



USP

Estruturas Associadas

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Estruturas Associadas

Apostila do prof. Lindenberg, capítulo 11
págs. 168 a 177

Livro do Sussekind, vol. 01, págs. 130 a 140

Estruturas Associadas

As estruturas associadas são correlatas às vigas Gerber

Mas ao invés de sistemas isostáticos apenas entre vigas, são conjuntos de sistemas estruturais diferentes

Estruturas Associadas

São estruturas complexas de sistemas isostáticos

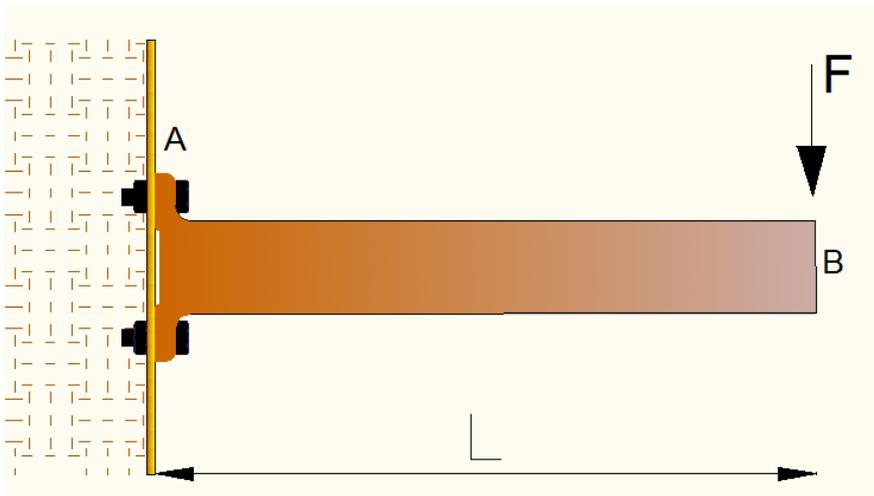
Existem as estruturas isostáticas independentes que apóiam outras estruturas sem apoio próprio, tornando-se as isostáticas também.

Estruturas Associadas

Dentre os vários objetivos de associarem diferentes sistemas estruturais, destaque para a vantagem de procurar potencializar cada um das subestruturas com respeito a suas características intrínsecas vantajosas.

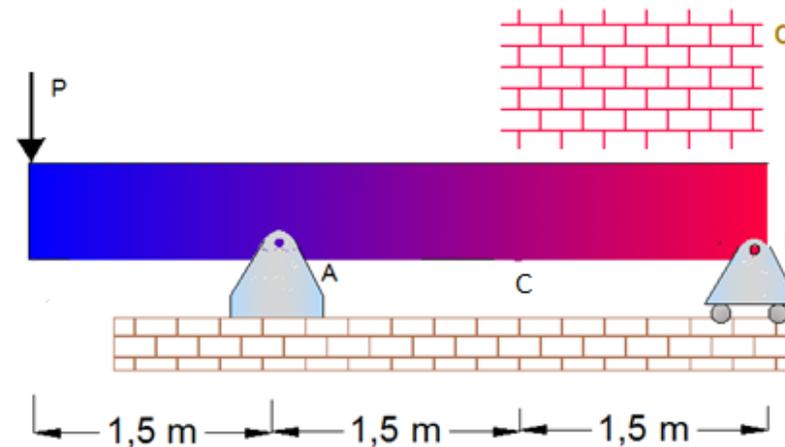
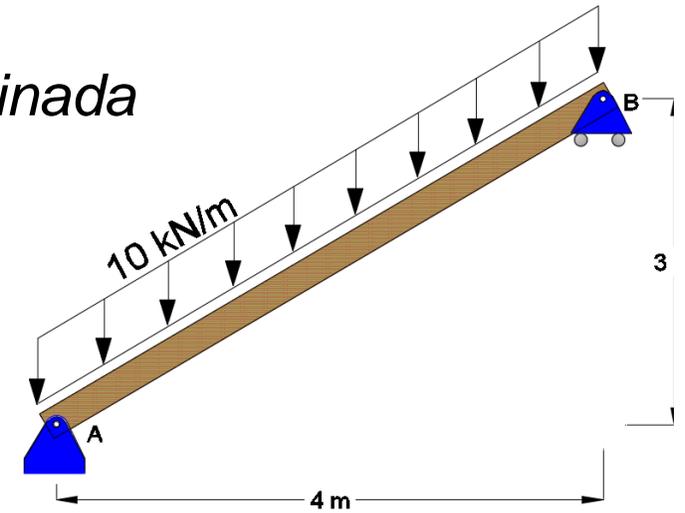
Sistemas isostáticos independentes conhecidos no curso:

Vigas



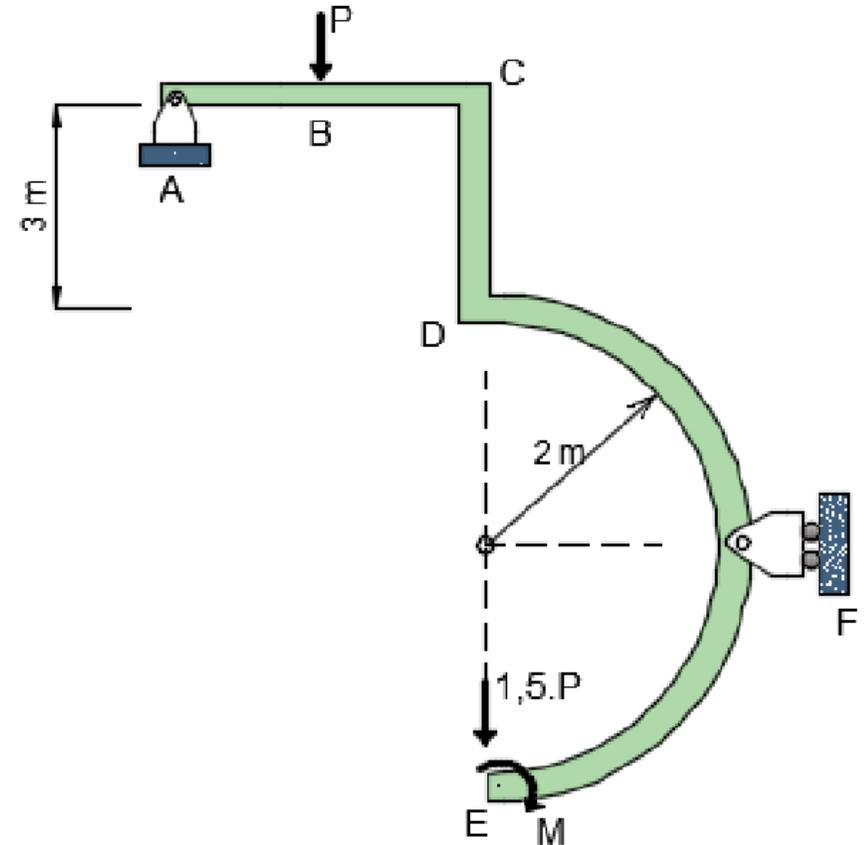
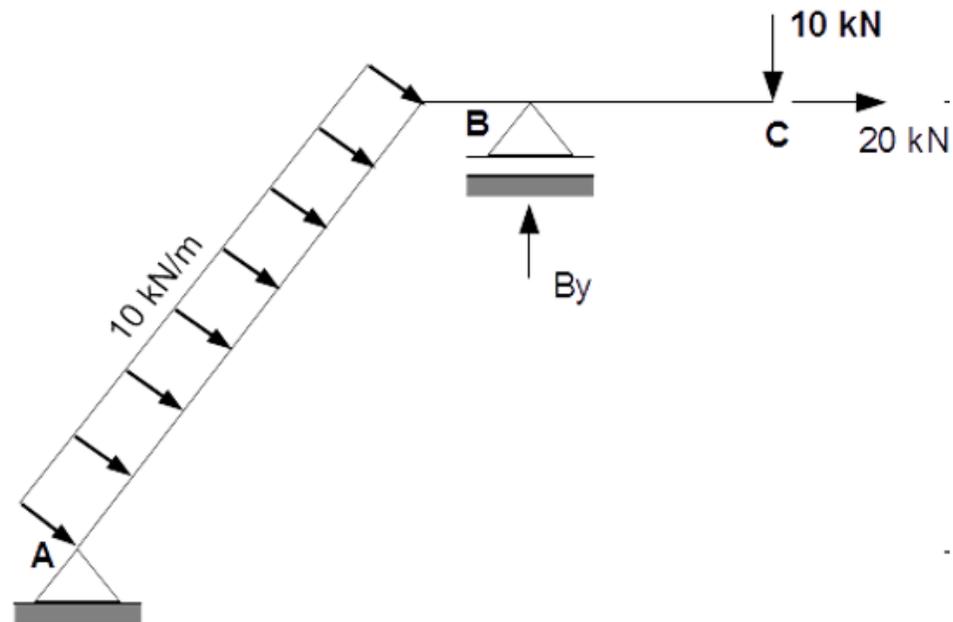
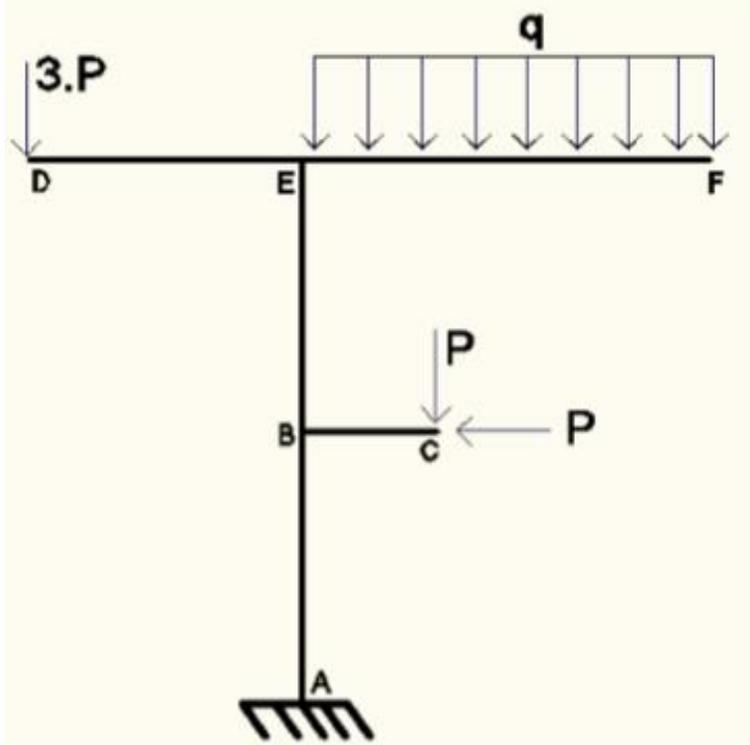
viga em balanço

viga inclinada

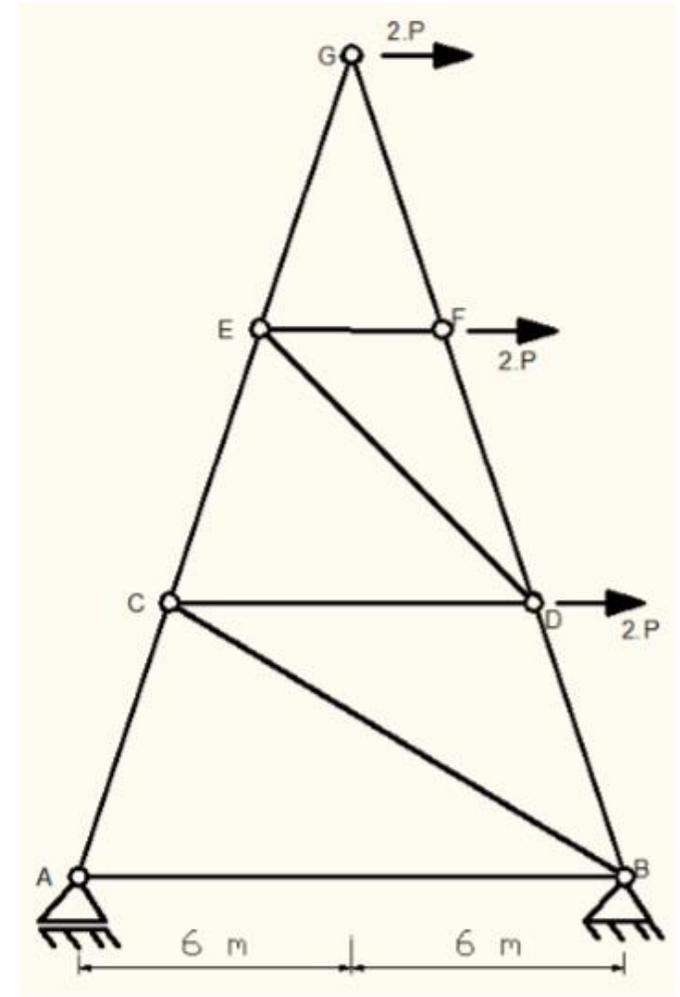
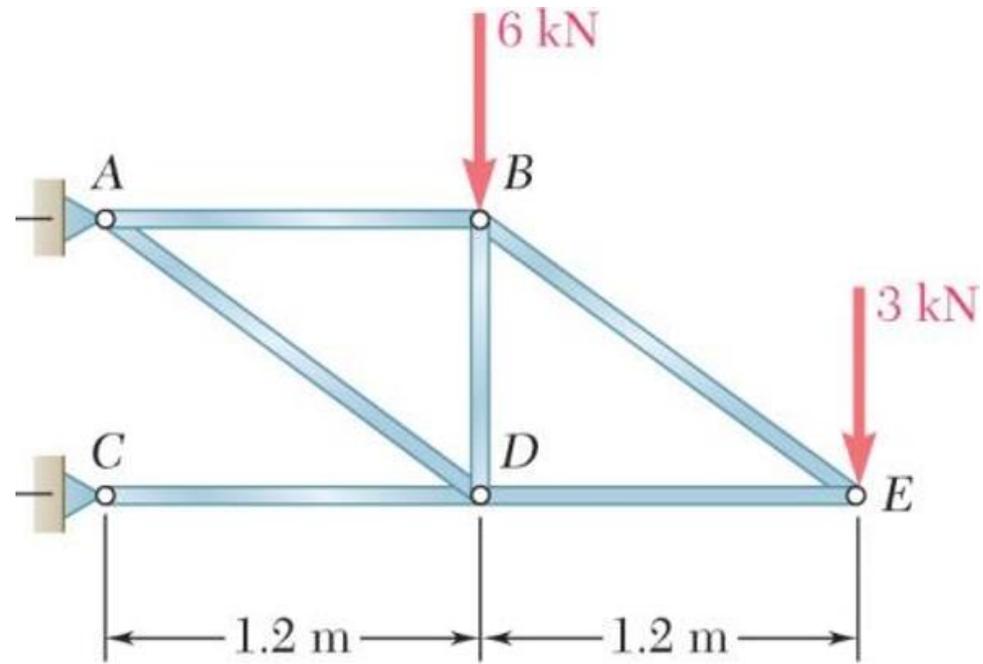
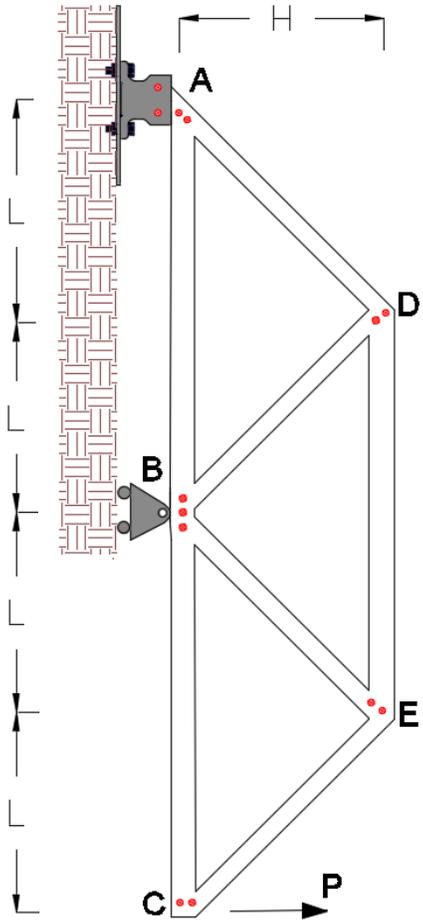


viga bi-apoiada com balanço

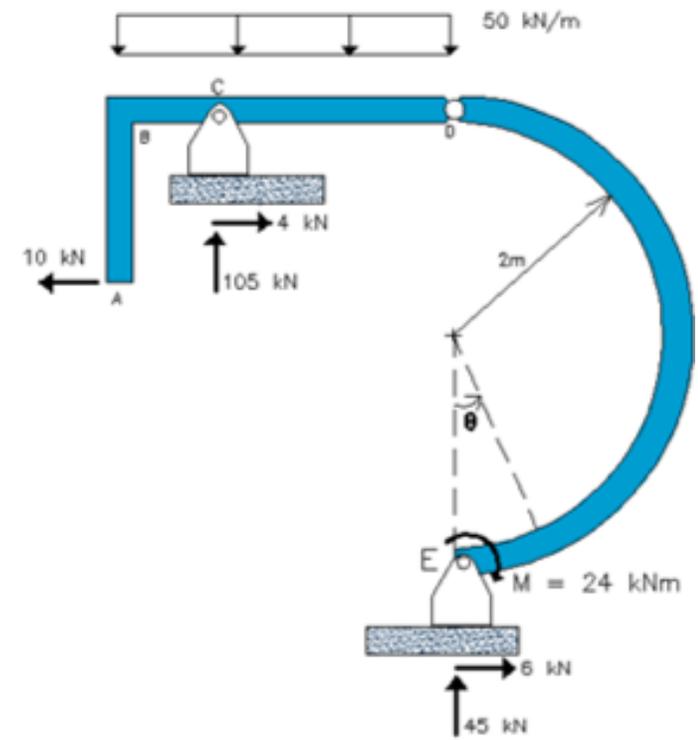
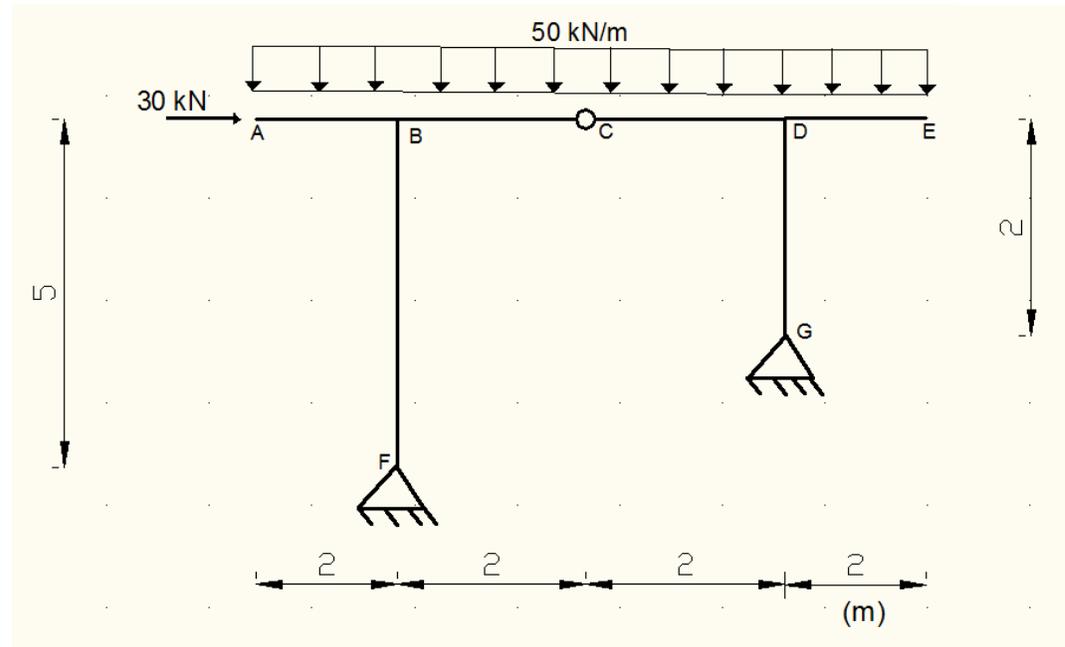
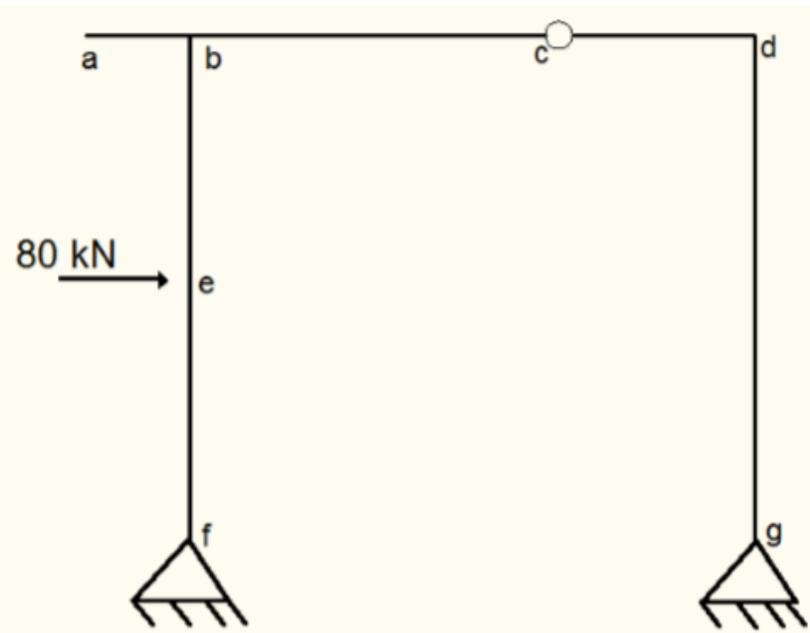
Vigas poligonais



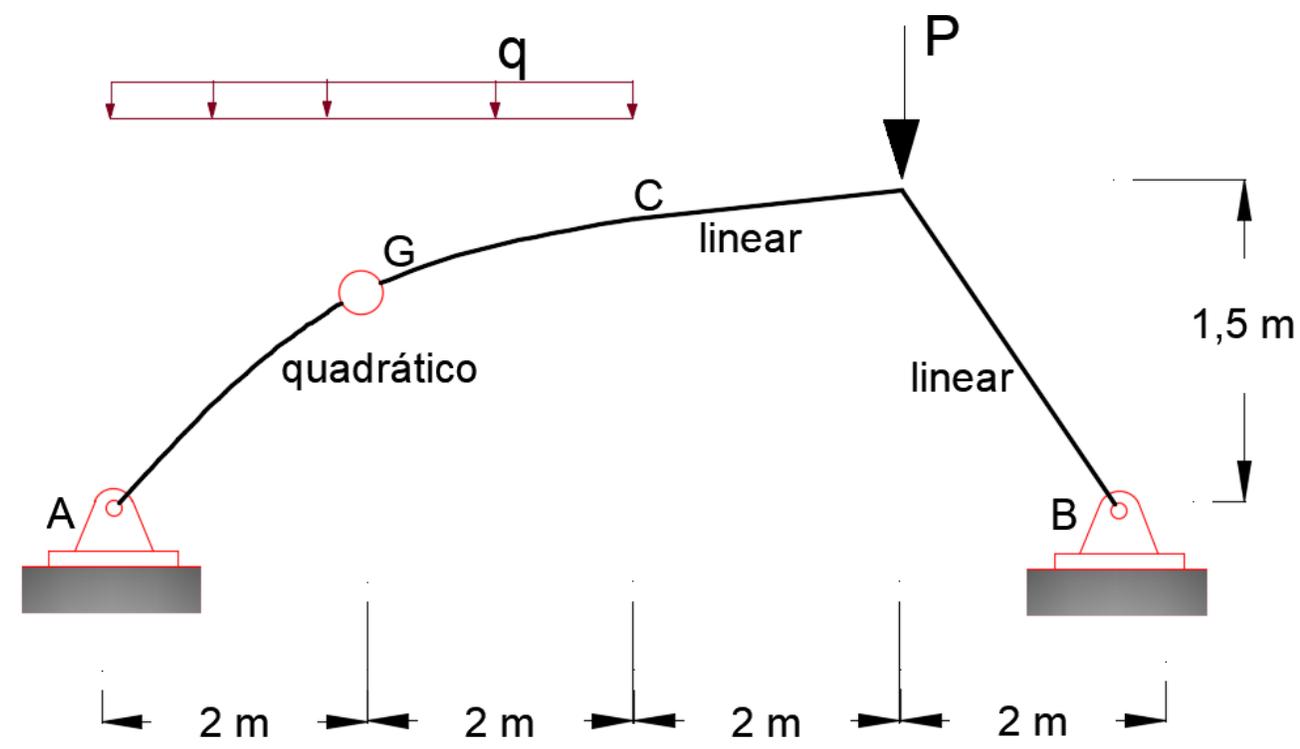
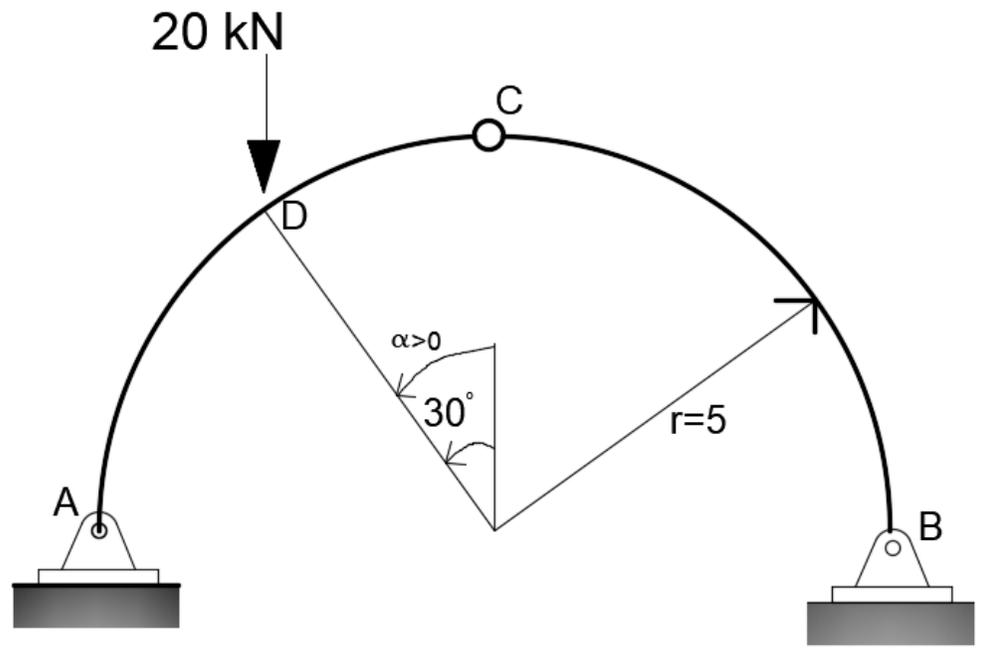
Treliças



Pórticos triarticulados

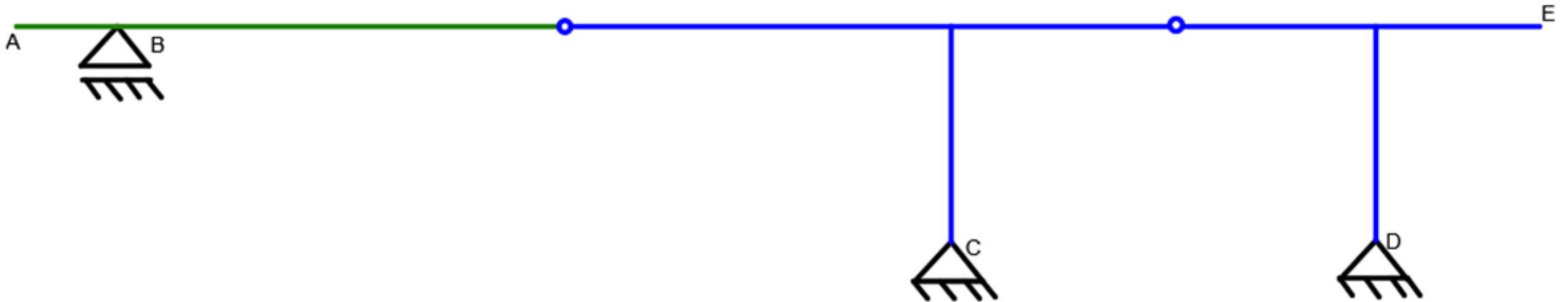


Arcos triarticulados



Estruturas Associadas

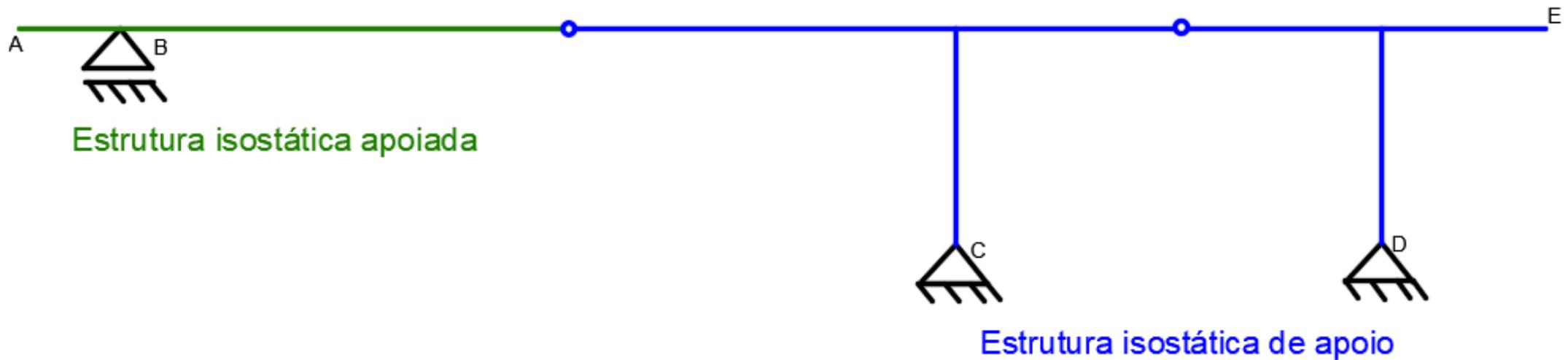
Associação de vários sistemas conhecidos (subestruturas), sendo que alguns são isostáticos por si só e outros se apóiam nestes.



Descobrir as Subestruturas

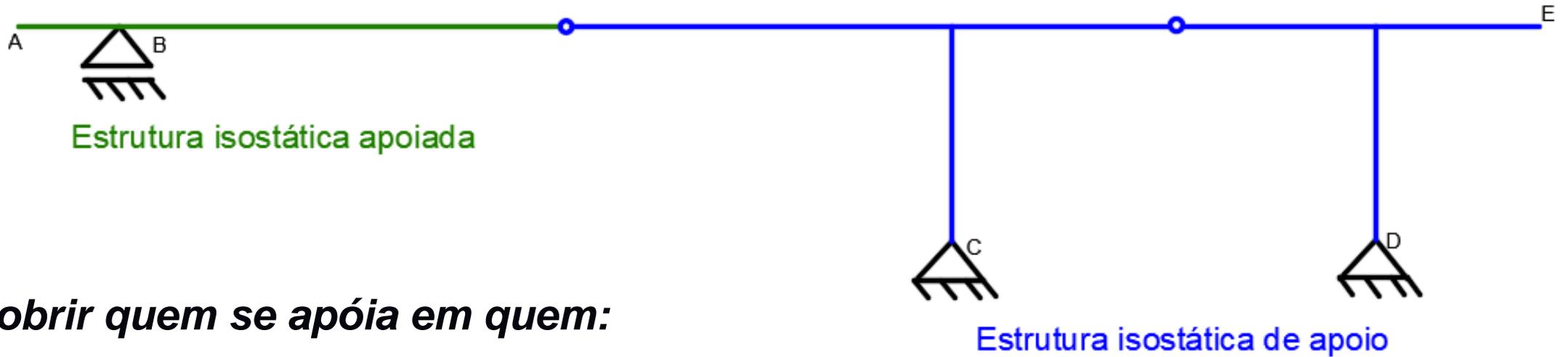
Estruturas Associadas

*Descobrir quem se
apóia em quem:*



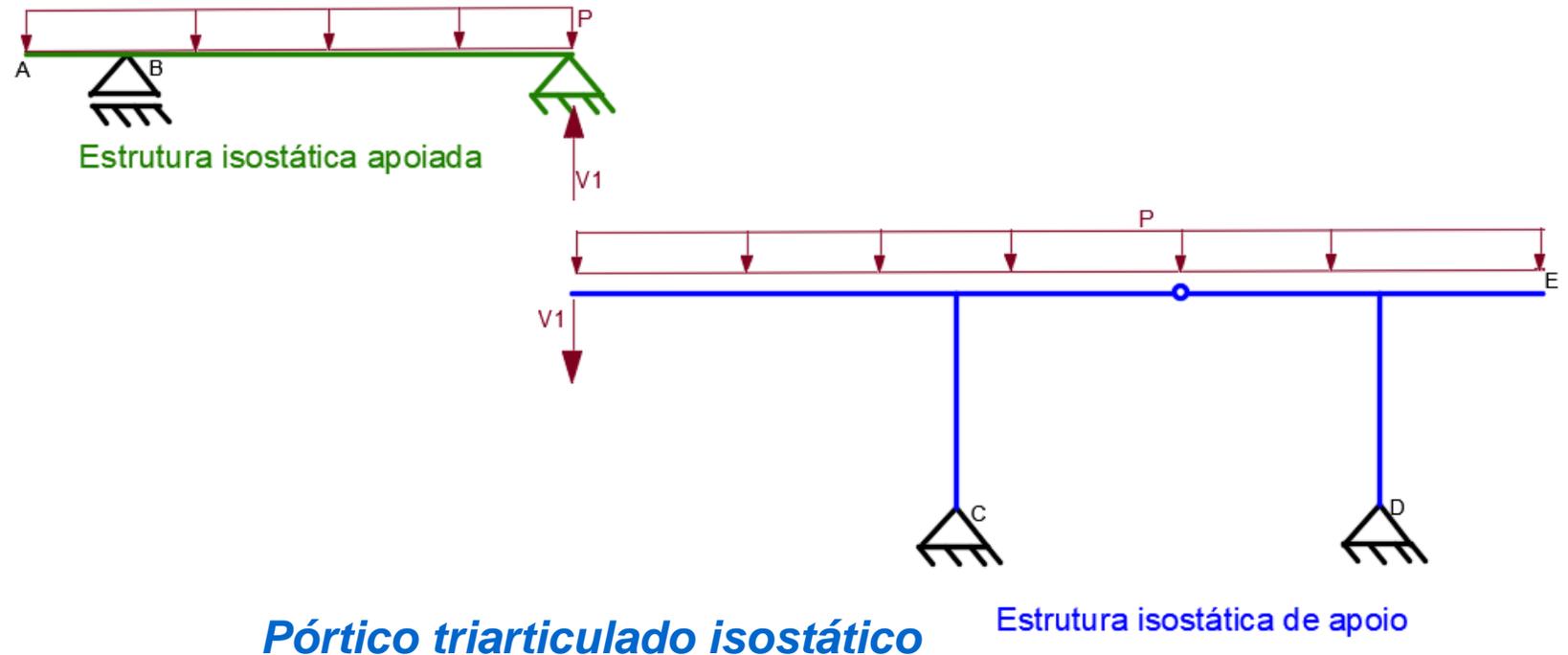
Identificar cada subestrutura

Estruturas Associadas

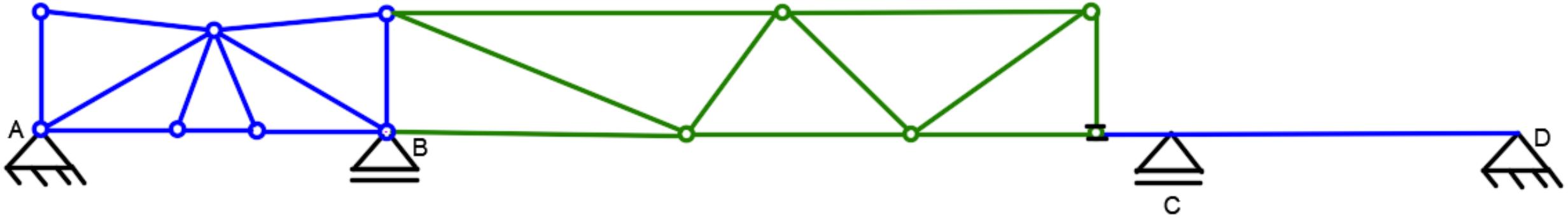


Descobrir quem se apóia em quem:

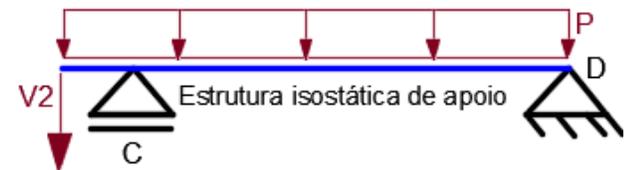
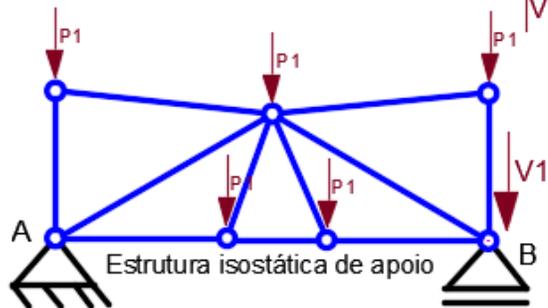
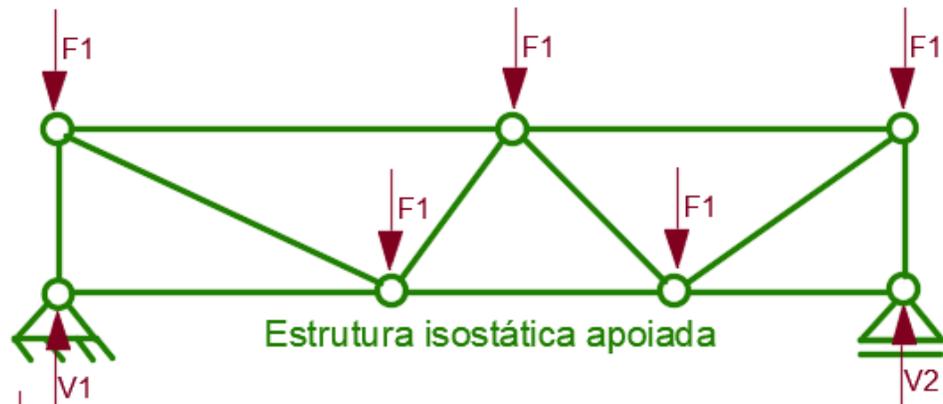
Viga que precisa se apoiar em alguém

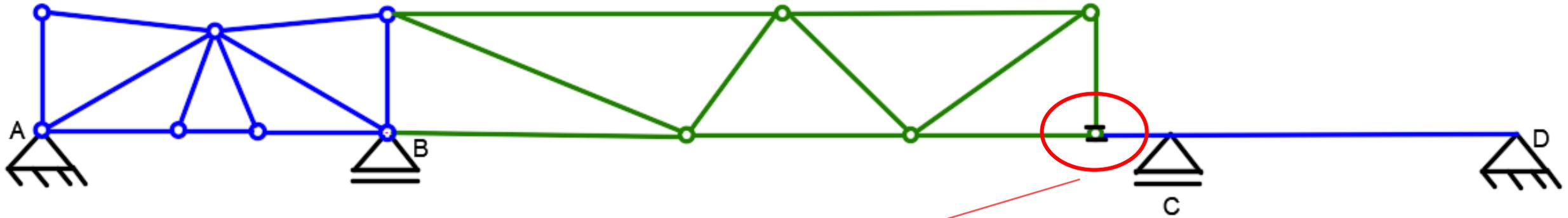


Estruturas Associadas



Descobrir quem se apóia em quem:

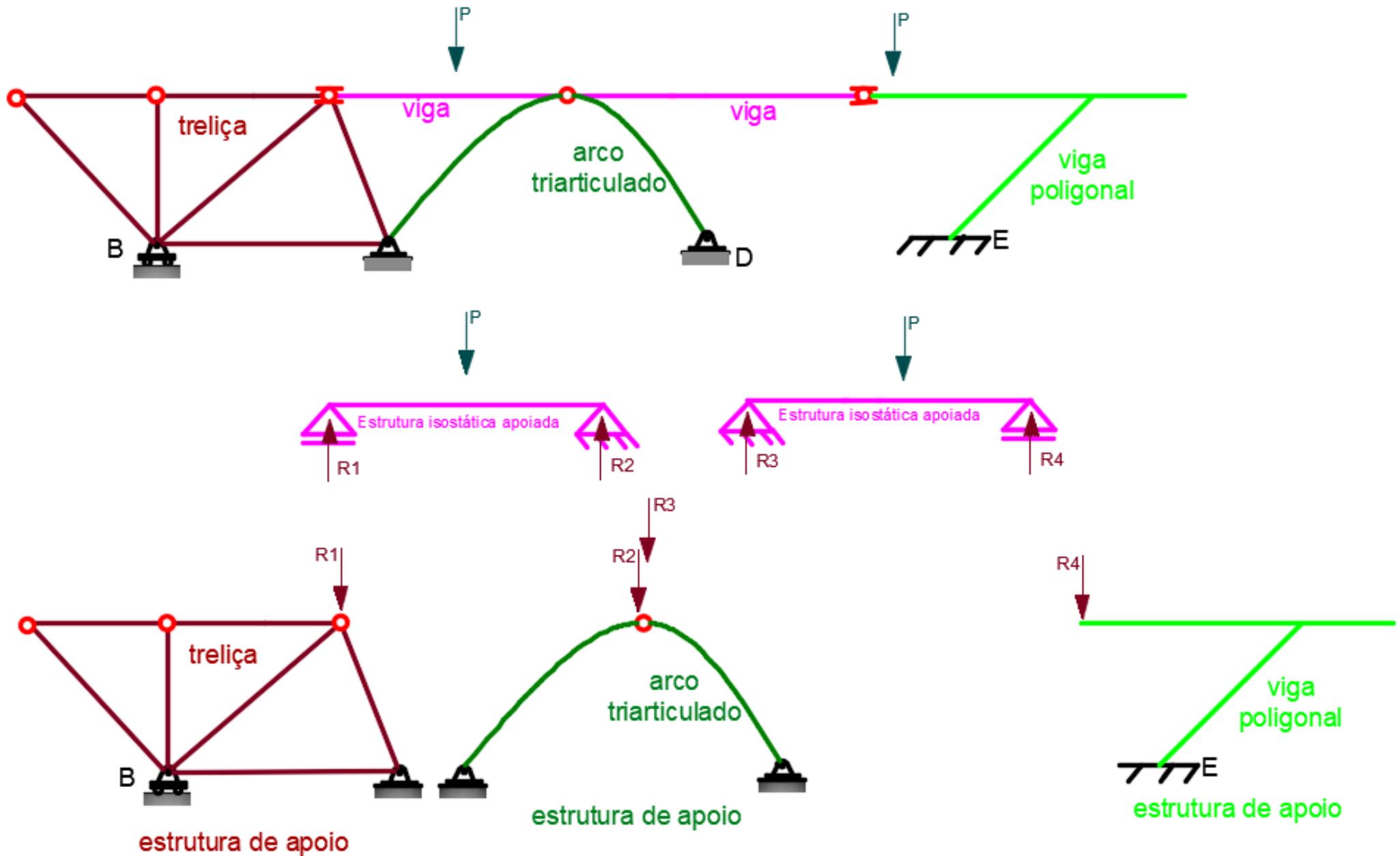





 transmite apenas V (esforço cortante)

Estruturas Associadas

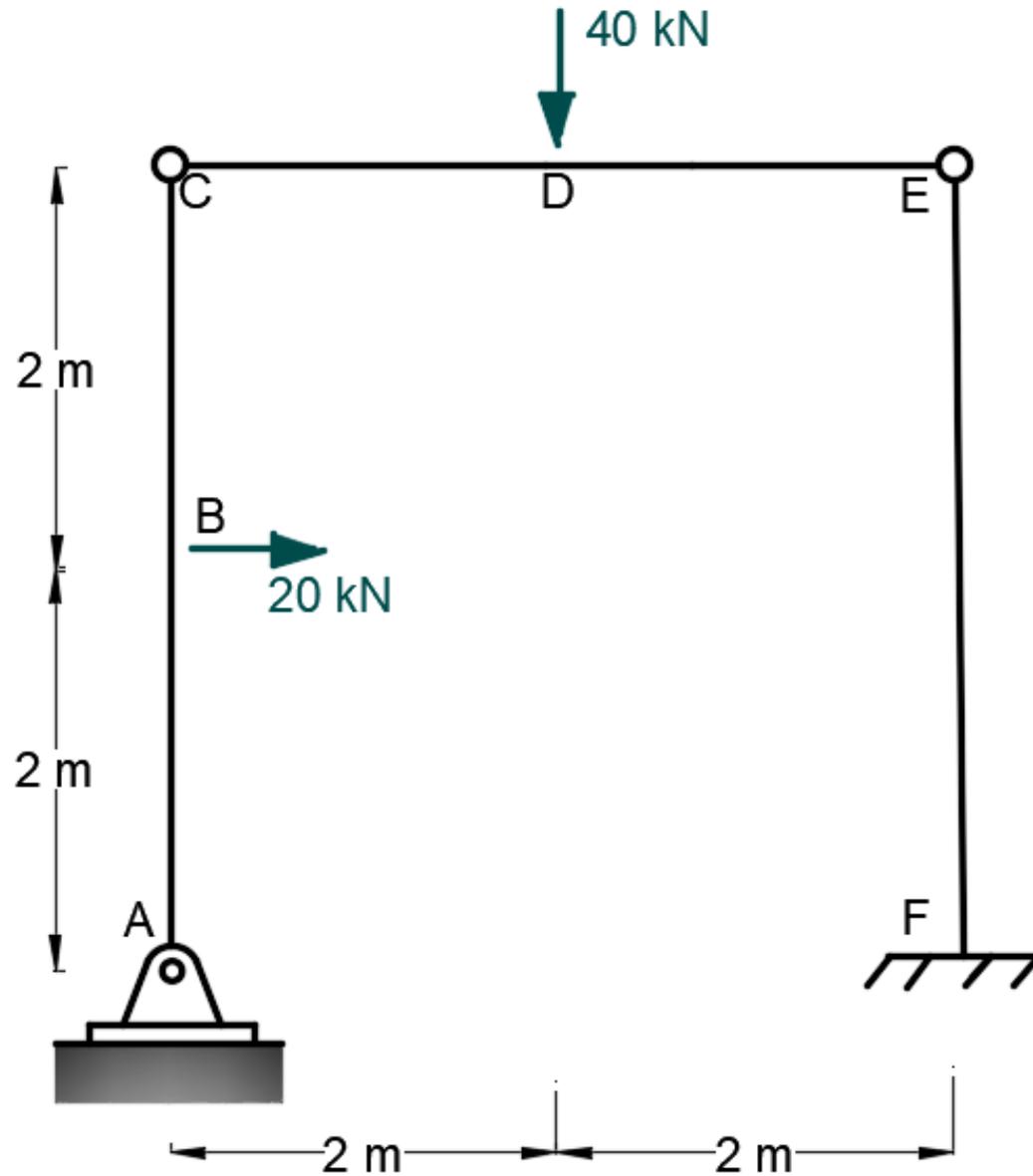
Identificar as subestruturas



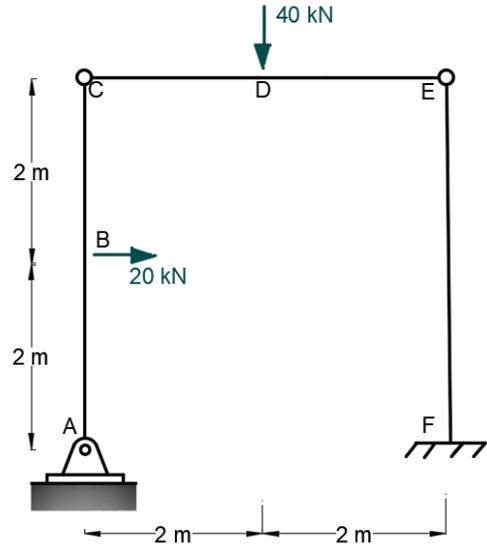
Exemplo 1 - Estruturas Associadas

Dividir em suas subestruturas

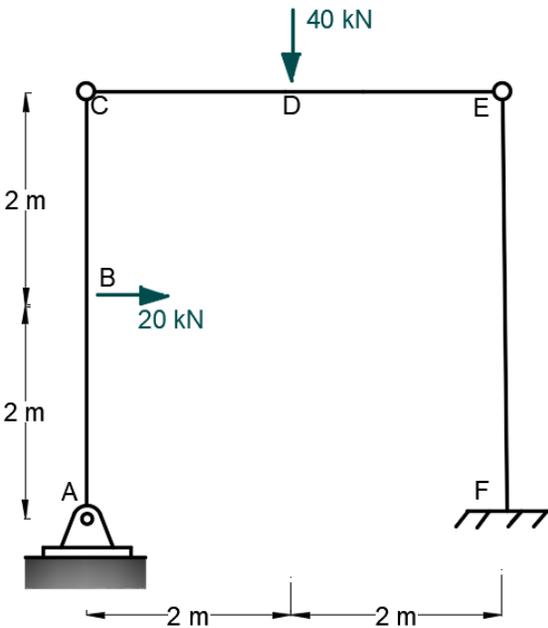
Calcular diagramas



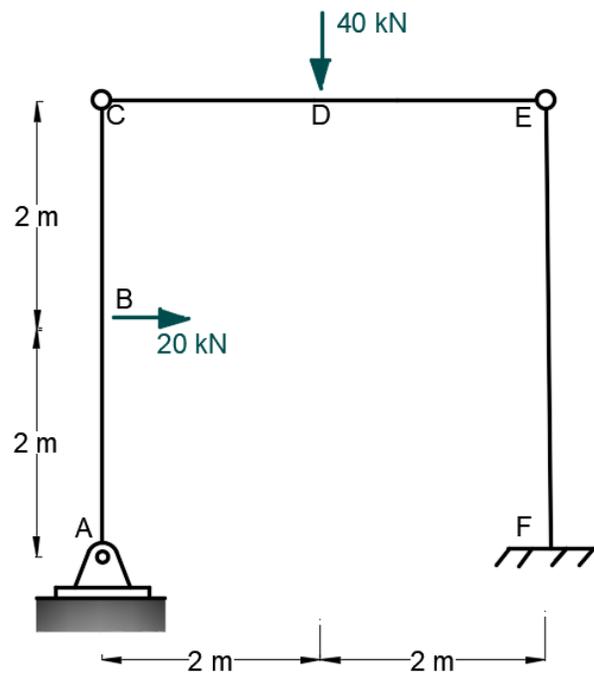
Exemplo 1 - Estruturas Associadas



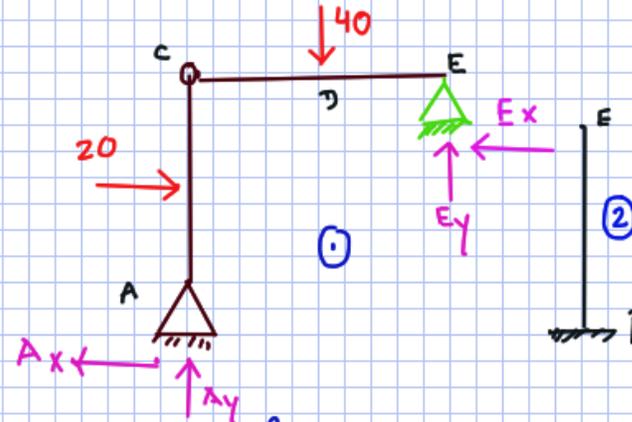
Exemplo 1 - Estruturas Associadas



Exemplo 1 - Estruturas Associadas



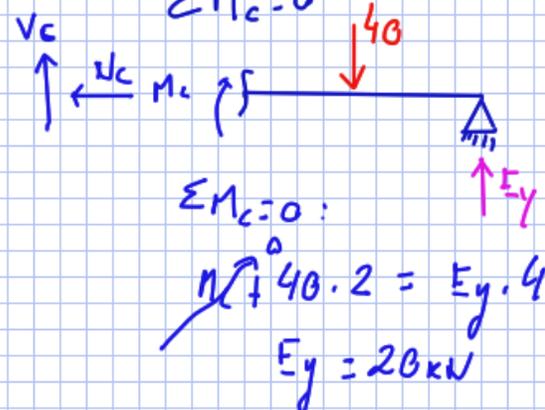
a) dividir a estrutura ASSOCIADA EM SISTEMAS ISOSTÁTICOS básicos



① Calcular REAÇÕES e fazer DIAGRAMAS

$$\sum F_x = 0: A_x + E_x = 20 \quad (1)$$

$$\sum M_c = 0$$



$$\sum M_c = 0:$$

$$40 \cdot 2 = E_y \cdot 4$$

$$E_y = 20 \text{ kN}$$

$$\sum F_y = 0: A_y + E_y = 40$$

$$A_y = 20 \text{ kN}$$

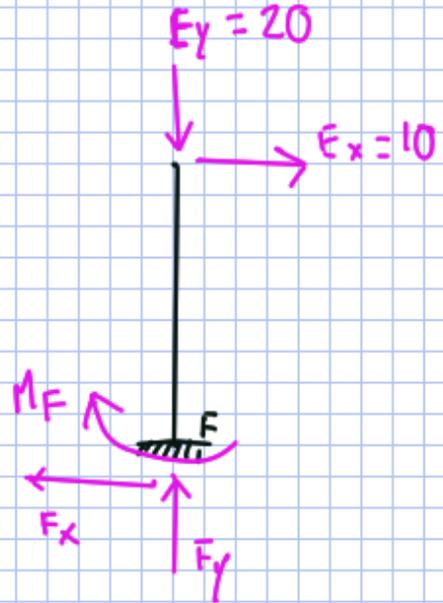
$$\sum M_A = 0: E_y \cdot 4 + E_x \cdot 4 = 20 \cdot 2 + 40 \cdot 2$$

$$E_x = (120 - 80) / 4 = 10 \text{ kN}$$

$$\text{Na eq. (1): } A_x = 10 \text{ kN}$$

Exemplo 1 - Estruturas Associadas

Na estrutura (2):



$$\sum F_x = 0$$

$$F_x = 10 \text{ kN}$$

$$\sum F_y = 0$$

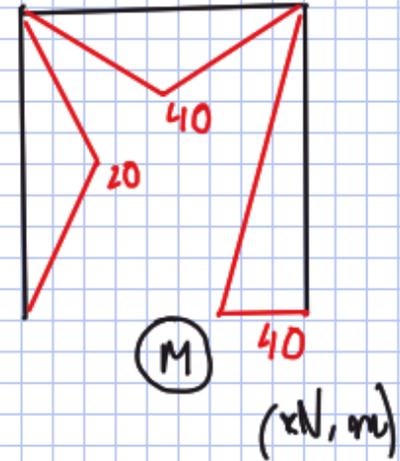
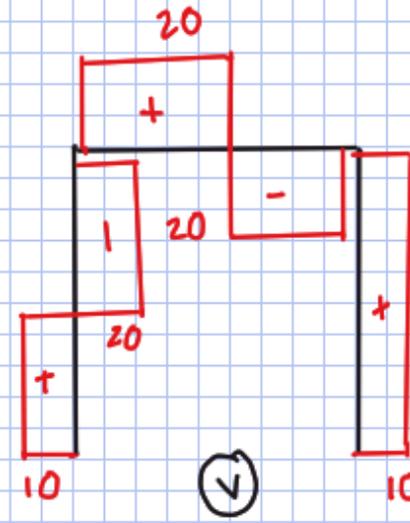
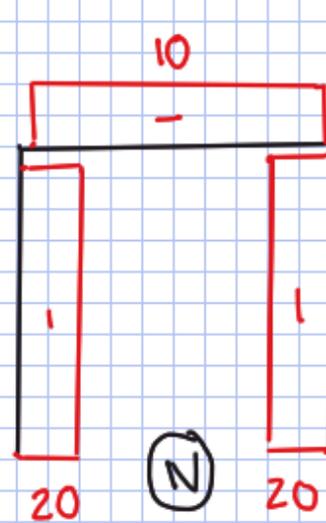
$$F_y = 20 \text{ kN}$$

$$\sum M_F = 0:$$

$$M_F + 10 \cdot 4 = 0$$

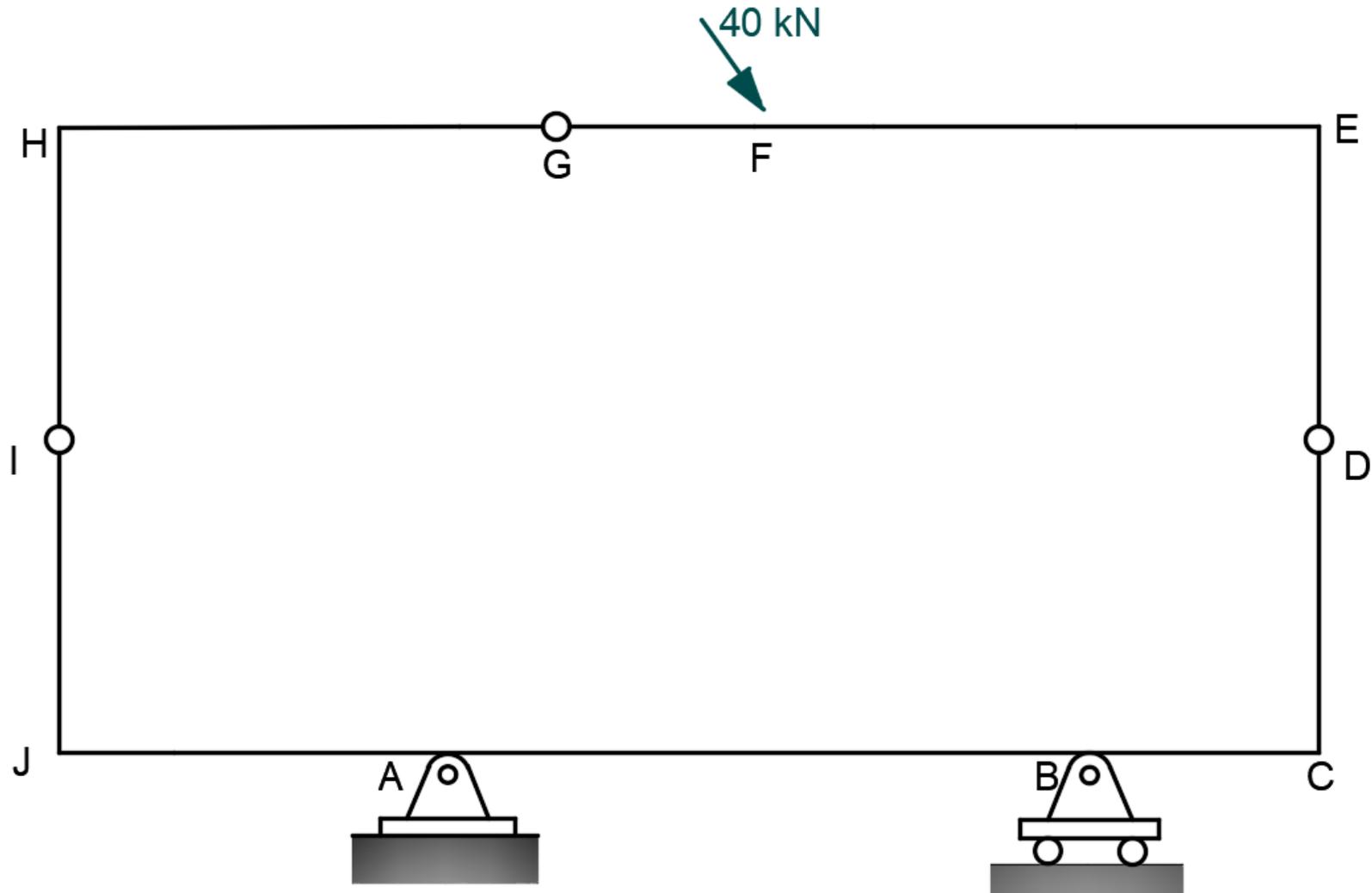
$$M_F = -40 \text{ kNm}$$

DIAGRAMAS:

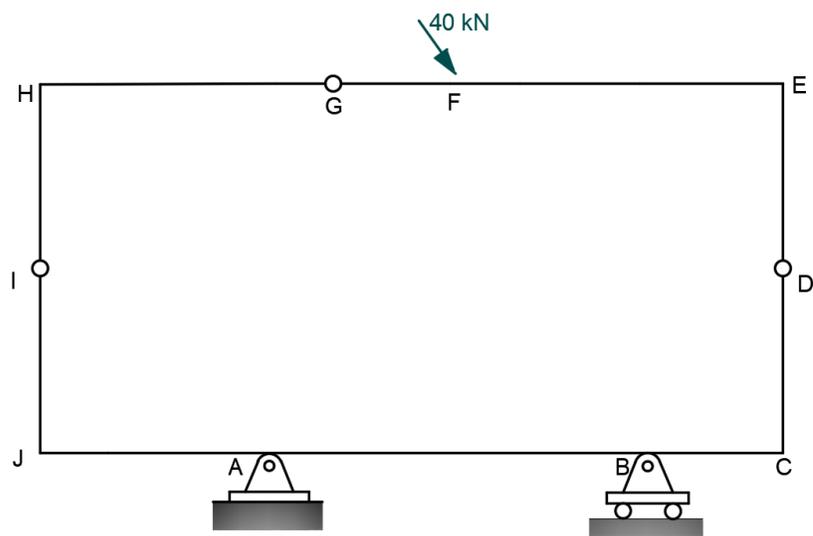


Estruturas Associadas

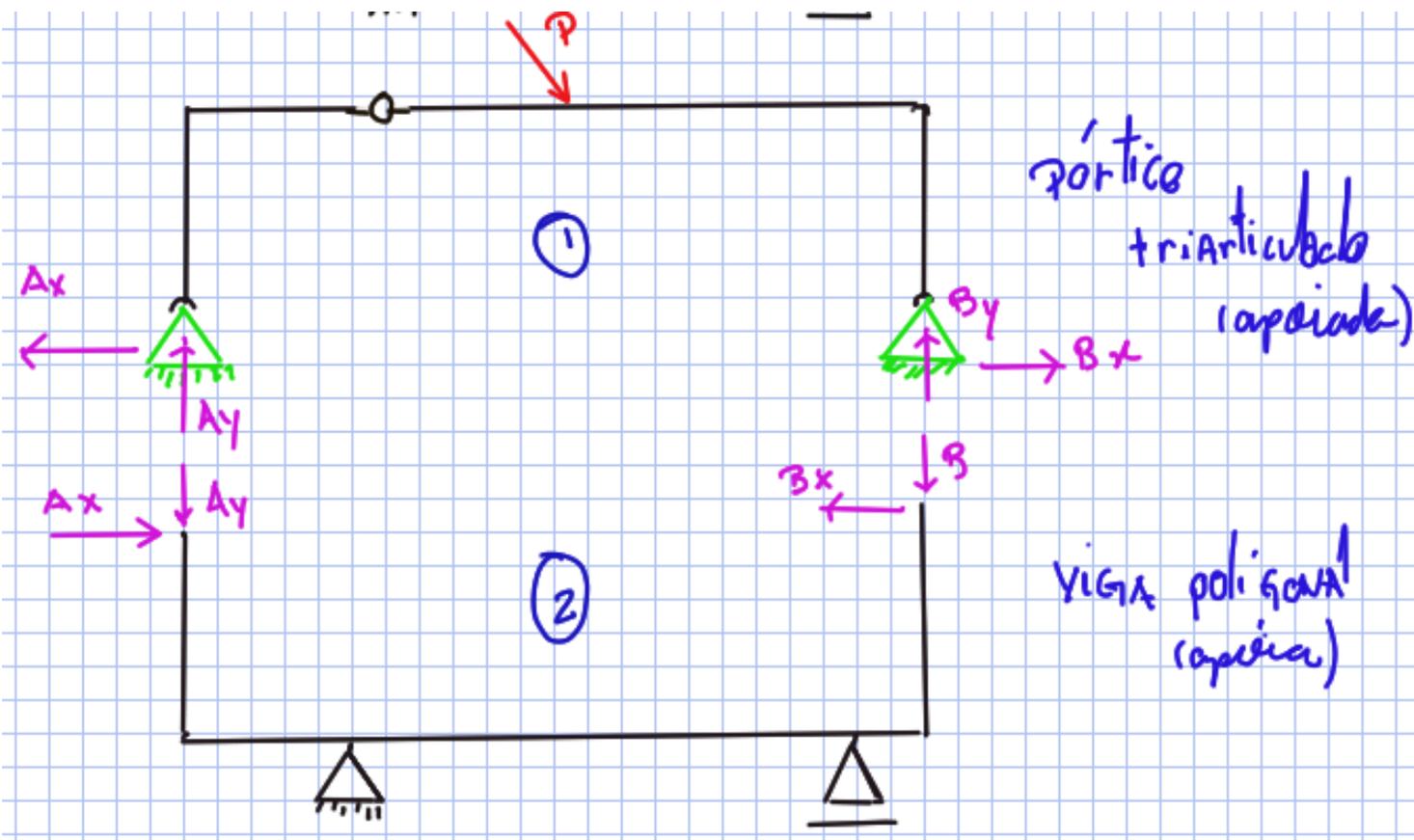
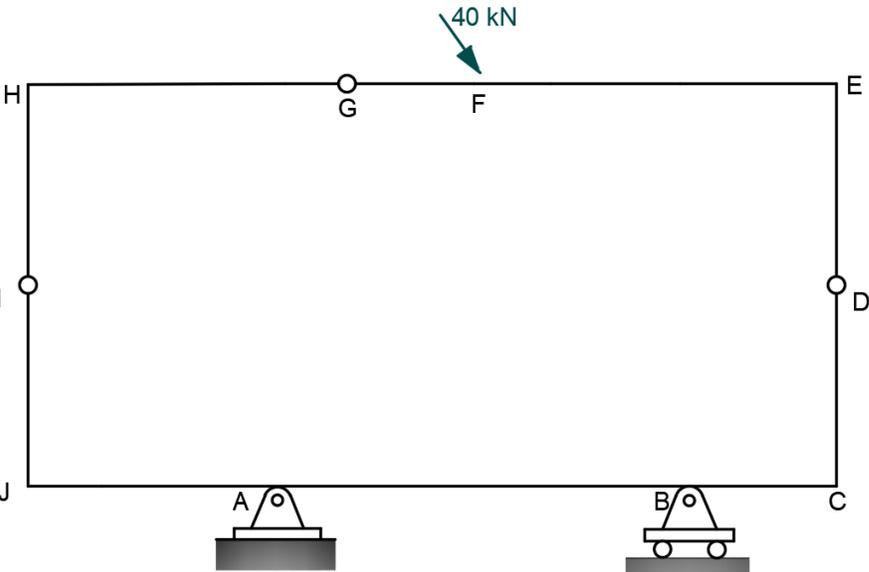
Dividir em suas subestruturas



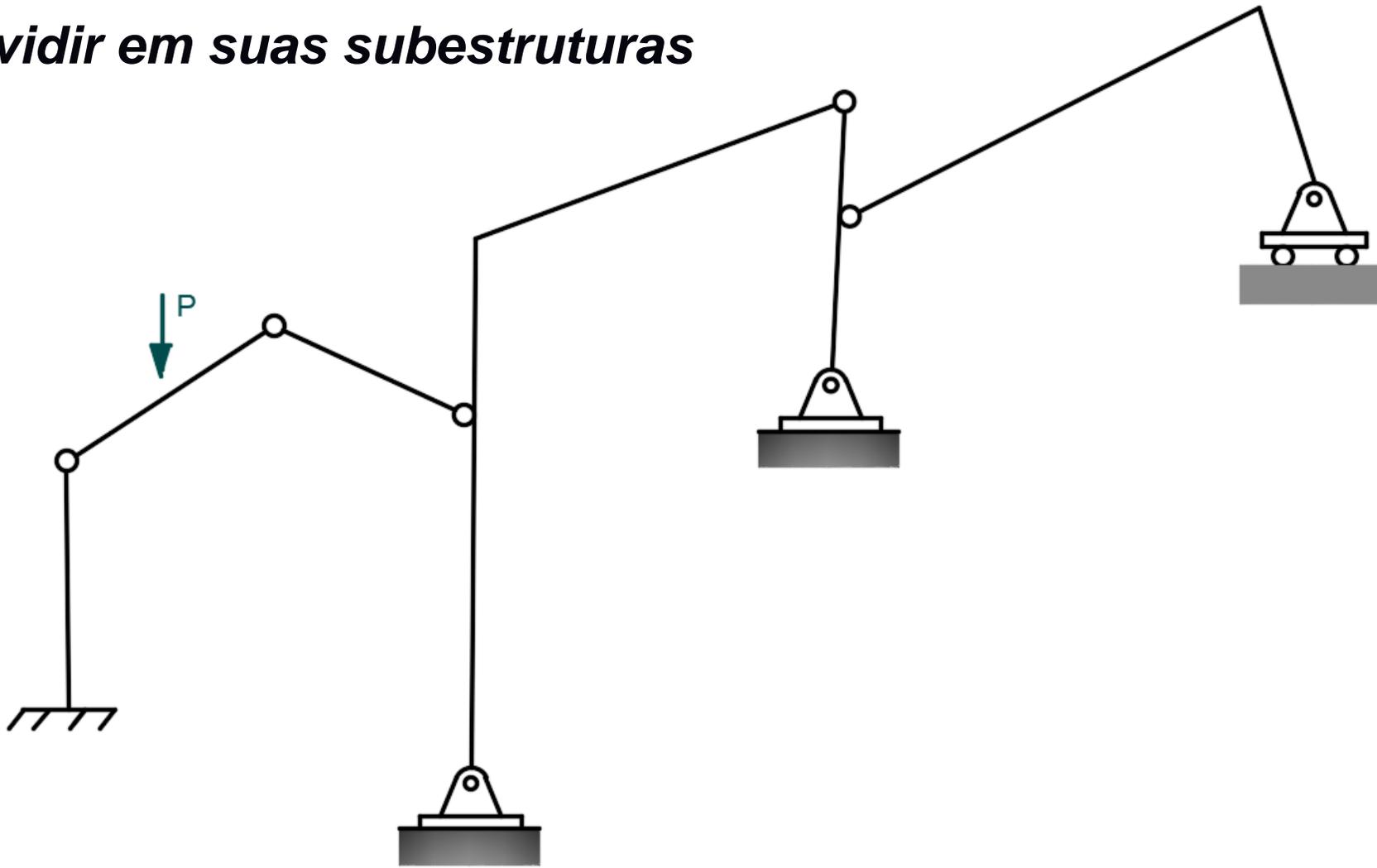
Dividir em suas subestruturas

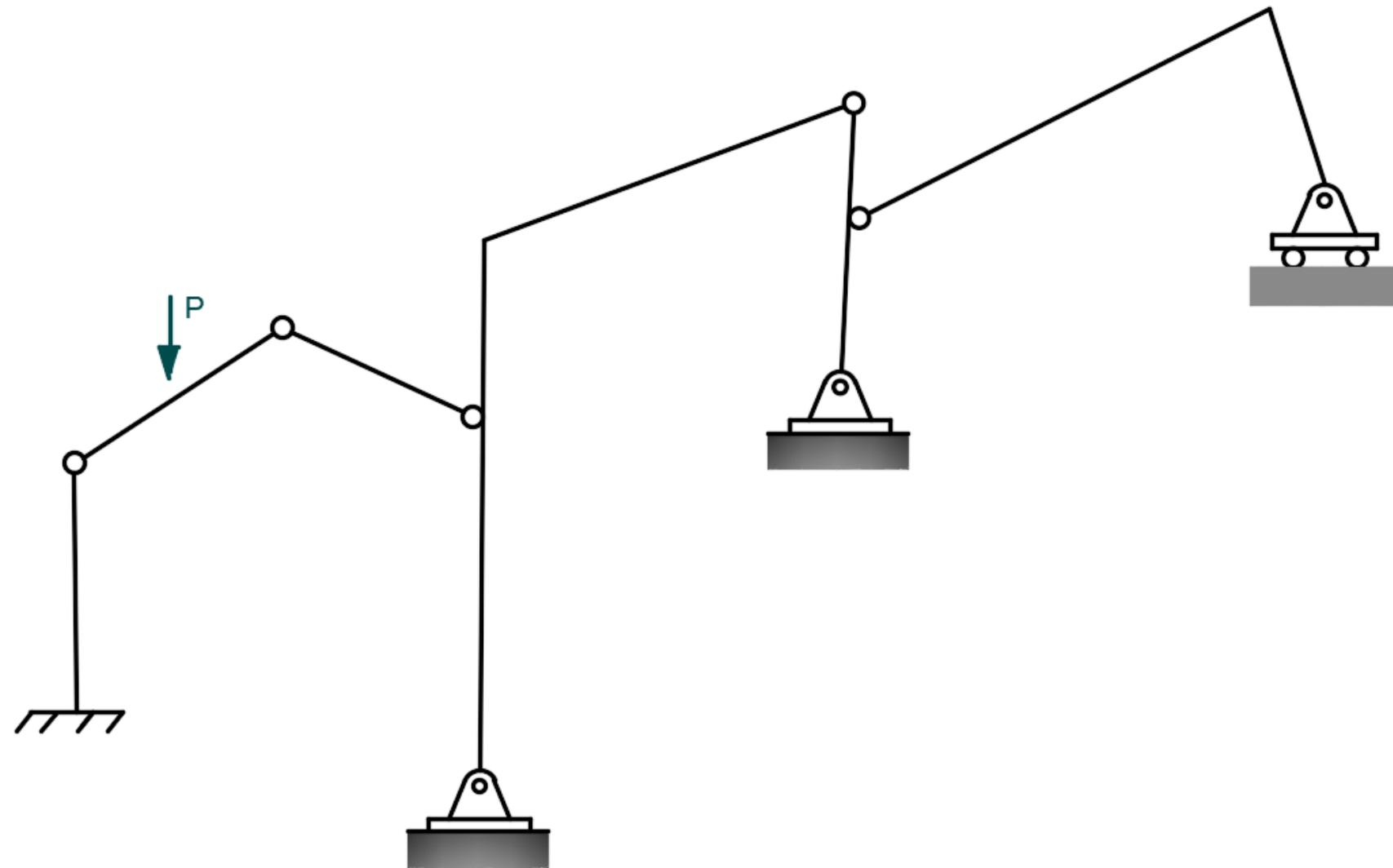


Estruturas Associadas

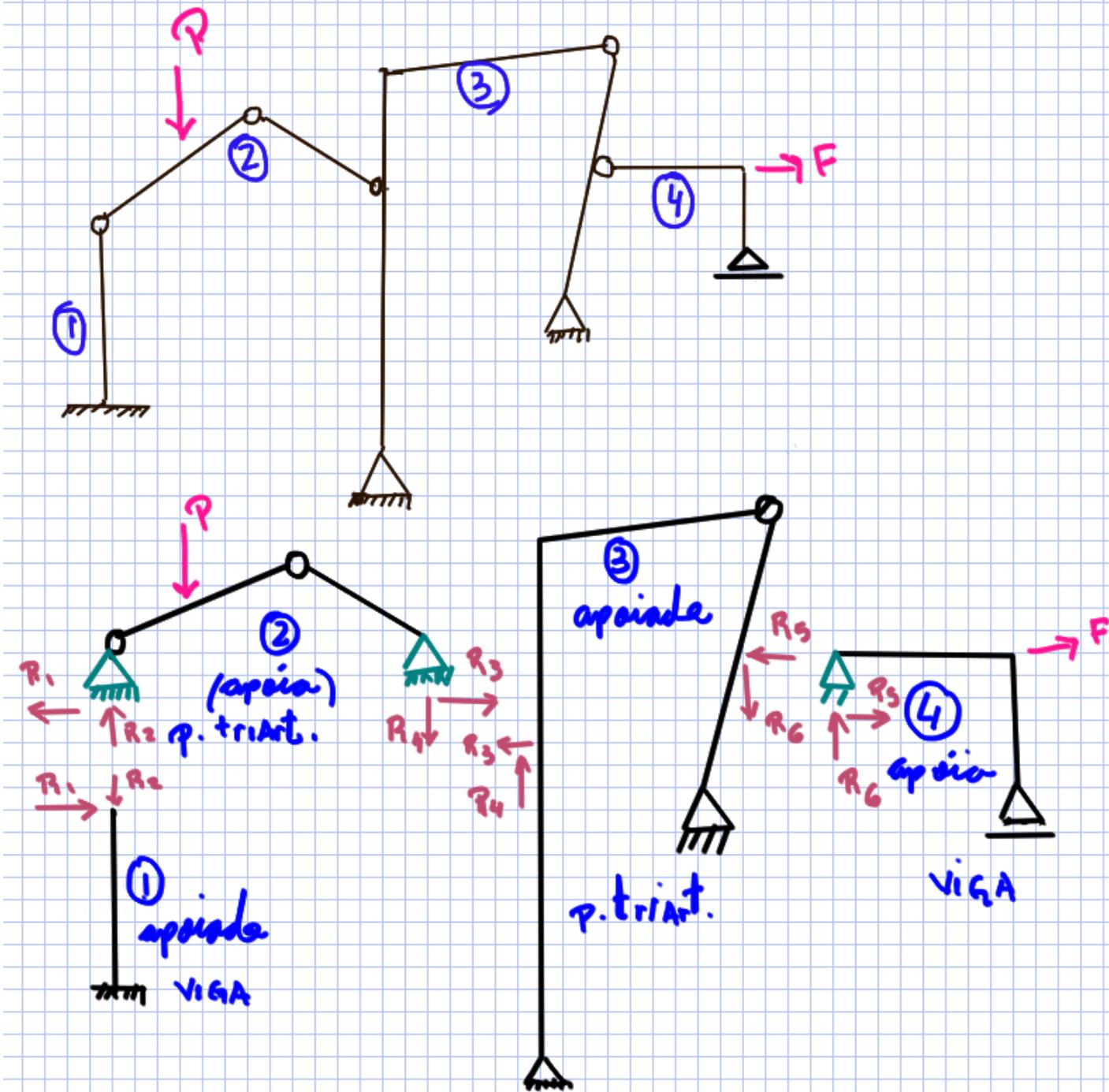


Dividir em suas subestruturas



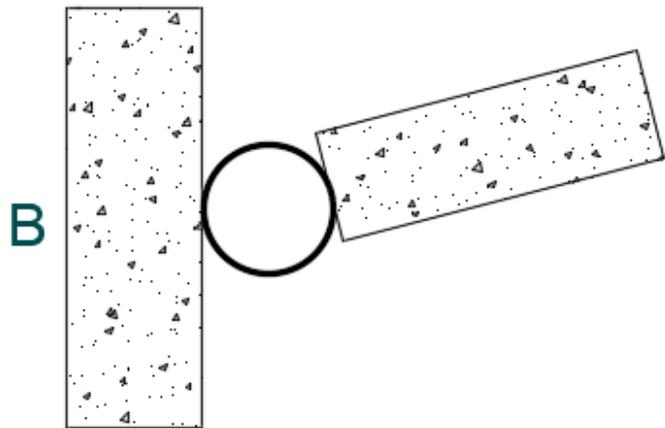


Estruturas Associadas



Atento as ligações distintas!

Rótula tangencia uma barra



≡

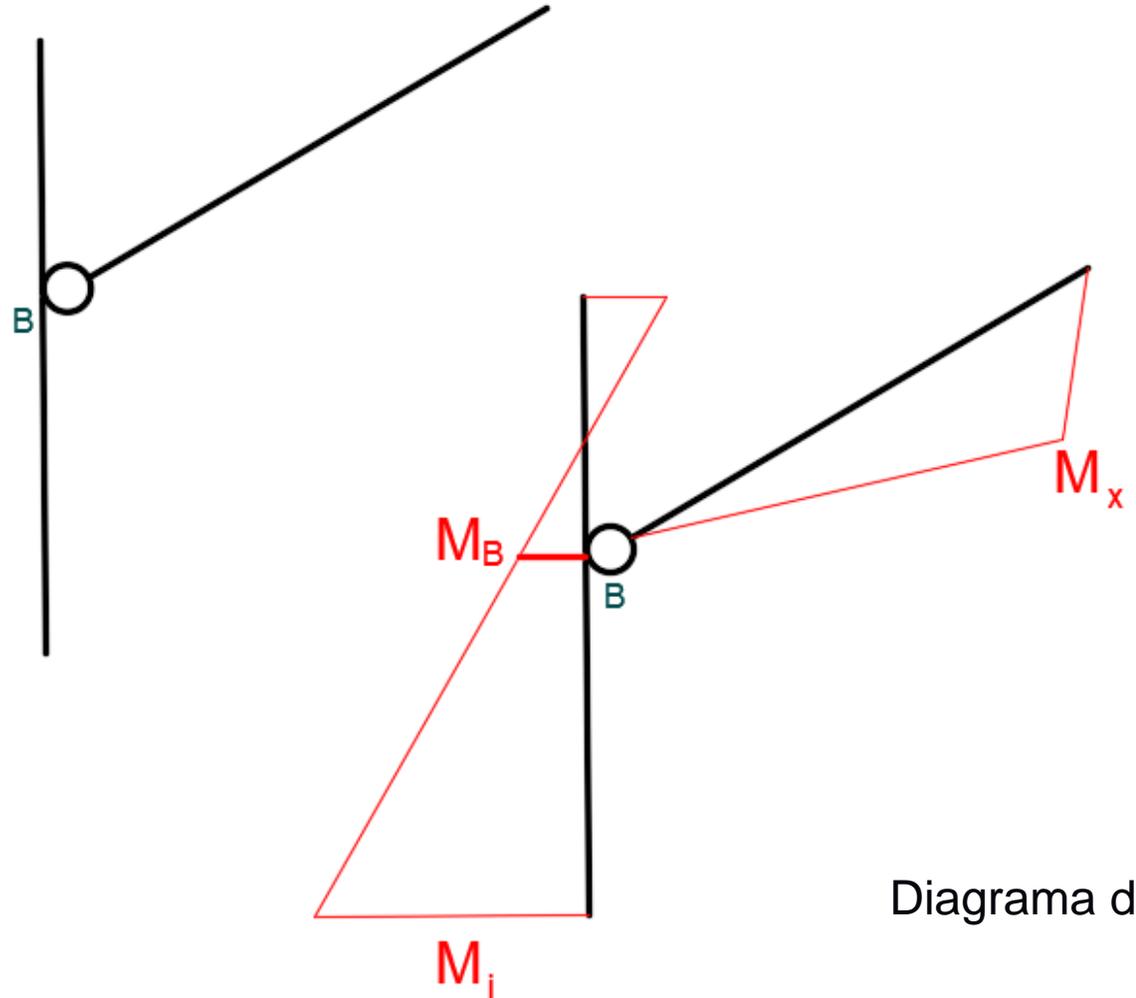
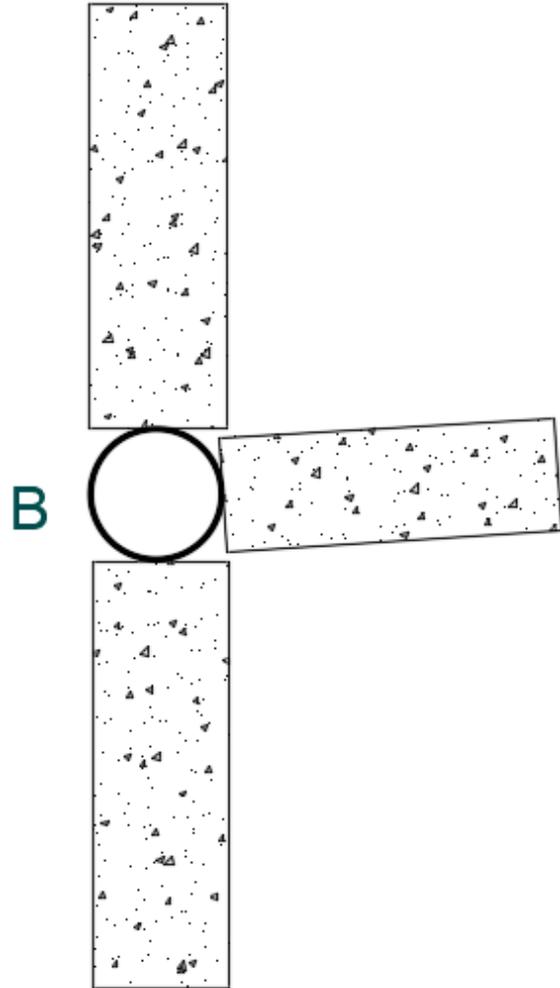


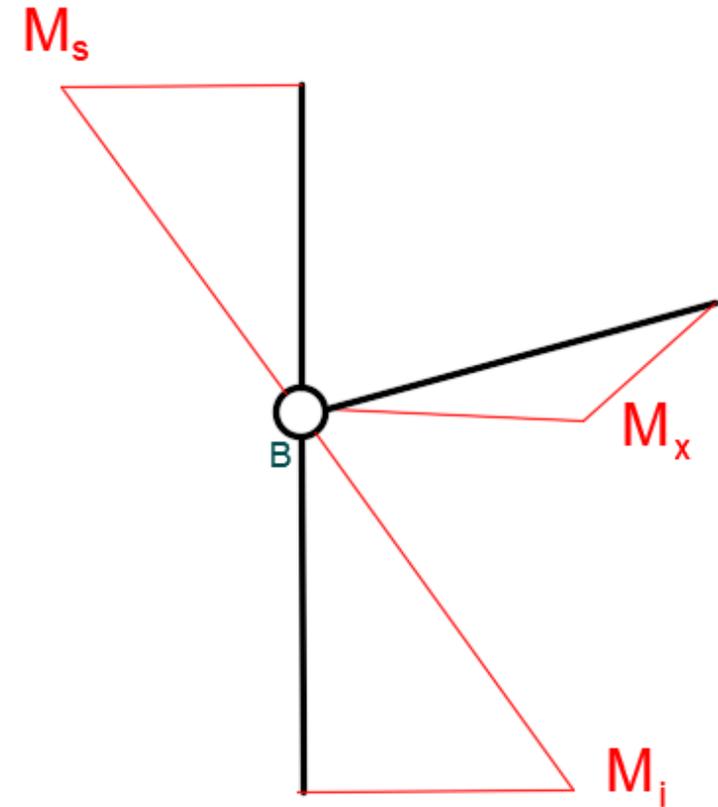
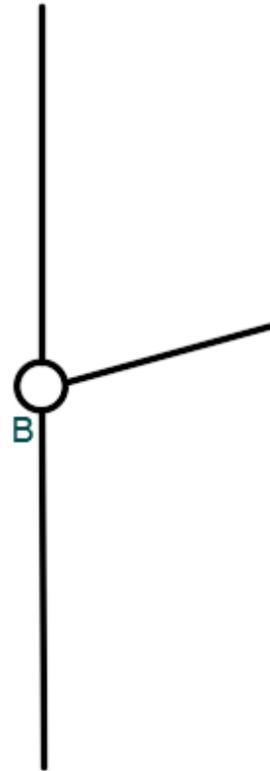
Diagrama de M

Atento as ligações distintas!

Rótula conecta todas as barras

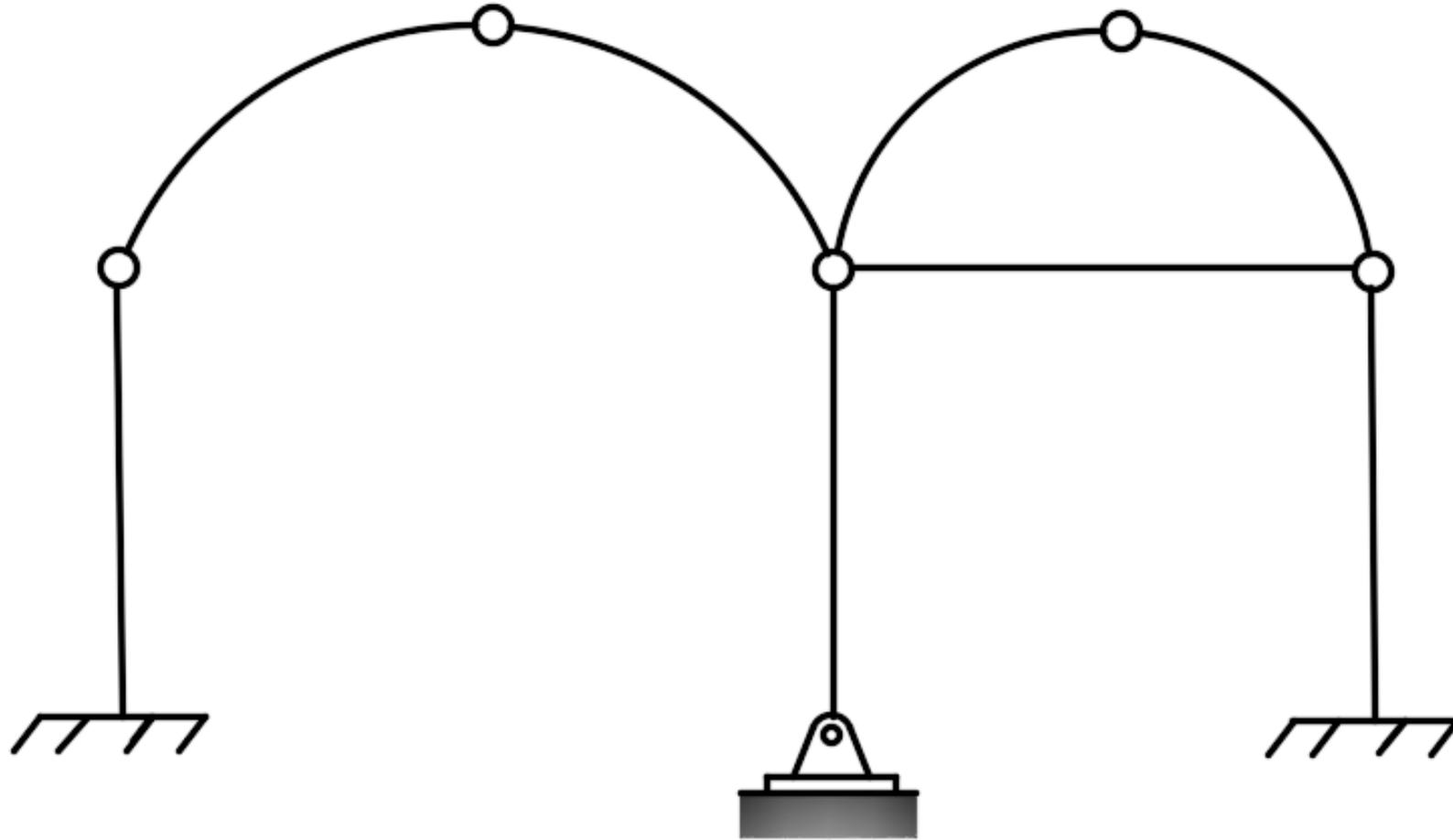


≡



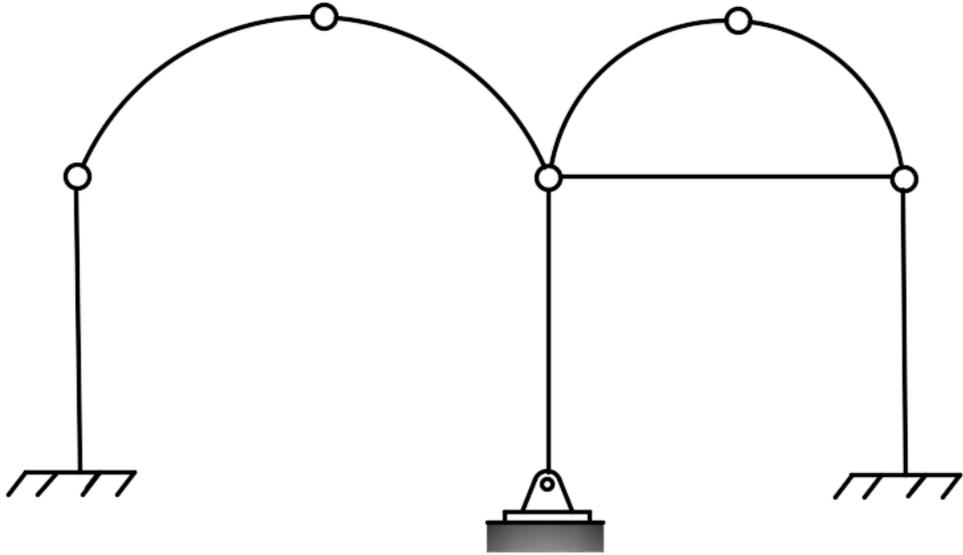
Estruturas Associadas

Dividir em suas subestruturas

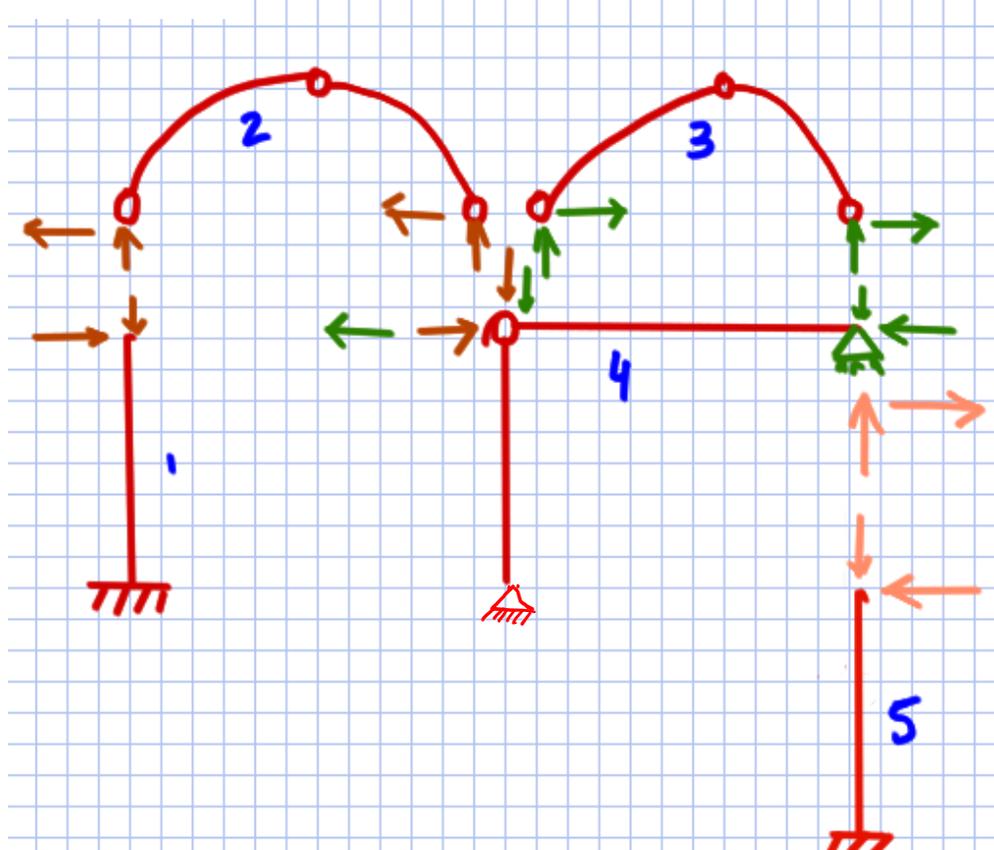
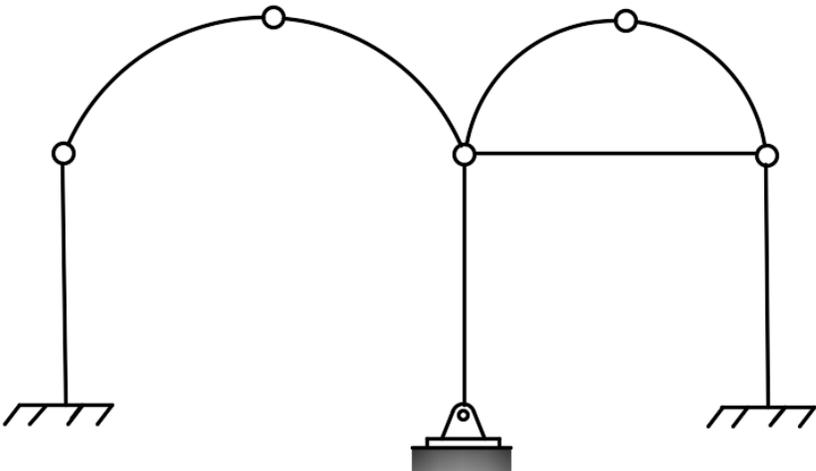


Estruturas Associadas

Dividir em suas subestruturas



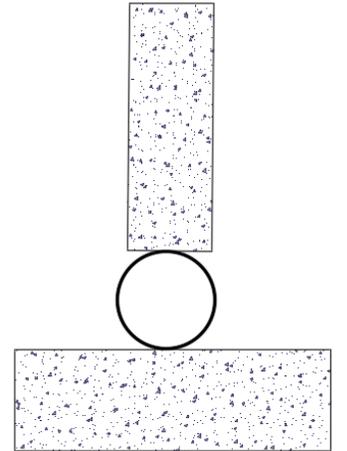
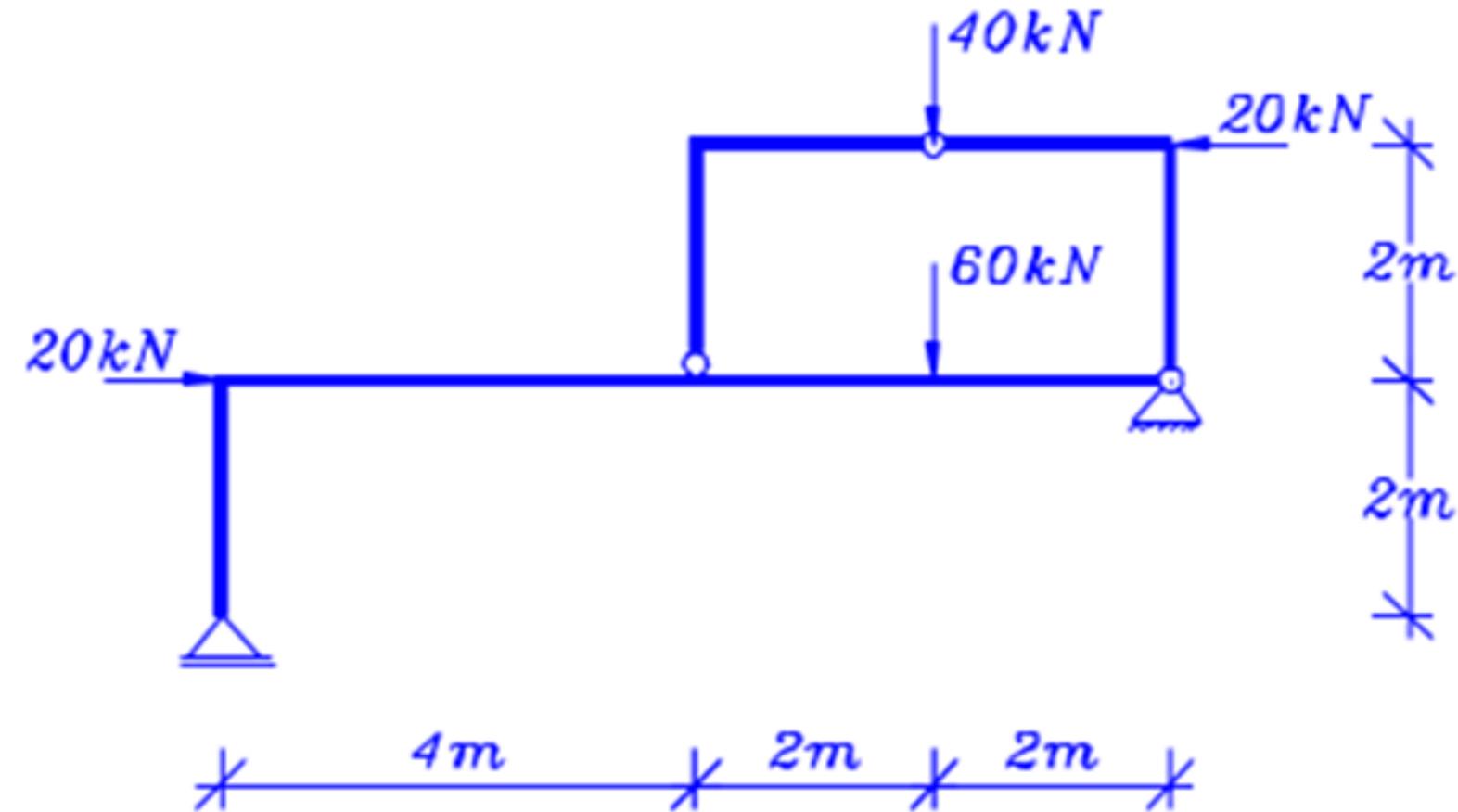
Estruturas Associadas



- ② opoia em ① e ④
- ③ " em ④ e ⑤
- ④ " em ⑤

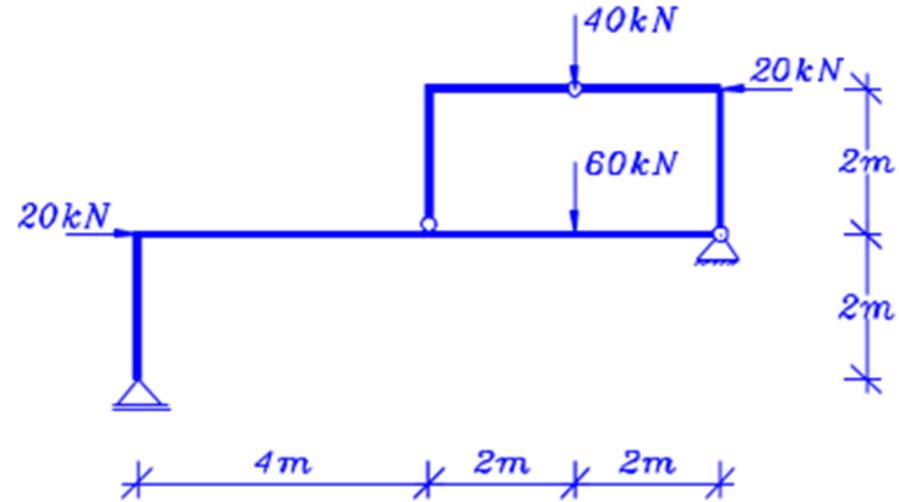
Exemplo 2 - Estruturas Associadas

Dividir em suas subestruturas



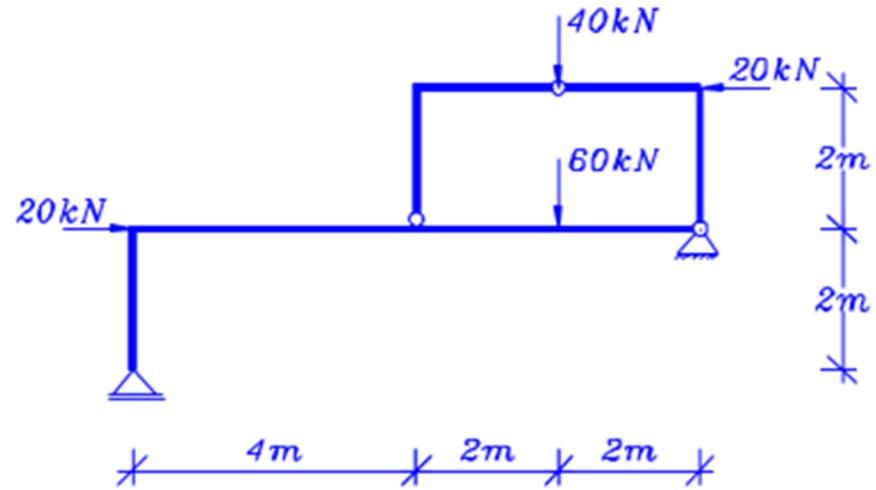
Exemplo 2 - Estruturas Associadas

Calcular reações/esforços em cada subestrutura

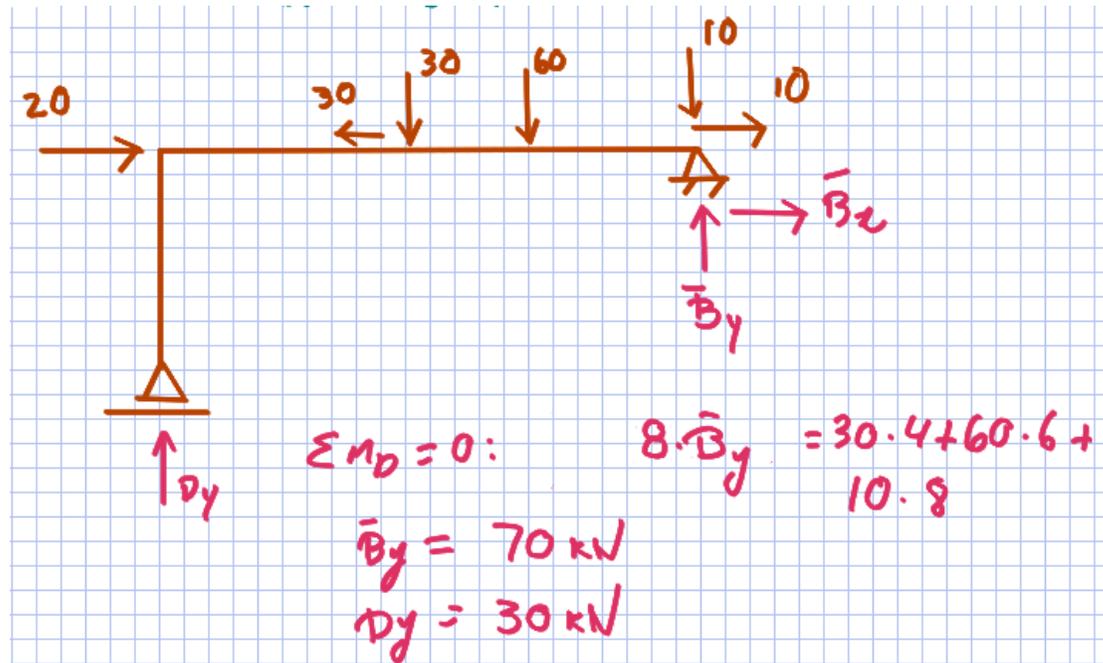
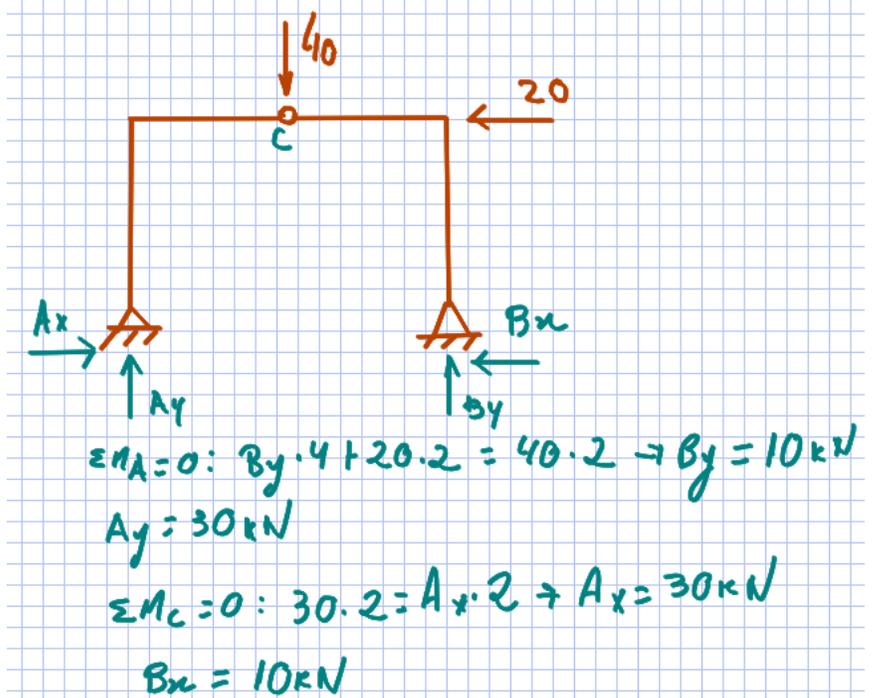
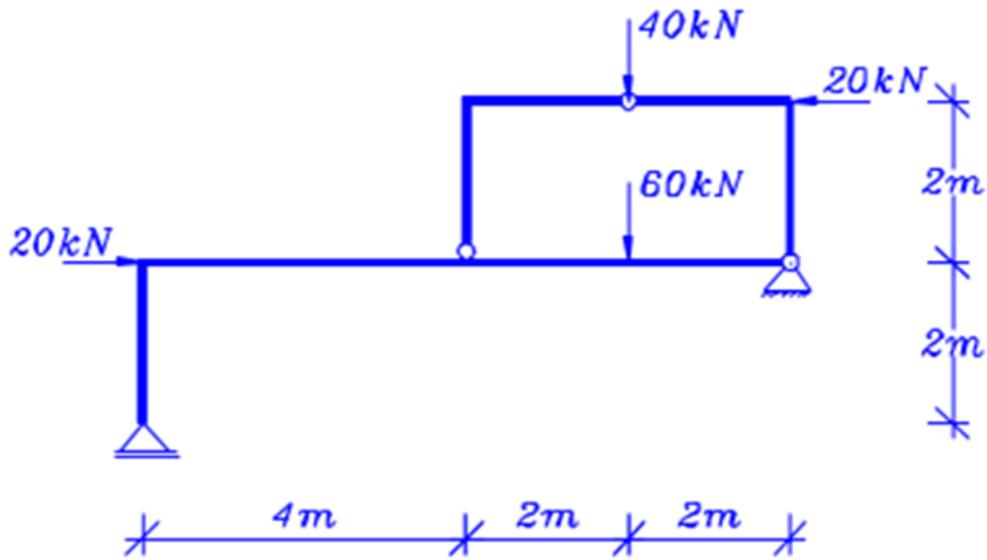


Exemplo 2 - Estruturas Associadas

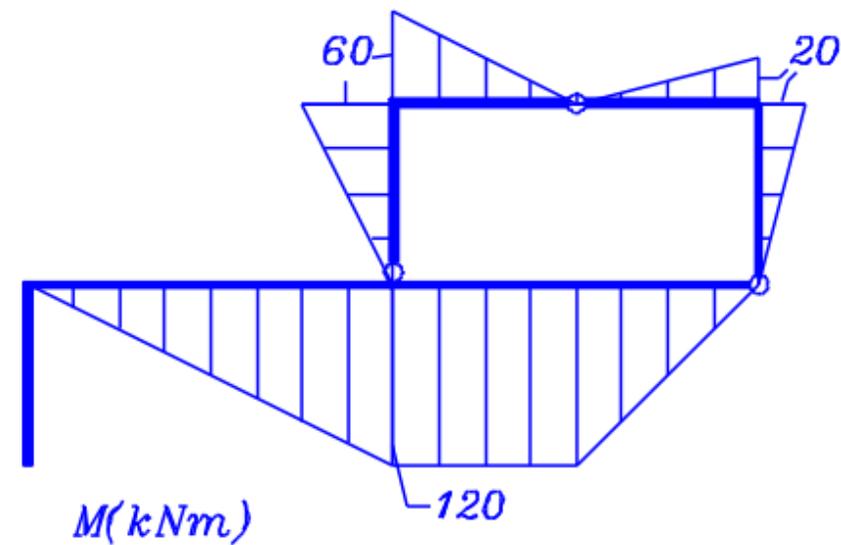
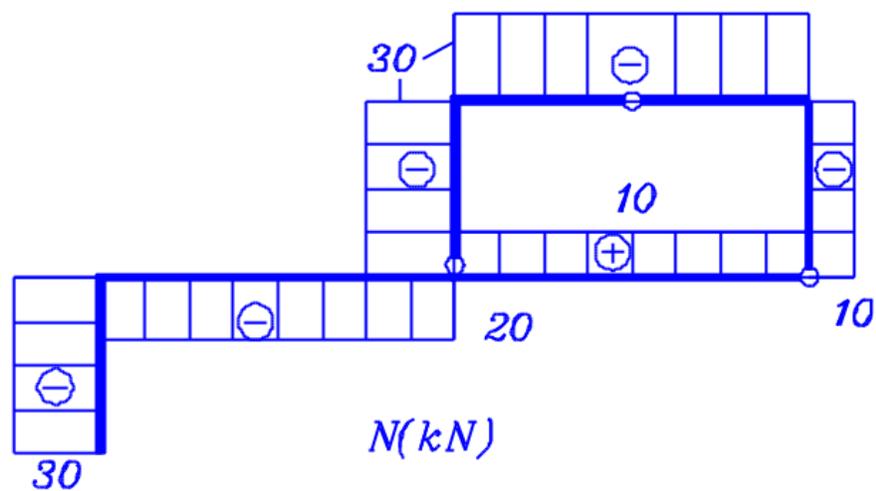
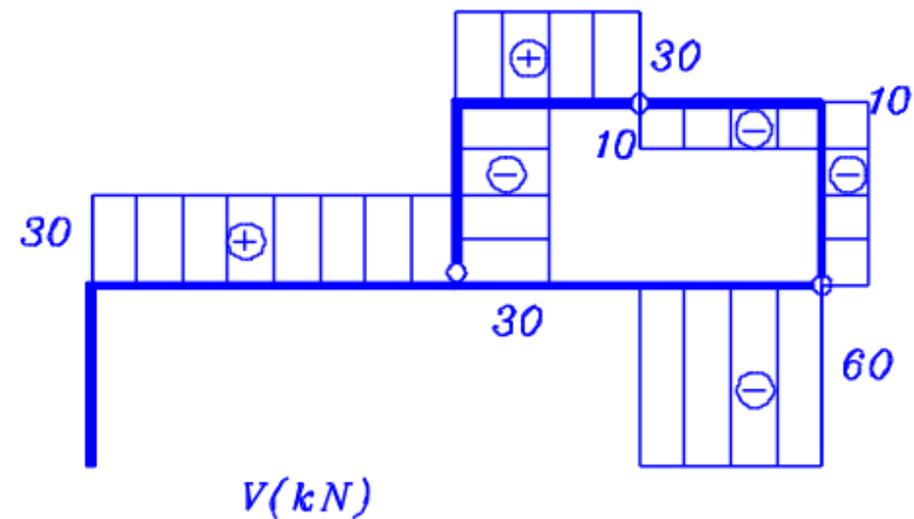
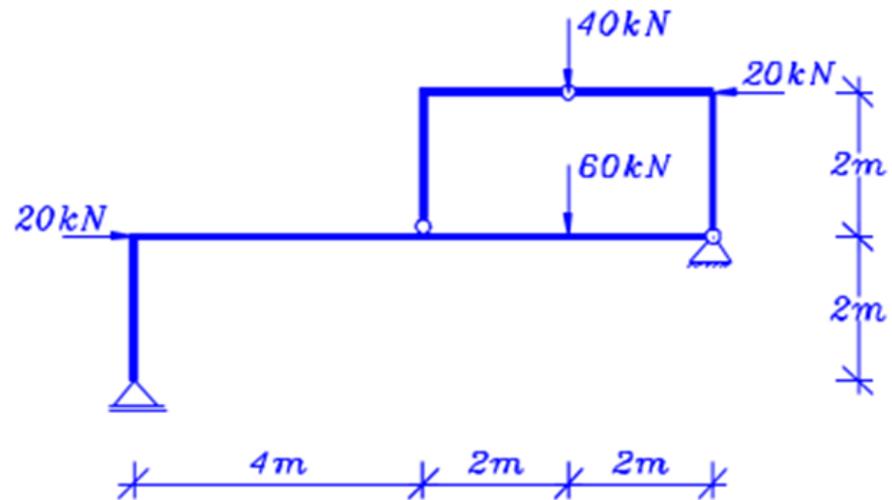
Calcular reações/esforços em cada subestrutura



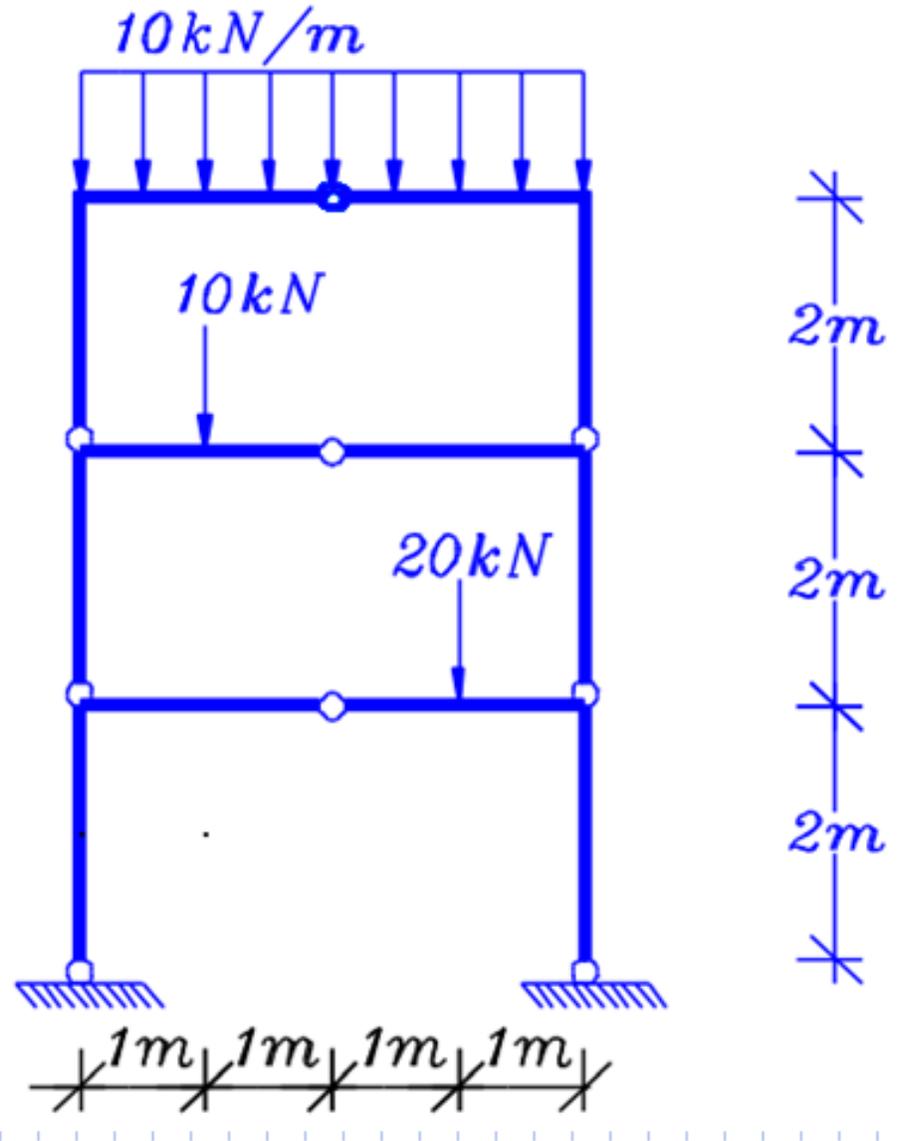
Exemplo 2 - Estruturas Associadas



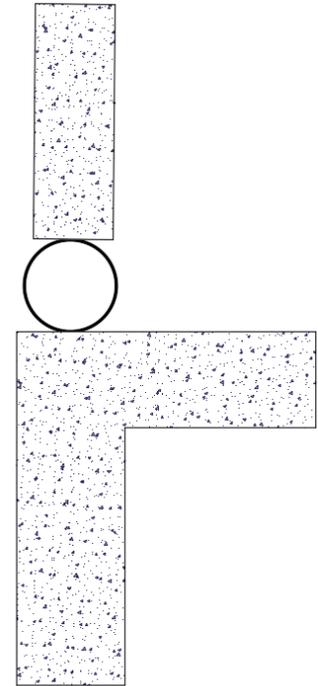
Exemplo 2 - Estruturas Associadas



Exemplo 3

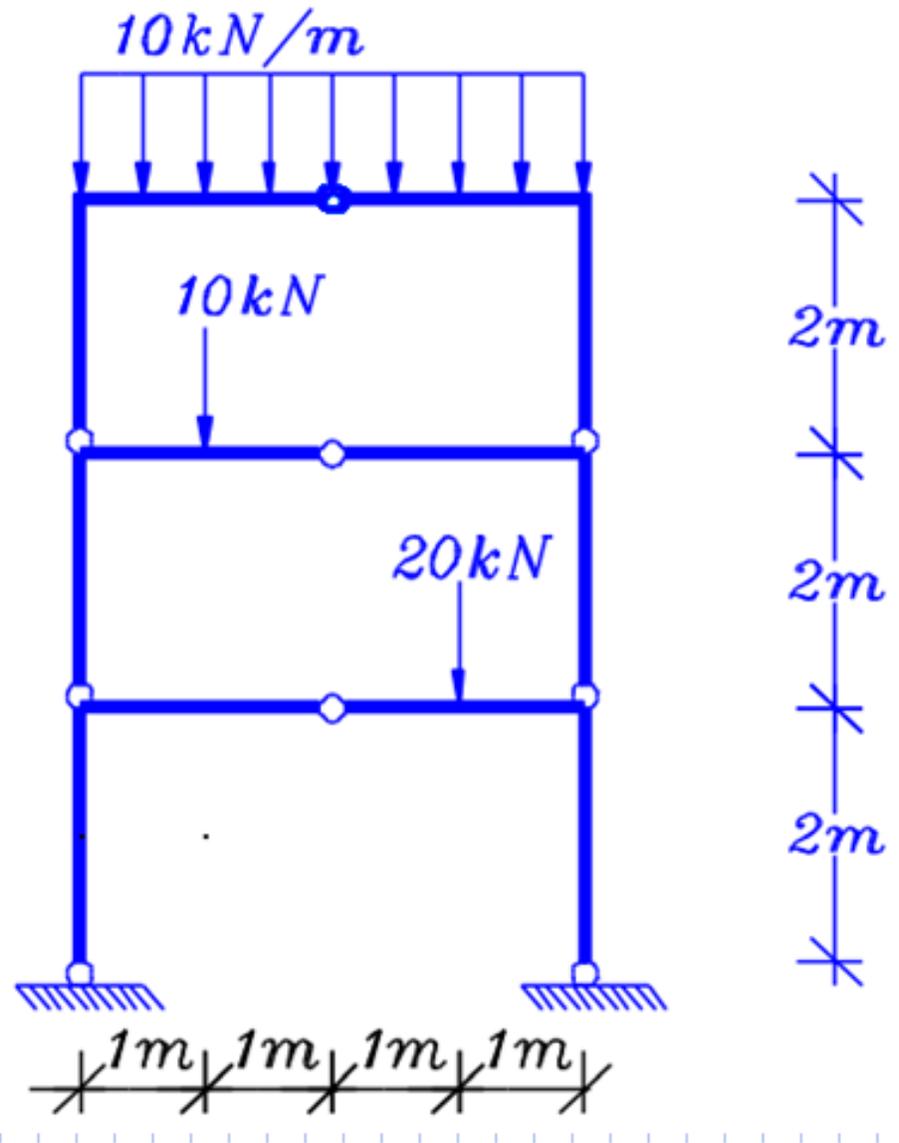


Dividir em suas subestruturas

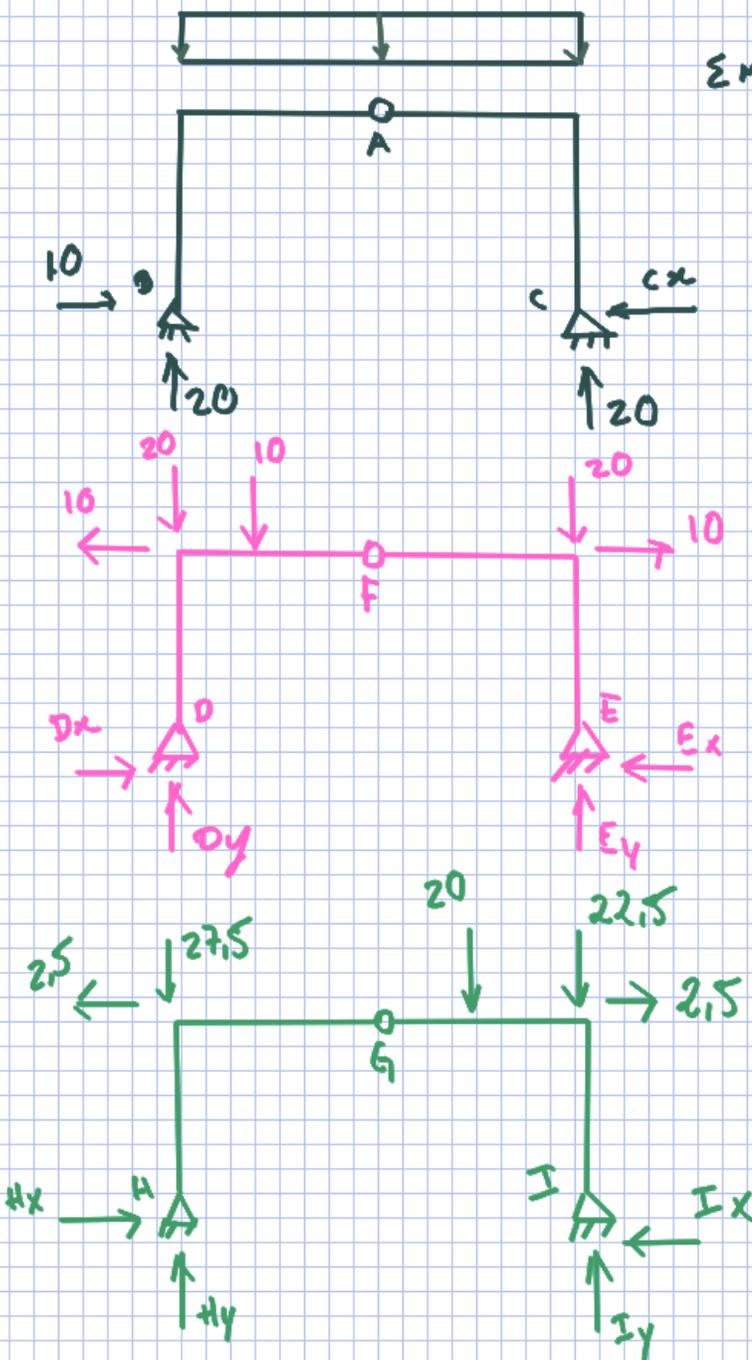
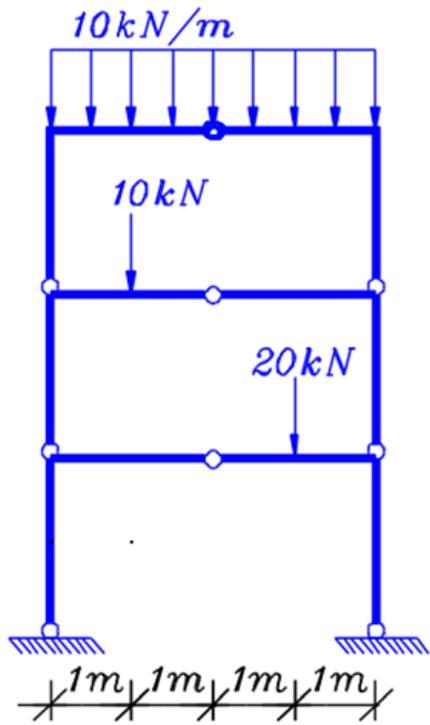


Exemplo 3

Dividir em suas subestruturas



E x e m p l o 3



$$\sum M_A = 0 : 20 \cdot 2 = c_x \cdot 2 + 20 \cdot 1$$

$$c_x = 10\text{ kN}; B_x = 10\text{ kN}$$

$$\sum M_D = 0 : 4E_y = 10 \cdot 1 + 20 \cdot 4$$

$$E_y = 22,5\text{ kN}$$

$$D_y = 27,5\text{ kN}$$

$$\sum M_F = 0 : 22,5 \cdot 2 = 20 \cdot 2 + E_x \cdot 2$$

$$E_x = 2,5\text{ kN}$$

$$D_x = 2,5\text{ kN}$$

$$\sum M_H = 0 : 4I_y = 20 \cdot 3 + 22,5 \cdot 4$$

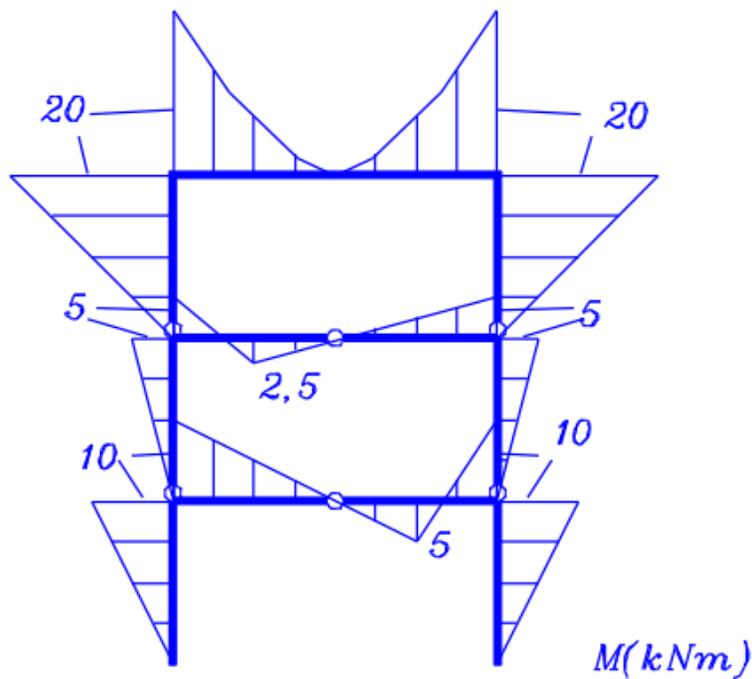
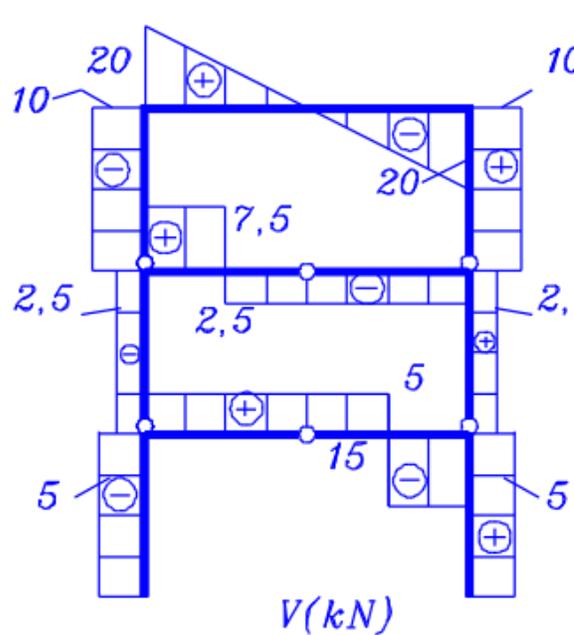
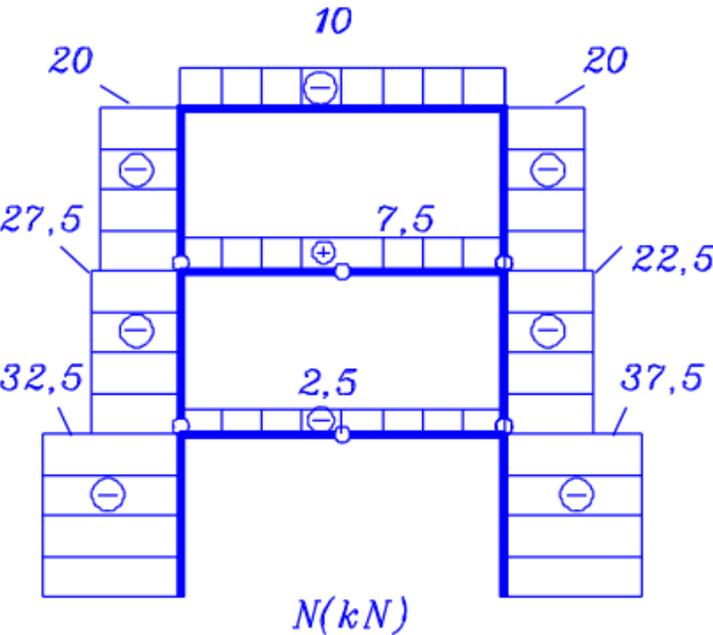
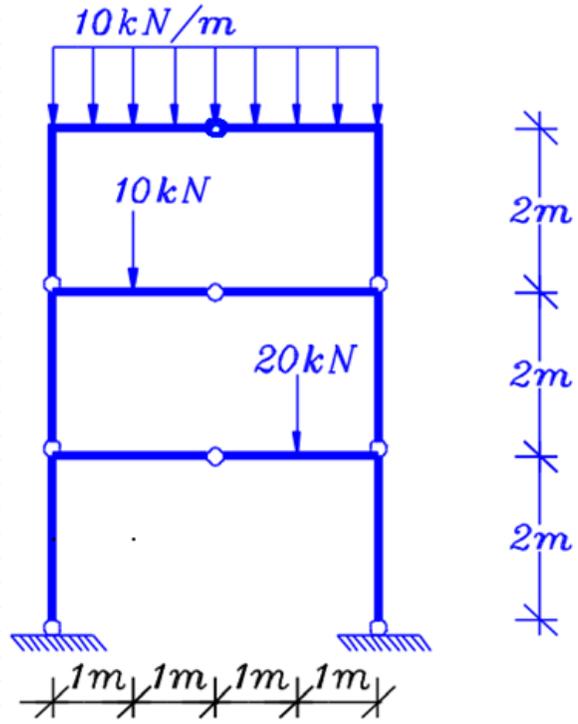
$$I_y = 37,5\text{ kN}; H_y = 32,5\text{ kN}$$

$$\sum M_G = 0 : 37,5 \cdot 2 = 20 \cdot 1 + 22,5 \cdot 2 + I_x \cdot 2$$

$$I_x = 5\text{ kN}$$

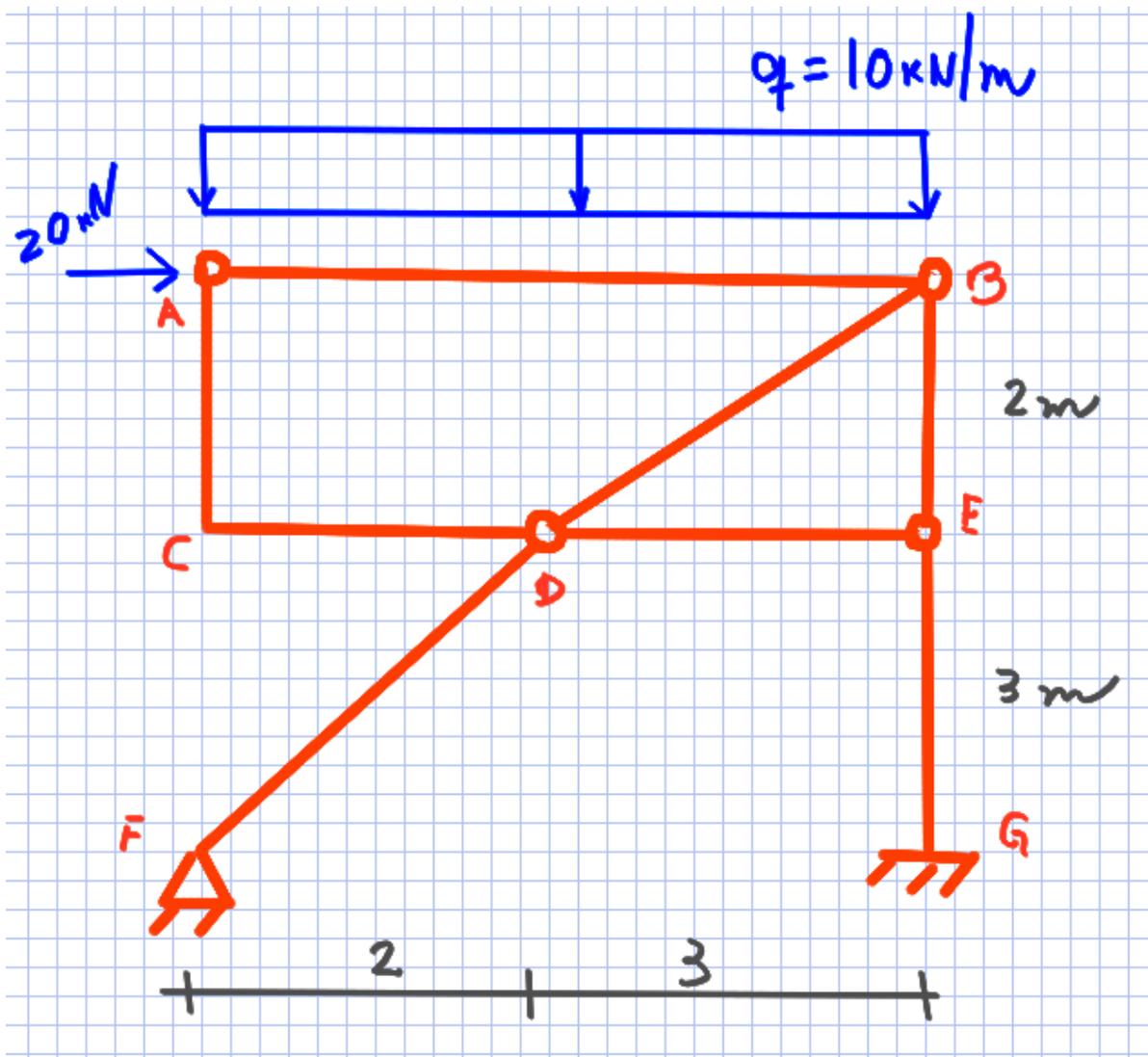
$$H_x = 5\text{ kN}$$

Exemplo 3 - Estruturas Associadas

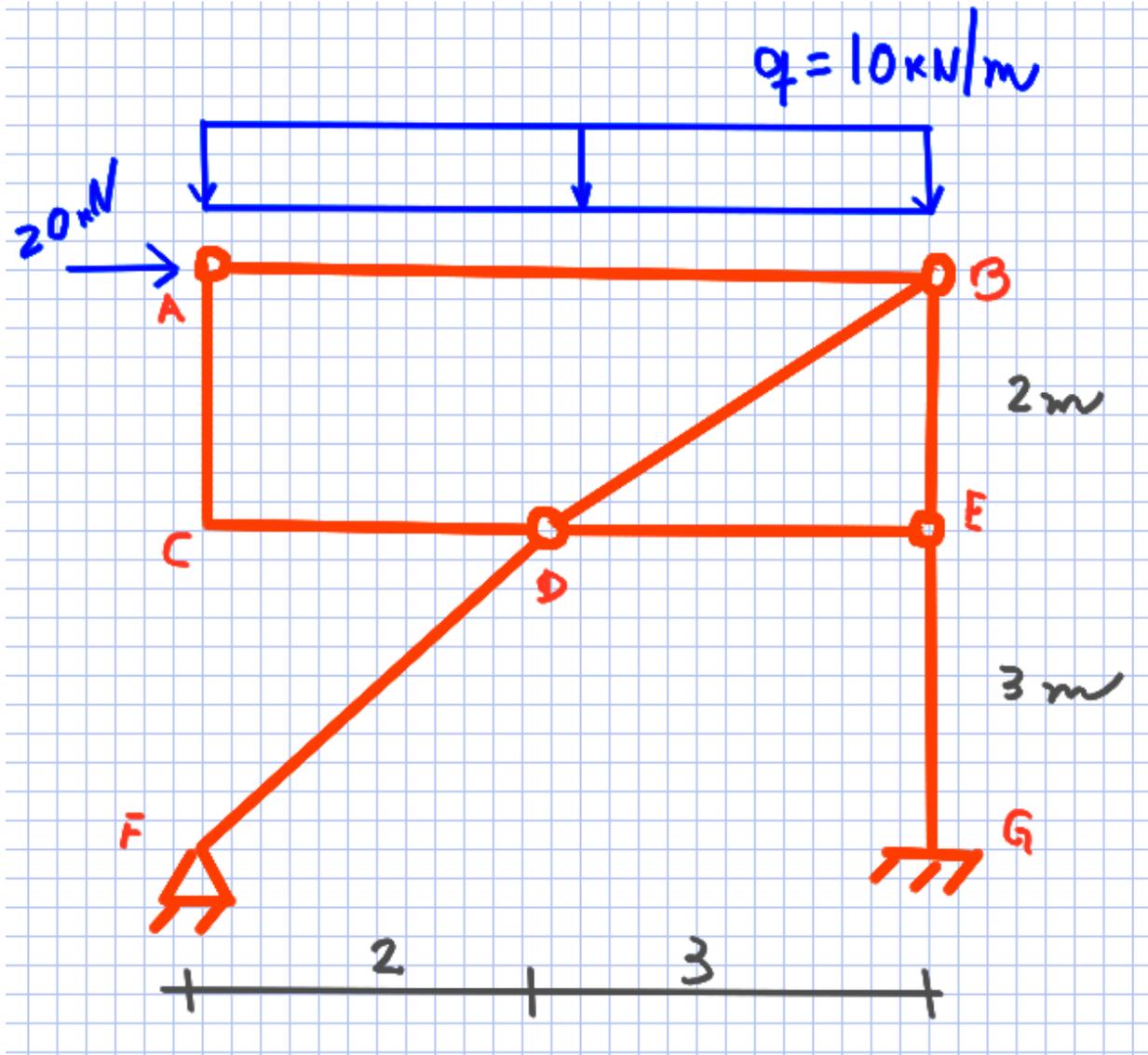


Exemplo 4 - Estruturas Associadas

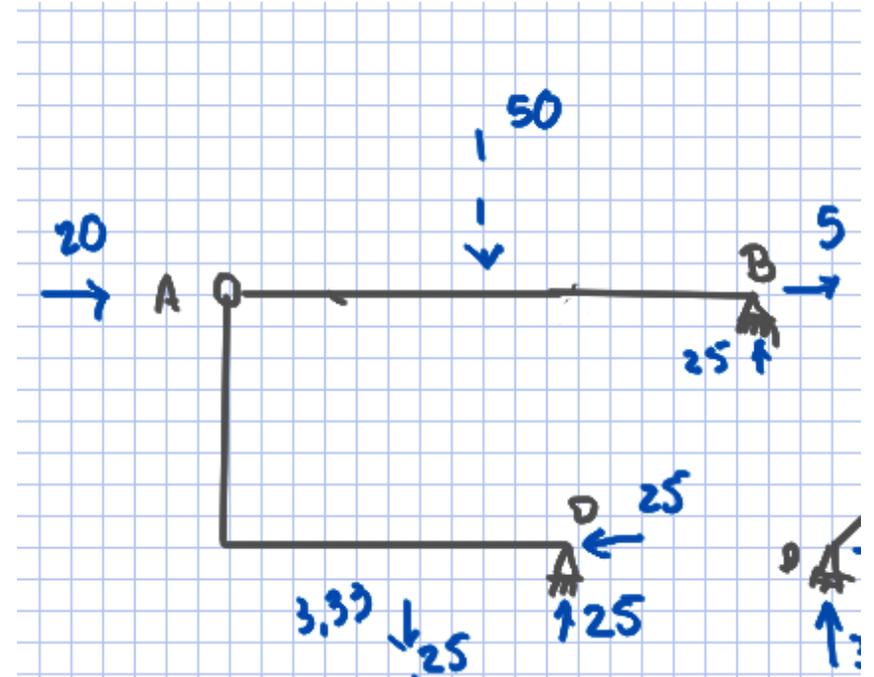
Dividir em suas subestruturas



Exemplo 4 - Estruturas Associadas

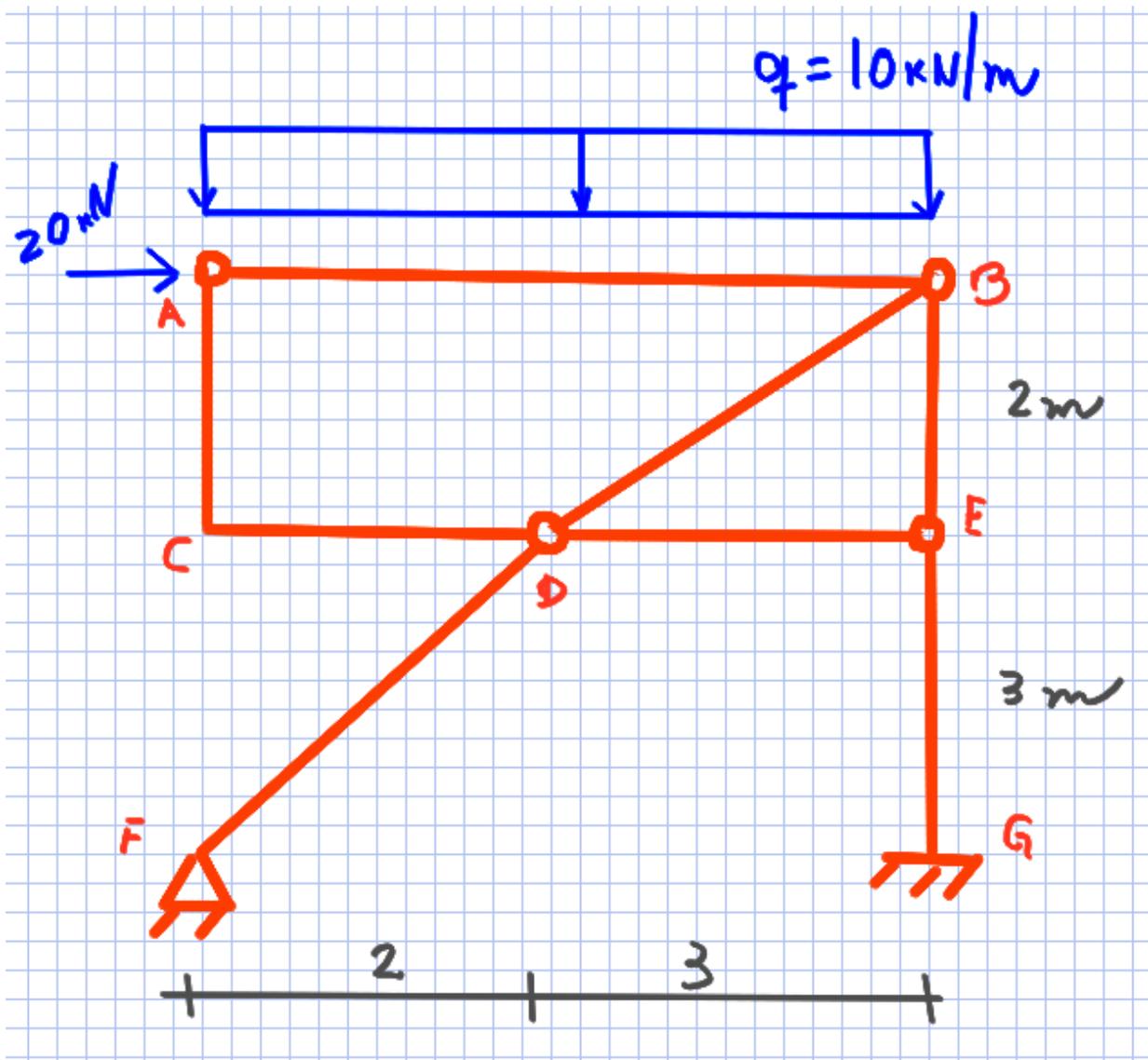


Dividir em suas subestruturas

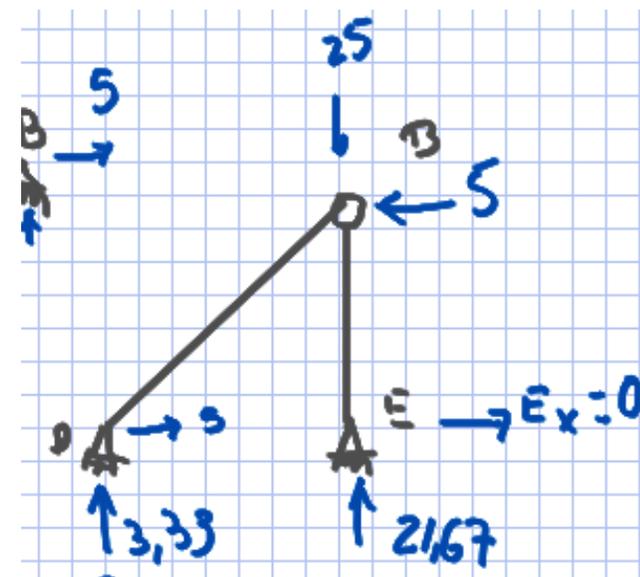


Pórtico triarticulado

Exemplo 4 - Estruturas Associadas

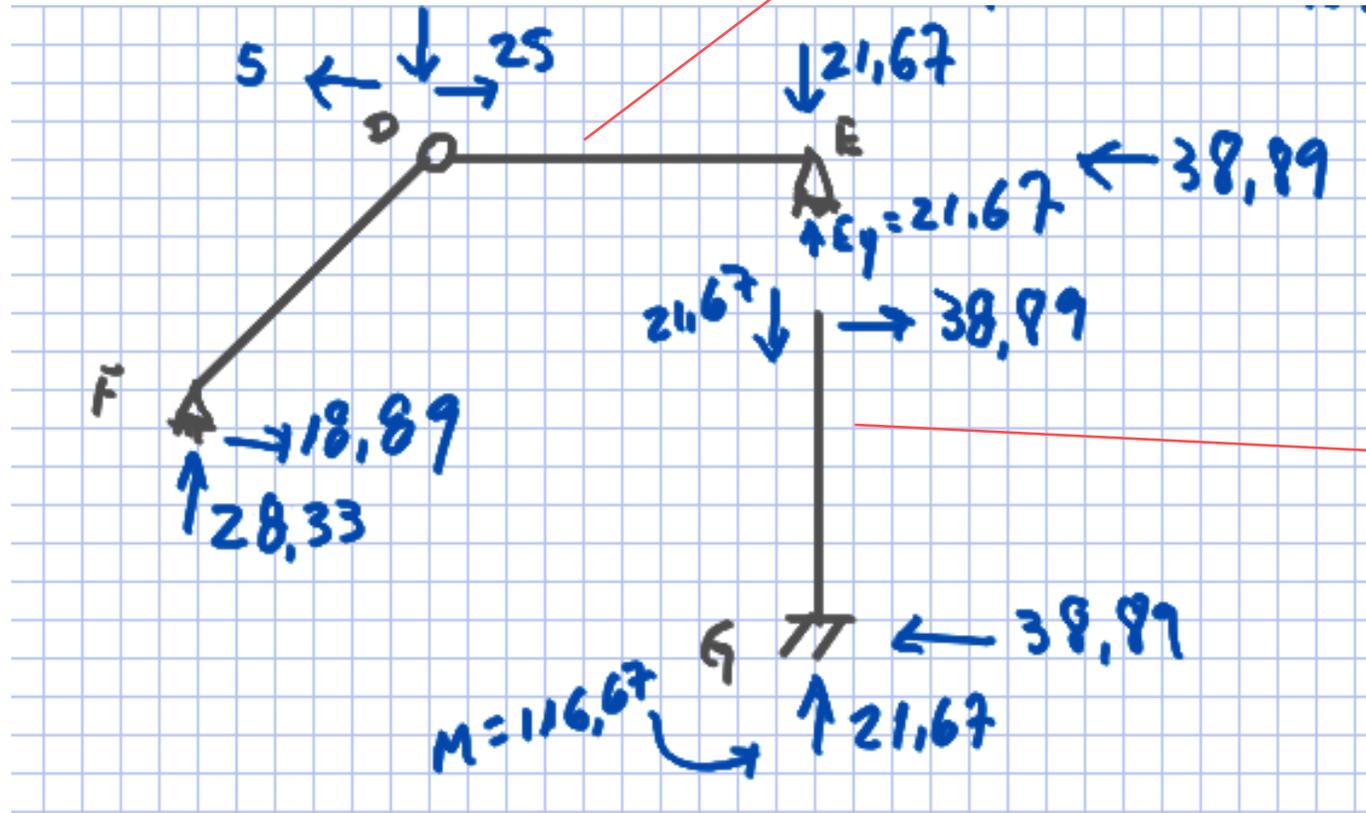


Dividir em suas subestruturas



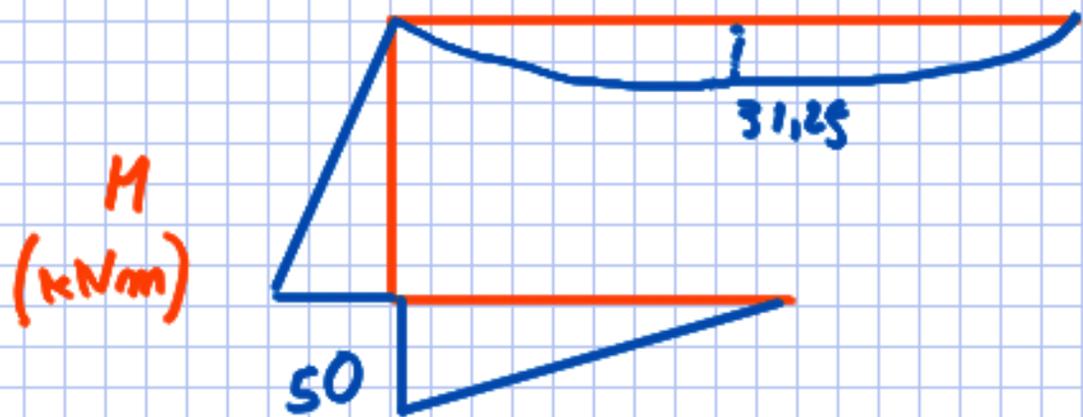
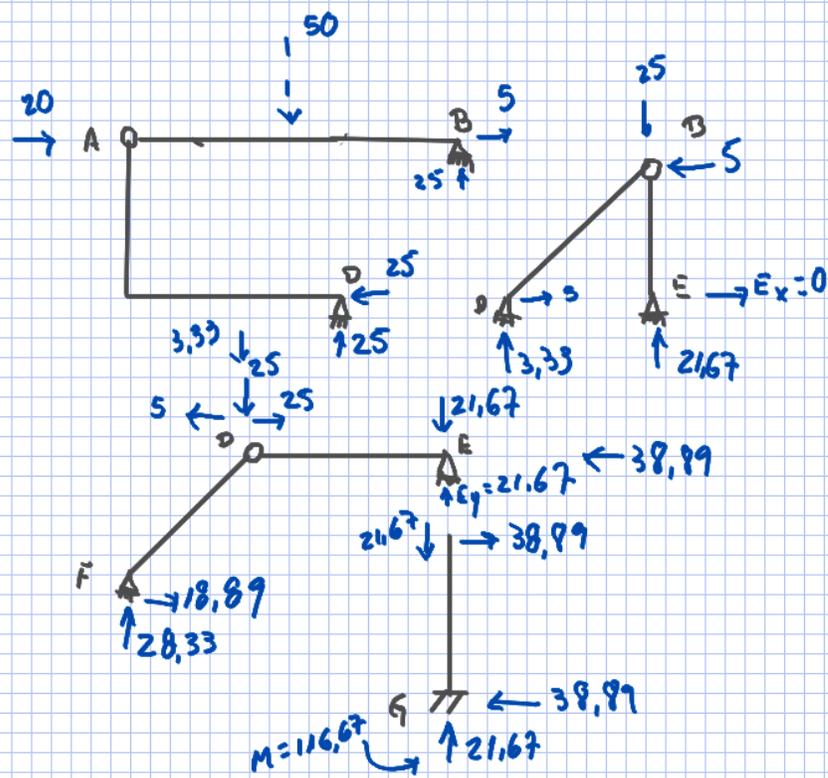
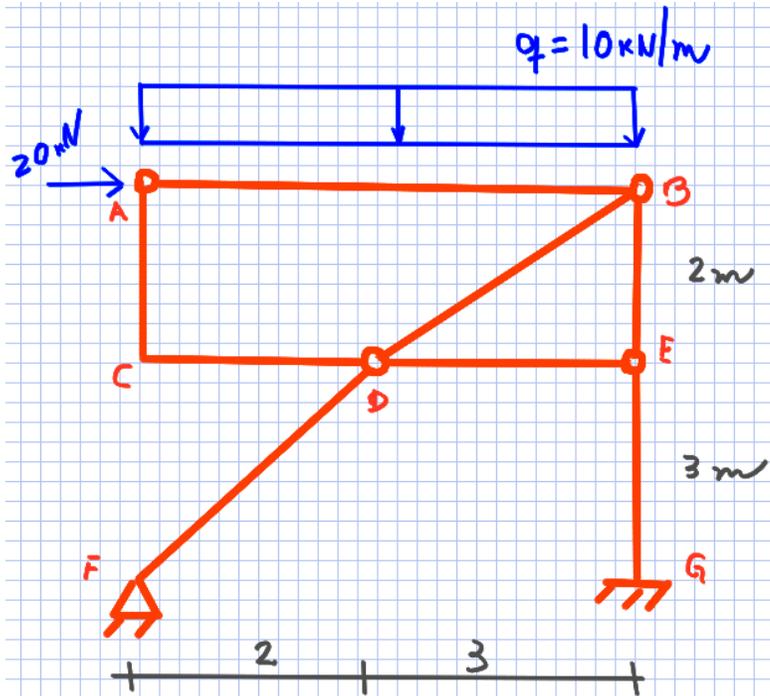
Treliça

Pórtico triarticulado



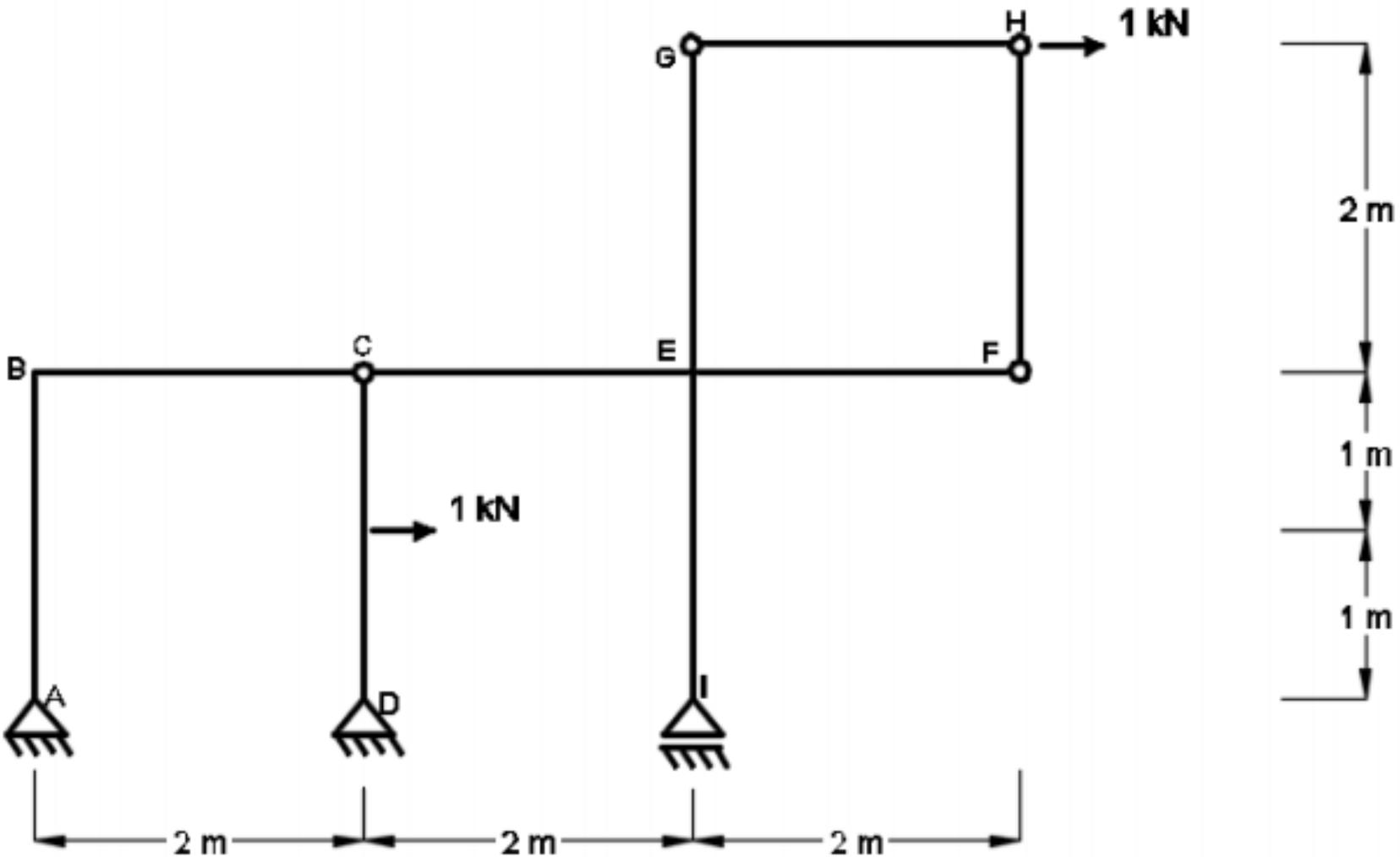
Viga em balanço

Exemplo 4 - Estruturas Associadas

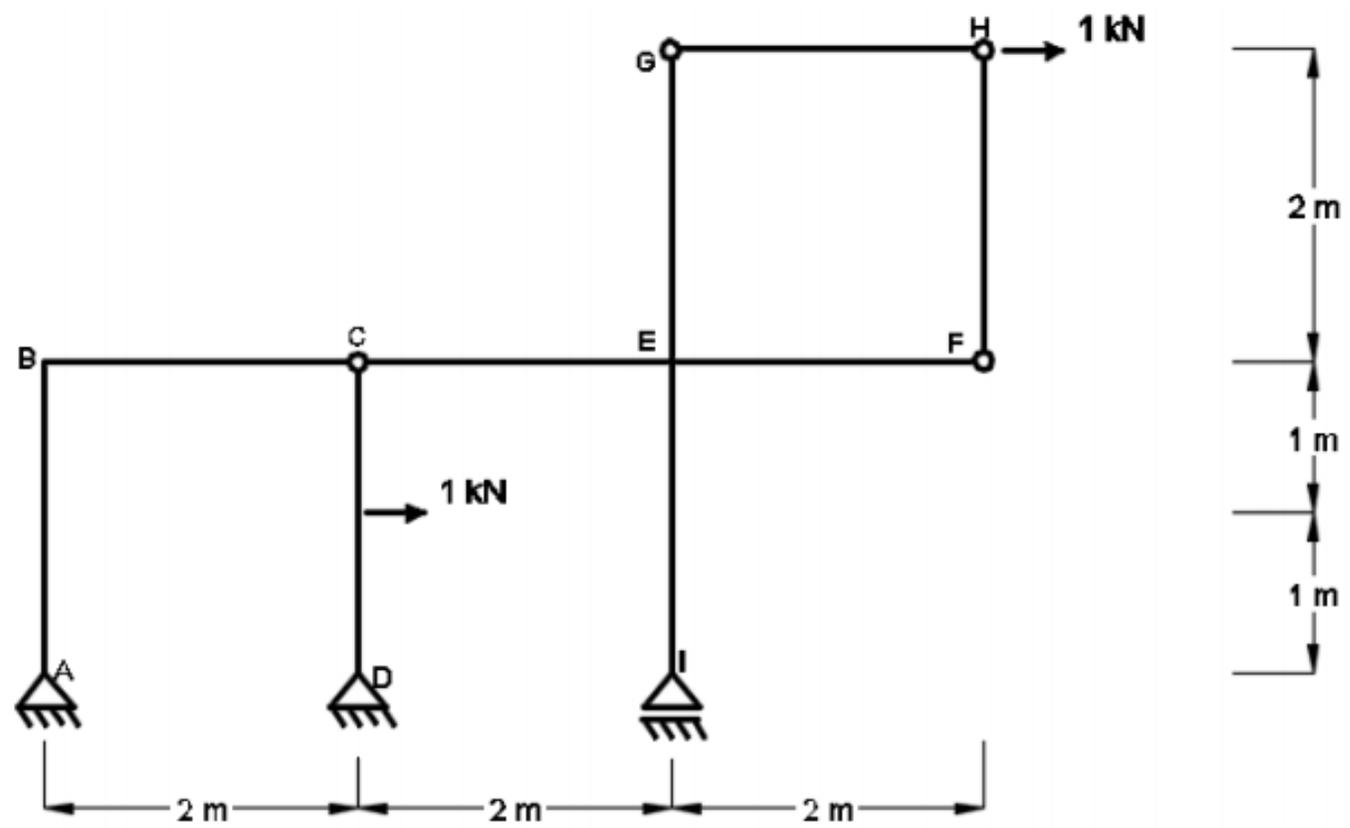


$\text{BAD} : \sum M_A = 0 : B_y \cdot 5 = 50 \cdot 2.5 \rightarrow B_y = 25 \text{ kN}$
 $\text{BDE} : \sum M_E = 0 : 5 \cdot 2 = D_y \cdot 3 \rightarrow D_y = 3.33 \text{ kN}$
 $\text{FDE} : \sum M_D = 0 : E_y \cdot 3 = 21.67 \cdot 3 \rightarrow E_y = 21.67 \text{ kN}$
 $\sum M_D = 0 : 28.33 \cdot 2 = F_x \cdot 3 \rightarrow F_x = 18.89 \text{ kN}$

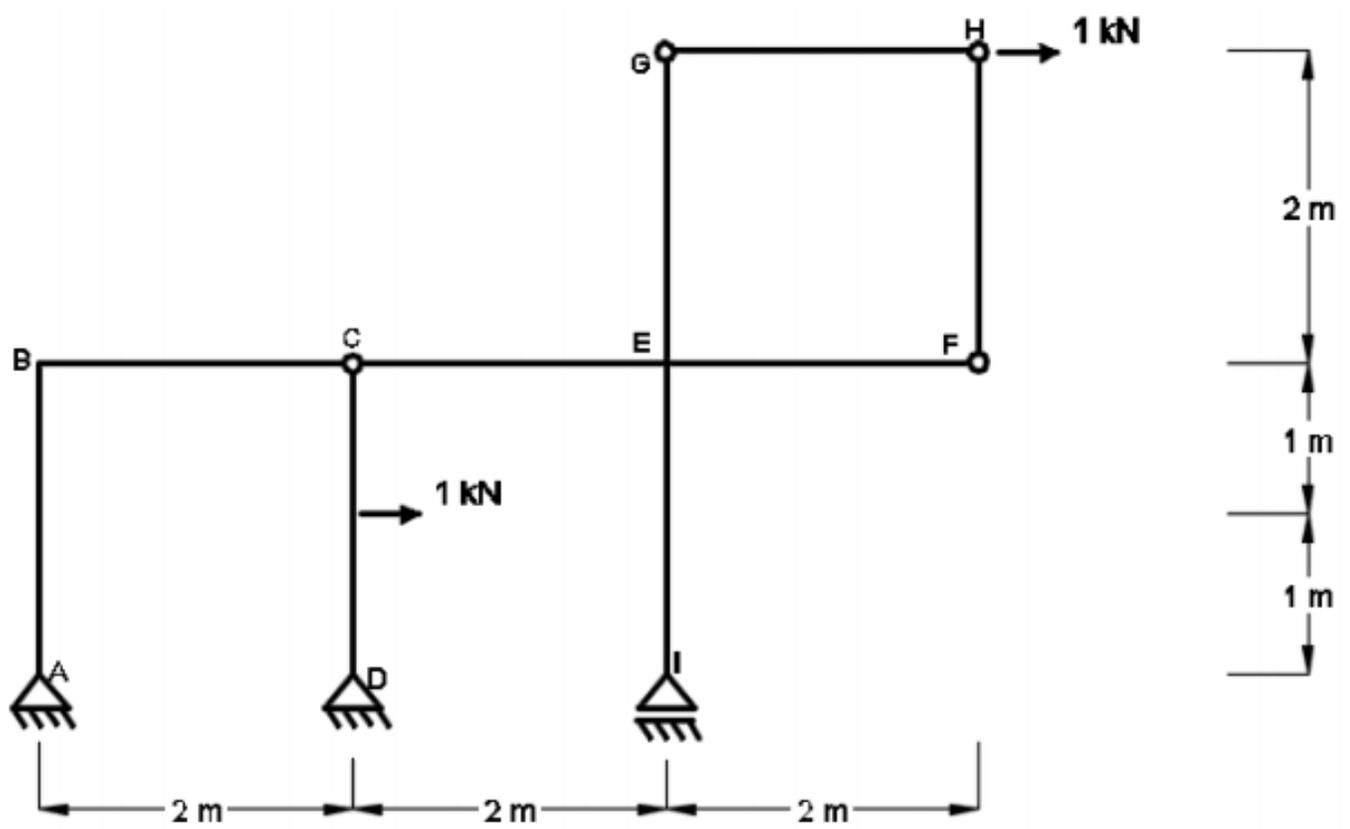
Para a estrutura associada, com forças concentradas na horizontal aplicadas em H e no ponto médio da barra CD no sentido indicado, conforme desenho, obtenha os diagramas de esforços normal e de momento fletor para toda a estrutura.



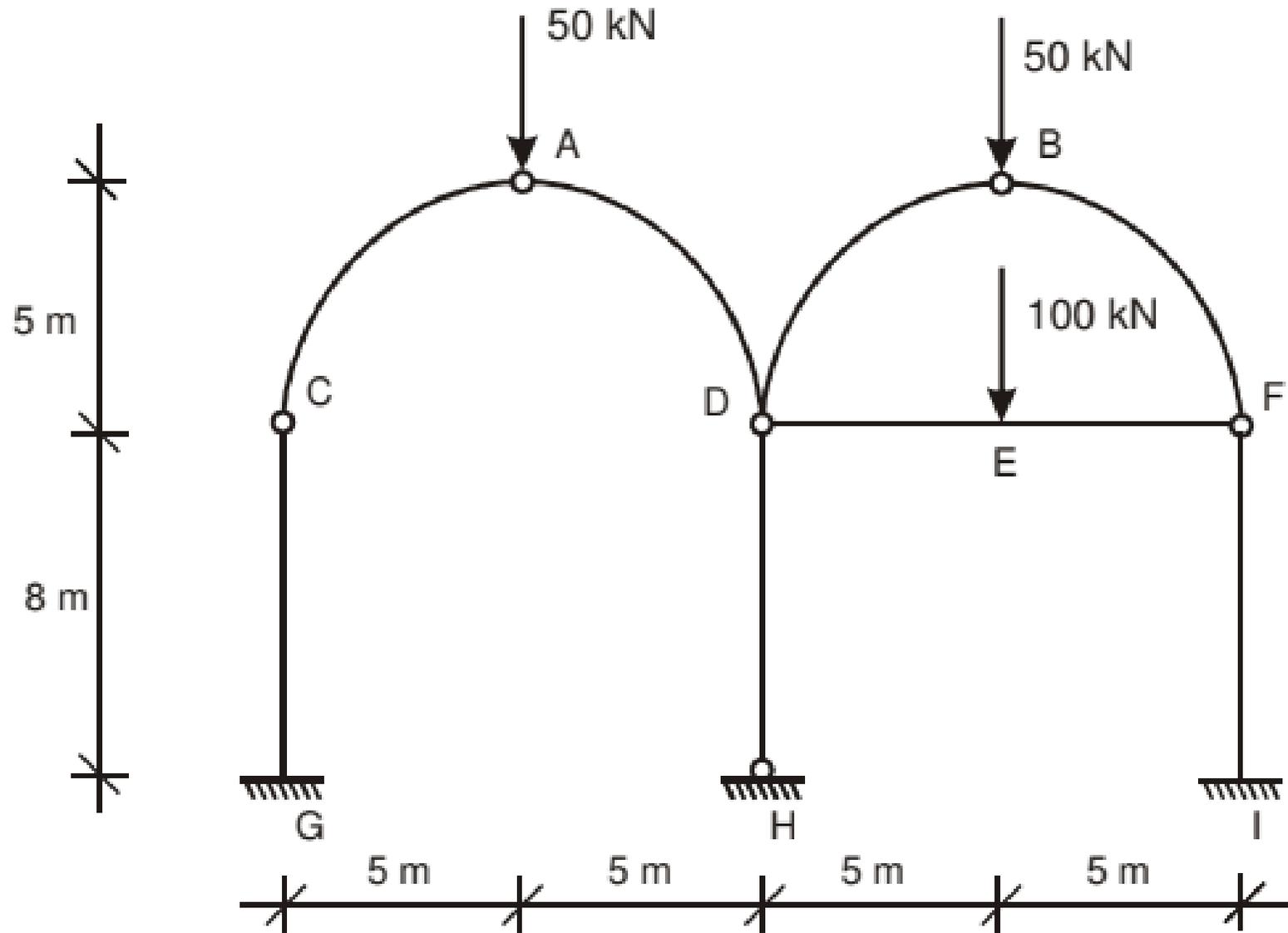
Dividir em suas subestruturas



Dividir em suas subestruturas

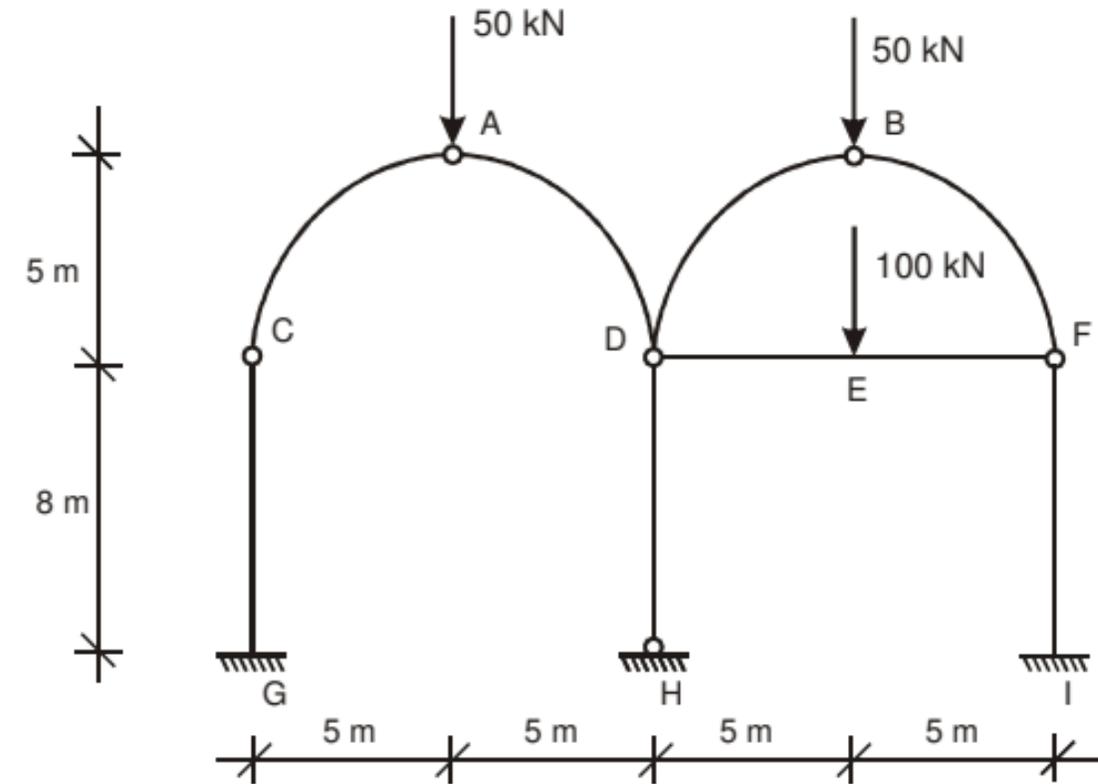


Exemplo 5 - Estruturas Associadas



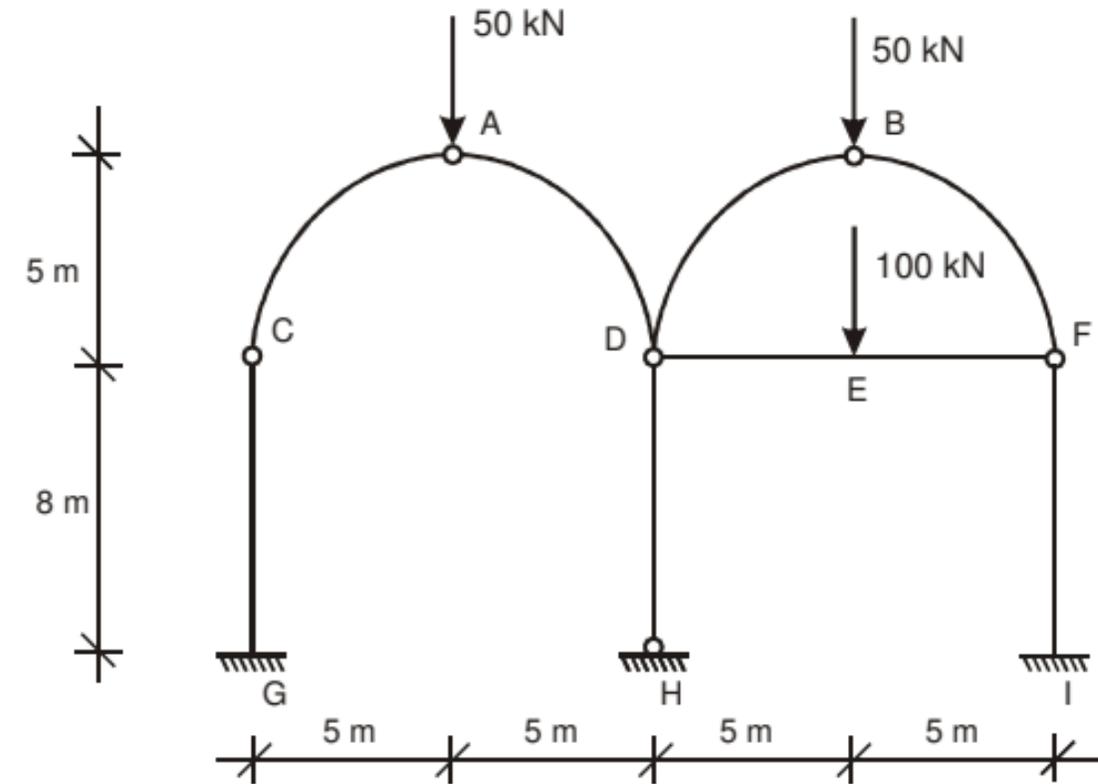
Exemplo 5 - Estruturas Associadas

Dividir em suas subestruturas

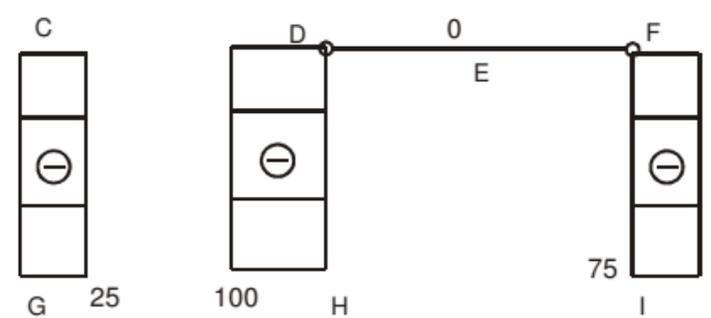
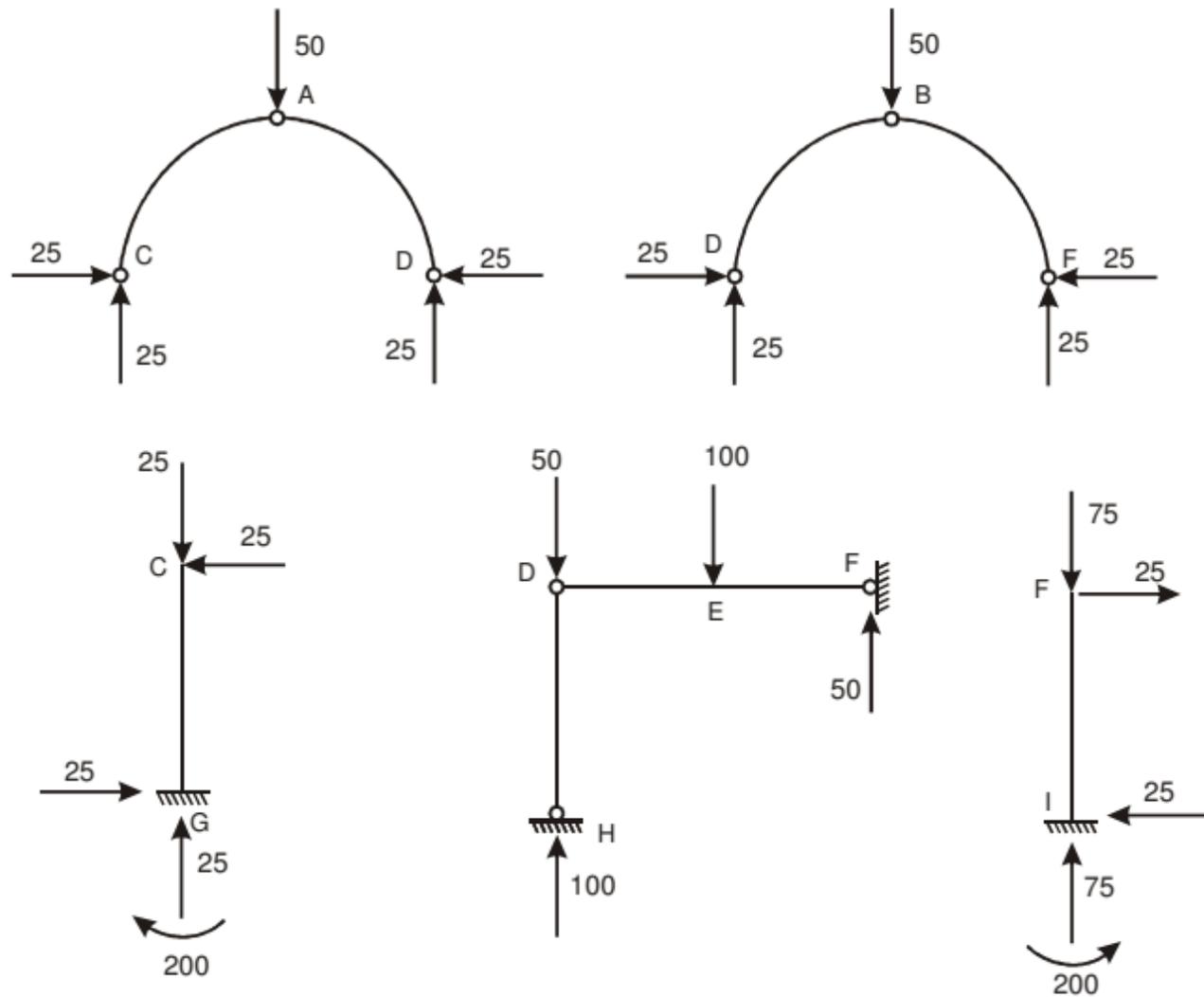


Exemplo 5 - Estruturas Associadas

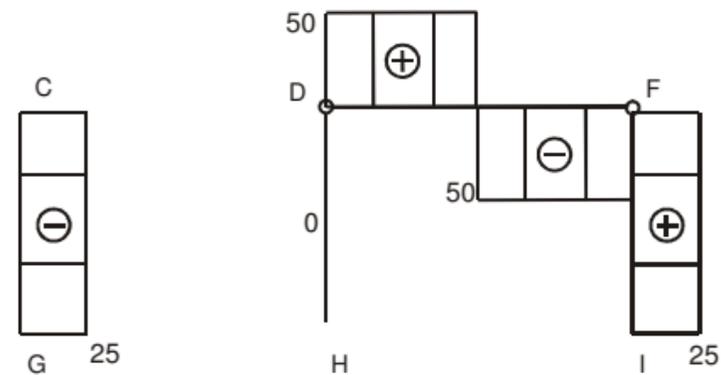
Dividir em suas subestruturas



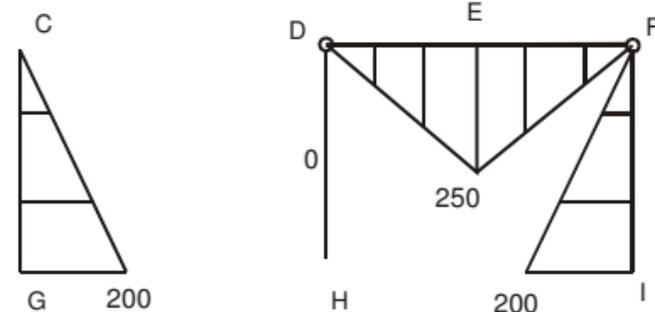
Exemplo 5 - Estruturas Associadas



N (kN)



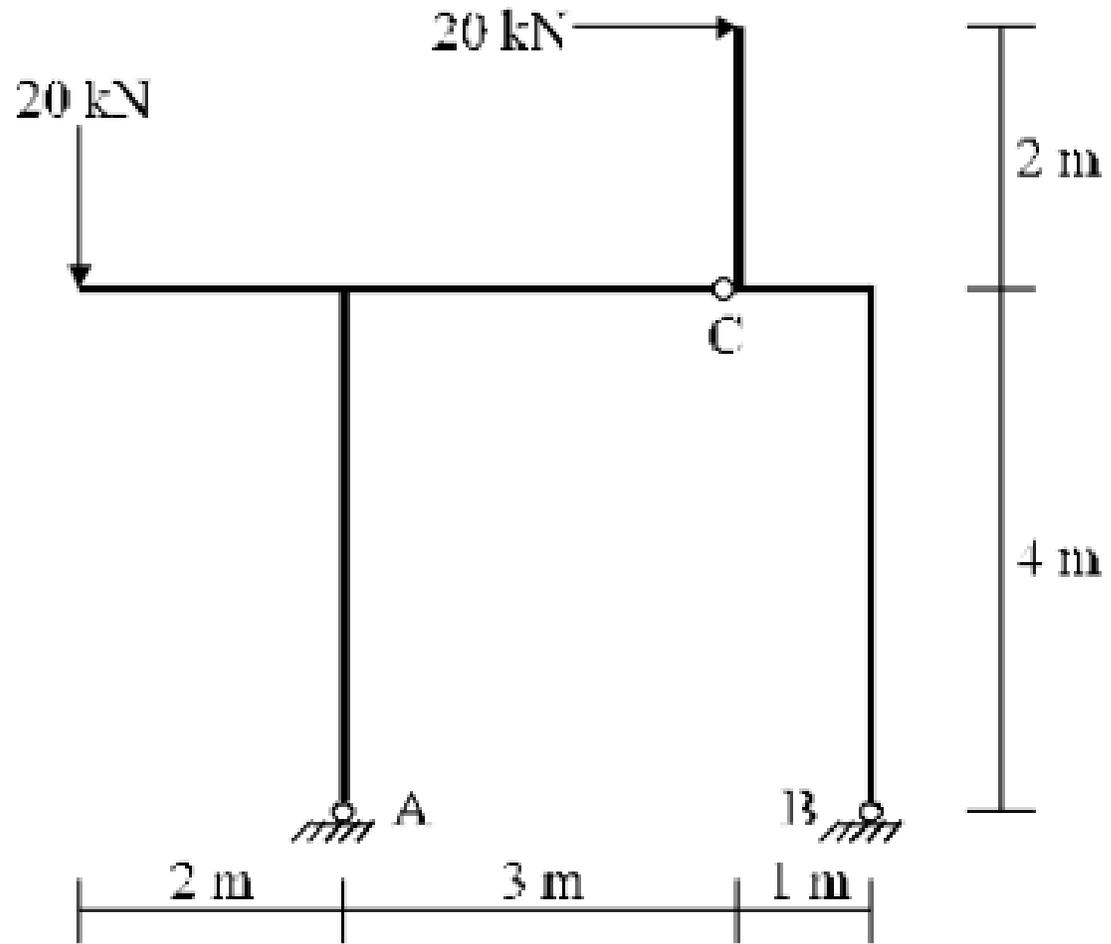
V (kN)



M (kN.m)

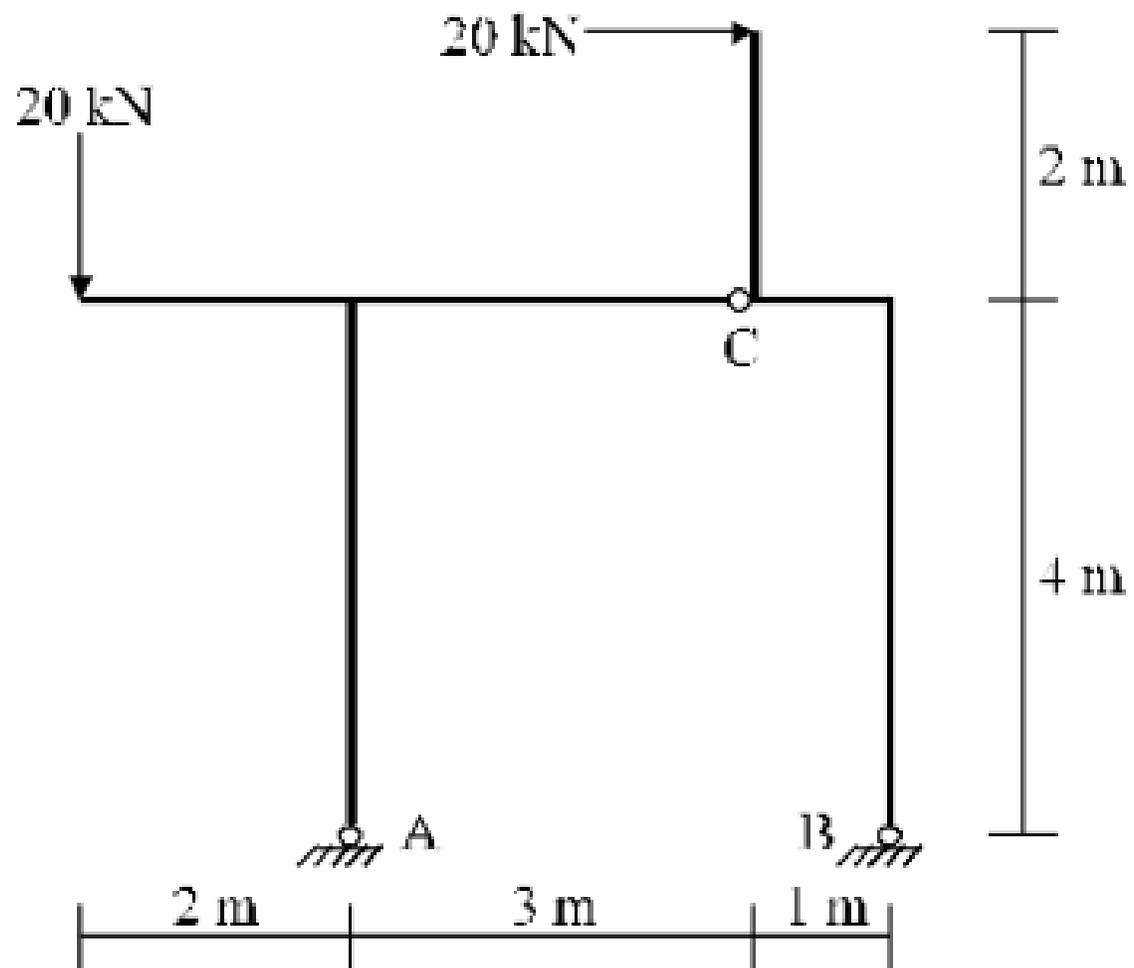
Exemplo 6 - Estruturas Associadas

Dividir em suas subestruturas



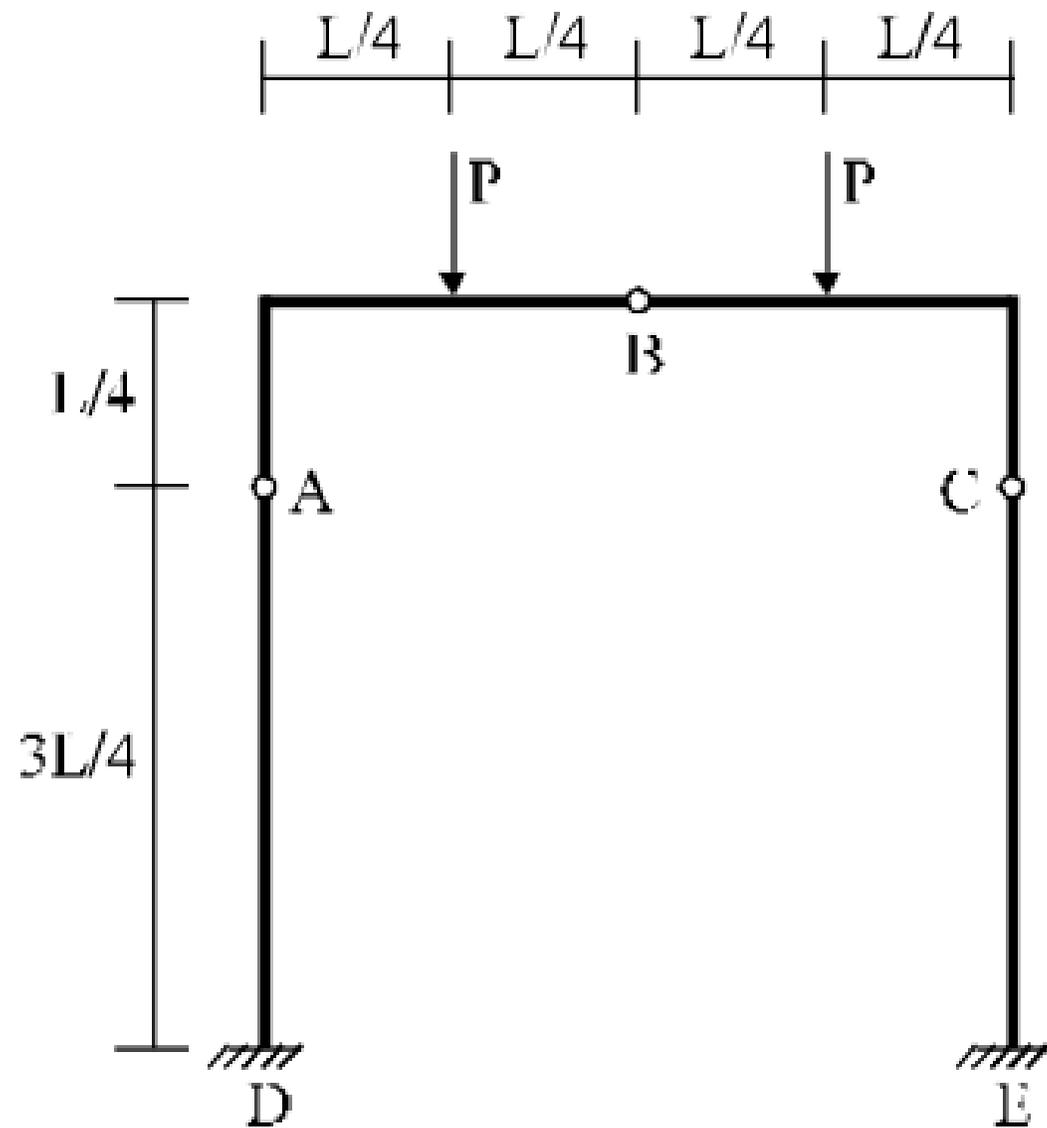
Exemplo 6 - Estruturas Associadas

Dividir em suas subestruturas



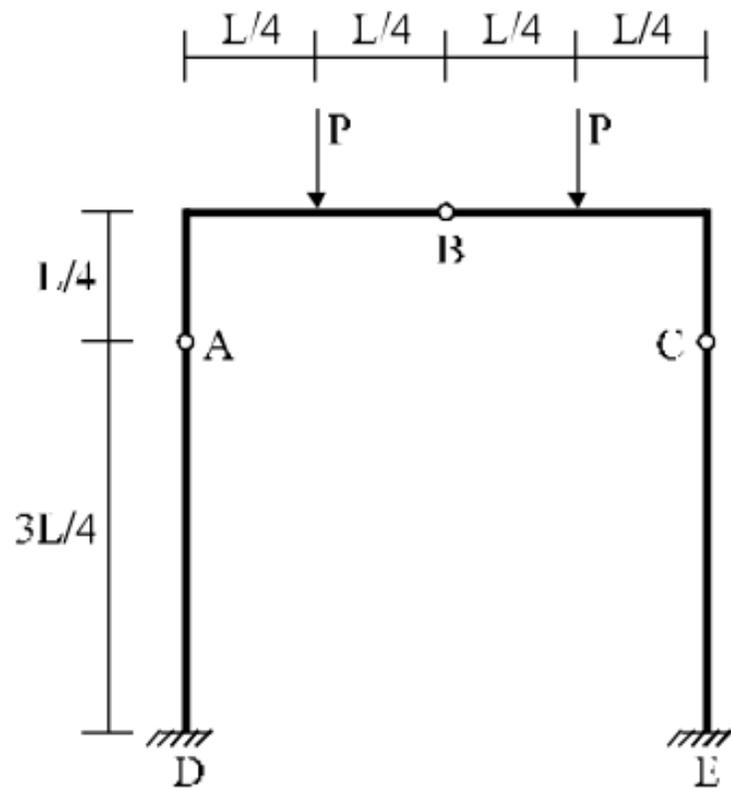
Exemplo 7 - Estruturas Associadas

Dividir em suas subestruturas

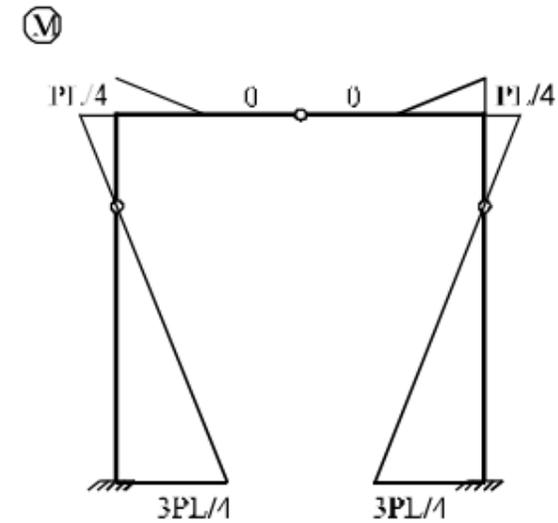
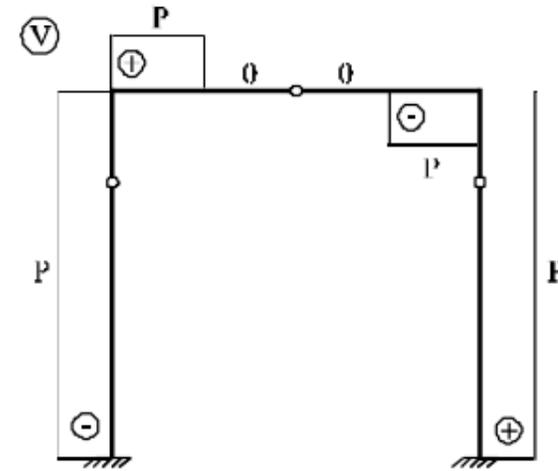
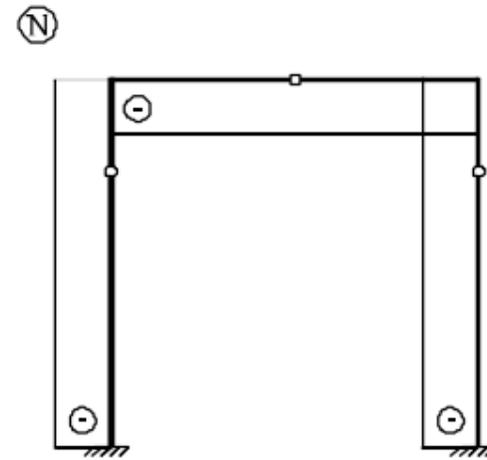
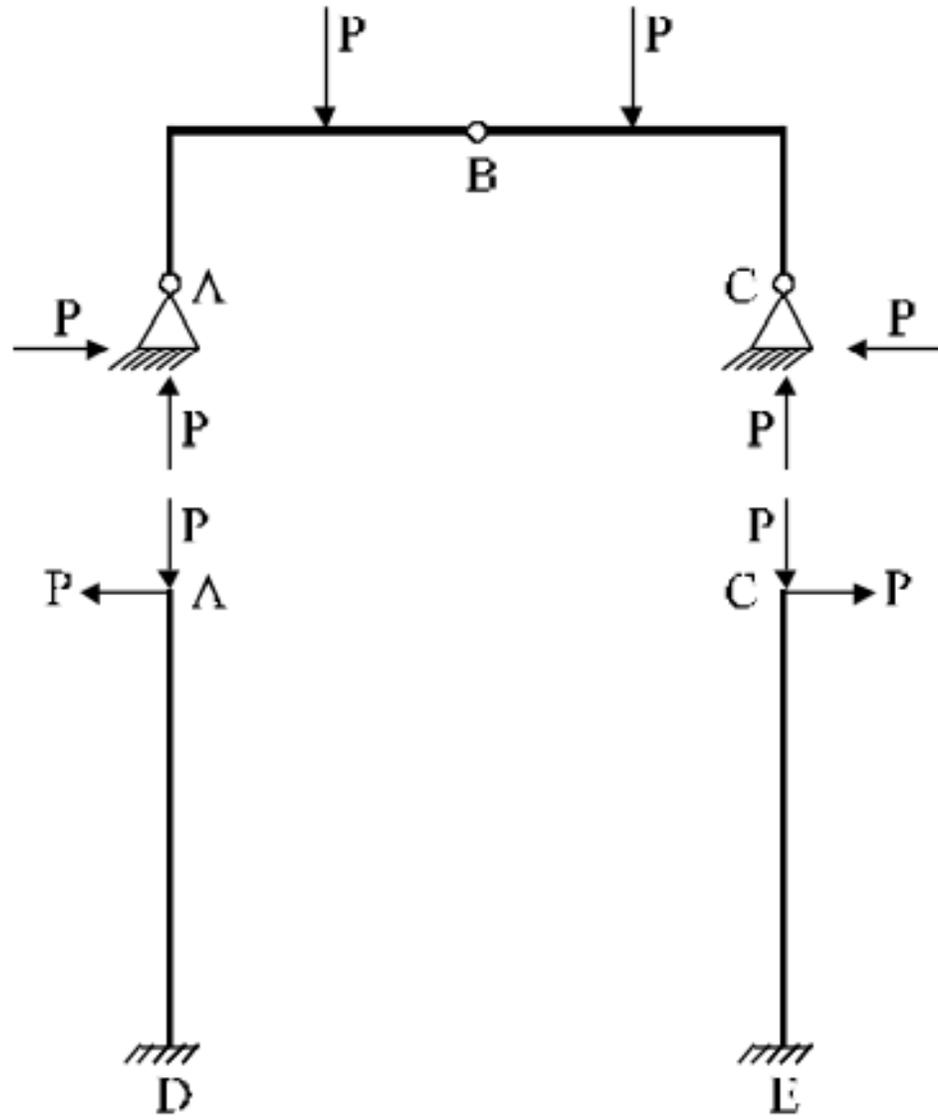


Exemplo 7 - Estruturas Associadas

Dividir em suas subestruturas



Exemplo 7 - Estruturas Associadas



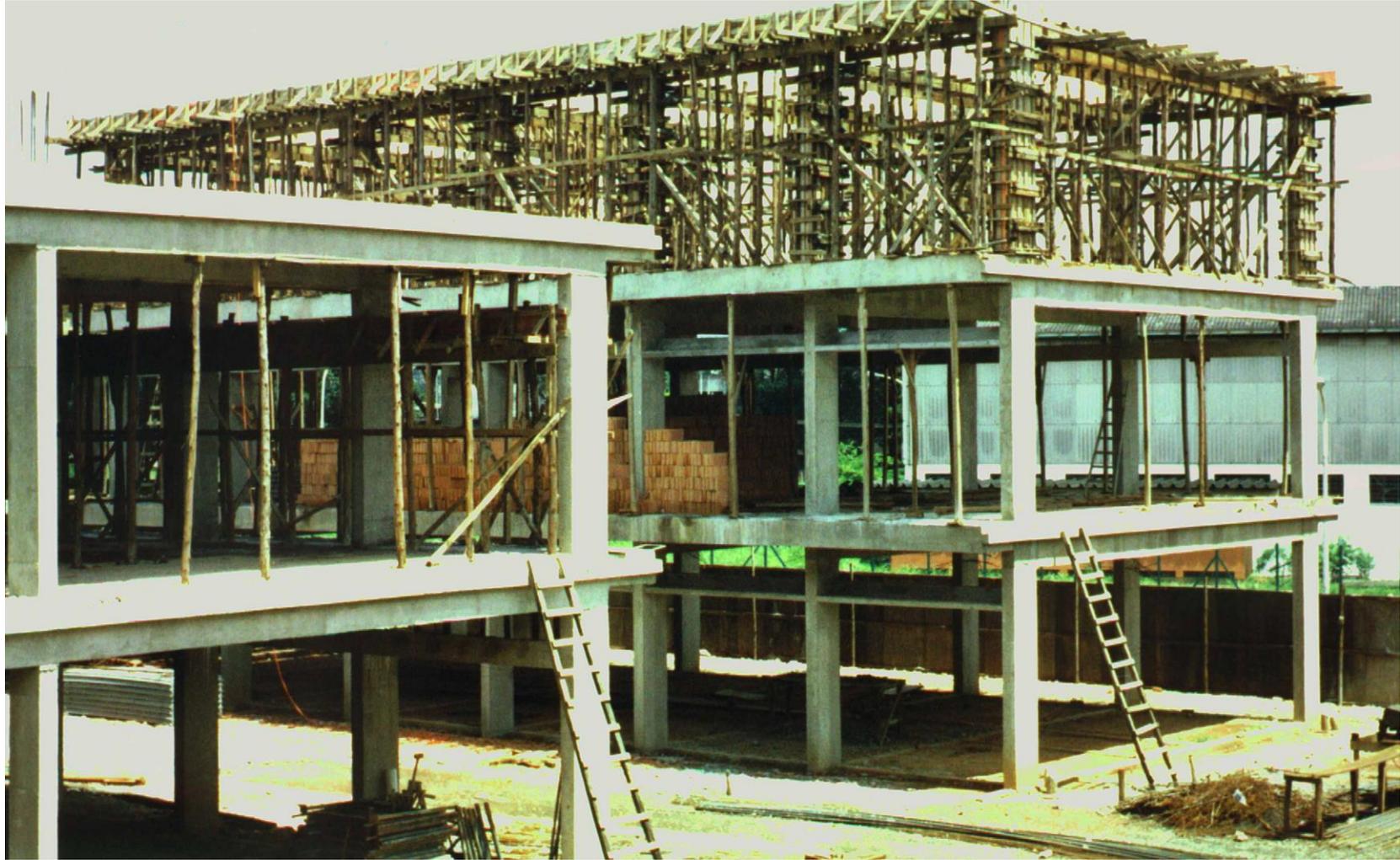


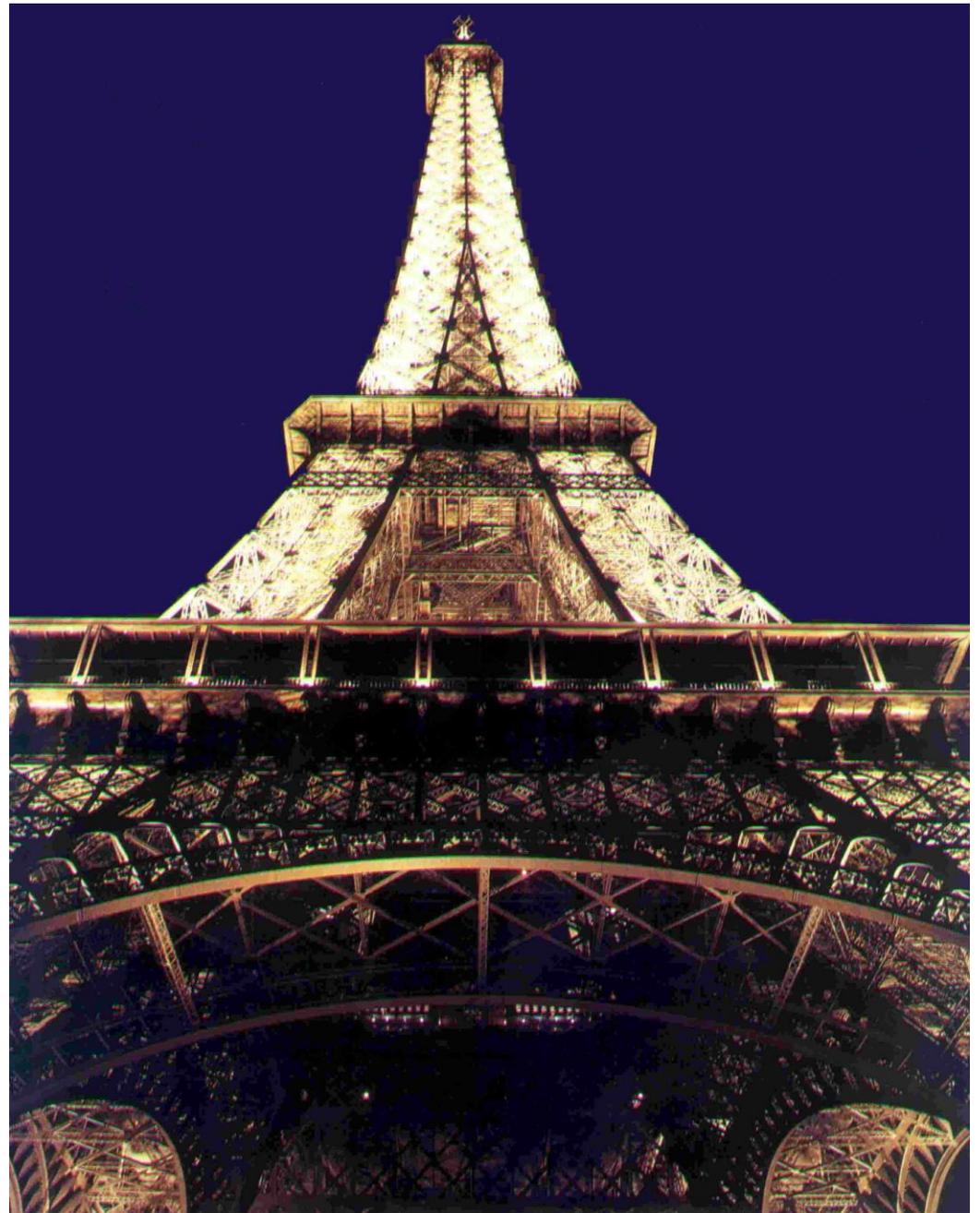














P3 (presencial)

Pórticos/Arcos triarticulados/linhas de pressão

Vigas Gerber

Estruturas Associadas

FIM

OBRIGADO!