PROVA 1

① $P(A) = 0{,}24$, $P(B) = 0{,}71$

ⓐ A e B indep $\Rightarrow P(A \cap B) = 0{,}24 \times 0{,}71 = 0{,}17$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0{,}78$

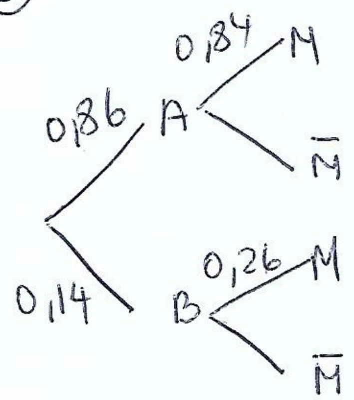
$\therefore \dfrac{P(A \cup B)}{P(A \cap B)} = \dfrac{0{,}78}{0{,}17} = \underline{4{,}59}$

ⓑ A e B m.e $\Rightarrow P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) = 0{,}95$

$\therefore \dfrac{P(A \cap B)}{P(A \cup B)} = \dfrac{0}{0{,}95} = \underline{0}$

②



$P(M) = 0{,}86(0{,}84) + 0{,}14(0{,}26) = 0{,}7224 + 0{,}0364$
$= \underline{0{,}7588}$

$P(A/M) = \dfrac{P(M/A)P(A)}{P(A)} = \dfrac{0{,}86(0{,}84)}{0{,}7588} = \underline{0{,}9520}$

③ p: prob de praticar esporte $p = 0{,}4$

ⓐ $n = 15$: X: # de estudantes que praticam esportes entre os 15 escolhidos

$X \sim B(n=15, p=0{,}4) \Rightarrow E(X) = np = (15)(0{,}4) = \underline{6}$

ⓑ X: # de entrevistas até encontrar o 5to que pratica esportes

$X \sim BN(r=5, p=0{,}4) \Rightarrow E(X) = \dfrac{r}{p} = \dfrac{5}{0{,}4} = \underline{12{,}5}$

④ $R \sim N(\mu = 0{,}05, \sigma = 0{,}25)$ $E(R) = 0{,}05$ $V(R) = 0{,}25^2$

$RF = 10 + 10R \Rightarrow E(RF) = 10 + 10E(R) = 10{,}5$

$V(RF) = 100V(R) = 6{,}25$

$RF \sim N(10{,}5; 6{,}25)$

$\therefore P(RF > V(RF)) = P(RF > 6{,}25) = P\left(Z > \dfrac{6{,}25 - 10{,}5}{\sqrt{6{,}25}}\right)$

$P(Z > -1{,}7) = P(Z < 1{,}7) = \underline{0{,}9554}$

outra $P(10 + 10R > 100V(R))$

$P(R > -0{,}375) = P(Z < 1{,}7) = \underline{0{,}9554}$

(5) Encontrando c

$$\int_0^1\int_0^2 (x^2+cxy)\,dy\,dx = \int_0^1 (2x^2+2cx)\,dx = \frac{2x^3}{3}+\frac{2cx^2}{2}\Big|_0^1 = \frac{2}{3}+c=1$$

$\rightarrow c=1/3$

Marginal de X

$$f(x)=\int_0^2 \left(x^2+\frac{xy}{3}\right)dy = 2x^2+\frac{2}{3}x \quad , \quad 0<x<1$$

$$E(X)=\int_0^1 x\left(2x^2+\frac{2}{3}x\right)dx = \frac{2x^4}{4}+\frac{2}{3}\left(\frac{x^3}{3}\right)=\frac{1}{2}+\frac{2}{9}=\frac{13}{18}=0{,}722$$

$$E(X^2)=\int_0^1 x^2\left(2x^2+\frac{2}{3}x\right)dx = \frac{2x^5}{5}+\frac{2}{3}\left(\frac{x^4}{4}\right)\Big|_0^1=\frac{2}{5}+\frac{1}{6}=\frac{17}{30}=0{,}567$$

$$V(X)=E(X^2)-E(X)^2=17/30-(13/18)^2=\frac{438}{9720}=\frac{73}{1620}=0{,}04506$$

Marginal de Y

$$f(y)=\int_0^1\left(x^2+\frac{xy}{3}\right)dx=\frac{x^3}{3}+y\left(\frac{x^2}{6}\right)\Big|_0^1=\frac{1}{3}+\frac{y}{6} \quad , \quad 0<y<2$$

$$E(Y)=\int_0^2 y\left(\frac{1}{3}+\frac{y}{6}\right)dy=\frac{y^2}{6}+\frac{y^3}{18}\Big|_0^2=\frac{4}{6}+\frac{8}{18}=\frac{20}{18}=1{,}111$$

$$E(Y^2)=\int_0^2 y^2\left(\frac{1}{3}+\frac{y}{6}\right)dy=\frac{8}{9}+\frac{16}{24}=\frac{112}{72}=\frac{14}{9}=\cancel{1{,}22}\ 1{,}55$$

$$V(Y)=E(Y^2)-E(Y)^2=\frac{112}{72}-\left(\frac{20}{18}\right)^2=\frac{208}{648}=\frac{26}{81}=0{,}32$$

$$E(XY)=\int_0^1\int_0^2 xy\left(x^2+\frac{xy}{3}\right)dy\,dx=\int_0^1\left(\frac{x^3y^2}{2}+\frac{x^2}{3}\left(\frac{y^3}{3}\right)\Big|_0^2\right)dx$$

$$=\int_0^1\left(2x^3+\frac{8}{9}x^2\right)dx=\frac{2x^4}{4}+\frac{8}{9}\left(\frac{x^3}{3}\right)\Big|_0^1=\frac{1}{2}+\frac{8}{27}=\frac{43}{54}=0{,}7963$$

$$\mathrm{cov}(X,Y)=E(XY)-E(X)E(Y)=\frac{43}{54}-\left(\frac{13}{18}\right)\left(\frac{20}{18}\right)=\frac{-2}{324}=-0{,}006172$$

$$r=\frac{\mathrm{cov}(X,Y)}{\sqrt{V(X)V(Y)}}=\frac{-0{,}06}{\sqrt{0{,}045\times0{,}32}}=-0{,}049$$

$$r=\frac{-1/16{,}2}{\sqrt{\frac{438}{9720}}\sqrt{\frac{468}{1458}}}=0{,}049$$