Dedução do retorno e da variância da carteira

Fórmulas base

$média = E[x]$

$variância = VAR[x] = E[x^2] - (E[x])^2$

$\tilde{r}_c = x_1 . \tilde{r}_1 + x_2 . \tilde{r}_2$

Retorno

$\bar{r}_c = E[\tilde{r}_c] = E[x_1 . \tilde{r}_1 + x_2 . \tilde{r}_2]$

$\bar{r}_c = E[x_1 . \tilde{r}_1] + E[x_2 . \tilde{r}_2]$

$\bar{r}_c = x_1 . E[\tilde{r}_1] + x_2 . E[\tilde{r}_2]$

$\bar{r}_c = x_1 . \bar{r}_1 + x_2 . \bar{r}_2$

Variância

$variância = \sigma_{r_c}{}^2 = VAR[\tilde{r}_c] = E[\tilde{r}_c{}^2] - (E[\tilde{r}_c])^2$

por partes

$1. E[\tilde{r}_c{}^2] = E[(x_1 . \tilde{r}_1 + x_2 . \tilde{r}_2)^2]$

$= E[(x_1 . \tilde{r}_1)^2 + (x_2 . \tilde{r}_2)^2]$

$= E[(x_1)^2 . (\tilde{r}_1)^2 + 2.\mathrm{x}_1 . x_2 . \tilde{r}_1 . \tilde{r}_2 + (x_2)^2 . (\tilde{r}_2)^2]$

$= E[(x_1)^2 . (\tilde{r}_1)^2] + E[2.\mathrm{x}_1 . x_2 . \tilde{r}_1 . \tilde{r}_2] + E[(x_2)^2 . (\tilde{r}_2)^2]$

$= x_1^2 . E[(\tilde{r}_1)^2] + 2.\mathrm{x}_1 . x_2 . E[\tilde{r}_1 . \tilde{r}_2] + x_2^2 . E[(\tilde{r}_2)^2]$

$2. (E[\tilde{r}_c])^2 = (E[x_1 . \tilde{r}_1 + x_2 . \tilde{r}_2])^2$

$= (x_1 . E[\tilde{r}_1] + x_2 . E[\tilde{r}_2])^2$

$= x_1^2 . (E[\tilde{r}_1])^2 + 2.\mathrm{x}_1 . x_2 . E[\tilde{r}_1] . E[\tilde{r}_2] + x_2^2 . (E[\tilde{r}_2])^2$

juntando tudo

$E[\tilde{r}_c{}^2] - (E[\tilde{r}_c])^2 = x_1^2 . E[(\tilde{r}_1)^2] + 2.\mathrm{x}_1 . x_2 . E[\tilde{r}_1 . \tilde{r}_2] \quad + x_2^2 . E[(\tilde{r}_2)^2]$

$- x_1^2 . (E[\tilde{r}_1])^2 - 2.\mathrm{x}_1 . x_2 . E[\tilde{r}_1] . E[\tilde{r}_2] - x_2^2 . (E[\tilde{r}_2])^2$

$= x_1^2 . (E[(\tilde{r}_1)^2] - (E[\tilde{r}_1])^2) + 2.\mathrm{x}_1 . x_2 . (E[\tilde{r}_1 . \tilde{r}_2] - E[\tilde{r}_1] . E[\tilde{r}_2]) + x_2^2 . (E[(\tilde{r}_2)^2] - (E[\tilde{r}_2])^2)$

lembrando

$E[(\tilde{r}_1)^2] - (E[\tilde{r}_1])^2 = VAR[\tilde{r}_1] = \sigma_{r_1}{}^2$

$E[(\tilde{r}_2)^2] - (E[\tilde{r}_2])^2 = VAR[\tilde{r}_2] = \sigma_{r_2}{}^2$

$E[\tilde{r}_1 . \tilde{r}_2] - E[\tilde{r}_1] . E[\tilde{r}_2] = \sigma_{r_1, r_2}$

temos

$\sigma_{r_c}{}^2 = x_1^2 . \sigma_{r_1}{}^2 + 2.\mathrm{x}_1 . x_2 . \sigma_{r_1, r_2} + x_2^2 . \sigma_{r_2}{}^2$