing the exact answer could harm students' self-esteem (see [25]): "We need to return to some absolutes in our society, the Bible does not say that the font was thirty-something cubits. Plain reading says thirty cubits. Period." The members of the state school board supported the change in value of π, but they believed that the old value should be retained as an alternative (see [25]): "...the value of π is only a theory, and we should be open to all interpretations." Their idea was that students would be given the freedom to decide for themselves what value π should have.

Somehow bewildering is the fact that this joke was probably inspired by true events. In 1897, the legislature of Indiana attempted to give π an exact value by law. The Indiana Pi Bill was fabrication of Edwin J. Goodwin. This man was convinced he had found the solution of "squaring the circle" - an ancient problem whose task is to construct a square with a pair of compasses and a pair of scissors. This problem had been infatuated many thinkers, but it had already been proven impossible in 1882. Among other, Goodwin's contradictory explanation contained argument regarding relating to the diameter of a circle (see [31]): "...the fourth important fact, that the ratio of the diameter and circumference is as five-fourths to four."

We know that the ratio of the diameter to the circumference is equal to π, so Goodwin was effectively dictating a value for π according to the following recipe:

$$\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{4}{\frac{5}{4}} = 3,2$$

Goodwin reportedly said (see [31]): "Indiana schools could use his discovery without charge, but that the state and he would share the profits from royalties charged to other schools who wished to adopt a value of 3,2 for π." The technical nature of the bill so baffled the politicians that it being passed without any objection.

[18] *New Mexicans for Science and Reason* is a non-profit group with the goals of promoting science, the scientific method, rational thinking, and critical examination of dubious or extraordinary claims (see [25]).

However, considerations of whether the number π can be expressed as an integer constant, are completely misleading. The number π, which is approximately $3,141\,592\,6$, is the length of the circumference of a circle whose diameter is exactly 1. Generally speaking, a circle of diameter d has a circumference of $\pi\, d$. The number π cannot be expressed as a fraction and it really is an irrational number. The simplest proof that p is irrational uses calculus, and it was founded by Johann Lambert [19] in 1770. More strongly, π is transcendental. It means that it does not satisfy any algebraic equation that relates it to rational numbers. In 1882, this property was proved by Ferdinand Lindemann, [20] also using calculus. The fact that π is transcendental implies that the classical geometric problem of 'squaring the circle' is impossible.

Although there were lots of evidences suggesting deception in the article, for example, the fictitious names of people who wanted to enforce the law, thanks to the Internet the article was passed on to many different countries. In a short time the article spreaded in the world. But, as the story was forwarded from person to person, all of the deliberate hints disappeared. The names, which were originally fictitious, began to be gradually replaced by the names of particular living people working at the university or in the Alabama state legislature. Gradually, the newer and better versions of the original article began to appear, cutting their way through the internet. All funny hints from the article disappeared and it became more and more credible. As the general public was more and more acquainted with the article, the Alabama state legislature began to receive hundreds of letters and phone calls from people who protested against this legislation.

7 Conclusion

The previous chapters contain cross-section of fool hoax articles from the past to the present. The fact is that both the daily press or in the serious magazine, the authors of fool hoax articles follow a similar goal - to entertain its readers and force them to critically assess the content of the report and while watching TV or reading newspapers to ask themselves whether the given report may be true or not. But the question is, if this is not a realistic goal.

Not all information can be verified and not all are connected to the lack of knowledge among the recipients. And thus created simple or complex judgments about the facts. Every person, even the great professionals, has many ideas about different things, people, institutions, countries, etc. Modern man can not cover all the knowledge and information that surround it. Expertise based on knowledge in a particular field then exposes humans

[19] Johann Heinrich Lambert (1728—1777) was a Swiss mathematician, physicist, philosopher and astronomer. He is best known for proving the Irrationality of π. He was the first to introduce hyperbolic function into trigonometry. Also, he made conjecture regarding non-Euclidean space. In Photometria Lambert also formulated the law of light absorption – the Beer-Lambert law (see [15]).

[20] Carl Louis Ferdinand von Lindemann (1852—1939) was a German mathematician, noted for his proof, published in 1882, that π is a transcendental number. His methods were similar to those used nine years earlier by Charles Hermite to show that , the base of natural logarithms, is transcendental (see [6]).

to receive and formed the only opinion in other fields. These opinions - unlike knowledge - have a stron irrational character and are not only view but also a conviction.

In everyday life we rarely verify the information that we hear from the media. Social life is based on trust that the task of verifying the report is commissioned by someone. If we read the report in the newspaper, we assume that it is proven, though for we have no proof. We rely on responsibility and professional duties of journalists.

The authors of the hoax articles are always surprised and shocked by the apparent lack of will verify the facts, which are given in the article (see [20]): "The journalistic profession the verifying, what is then further distributed to thousands of people, is a fundamental requirement. Leading French journalist Jean Lacouture reminds aptly that the role of the journalist is not so much to spread the message about the birth or death of the king, but rather to refute or confirm rumors that one or the other would accelerate, accompanied or distorted."The fact that it is necessary to inculcate future journalists verification reflex, mean that this kind of reaction is not spontaneous. Why do we take for normal that nobody – or almost nobody – not verifies the information in the newspaper? There are some situations when we try to verify information from a newspaper or other media, especially if the report is born on us, for example stock ticker, military or other major decisions. On the other hands, the cases if we act without apparent risk eliminate the need for verification. If we are not forced to make any decisions, the motivation for authentication is missing. Only the professional skeptics (such as journalists) or those with their own personal interests can make an effort to explore more information.

Moreover, we cannot argue that the people believe everything what the media give them. If we read the report in the newspaper or magazine, tacitly assume that the news passed the filter in some groups, such as the editorial board, before the news came to us. If it was a hoax, it would have not believed so many people would not get to us.

For the media, which created a ethical code and behave according to him, this code can be considered a guarantee objectivity and proper etiquette. Still, however, the rules of behavior by the ethical code are not in principle for the public enforceable. It depends only on the media; whether they will act as powerful without liability or whether they will aspire higher and look for what is one of the deepest level of humanity – the good and the associated truth. The problem of the April articles is that although the media behave unethically in that moment, it still remains within the law. Then the public can do nothing else but be a little indulgent and taken the April cells with reserve.

The question is whether, in the upcoming years, the meaning of April articles will tend to decrease, as even completely serious news often seem unbelievable and people are confused whether the April Fool's Day is not in December. But now, we can do nothing but to look forward to April and new kinds of joke the professionals, for sure, are coming up with.

References

[1] APPEL, K., HAKEN, W.: *Every planar map is four colorable, Parts I, II.* Illinois J. Math. *21* (1977), 429–490, 491–567.

[2] ACHESON, D. *1089 and all that, a journey into mathematics.* New York: Oxford university press, c2002, p. 147–158.

[3] APRIL HOLIDAY. Alabama Legislature Lays Siege to Pi. *New Mexicans for Science and Reason*, April 1998, vol. 4, no. 4.

[4] *Edgar Allan Poe.* [online], last modified on 7. March 2014 [cit. 2014-03-09]. Dostupné na World Wide Web: <http://cs.wikipedia.org/wiki/Edgar_Allan_Poe>

[5] ELLMORE, F. S. It is only hypnotism. *Chicago Daily Tribune*, 9th August 1890, p. 9.

[6] Ferdinand von Lindemann [online], last modified on 1 March 2014 [cit. 2014-03-15]. Available from: `http://en.wikipedia.org/wiki/Ferdinand_von_Lindemann`

[7] FRAZIER, K.: *A Mind at Play: An Interview with Martin Gardner* [online], [cit. 2014-2-10]. Available from: `http://www.csicop.org/si/show/mind_at_play_an_interview_with_martin_gardner/`

[8] GARDNER, M.: *Six sensational descoveries that somehow or another have escaped public attention.* Scientific American *April* (1975), 126–133.

[9] GARDNER, M.: *The Colosall Book of Mathematics: Classic Puzzles, Paradoxes, and Problems: Number Theory, Algebra, Geometry, Probability, Topology, Game Theory, Infinity, and Other Topics of Recreational Mathematics.* W. W. Norton & Company Ltd., New York, 2001, 580–591.

[10] GERNSBACKER, H. *Gravity Nullified: Quartz Crystals Charged by High Frequency Current Lose their Weight.* Science and Invention, (September Issue) p. 398. Dostupné na World Wide Web: <www.theorionproject.org/en/research.html >

[11] GERNSBACK, H.: *$50.00 Prize Contest - Psychic Motor.* Science and Invention, *November* (1923), 651.

[12] Gravity Nullified – A Hoax. *Science and Invention*, October 1972, p. 509.

[13] *Indian rope trick.* [online], last modified on 5. March 2014 [cit. 2014-03-09]. Dostupné na World Wide Web: <http://en.wikipedia.org/wiki/Indian_rope_trick>

[14] JIRÁK, J.; KÖPPLOVÁ, B. *Média a společnost.* Praha: Portál, c2007, p. 31, ISBN 80-7367-287-1.

[15] Johann Heinrich Lambert [online], last modified on 8 March 2014 [cit. 2014-03-15]. Available from: `http://en.wikipedia.org/wiki/Johann_Heinrich_Lambert`

[16] *John Herschel.* [online], last modified on 11. November 2013 [cit. 2014-03-09]. Dostupné na World Wide Web:

[17] *John Wilkie.* [online], last modified on 14. April 2013 [cit. 2014-03-22]. Dostupné na World Wide Web:

[18] *Joseph Nicollet.* [online], last modified on 8. December 2013 [cit. 2014-03-09]. Dostupné na World Wide Web: <http://en.wikipedia.org/wiki/Joseph_Nicollet>

[19] *Hugo Gernsback* [online], c2013 [cit. 2013-12-28]. Dostupné na World Wide Web: <http://www.nndb.com/people/381/000045246/>

[20] KAPFERER, J.-N. *Fáma - nejstarší médium světa.* 1. vydání, Praha: Nakladatelství Práce, c1992, ISBN ???.

[21] KÖPPLOVÁ, B.; KÖPPL, L. *Dějiny světové žurnalistiky. Celý svět v novinách.* 1. vydání, Brno: Vydavatelství a nakladatelství Novinář, c1989, ISBN 80-7077-216-6.

[22] KRISTELLER, P. O. *Studies in Renaissance and Letters. Scientific American*, October 1977, vol. 237, no. 4, p. 108–121.

[23] Kurt Heegner [online], last modified on 21 October 2013 [cit. 2014-03-22]. Available from: `http://en.wikipedia.org/wiki/Kurt_Heegner`

[24] LAMONT, P. *The rise of the indian rope trick.* First Edition, 2005, Great Britain: Abacus, c2004, ISBN 0-349-11824-8.

[25] New Mexicans for Science and Reason [online], last modified on 8 March 2014 [cit. 2014-03-15]. Available from: `http://www.nmsr.org/`

[26] *Newsstand: 1925: Science and Invention*[online], [cit. 2014-03-22]. Dostupné na World Wide Web: <http://uwf.edu/dearle/enewsstand/enewsstand_files/Page3172.htm>

[27] O'BRIEN, F. M. *The Story of the Sun.* [online]. c2006, [cit. 2006-02-27] Dostupné na World Wide Web: <https://archive.org/details/storysunnewyork00obrgoog>

[28] OSVALDOVÁ, B.; HALADA J. a kol. *Encyklopedie praktické žurnalistiky.* 1. vydání, Praha: Nakladatelství Libri, c1999, ISBN 80-85983-76-1.

[29] *Peter Lamont (historian).* [online], last modified on 19. February 2014 [cit. 2014-03-15]. Dostupné na World Wide Web:
<http://en.wikipedia.org/wiki/Peter_Lamont_%28historian%29>

[30] RAMANUJAN, S.: *Modular Equations and Approximations to* π. The Quarterly Journal of Pure and Applied Mathematics *45* (1994), 350–372.

[31] SINGH, S.: *The Simpsons and their mathematical secrets.* Bloomsbury, New York, 2013.

[32] Srinivasa Ramanujan [online], last modified on 13 March 2014 [cit. 2014-03-22]. Available from: `http://cs.wikipedia.org/wiki/Srinivasa_Ramanujan`

[33] STEWART, I. *Kabinet matematických kuriozit profesora Stewarta* 1. vydání v českém jazyce, Praha: Dokořán, c2013, ISBN 978-80-7363-292-2.

[34] *Tajemní fakíři a jogíni* [online]. c2010, [cit. 2014-03-22] Dostupné na World Wide Web: <http://www.gamepark.cz/tajemni_fakiri_a_jogini_277941.htm>

[35] *The Great Moon Hoax of 1835* [online]. c2011, [cit. 2006-02-27] Dostupné na World Wide Web:
<http://www.museumofhoaxes.com/hoax/archive/permalink/the_great_moon_hoax>

[36] *The Times.* [online], last modified on 8. March 2013 [cit. 2014-03-14]. Dostupné na World Wide Web: <http://en.wikipedia.org/wiki/The_Times>

[37] TOLAN, J. Queries and answers. *Chicago Daily Tribune*, 6th December 1890, p. 4.

[38] User:Babajobu/European toilet paper holder [online], last modified on 31 January 2013 [cit. 2014-02-27]. Available from:
`http://en.wikipedia.org/?title=User:Babajobu/European_toilet_paper_holder`

[39] WILSON, M. R. *Chicago Tribune.* [online], c2004 [cit. 2014-03-15]. Dostupné na World Wide Web: <http://www.encyclopedia.chicagohistory.org/pages/275.html>

Paper V

T. Bártlová: *New ways in teaching of mathematics*, Manuscript submitted, 2016.

The preface to the article *Science center*

The article on the science center refers to the educational aspect of recreational mathematics. Although generally the teaching of mathematics should be fun, we cannot just have fun at the school. Recreational mathematics is primarily realized by the professional mathematicians who know that if we want to study a fun mathematical application, we must also learn something. A book on recreational mathematics does not primarily aim to remake the school curriculum, it aims to motivate students to study mathematics.

Science centers are one of the ways to achieve it. Children can spend their free time, beyond normal school hours. A fun way can verify their knowledge acquired in school. The aim of the science center is the idea that everything that kids learn in school can be used in practice, and therefore it is important to learning math.

New ways in teaching of mathematics

Tereza Bártlová

Charles University in Prague

Mathematics education is usually associated with the school teaching. The education is not just about the volume of knowledge, but a permanent mental mastery of skills and abilities to use them. The mathematics education is possible in addition to the classroom to develop in completely unconventional environments such as interactive science centers, which formed in a quarter of a century around the world. In these centers are created favorable conditions for education of children and they can learn while enjoying themselves in their spare time.

1 WHAT IS A SCIENCE CENTER?

A science center, called also science museum, interactive science center or hands-on center is an environment designed for experimenting – essentially experiment-based and usually in an exhibition form. There are dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of technology and natural science. "Hands-on" and "interactive" are the essential qualities of the science center experience. As science center offers direct experience with natural phenomena, a first step on the path towards scientific reasoning. Visitors are not expected to be passive spectators but are presented with countless opportunities to have an experience that stimulates their senses and intellect. It means that all of the exhibits invite participation. The traditional museum "don't touch", sign is replaced by "try it"! In other words: Nothing will happen unless you make it happen!

In the science center you can choose the experiment as you want. You decide whether you want to focus on a particular area or simply to try something from everything. It is not critical whether you see the world through the eyes of a scientist of an artist, because the art and science complement each other in intriguing ways, as evident in many of the exhibits. The visitors are in full control of the exhibits. Interaction in the science center also implies dealing with actual things. Struggling to understand is rendered much easier through hands-on activity, through individual participation. Above all, it requires time and leisure. The hands-on, individual working with experiments, watching things happen, leads to the development of a real fascination with technology and natural sciences. Again and again, one will experience the joy of a real "Eureka!"- moment!

In the science center you can usually spend as much time as you like on your own experimenting at an exhibit. None will interfere and you are not dependent on the instruction of an expert. However, if you own attempts are unsuccessful, one of the attentive but unobtrusive staff assistants is available to help with useful tips and suggestion.

In the science center, it is not unusual that the participants mutually exchange their experience of dealing with exhibits. Science center's unique, informal atmosphere lowers the barriers between younger and older generations. The all-ages spontaneous exchanges of opinion and even full-blow scientific discussions, is an integral part of the science center experience. Visitors find themselves engrossed in conversation with complete strangers whom they would ordinarily never speak with. And the understanding thrives as a result of social interaction and social context.

2 THE IMPORTANCE SCIENCE CENTER IN EDUCATION

Importance of science centers is increasing in these days when we realize that the education of natural sciences should not be associated only with school teaching. An additional education outside the classroom is also meaningful.

The science teaching cannot lay much emphasis on experiencing the phenomena themselves. The learning must begin with real observation and not with verbal description of things. Only this way can you develop a secure knowledge. The extensive range of interactive exhibits in science center can help children and adolescents to think independently and to reason. Prof. Kurt Reusser and Dr. Urs Aeschbacher of the Pedagogical Institute of the University of Zurich claim that [7]: "Experimental environments, unlike schools, are truly greated towards transmitting a perfect educational experience – they render any external learning motivation unnecessary."

This idea was also supported by several researches. The conditions, which are created for the education of children in the science center, have been investigated in detail (for example see [4], [5] or [6].) Falk and Storksdieck observed different types of learning on a wide variety of respondents who visited a life science exhibition at a major science center (in this case it was an biological theme exhibition), see [5]. They observed the process of learning from different viewpoints and concluded that environment developed to support real world, such as the Science Centre, are not mere props to support knowledge transfer, but it is what the authors Barab and Kirshner in his article [4] called dynamic learning environment.

3 MATHEMATICS IN THE SCIENCE CENTER

The critical question, whether is a similar way, how it is possible to get mathematics to the science center, was asked by the founders of the science museums already in the early 90s of the 20th century. Many of them thought that it is a very difficult task. For example, Michel Demazure, French mathematician and later a director of two French science museums claimed: “the development of mathematical exhibits is one of the most demanding tasks of a science center,” but absolutely necessary.

The mathematical learning process is not limited to memorization or drills. Memorizing formulas without understanding brings dislike of mathematics. We know that abstract concepts which have not grown out of actual sense experience are likely to be misunderstood and treated almost like magical notions and mathematics is more than arithmetic. We need some out-of-school opportunity to consolidate students' knowledge so, exactly what science center provides, is fundamentally indispensable. This, of course, is parallel to science center's usage as a leisure time activity, which stimulates learning, making it so much easier and more permanent. And the science center is one of the possibility how we can enthuse children, students for mathematics. For example, we can to demonstrate what happens when you cut into a cone or other solid. Where do the various remarkable and useful curves originate. How can we create a Möbius strip and what arises when you start to cut the strip with scissors in different directions. How does the Pythagorean theorem work, where does the number π take. What are Platonic or Archimedean solids and so others. A visitor who regards mathematics as a dull domain of chalkboards and calculators will see the colors and shapes and dynamism of science center and will have his eyes opened to the unexpected possibilities of mathematics. Moreover, it is very interesting that many a great scientist has stressed that they were puzzlists in their early days, like Tyndall, Huxley, Humboldt, Darwin, Edison or Euler, so we can say that their early puzzle training gave the bent to their minds which in after years inclined them to grapple with problems of greater magnitude. Sam Loyd, American chess player, puzzle author and recreational mathematician said [3]: “We see how the average boy, who abhors square root or algebra, will find delight in working out puzzles which involve identically the same principles. It makes one think that millions of earnest students who would really have loved to learn, have been abandoned as incorrigible blockheads, because those who had charge of their education did not know how to interest them in their studies. An aversion to figures and desire to forget all about mathematics as soon as one leaves school is almost universal, and yet, if the subject had been taught in more congenial way, the mathematics and inventive bumps might have developed in a way to astonish the family phrenologist.”

In the following years, it showed that it is not uphill battle to develop mathematical exhibits thanks to recreational mathematics. One of the leading protagonists of recreational mathematics is considered Martin Gardner who defined the recreational mathematics as a mathematics which [1]: “include anything that has a spirit of play about it”.

Nowadays the development of the mathematical exhibits is based on cooperation the recreational mathematicians with designers and engineers. Moreover it appears that a future have also separate centers focus only on mathematics.

4 THE MOST FAMOUS SCIENCE CENTERS WITH MATHEMATICAL EXHIBITS IN THE WORLD

The beginnings of the development of science centers are associated with two important science centers.

In 1888 it was realized quite revolutionary project in Berlin, which should to present the scientific findings and inventions not only students, but also to a wide audience. It was the first world science center, called **Urania**.

Nearly a hundred years later, in the fall of 1969 was opened **Exploratorium** in San Francisco.

The most important mathematically-oriented science center in the world was **Goudreau Museum of Mathematics in Art and Science** in New York. The museum was founded in 1980 and it was named after teacher of mathematics Bernhard Goudreau. Unfortunately, it was closed in 2006. Many mathematicians and educators of mathematics were disappointed because of this. They perceived that the closing of this museum was not just about the closing one of the many museums, but it pointed out the problem with the view and attitude toward mathematics and the role it plays in the culture. In August 2008, the group of interested people met to explore the creation of a new museum of mathematics. They discovered that there was no museum of mathematics in the United States, so they opened big science center in Manhattan called **MoMath**. And today we can say that it is attracted over 170 000 visitors in one year.

Other mathematical exhibits we can find also in other museum in the world [2]. These include, for example, the **German Museum** in Munich, **Mathematikum** in German, **Technorama** in Switzerland, **Palais de Découverte** in France or **Heureka** in Finland and others.

References

[1] Barcellos, A, Gardner, M.: *A Conversation with Martin Gardner. The Two-Year College Mathematics Journal,* 10(4), 1979, 233-244.

[2] Ecsite. The European Network of Science Centers [online]. Available from World Wide Web: http://www.ecsite.eu/

[3] Sam Loyd. Sam Loyd official site [online]. Available from World Wide Web: http://www.samloyd.com/

[4] Barab, S. A., Kirshner, D.: *Rethinking methodology in the learning sciences,* Journal of the Learning Sciences, 10(1/2), 2001, 5-15.

[5] Falk, J. H., Storksdieck, M.: *Using the contextual model of learning to understand visitor learning from a science center exhibition,* Sci. Ed., 2005, 745-778.

[6] Falk, J. H., Needham, M. D.: *Measuring the impact of a science center on its community,* J. Res. Sci. Teach., 48(1), 2011, 1-12.

[7] Technorama [online]. Available from World Wide Web: http://www.technorama.ch/en/about-us/a-science-centre/