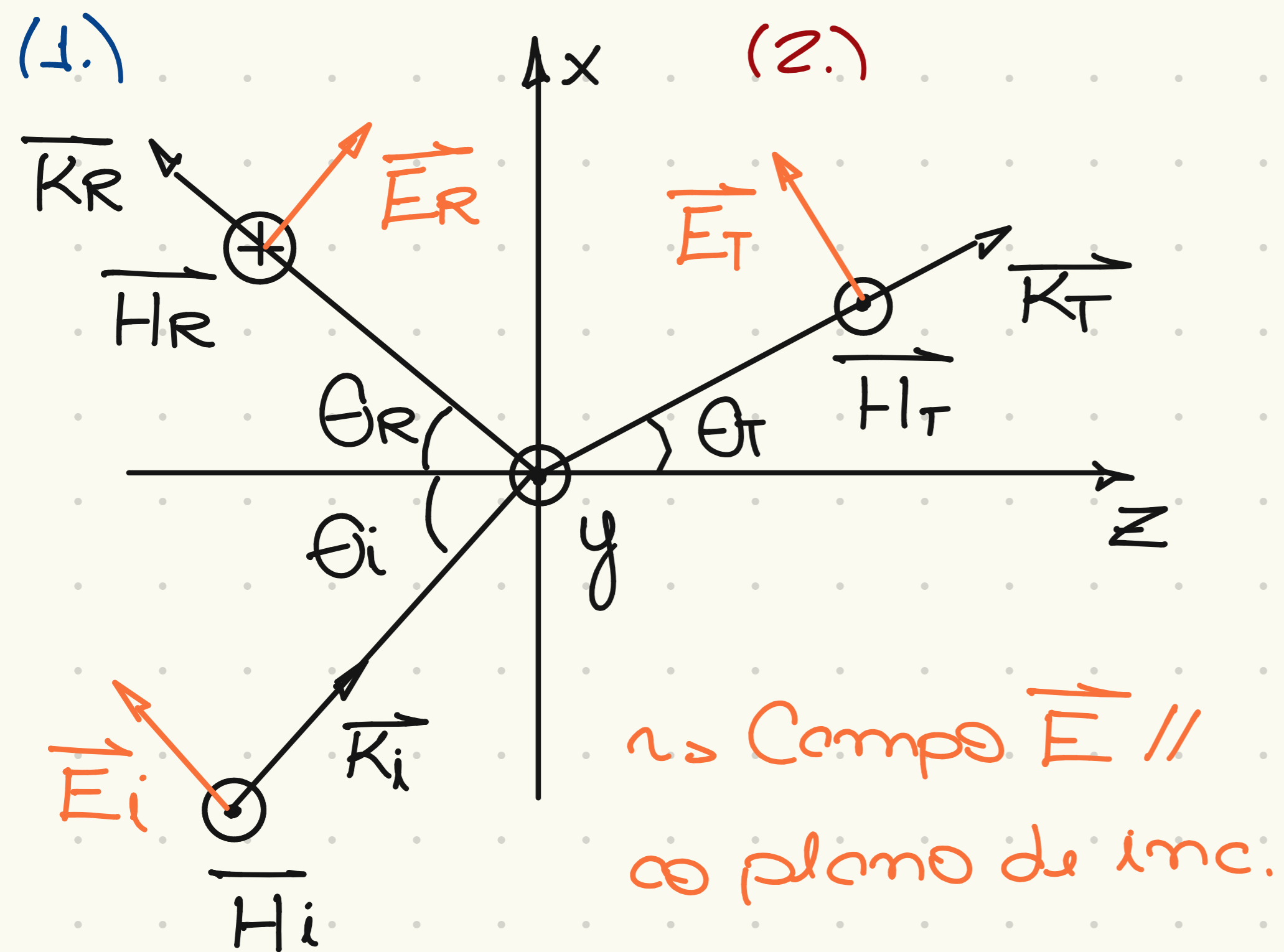
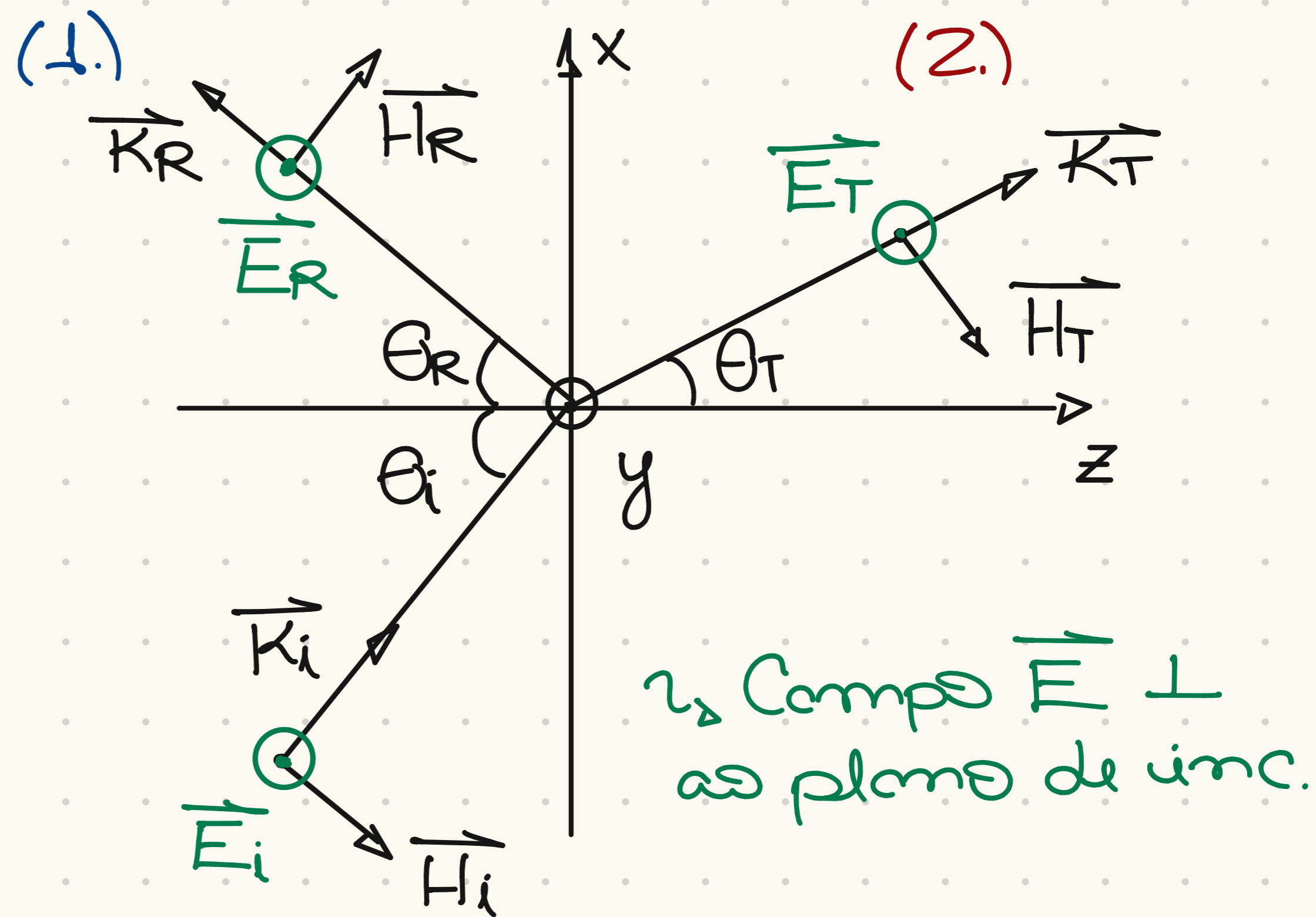


# Polarizācijas otra 3.

## Polarizācijas TM (onda p)



## Polarizācijas TE (onda s)



Onda p (pol. TM)

$$R_p = \frac{\hat{E}_{OR}}{\hat{E}_{OI}} = \frac{-n_2/n_1 \cos \theta_i + \cos \theta_T}{n_2/n_1 \cos \theta_i + \cos \theta_T} = \frac{-(n_2/n_1)^2 \cos \theta_i - \sqrt{(n_2/n_1)^2 - \sin^2 \theta_i}}{(n_2/n_1)^2 \cos \theta_i + \sqrt{(n_2/n_1)^2 - \sin^2 \theta_i}}$$

Usando a lei de Snell podemos demonstrar:  $\left( \frac{n_2}{n_1} = \frac{\sin \theta_i}{\sin \theta_T} \right)$

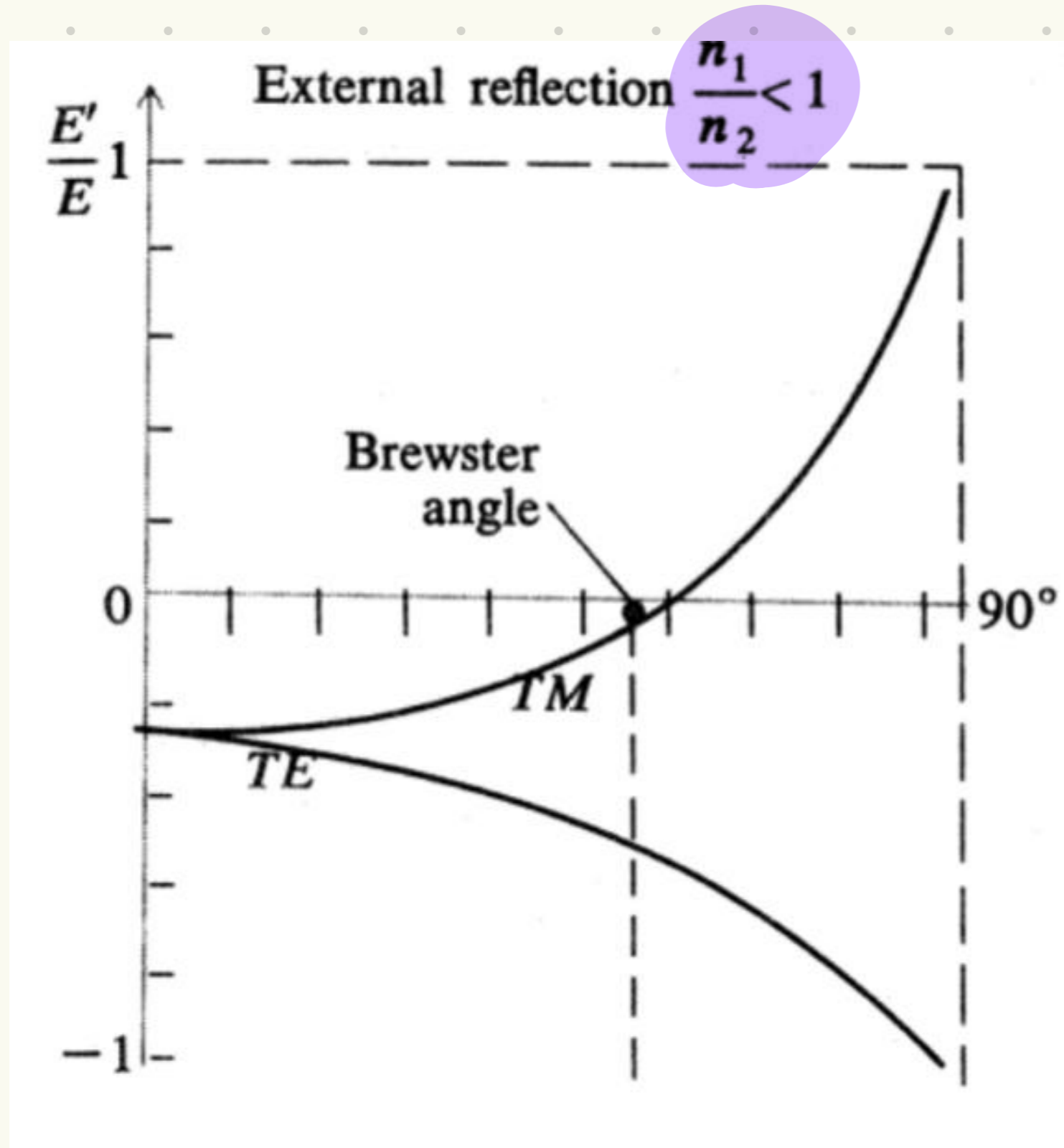
$$R_p = -\frac{\operatorname{tg}(\theta_i - \theta_T)}{\operatorname{tg}(\theta_i + \theta_T)}$$

De forma análoga, p/a onda s (pol. TE):

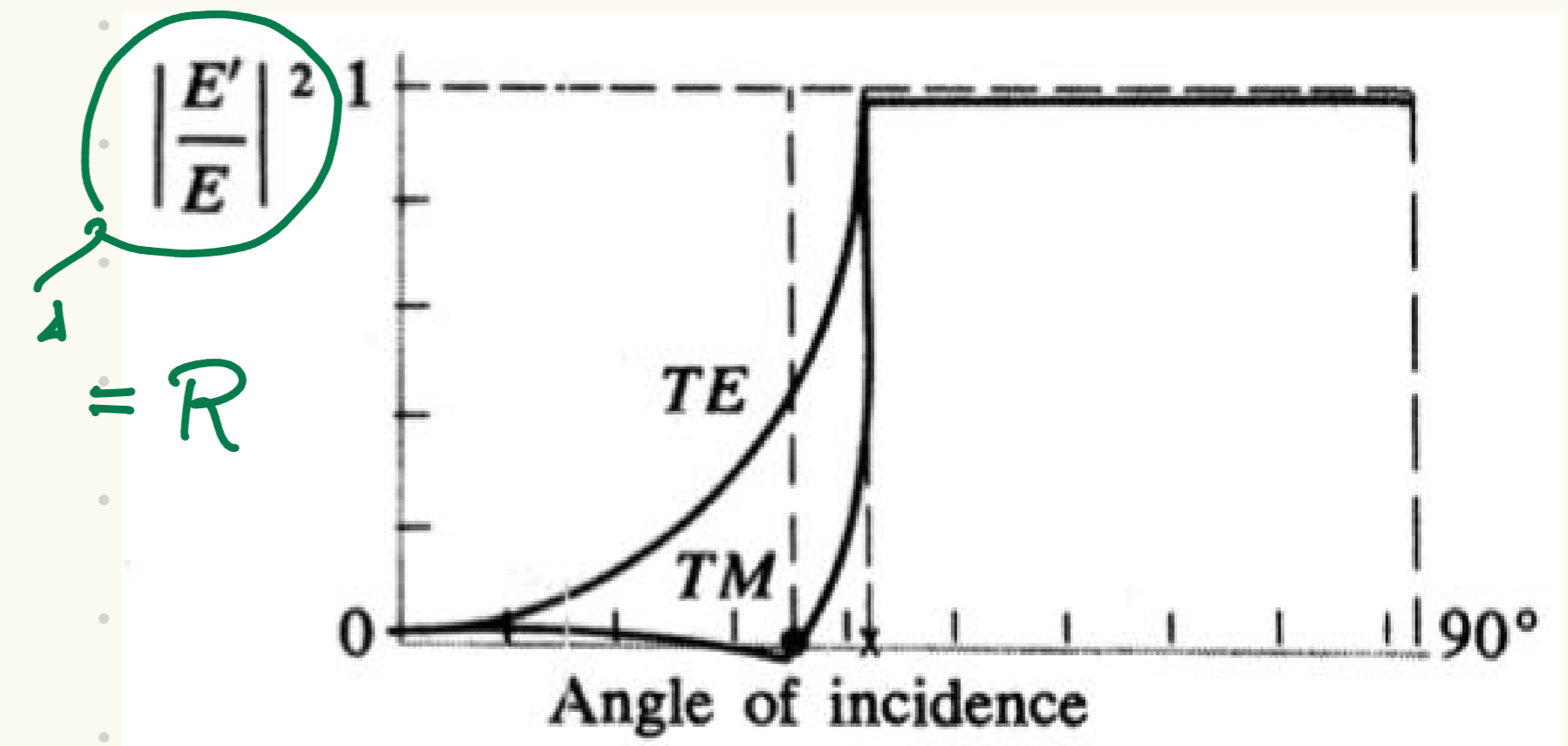
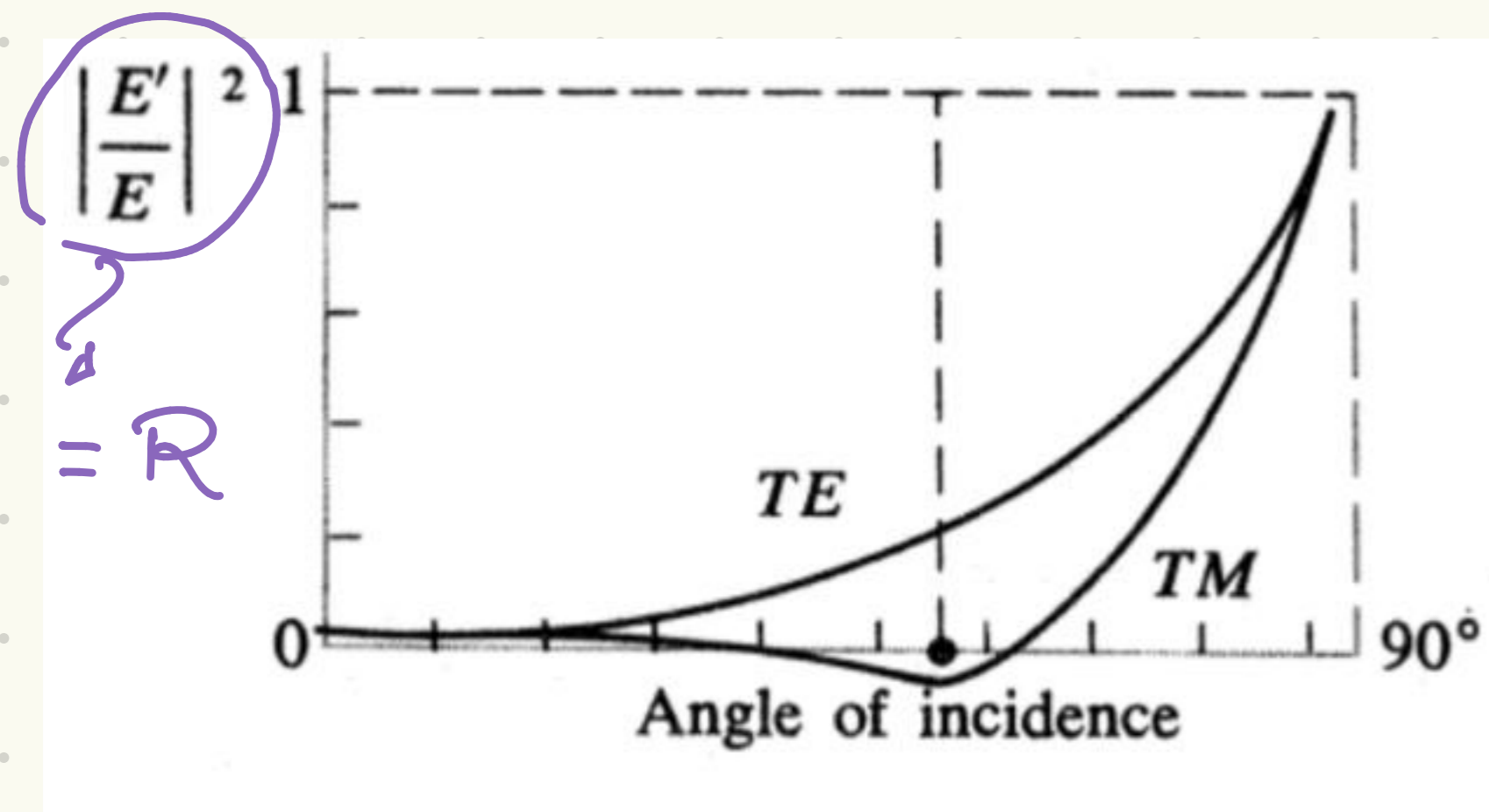
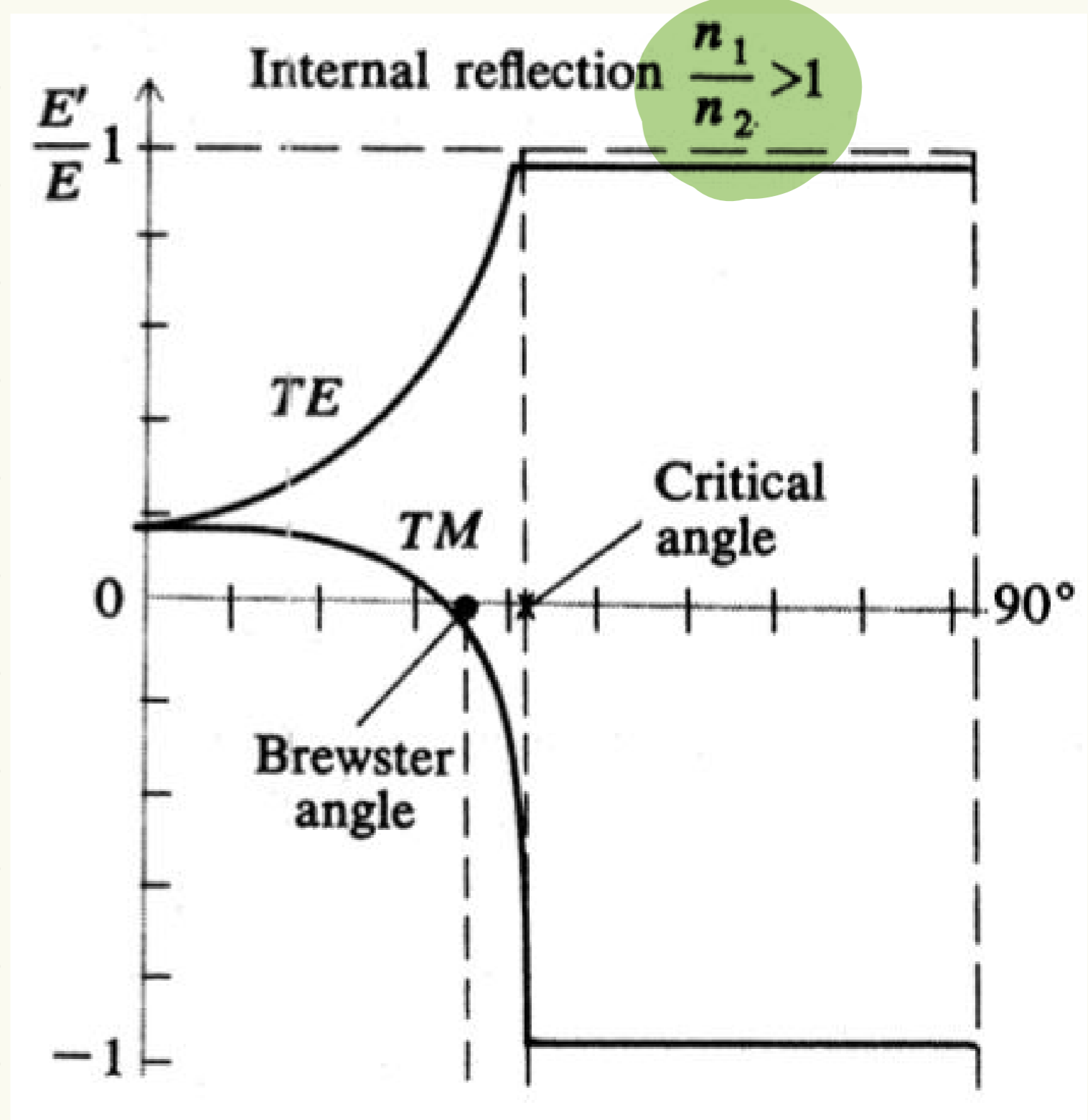
$$R_s = \frac{\hat{E}_{OR}}{\hat{E}_{OI}} = \frac{\cos \theta_i - n_2/n_1 \cos \theta_T}{\cos \theta_i + n_2/n_1 \cos \theta_T} = \frac{\cos \theta_i - \sqrt{(n_2/n_1)^2 - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{(n_2/n_1)^2 - \sin^2 \theta_i}}$$

$$R_s = -\frac{\sin(\theta_i - \theta_T)}{\sin(\theta_i + \theta_T)}$$

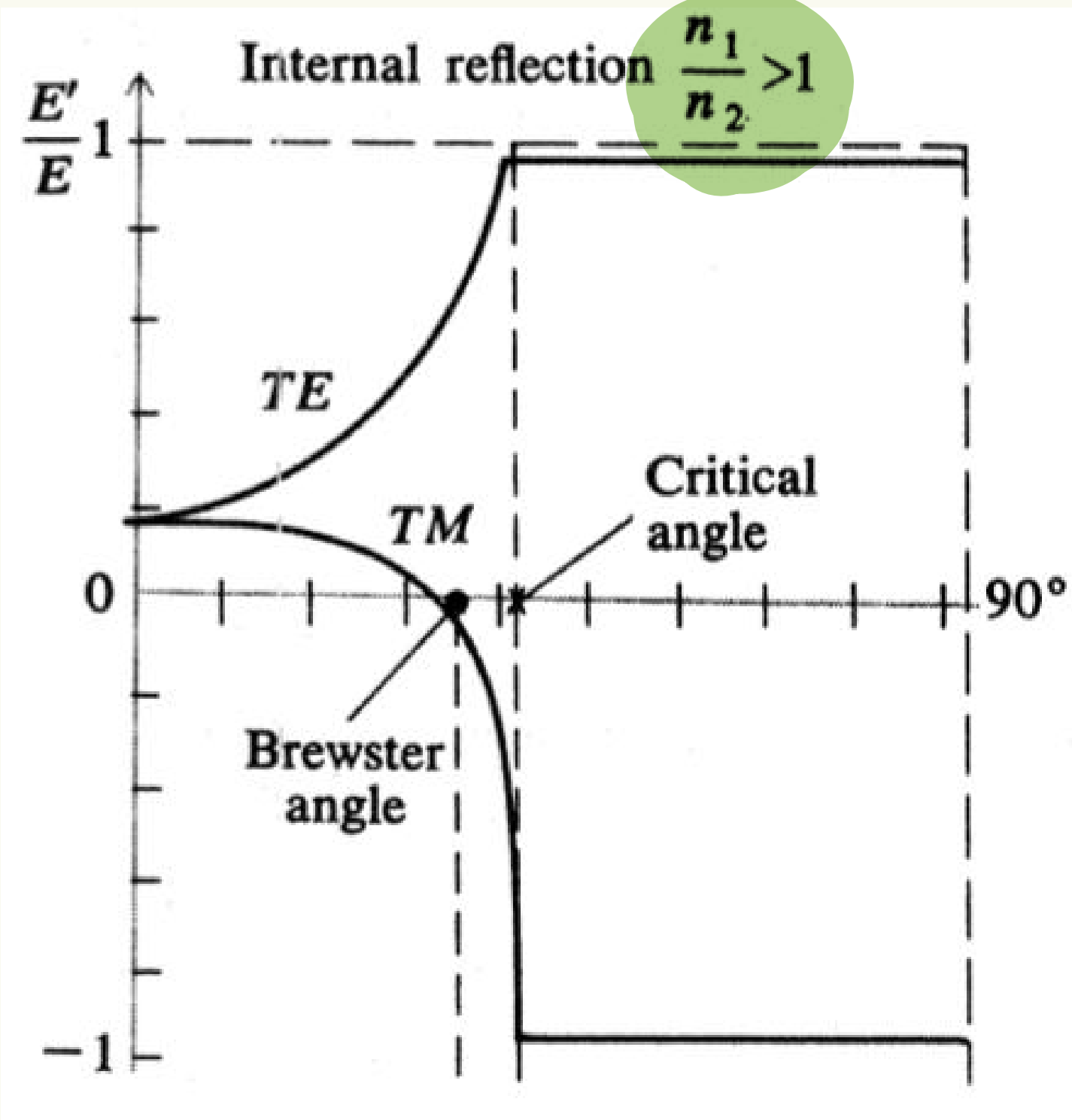
case:  $n_2 > n_1$



case:  $n_2 < n_1$



caso:  $n_2 < n_1$



Coef. de reflexão p/ a onda p (TM):

$$R_p = \frac{-(n_2/n_1)^2 \cos \theta_i - \sqrt{(n_2/n_1)^2 - \sin^2 \theta_i}}{(n_2/n_1)^2 \cos \theta_i + \sqrt{(n_2/n_1)^2 - \sin^2 \theta_i}} \quad (I)$$

Ang. crítico:  $\theta_c \rightarrow$  ang. a partir do qual toda a luz é refletida

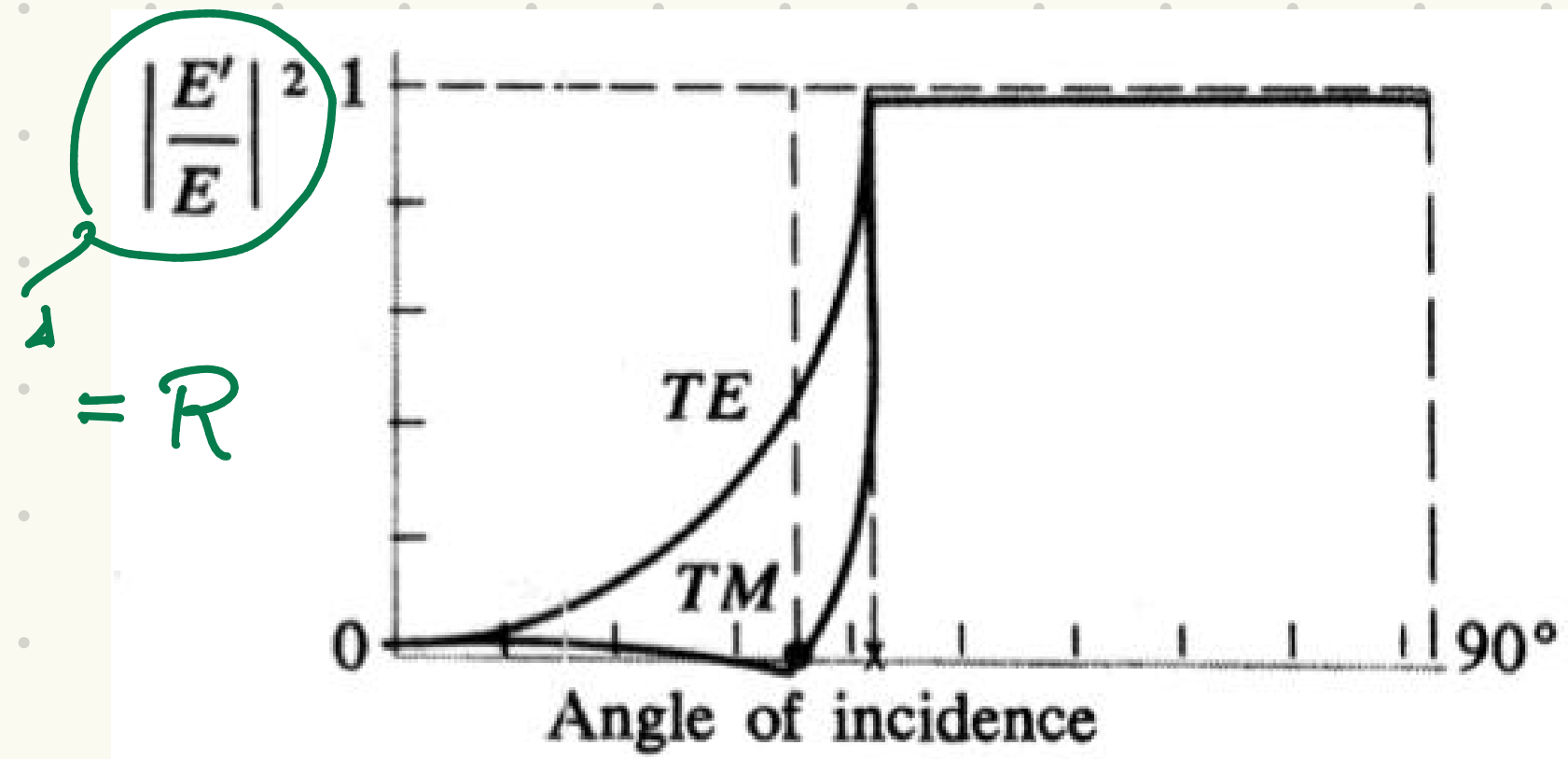
Lei de Snell:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_c = \frac{n_2 \sin(\pi/2)}{n_1} = \frac{n_2}{n_1}$$

A luz sofre reflexão total interna (RTI) quando

$$\theta_i \geq \theta_c \quad \text{ou} \quad \sin \theta_i \geq \sin \theta_c = \frac{n_2}{n_1}$$

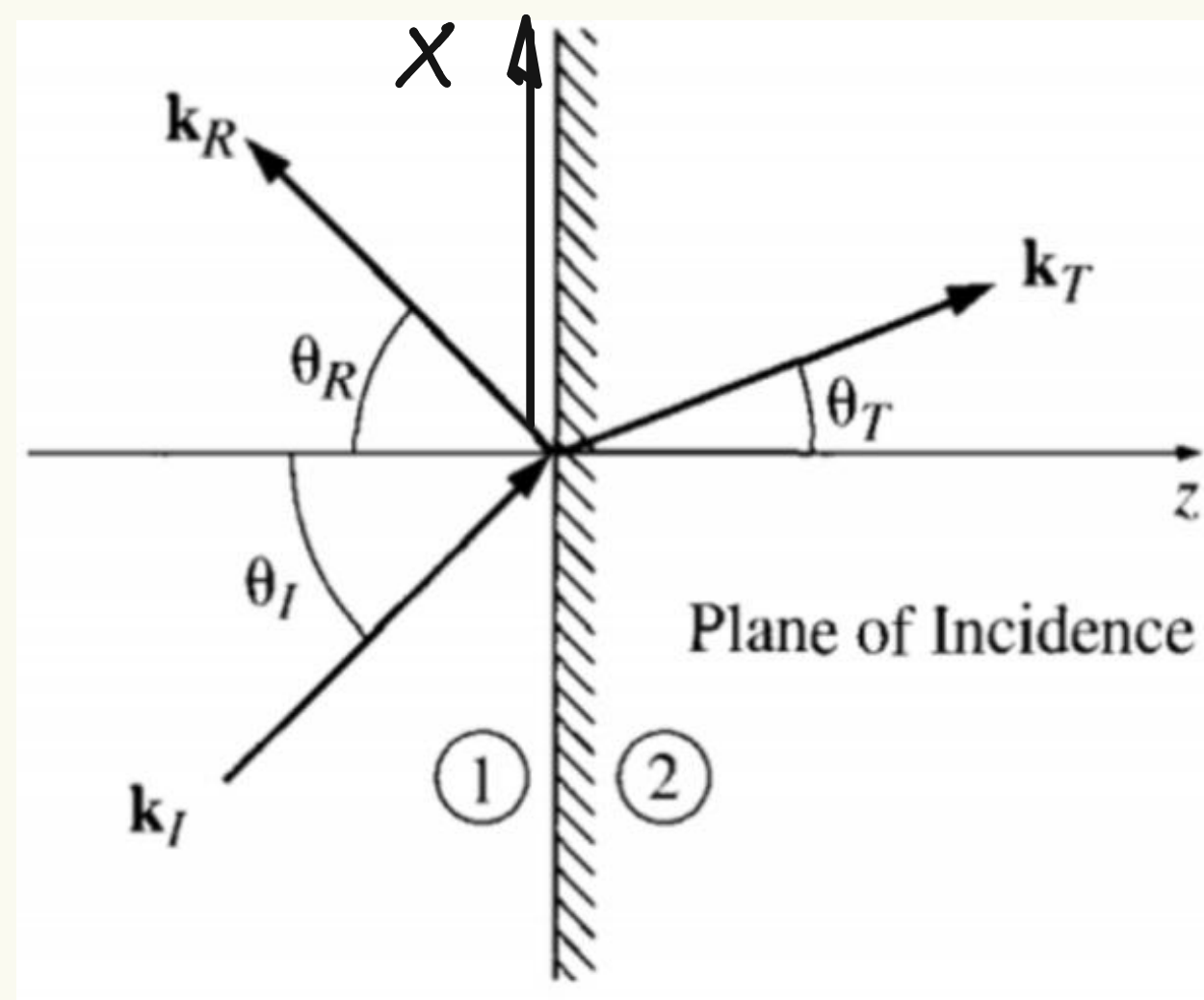


Caso  $n_2 < n_1$  (RTI):

$$\rho_p = \frac{-(n_2/n_1)^2 \cos \theta_i - \sqrt{(n_2/n_1)^2 - \sin^2 \theta_i}}{(n_2/n_1)^2 \cos \theta_i + \sqrt{(n_2/n_1)^2 - \sin^2 \theta_i}} = \frac{-(n_2/n_1)^2 \cos \theta_i - i \sqrt{\sin^2 \theta_i - (n_2/n_1)^2}}{(n_2/n_1)^2 \cos \theta_i + i \sqrt{\sin^2 \theta_i - (n_2/n_1)^2}}$$

É possível mostrar que  $|\rho_p| = |\rho_s| = 1$

Ainda que a onda (a energia) seja totalmente refletida, uma parte finita do campo EM penetra no meio 2.



Onda transmitida.

$$\vec{E}_T(\vec{r}, t) = \vec{E}_{0T} e^{i(\vec{k}_T \cdot \vec{r} - \omega t)}$$

$$= k_{Tx} \cdot x + k_{Tz} z$$

$$= k_T \sin \theta_T x + k_T \cos \theta_T z$$

Usando a lei de Snell:

$$\cos \theta_T = \sqrt{1 - \frac{\sin^2 \theta_i}{(n_2/n_1)^2}}$$

$$\sin \theta_T = \frac{\sin \theta_i}{n_2/n_1}$$

ou seja:

$$\vec{k}_T \cdot \vec{r} = \underbrace{k_T \frac{\sin \theta_i}{n_2/n_1}}_{k_{Tx}} x + \underbrace{k_T \sqrt{1 - \frac{\sin^2 \theta_i}{(n_2/n_1)^2}}}_{k_{Tz}} z$$

; na RTI,  $\sin \theta_i \geq \sin \theta_c = \frac{n_2}{n_1}$   
 c/ isso,

$$k_{Tz} = i k_T \underbrace{\sqrt{\frac{\sin^2 \theta_i}{(n_2/n_1)^2} - 1}}_B$$

valor usado é onde transmitida:

$$\vec{E}_T(\vec{r}, t) = \vec{E}_{0T} e^{i k_T \cdot A x - i \omega t} e^{-k_T B z}$$

propagação  
na direção x

atenuação na  
direção z

Comp. característicos de penetração  
no meio:

$$\alpha = \frac{1}{k_T B} \approx \lambda_2$$

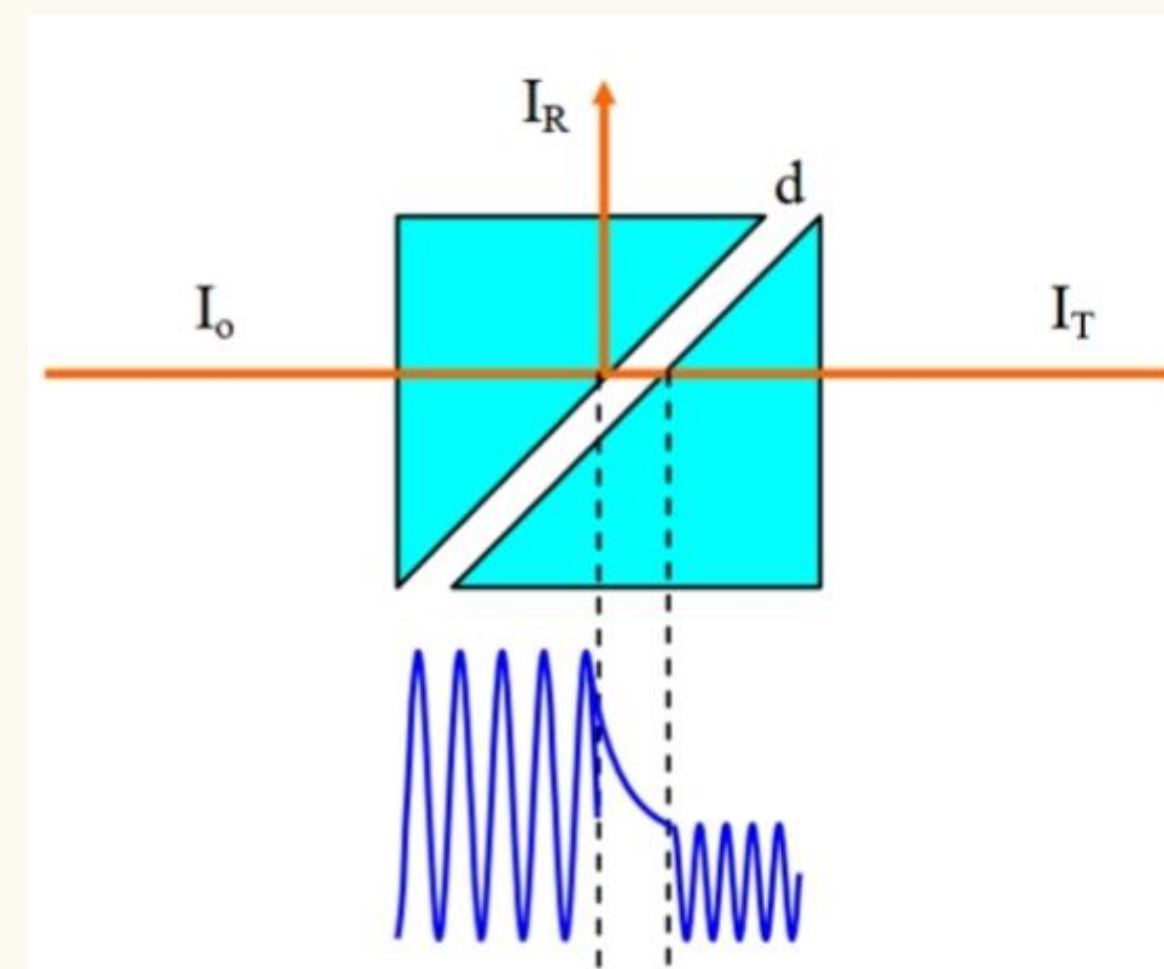


Fig. 5.17 – Tunelamento fotônico.

Caso  $n_2 < n_1$  (RTI)

Def. de fase das ondas s e onda p:

nós vimos que  $|R_s| = |R_p| = 1$ , então:

$$R_s = 1 \cdot e^{-i\delta_s}$$
$$R_p = -1 \cdot e^{-i\delta_p}$$

Essas mudanças de fase sofridas pelo onda

EM. na reflexão podem ser calculadas como:

Esc. onda s:  $\delta_s = \text{tg}^{-1} \left[ \frac{\text{Im}(R_s)}{\text{Re}(R_s)} \right] = 2 \text{tg}^{-1} \left\{ \frac{\sqrt{\sin^2 \epsilon_i - n^2}}{\cos \epsilon_i} \right\}$ ; onde  $n = \frac{n_2}{n_1}$

e  $\delta_p = \quad = \quad = 2 \text{tg}^{-1} \left\{ \frac{\sqrt{\sin^2 \epsilon_i - n^2}}{n^2 \cos \epsilon_i} \right\}$

fase adquirida pelas ondas s e p na RTI

$\odot \vec{E}_s$ 

$$\delta_s = 2 \operatorname{tg}^{-1} \left\{ \frac{\sqrt{\operatorname{sen}^2 \theta_i - n^2}}{\operatorname{cos} \theta_i} \right\}; \quad n = \frac{n_2}{n_1}$$

$\uparrow \vec{E}_p$ 

$$\delta_p = 2 \operatorname{tg}^{-1} \left\{ \frac{\sqrt{\operatorname{sen}^2 \theta_i - n^2}}{n^2 \operatorname{cos} \theta_i} \right\}$$

É possível manipular o estado de polarização da luz a partir de RTI!

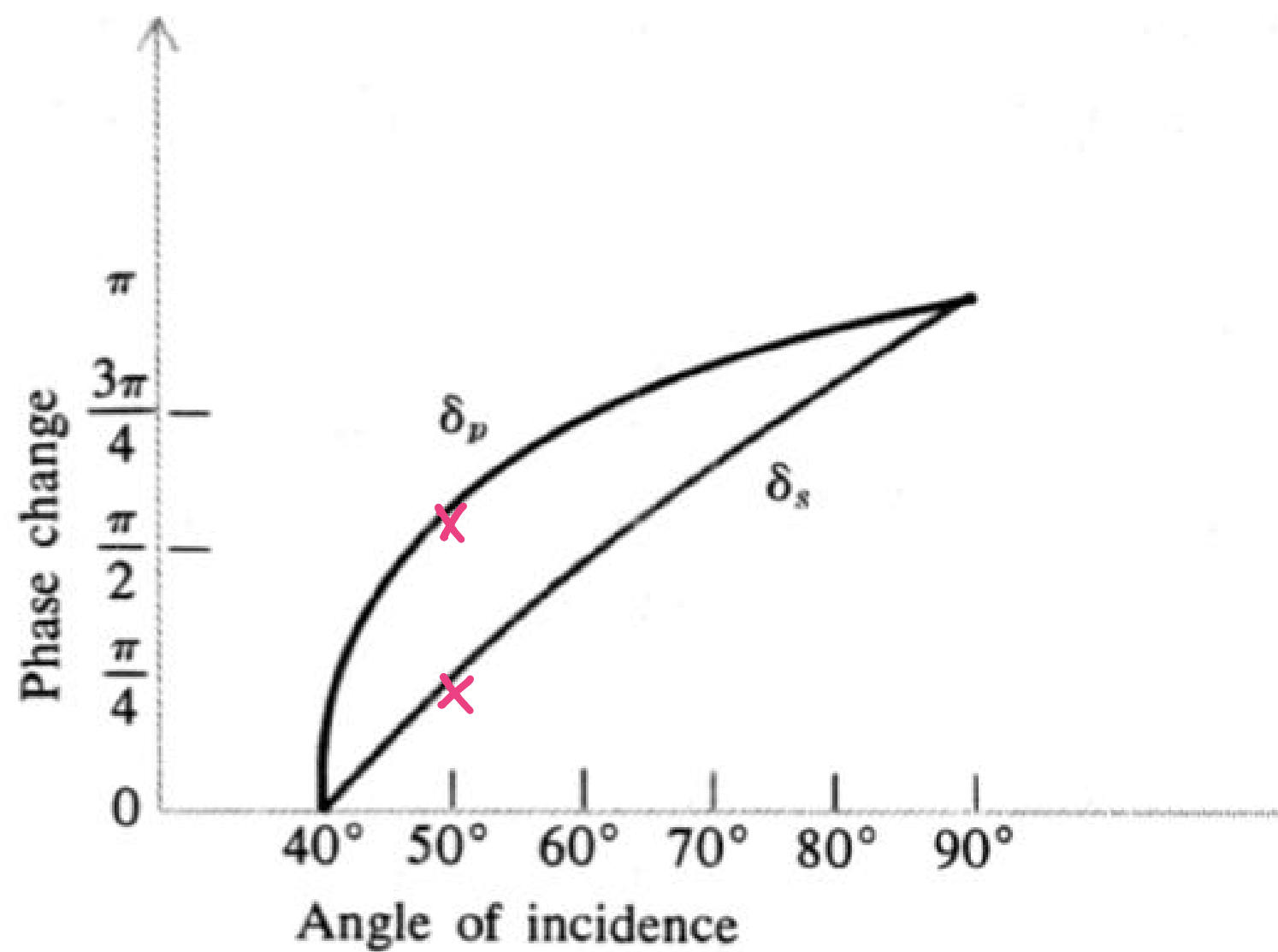
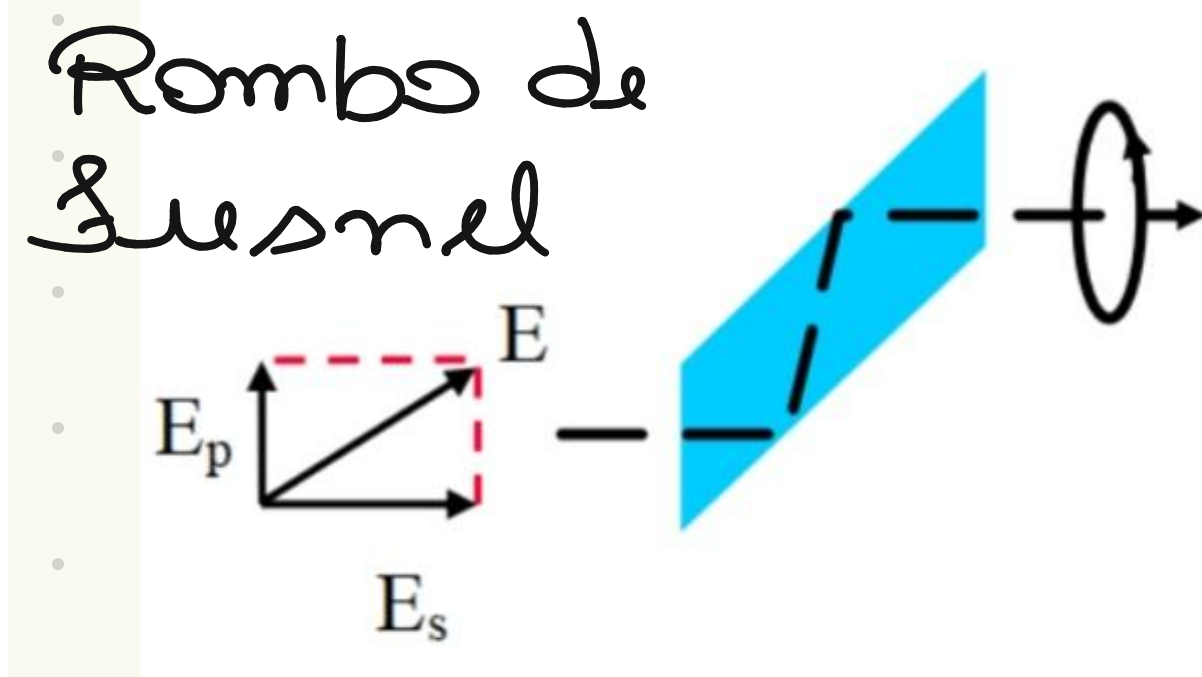
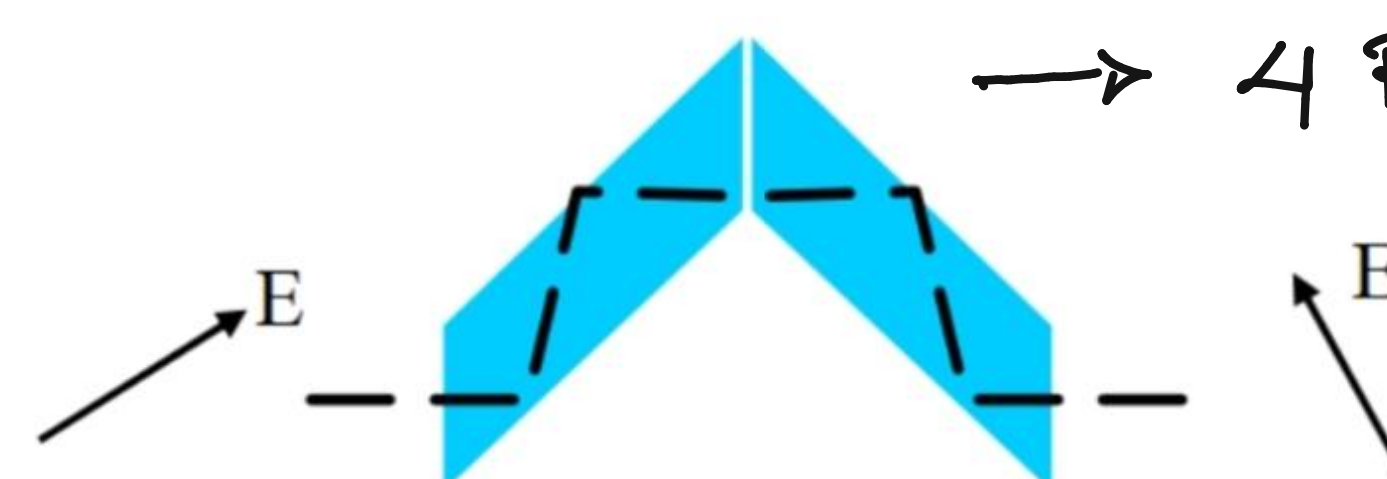


Figure 2.17. Phase changes occurring in total internal reflection. (Approximate curves for glass of index 1.5.)



2 RTIs:  
Obtenção de luz circularmente polarizada



4 RTIs:  
gira o plano de polarização



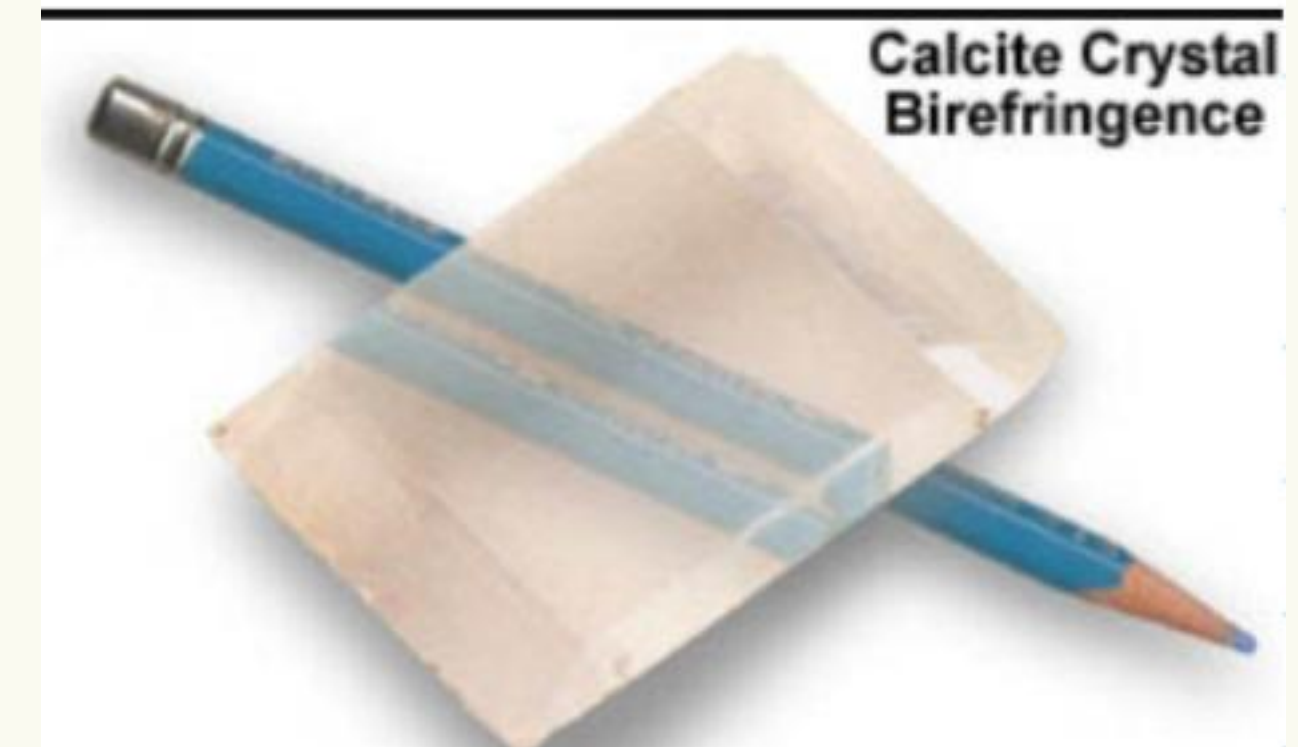
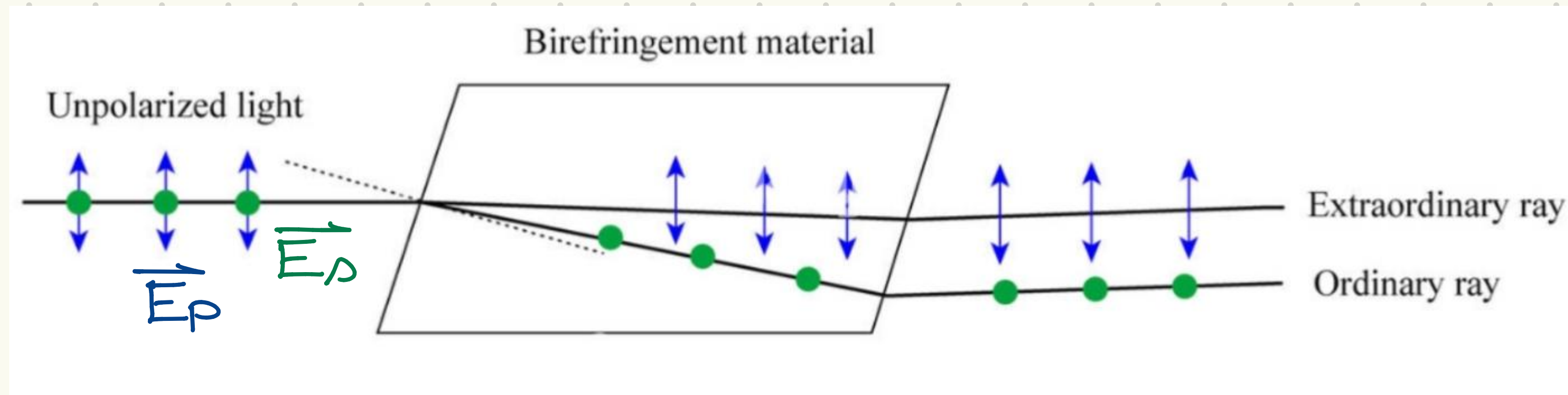
lâminas (placas) de quarto de onda e meia onda

→ cristal anisotrópico

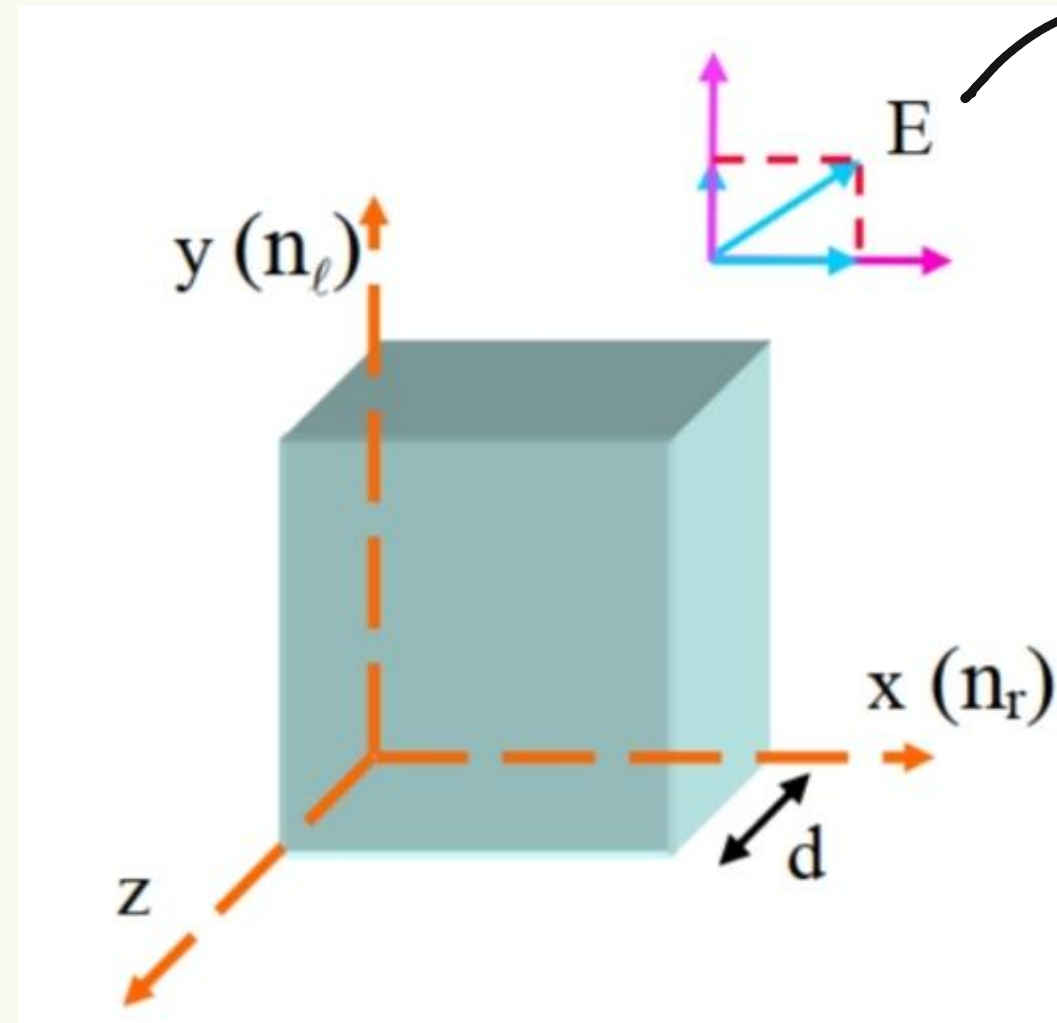
cujos ind. de refração depende de direção ;  
(birrefringência)

Exemplos:

mica, quartzo, calcita



lâminas (placas) de **quarto de onda** e **meia onda**



$$\vec{E}(\vec{r}, t) = E_p e^{i(K_r z - \omega t)} \hat{x} + E_s e^{i(K_e z - \omega t)} \hat{y}; \quad K_j = n_j k$$

onda atinge o cristal em  $z = 0$ :

$$\vec{E}(\vec{r}, t) = (E_p \hat{x} + E_s \hat{y}) e^{-i\omega t}$$

e deixa o cristal em  $z = d$ :

$$\vec{E}(\vec{r}, t) = E_p e^{i(K_r d - \omega t)} \hat{x} + E_s e^{i(K_e d - \omega t)} \hat{y}$$

lembrando que  
 $K_j = n_j k_0$

→ Qual é a diferença de fase entre os componentes s e p?

$$S = (K_e - K_r) d = \frac{2\pi}{\lambda_0} (n_e - n_r) \cdot d$$

luz circularmente polarizada:  $S = \pi/2$

$$\left. \begin{aligned} \frac{\pi}{2} &= \frac{2\pi}{\lambda_0} (n_e - n_r) d \\ (n_e - n_r) d &= \lambda_0/4 \end{aligned} \right\}$$

p/roda o plano de pol. de luz de  $\pi/2$ :  $S = \pi$

$$\pi = \frac{2\pi}{\lambda_0} (n_e - n_r) d$$

$$(n_e - n_r) \cdot d = \lambda_0/2$$

Lâminas (placas) de **quarto de onda** e **meia onda**

dif. de fase entre ondas s e p  $\left\{ \begin{aligned} \delta &= (k_e - k_r) d = \frac{2\pi}{\lambda_0} (n_e - n_r) \cdot d \end{aligned} \right.$

luz circularmente polarizada:  $\delta = \pi/2$

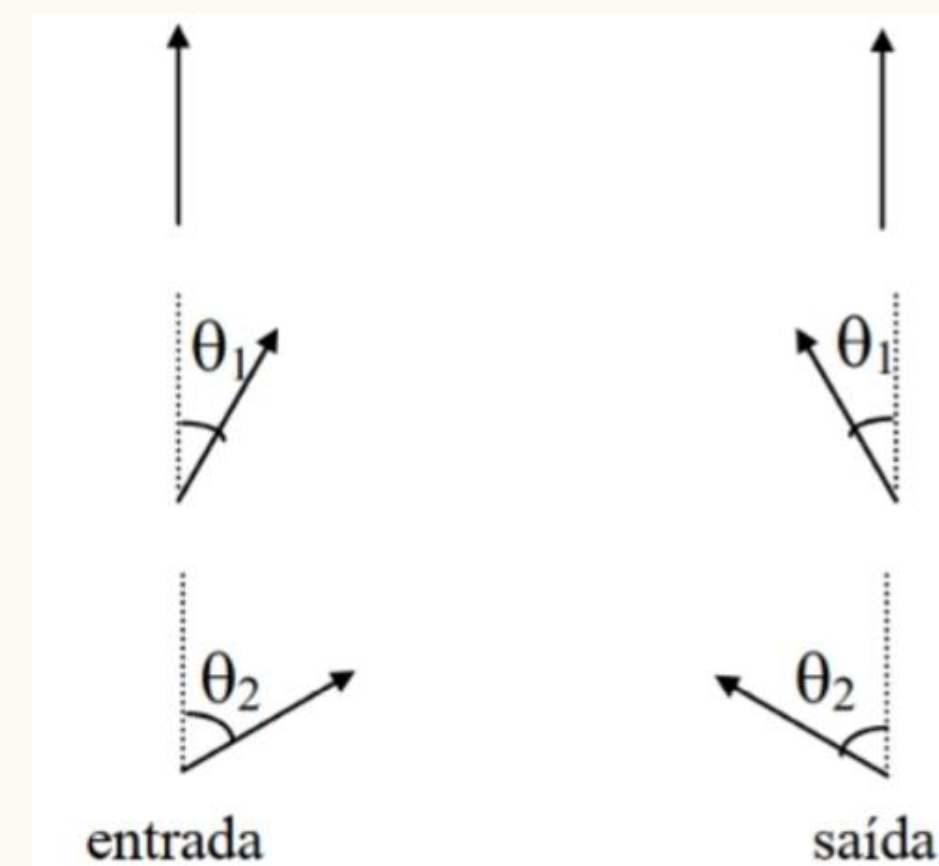
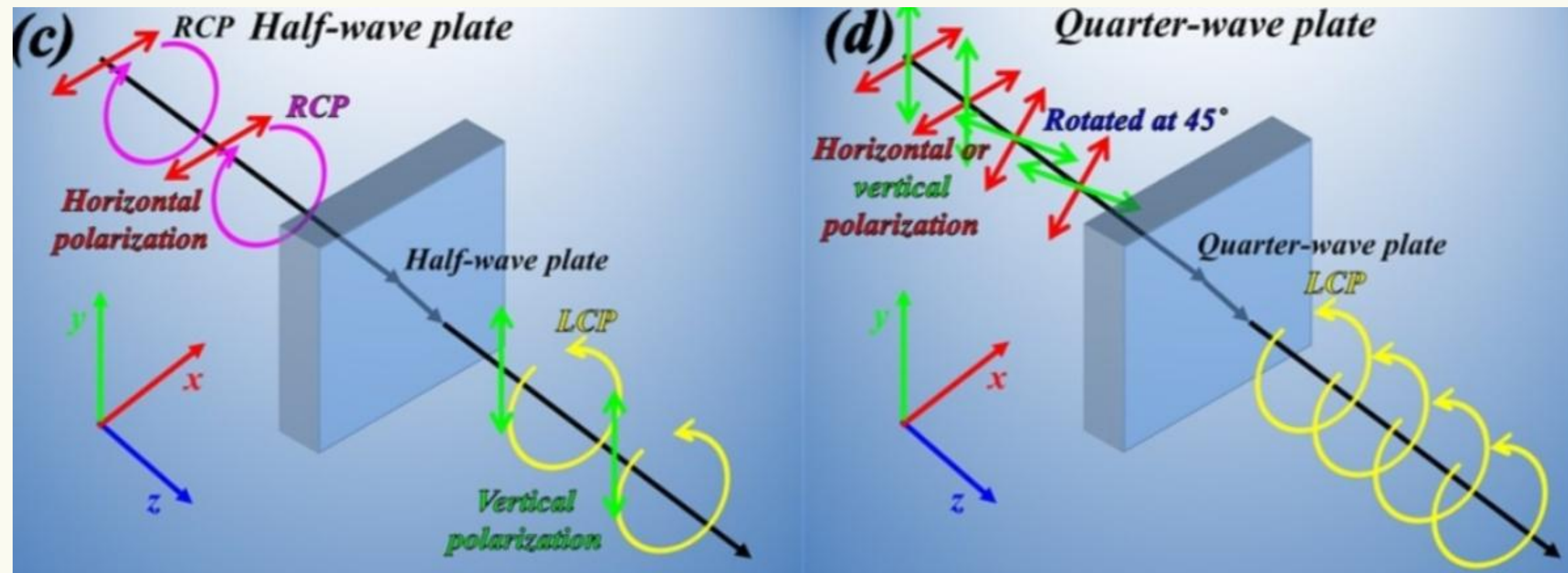
$\left. \begin{aligned} \frac{\pi}{2} &= \frac{2\pi}{\lambda_0} (n_e - n_r) d \\ (n_e - n_r) d &= \lambda_0/4 \end{aligned} \right\}$

para rodar o plano de pol. de luz de  $\pi/2$ :  $\delta = \pi$

$\pi = \frac{2\pi}{\lambda_0} (n_e - n_r)$

$(n_e - n_r) \cdot d = \lambda_0/2$

Se a pol. de entrada,  $\theta$ ,  $\neq 45^\circ$  o ângulo de rotação será  $2\theta$



## Referências:

- David J. Griffiths - Introduction to Electrodynamics
- Grant R. Sowers - Modern Optics
- Sérgio C. Zilio - Óptica Moderna: Fundamentos e aplicação

## Animações mostradas em aula:

[https://www.met.reading.ac.uk/clouds/maxwell/total\\_internal\\_reflection.html](https://www.met.reading.ac.uk/clouds/maxwell/total_internal_reflection.html)

[https://www.met.reading.ac.uk/clouds/maxwell/frustrated\\_total\\_internal\\_reflection.html](https://www.met.reading.ac.uk/clouds/maxwell/frustrated_total_internal_reflection.html)