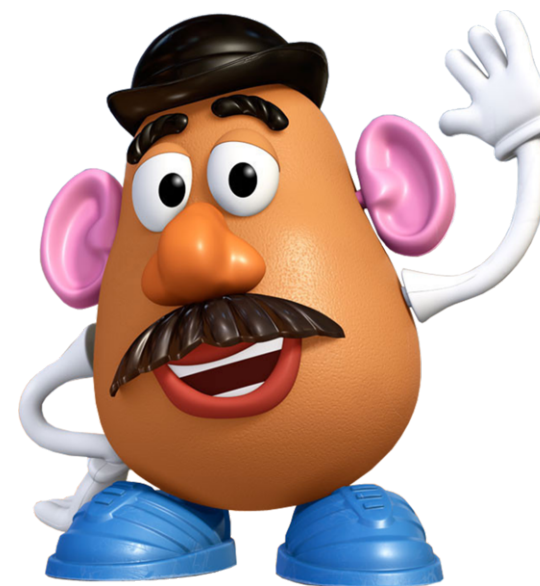
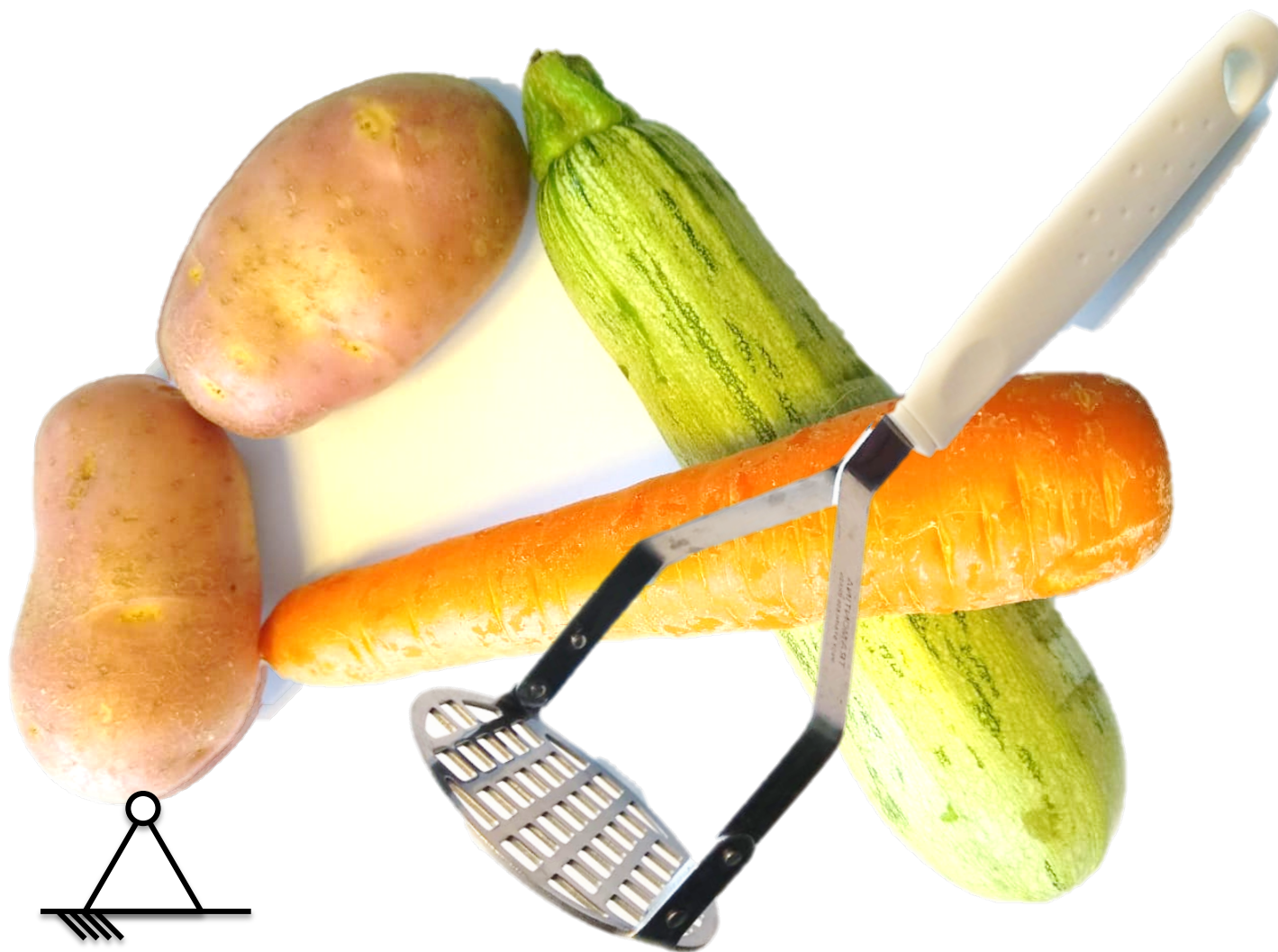




# Equilíbrio de Corpo Rígido





## Definição:

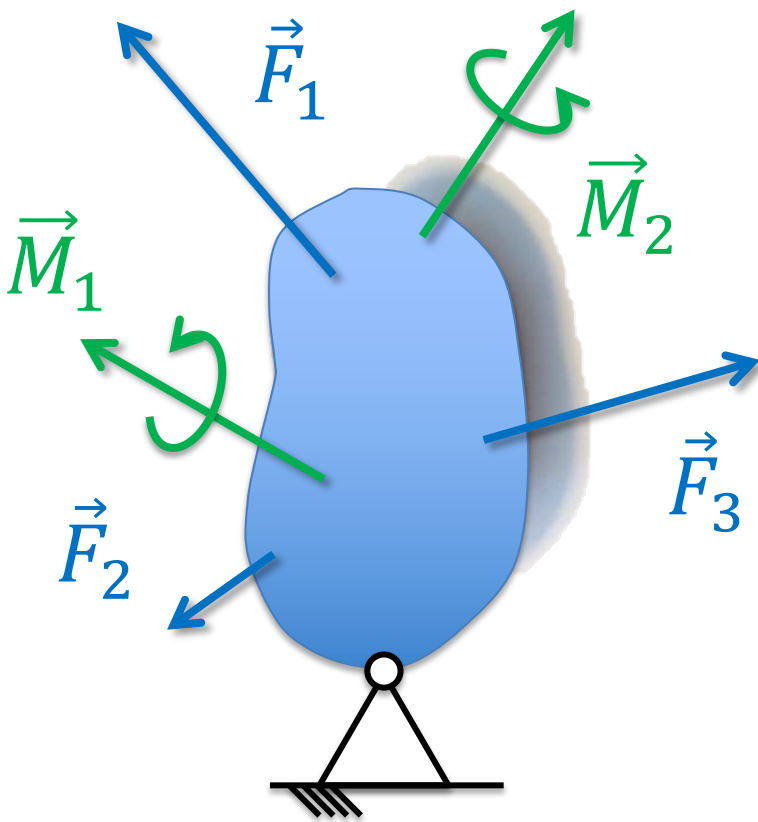
- ❖ Corpo Rígido  
(*rígido mesmo?*)
- ❖ Conjuntos de Corpos  
(*vínculos / juntas*)
- ❖ Condições de Contorno  
(*restrições*)

## Natureza das Forças:

- ❖ Campos, magnético, elétrico, gravitacional
- ❖ Contatos com outros corpos
- ❖ Vínculos com a terra

## Definição:

- ❖ Corpo Rígido (*rígido mesmo?*)
- ❖ Conjuntos de Corpos (*vínculos / juntas*)
- ❖ Condições de Contorno (*restrições*)



*interpretação / representação física do problema  
resolver as equações*



## interpretação física do problema

### ❖ 1ª lei:

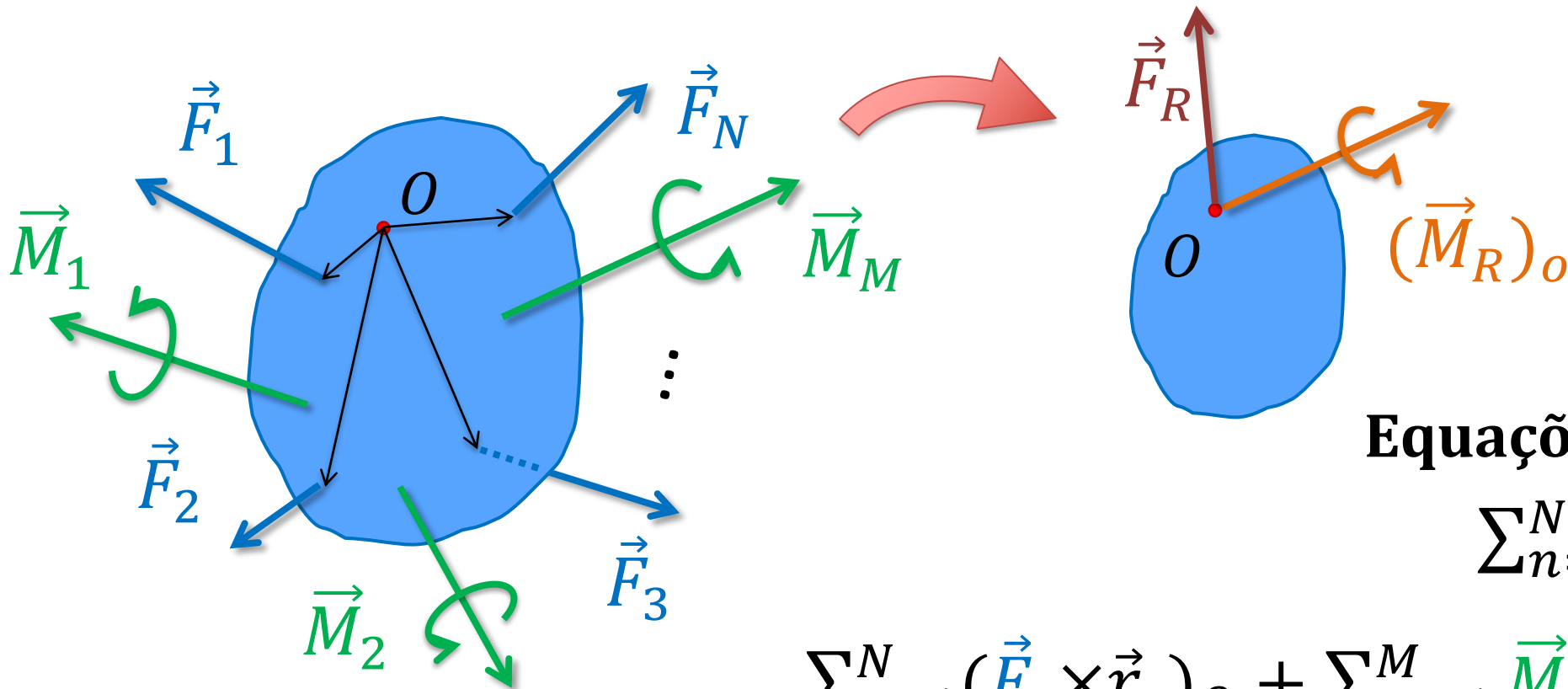
*“todo corpo continua em seu estado de repouso ou movimento uniforme não acelerado ...”*

### ❖ 2ª lei:

*“a mudança de movimento é proporcional à força motora imprimida ...”*

### ❖ 3ª lei:

*“para toda ação há sempre uma reação oposta e de mesma intensidade...”*



❖ 1ª lei:

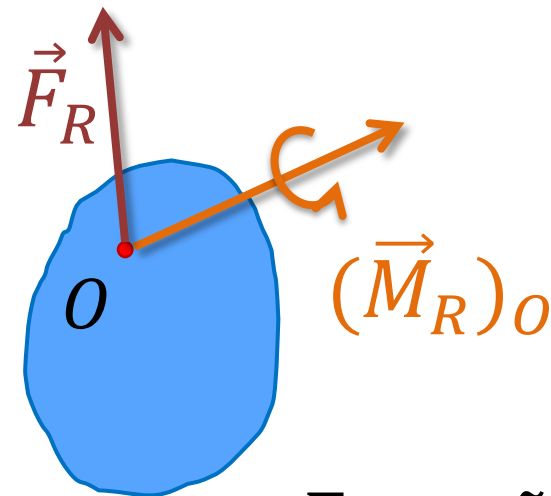
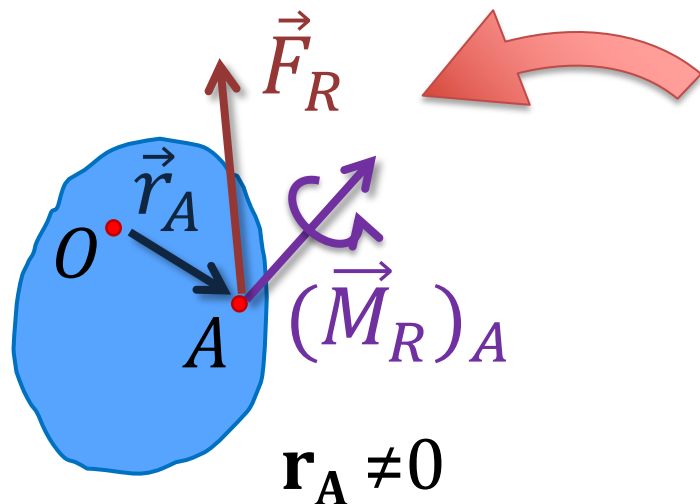
### Equações de Equilíbrio

$$\sum_{n=1}^N \vec{F}_n = \vec{F}_R = 0$$

$$\underbrace{\sum_{n=1}^N (\vec{F}_n \times \vec{r}_n)_O}_{\text{Momento das Forças}} + \underbrace{\sum_{m=1}^M \vec{M}_m}_{\text{Binários}} = (\vec{M}_R)_O = 0$$

Momento das Forças  
(depende de  $O$ )

Binários  
(não depende de  $O$ )



❖ 1ª lei:

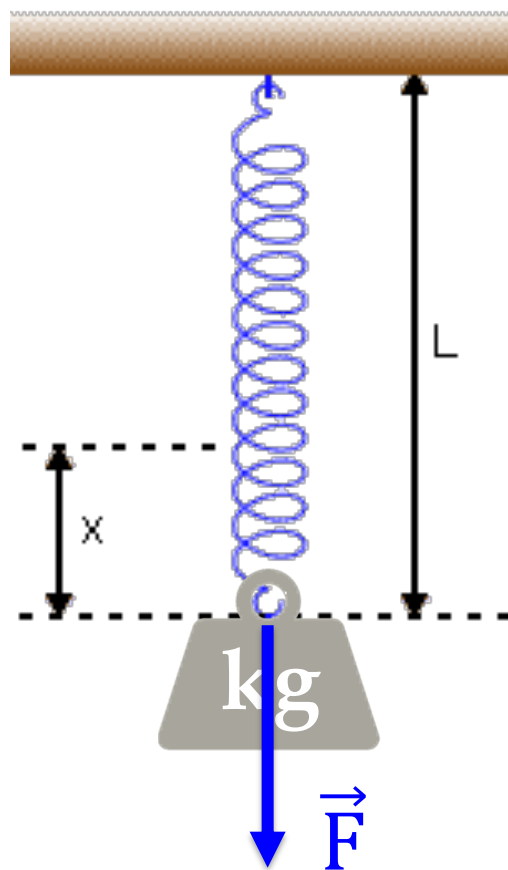
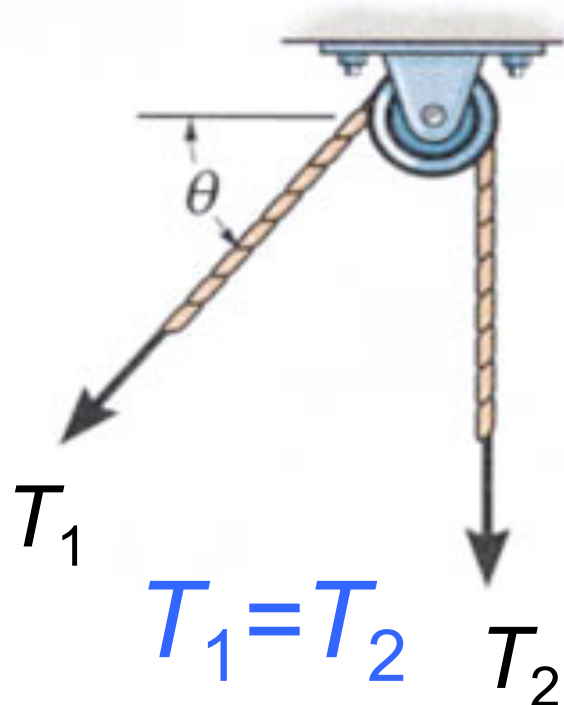
Equações de Equilíbrio

Nesta nova posição

$$(\vec{M}_R)_A = \vec{F}_R \times \vec{r}_A + (\vec{M}_R)_O = 0 \quad \left\{ \begin{array}{l} \vec{F}_R = 0 \\ (\vec{M}_R)_O = 0 \end{array} \right.$$

mas de onde vêm essas forças e momentos?

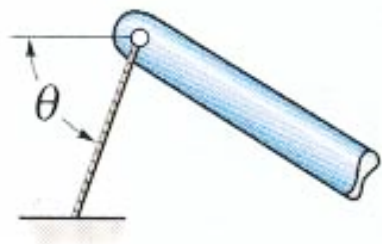
condição suficiente!



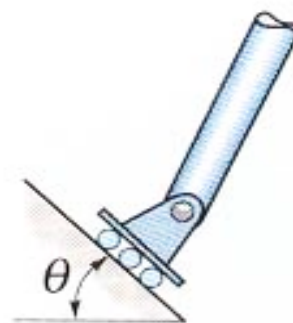
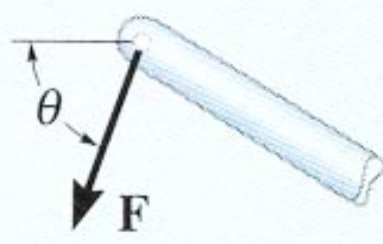
interpretação física do problema

Já começamos a fazer interpretação física de sistemas reais (*modelos*)

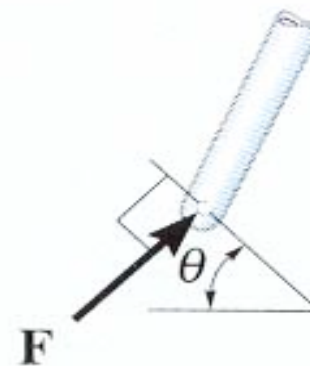
representação física do problema



cable



roller



*A regra é clara: Removeu um grau de liberdade, aparece uma força de restrição!!!*

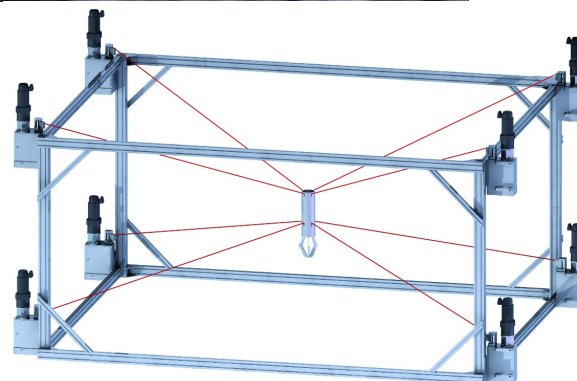
Será que tem uma REGRA?  
... mas o que é GdL?

**Definição:** é o número de parâmetros independentes que são necessários para se definir a posição de um corpo no espaço em qualquer instante.




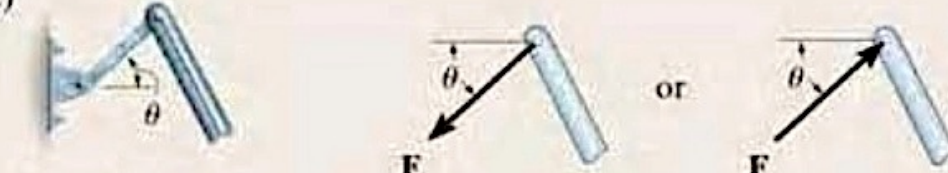

Quantos GdL tem esse robô?

Quantos motores tem esse robô?





**TABLE 5-1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems**

Types of Connection	Reaction	Number of Unknowns
<p>(1)</p>  <p>cable</p>	<p>One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.</p>	
<p>(2)</p>  <p>weightless link</p>	<p>One unknown. The reaction is a force which acts along the axis of the link.</p>	
<p>(3)</p>  <p>roller</p>	<p>One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.</p>	



roller or pin in confined smooth slot

One unknown. The reaction is a force which acts perpendicular to the slot.



rocker

One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.



smooth contacting surface







One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.



member pin connected to collar on smooth rod

One unknown. The reaction is a force which acts perpendicular to the rod.

TABLE 5-1 Continued

Types of Connection	Reaction	Number of Unknowns
<p>(8)</p>  <p>smooth pin or hinge</p>	<p>or</p> 	<p>Two unknowns. The reactions are two components of force, or the magnitude and direction <math>\phi</math> of the resultant force. Note that <math>\phi</math> and <math>\theta</math> are not necessarily equal [usually not, unless the rod shown is a link as in (2)].</p>
<p>(9)</p>  <p>member fixed connected to collar on smooth rod</p>		<p>Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.</p>
<p>(10)</p>  <p>fixed support</p>	<p>or</p> 	<p>Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction <math>\phi</math> of the resultant force.</p>

# Diagrama de Corpo Livre (DCL)

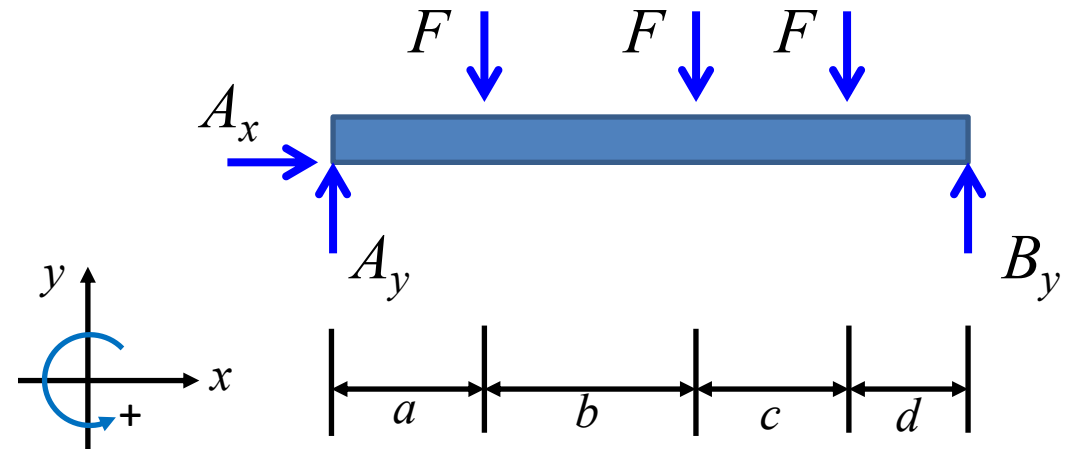
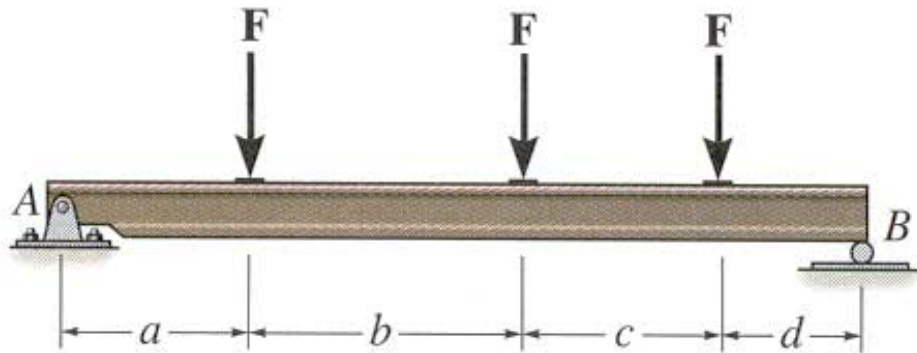


**QQR  
CORPO  
MINHAS  
REGRAS**

- ❖ esboçar um corpo livre;
- ❖ representar os esforços externos no corpo;
- ❖ substituir todos os vínculos pelos respectivos esforços;
- ❖ indicar o sistema de coordenadas e dimensões principais.

# Diagrama de Corpo Livre (DCL)

- DCL deste exemplo:

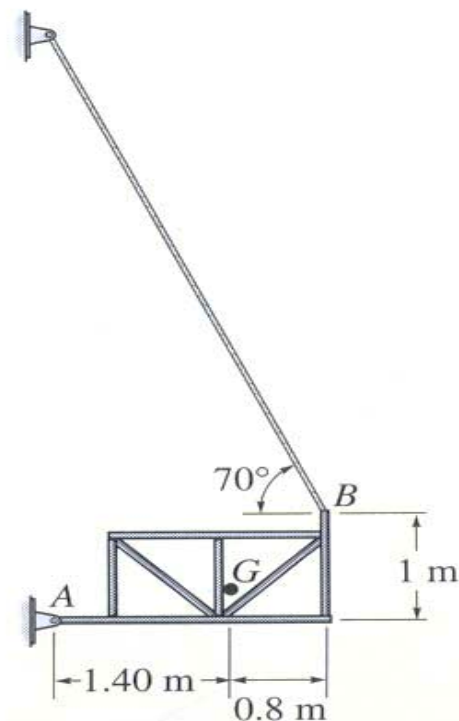


# Diagrama de Corpo Livre (DCL)

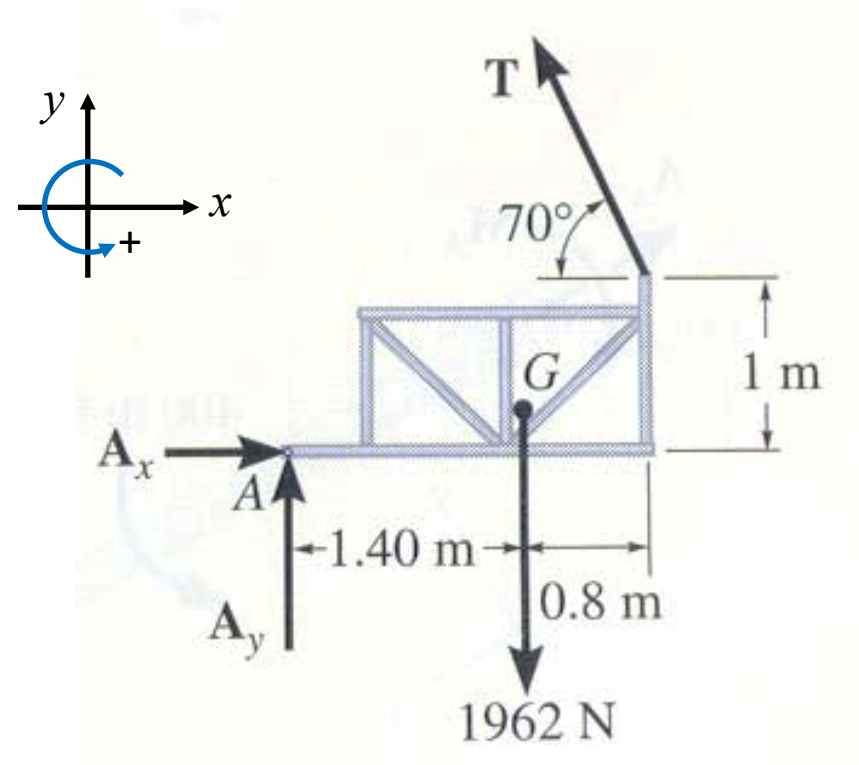
- Vamos tentar desenhar o DCL desses exemplos:



fotografia



esboço

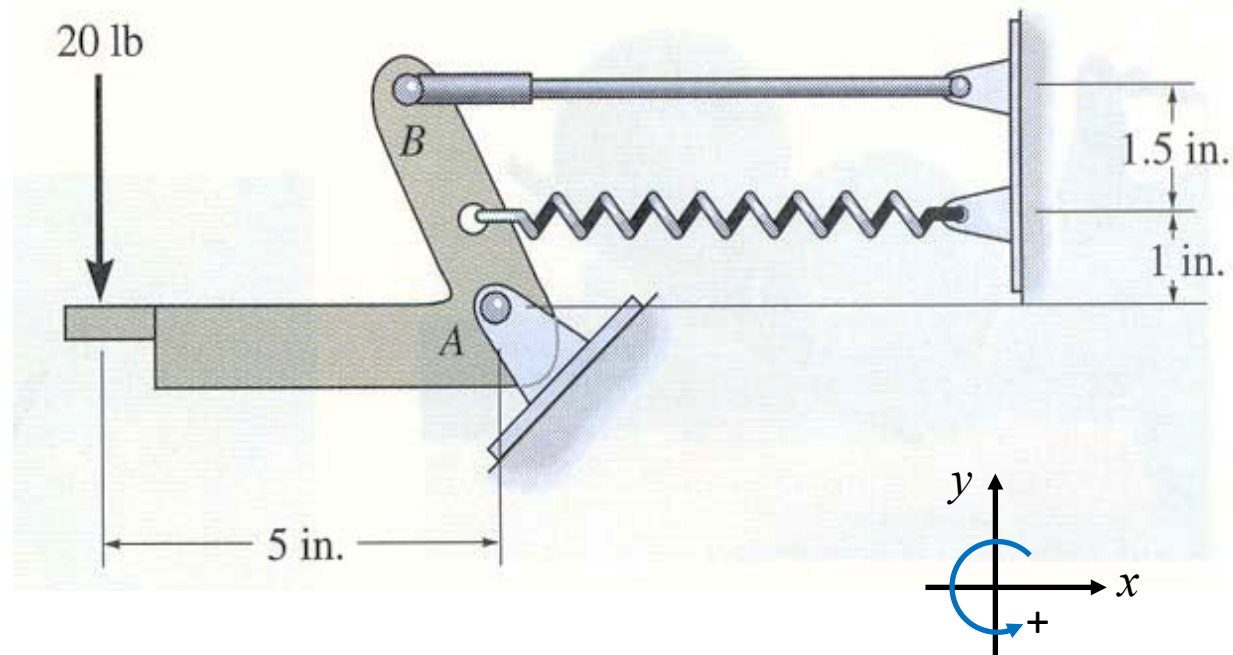
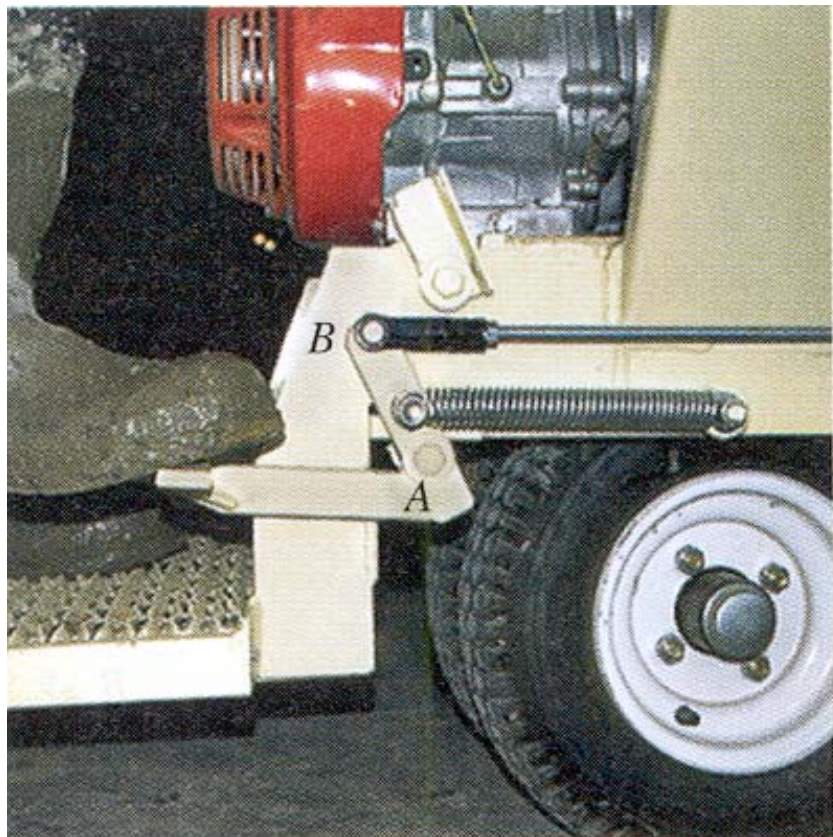


(DCL)



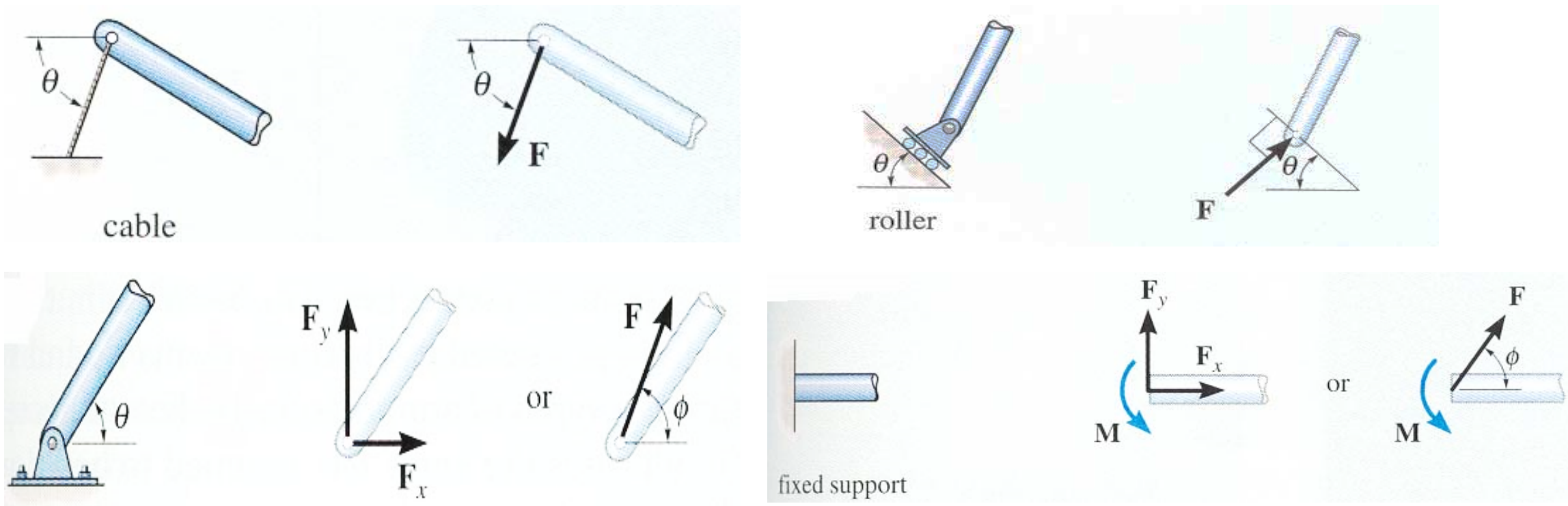
# Diagrama de Corpo Livre (DCL)

- Vamos tentar desenhar o DCL desses exemplos:



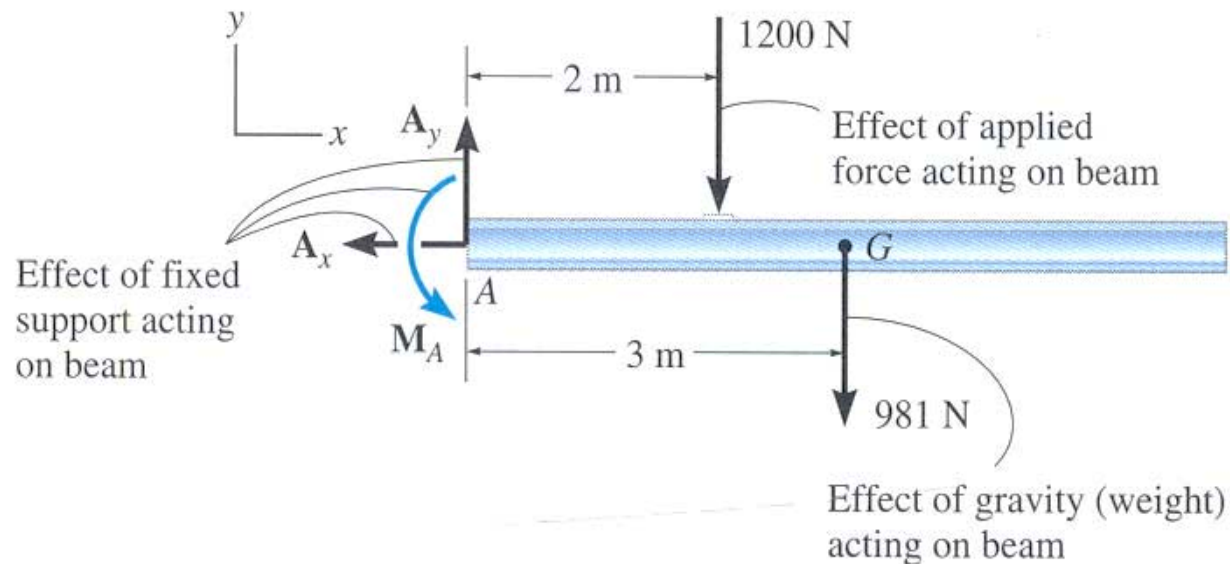
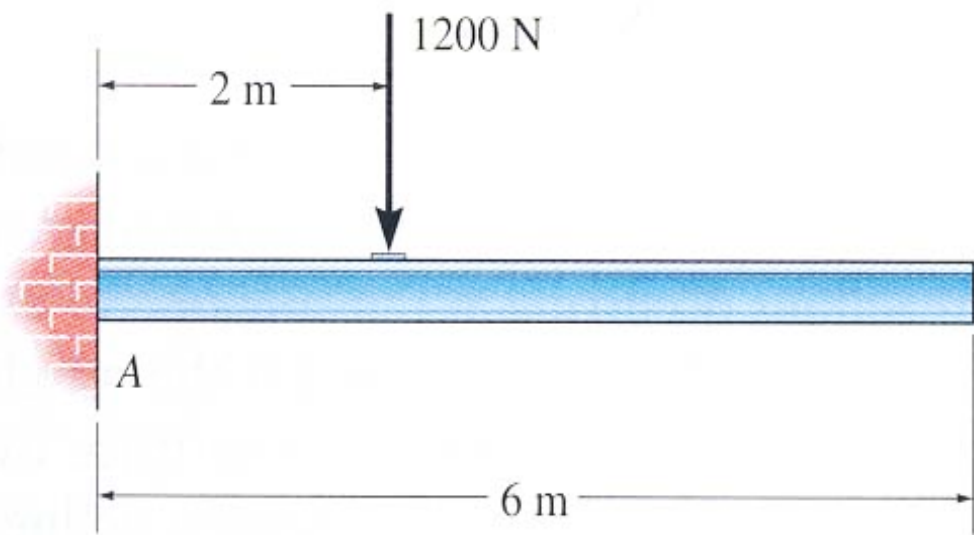
# Diagrama de Corpo Livre (DCL)

- Representações clássicas (as mais utilizadas):



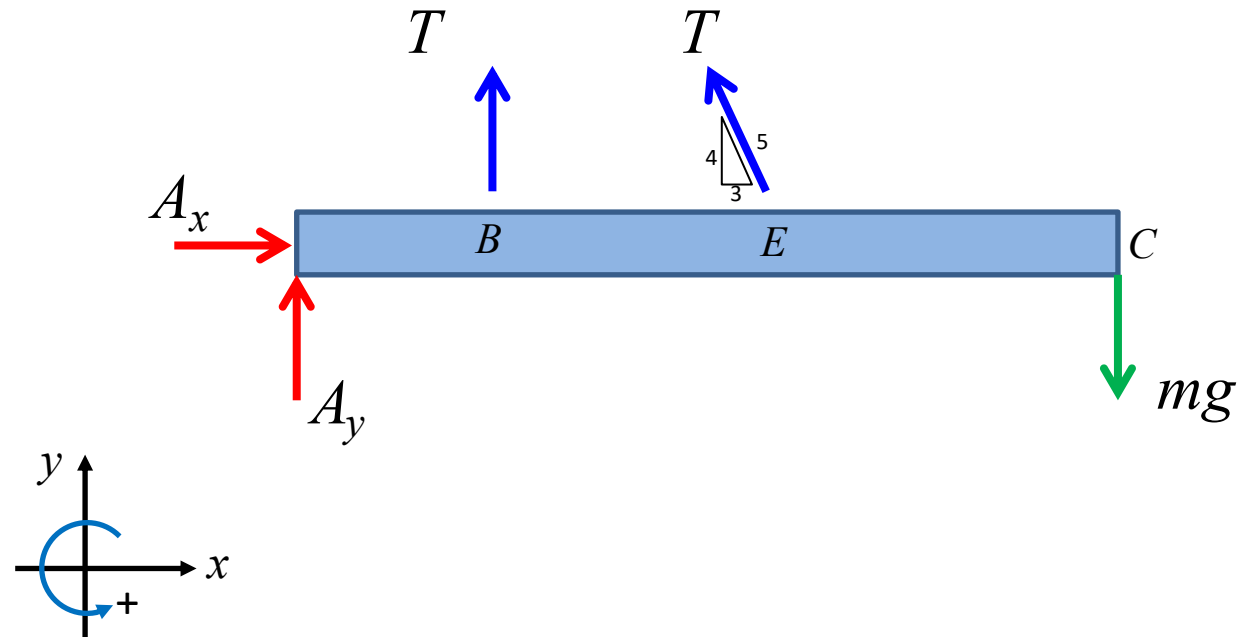
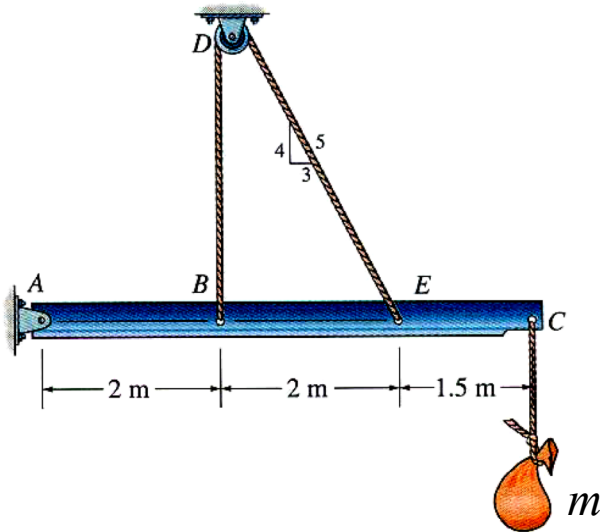
# Diagrama de Corpo Livre (DCL)

- Representações clássicas (as mais utilizadas):



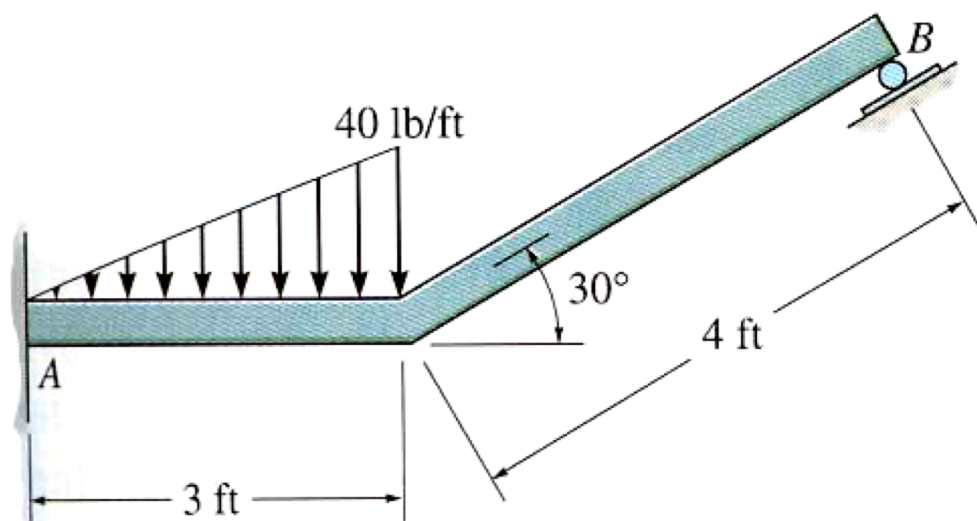
# Diagrama de Corpo Livre (DCL)

- Representações clássicas (as mais utilizadas):

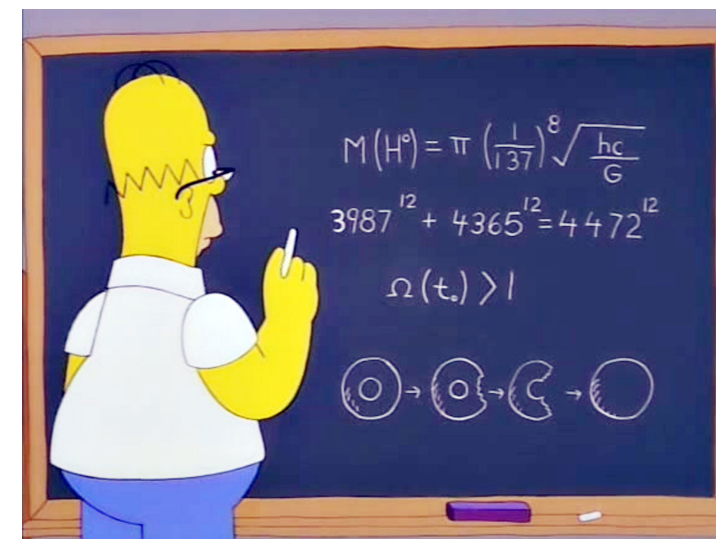


# Diagrama de Corpo Livre (DCL)

- problema: faça o DCL da estrutura ao lado e diga se ele pode ser resolvido como um problema de (iso)estática.

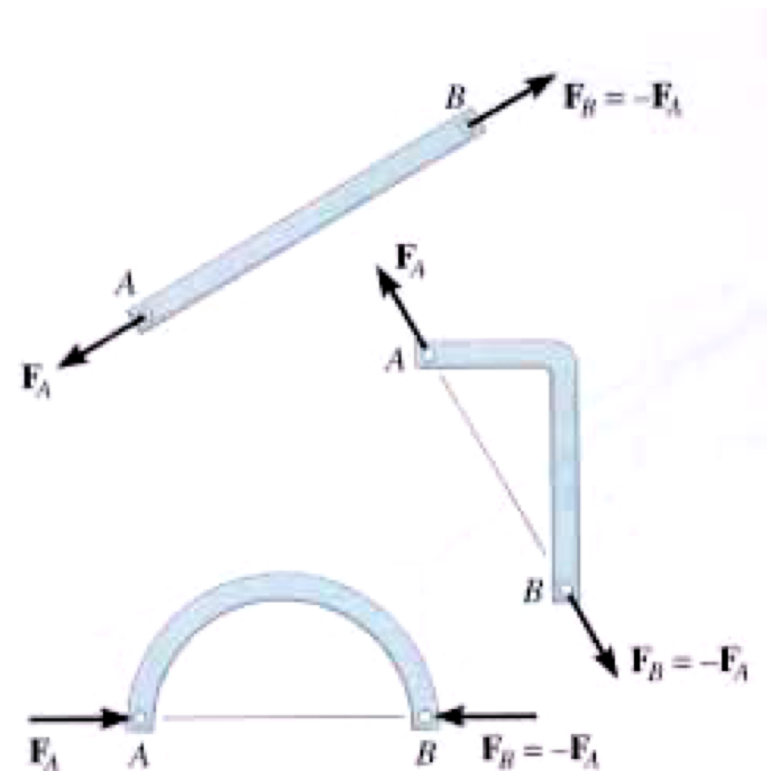


Resolvendo na lousa ....



# Casos Especiais

- Elementos de 2 Forças



Two-force members

## Casos Especiais

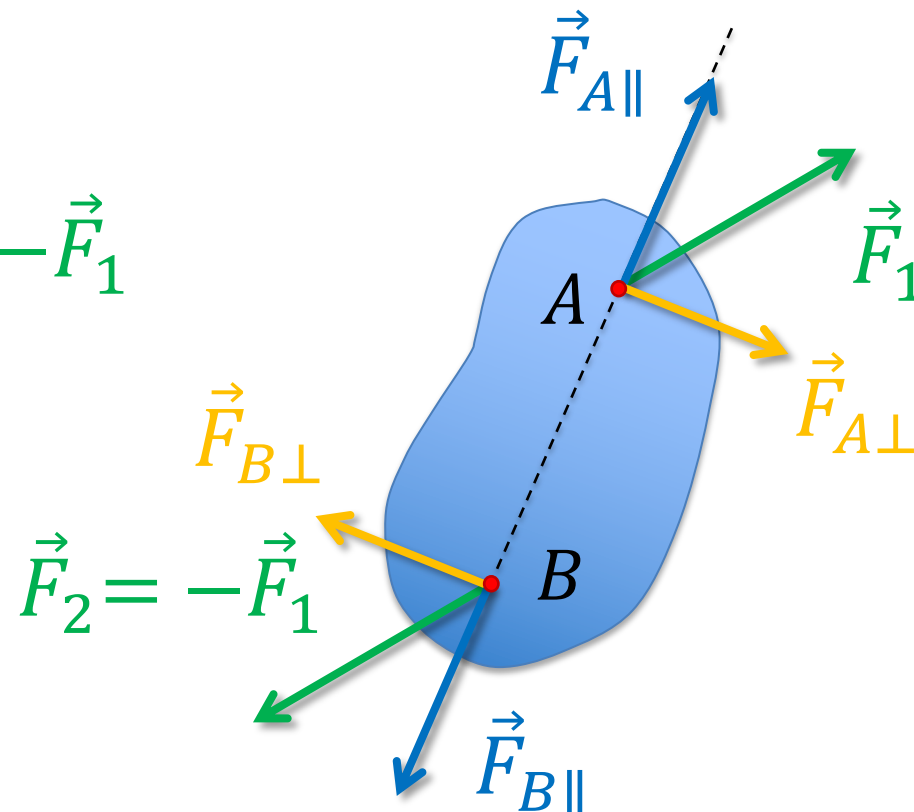
- Elementos de 2 Forças

### Equações de Equilíbrio

$$\sum_{n=1}^N \vec{F}_n = 0 \rightarrow \vec{F}_1 + \vec{F}_2 = 0 \rightarrow \vec{F}_2 = -\vec{F}_1$$

$$\vec{F}_{A\parallel} = \vec{F}_{B\parallel} \quad e \quad \cancel{\vec{F}_{A\perp}} = \cancel{\vec{F}_{B\perp}}$$

$$(\vec{M}_R)_B = 0 \rightarrow \vec{F}_{A\perp} = 0$$



*As 2 forças atuam na direção da reta que passa por A e B.*

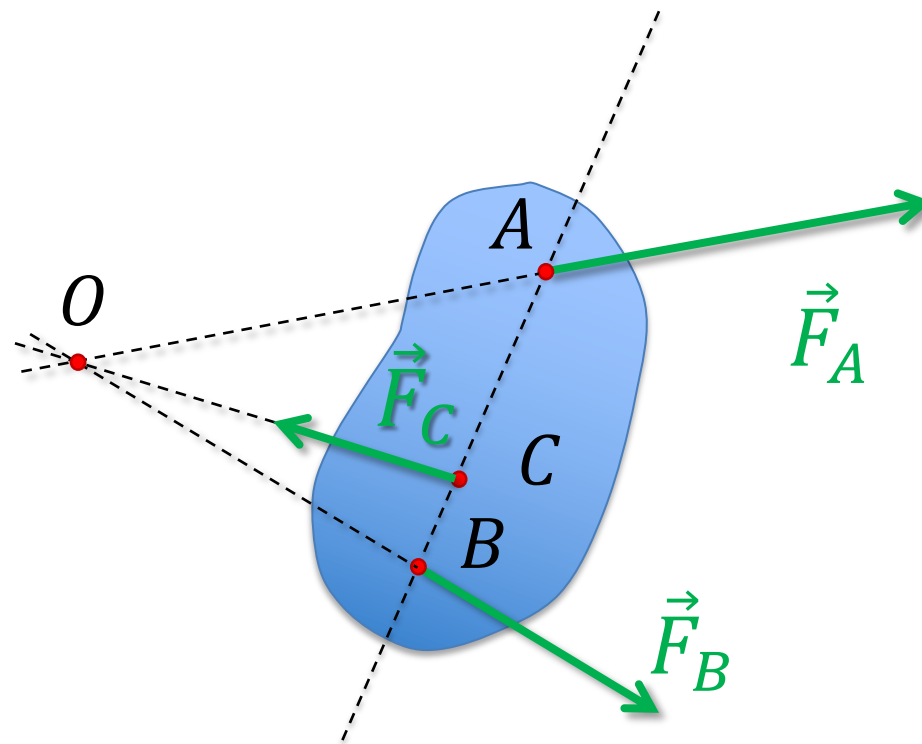
## Casos Especiais

- Elementos de 3 Forças

### Equações de Equilíbrio

$$\sum_{n=1}^N \vec{F}_n = 0 \rightarrow \vec{F}_A + \vec{F}_B + \vec{F}_C = 0$$

$$(\vec{M}_R)_O = 0 \rightarrow \text{As 3 forças devem ser concorrentes em } O$$





## Casos Especiais

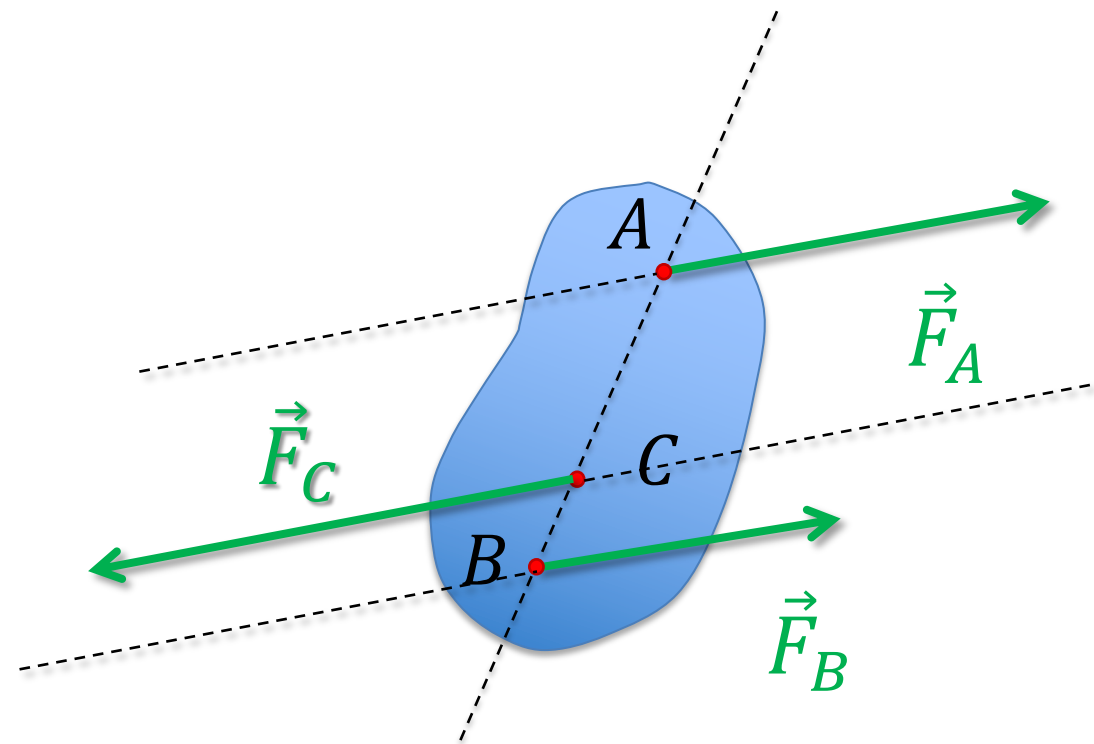
- Elementos de 3 Forças

### Equações de Equilíbrio

$$\sum_{n=1}^N \vec{F}_n = 0 \rightarrow \vec{F}_A + \vec{F}_B + \vec{F}_C = 0$$

$$(\vec{M}_R)_O = 0 \rightarrow \text{As 3 forças devem ser concorrentes em } O$$

→ As 3 forças devem ser paralelas



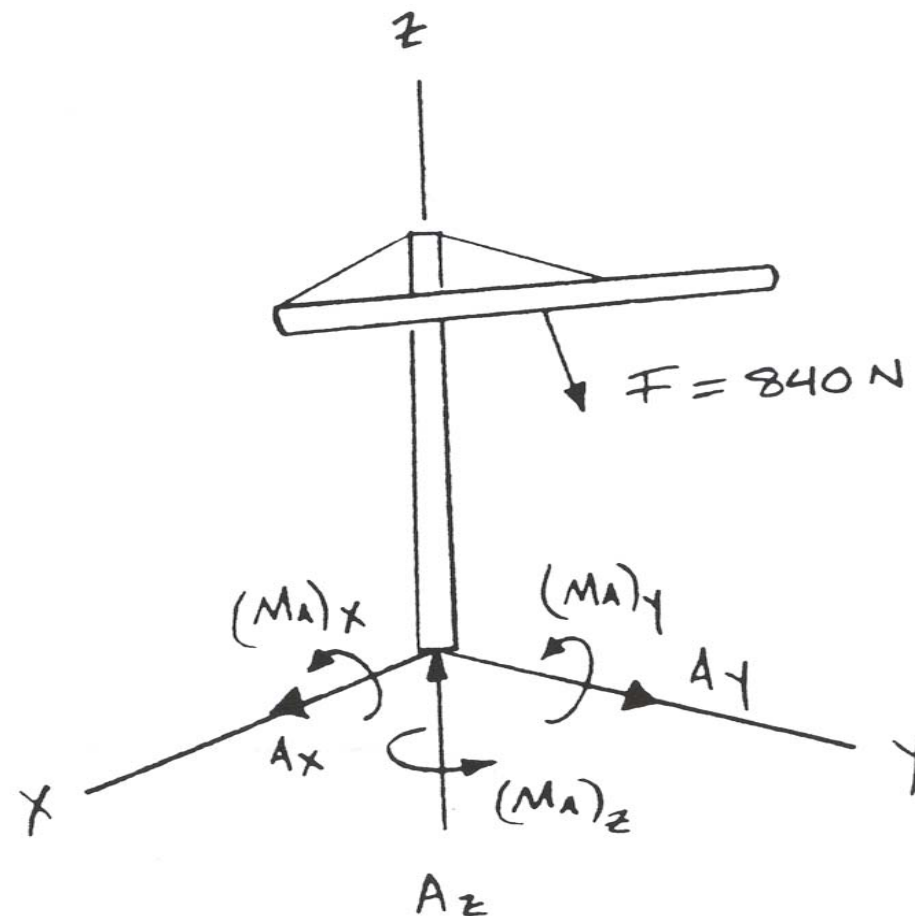
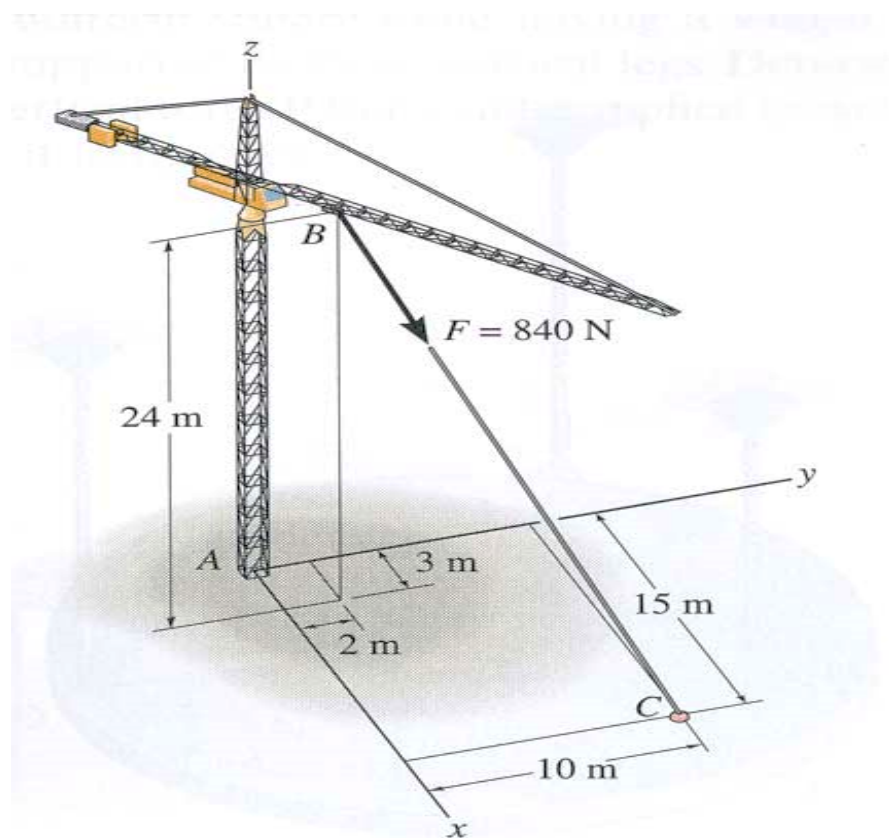
ATENÇÃO para o exemplo (vídeo).



# Diagrama de Corpo Livre em 3D

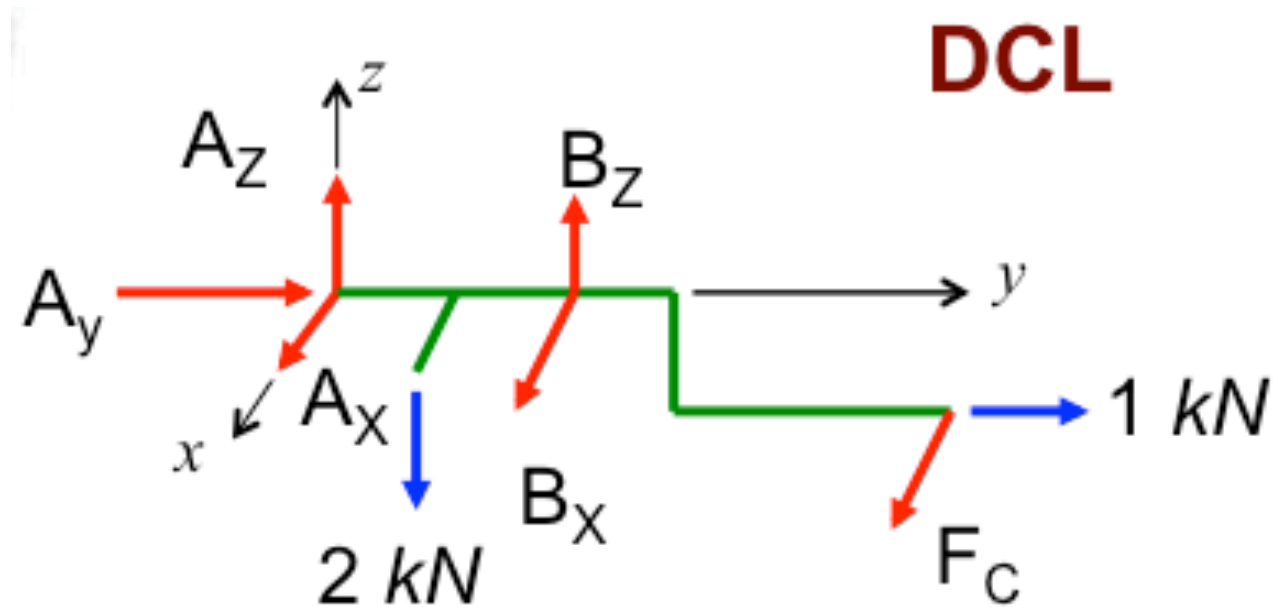
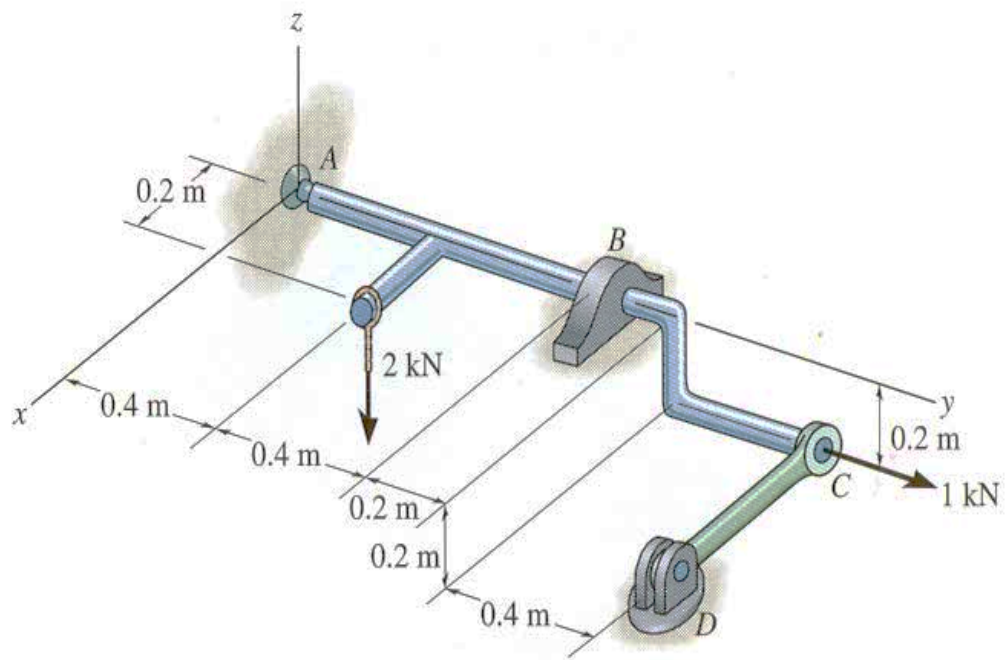
# DCL – problemas tridimensionais

- pode parecer trivial...



# DCL – problemas tridimensionais

... mas há peculiaridades

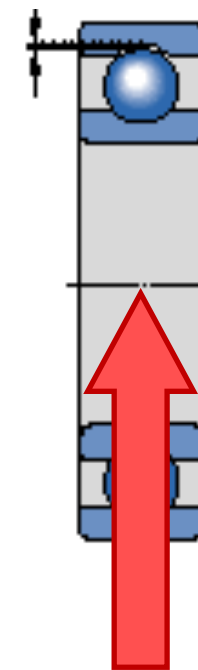


## DCL – problemas tridimensionais

- Mancal Radial



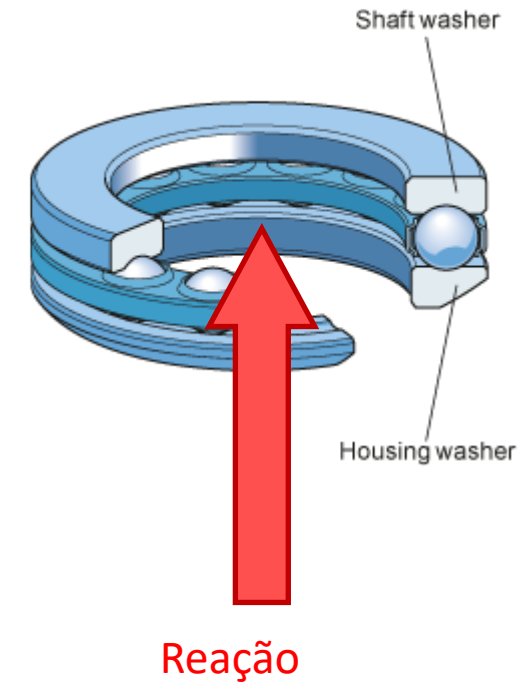
Mancal



Reação  
(caso ideal)

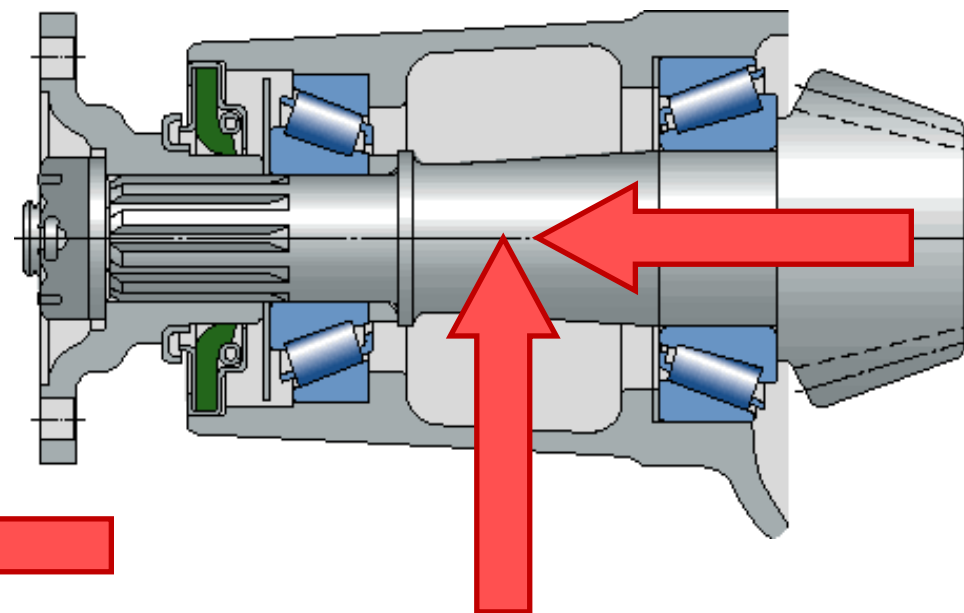
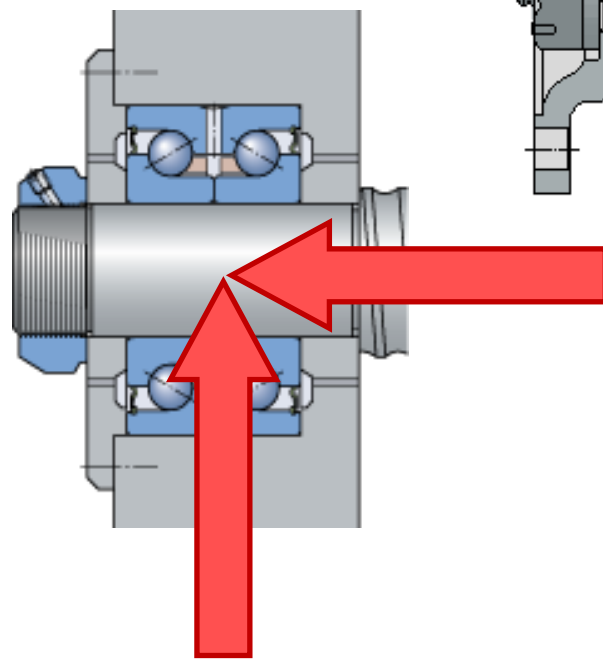
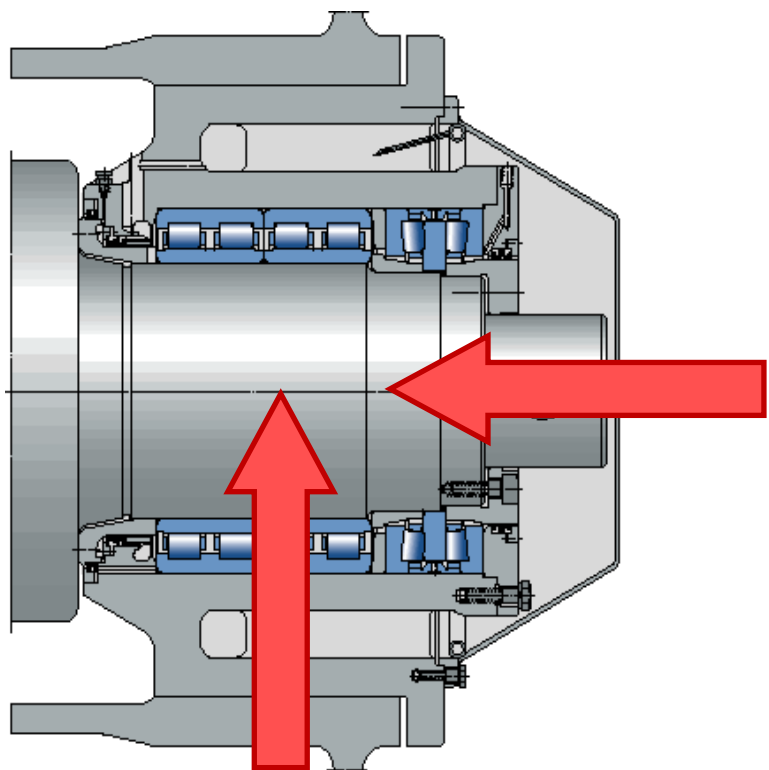
## DCL – problemas tridimensionais

- Mancal Axial (de encosto)



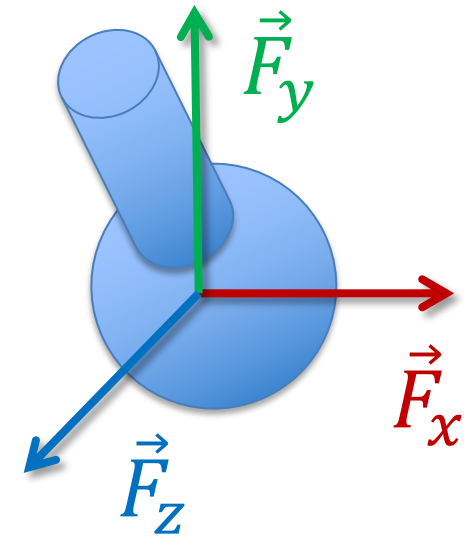
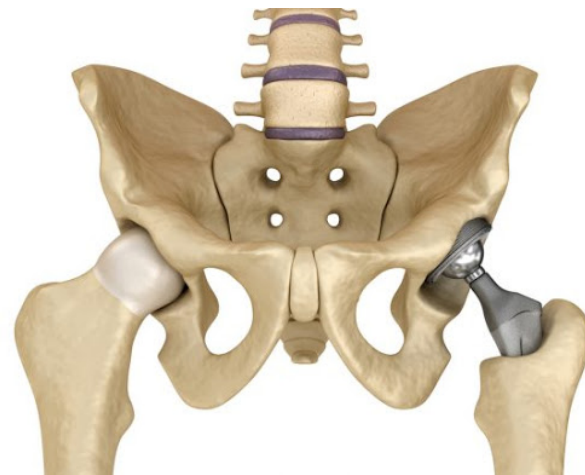
## DCL – problemas tridimensionais

- Mancal Radial/Axial



## DCL – problemas tridimensionais

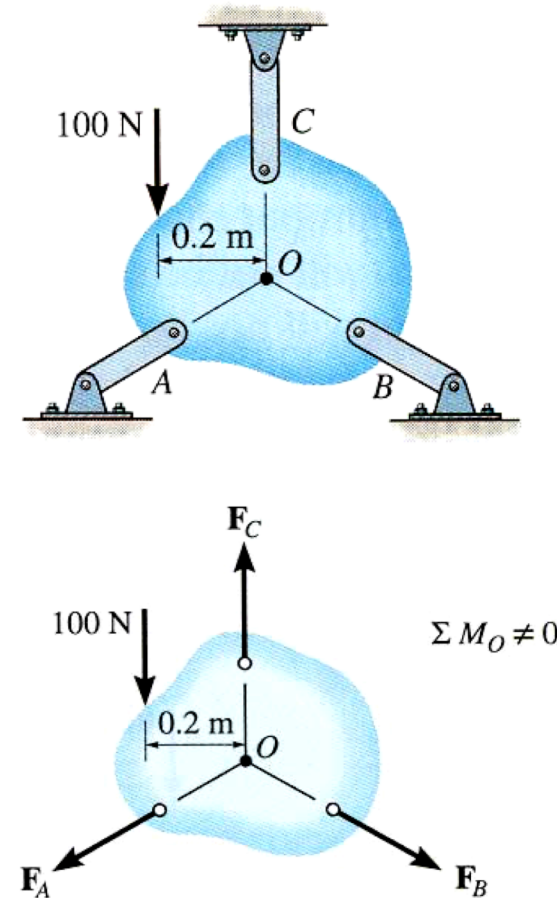
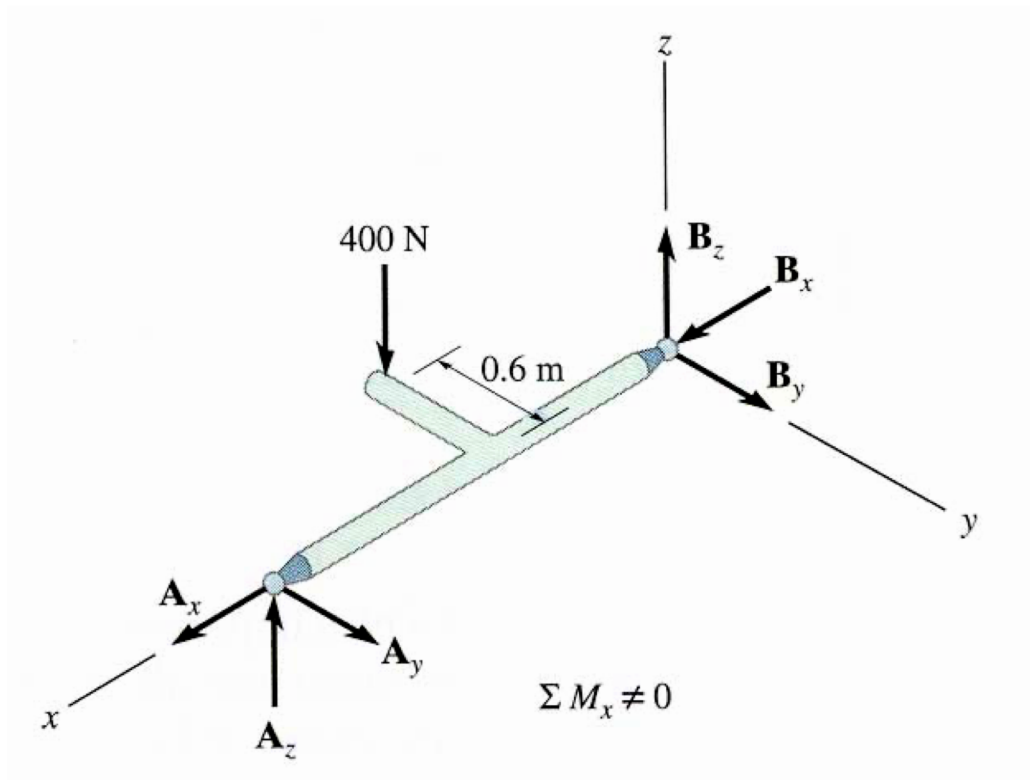
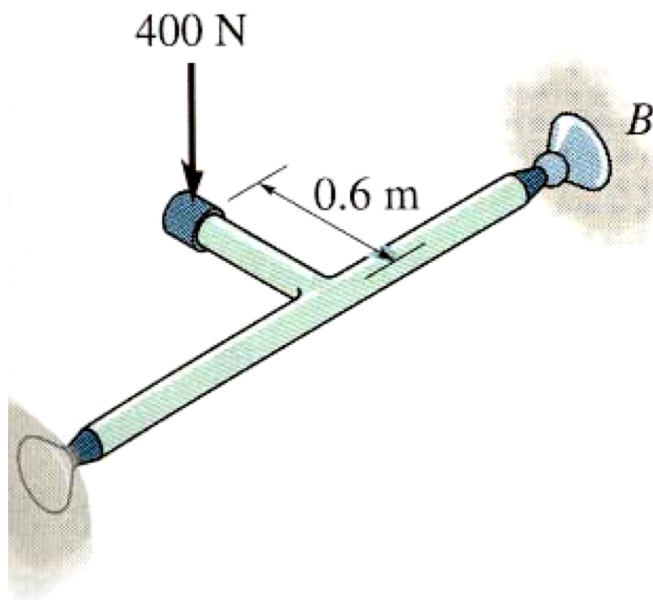
- Junta esférica (rótula)





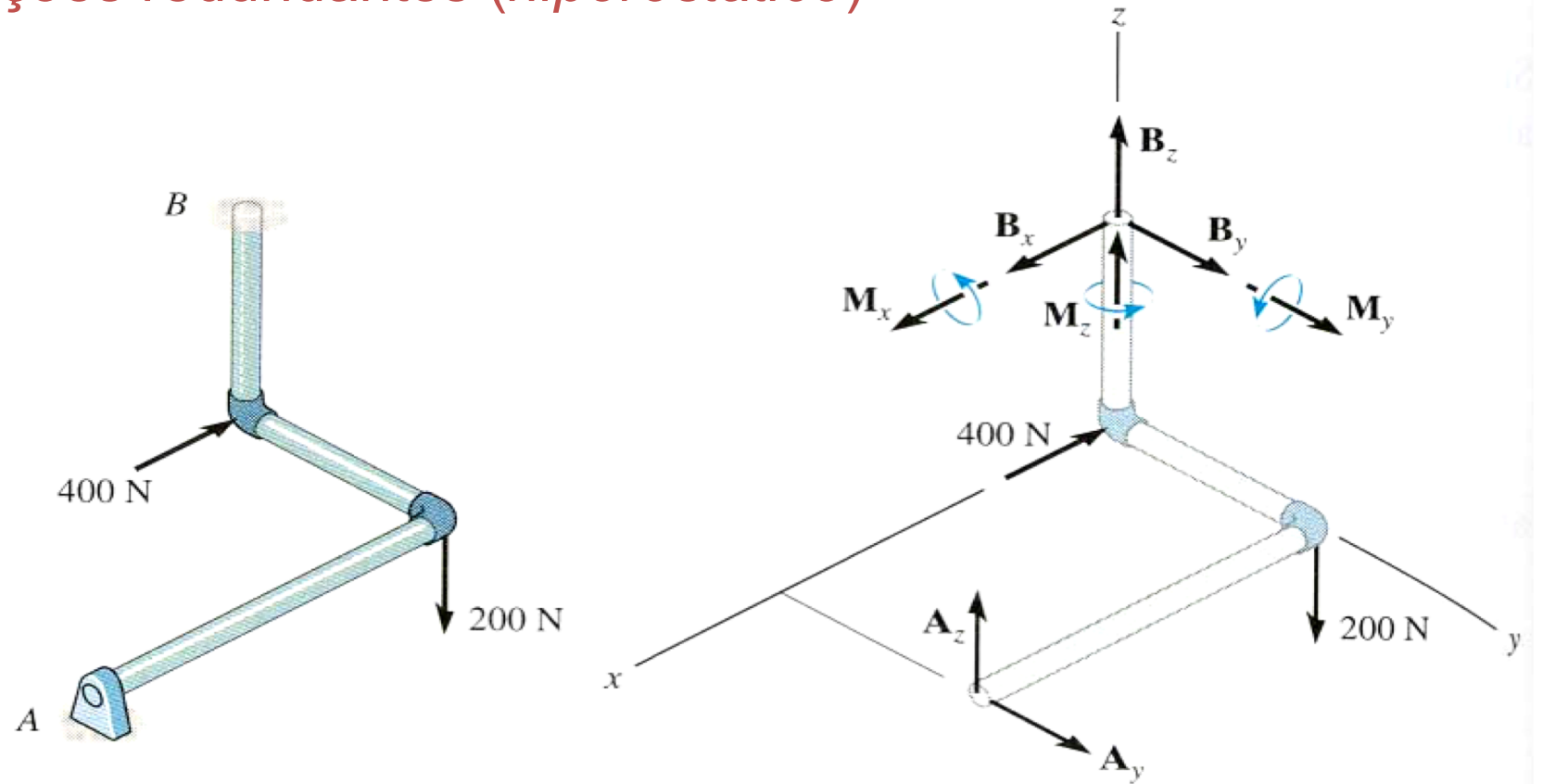
# DCL – problemas tridimensionais

- CUIDADO: Restrições Impróprias



## DCL – problemas tridimensionais

- CUIDADO: Restrições redundantes (*hiperestático*)





Definir um  
SISTEMA DE  
REFERÊNCIA

DCL

Desenhar o  
DIAGRAMA DE  
CORPO LIVRE



$$\Sigma f = 0$$

Escrever as  
EQUAÇÕES DO  
EQUILÍBRIO



Por hora, é isso meus caros!

# Engineering Flowchart

