

**MAT2352 - Cálculo para funções de várias variáveis II**  
**Lista de Exercícios 0 - Revisão de integrais**

Calcule as integrais indefinidas abaixo (de 1 a 73):

- |  |   |   |
|--|---|---|
| 1. $\int \frac{x^7 + x^2 + 1}{x^2} dx$                       | 2. $\int e^{2x} dx$                                   | 3. $\int \cos 7x dx$  |
| 4. $\int \operatorname{tg}^2 x dx$                           | 5. $\int \frac{7}{x-2} dx$                            | 6. $\int \operatorname{tg}^3 x \sec^2 x dx$   |
| 7. $\int \frac{\operatorname{sen}^3 x}{\sqrt{\cos x}} dx$    | 8. $\int \operatorname{tg} x dx$                      | 9. $\int \operatorname{tg}^3 x dx$  |
| 10. $\int \frac{x}{1+x^2} dx$                                | 11. $\int \frac{x}{1+x^4} dx$                         | 12. $\int \frac{x^2}{1+x^2} dx$   |
| 13. $\int x \sqrt{1-x^2} dx$                                 | 14. $\int \sec x dx$                                  | 15. $\int \frac{1}{x \sqrt{1+\ln x}} dx$  |
| 16. $\int x^2 \sqrt[5]{x^3+1} dx$                            | 17. $\int \frac{4x+8}{2x^2+8x+20} dx$                 | 18. $\int \frac{\sqrt{\ln x}}{x} dx$  |
| 19. $\int \frac{dx}{(\operatorname{arcsen} x) \sqrt{1-x^2}}$ | 20. $\int \frac{e^x}{1+e^x} dx$                       | 21. $\int \frac{\operatorname{sen} 2x}{1+\cos^2 x} dx$                                    |
| 22. $\int e^{x^3} x^2 dx$                                    | 23. $\int e^x \sqrt[3]{1+e^x} dx$                     | 24. $\int \frac{\operatorname{sen} \sqrt{x}}{\sqrt{x}} dx$                                |
| 25. $\int \frac{e^{\operatorname{arctg} x}}{1+x^2} dx$       | 26. $\int 2x(x+1)^{2006} dx$                          | 27. $\int x \operatorname{sen} x dx$  |
| 28. $\int e^x \cos x dx$                                     | 29. $\int x^r \ln x dx, r \in \mathbb{R}$             | 30. $\int (\ln x)^2 dx$   |
| 31. $\int x e^{-x} dx$                                       | 32. $\int x \operatorname{arctg} x dx$                | 33. $\int \operatorname{arcsen} x dx$   |
| 34. $\int \sec^3 x dx$                                       | 35. $\int \cos^2 x dx$                                | 36. $\int \operatorname{sen}^2 x \cos^3 x dx$   |
| 37. $\int \operatorname{sen}^2 x \cos^2 x dx$                | 38. $\int \frac{1-\operatorname{sen} x}{\cos x} dx$   | 39. $\int \frac{3x^2+4x+5}{(x-1)(x-2)(x-3)} dx$   |
| 40. $\int \frac{1}{2x^2+8x+20} dx$                           | 41. $\int \frac{3x^2+4x+5}{(x-1)^2(x-2)} dx$          | 42. $\int \frac{x^5+x+1}{x^3-8} dx$   |
| 43. $\int \frac{x^2}{\sqrt{1-x^2}} dx$                       | 44. $\int x^2 \sqrt{1-x^2} dx$                        | 45. $\int e^{\sqrt{x}} dx$  |
| 46. $\int \ln(x + \sqrt{1+x^2}) dx$                          | 47. $\int \frac{dx}{\sqrt{5-2x+x^2}}$                 | 48. $\int \sqrt{x} \ln x dx$  |
| 49. $\int \operatorname{sen}(\ln x) dx$                      | 50. $\int \frac{x}{x^2-4} dx$                         | 51. $\int \frac{3x^2+5x+4}{x^3+x^2+x-3} dx$   |
| 52. $\int \sqrt{a^2+b^2x^2} dx$                              | 53. $\int \frac{1}{\sqrt{a^2+b^2x^2}} dx$             | 54. $\int \sqrt{x^2-2x+2} dx$   |
| 55. $\int \sqrt{3-2x-x^2} dx$                                | 56. $\int \frac{1}{(1+x^2)\sqrt{1-x^2}} dx$           | 57. $\int \cos^3 x dx$  |
| 58. $\int \operatorname{sen}^5 x dx$                         | 59. $\int \frac{\cos^5 x}{\operatorname{sen}^3 x} dx$ | 60. $\int \operatorname{sen}^3\left(\frac{x}{2}\right) \cos^5\left(\frac{x}{2}\right) dx$ |

61.  $\int \frac{1}{\operatorname{sen}^5 x \cos^3 x} dx$

62.  $\int \operatorname{sen}^4 x dx$

63.  $\int \operatorname{sen}^2 x \cos^5 x dx$

64.  $\int \operatorname{sen}^2 x \cos^4 x dx$

65.  $\int \cos^6(3x) dx$

66.  $\int \frac{\cos^2 x}{\operatorname{sen}^6 x} dx$

67.  $\int \frac{1}{\operatorname{sen}^2 x \cos^4 x} dx$

68.  $\int \sqrt{\frac{1-x}{1+x}} dx$

69.  $\int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx$   
(Sugestão: Faça  $u = \sqrt[6]{x}$ )

70.  $\int \frac{x+1}{x^2(x^2+4)^2} dx$

71.  $\int \frac{\operatorname{arctg} x}{x^2} dx$

72.  $\int \frac{x^2 dx}{\sqrt{2x-x^2}} dx$

73.  $\int \frac{4x^2 - 3x + 3}{(x^2 - 2 + 2)(x+1)} dx$

74. Determine condições sobre  $a, b, c, d \in \mathbb{R}$  para que as primitivas de

$$f(x) = \frac{(x-a)(x-b)}{(x-c)^2(x-d)^2}$$
 sejam funções racionais.

(Resp.:  $d = c$  ou  $(a+b)(c+d) = 2(ab+cd)$ )75. Calcule  $\int \frac{x^2}{(\cos x + x \operatorname{sen} x)^2} dx$ . **Sugestão:** Calcule a derivada de  $v(x) = \frac{-1}{\cos x + x \operatorname{sen} x}$ . Use integração por partes. (Resp.:  $\frac{\operatorname{sen} x - x \cos x}{\cos x + x \operatorname{sen} x} + C$ )

## RESPOSTAS

1)  $\frac{x^6}{6} + x - \frac{1}{x} + k$

3)  $\frac{\operatorname{sen} 7x}{7} + k$

5)  $7 \ln|x-2| + k$

7)  $2\sqrt{\cos x} \left( \frac{\cos^2 x}{5} - 1 \right) + k$

9)  $\frac{\operatorname{tg}^2 x}{2} + \ln|\cos x| + k$

11)  $\frac{1}{2} \operatorname{arctg} x^2 + k$

13)  $-\frac{1}{3} \sqrt{(1-x^2)^3} + k$

15)  $2\sqrt{1+\ln x} + k$

17)  $\ln(2x^2 + 8x + 20) + k$

19)  $\ln|\operatorname{arcsen} x| + k$

21)  $-\ln(1 + \cos^2 x) + k$

23)  $\frac{3}{4} \sqrt[3]{(1+e^x)^4} + k$

25)  $e^{\operatorname{arctg} x} + k$

27)  $-x \cos x + \operatorname{sen} x + k$

29)  $\begin{cases} \frac{x^{r+1}}{r+1} \ln x - \frac{x^{r+1}}{(r+1)^2} + k & \text{se } r \neq -1 \\ \frac{1}{2} (\ln x)^2 + k & \text{se } r = -1 \end{cases}$

31)  $(-x-1)e^{-x} + k$

33)  $x \operatorname{arcsen} x + \sqrt{1-x^2} + k$

35)  $\frac{1}{2}(x + \operatorname{sen} x \cos x) + k$

37)  $\frac{1}{8} \left( x - \frac{\operatorname{sen} 4x}{4} \right) + k$

2)  $\frac{e^{2x}}{2} + k$

4)  $\operatorname{tg} x - x + k$

6)  $\frac{\operatorname{tg}^4 x}{4} + k$

8)  $-\ln|\cos x| + k$

10)  $\frac{1}{2} \ln(1+x^2) + k$

12)  $x - \operatorname{arctg} x + k$

14)  $\ln|\sec x + \operatorname{tg} x| + k$

16)  $\frac{5}{18} \sqrt[5]{(x^3+1)^6} + k$

18)  $\frac{2}{3} \sqrt{(\ln x)^3} + k$

20)  $\ln(1+e^x) + k$

22)  $\frac{1}{3} e^{x^3} + k$

24)  $-2 \cos \sqrt{x} + k$

26)  $2(x+1)^{2005} \left( \frac{x+1}{2006} - \frac{1}{2005} \right) + k$

28)  $\frac{1}{2} e^x (\operatorname{sen} x + \cos x) + k$

30)  $x(\ln x)^2 - 2(x \ln x - x) + k$

32)  $\frac{x^2}{2} \operatorname{arctg} x - \frac{x}{2} + \frac{1}{2} \operatorname{arctg} x + k$

34)  $\frac{1}{2} \sec x \operatorname{tg} x + \frac{1}{2} \ln|\sec x + \operatorname{tg} x| + k$

36)  $\frac{\operatorname{sen}^3 x}{3} - \frac{\operatorname{sen}^5 x}{5} + k$

38)  $\ln|1 + \operatorname{sen} x| + k$

- 39)  $6 \ln |x - 1| - 25 \ln |x - 2| + 22 \ln |x - 3| + k$
- 40)  $\frac{\sqrt{6}}{12} \operatorname{arctg} \left( \frac{x + 2}{\sqrt{6}} \right) + k$
- 41)  $-22 \ln |x - 1| + \frac{12}{x - 1} + 25 \ln |x - 2| + k$
- 42)  $\frac{x^3}{3} + \frac{35}{12} \ln |x - 2| + \frac{61}{24} \ln \left[ 1 + \left( \frac{x + 1}{\sqrt{3}} \right)^2 \right] + \frac{\sqrt{3}}{12} \operatorname{arctg} \left( \frac{x + 1}{\sqrt{3}} \right) + k$
- 43)  $\frac{1}{2} \operatorname{arcsen} x - \frac{1}{2} x \sqrt{1 - x^2} + k$
- 44)  $\frac{x}{8} (2x^2 - 1) \sqrt{1 - x^2} + \frac{1}{8} \operatorname{arcsen} x + k$
- 45)  $2(\sqrt{x} - 1)e^{\sqrt{x}} + k$
- 46)  $x \ln(x + \sqrt{1 + x^2}) - \sqrt{1 + x^2} + k$
- 47)  $\ln |\sqrt{5 - 2x + x^2} + x - 1| + k$
- 48)  $\frac{2}{3} x \sqrt{x} \left( \ln x - \frac{2}{3} \right) + k$
- 49)  $\frac{x}{2} [\operatorname{sen}(\ln x) - \cos(\ln x)] + k$
- 50)  $\frac{1}{2} \ln |x^2 - 4| + k$
- 51)  $2 \ln |x - 1| + \frac{1}{2} \ln(x^2 + 2x + 3) + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x + 1}{\sqrt{2}} + k$
- 52)  $x \sqrt{a^2 + b^2 x^2} + \frac{a^2}{2b} \ln \left[ \frac{bx}{a} + \frac{\sqrt{a^2 + b^2 x^2}}{a} \right] + k$
- 53)  $\frac{1}{b} \ln \left[ \frac{bx}{a} + \frac{\sqrt{a^2 + b^2 x^2}}{a} \right] + k$
- 54)  $\frac{x - 1}{2} \sqrt{x^2 - 2x + 2} + \frac{1}{2} \ln(x - 1 + \sqrt{x^2 - 2x + 2}) + k$
- 55)  $\left( \frac{x + 1}{2} \right) \sqrt{3 - 2x - x^2} + 2 \operatorname{arcsen} \left( \frac{x + 1}{2} \right) + k$
- 56)  $\frac{1}{\sqrt{2}} \operatorname{arctg} \left( \frac{x \sqrt{2}}{\sqrt{1 - x^2}} \right) + k$
- 57)  $\operatorname{sen} x - \frac{1}{3} \operatorname{sen}^3 x + k$
- 58)  $-\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + k$
- 59)  $\frac{\operatorname{sen}^2 x}{2} - \frac{1}{2 \operatorname{sen}^2 x} - 2 \ln |\operatorname{sen} x| + k$
- 60)  $\frac{1}{4} \cos^8 \left( \frac{x}{2} \right) - \frac{1}{3} \cos^6 \left( \frac{x}{2} \right) + k$
- 61)  $\frac{\operatorname{tg}^2 x}{2} + 3 \ln |\operatorname{tg} x| - \frac{3}{2 \operatorname{tg}^2 x} - \frac{1}{4 \operatorname{tg}^4 x} + k$
- 62)  $\frac{3x}{8} - \frac{\operatorname{sen}(2x)}{4} + \frac{\operatorname{sen}(4x)}{32} + k$
- 63)  $\frac{\operatorname{sen}^3 x}{3} - 2 \frac{\operatorname{sen}^5 x}{5} + \frac{\operatorname{sen}^7 x}{7} + k$
- 64)  $\frac{x}{16} - \frac{\operatorname{sen}(4x)}{64} + \frac{\operatorname{sen}^3(2x)}{48} + k$
- 65)  $\frac{5}{16} x + \frac{1}{12} \operatorname{sen}(6x) + \frac{1}{64} \operatorname{sen}(12x) - \frac{\operatorname{sen}^3(6x)}{144} + k$
- 66)  $-\frac{\operatorname{cotg}^3 x}{3} - \frac{\operatorname{cotg}^5 x}{5} + k$
- 67)  $\operatorname{tg} x + \frac{\operatorname{tg}^3 x}{3} - 2 \operatorname{cotg}(2x) + k$
- 68)  $\operatorname{arcsen} x + \sqrt{1 - x^2} + k$
- 69)  $2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6 \ln |\sqrt[6]{x} - 1| + k$
- 70)  $\frac{1}{4} \ln |x| - \frac{1}{4x} - \frac{1}{4} \left[ \frac{1}{2} \ln(x^2 + 4) + \frac{1}{2} \operatorname{arctg} \left( \frac{x}{2} \right) \right] + k$
- 71)  $\frac{-\operatorname{arctg} x}{x} + \ln |x| - \ln \sqrt{1 + x^2} + k$
- 72)  $\frac{3}{2} \operatorname{arcsen}(x - 1) - \left( \frac{x + 3}{2} \right) \sqrt{2x - x^2} + k$
- 73)  $2 \ln |x + 1| + \ln(x^2 - 2x + 2) + 3 \operatorname{arctg}(x - 1) + k.$