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Utility Function for modeling Group Multicriteria Decision Making problems as games

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ABSTRACT

To assist in the decision making process, several multicriteria methods have been proposed. However, the existing methods assume a single decision-maker and do not consider decision under risk, which is better addressed by Game Theory. Hence, the aim of this research is to propose a Utility Function that makes it possible to model Group Multicriteria Decision Making problems as games. The advantage of using Game Theory for solving Group Multicriteria Decision Making problems is to evaluate the conflicts between the decision makers using a strategical approach.

1. Introduction

From time to time individuals face the task of choosing from a set of outcomes that which best meets their preferences on the basis of criteria evaluation. To assist individuals in this process, several methods have been proposed, including those for situations under certainty, such as Linear Programming — LP, Multiobjective Programming — MOP and Multicriteria Decision-Making — MCDM, those under risk, such as Game Theory — GT and Multiattribute Utility Theory — MAUT, and those under the realm of uncertainty, such as Statistics and Simulation.

Within the domain of certainty, the MCDM approach is currently used by individuals in several knowledge areas [1]. However, MCDM may have reduced efficiency due to problems with the aggregation of preferences when the decision-making process involves more than one individual [2,3] and in situations under risk [4]. In this scenario, GT allows to better deal with strategic analysis of group decision-making [5,6].

Some studies have proposed the use of the GT approach for modeling LP and MCDM problems. A pioneer study was performed by Szidarovszky and Duckstein [7], which demonstrates how a multiobjective programming model representing an aquifer management problem can be solved by means of a game theoretical approach. Recently, Madani and Lund [8] proposed modeling MCDM problems as a strategic game, and solved this using non-cooperative GT concept. In their approach, the payoff values are obtained by a transition matrix, which includes both cooperative and non-cooperative outcomes.

However, for generalization of the methodology, a Utility Function – UF – is necessary to translate into a real number all the possible combinations of choices (strategies) in the group MCDM–GMCDM – process. According to Luce and Raiffa [4], the UF would be a reasonable way to describe the preferences of the individual, in order to analyze their choice. Hence, the aim of this research is to propose a UF for modeling GMCDM problems as games.

The UF proposed in this research uses the concept of pairwise comparison in the Euclidian Space to determine the payoffs for all the different strategies of the players. The use of relations in the Euclidian Space has been previously reported by other authors to propose or evaluate MCDM methods [9,10]. Here, the pairwise comparison is an intermediate step for the creation of the UF with the aim of measuring “player satisfaction” [11]. Finally, the UF is applied for modeling the classic game “Battle of the Sexes” and for modeling a travel destination GMCDM problem as a game.

2. The utility function — UF

Let us define a strategic game as $\langle N, A, \succsim_i \rangle$, where N is the set of n players (decision makers), A is the set of m actions (alternatives) and $\succsim_i$ is the preference set over A for each player $i \in N$. In the game proposed here, three strategies for each player are defined, being: (I) keeping the initial alternative when another is offered by another player; (II) changing the initial alternative for the one offered by another player; or (III) changing the initial alternative for another alternative different from that offered by another player.

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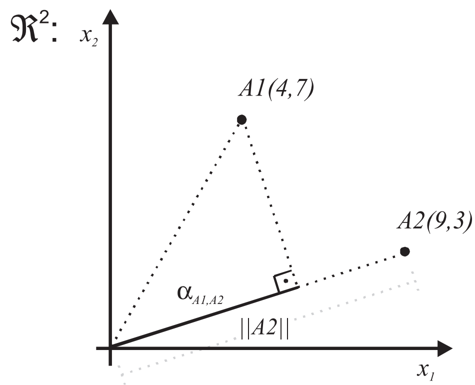

Fig. 1. Scalar projection and the relative measure in the $\Re^2$ Euclidian space.

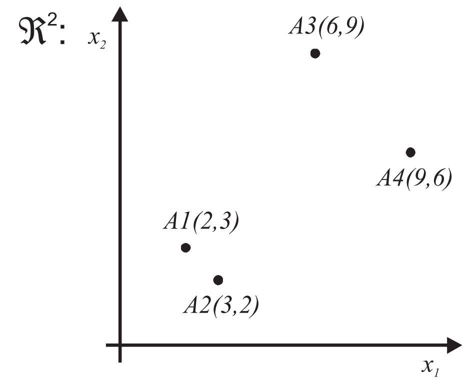

Fig. 2. Example with four alternatives under evaluation against two criteria.

Therefore, the function $\pi\colon \Re_+^{c\times n} \to [0,1]$, which is a numeric representation of the set of preferences $\succsim_i$ jointly, estimates the payoff for every joint strategy of the n players, considering that a player starts with an alternative $\boldsymbol{x}$ and needs to decide either to keep or to change for another alternative $\boldsymbol{y}$, when it is offered by another player,[1] according to his/her preferences over c criteria.

As an intermediate step for the UF, a Pairwise Comparison Function – PCF $\varphi\colon \Re_+^c \to [0,1]$, based on the angles and distance (Norm) between the alternatives [9] plotted in the Euclidian space, is proposed. The PCF aims to estimate the subjective pairwise evaluation of decision makers in order to maintain rationality conditions. The PCF proposed here has two main components: (i) a relative component, calculated by the proportional projection of one alternative onto another; and (ii) a direction component, based on the angle between the alternatives. The relative component is calculated using the Scalar Projection ($\alpha_{xy} = \|x\| \cos\theta_{xy}$) of one alternative onto the Norm ($\|y\|$) of another. Fig. 1 illustrates the concept of the relative measure using the Scalar Projection of one alternative $A1$ (defined by the vector [4,7]) onto the Norm of another alternative $A2$ (defined by the vector [9,3]), when considering two criteria, x_1 and x_2, in the Euclidian Space $\Re^2$.

In Fig. 1, the comparison of $A1$ (in relation to) and $A2$ is given by the Scalar Projection of $A1$ onto $A2(\alpha_{A1,A2})$ divided by the Norm of $A2(\|A2\|)$. In other words, the relative measure is the proportional measurement of how much $A1$ is worth in relation to $A2$, on the $A2$ basis. This is the first component of the PCF, as shown in Eq. (1).

$$\frac{\alpha_{xy}}{\|y\|}. \tag{1}$$

The direction component is proposed based on the angle between the alternatives. This measurement indicates whether the alternatives lie in the same direction, one being when the angle between them is 0° ($\cos 0° = 1$), or in a different direction, varying from zero to one, when the angle between them is more than zero [9]. The purpose of the direction component, which is the angle between the vectors ($\cos\theta_{xy}$), is to incorporate into the value of the PCF a measure of how much the alternative $A1$ is in accordance with the alternative $A2$.

With the components of relation and direction, the PCF, for comparing each pair of alternatives from the set of actions A against the set of criteria C, is defined as shown in Eq. (2).

$$\varphi(x,y) = \left[\frac{\alpha_{xy}}{\|y\|}\right]^{\delta} \cdot \cos\theta_{xy}, \quad \text{where } \delta = \begin{cases} 1, & \text{if } \alpha_{xy} \le \|y\| \\ -1, & \text{otherwise.} \end{cases} \tag{2}$$

Eq. (2) shows the calculations for the PCF proposed, where $\varphi(x,y)$ is the measurement of the pairwise comparison between the alternatives $\boldsymbol{x}$ and $\boldsymbol{y}$ on $\Re^c$ (with c being the number of criteria[2]), $\cos\theta_{xy}$ is the angle between the two alternatives, $\|y\| = \sqrt{y_1^2 + y_2^2 + \cdots + y_c^2}$ is the Norm of the respective vector, and $\alpha_{xy} = \|x\| \cdot \cos\theta_{xy}$ is the Scalar Projection of the vector $\boldsymbol{x}$ onto the vector $\boldsymbol{y}$. The image (range of the function values) varies between zero and one (due to the conditional δ), meaning the closer it is to one the more similar are the alternatives.

In mathematical terms, the PCF satisfies the following properties: (i) $0 \le \varphi(x,y) \le 1$, it establishes values between zero and one for the pairwise comparisons; and (ii) $\varphi(x,y) \ne \varphi(y,x)$, it is asymmetric, i.e., it establishes different values when it has at the beginning one alternative instead of another. A necessary condition is that the criteria must be independent due to the fact that in the Euclidian Space the orthogonality condition is necessary.

In practice, the PCF provides ordinal preference information over the alternatives, which is used to estimate decision makers' pairwise alternative assessment. In fact, the PCF satisfies all properties of preference, that is: (i) comprehensive, since it is possible to compare any pair of alternatives in the Euclidian Space; (ii) it is monotone, since larger values are preferred to smaller values (it is necessary that all criteria be benefiting criteria); (iii) it is reflexive, since if any two alternatives $\boldsymbol{x}$ and $\boldsymbol{y}$ are equal, then $\varphi(x,y) \sim \varphi(y,x)$; and (iv) it is homothetic, since for the same equal two alternatives $\boldsymbol{x}$ and $\boldsymbol{y}$, $k \cdot \varphi(x,y) \sim k \cdot \varphi(y,x)$ for any $k \ge 0$. The transitive property is conditional, given the initial alternative chosen.

To illustrate the preference information provided by the PCF, let us take the example of Fig. 1. In this example, the comparison between the alternative $A1$ and $A2$, given by $\varphi(A1, A2)$, is 0.472, whereas the comparison between the alternative $A2$ and $A1$, given by $\varphi(A2, A1)$, is 0.654. From these results, it can be induced that $A1 \succ A2$ ($A1$ is preferred to $A2$), because when starting with the alternative $A1$ the PCF value is 0.472 for the comparison with the alternative $A2$, while it is 0.654 when starting with the alternative $A2$ in comparison to the alternative $A1$. Other examples of the PCF preference interpretation can be seen in Table 1, recalling that the closer to one the more similar are the alternatives.

From Examples 1 and 2 of Table 1, one can see that distinguishing the preference information provided by the PCF is necessary. To illustrate this need, let us consider now four alternatives, $A1, A2, A3$, and $A4$, being evaluated against two criteria, x_1 and x_2, on $\Re^2$. Let us suppose that $A1$ and $A2$ have lower values for these two criteria, while $A3$ and $A4$ have higher values for them, as shown in Fig. 2.

[1] It is considered that a decision always must be made and no player has veto power.

[2] From the calculation of α_{xy} and $\|y\|$ it can be seen that the number of criteria can be straightforwardly changed without structural modifications to the PCF.

Table 1
Examples for the proposed pairwise comparison function — PCF on $\Re^2$.

Example	Alternative x	Alternative y	Pairwise comparison $\varphi(x, y)$	Preference interpretation
1	[10,10]	[5,5]	0.5	$x \succ y$
2	[5,5]	[10,10]	0.5	$x \prec y$
3	[5,5]	[5,5]	1.0	$x \sim y$
4	[10,1]	[1,10]	≈0.0	$x \sim y$
5	[1,10]	[10,1]	≈0.0	$x \sim y$
6	[0,0]	[0,0]	N/R	N/R

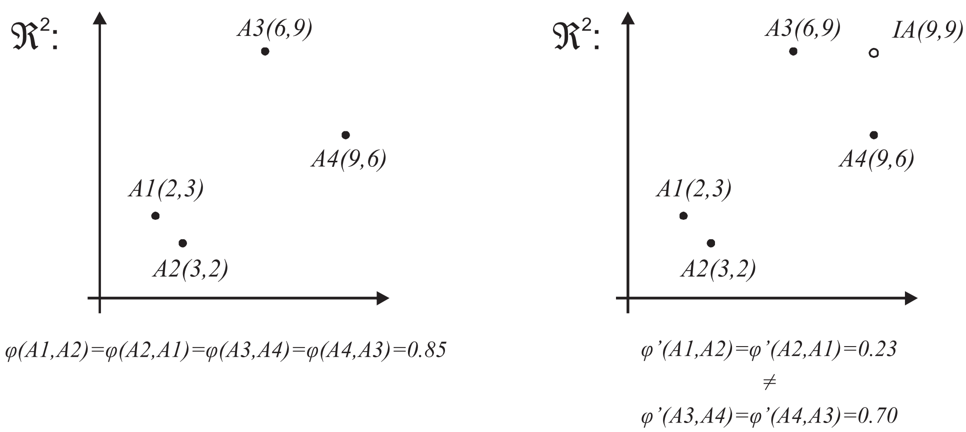

Fig. 3. Difference between the pairwise comparison function — PCF (left) and the adjusted pairwise comparison function — APCF (right).

Considering that criteria x_1 and x_2 are benefiting criteria, i.e., the larger the score the more preferable is the alternative, it would be desirable that the PCF differentiates the alternatives preferences. One way of differentiating them in a Euclidian Space is the use of an Ideal Alternative — *IA*, which incorporates the best scores of all criteria [9,12]. Still considering the example, the *IA* would be the vector [9,9], since it has the highest scores for criteria x_1 and x_2 among all alternatives. Consequently, the values of the PCF between each alternative and the *IA*, are: $\varphi(A1, IA) = \varphi(A2, IA) = 0.27$ and $\varphi(A3, IA) = \varphi(A4, IA) = 0.82$, which here are used to adjust the original PCF. Without this adjustment, the PCF might give incorrect preference information as can be seen, for example, for $\varphi(A1, A2) = \varphi(A2, A1) = 0.85$ and $\varphi(A3, A4) = \varphi(A4, A3) = 0.85$, from which one can incorrectly interpret that $A1 \sim A2 \sim A3 \sim A4$. Fig. 3 depicts graphically the difference between the PCF and the Adjusted Pairwise Comparison Function — APCF (Eq. (3)).

$$\varphi'(x, y) = \varphi(x, y) \cdot \varphi(y, IA). \tag{3}$$

From Fig. 3 it can be seen that, due to the asymmetric property, depending on the movement (the alternative that the decision maker starts), the value for the APCF will be different, which here is used to determine the strategies of the players. In order to illustrate this assertion, and still considering the previous example, $\varphi'(A3, A1) = 0.09$ and $\varphi'(A1, A3) = 0.27$, which means that $A3 \succ A1$ and that, from a strategic point of view, it would be worth trading the alternative $A1$ (far from *IA*) for alternative $A3$ (close to *IA*), while the opposite is false.

Finally, the UF proposed in this research represents numerically the joint utility for each player in order to reach higher scores for keeping (Strategy I) or for trading for (Strategies II or III) alternatives close to the *IA* than for the alternatives far away from it. According to Keeney [2], it is possible to define a UF for every joint strategy of a set of players combining their individual UF values.[3]

Here, the combination of the individual APCF values is used to define a numerical value for the joint strategy of the players, which is used to propose the UF $\pi: \Re_+^{c\times n} \to [0, 1]$ for the game $\langle N, A, \succsim_i \rangle$ as can be seen on Eq. (4).

$$\pi(x, y_i) = \varphi'(x, IA) \cdot \prod_{i=1}^{n-1} \varphi'(x, y_i). \tag{4}$$

Eq. (4) presents the UF proposed in this study for the game $\langle N, A, \succsim_i \rangle$, where $\succsim_i$ is given by $\pi(x, y)$, which defines, for a determined player, the payoff for all strategies (I, II and III) for alternative $\boldsymbol{x}$ when trading with another set of alternatives $\boldsymbol{y}_i$ proposed by all other $(n-1)$ players. With a little algebraic development, one can see that the UF can be calculated using Eq. (5) for $n = 2$ players and using Eq. (6) for $n > 2$ players.

$$\pi(x, y) = \varphi(x, IA) \cdot \varphi(x, y) \cdot \varphi(y, IA) \tag{5}$$

$$\pi(x, y_i) = \varphi(x, IA) \cdot \prod_{i=1}^{n-1} \varphi(x, y_i)\varphi(y_i, IA). \tag{6}$$

Mathematically, if one of the terms of the utility function is close to zero (low similarity between any pair of alternative), then $\pi(x, y)$ tends to be zero also, which means that only similar alternatives close to *IA* are going to be considered in what is called "kernell" of the game. The use of the UF will generate the Payoff Tables – PoT – for the players, which estimate a measure of satisfaction for every possible joint strategy in the set of actions *A*. Table 2 presents the values of the UF for the player of the aforementioned example.

[3] Keeney [2] states that the frame for group decision making must be necessarily different from the frame for individual decision analysis. For the author, the first necessary step for group decision making is that the set of alternatives *A* must be the same for each player. The author affirms that each decision-maker may have his/her own set of criteria for alternative evaluation, which, eventually can be joined in the form of vectors that express the various possible combinations of criteria values. In this direction, Keeney [2] proposes that individual utilities can be grouped to represent each possible combination of criteria values, when assuming that the changes of utilities from 0 to 1 are equally significant for all decision-makers. This procedure includes and maintains the integrity of each decision-maker's view about the group decision-making problem.

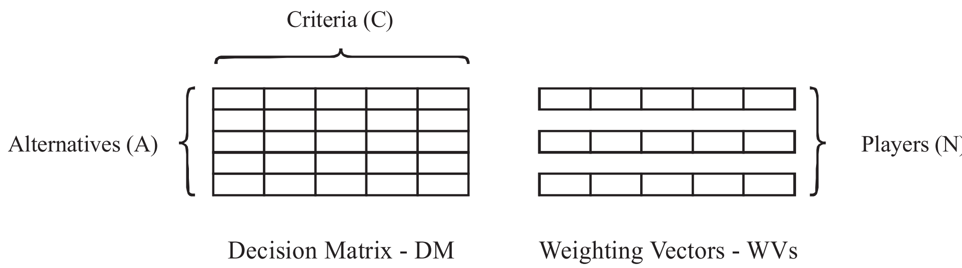

Fig. 4. General structure of the GMCDM problem.

Table 2
Example using the utility function — UF, in a game with two players.

Initial action of Player 1 (x)	Action offered by another player (y)	$\pi_1(x, y)$
A1	**A1**	0.074
A2	**A1**	0.063
A3	**A1**	0.074
A4	**A1**	0.074
A1	**A2**	0.063
A2	**A2**	0.074
A3	**A2**	0.074
A4	**A2**	0.074
A1	**A3**	0.074
A2	**A3**	0.063
A3	**A3**	0.668
A4	**A3**	0.569
A1	**A4**	0.063
A2	**A4**	0.074
A3	**A4**	0.569
A4	**A4**	0.668

In Table 2 it is possible to see all the payoffs of a determined player when trading $\boldsymbol{x}$ for any $\boldsymbol{y}$, considering $\boldsymbol{x}$ as his/her initial choice. For example, if Player 1 of the example starts with alternative $A3$ or $A4$, and the other player proposes the alternative $A3$ or $A4$ (making a pair of $A3$ and $A3$ or $A4$ and $A4$), the player will have the highest payoff, since these alternatives are the closest to IA (Fig. 3, right) and a change was not necessary (Strategy I). On the other hand, if the player starts with alternative $A3$ or $A4$, and the other player proposes the alternative $A4$ or $A3$, respectively, the player might use Strategies II or III. If the player uses Strategy II, for example, he/she will still have higher payoffs, but not the highest, since, although the player ended up with one of the best alternatives, it was necessary to open up his/her initial choice. Choosing Strategy III will give him the lowest payoffs, when trading for alternatives $A1$ or $A2$, for instance. The PoT can be used to aid the decision-maker in choosing strategically the preferable alternative considering the choice of the other players.

3. Using the UF for modeling GMCDM problems as games

Before starting this section, it is worth recalling that the process of modeling GMCDM problems might be designed with the aid of a matrix involving the alternatives and criteria selected. This matrix is known as the Decision Matrix — DM, and it is usually normalized, and then weighted by the weighting vectors — WVs of the players, generating Weighted Decision Matrixes — WDMs (Fig. 4).

Therefore, the weight elicitation process of DM is a fundamental characteristic of GMCDM problems, since decision makers have different priorities and preferences over the criteria. Considering that for group decision making the use of rank-order methods is preferred than ratio weight methods for the eliciting process [13], in this approach, it is suggested the use of the Rank-Ordered Centroid – ROC [14] – method for eliciting the decision makers' preferences. To illustrate the use of ROC, let us consider a DM with the four alternatives, $A1, A2, A3$ and $A4$, presented in Fig. 2, evaluated against two criteria, x_1 and x_2, on $\Re^2$. Let us also suppose two players, P1 and P2, who have different preferences made explicit by their ranking over the two criteria x_1 and x_2, being [2nd;1st] for P1 and [1st;2nd] for P2. By applying the ROC procedures (Appendix) the following WVs are found: [0.25;0.75] for P1 and [0.75;0.25] for P2. These different WVs will generate different WDMs for players P1 and P2, which are presented in Table 3, respectively.

Table 3
Weighted Decision Matrix – WDM – for players P1 and P2.

	P1			***P2***	
	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$		$\boldsymbol{x_1}$	$\boldsymbol{x_2}$
A1	0.50	2.25	**A1**	1.50	0.75
A2	0.75	1.50	**A2**	2.25	0.50
A3	1.50	6.75	**A3**	4.50	2.25
A4	2.25	4.50	**A4**	6.75	1.50

Applying the UF (Eq. (5)) to each WDM will generate the payoffs for all possible sets of strategies of the game (Strategies I, II and III) for each player. These payoffs will compose what is called the PoT, which are the basis of the game translated from the original GMCDM problem. These PoT contain the group utilities of each player based on their respective individual evaluation over the criteria. Table 4 presents the PoT for both the P1 and for the P2, which are the framework of the game. Fig. 5 depicts graphically the UF values for each player (P1 on left and P2 on right) for the GMCDM problem with two players presented in Table 4.

It can be seen in Fig. 5 that for P1, the highest group utility occurs when P1 starts with alternative $A3$ and the opposite player (P2) agrees with this alternative (Strategy I); while the opposite is not true for P2. For this reason, the actions must be evaluated strategically, and a solution can be found, i.e., seeking the Nash equilibrium (in this case it would be the alternative $A3$).

4. Numerical examples

4.1. Modeling the "Battle of the Sexes" game

To illustrate the use of the UF for modeling GMCDM as games, let us firstly describe the classic game "Battle of the Sexes" as a GMCDM problem. In this sense, let us consider a group of two persons: "Husband" — P1, and "Wife" — P2, who should decide together which event to attend: "Cinema"–$A1$, or "Football" — $A2$. Let us also consider two criteria to differentiate the alternatives $A1$ and $A2$, which are "Adventure" — C1, and "Romance" — C2. If we suppose that the levels for the criteria C1 and C2 of the alternative $A1$ are zero and ten, respectively, and for the alternative $A2$, ten and zero, we will have the DM shown in Table 5.

Here, both players have the same two actions, which are either to go to the cinema or go to the football match, but they might have different preferences, since men would usually rather go to a football match than to the cinema, while for women it is the opposite. In this sense, let us suppose that the WV for player P1 is [0.6;0.4], meaning that the "Husband" prefers C1 (adventure) to

Table 4
Payoff Tables – PoT – for the GMCDM problem for player P1 (left) and player P2 (right).

	P1					***P2***			
	A1	**A2**	**A3**	**A4**		**A1**	**A2**	**A3**	**A4**
A1	0.103	0.051	0.103	0.102	**A1**	0.053	0.054	0.053	0.054
A2	0.054	0.053	0.054	0.053	**A2**	0.051	0.103	0.102	0.103
A3	0.103	0.051	0.925	0.456	**A3**	0.053	0.096	0.480	0.485
A4	0.096	0.053	0.485	0.480	**A4**	0.051	0.103	0.456	0.925

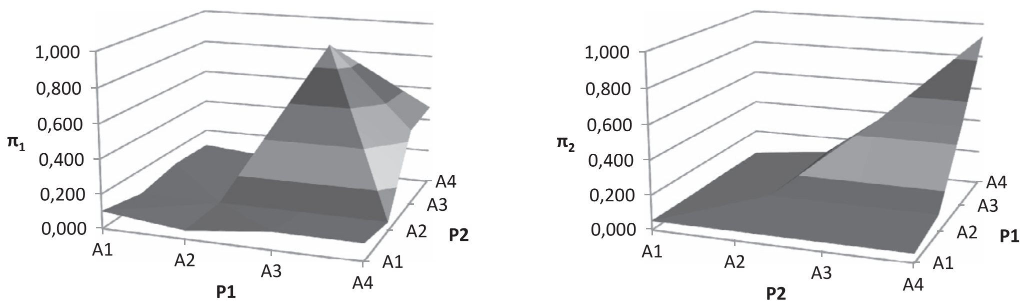

Fig. 5. UF values for the GMCDM problem in the view of player P1 (left) and player P2 (right).

Husband

	Football	**Cinema**
Football	0.33	0.00
Cinema	0.00	0.03

Wife

	Football	**Cinema**
Football	0.03	0.00
Cinema	0.00	0.33

Fig. 6. Payoff tables for the game based on the “Battle of the Sexes.”.

Table 5
Decision matrix – DM – for the “Battle of the Sexes” GMCDM problem.

	Adventure	Romance
Football	10	0
Cinema	0	10

Table 6
Decision matrix – DM – for the travel destination GMCDM problem.

	C1	C2*	C3	C4*
A1	3	2.5	2	4850
A2	3.5	12	6	3700
A3	4	4	5	2600

* The criteria C2 and C4 must be converted to benefit criteria using the transformations $1/C2$ and $1/C4$.

C2 (romance), while the WV for P2 is [0.4;0.6], meaning that the “Wife” prefers C2 (romance) to C1 (adventure). These WVs will be used to weight the DM before applying the UF to create the game framework.

Thereby, the UF applied to the two WDMs generated from DM using each individual WV, creates the PoTs, presented in Fig. 6, which are the framework for the decision game.

Fig. 6 presents the game for the “Battle of the Sexes” GMCDM problem, which is defined by $\langle N, A, \succsim_i \rangle$, where N is the set of decision makers: P1 and P2, A is the set of actions for each player: $A1$ and $A2$, and $\succsim_i$ is calculated by the UF proposed in this research (Eq. (5)). For the aforementioned example, if the arithmetic mean were used to aggregate the players' WV in order to use a MCDM approach, the set of average weights would be [0.5;0.5], which would not reflect the actual players' preferences. Therefore, using the UF presented in this study makes it possible to find the solution of the GMCDM problem by means of a theoretical game approach.

4.2. Modeling a travel destination GMCDM problem as a game

A common group problem is to decide on a travel destination. Each of the group's members needs to agree with a joint decision that is usually reached by means of negotiation. For modeling such problem, let us consider a group of three persons: P1, P2 and P3, who should decide together which destination to travel: $A1, A2$, or $A3$. Let us also consider four criteria to evaluate the alternatives, which are: (i) C1 — Hotel evaluation: the hotel evaluation grades; (ii) C2 — Travel time (in hours): duration of journey; (iii) C3 — Length of stay (number of nights): number of nights included; and (iv) C4 — Cost: cost in dollars of the package that includes accommodation and breakfast. Table 6 presents the DM for the GMCDM problem.

Let us also suppose that the players have different ranking preferences over the criteria C1 to C4, being: P1 [1st;2nd;3rd;4th]; P2 [4th;3rd;2nd;1st]; and P3 [3rd;4th;1st;2nd]. By applying the ROC procedures the following WVs are found: [0.52;0.27;0.15;0.06] for P1; [0.06;0.15;0.27;0.52] for P2; and [0.15;0.06;0.52;0.27] for P3. These WVs are used to weight the DM in order to apply the UF (Eq. (6)) to create the game (Table 7).

It can be seen in Table 7 that the combination that generates the largest payoffs for the three players simultaneously is when P1, P2 and P3 choose the alternative $A3$ (a consensus solution). It also can be seen that there are coalitions that also lead to high payoffs as, for instance, when P1 and P2 choose alternative $A3$ and P3 chooses alternative $A2$ (a coalition solution). On the other hand, the lowest payoffs always include alternative $A1$, as can be seen, for instance, when the three players choose alternative $A1$ (consensus) or when P1 and P2 choose alternative $A1$ and P3 chooses alternative $A2$. One can verify that, in fact, the two Nash equilibria for the game are: (i) a consensus solution with players agreeing with alternative $A2$; and (ii) a consensus solution with players agreeing with alternative $A3$. Most importantly, one can verify that it is now possible to know every possible strategy combination and to choose the one that can increase the chance for a group agreement.

Table 7
Payoff tables for the travel destination GMCDM problem.

P1	P2	P3	$\pi_1(x, y_i)$	$\pi_2(x, y_i)$	$\pi_3(x, y_i)$
A1	**A1**	**A1**	0.463	0.133	0.056
A2	**A1**	**A1**	0.155	0.100	0.027
A3	**A1**	**A1**	0.420	0.083	0.030
A1	**A2**	**A1**	0.276	0.101	0.049
A2	**A2**	**A1**	0.199	0.187	0.144
A3	**A2**	**A1**	0.323	0.155	0.131
A1	**A3**	**A1**	0.411	0.126	0.057
A2	**A3**	**A1**	0.216	0.184	0.128
A3	**A3**	**A1**	0.524	0.260	0.139
A1	**A1**	**A2**	0.276	0.101	0.049
A2	**A1**	**A2**	0.199	0.187	0.144
A3	**A1**	**A2**	0.323	0.155	0.131
A1	**A2**	**A2**	0.165	0.077	0.042
A2	**A2**	**A2**	0.255	0.349	0.759
A3	**A2**	**A2**	0.248	0.291	0.577
A1	**A3**	**A2**	0.245	0.096	0.049
A2	**A3**	**A2**	0.277	0.345	0.677
A3	**A3**	**A2**	0.403	0.487	0.615
A1	**A1**	**A3**	0.411	0.126	0.057
A2	**A1**	**A3**	0.216	0.184	0.128
A3	**A1**	**A3**	0.524	0.260	0.139
A1	**A2**	**A3**	0.245	0.096	0.049
A2	**A2**	**A3**	0.277	0.345	0.677
A3	**A2**	**A3**	0.403	0.487	0.615
A1	**A3**	**A3**	0.365	0.120	0.058
A2	**A3**	**A3**	0.300	0.341	0.603
A3	**A3**	**A3**	0.655	0.817	0.655

5. Conclusions

The Utility Function proposed in this research allows modeling Group Multicriteria Decision Making problems as games taking into account individual preferences over alternatives for estimating the satisfaction with the group's decision. A game with three strategies (keeping the initial alternative when another is offered by another player; changing it for the one offered by another player; or changing it for another alternative different from that offered by another player) can be modeled using the Utility Function. The basis of the Utility Function is pairwise comparisons among all alternatives relative to an Ideal Alternative in the Euclidian Space with multiple dimension (one dimension for each criterion used to assess the alternatives).

The Utility Function presented here is unprecedented in the literature. The advantage of using this function for modeling Group Multicriteria Decision Making problems as games is the use of Game Theory approach to circumvent the limitation of the aggregation procedure that is necessary for group decision making in the traditional Multicriteria Decision Making approach. It is also noteworthy to mention that Game Theory better addresses problems where a decision should be made without guarantee of collaboration agreements. One limitation of the present Utility Function is that the Pairwise Comparison Function, used to estimate the player preference over the alternatives, is modeled in terms of Linear Algebra. For future studies the use of analytical methods in the calculation of the Pairwise Comparison Function is suggested.

In order to illustrate the application of the Utility Function, two numerical examples were successfully modeled. The classic problem "Battle of the Sexes" was modeled from a Group Multicriteria Decision Making perspective and a travel destination Group Multicriteria Decision Making problem was modeled as a game using the Utility Function. For the latter example, analyses were made in order to highlight the advantage of making a group decision using the Utility Function proposed here.

By establishing a Utility Function for modeling Group Multicriteria Decision Making problems as games, the main contribution of the present research is to allow decision-makers to choose the strategies that will give them the highest payoff, taking into consideration the choices of the other players involved. Our research group has been exploring the use of the Utility Function presented in this paper in real-world applications in games with more than 2 players and 2 alternatives. The results are promising and will be published in an extended article.

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Appendix

The Rank-Ordered Centroid – ROC – method for eliciting the decision makers' preferences is a type of Rank-Order method, which establishes ratios among criteria by applying a transformation of ranks into ratios. It is a useful approach when little quantitative information is known about the criteria and/or for the group decision making process [13]. The calculation is given by the equation below, for which are presented some examples in the following table.

$$w(i) = \frac{1}{n}\sum_{k=1}^{n}\frac{1}{k}, \quad \text{where } k = 1, 2, \ldots, n \text{ criteria.}$$

Criteria number (n)	$w(1)$	$w(2)$	$w(3)$	$w(4)$	$w(5)$	$w(6)$	$w(7)$	…
1	1							
2	0,75	0,25						
3	0,611	0,278	0,111					
4	0,521	0,271	0,146	0,063				
5	0,457	0,257	0,157	0.090	0,040			
6	0,408	0,242	0,158	0,103	0,061	0,028		
7	0,370	0,228	0,156	0,109	0,073	0,044	0,020	
…	…	…	…	…	…	…	…	…

Source: Barron and Barrett [14].

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