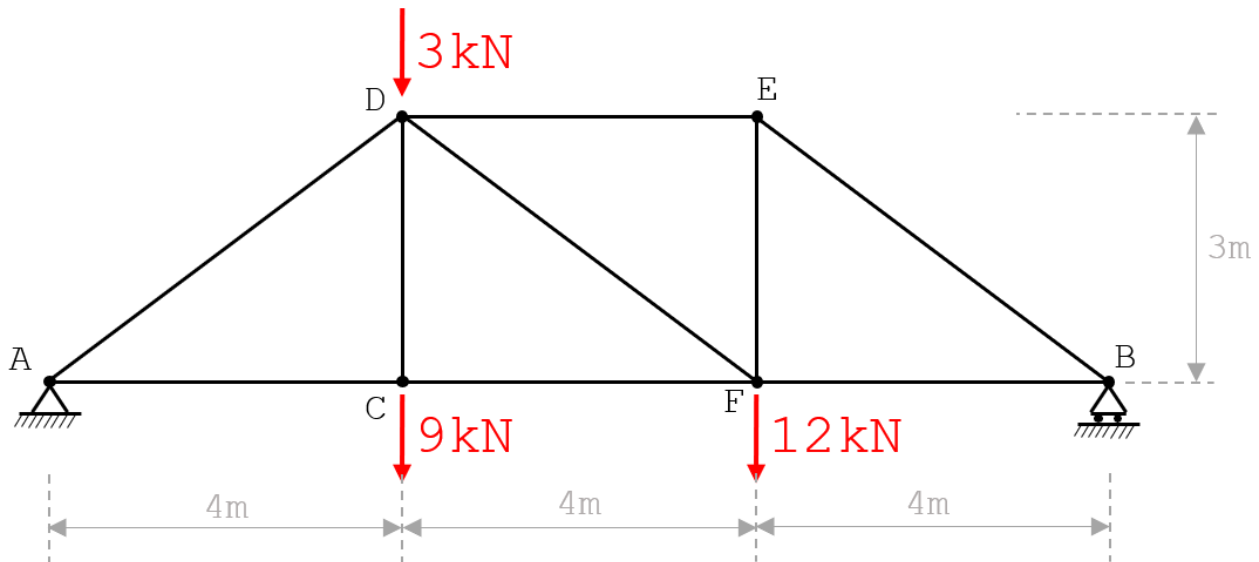




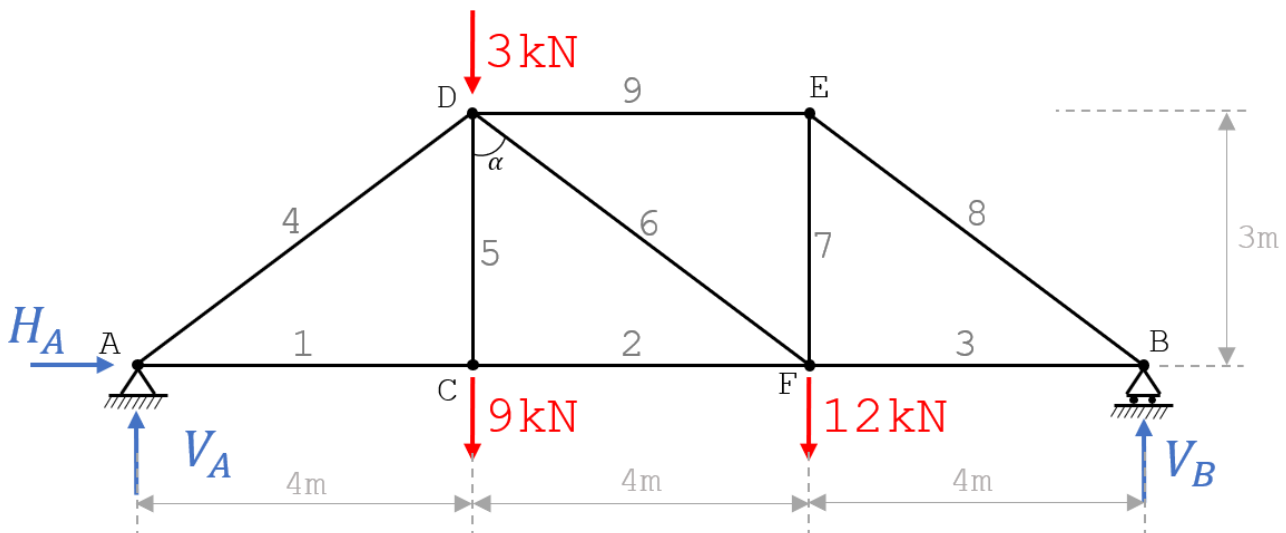
Dimensionar as barras das treliças:

Considere  $E = 210 \text{ GPa}$ ,  $\sigma_{lim} = \sigma_e = 250 \text{ MPa}$  e  $s = 2$ .

a)



Resolução:



$$\boxed{\text{sen } \alpha = \frac{4}{5}}$$

$$\boxed{\text{cos } \alpha = \frac{3}{5}}$$



1) Reações de apoio

$$\sum M_A = 0$$

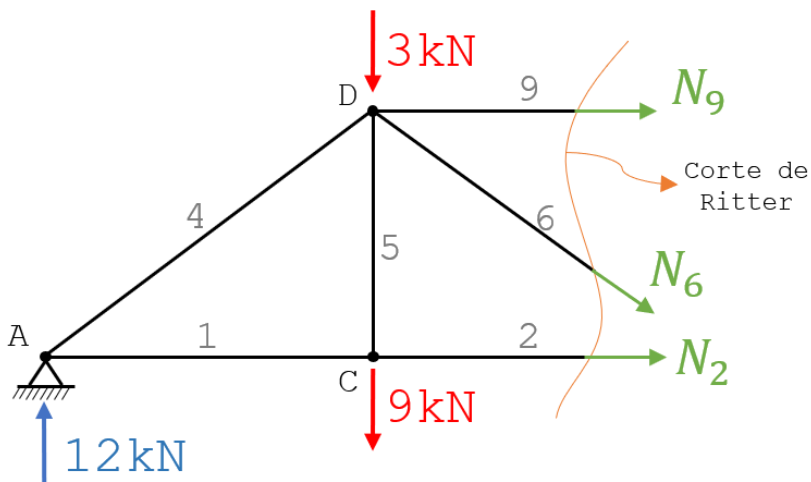
$$-3kN \times 4m - 9kN \times 4m - 12kN \times 8m + V_B \times 12m = 0 \quad \rightarrow \quad \boxed{V_B = 12kN}$$

$$\sum F_x = 0 \quad \rightarrow \quad \boxed{H_A = 0}$$

$$\sum F_y = 0$$

$$V_A + V_B - 3kN - 9kN - 12kN = 0 \quad \rightarrow \quad \boxed{V_A = 12kN}$$

2) Cálculo dos esforços nas barras



$$\sum M_D = 0$$

$$N_2 \times 3m - 12kN \times 4m = 0$$

$$\boxed{N_2 = 16kN}$$

$$\sum F_y = 0$$

$$12kN - 3kN - 9kN - N_6 \times \cos \alpha = 0$$

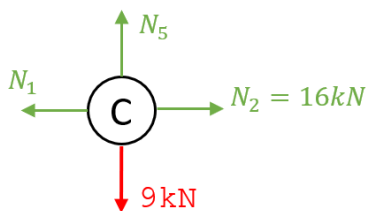
$$\boxed{N_6 = 0}$$

$$\sum F_x = 0$$

$$N_9 + N_2 = 0 \rightarrow N_9 + 16kN = 0$$

$$\boxed{N_9 = -16kN} \quad (\text{Compressão})$$

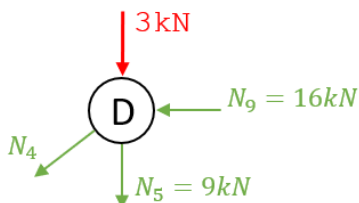
o Equilíbrio nó C:



$$\sum F_y = 0 \rightarrow \boxed{N_5 = 9kN}$$

$$\sum F_x = 0 \rightarrow \boxed{N_1 = 16kN}$$

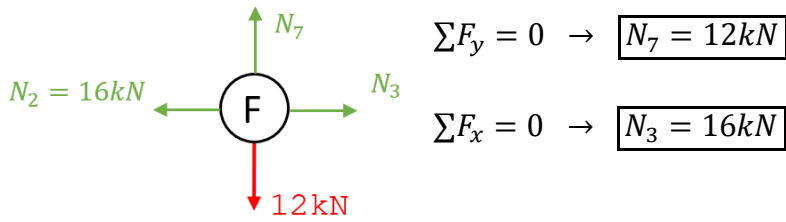
o Equilíbrio nó D:



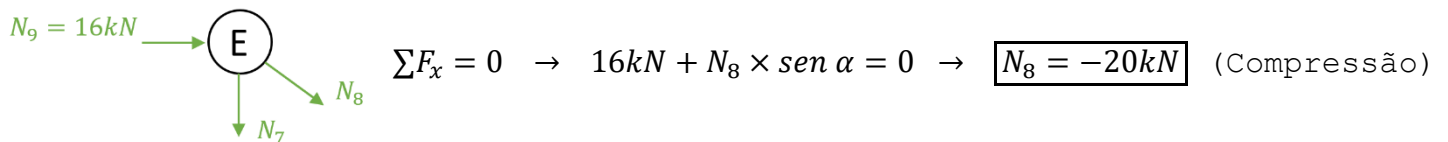
$$\sum F_x = 0 \rightarrow -16kN - N_4 \times \text{sen } \alpha = 0 \rightarrow \boxed{N_4 = -20kN} \quad (\text{Compressão})$$



o Equilíbrio nó F:



o Equilíbrio nó E:



Barra	[kN]
N <sub>1</sub>	16
N <sub>2</sub>	16
N <sub>3</sub>	16
N <sub>4</sub>	-20
N <sub>5</sub>	9
N <sub>6</sub>	0
N <sub>7</sub>	12
N <sub>8</sub>	-20
N <sub>9</sub>	-16

### 3) Dimensionamento

1º Critério: Tensão Normal

$$|\sigma_{max}| = \frac{|N_{max}|}{A} \leq \frac{\sigma_e}{s} \rightarrow A \geq \frac{s \times |N_{max}|}{\sigma_e}$$

$$A \geq \frac{2 \times 20 \times 10^3}{250 \times 10^6} \rightarrow A \geq 1,6 \times 10^{-4} m^2 \rightarrow A \geq 1,6 cm^2$$

2º Critério: Estabilidade

$$|N_{max}^c| \leq \frac{1}{s} \times \frac{\pi^2 \times E \times I}{\ell^2} \rightarrow I \geq \frac{s \times \ell^2 |N_{max}^c|}{\pi^2 E}$$

$$I \geq \frac{2 \times 5^2 \times 20 \times 10^3}{\pi^2 \times 210 \times 10^9} \rightarrow I \geq 4,82 \times 10^{-7} m^4 \rightarrow I \geq 48,2 cm^4$$



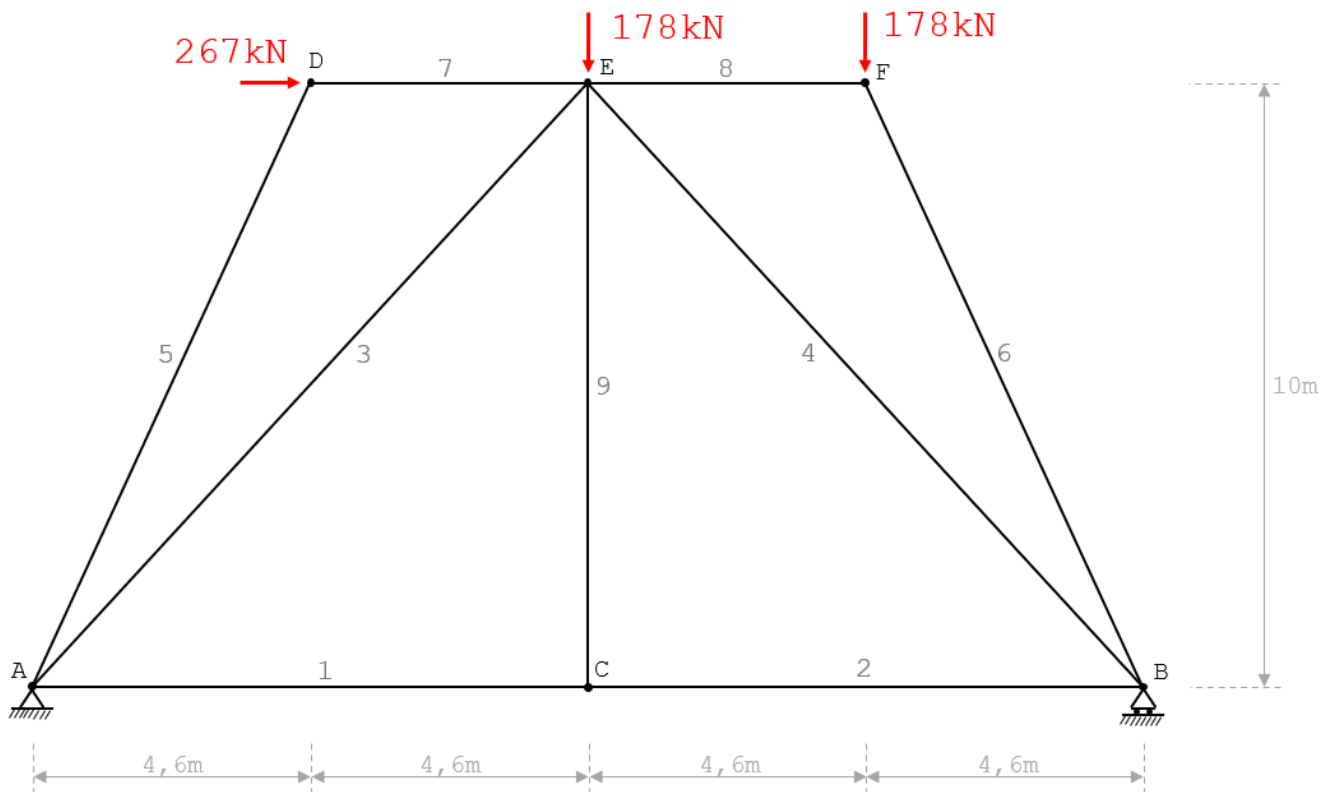
4) Escolha do perfil comercial (Catálogo Vallourec & Mannesmann)

Dimensões externas	Espessura da parede	Massa por Unidade de Comp.	Área da Seção Transversal	Momento de Inércia
d (mm)	t (mm)	m (kg/m)	A (cm <sup>2</sup> )	I <sub>x</sub> = I <sub>y</sub> (cm <sup>4</sup> )
60,3	3,6	5,03	6,41	25,9
	4,0	5,55	7,07	28,2
	4,5	6,19	7,89	30,9
	5,0	6,82	8,69	33,5
	5,6	7,55	9,62	36,4
	6,4	8,51	10,8	39,9
	7,1	9,32	11,9	42,7
	8,0	10,3	13,1	46,0
	8,8	11,2	14,2	48,6

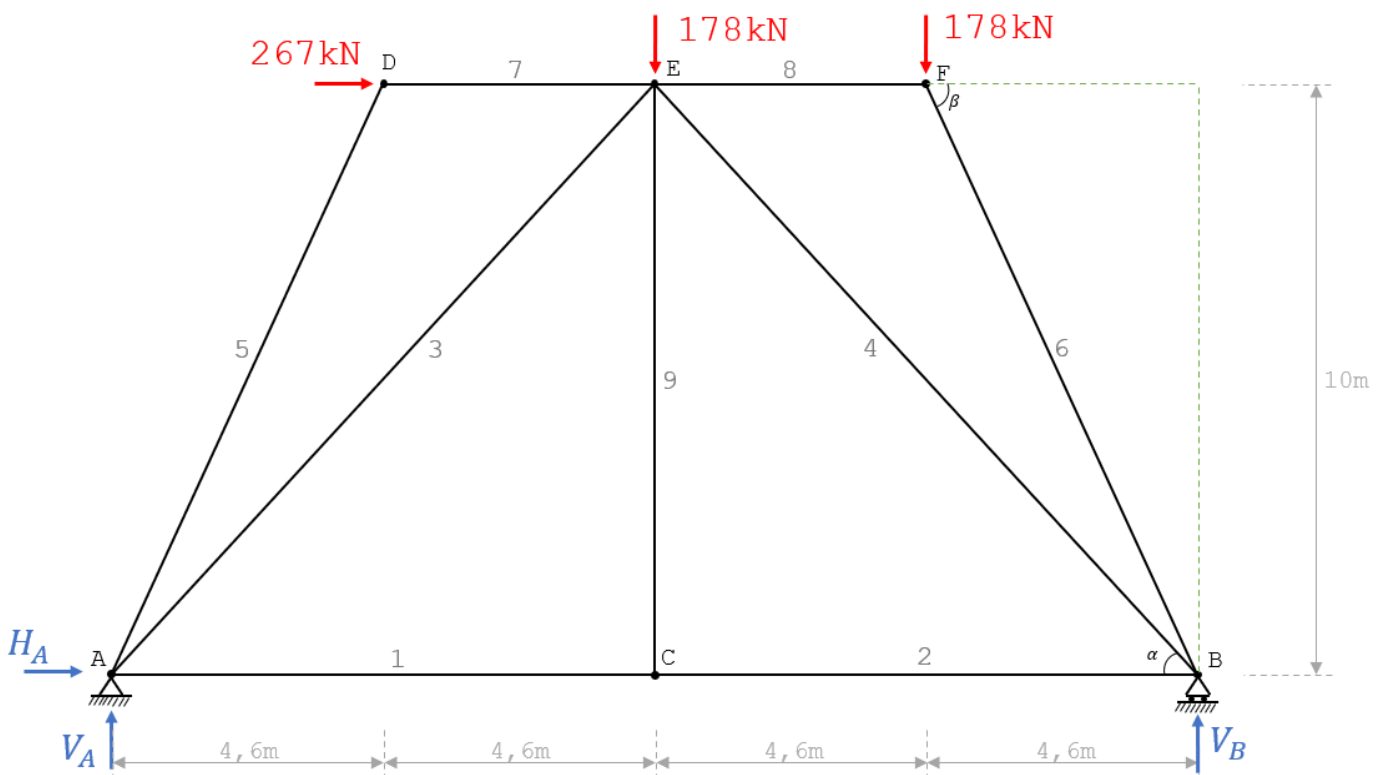


Dimensões externas	Espessura da parede	Massa por Unidade de Comp.	Área da Seção Transversal	Momento de Inércia
d (mm)	t (mm)	m (kg/m)	A (cm <sup>2</sup> )	I <sub>x</sub> = I <sub>y</sub> (cm <sup>4</sup> )
73,0	3,6	6,16	7,85	47,4
	4,0	6,81	8,67	51,8
	4,5	7,60	9,68	57,0
	5,0	8,38	10,7	62,1
	5,6	9,31	11,9	67,8
	6,4	10,5	13,4	74,9
	7,1	11,5	14,7	80,7
	8,0	12,8	16,3	87,6
	8,8	13,9	17,7	93,2
	10,0	15,5	19,8	101
	11,0	16,8	21,4	106

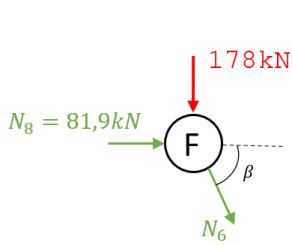
b)



**Resolução:**



$$\overline{BE}^2 = 9,2^2 + 10^2 \rightarrow \overline{BE} = 13,59m$$



$$\boxed{\operatorname{sen} \alpha = \frac{10}{13,59}} \quad \boxed{\cos \alpha = \frac{9,2}{13,59}}$$

$$\overline{BF}^2 = 4,6^2 + 10^2 \rightarrow \overline{BF} = 11,01\text{m}$$

$$\boxed{\operatorname{sen} \beta = \frac{10}{11,01}} \quad \boxed{\cos \beta = \frac{4,6}{11,01}}$$

1) Reações de apoio

$$\sum M_A = 0$$

$$-267\text{kN} \times 10\text{m} - 178\text{kN} \times 9,2\text{m} - 178\text{kN} \times 13,8\text{m} + V_B \times 18,4\text{m} = 0 \rightarrow \boxed{V_B = 367,6\text{kN}}$$

$$\sum F_x = 0$$

$$267\text{kN} + H_A = 0 \rightarrow \boxed{H_A = 267\text{kN} \leftarrow}$$

$$\sum F_y = 0$$

$$-178\text{kN} - 178\text{kN} + V_B + V_A = 0 \rightarrow \boxed{V_A = 11,6\text{kN}}$$

2) Cálculo dos esforços nas barras

$$\sum F_y = 0$$

$$-178\text{kN} + 367,6\text{kN} + N_4 \times \operatorname{sen} \alpha = 0$$

$$\boxed{N_4 = -257,6\text{kN}} \text{ (Compressão)}$$

$$\sum M_B = 0$$

$$178\text{kN} \times 4,6\text{m} + N_8 \times 10 = 0$$

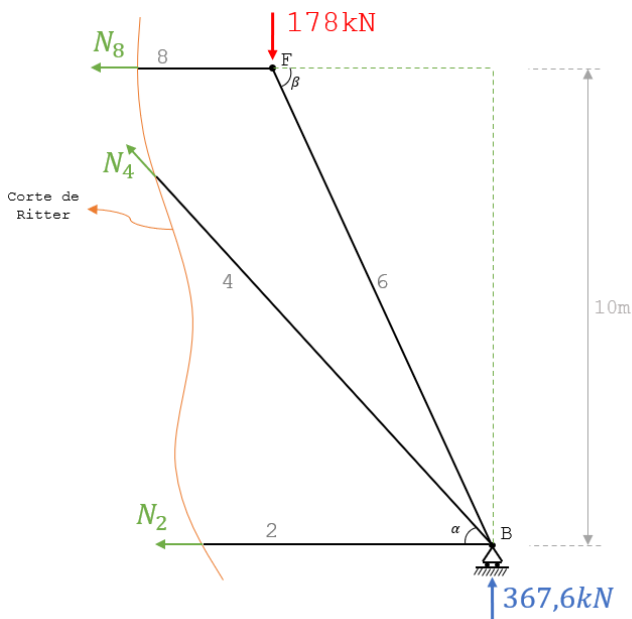
$$\boxed{N_8 = -81,9\text{kN}} \text{ (Compressão)}$$

$$\sum F_x = 0$$

$$-N_2 - N_4 \times \cos \alpha - N_8 = 0$$

$$-N_2 - (-257,6\text{kN}) \times \cos \alpha - (-81,9\text{kN}) = 0$$

$$\boxed{N_2 = 256,3\text{kN}}$$



o Equilíbrio nó F:

$$\sum F_y = 0$$

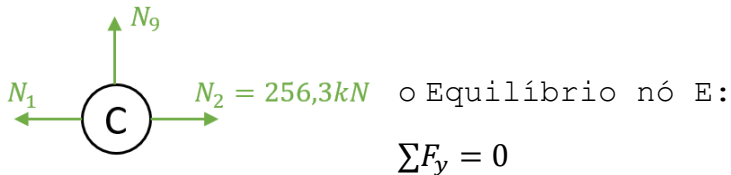


$$-178kN - N_6 \times \text{sen } \beta = 0 \rightarrow \boxed{N_6 = -195,9kN} \text{ (Compressão)}$$

o Equilíbrio nó C:

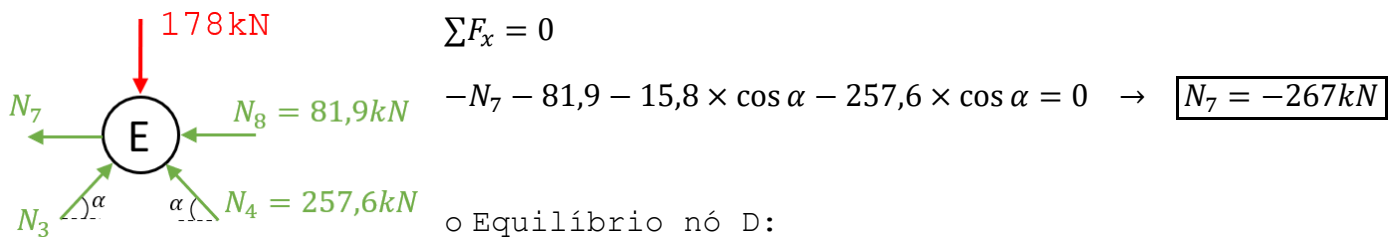
$$\sum F_y = 0 \rightarrow \boxed{N_9 = 0}$$

$$\sum F_x = 0 \rightarrow \boxed{N_1 = 256,3kN}$$



$$\sum F_y = 0$$

$$-178kN - N_3 \times \text{sen } \alpha + 257,6 \times \text{sen } \alpha = 0 \rightarrow \boxed{N_3 = 15,8kN}$$

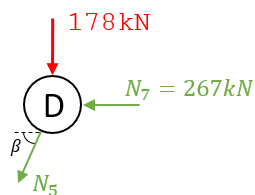


$$\sum F_x = 0$$

$$-N_7 - 81,9 - 15,8 \times \cos \alpha - 257,6 \times \cos \alpha = 0 \rightarrow \boxed{N_7 = -267kN}$$

$$\sum F_x = 0 \rightarrow \boxed{N_5 = 0}$$

(Compressão)



Barra	[kN]
N <sub>1</sub>	256,3
N <sub>2</sub>	256,3
N <sub>3</sub>	15,8
N <sub>4</sub>	-257,6
N <sub>5</sub>	0
N <sub>6</sub>	-195,9
N <sub>7</sub>	-267
N <sub>8</sub>	-81,9
N <sub>9</sub>	0

### 3) Dimensionamento

1º Critério: Tensão Normal



$$|\sigma_{max}| = \frac{|N_{max}|}{A} \leq \frac{\sigma_e}{s} \rightarrow \boxed{A \geq \frac{s \times |N_{max}|}{\sigma_e}}$$

$$A \geq \frac{2 \times 257,6 \times 10^3}{250 \times 10^6} \rightarrow A \geq 2,06 \times 10^{-3} m^2 \rightarrow \boxed{A \geq 20,6 cm^2}$$

2º Critério: Estabilidade

$$|N_{max}^c| \leq \frac{1}{s} \times \frac{\pi^2 \times E \times I}{\ell^2} \rightarrow \boxed{I \geq \frac{s \times \ell^2 |N_{max}^c|}{\pi^2 E}}$$

i. Barra 4

$$I \geq \frac{2 \times 13,59^2 \times 257,6 \times 10^3}{\pi^2 \times 210 \times 10^9} \rightarrow I \geq 4,59 \times 10^{-5} m^4 \rightarrow \boxed{I \geq 4590 cm^4}$$

ii. Barra 7

$$I \geq \frac{2 \times 4,6^2 \times 267 \times 10^3}{\pi^2 \times 210 \times 10^9} \rightarrow I \geq 5,45 \times 10^{-6} m^4 \rightarrow \boxed{I \geq 545 cm^4}$$

4) Escolha do perfil comercial (Catálogo Vallourec & Mannesmann)

Dimensões externas	Espessura da parede	Massa por Unidade de Comp.	Área da Seção Transversal	Momento de Inércia
d (mm)	t (mm)	m (kg/m)	A (cm²)	I <sub>x</sub> = I <sub>y</sub> (cm⁴)
	6,4	33,6	42,8	2.421
	7,1	37,1	47,3	2.660
	8,0	41,6	53,1	2.960
	8,8	45,6	58,1	3.220
	10,0	51,6	65,7	3.598
	11,0	56,5	71,9	3.904
	12,5	63,7	81,1	4.345
219,1	14,2	71,8	91,4	4.820
	16,0	80,1	102	5.297
	17,5	87,0	111	5.673
	20,0	98,2	125	6.261
	25,0	120	152	7.298



Dimensões externas	Espessura da parede	Massa por Unidade de Comp.	Área da Seção Transversal	Momento de Inércia
d (mm)	t (mm)	m (kg/m)	A (cm²)	I <sub>x</sub> = I <sub>y</sub> (cm⁴)
	6,4	42,1	53,6	4.765
	7,1	46,6	59,3	5.245
	8,0	52,3	66,6	5.852
	8,8	57,3	73,0	6.380
	10,0	64,9	82,6	7.154
	11,0	71,1	90,5	7.783
	12,5	80,3	102	8.697
	14,2	90,6	115	9.695
	16,0	101	129	10.707
	17,5	110	140	11.516
	20,0	125	159	12.798
	25,0	153	195	15.127
	30,0	180	229	17.162

273,0