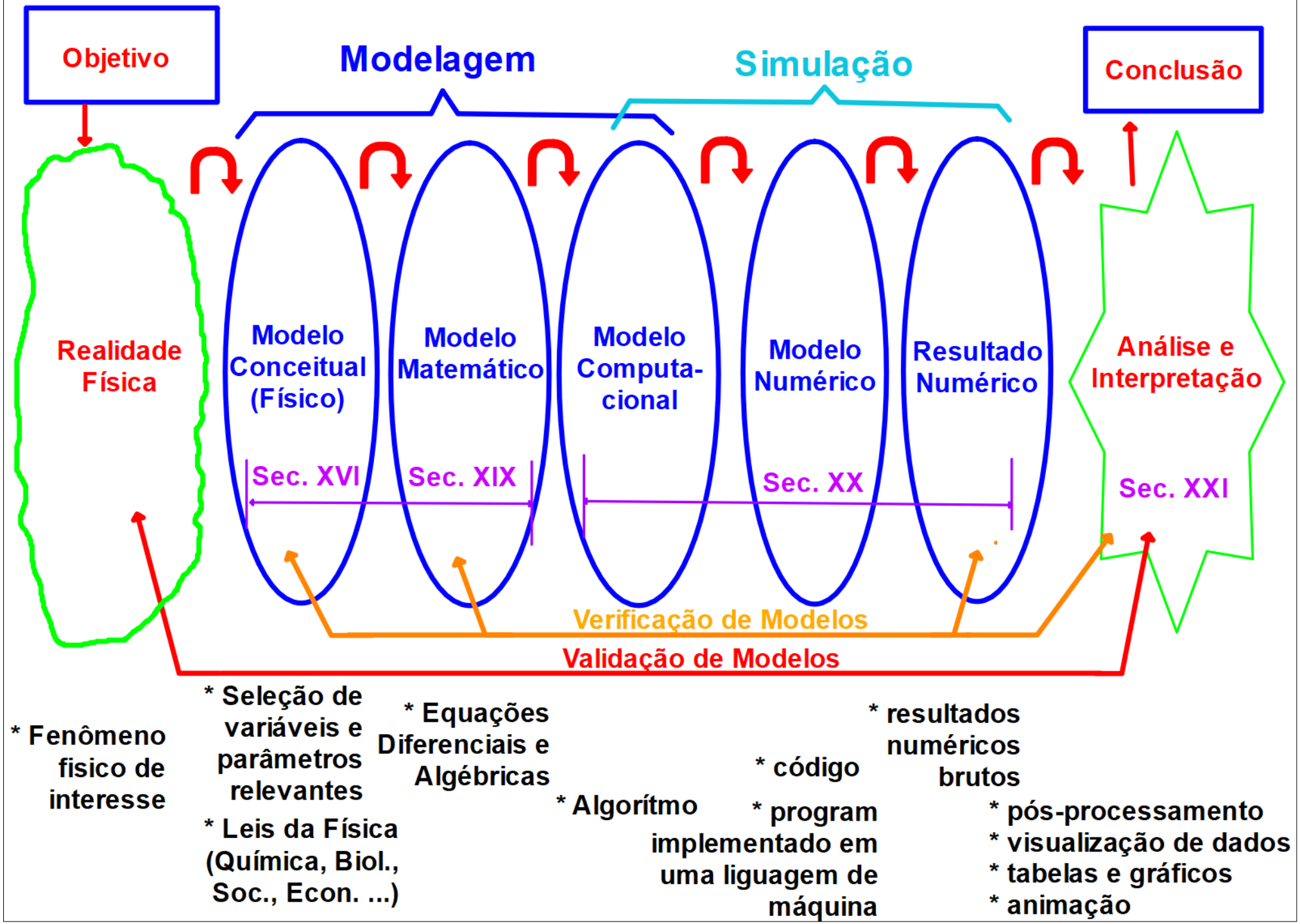
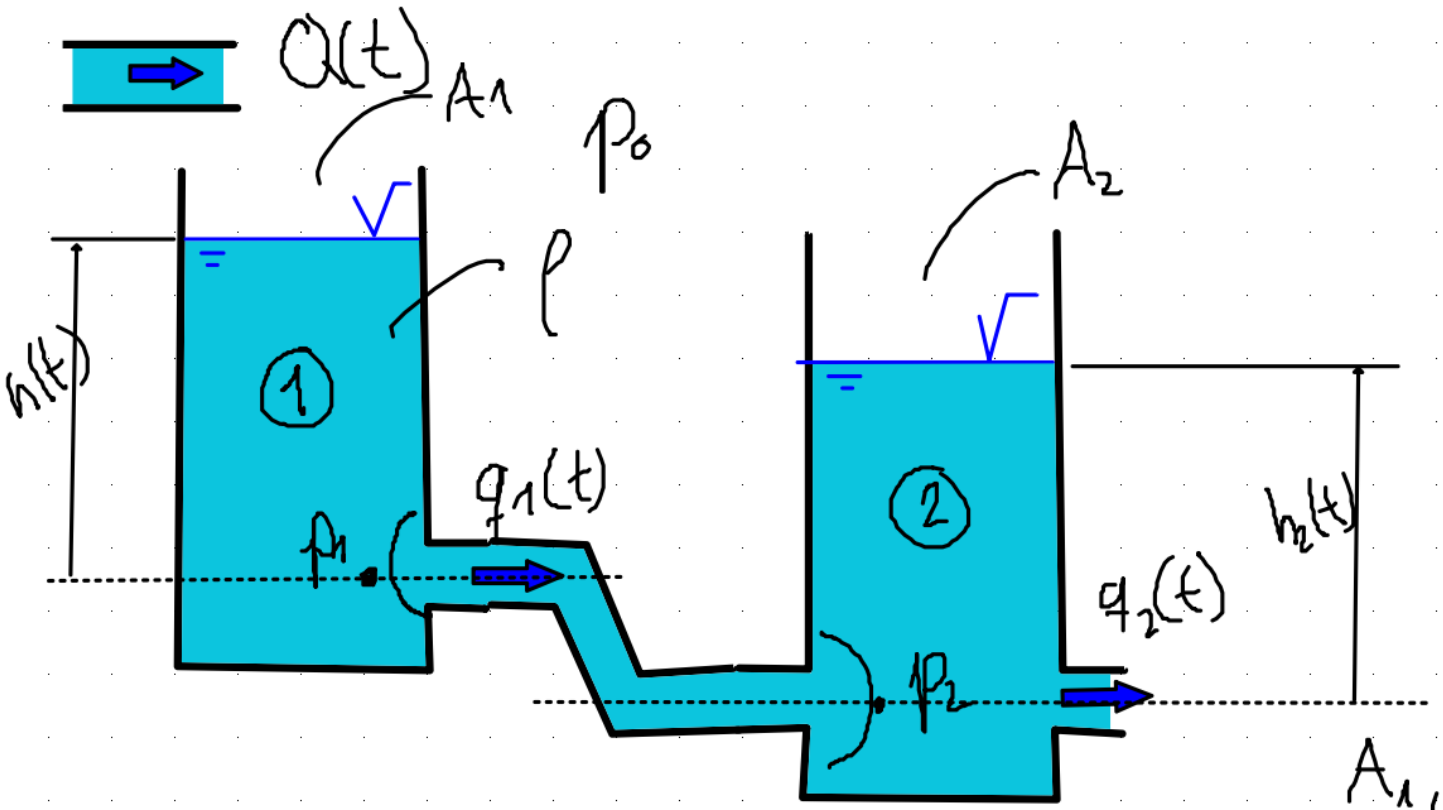

Objetivo
Modelagem
Simulação
Conclusão
Realidade Física
Modelo Conceitual (Físico)
Modelo Matemático
Modelo Computa-cional
Modelo Numérico
Resultado Numérico
Análise e Interpretação
Sec. XVI
Sec. XIX
Sec. XX
Sec. XXI
Verificação de Modelos
Validação de Modelos
* Fenômeno fisico de interesse
* Seleção de variáveis e parâmetros relevantes
* Leis da Física (Química, Biol., Soc., Econ. ...)
* Equações Diferenciais e Algébricas
* Algorítmo
* código
* program implementado em uma liguagem de máquina
* resultados numéricos brutos
* pós-processamento
* visualização de dados
* tabelas e gráficos
* animação



A_1, A_2 grandes

$$A_1 \frac{dh_1}{dt} = Q(t) - q_1(t)$$

$$A_2 \frac{dh_2}{dt} = q_1(t) - q_2(t)$$

$$p_1 = p_0 + \rho g h_1$$

$$p_2 = p_0 + \rho g h_2$$

$$K = f \frac{L}{D}$$

$$f = f(Re, \epsilon)$$

$$|p_1 - p_2| = R_1 q_1^2$$

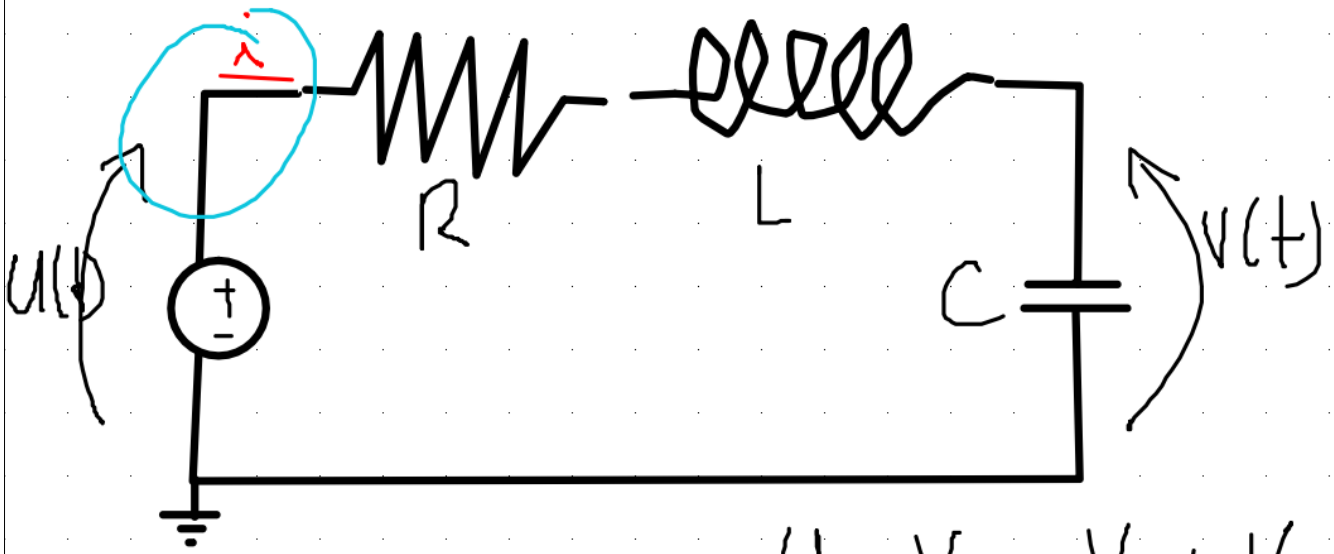
$$|p_2 - p_0| = R_2 q_2^2$$

$$R = f(Re, \epsilon)$$

$$R = \frac{K}{2g A^2}$$

$$K = \sum K_S + K_L$$

$$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{\epsilon}{3{,}70} + \frac{2{,}51}{Re\sqrt{f}}\right)$$



$$U = V_R + V_L + V$$

$$V_R = Ri$$

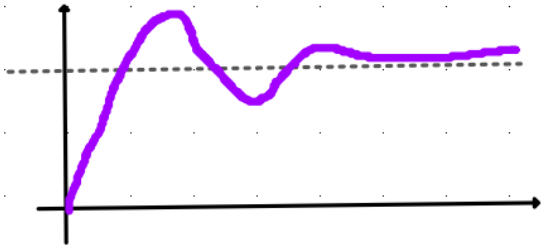
$$V_L = L\frac{di}{dt}$$

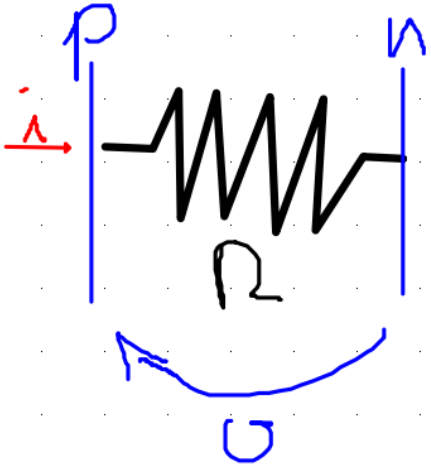
$$Ri + L\frac{di}{dt} + V = U$$

$$i = C\frac{dV}{dt}$$

$$LC\frac{d^2V}{dt^2} + RC\frac{dV}{dt} + V = U$$

$$U(t) = \begin{cases} 0 & t < t_0 \\ U_0 & t \geq t_0 \end{cases}$$



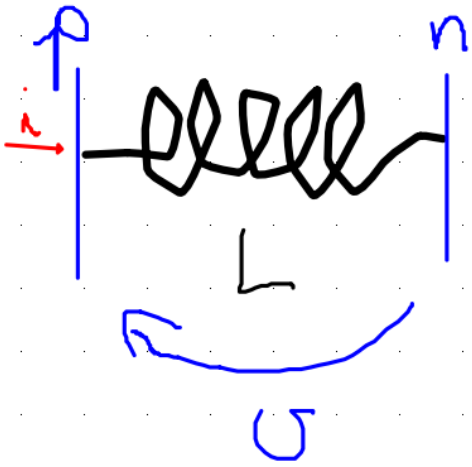


$$u = Ri$$

$$i = p_0 i$$

$$u = p_0 u - n_0 u$$

$$p_0 i + n_0 \tilde{i} = 0$$

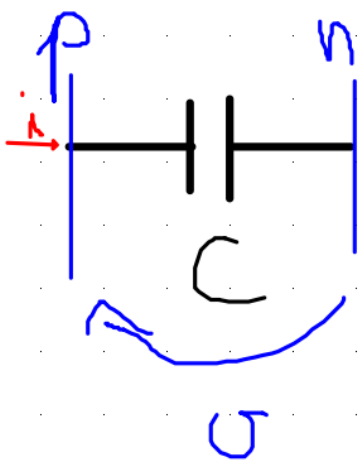


$$u = L\frac{di}{dt}$$

$$i = p_0 i$$

$$u = p_0 u - n_0 u$$

$$p_0 i + n_0 i = 0$$

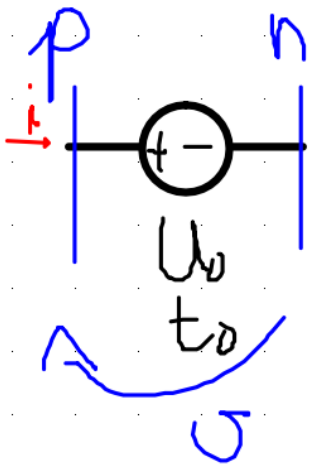


$$i = C\frac{du}{dt}$$

$$i = p_1 i$$

$$u = p_0 u - n_0 u$$

$$p_0 i + n_0 i = 0$$



$$u = U(t)$$

$$i = p_0 i$$

$$u = p_0 u - n_0 u$$

$$p_0 i + n_0 i = 0$$

$$u = 0$$

$$u = C_0 u$$

$$A_1 \frac{dh_1}{dt} = Q - q_1$$

$$A_2 \frac{dh_2}{dt} = q_1 - q_2$$

$$p_1 - p_2 = R_1 q_1^2$$

$$p_2 - p_0 = R_2 q_2^2$$

$$p_1 - p_0 = \rho g h_1 = R_1 q_1^2$$

$$p_2 - p_0 = \rho g h_2 = R_2 q_2^2$$

$$\frac{dh_1}{dt} = \frac{R_1}{\rho g} 2 q_1 \frac{dq_1}{dt}$$

$$\frac{dh_2}{dt} = \frac{R_2}{\rho g} 2 q_2 \frac{dq_2}{dt}$$

$$\frac{2 A_1 R_1}{\rho g} q_1 \frac{dq_1}{dt} = Q - q_1$$

$$\frac{2 A_2 R_2}{\rho g} q_2 \frac{dq_2}{dt} = q_1 - q_2$$

$$\begin{cases} \dfrac{2A_1R_1}{\ell g} q_1 \dfrac{dq_1}{dt} + q_1 = Q(t) \\ \dfrac{2A_2R_2}{\ell g} q_2 \dfrac{dq_2}{dt} - q_1 + q_2 = 0 \end{cases}$$

Solução: dado $Q(t) \Rightarrow$ $q_1(t) = ?$

$q_2(t) = ?$

$$\begin{cases} \dfrac{2A_1R_1}{\rho g} q_1 \dfrac{dq_1}{dt} + q_1 = Q(t) \\ \dfrac{2A_2R_2}{\rho g} q_2 \dfrac{dq_2}{dt} - q_1 + q_2 = 0 \end{cases}$$

Caso particular:

$Q(t) = 0$ $\qquad A_1 = A_2 = A$

$q_2(t) = 0$

$$\frac{2AR}{\rho g} q \frac{dq}{dt} + q = 0$$

$$q\frac{dq}{dt} = \frac{-\rho_0}{2AR} q \qquad q(t) \neq 0$$

$$\frac{dq}{dt} = -\frac{\rho_0}{2AR} \Rightarrow q(t) = q_0 - \frac{\rho_0}{2AR} t$$

$$\frac{2AR}{\rho_0} q\frac{dq}{dt} + q = 0$$

$$\Delta p = R q^2$$

$$f(q) = R q^2 \Rightarrow \frac{\partial f}{\partial q} = 2Rq$$

$$f(q) = f(Q_0) + \frac{1}{1!}\frac{\partial f}{\partial q}\Big|_{Q_0}(q - Q_0) + \frac{1}{2!}\frac{\partial^2 f}{\partial q^2}\Big|_{Q_0}(q - Q_0)^2 + \ldots$$

≈ 0

$$f(q) = R Q_0^2 + 2RQ_0(q - Q_0)$$

$$f(q) = -RQ_0^2 + 2RQ_0 q$$

$$A_1 \frac{dh_1}{dt} = Q - q_1 \qquad \rho g h_1 = p_1 - p_0 = -RQ_0^2 + 2RQ_0 q_1$$

$$A_2 \frac{dh_2}{dt} = q_1 - q_2 \qquad \rho g h_2 = p_2 - p_0 = -RQ_0^2 + 2RQ_0 q_2$$

$$\rho g \frac{dh_1}{dt} = 2RQ_0 \frac{dq_1}{dt}$$

$$\rho g \frac{dh_2}{dt} = 2R_2 Q_0 \frac{dq_2}{dt}$$

$$\frac{A_1 2RQ_0}{\rho g} \frac{dq_1}{dt} = Q - q_1 \quad \Big| \quad \frac{A_2 2R_2 Q_0}{\rho g} \frac{dq_2}{dt} = q_1 - q_2$$

$$\begin{cases} \dfrac{dq_1}{dt} = -\dfrac{\rho g}{2R_1 A_1 Q_0} q_1 + \dfrac{\rho g}{2R_1 A_1 Q_0} Q \\ \dfrac{dq_2}{dt} = +\dfrac{\rho g}{2R_2 A_2 Q_0} q_1 - \dfrac{\rho g}{2R_2 A_2 Q_0} q_2 \end{cases}$$

$$\tau_1 = \frac{2R_1 A_1 Q_0}{\rho g} \qquad \tau_2 = \frac{2R_2 A_2 Q_0}{\rho g}$$

$$\frac{A_1 2R_1 Q_0}{\rho g}\frac{dq_1}{dt} = Q - q_1 \;\Big|\; \frac{A_2 2R_2 Q_0}{\rho g}\frac{dq_2}{dt} = q_1 - q_2$$