Power-Efficiency-Dissipation Relations in Linear Thermodynamics

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(Received 31 March 2016; revised manuscript received 6 May 2016; published 2 June 2016)

We derive general relations between the maximum power, maximum efficiency, and minimum dissipation regimes from linear irreversible thermodynamics. The relations simplify further in the presence of a particular symmetry of the Onsager matrix, which can be derived from detailed balance. The results are illustrated on a periodically driven system and a three-terminal device subject to an external magnetic field.

DOI: 10.1103/PhysRevLett.116.220601

Introduction.—Thermodynamic machines transform different forms of energy into one another. For such a machine, it would be of obvious interest to maximize the power P and the efficiency η, and to minimize the dissipation $\dot{S}$ [1–37]. The extrema (maximum or minimum) here are understood with respect to a variation of the engine's load parameters, which are often the ones that are easy to tune. In general, the above goals are incompatible. For example, the efficiency when operating at maximum power is (in a time-symmetric setting) limited to half of the reversible efficiency $\eta_r = 1$. The latter efficiency, being an overall upper bound, can only be reached when operating reversibly, hence infinitely slowly. Consequently, the corresponding power vanishes. More generally, one may wonder whether there exist specific relationships between the regimes of maximum power (which will be denoted by the subscript MP), maximum efficiency (subscript ME), and minimum dissipation (subscript mD). Recently, such relations have been discovered between the MP and ME in the context of two case studies [23,36].

In this Letter, we derive general relations between the three regimes, within the framework of linear irreversible thermodynamics. Two results stand out. The first one is a remarkably simple relation linking the MP to the ME:

$$\eta_{\mathrm{MP}} = \frac{P_{\mathrm{MP}}}{2P_{\mathrm{MP}} - P_{\mathrm{ME}}}\eta_{\mathrm{ME}}. \tag{1}$$

As an implication, note that, since the power output $P_{\mathrm{ME}} > 0$ and efficiency $\eta_{\mathrm{MP}} > 0$ are positive, the efficiency at maximum power is at least half the maximum efficiency, $\eta_{\mathrm{MP}} \geq \eta_{\mathrm{ME}}/2$. The second result links the regimes of MP and mD by two equally simple equations:

$$T\dot{S}_{\mathrm{mD}} = \left(\frac{1}{\eta_{\mathrm{MP}}} - \frac{1}{\eta_{\mathrm{ME}}^2} - 1\right)P_{\mathrm{MP}} + \frac{1}{\eta_{\mathrm{ME}}^2}P_{\mathrm{ME}}, \tag{2}$$

$$P_{\mathrm{mD}} = P_{\mathrm{MP}} - \frac{1}{\eta_{\mathrm{ME}}^2}(P_{\mathrm{MP}} - P_{\mathrm{ME}}), \tag{3}$$

where T is the reference temperature of the system. As a consequence, note that when the minimum dissipation coincides with a reversible operation, i.e., $\dot{S}_{\mathrm{mD}} = 0$ and $P_{\mathrm{mD}} = 0$, one finds from Eqs. (2) and (3) that $\eta_{\mathrm{MP}} = 1/2$. The above relations become more specific when the Onsager matrix, which links the thermodynamic fluxes and forces, satisfies a generalized Onsager symmetry condition, which we discuss in more detail below. The "standard" Onsager symmetry, which applies to time-symmetric machines, is a particular case. Under this extra condition, the link between the maximum power and efficiency, cf. Eq. (1), splits into two separate relations, in agreement with the special cases discussed in Refs. [23,36]:

$$\frac{P_{\mathrm{ME}}}{P_{\mathrm{MP}}} = 1 - \eta_{\mathrm{ME}}^2, \qquad \eta_{\mathrm{MP}} = \frac{\eta_{\mathrm{ME}}}{1 + \eta_{\mathrm{ME}}^2}. \tag{4}$$

To mention some further implications of these results, reversible efficiency, $\eta_{\mathrm{ME}} = 1$, can only be reached when the power goes to zero, $P_{\mathrm{ME}} = 0$. Furthermore, $0 \leq \eta_{\mathrm{ME}} \leq 1$ implies $0 \leq \eta_{\mathrm{MP}} \leq 1/2$, as first noted in Ref. [1] (for a symmetric Onsager matrix). Note also that the equality sign in $P_{\mathrm{ME}} \leq P_{\mathrm{MP}}$ is only reached for $\eta_{\mathrm{ME}} = 0$, hence $\eta_{\mathrm{MP}} = 0$, illustrating the conflict between maximizing efficiency and maximizing power.

Under the same generalized Onsager symmetry condition, the links between the maximum power and minimum dissipation, Eqs. (2) and (3), simplify as follows [38]:

$$P_{\mathrm{mD}} = 0, \qquad T\dot{S}_{\mathrm{mD}} = \left(\frac{1}{\eta_{\mathrm{MP}}} - 2\right)P_{\mathrm{MP}}. \tag{5}$$

A zero minimum dissipation (with $P_{\mathrm{MP}} > 0$) implies $\eta_{\mathrm{MP}} = 1/2$, $\eta_{\mathrm{ME}} = 1$, and $P_{\mathrm{ME}} = 0$. Note the close interconnection between the results (4) and (5), since all of them follow from Eqs. (1)–(3), if any one of them is valid.

We close the introduction with an important comment concerning the mathematical and physical content of the above relations. We will derive the above results first in the simple setting of two thermodynamic fluxes and forces, linked by a 2×2 Onsager matrix $\mathbf{L}$. The relations (1)–(3)

follow from straightforward algebra applied to the standard expressions from linear irreversible thermodynamics. No additional assumptions are needed. Equations (4) and (5) on the other hand require Onsager symmetry or antisymmetry [39], i.e., $L_{12} = \pm L_{21}$. We next will show that both sets of results remain valid when the thermodynamic driving and loading force and flux are vectorial, i.e., they are composed of subforces and subfluxes, provided one performs the "full" optimization, i.e., with respect to all the components of the loading force. The validity of Eqs. (4) and (5) then rests in addition on a generalized Onsager symmetry $\mathbf{L}_{12} = \pm \mathbf{L}_{21}^T$ (T standing for the transpose) or $\mathbf{L}_{12} = \pm \mathbf{L}_{21}$. This property can be derived from time reversibility and detailed balance of the underlying microdynamics, and is therefore expected to have a very wide range of validity. We will illustrate this state of affairs on a system subject to a time-asymmetric periodic driving and a three-terminal device with an external magnetic field.

Linear irreversible thermodynamics.—The thermodynamic processes that drive machines are generally induced by a spatial or temporal variation in quantities such as (inverse) temperature, chemical potential, pressure, etc. These differences are responsible for so-called thermodynamic forces, which we will denote by F. With every thermodynamic force, one can associate a flux, for example, a heat flux or a particle flux, denoted as J. The generic function of a machine is to transform one type of energy into another one. The simplest such construction thus features two forces, one playing the role of a load force, say F_1, and another functioning as a driving force F_2. With proper definitions of fluxes and forces, the entropy production or dissipation $\dot{S}$ can be written as a bilinear form [40,41]:

$$\dot{S} = F_1 J_1 + F_2 J_2. \tag{6}$$

The working regime is defined as a driving entropy producing a flux, say J_2 with $F_2 J_2 \geq 0$, generating another flux J_1 against its own thermodynamic force, $F_1 J_1 \leq 0$. The standard example is that of a thermal machine, where a downhill heat flux pushes particles up a potential. The quantities of interest are the net dissipation $\dot{S}$, given in Eq. (6), the power output P, which we define as [42]

$$P = -T F_1 J_1, \tag{7}$$

and the efficiency η,

$$\eta = -\frac{F_1 J_1}{F_2 J_2}. \tag{8}$$

The power output and efficiency are both positive by definition of the working regime. In addition, the second law $\dot{S} \geq 0$ implies that, in the working regime, $\eta \leq \eta_r = 1$, with the reversible limit $\eta = \eta_r$ reached for zero entropy production, $\dot{S} = 0$. Hence, one has

$$\dot{S} \geq 0, \qquad P \geq 0, \qquad 0 \leq \eta \leq \eta_r = 1. \tag{9}$$

Finally, by their definitions, power, efficiency and entropy production are not independent quantities but obey the following relation:

$$T\dot{S} = P\left(\frac{1}{\eta} - 1\right). \tag{10}$$

Focusing on the regime of linear irreversible thermodynamics, one assumes that the thermodynamic forces are small, so that the associated thermodynamic fluxes are linear in the forces:

$$\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}. \tag{11}$$

The coefficients L_{ij} are known as the Onsager coefficients. For a given thermodynamic process, one can consider its time inverse, denoted by a tilde. It is obtained by reversing the time dependencies and inverting the variables, such as speed and magnetic field, which are odd under time inversion. The above coefficients satisfy the so-called Onsager-Casimir symmetry $\tilde{L}_{ij} = L_{ji}$, [43]. This relation is particularly useful in the time-symmetric scenario with even variables, for which it reduces to the celebrated Onsager symmetry, $L_{ij} = L_{ji}$ [44,45].

We are now ready to calculate the values of the three key quantities power, efficiency, and dissipation when performing the extremum of one of them with respect to the loading force F_1. In calculating the maximum efficiency and power, we will assume them to be in the working regime. This leads to nine expressions P_{MP}, P_{ME}, P_{mD}, η_{MP}, η_{ME}, η_{mD}, $\dot{S}_{\mathrm{MP}}$, $\dot{S}_{\mathrm{ME}}$, $\dot{S}_{\mathrm{mD}}$, of which, in view of Eq. (10), six are *a priori* independent. Straightforward algebra leads to the following explicit expressions:

$$P_{\mathrm{MP}} = T\frac{L_{12}^2 F_2^2}{4L_{11}}, \qquad \eta_{\mathrm{MP}} = \frac{L_{12}^2}{4L_{11}L_{22} - 2L_{12}L_{21}}, \tag{12}$$

$$\begin{aligned} P_{\mathrm{mD}} &= T\frac{(L_{12}^2 - L_{21}^2)F_2^2}{4L_{11}}, \\ \dot{S}_{\mathrm{mD}} &= F_2^2\left(L_{22} - \frac{(L_{12} + L_{21})^2}{4L_{11}}\right), \end{aligned} \tag{13}$$

$$P_{\mathrm{ME}} = -TF_2^2\left(L_{11}L_{22} - \sqrt{L_{11}L_{22}(L_{11}L_{22} - L_{12}L_{21})}\right) \times \frac{\left(L_{11}L_{22} - L_{12}L_{21} - \sqrt{L_{11}L_{22}(L_{11}L_{22} - L_{12}L_{21})}\right)}{L_{11}L_{21}^2}, \tag{14}$$

$$\eta_{\mathrm{ME}} = -(L_{11}L_{22} - \sqrt{L_{11}L_{22}(L_{11}L_{22} - L_{12}L_{21})}) \times \frac{(L_{11}L_{22} - L_{12}L_{21} - \sqrt{L_{11}L_{22}(L_{11}L_{22} - L_{12}L_{21})})}{L_{21}^2\sqrt{L_{11}L_{22}(L_{11}L_{22} - L_{12}L_{21})}}. \tag{15}$$

The surprise is that there are, in fact, only three independent quantities: one verifies by inspection the validity of the relations (1) and (3). In the case of Onsager symmetry or antisymmetry, these equations further simplify with the appearance of one additional relation, cf. Eqs. (4) and (5). Hence, we are left with only two independent quantities out of the original nine, for example, any pair of power and efficiency, $\dot{S}_{\mathrm{mD}}$ and η_{MP}, $\dot{S}_{\mathrm{mD}}$ and P_{MP}, etc.

Multiple processes.—In a more general setting, a thermodynamic machine can involve many processes with input and output flux combinations of multiple subfluxes. Keeping the notation of subindices $i = 1, 2$ for loading and driving quantities, respectively, the corresponding fluxes $\mathbf{J}_i$, forces $\mathbf{F}_i$, and Onsager coefficients $\mathbf{L}_{ij}$ are no longer scalars but vectors and matrices, respectively. Onsager-Casimir symmetry predicts $\tilde{\mathbf{L}}_{ij} = \mathbf{L}_{ji}^T$. Although the proof now requires some more involved matrix algebra (cf. the Supplemental Material [46]), one can show that the first set of power-efficiency-dissipation relations, Eqs. (1)–(3), remain valid provided the optimum is carried out with respect to all components of the loading force $\boldsymbol{F}_1$. Under the same optimization, the second set of relations (4) and (5) follows for Onsager matrices obeying the following generalized Onsager condition:

$$\mathbf{L}_{12;s}\mathbf{L}_{11;s}^{-1}\mathbf{L}_{12;s} = \mathbf{L}_{21;s}\mathbf{L}_{11;s}^{-1}\mathbf{L}_{21;s}, \quad \mathbf{L}_{12;a}\mathbf{L}_{11;s}^{-1}\mathbf{L}_{12;a} = \mathbf{L}_{21;a}\mathbf{L}_{11;s}^{-1}\mathbf{L}_{21;a} \tag{16}$$

with $\mathbf{L}_{ij;s} = (\mathbf{L}_{ij} + \mathbf{L}_{ij}^T)/2$, the symmetric part of the matrix and $\mathbf{L}_{ij;a} = (\mathbf{L}_{ij} - \mathbf{L}_{ij}^T)/2$ the antisymmetric part of the matrix. We make the important observation that this condition is satisfied for matrices obeying

$$\mathbf{L}_{12} = \pm\mathbf{L}_{21}^T, \qquad \mathbf{L}_{12} = \pm\mathbf{L}_{21}. \tag{17}$$

It is clear from Onsager symmetry that systems with time-symmetric driving satisfy this condition, but it may also hold for systems violating time-reversal symmetry. Indeed, it has been shown that Onsager matrices of this form arise as a consequence of detailed balance [27,35], even though the setup itself might break time-reversal symmetry, cf. the Supplemental Material [46]. Consequently, Eqs. (4) and (5) are expected to have a wide range of validity, including systems that break time symmetry. We stress again that the optimization needs to be carried out with respect to all components of the loading force. In the case of partial optimization, the corresponding effective Onsager matrix of lower rank no longer satisfies Eq. (16), and therefore Eqs. (4) and (5) break down. On the other hand, Eqs. (1) and (3) remain valid when the system is optimized with respect to the reduced set of variables, since the latter results are algebraic in nature, and do not require additional physical input.

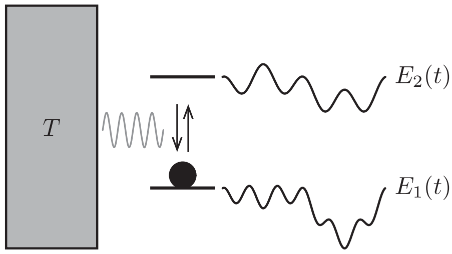

FIG. 1. Schematic representation of a periodically driven two-level system in contact with a heat reservoir.

Two examples.—We illustrate the above results on two systems that do not satisfy time-reversal symmetry: a thermodynamic machine subject to explicit time-periodic driving [27–29,35–37,47–50] and a three-terminal device in an external magnetic field [15,51–58].

The first example is a work-to-work converter consisting of a particle that can hop between two discrete energy levels, cf. Fig. 1. Transitions are induced by a thermal bath, while the periodic modulation (period $\mathcal{T}$) of the energy levels via two external work mechanisms allows the conversion of work extracted from the second source, driving the second energy level, and delivered to the first source, loading the first energy level. The time dependence of the energy in each level $i = 1, 2$ can be developed in terms of its Fourier components:

$$E_i(t) = \sum_n F_{(i,n,s)} \sin\left(\frac{2\pi nt}{\mathcal{T}}\right) + F_{(i,n,c)} \cos\left(\frac{2\pi nt}{\mathcal{T}}\right), \tag{18}$$

where the amplitudes $F_{(i,n,c)}$ and $F_{(i,n,s)}$ play the role of thermodynamic forces, n refers to the Fourier mode, and c and s refer to cosine and sine, respectively. Following standard techniques from stochastic thermodynamics [59–64], one can determine the explicit expression for the elements of the associated Onsager matrix [27] (cf. the Supplemental Material [46]):

$$L_{(1,n,\sigma),(2,n,\sigma)} = -\frac{4\pi^3 n^3 (4\pi^2 n^2 \mathbf{1} + \mathcal{T}^2 \mathbf{W}^{(0)2})_{12}^{-1} p_2^{\mathrm{eq}}}{\mathcal{T}}, \tag{19}$$

where $\mathbf{W}^{(0)}$ and $\mathbf{p}^{\mathrm{eq}}$ are the transition matrix and equilibrium probability distribution associated with the state of the particle in the absence of time-dependent driving, and $\sigma = s, c$. As a direct consequence of detailed balance, $\mathbf{W}_{12}^{(0)}\mathbf{p}_2^{\mathrm{eq}} = \mathbf{W}_{21}^{(0)}\mathbf{p}_1^{\mathrm{eq}}$, one finds

$$L_{(1,n,\sigma),(2,n,\sigma)} = L_{(2,n,\sigma),(1,n,\sigma)}. \tag{20}$$

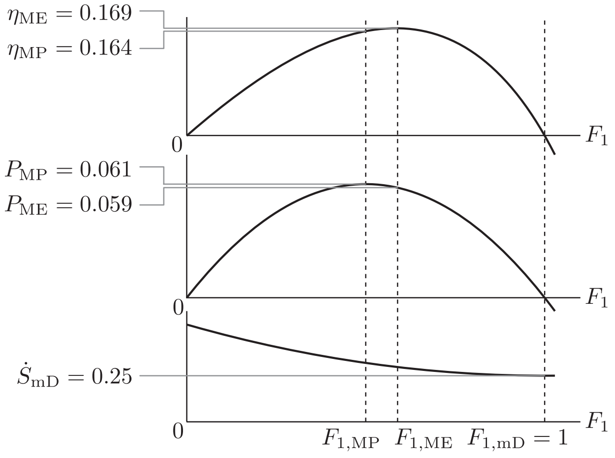

FIG. 2. Efficiency, power, and dissipation of a driven two-level system, $E_1(t) = F_1 \cos(2\pi t/\mathcal{T})$ and $E_2(t) = F_2[\cos(2\pi t/\mathcal{T}) + \cos(4\pi t/\mathcal{T})]$, with $\mathcal{T} = 1$, $T = 1$, and $F_2 = 1$. The Onsager coefficients are given by Ref. [27]: $L_{11} = -L_{12} = -L_{21} = 0.244$, $L_{22} = 0.492$. $P_{\rm mD} = 0$, as can be seen by visual inspection. One also verifies that $\eta_{\rm ME}/(1+\eta_{\rm ME}^2) = 0.164 = \eta_{\rm MP}$, $P_{\rm ME}/P_{\rm MP} = 0.971 = 1 - \eta_{\rm ME}^2$, $(1/\eta_{\rm MP} - 2)P_{\rm MP} = 0.25 = \dot{S}_{\rm mD}$, in agreement with Eqs. (4) and (5).

Analogous relations are found for $L_{(1,n,\sigma),(2,m,\rho)}$, with $\rho \neq \sigma$ and $m \neq n$. We conclude that the following symmetry relation holds:

$$L_{(1,n,\sigma),(2,m,\rho)} = L_{(2,n,\sigma),(1,m,\rho)}, \tag{21}$$

which satisfies Eq. (17). Hence, the second set of power-efficiency-dissipation relations, Eqs. (4) and (5), will be verified, see also Ref. [36] for a similar conclusion in a different model, and Fig. 2 for an illustration in case of a time-symmetric driving.

As another example of a system with broken time-reversal symmetry we consider a three-terminal thermoelectric device in a magnetic field, cf. Fig. 3. In this setup, three terminals are connected with each other via a central scattering region, inducing a particle flux $\mathbf{J}_\rho$ and a heat flux $\mathbf{J}_q$. In the working regime, the heat flux is from high to low temperature, while the particle flux is from low to high chemical potential. We assume that both fluxes are in the

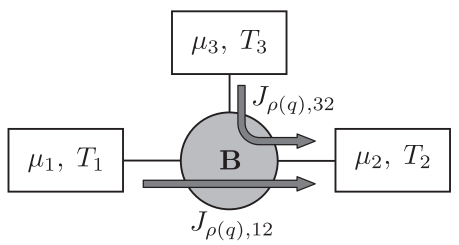

FIG. 3. Schematic representation of the three-terminal thermoelectric device.

direction of the second reservoir in Fig. 3. In this way heat is converted into chemical energy. A magnetic field $\mathbf{B}$ can be added to interact with the scattering region and break the time-reversal symmetry. An additional constraint that is often imposed is that the particle and heat flux through the third terminal vanish. The resulting 2×2 Onsager matrix, associated with the heat and particle flux between reservoir 1 and 2, is generally not symmetric, and the efficiency at maximum power can reach values up to $\eta_{\rm MP} = 4/7$ [15], clearly violating the second set of power-efficiency-dissipation relations, Eqs. (4) and (5), cf. the Supplemental Material [46]. Crucial to this analysis, however, is the constraint that the fluxes through the third terminal are zero, which makes it impossible to fully optimize the power output. Dropping the flux constraints will introduce thermodynamic subfluxes associated with the third terminal, and therefore $\mathbf{L}_{\rho q}$ and $\mathbf{L}_{q\rho}$ become 2×2 matrices.

In the present context of linear thermodynamics, we set the reference values for the temperature and chemical potential equal to those of the second reservoir, $T = T_2$ and $\mu = \mu_2$. The fluxes can be decomposed into a net flux from the first to the second terminal and from the third to the second terminal, $\mathbf{J}_{\rho(q)} = (J_{\rho(q),12}, J_{\rho(q),32})$ with the associated thermodynamic forces $\mathbf{F}_\rho = (e/T)(\mu_1 - \mu, \mu_3 - \mu)$ and $\mathbf{F}_q = (1/T^2)(T_1 - T, T_3 - T)$, where e is the charge of one electron. The behavior of the central region is described by the scattering matrix $\mathbf{S}(E, \mathbf{B})$, which gives the fluxes of electrons with energy E between the different terminals, when an external magnetic field $\mathbf{B}$ is applied to the central region. The resulting Onsager matrix is given by [65]

$$\mathbf{L}_{\alpha\beta} = \int_{-\infty}^{\infty} dE f_{\alpha\beta}(E)[\mathbf{1} - \mathbf{S}_{(1,3)}(E, \mathbf{B})] \tag{22}$$

with $\alpha, \beta = \rho$ or q, $\mathbf{S}_{(1,3)}(E, \mathbf{B})$ the scattering matrix associated with the first and the third terminal only, and $f_{\alpha\beta}(E)$ a function independent of the central scattering region, and in particular of the presence of a magnetic field (cf. the Supplemental Material [46]). Hence, it is invariant under time-reversal symmetry and satisfies $f_{\rho q}(E) = f_{q\rho}(E)$, implying $\mathbf{L}_{\rho q} = \mathbf{L}_{q\rho}$. We conclude that Eqs. (4) and (5) will be valid when the optimization is carried out without constraints on the third terminal. In particular, the efficiency at maximum power will drop to a value below 1/2.

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