



# PEF3200 – Introdução à Mecânica das Estruturas

Aula 13 - 28/06/2023

Estruturas Associadas.

Prof. Martin Paul Schwark

Prof. Osvaldo Shigueru Nakao

Prof. Valério S. Almeida

# O que vimos nas aulas 1 a 12:

- Como é a disciplina, materiais de apoio, programação
- Mecânica dos sólidos deformáveis, o que são estruturas, estão em tudo, modelos físicos e matemáticos, classificações das estruturas, ações que atuam sobre elas e alguns tópicos da mecânica
- Deformadas, movimentos em sistemas materiais, vínculos, estaticidade, estruturas hipostáticas, isostáticas e hiperestáticas, grau de hiperestaticidade, as simplificações adotadas nesta disciplina
- Cálculo de reações de apoio, tensões, esforços solicitantes, o Teorema Fundamental da Resistência dos Materiais
- Diagramas de esforços solicitantes em estruturas planas e espaciais
- Linha de Influência
- Treliças
- Pórticos triarticulados
- Arcos triarticulados
- Vigas Gerber

Nesta aula vamos ver: Estruturas Associadas









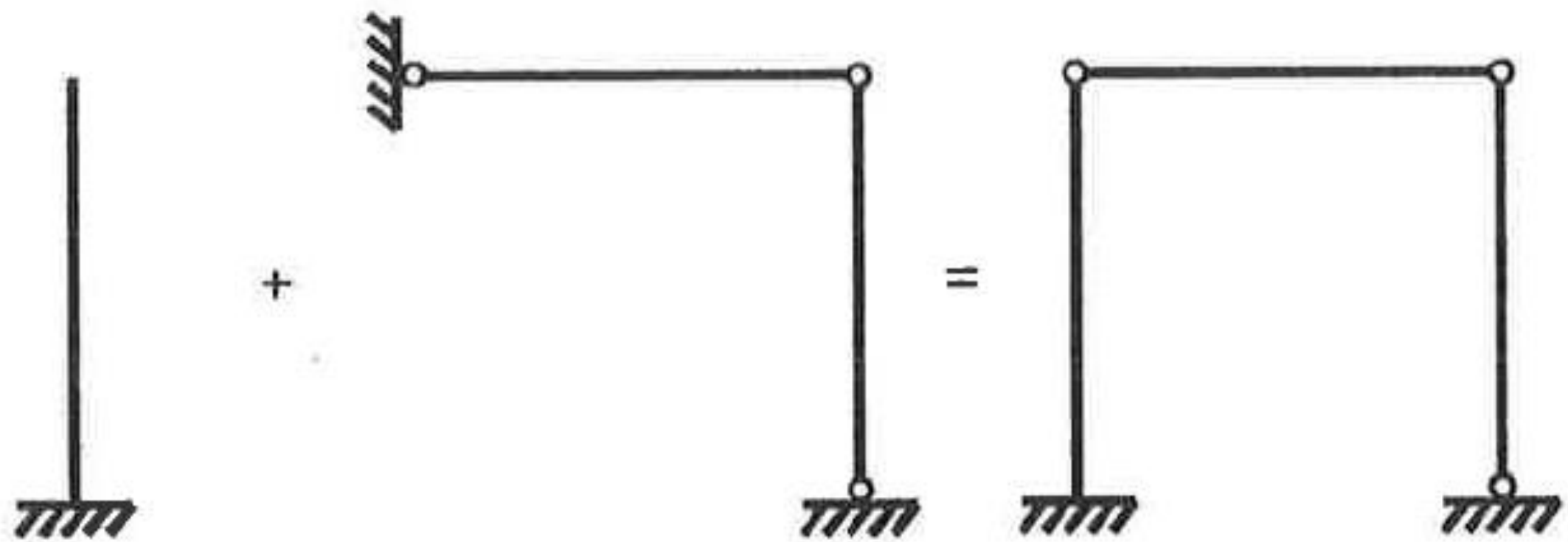
Camboriú - SC

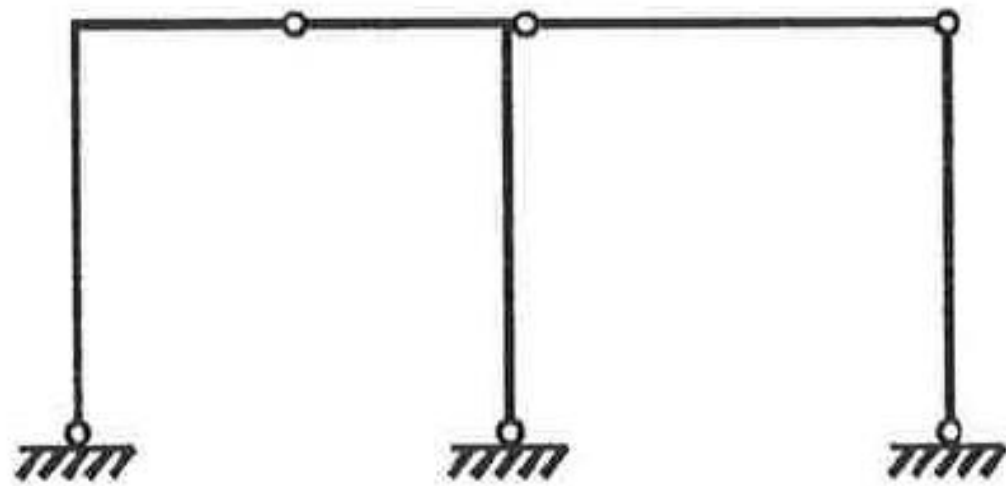
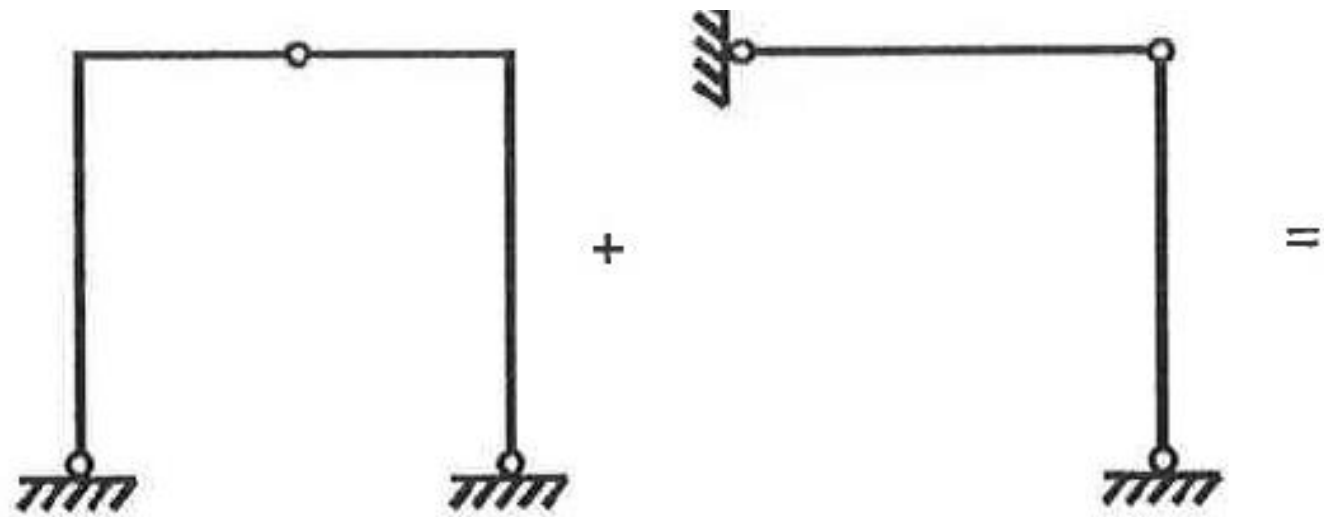


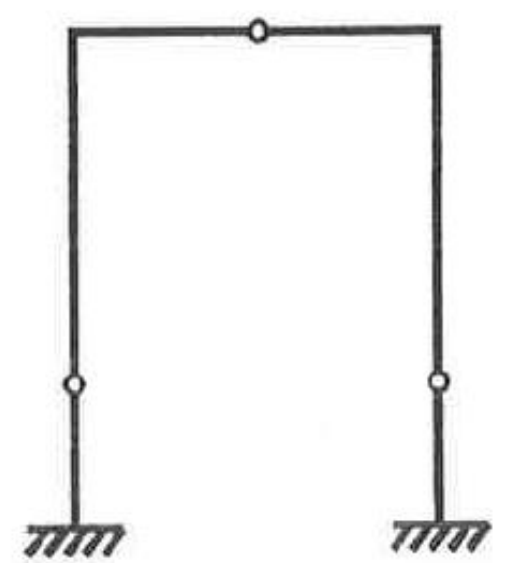
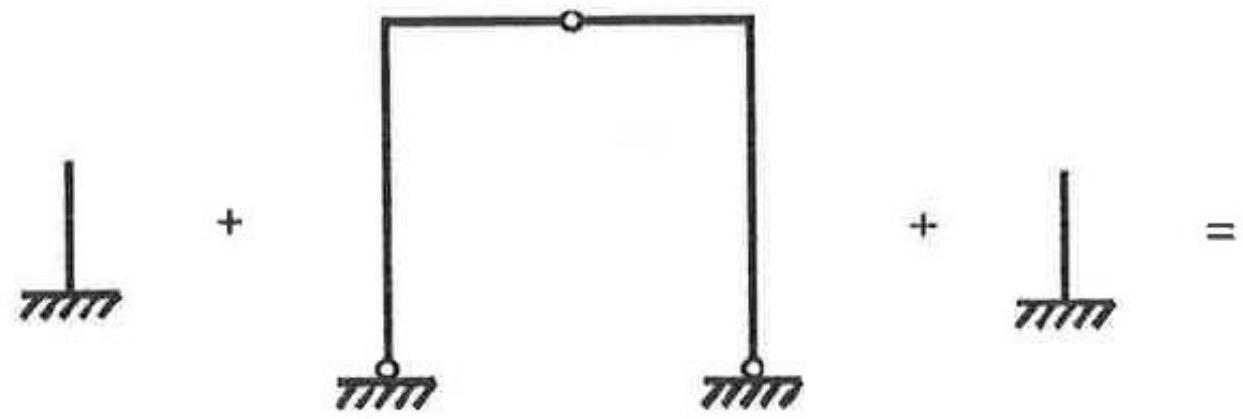
Santiago - Chile

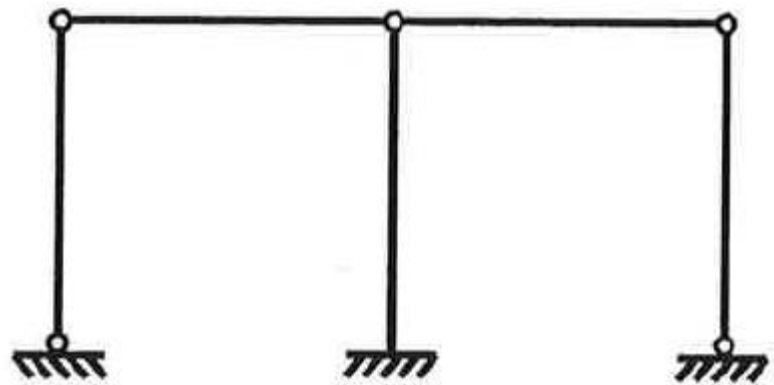
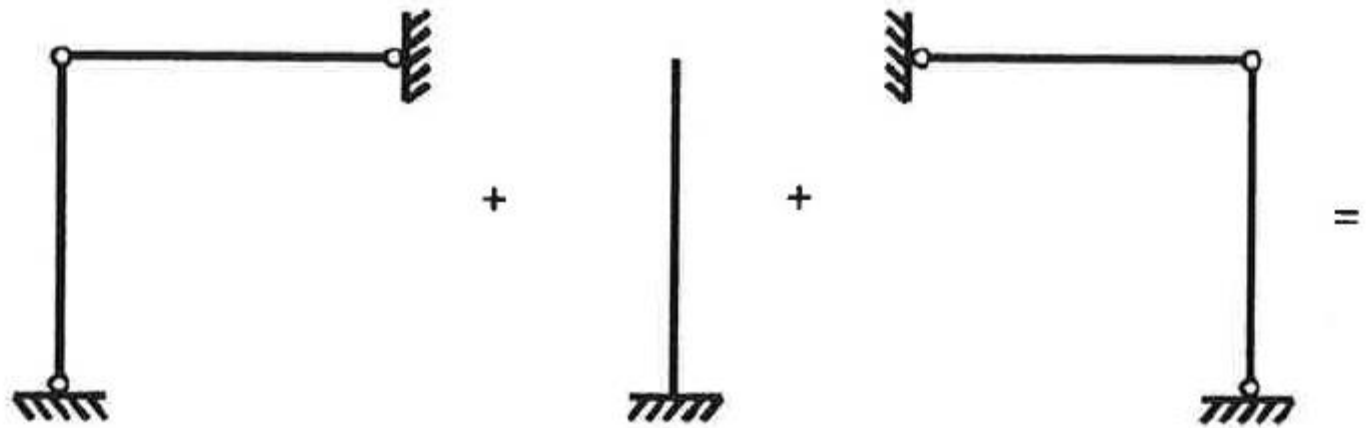


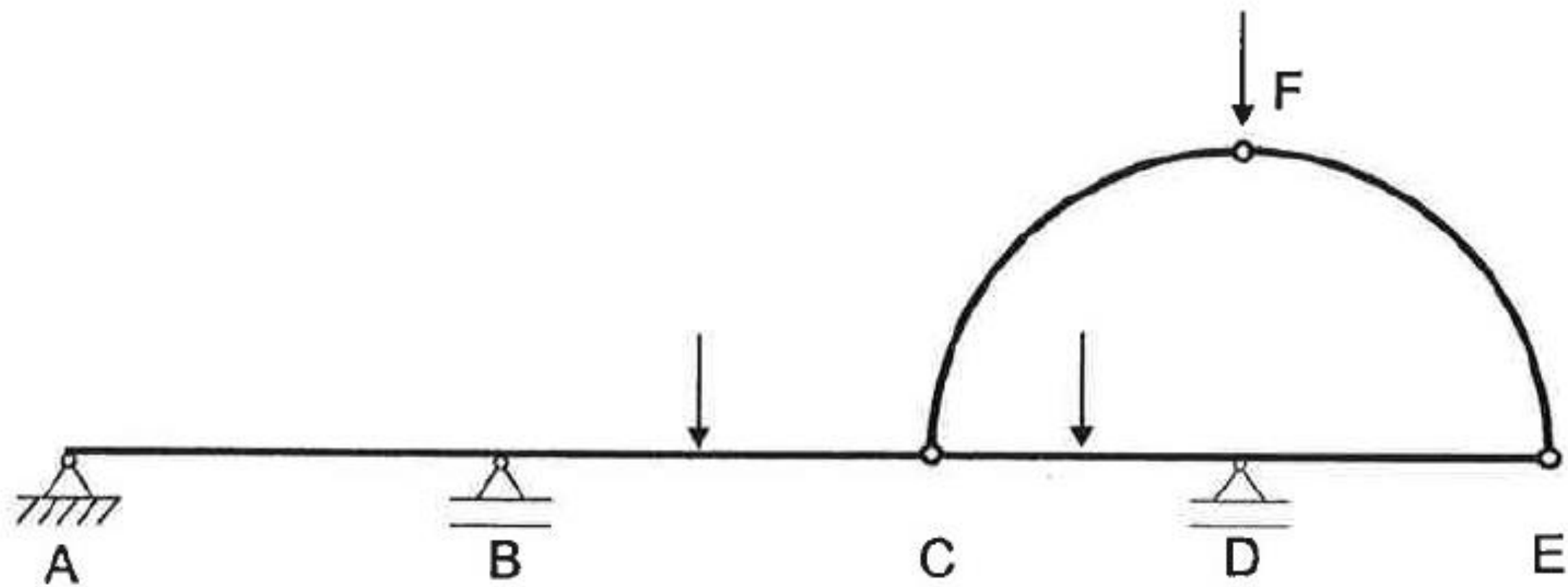


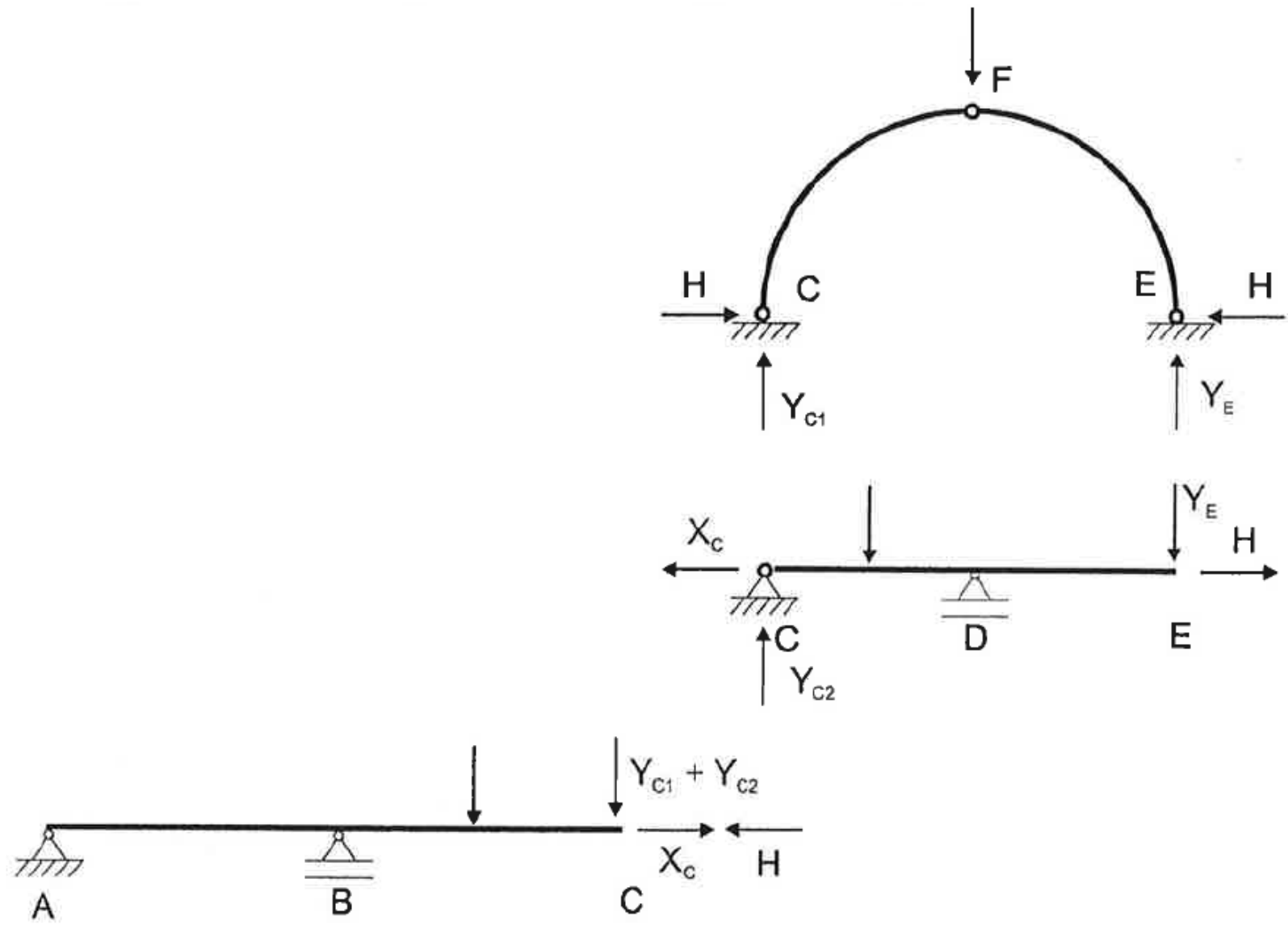


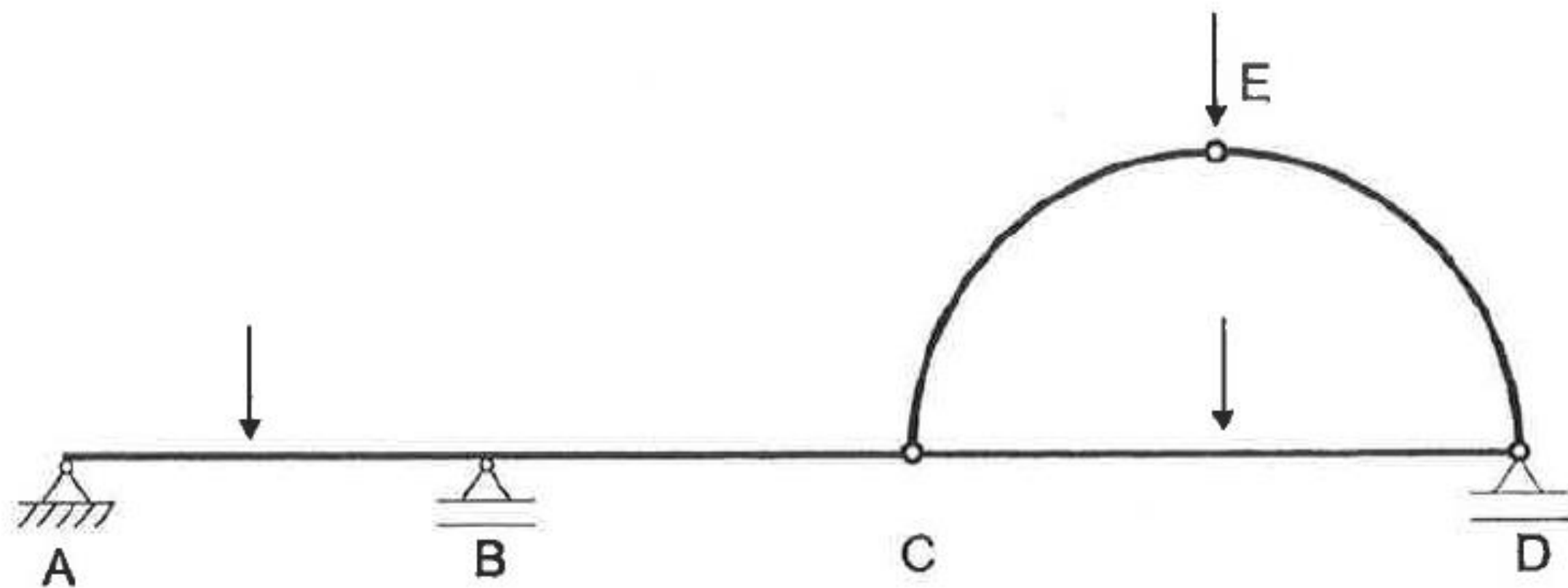


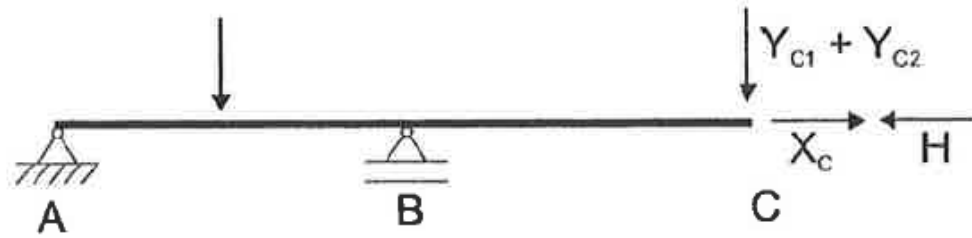
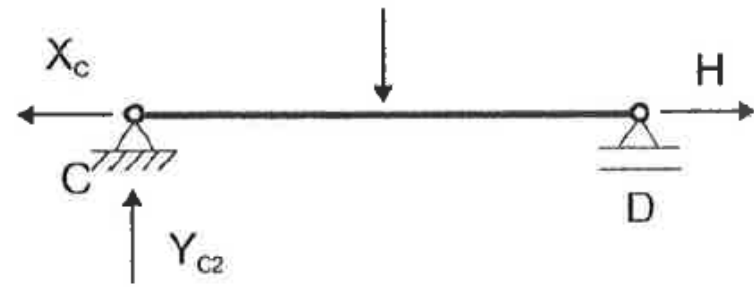
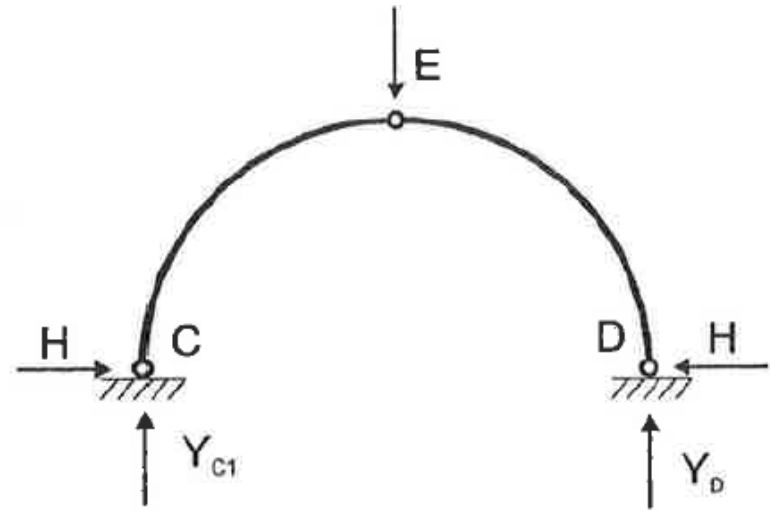




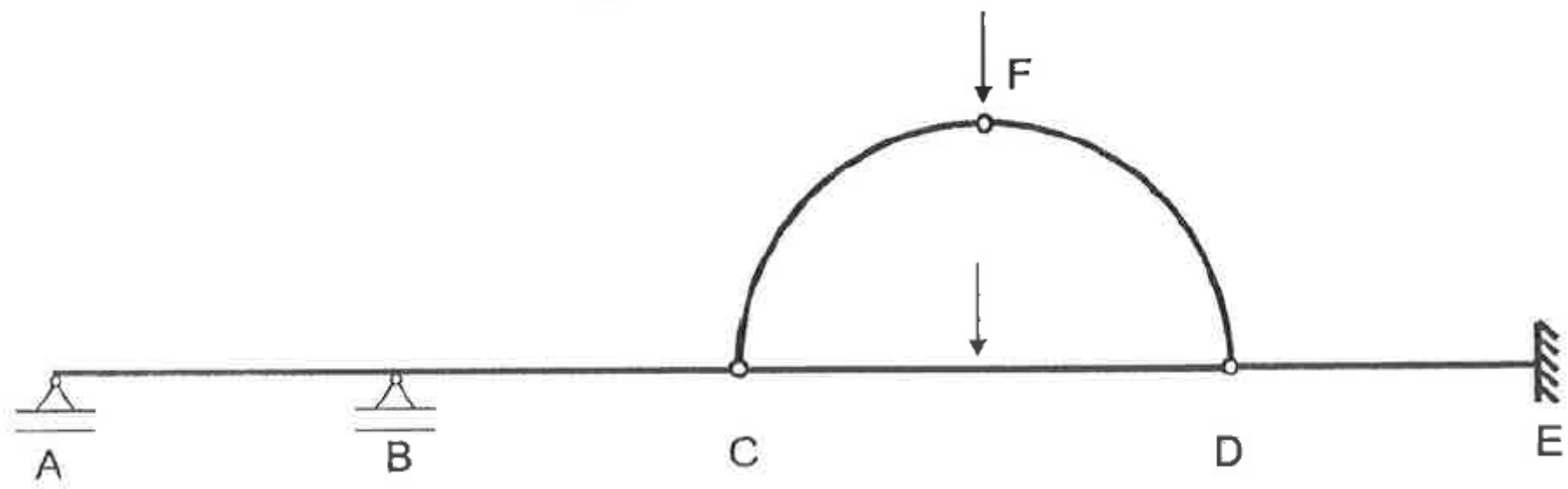


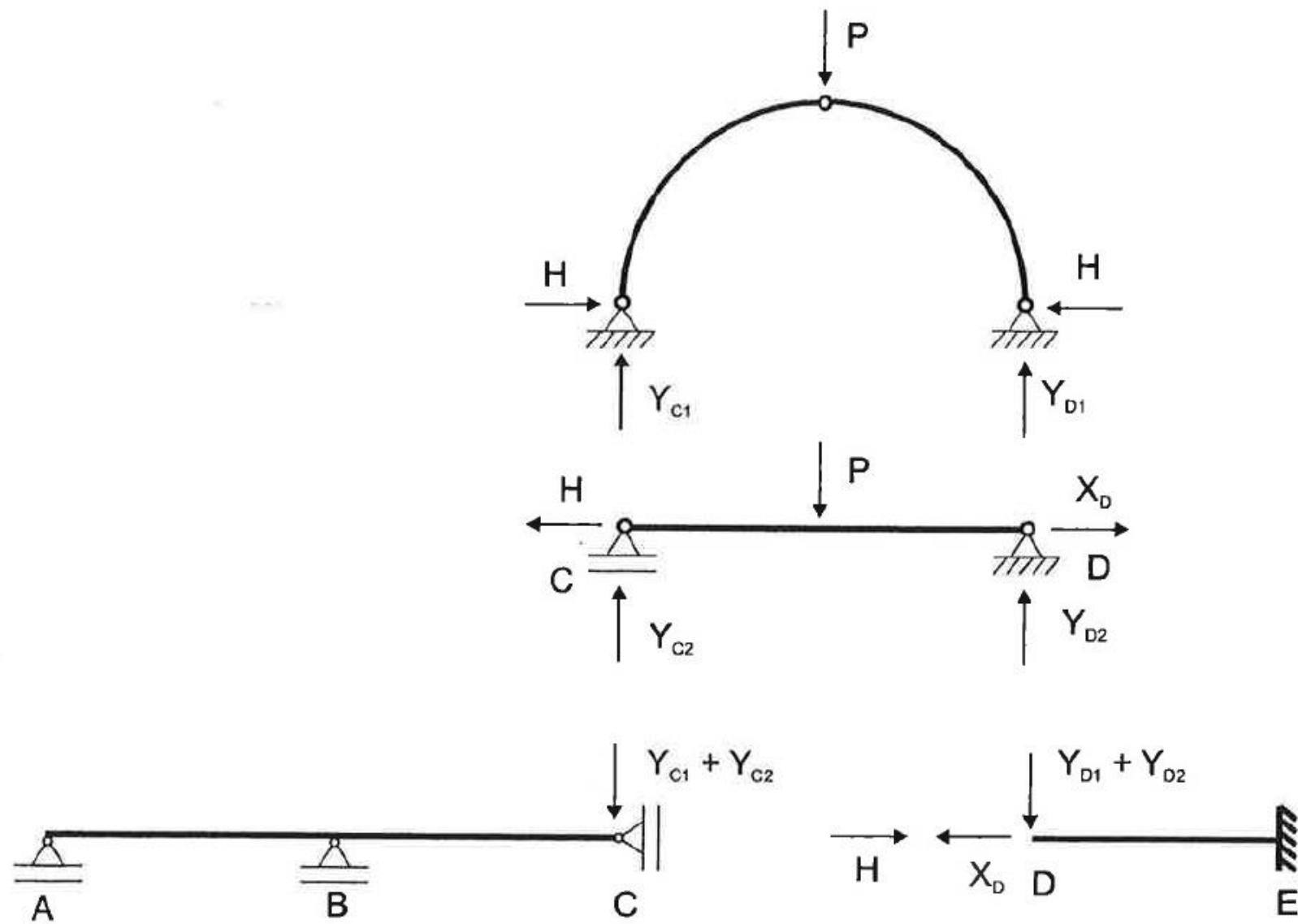




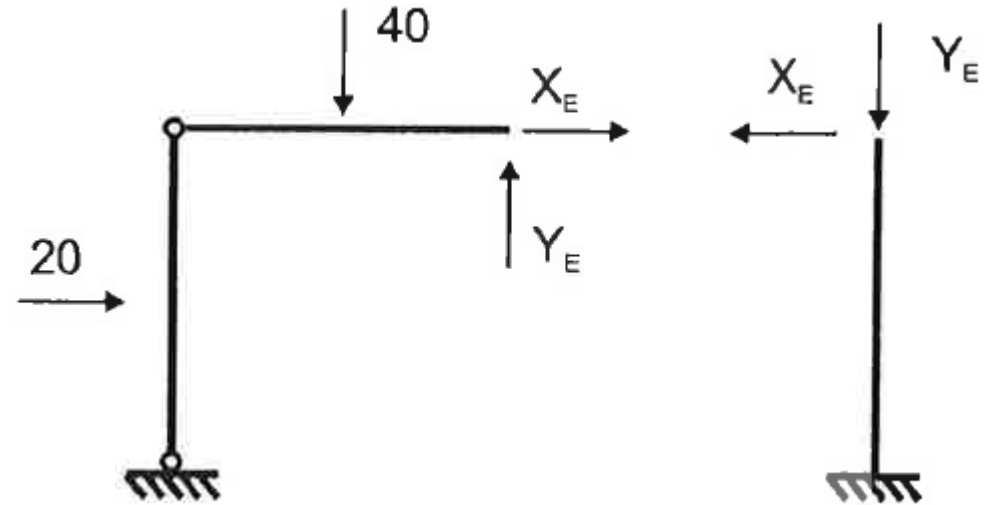
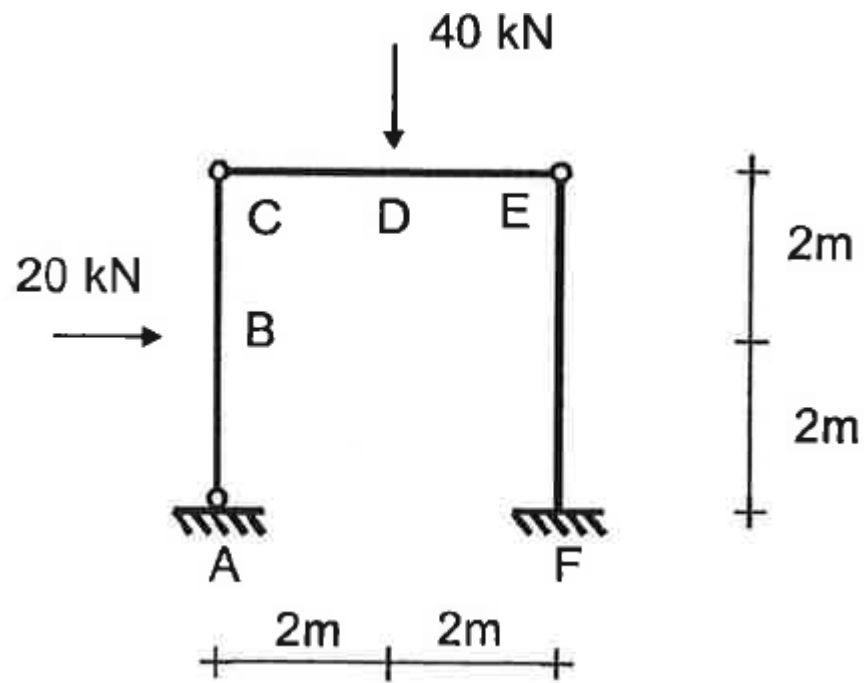


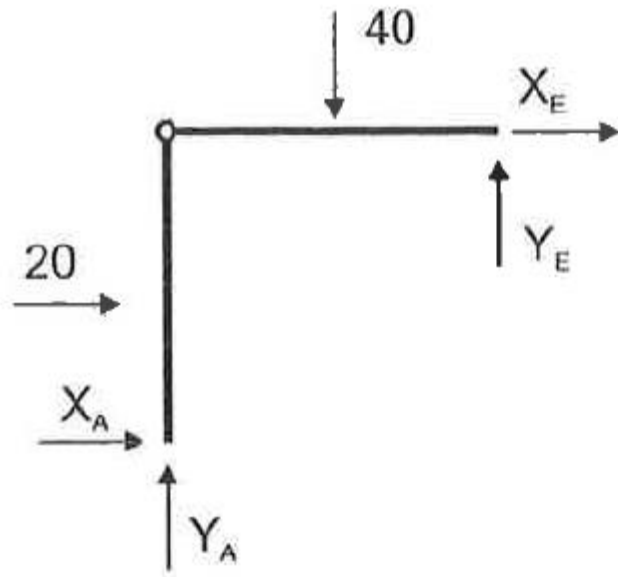




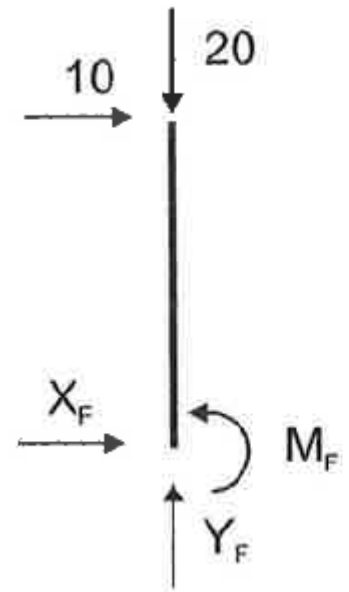


# Exemplo 1

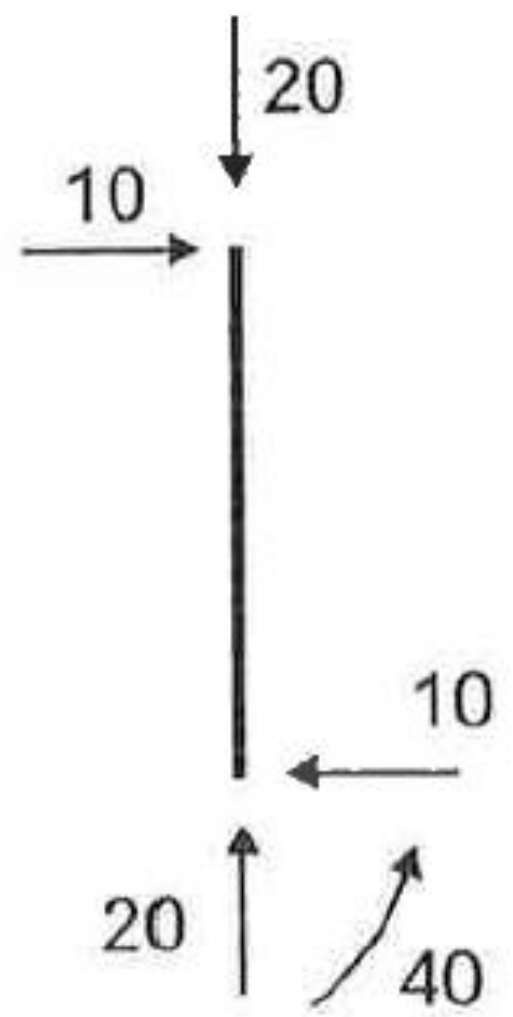
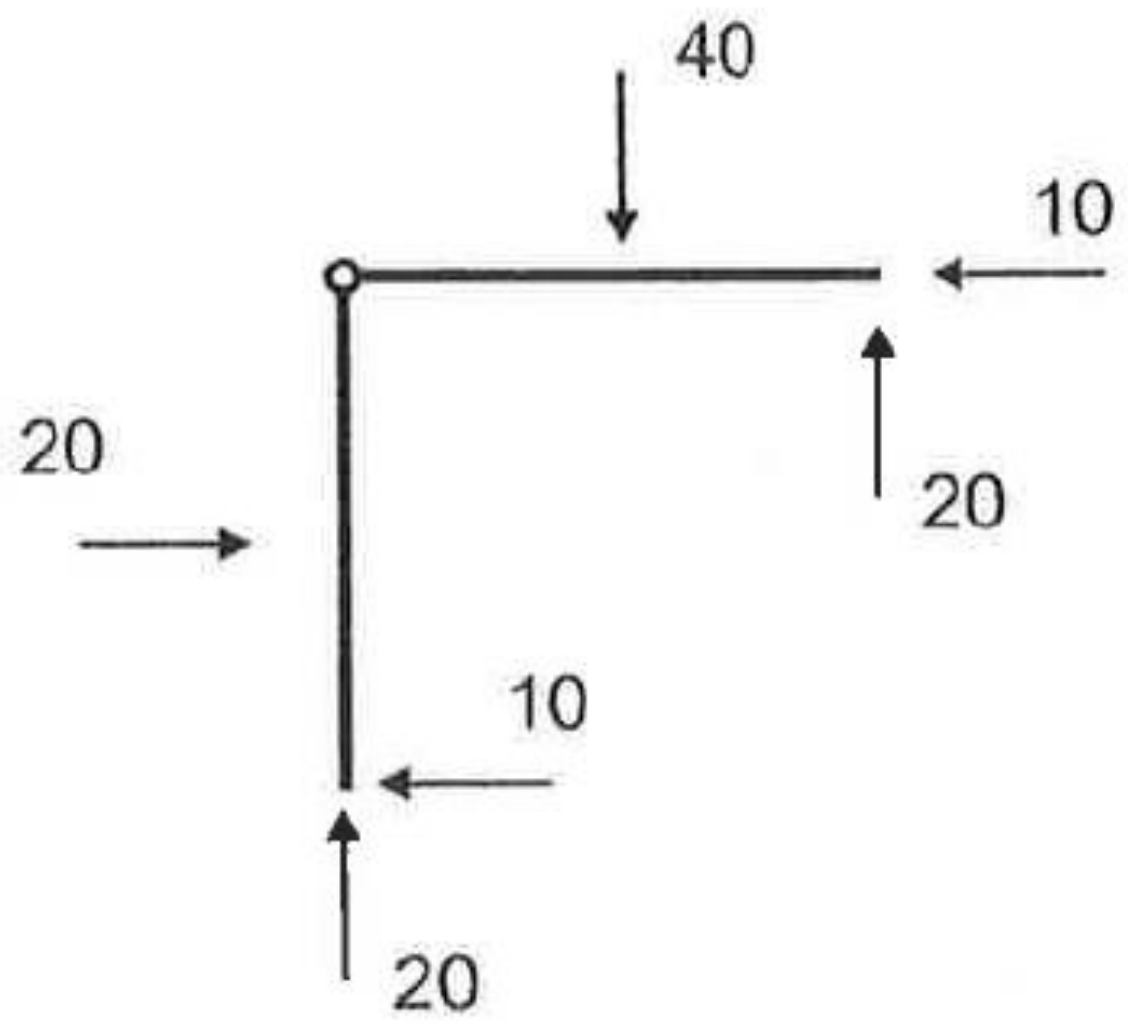


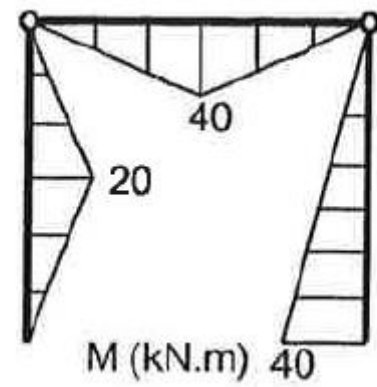
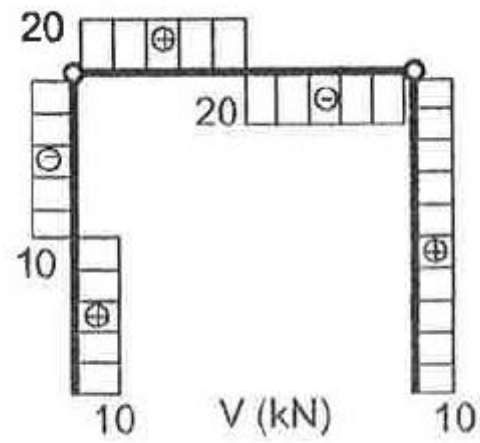
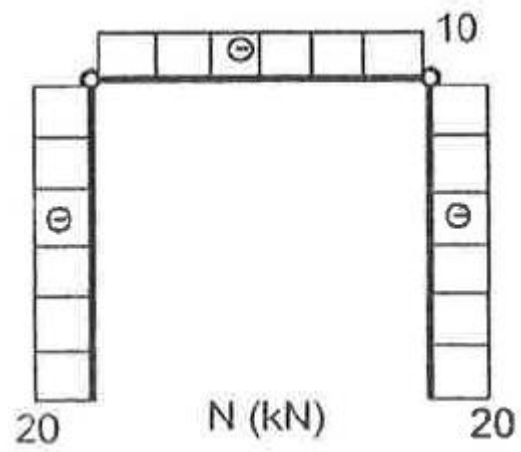


$$\begin{aligned} \sum X = 0 & \quad X_A + X_E + 20 = 0 \Rightarrow X_E = -10 \\ \sum Y = 0 & \quad Y_A + Y_E = 40 \Rightarrow Y_A = 20 \\ \sum M_A = 0 & \quad -20 \cdot 2 - 40 \cdot 2 - X_E \cdot 4 + Y_E \cdot 4 = 0 \Rightarrow Y_E = 20 \\ M_C = 0 & \quad X_A \cdot 4 + 20 \cdot 2 = 0 \Rightarrow X_A = -10 \end{aligned}$$

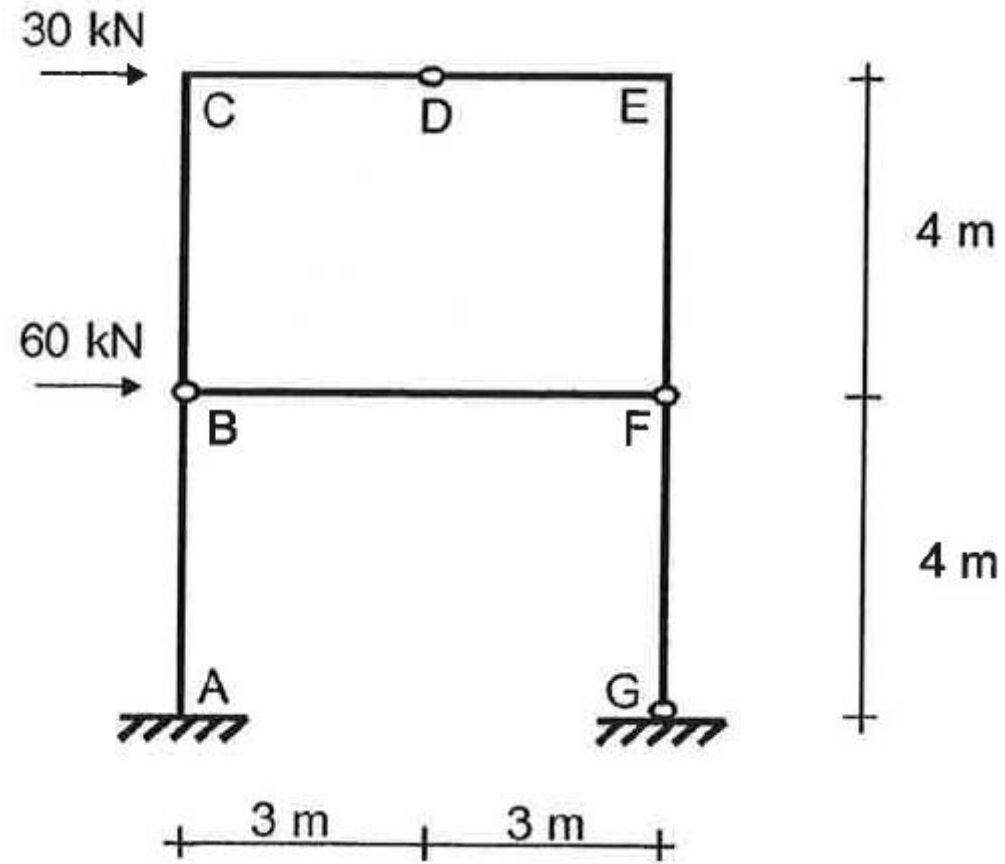


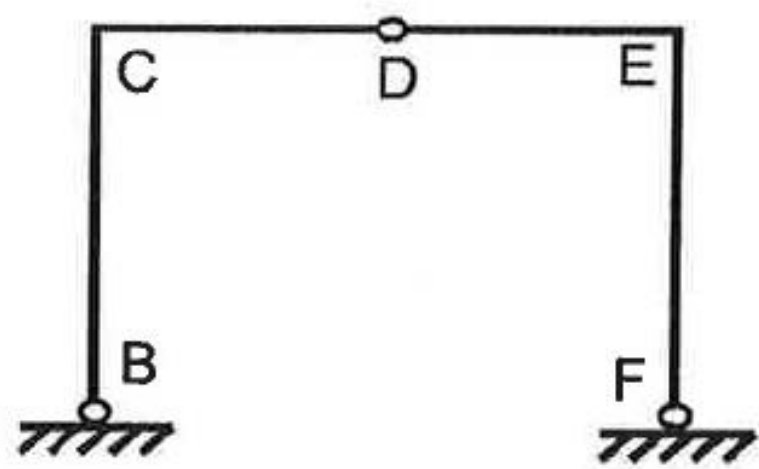
$$\begin{aligned} \sum X = 0 & \quad X_F + 10 = 0 \Rightarrow X_F = -10 \\ \sum Y = 0 & \quad Y_F - 20 = 0 \Rightarrow Y_F = 20 \\ \sum M_F = 0 & \quad M_F - 10 \cdot 4 = 0 \Rightarrow M_F = 40 \end{aligned}$$





# Exemplo 2

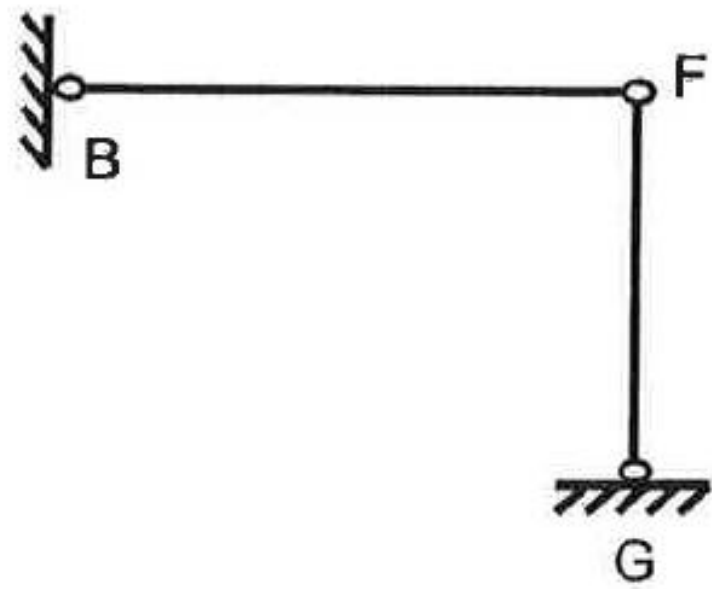




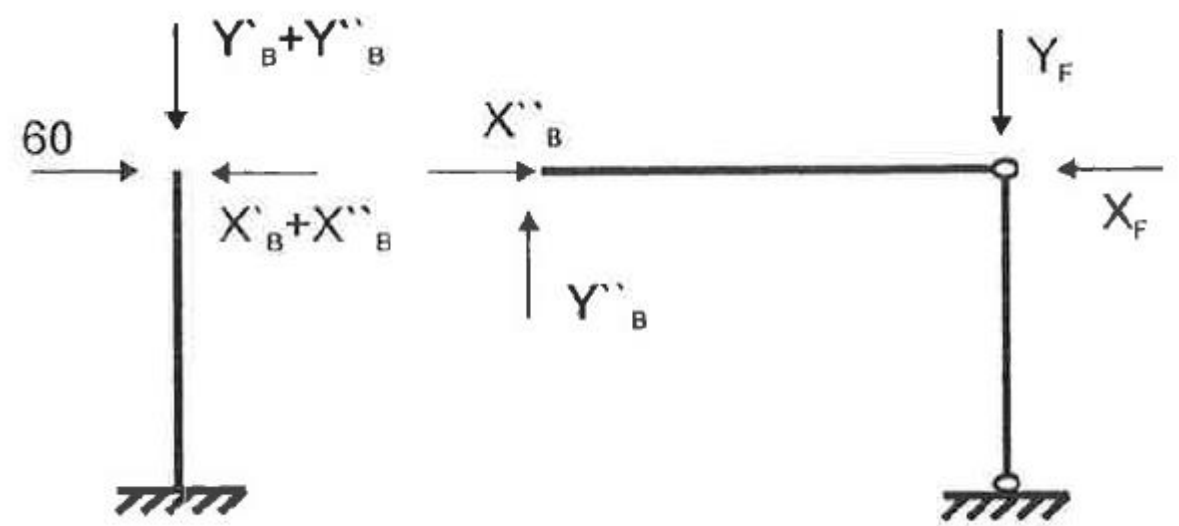
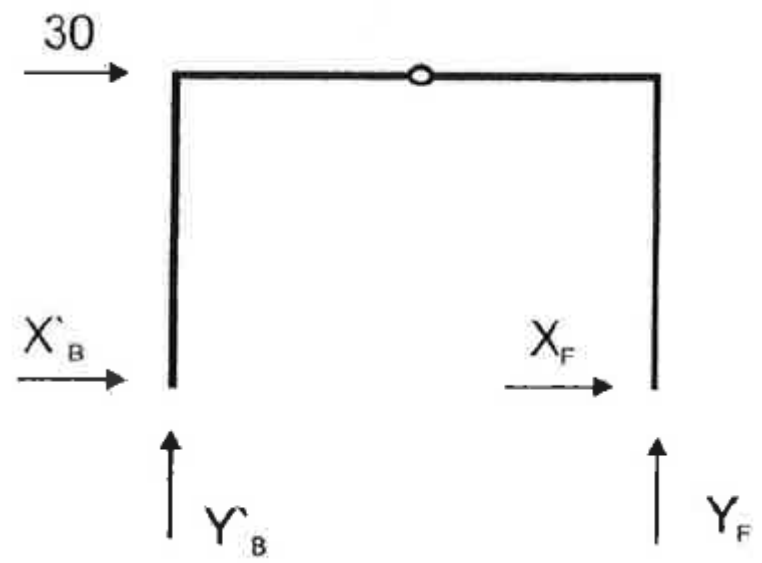
+

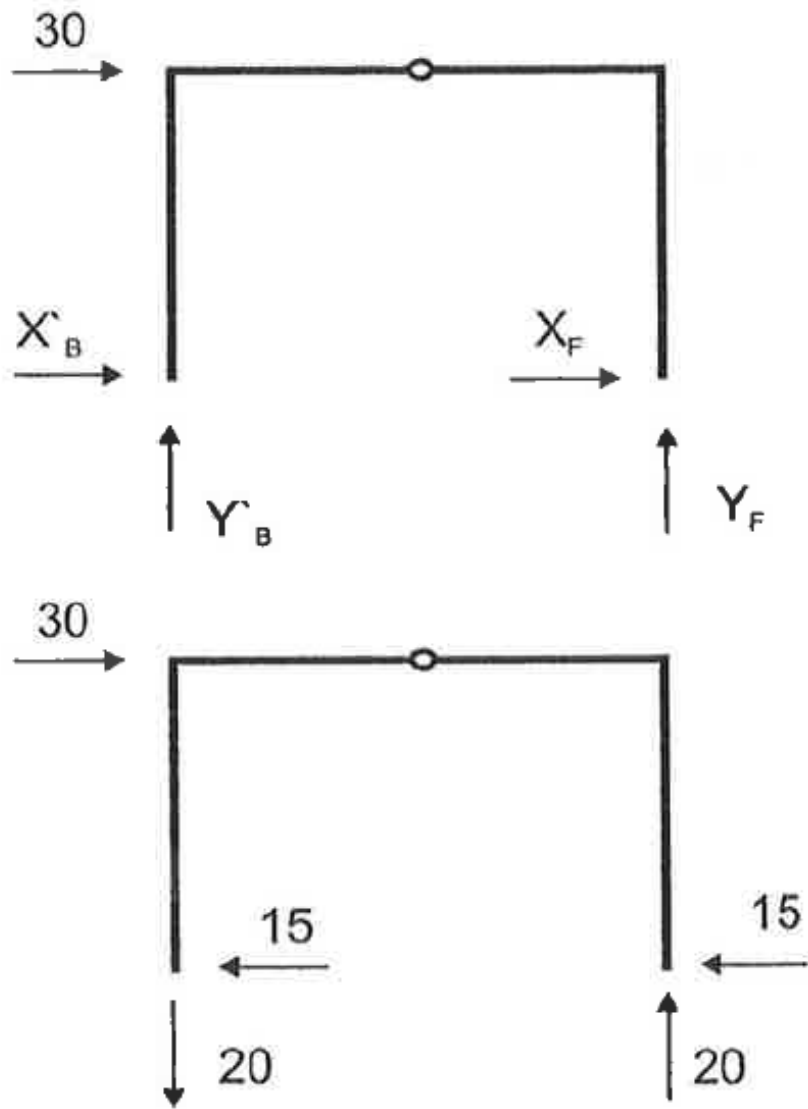


+

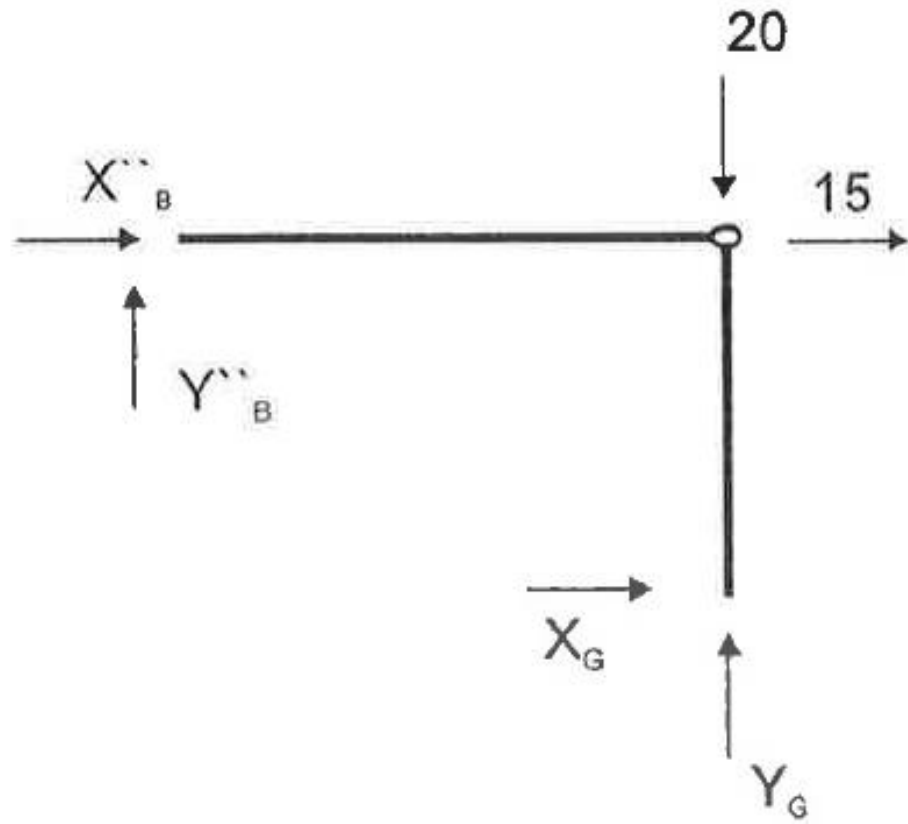








$\Sigma X = 0$	$X'_B + X_F + 30 = 0$	$\Rightarrow X'_B = -15$
$\Sigma Y = 0$	$Y'_B + Y_F = 0$	$\Rightarrow Y'_B = -20$
$\Sigma M_B = 0$	$-30 \cdot 4 + Y_F \cdot 6 = 0$	$\Rightarrow Y_F = 20$
$M_D = 0$	$Y_F \cdot 3 + X_F \cdot 4 = 0$	$\Rightarrow X_F = -15$



$$\sum X = 0$$

$$\sum Y = 0$$

$$\sum M_B = 0$$

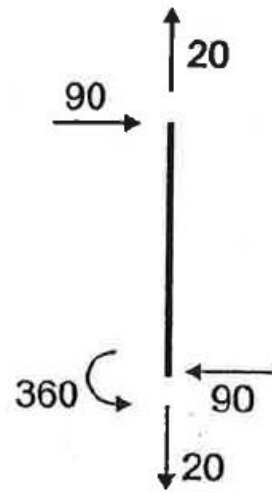
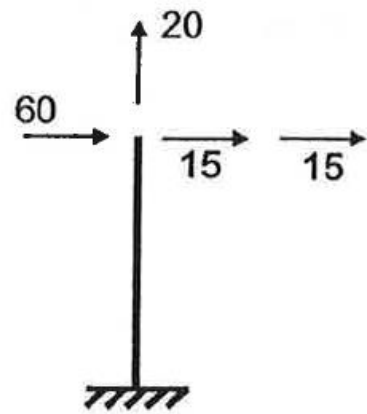
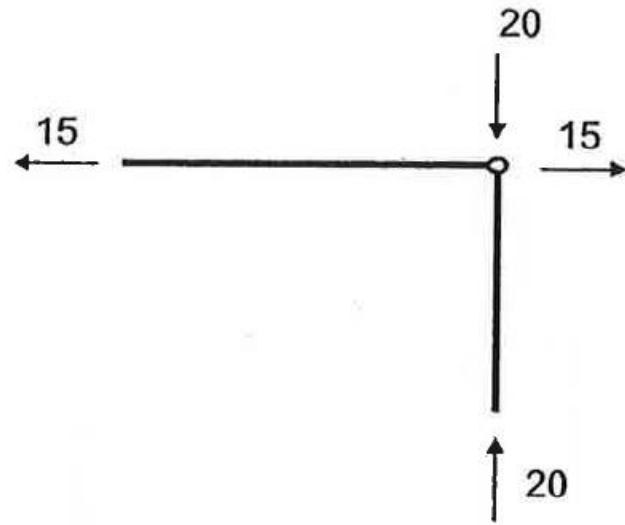
$$M_F = 0$$

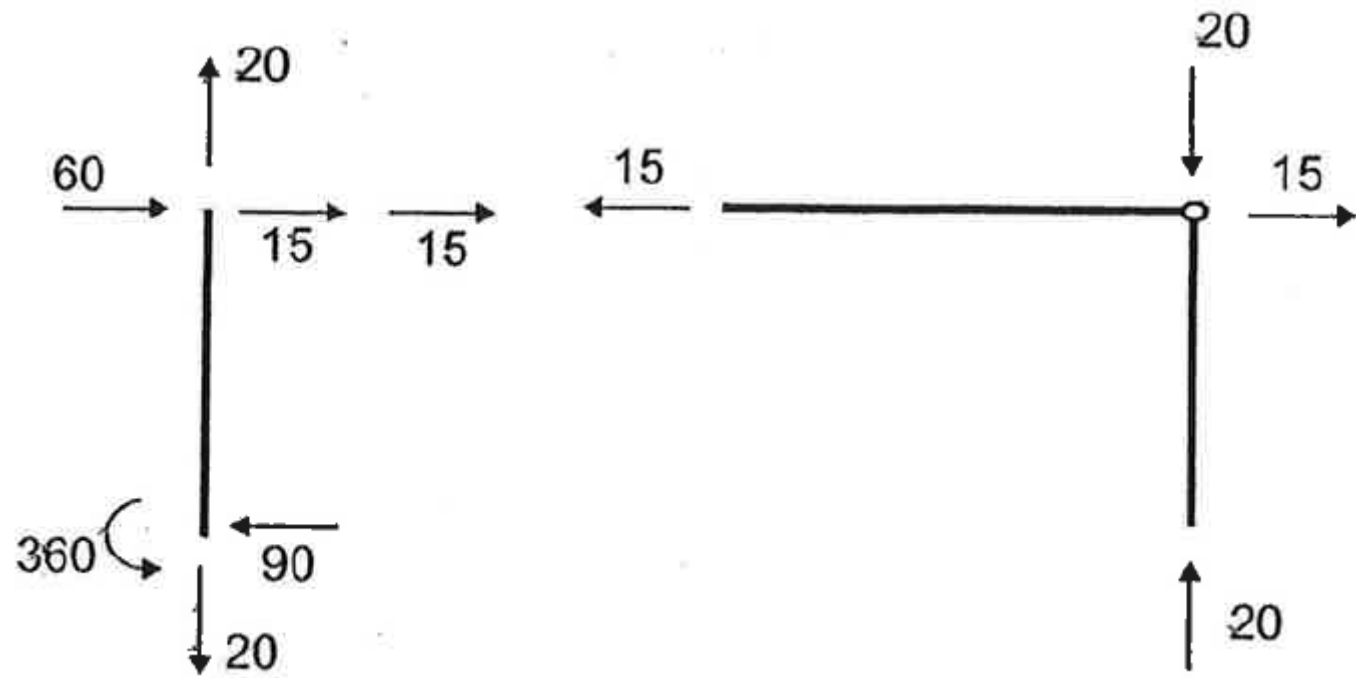
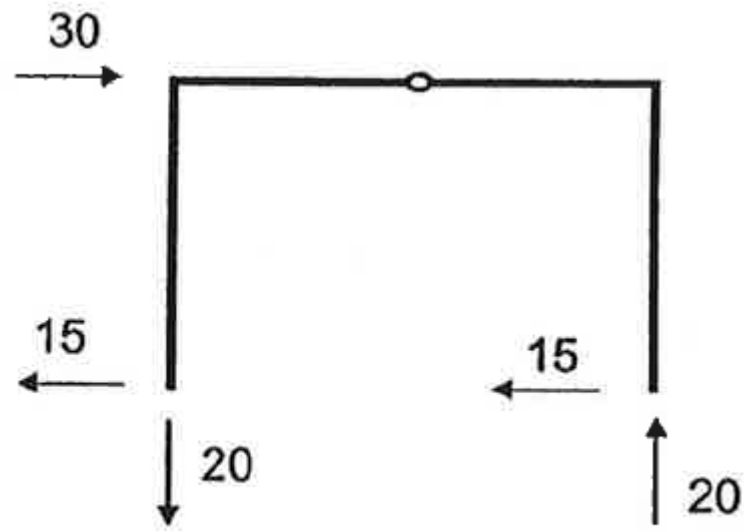
$$X''_B + X_G + 15 = 0 \Rightarrow X''_B = -15$$

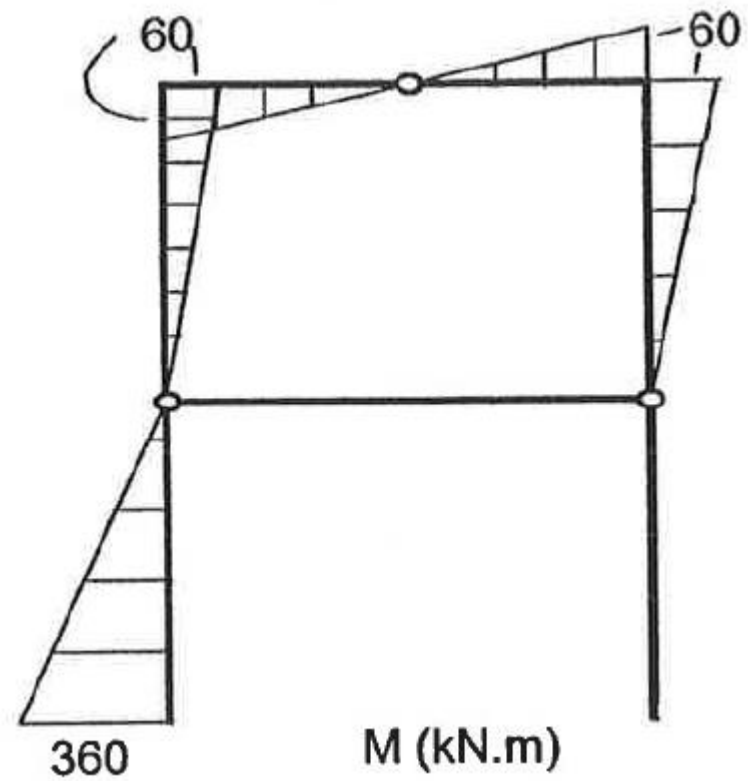
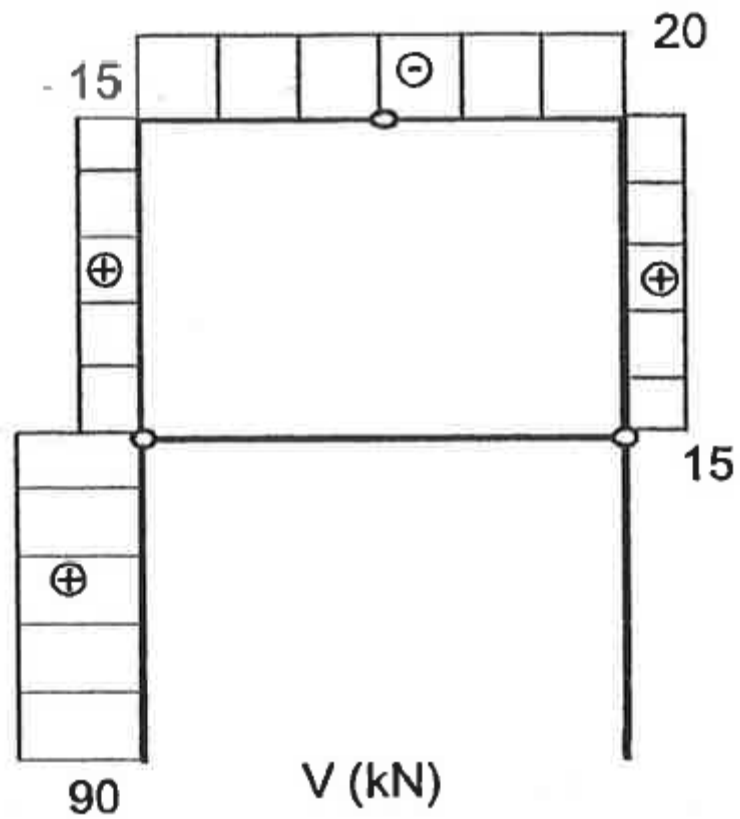
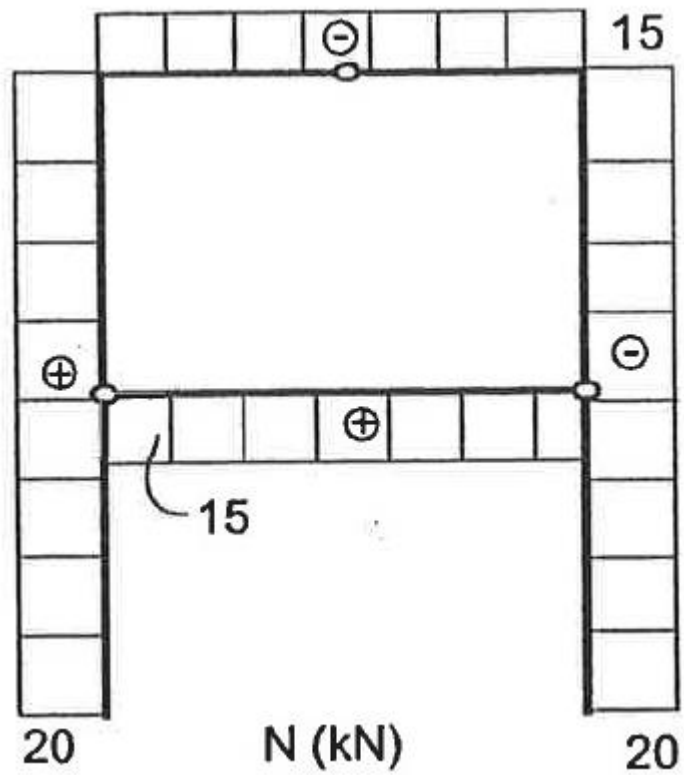
$$Y''_B + Y_G = 20 \Rightarrow Y''_B = 0$$

$$X_G \cdot 4 + Y_G \cdot 6 - 20 \cdot 6 = 0 \Rightarrow Y_G = 20$$

$$X_G \cdot 4 = 0 \Rightarrow X_G = 0$$

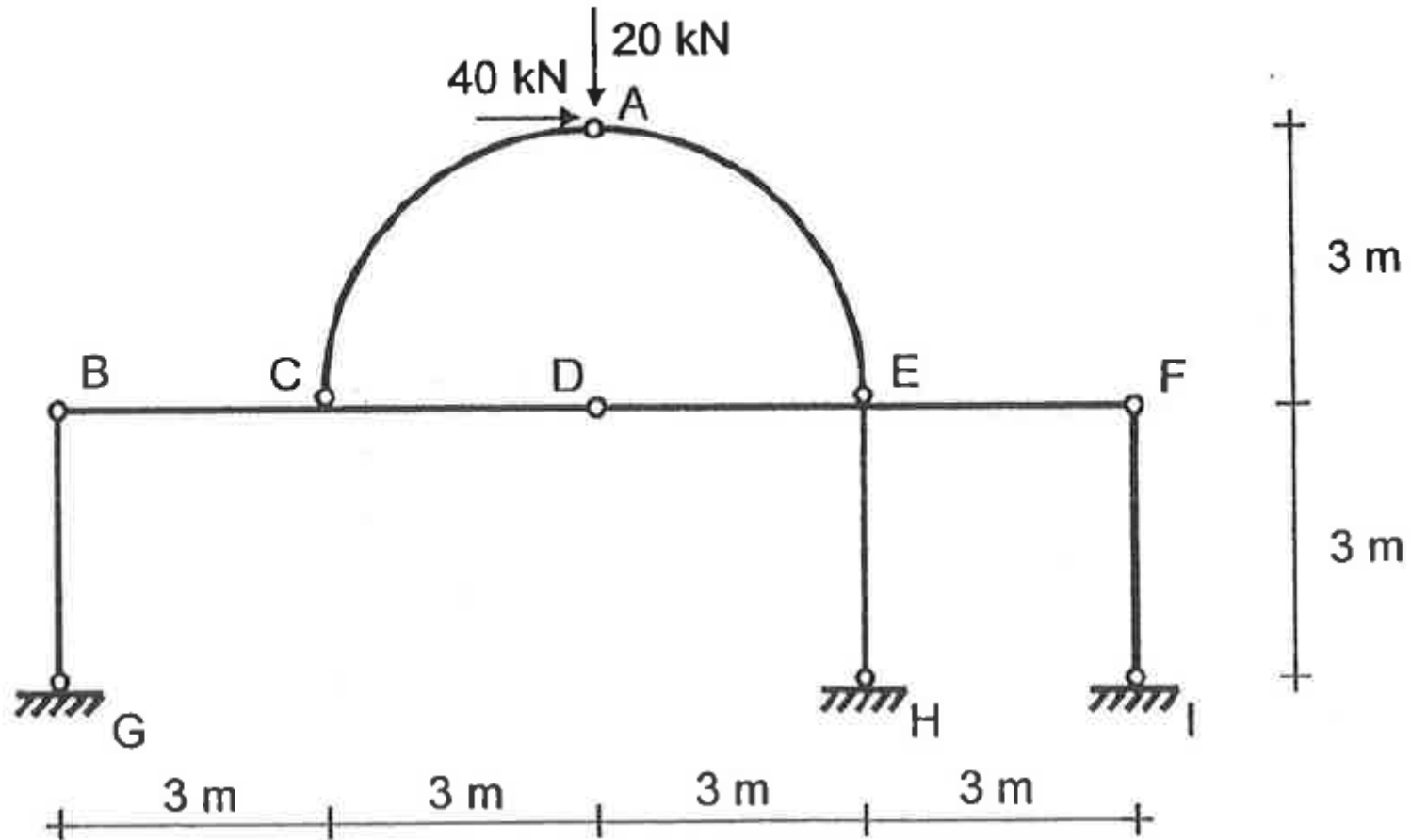


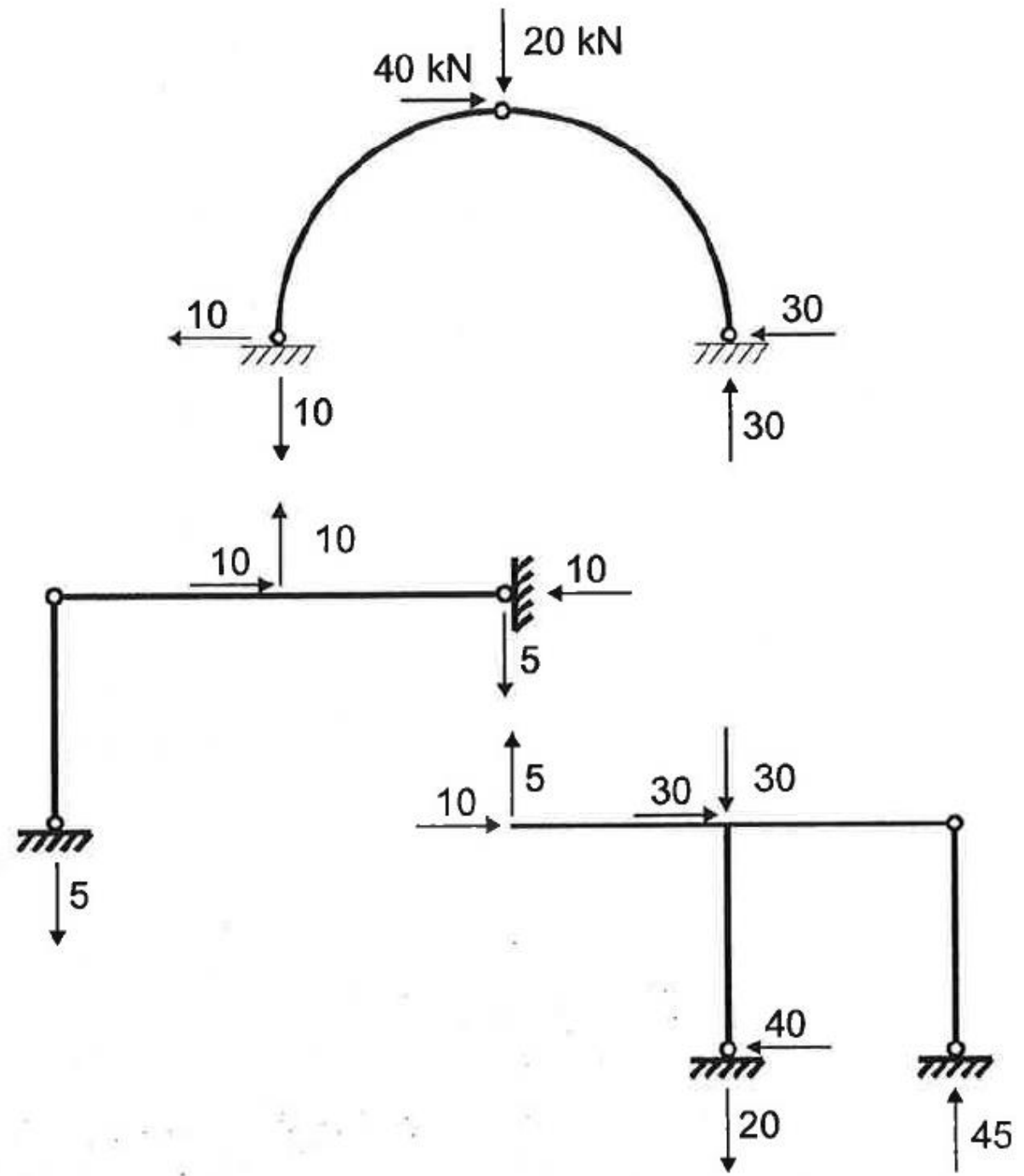




# Exemplo 3

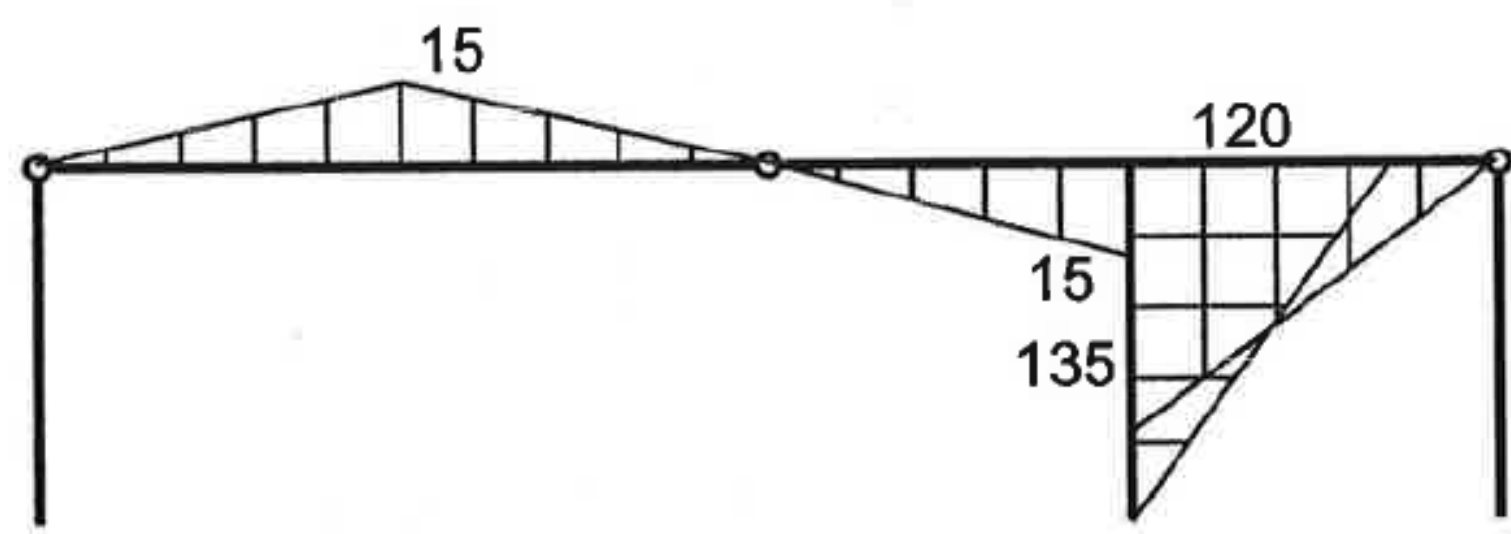
Traçar o diagrama de momentos fletores dos trechos retos da estrutura da figura:







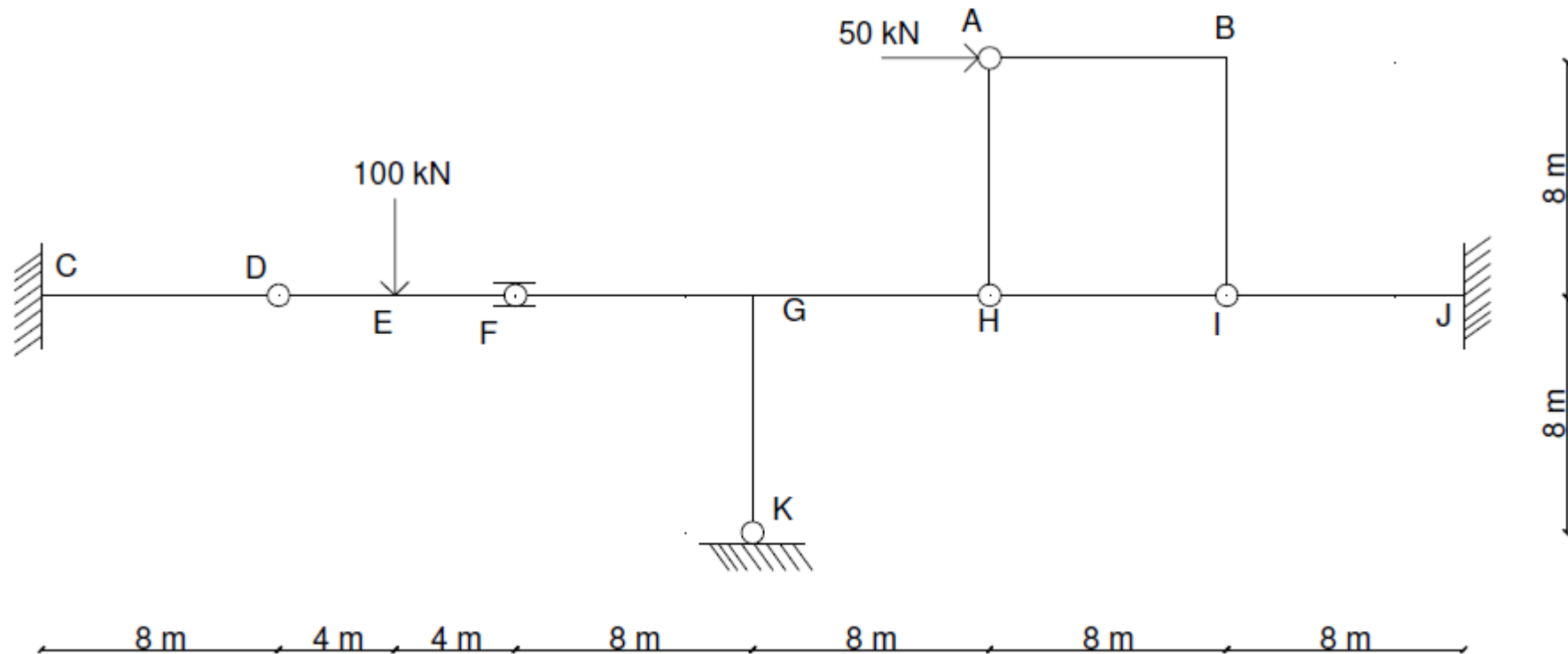
M (kNm)

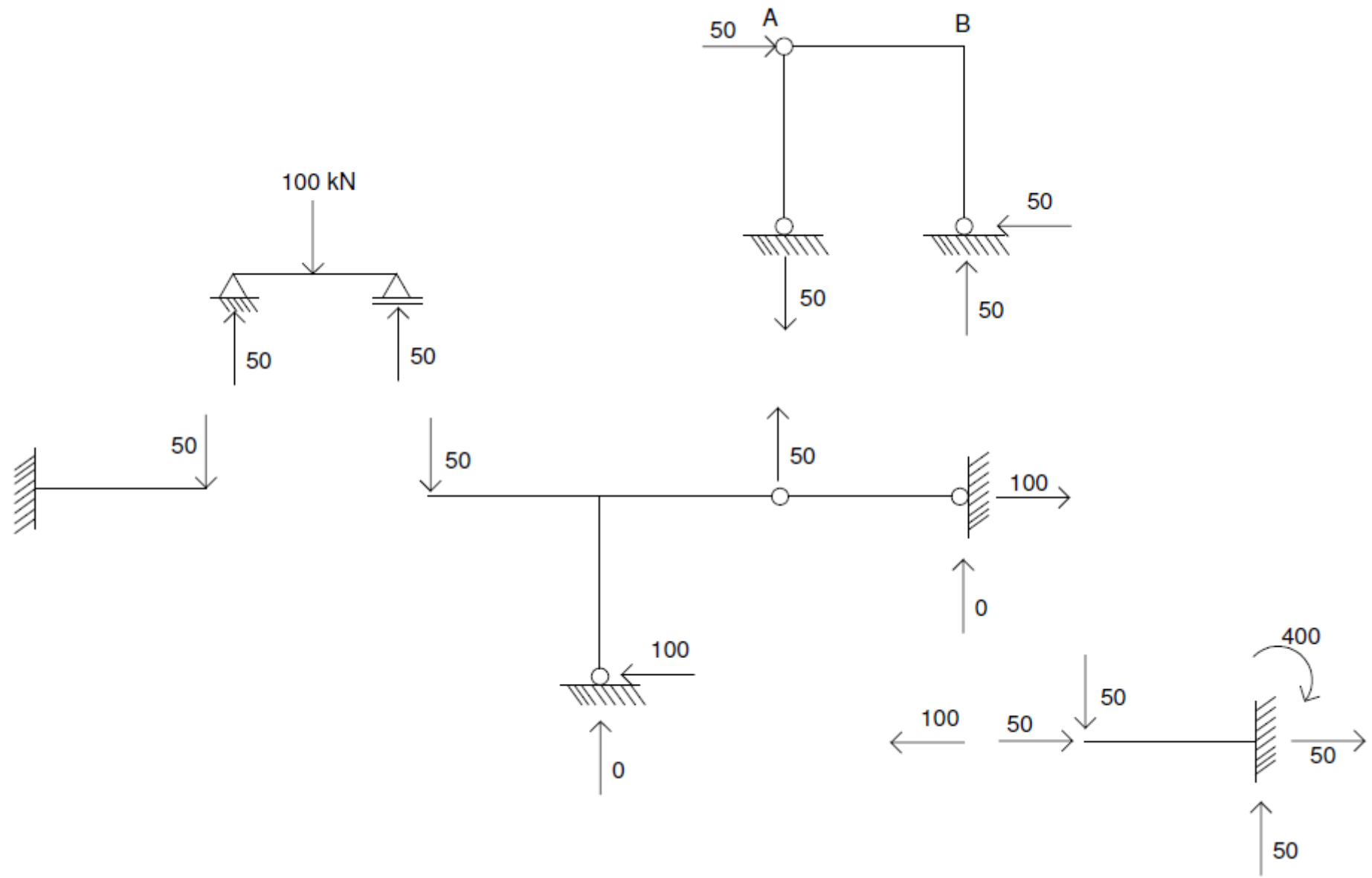


# Exemplo 4: SUB 2009

Dada a estrutura associada da figura:

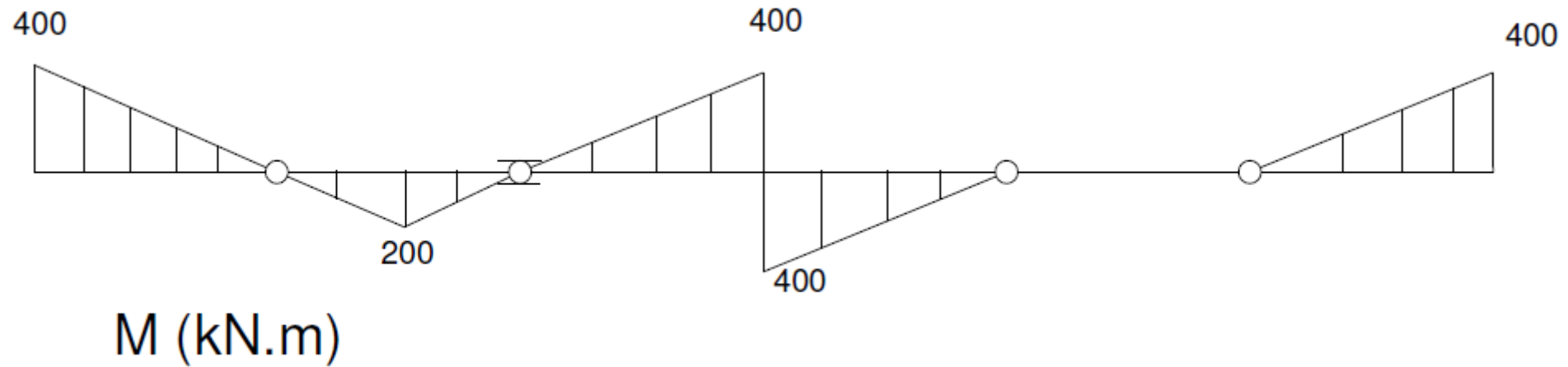
- Decompô-la nas subestruturas que a formam;
- Obter as reações de apoio em J e K;
- Traçar o diagrama de momentos fletores do trecho CDEFGHIJ.





$$X_J = +50 \text{ kN } (\rightarrow) \quad Y_J = +50 \text{ kN } (\uparrow) \quad M_J = -400 \text{ kN.m (horário)}$$

$$X_K = -100 \text{ kN } (\leftarrow) \quad Y_K = 0$$

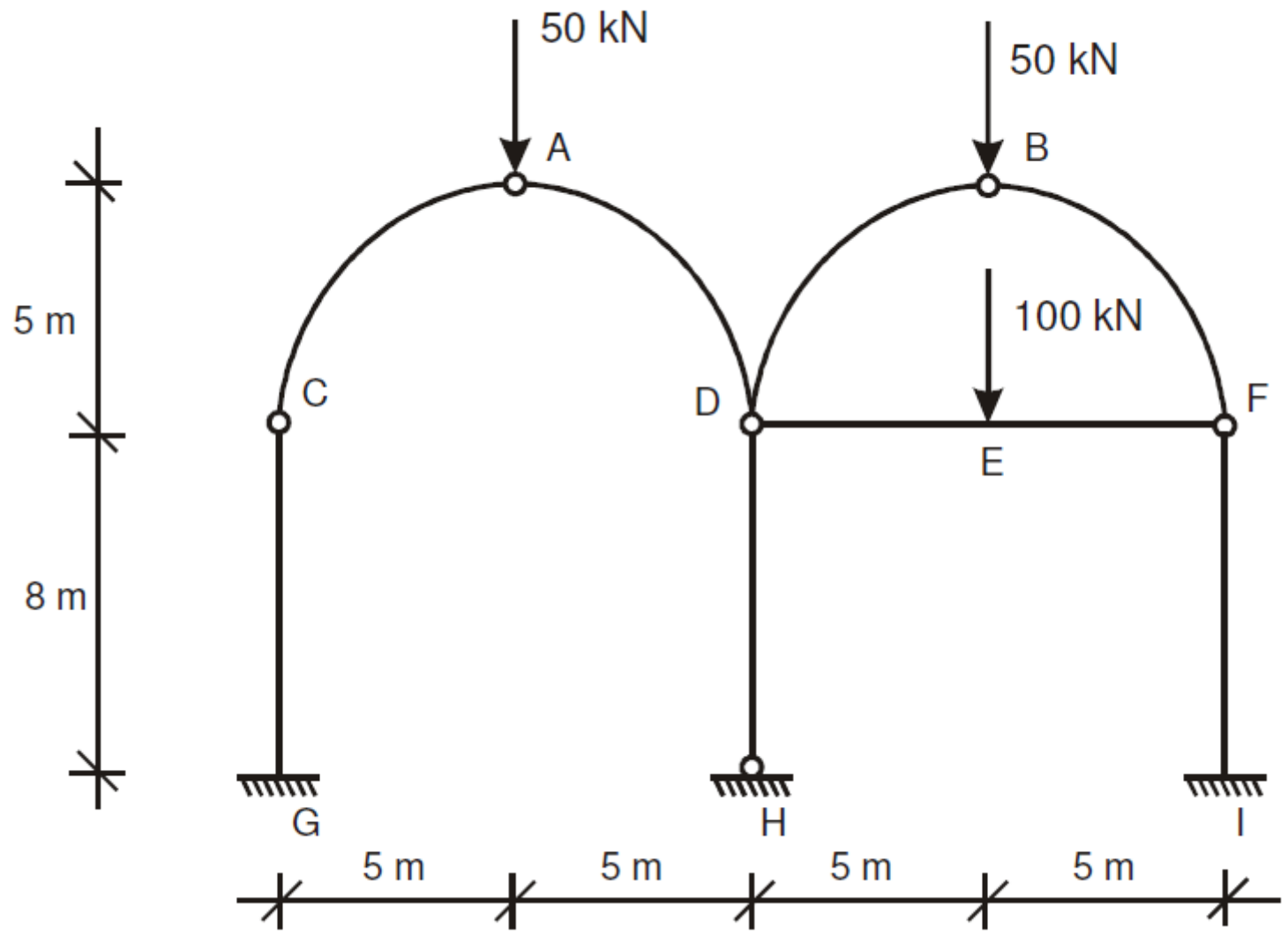


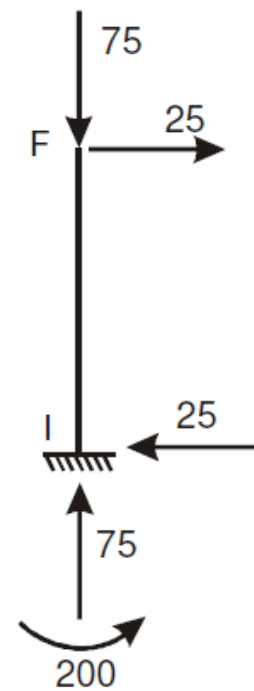
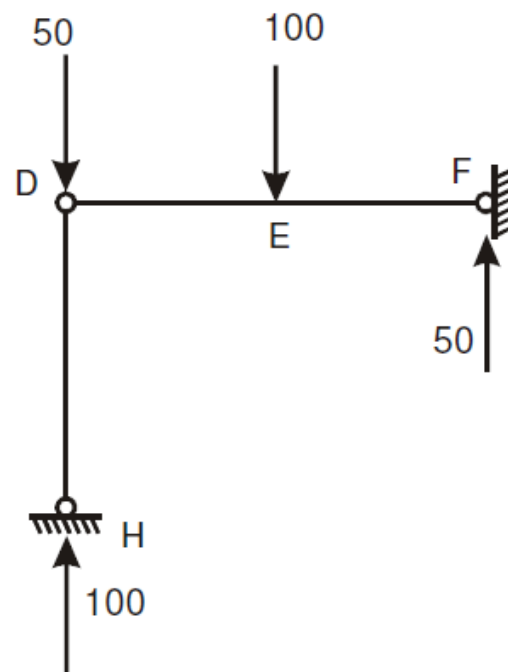
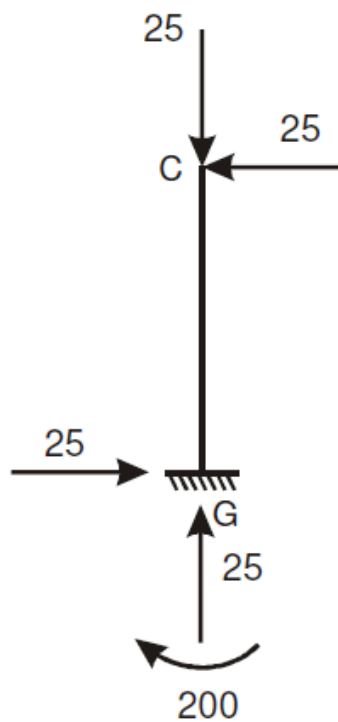
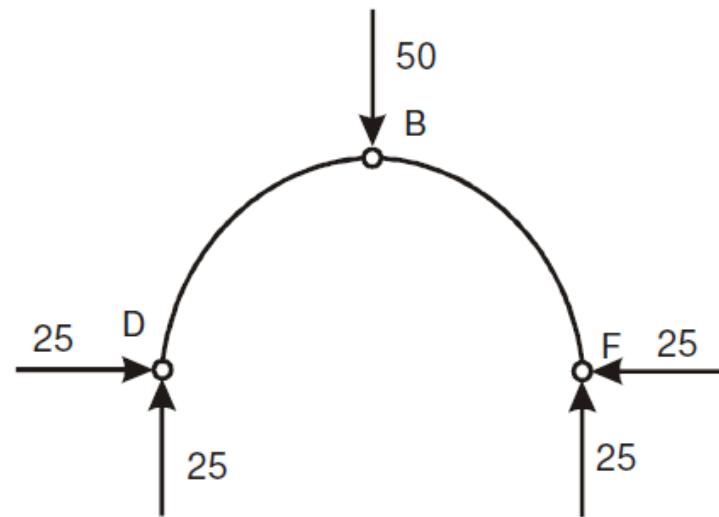
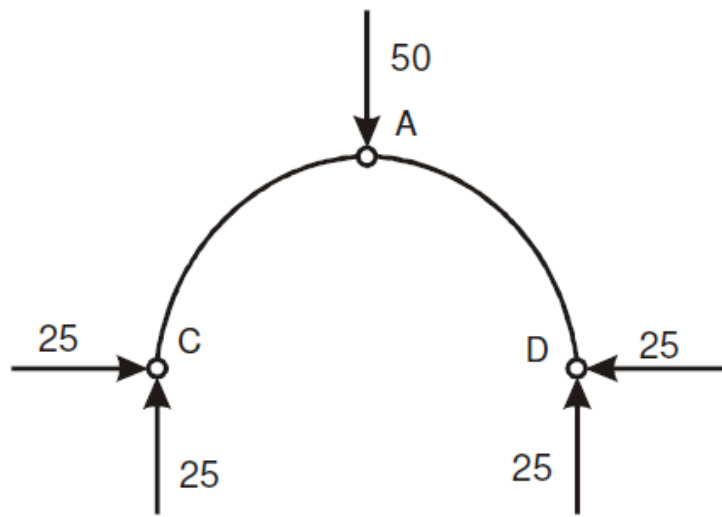
# Exemplo 5: REC 2009

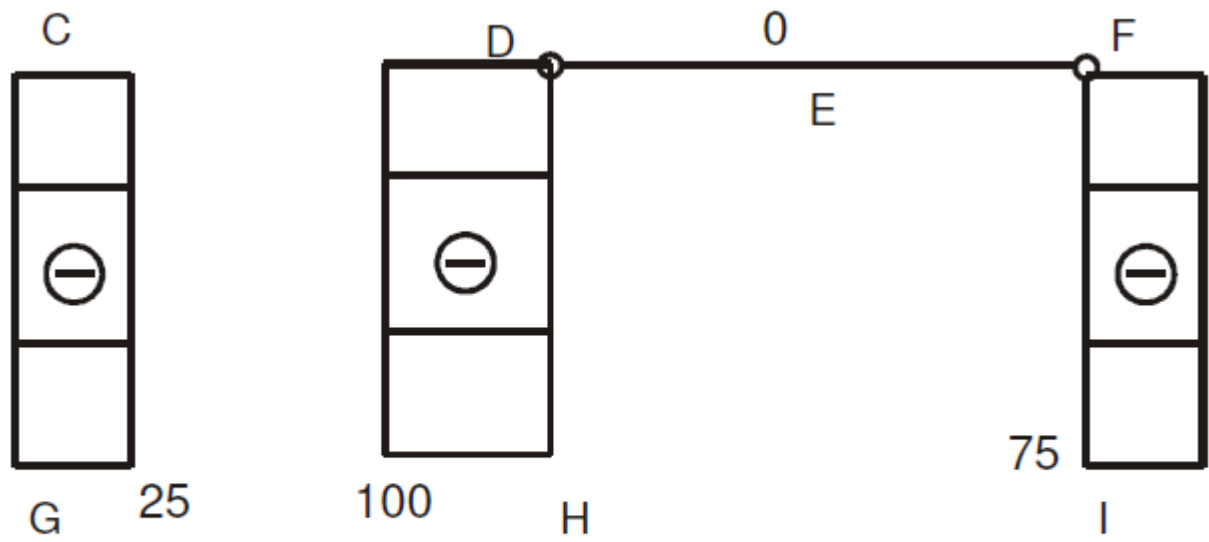
Na figura, mostra-se um corte transversal de uma loja, que possui dois ambientes cobertos por abóbadas semicirculares. O ambiente da direita possui um depósito situado no espaço criado pela abóbada de cobertura.

Observa-se que a estrutura transversal desta loja é uma estrutura associada, para a qual pede-se:

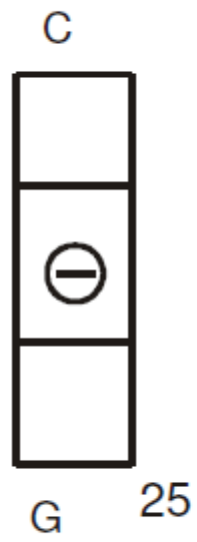
- a) Decompor a estrutura associada nas subestruturas que a compõem;
- b) Determinar as reações de cada uma destas subestruturas;
- c) Traçar os diagramas de forças normais, de forças cortantes e de momentos fletores **apenas das subestruturas formadas exclusivamente por barras retas.**





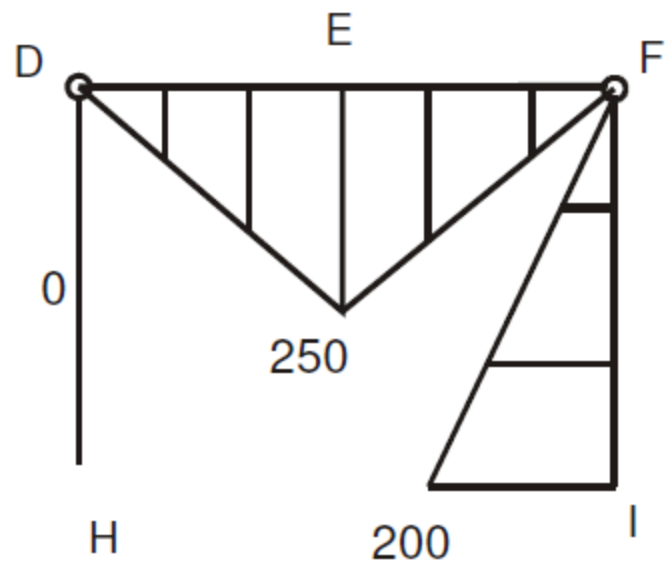
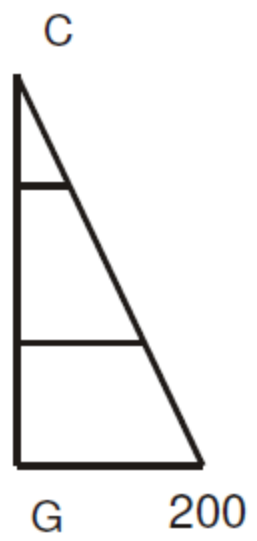


**N (kN)**



**V (kN)**



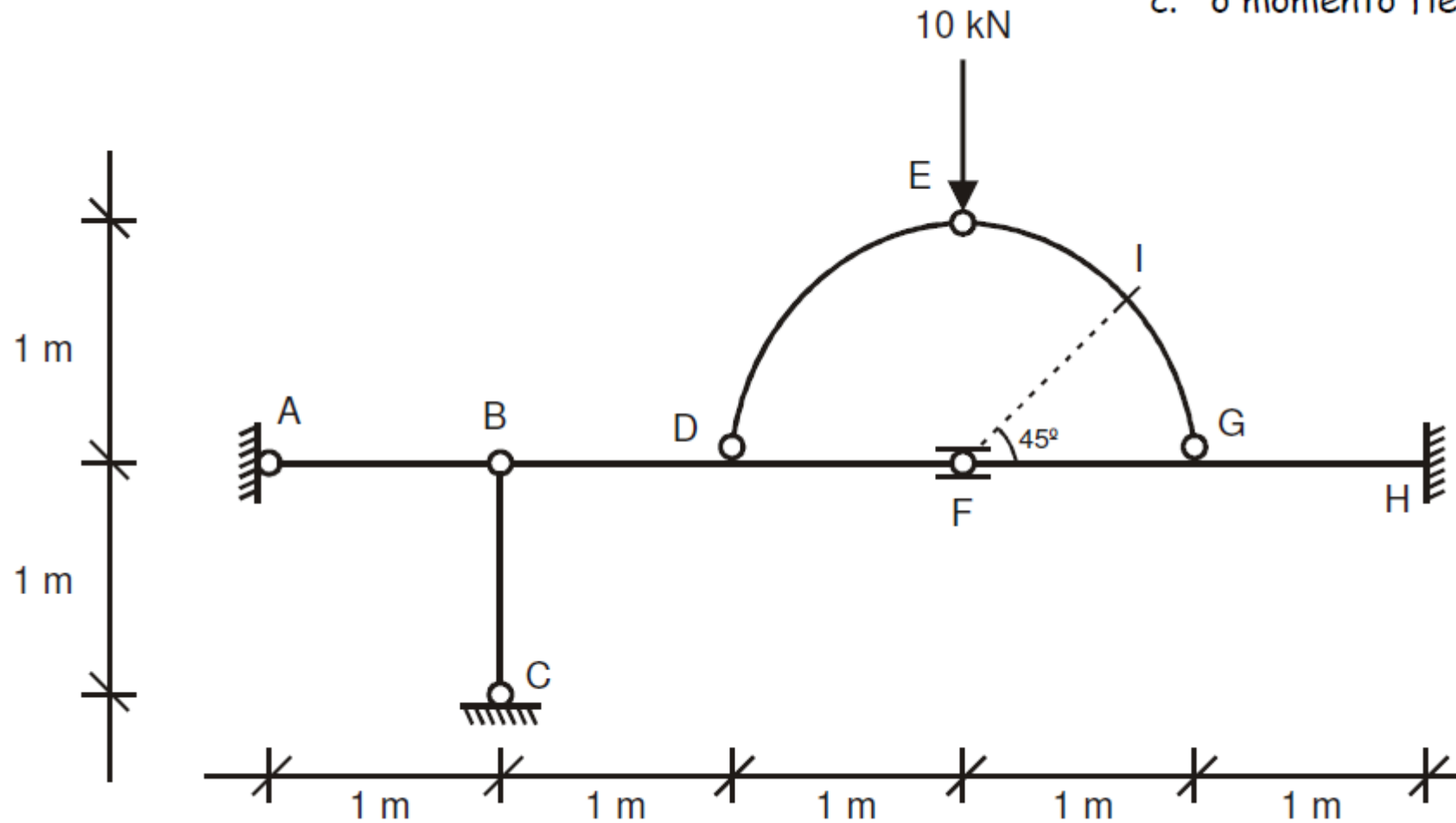


**M (kN.m)**

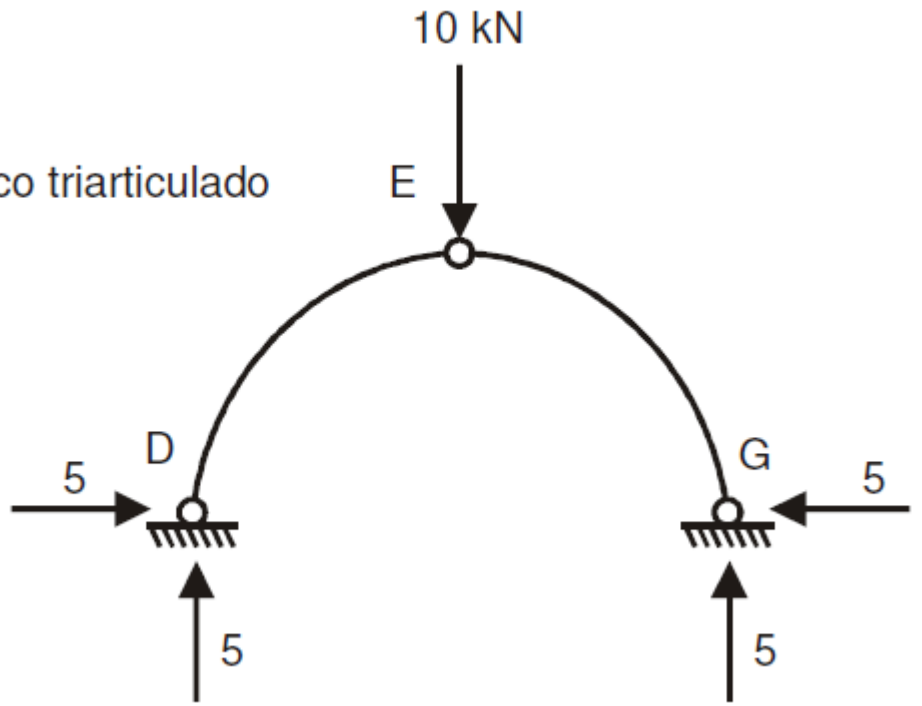
# Exemplo 6: P3 2008

Na estrutura associada da figura, determine:

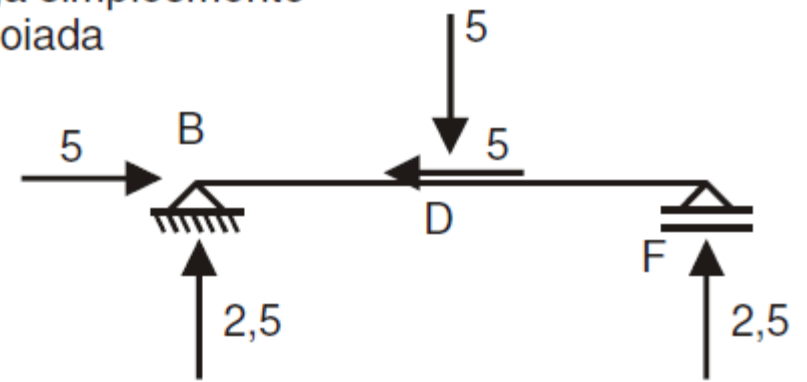
- as subestruturas e as suas denominações;
- as reações no apoio **H**;
- o momento fletor na seção **I**.



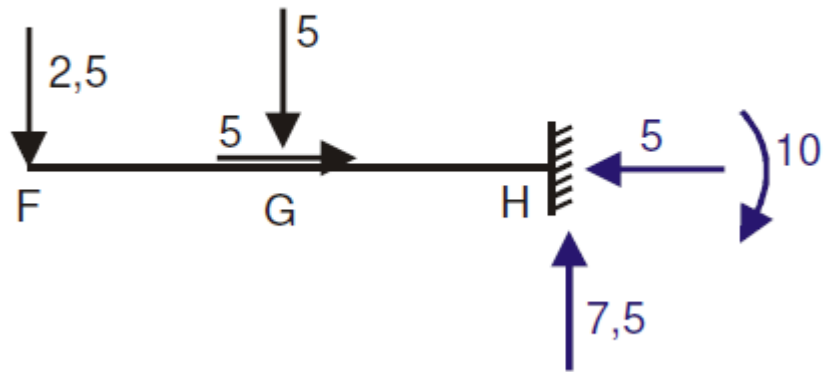
1) Arco triarticulado



2) Viga simplesmente apoiada



### 3) Viga engastada



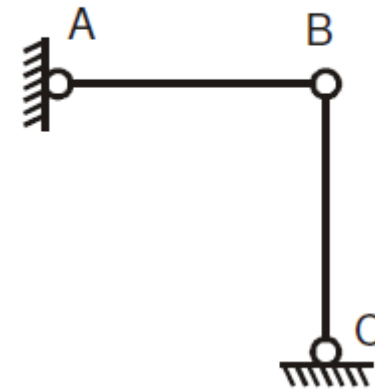
Reações em H:

$$X_H = -5 \text{ kN}$$

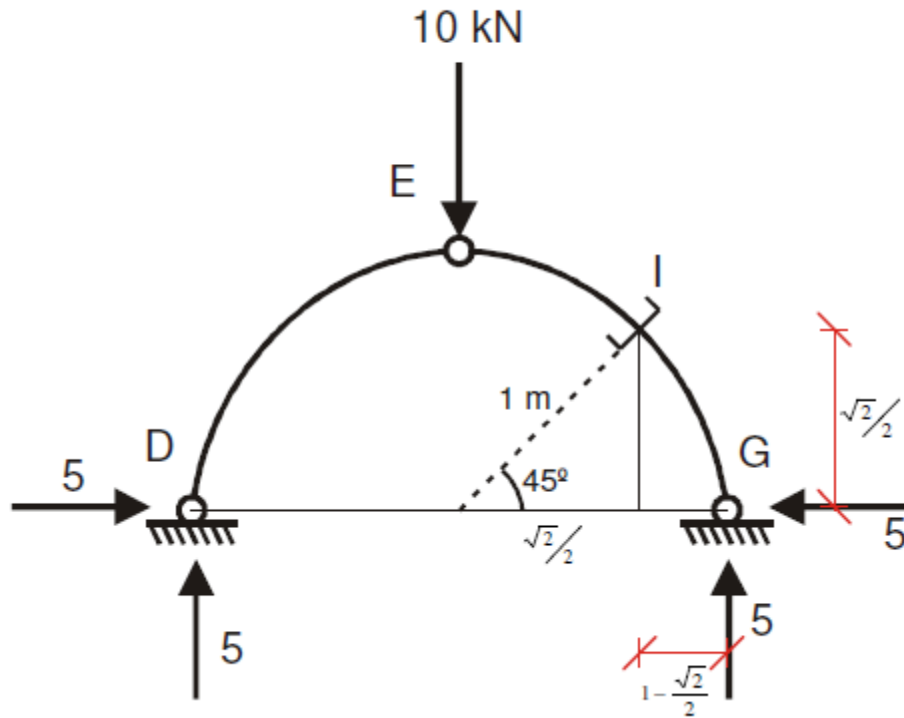
$$Y_H = +7,5 \text{ kN}$$

$$M_H = -10 \text{ kN.m}$$

### 4) Pórtico triarticulado



c) Momento fletor na seção I:



$$M = 5 \times \left( 1 - \frac{\sqrt{2}}{2} \right) - 5 \times \frac{\sqrt{2}}{2} = 5 \times (1 - \sqrt{2}) \text{ kN} \cdot \text{m}$$

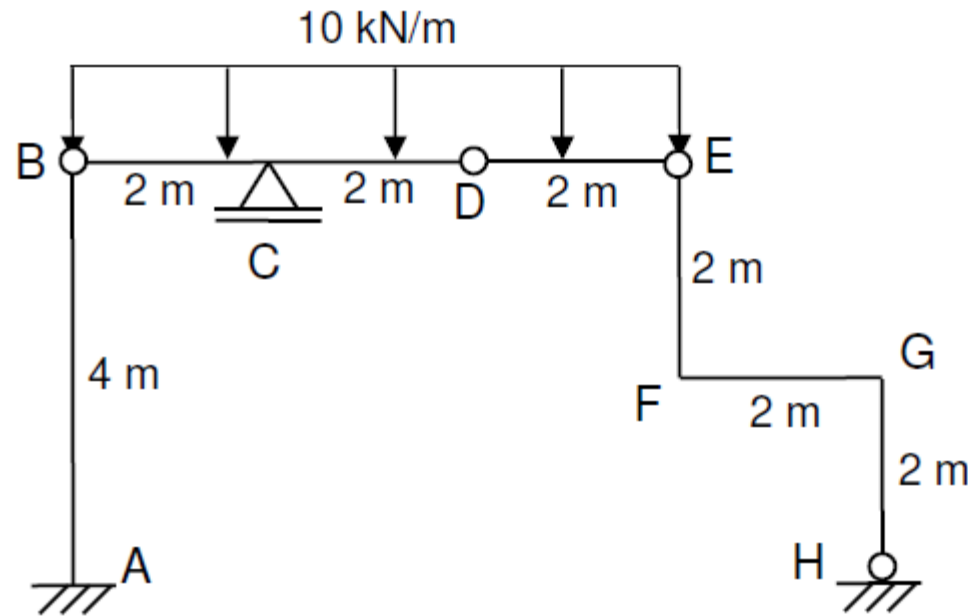
$$M = -2,07 \text{ kN} \cdot \text{m}$$

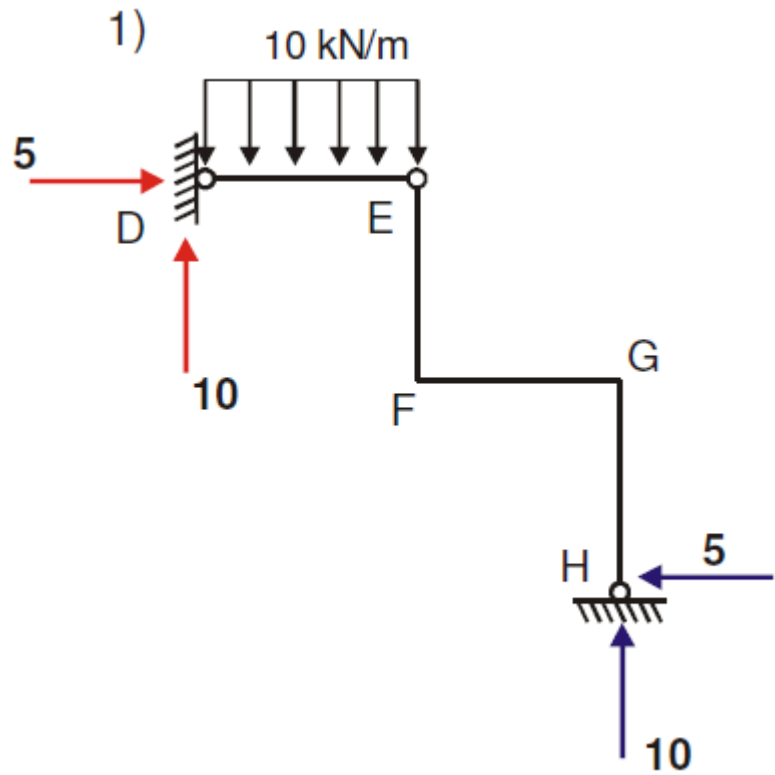
(traciona fibras superiores/externas)

# Exemplo 7: REC 2007

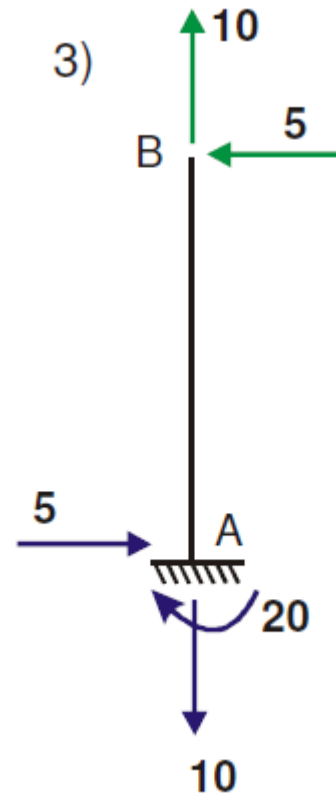
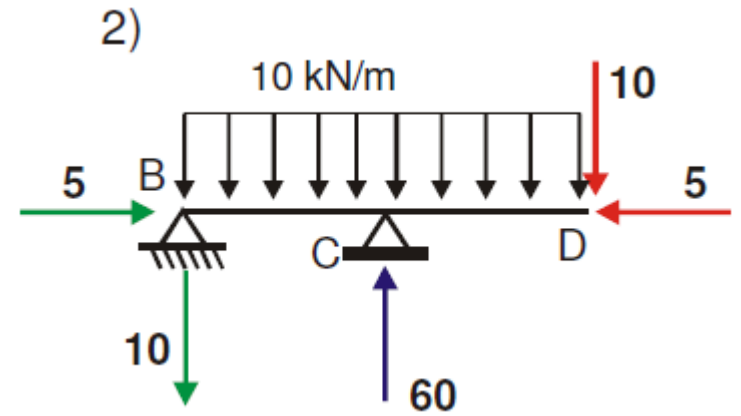
Para a estrutura associada da figura:

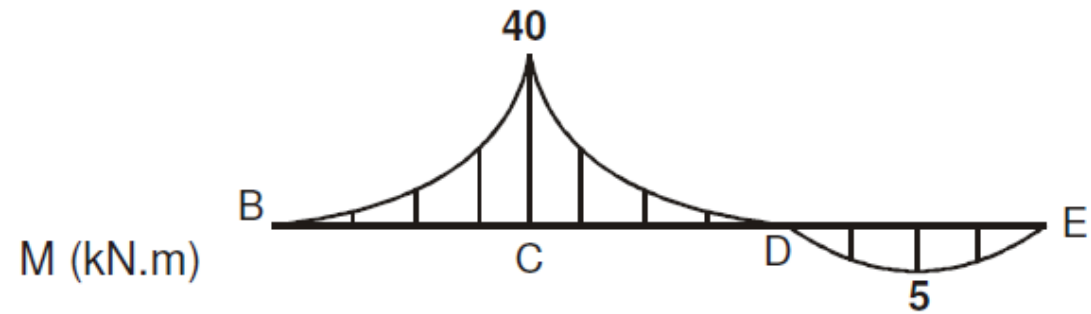
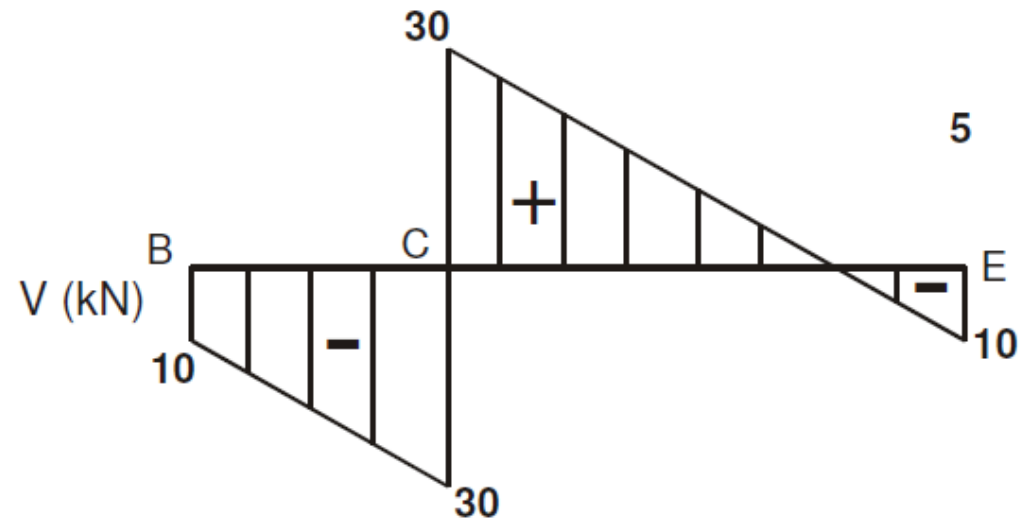
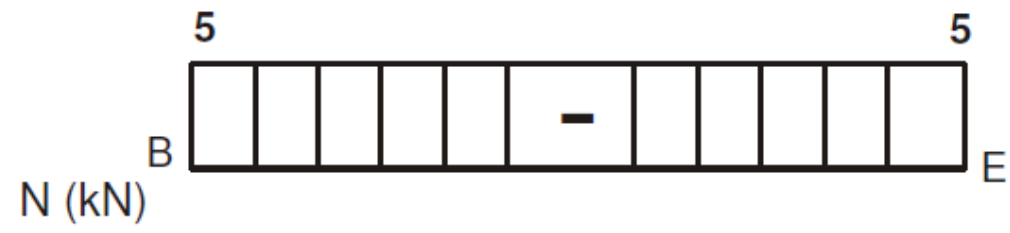
1. Divida a estrutura em subestruturas.
2. Obtenha os carregamentos e as reações de cada subestrutura.
3. Desenhe os diagramas de esforços solicitantes do trecho BCDE.





Divisão em 3 subestruturas:



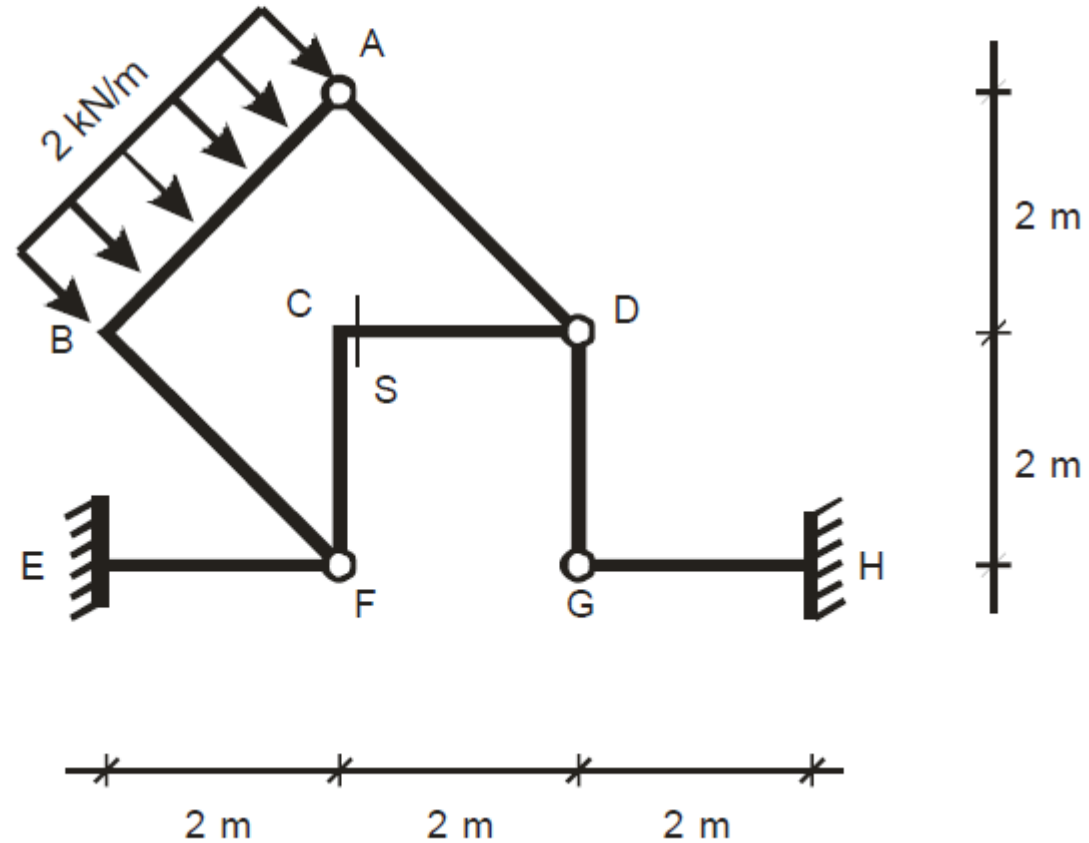




# Exemplo 8: SUB 2005

Para a estrutura da figura:

- Planejar, por meio de figuras, sua resolução;
- Obter as reações de apoio;
- Calcular os esforços solicitantes na seção S imediatamente à direita de C.



$$X_E = -4kN$$

$$R: Y_H = 4kN$$

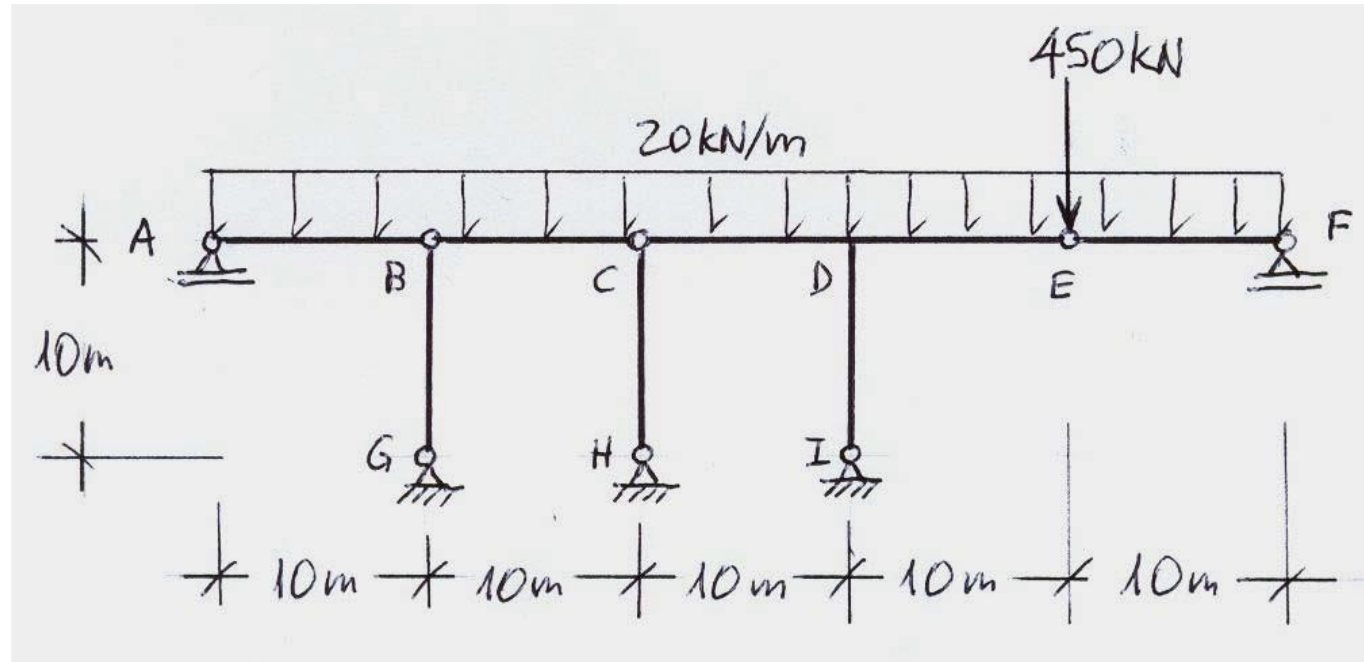
$$M_H = -8kNm$$

$$M = 4kNm$$

$$\text{Seção S [ : } N = 2kN$$

$$V = -2kN$$

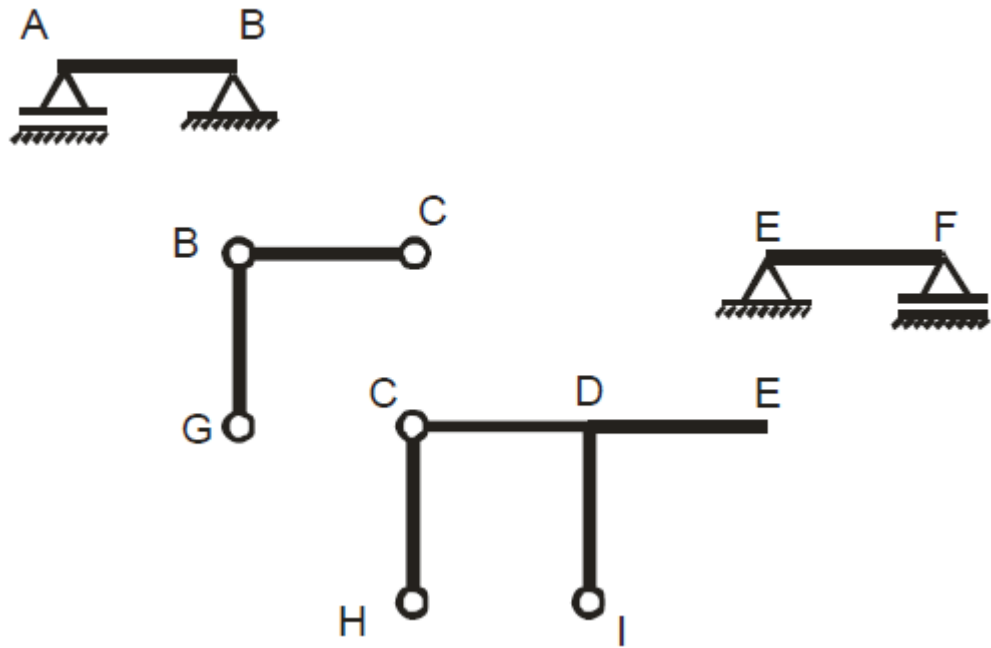
# Exemplo 9: REC 2005



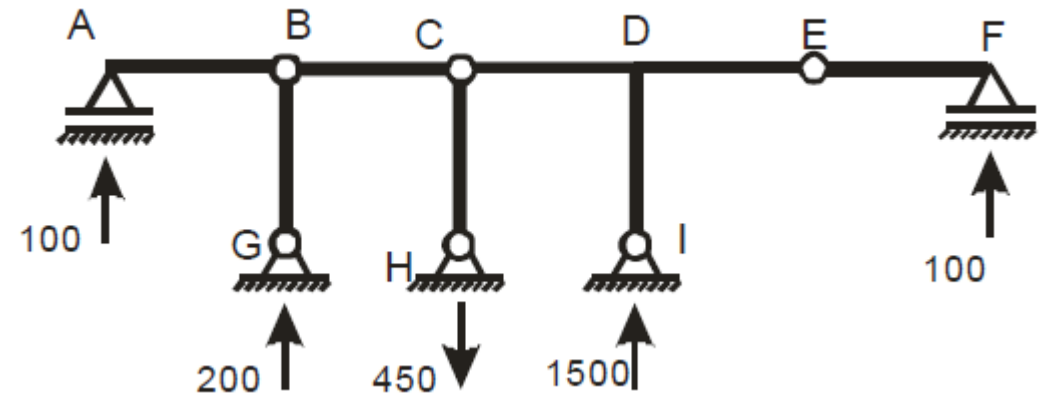
Para a estrutura ilustrada:

- conceba e ilustre uma estratégia de resolução;
- obtenha as reações de apoio;
- calcule e ilustre todos os esforços solicitantes atuantes nas seções em torno do nó D.

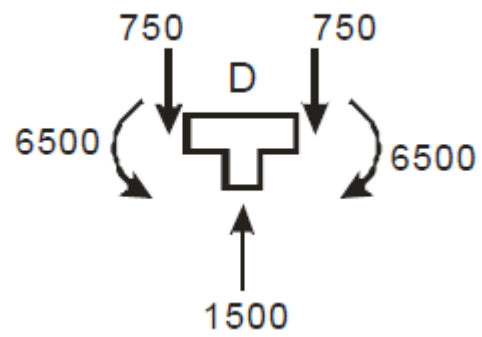
a.



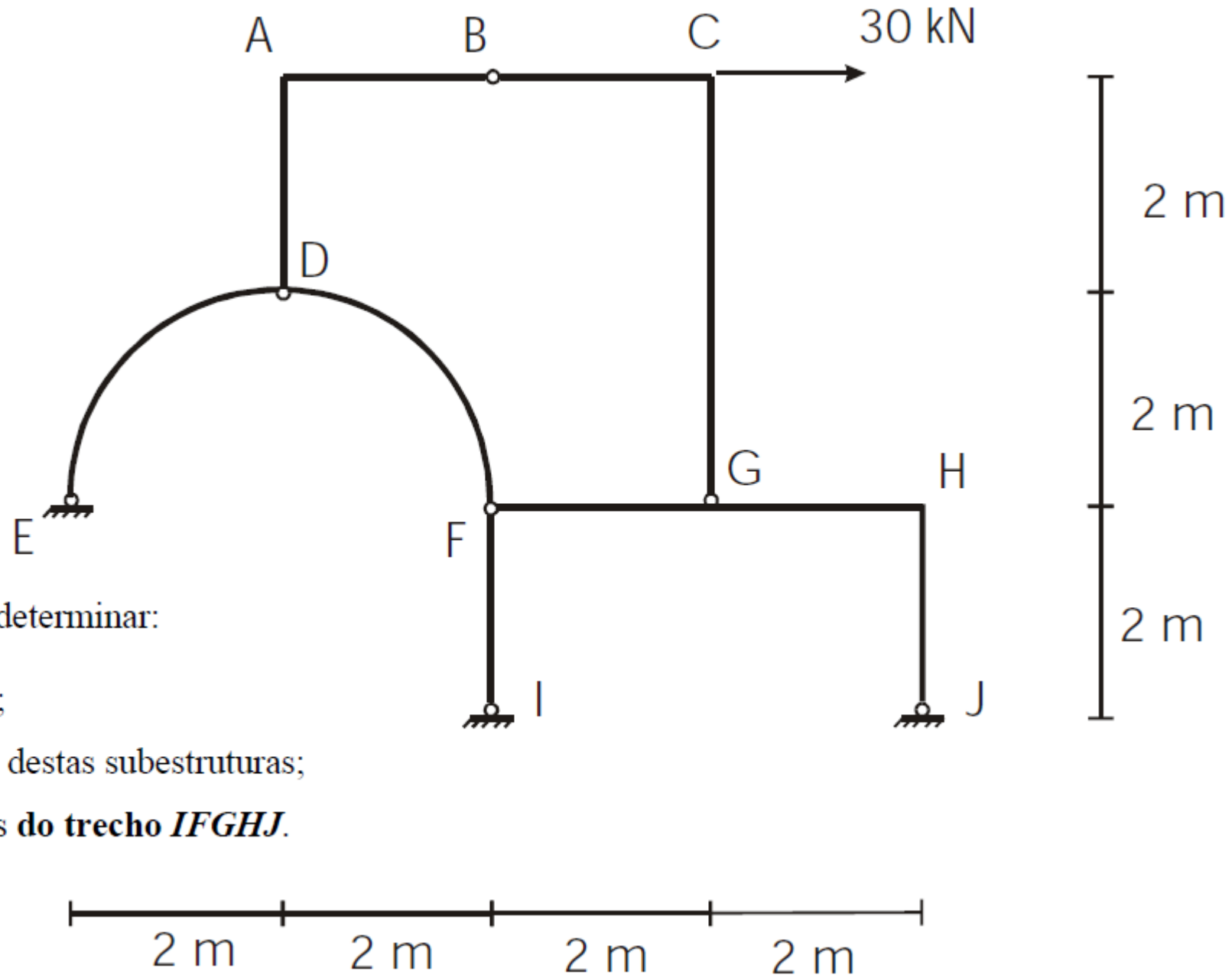
b.



c.

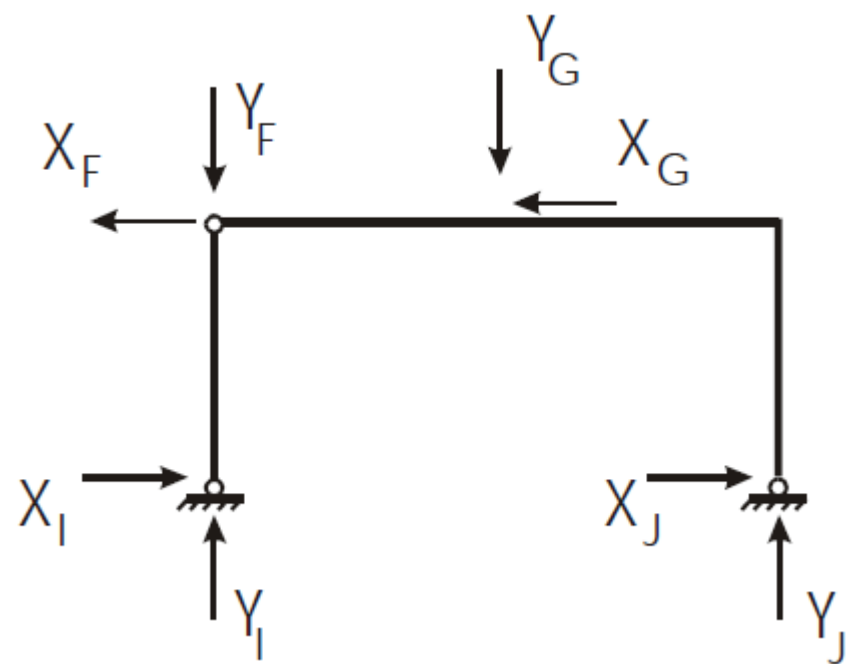
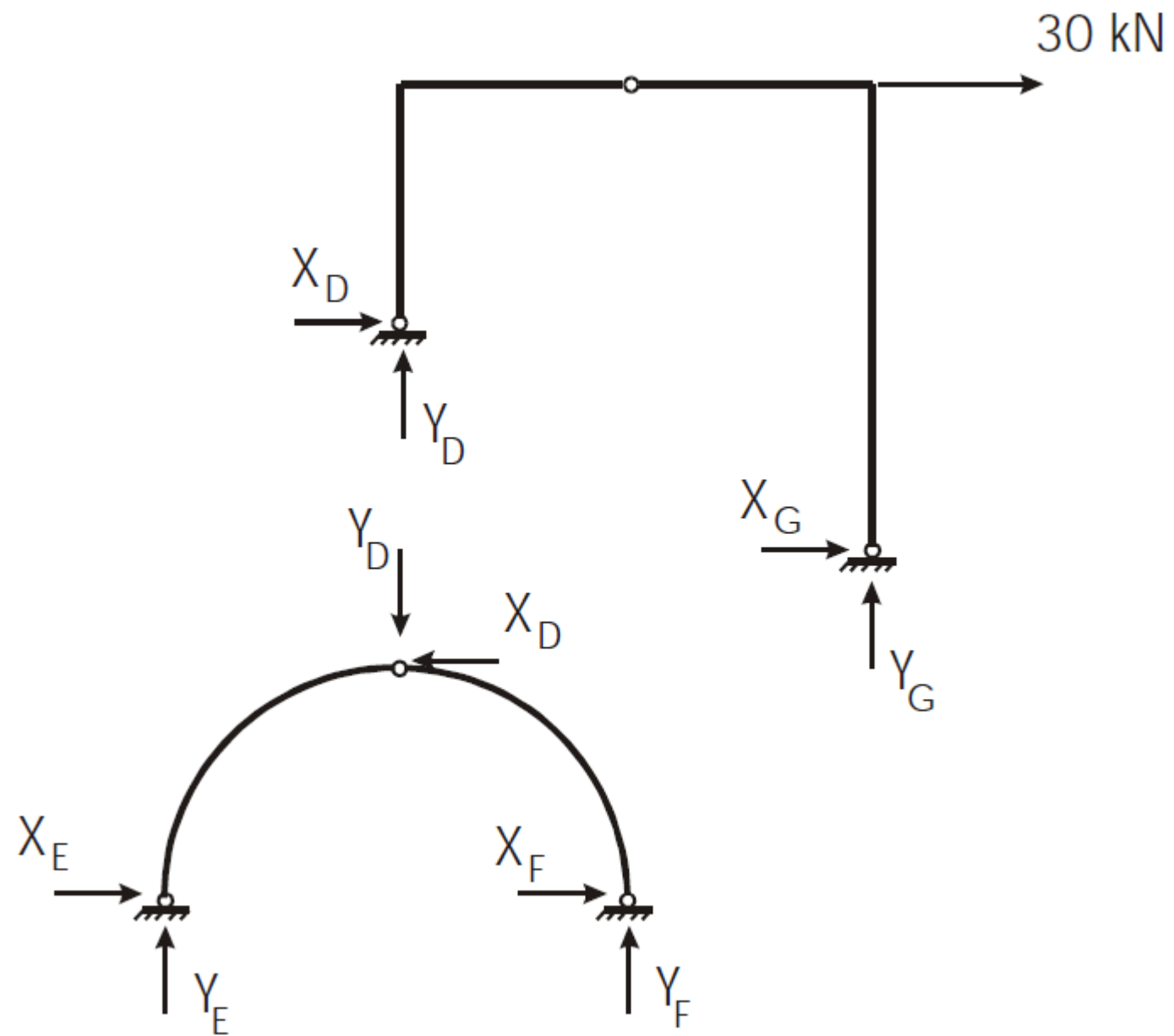


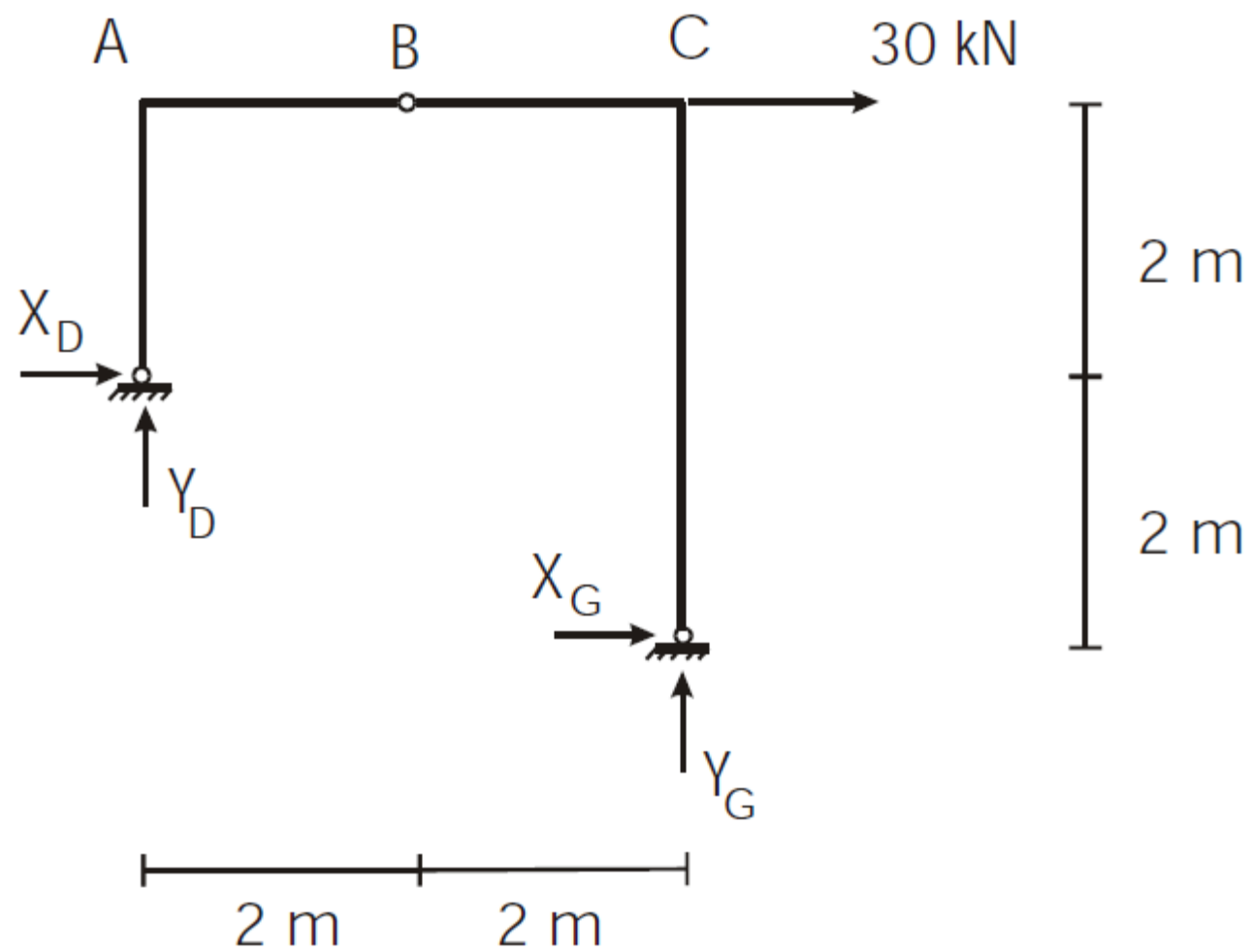
# Exemplo 10: REC 2004



Para a estrutura associada da figura, determinar:

- As subestruturas que a compõem;
- As reações de apoio de cada uma destas subestruturas;
- O diagrama de momentos fletores **do trecho *IFGHJ***.





$$\Sigma X = 0 \quad X_D + X_G + 30 = 0$$

$$\Sigma Y = 0 \quad Y_D + Y_G = 0$$

$$\Sigma M_G = 0 \quad -X_D \cdot 2 - Y_D \cdot 4 - 30 \cdot 4 = 0$$

$$M_{\text{fletor em B}} = 0 \quad Y_D \cdot 2 - X_D \cdot 2 = 0$$

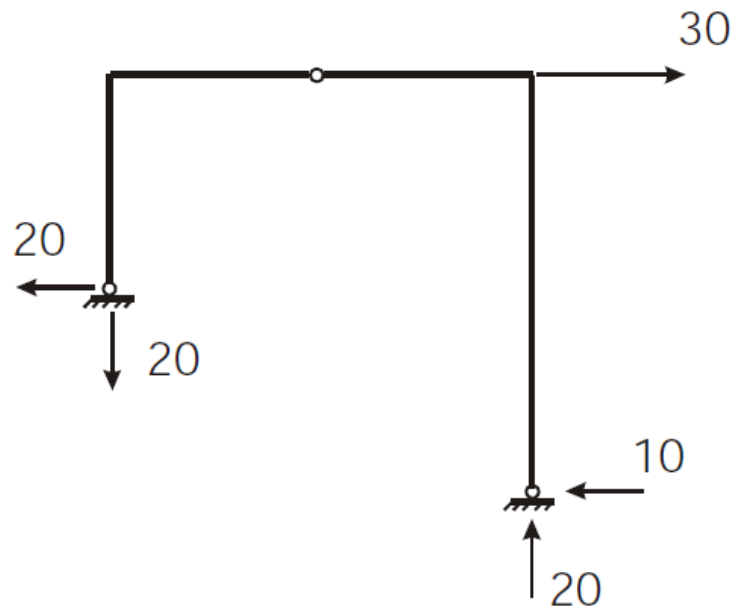
$$Y_D = X_D$$

$$-2 \cdot Y_D - 4 \cdot Y_D = 120$$

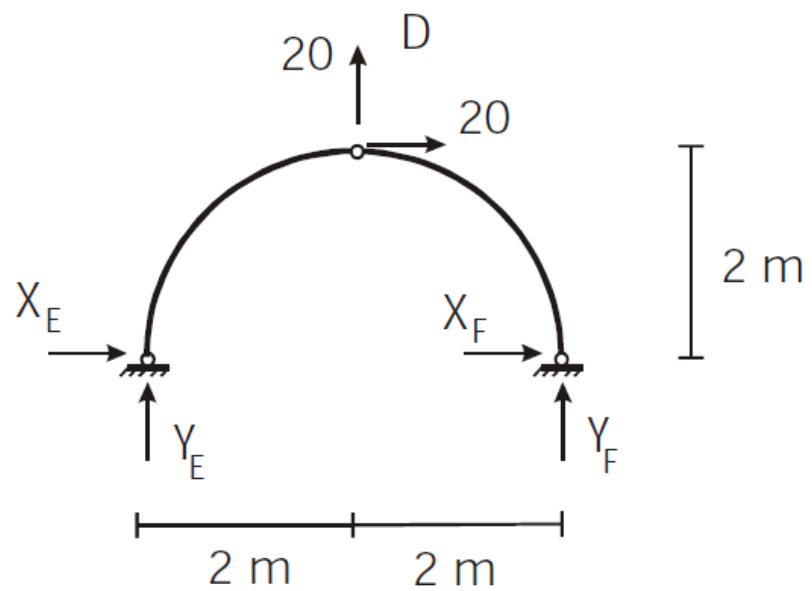
$$Y_D = -20 = X_D$$

$$Y_G = 20$$

$$-20 + X_G + 30 = 0 \quad X_G = -10$$



$$\begin{aligned} \Sigma X = 0 & \quad X_E + X_F + 20 = 0 \\ \Sigma Y = 0 & \quad Y_E + Y_F + 20 = 0 \\ \Sigma M_G = 0 & \quad 20 \cdot 2 - 20 \cdot 2 + Y_F \cdot 4 = 0 \\ M_{\text{fleitor em D}} = 0 & \quad Y_F \cdot 2 + X_F \cdot 2 = 0 \end{aligned}$$

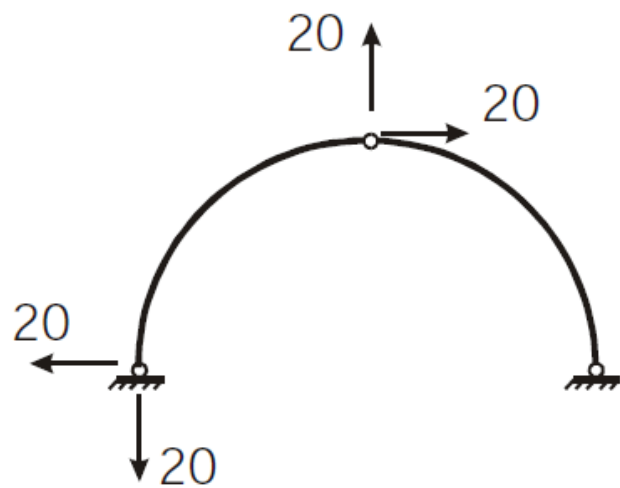


$$Y_F = 0$$

$$X_F = 0$$

$$Y_E = -20$$

$$X_E = -20$$

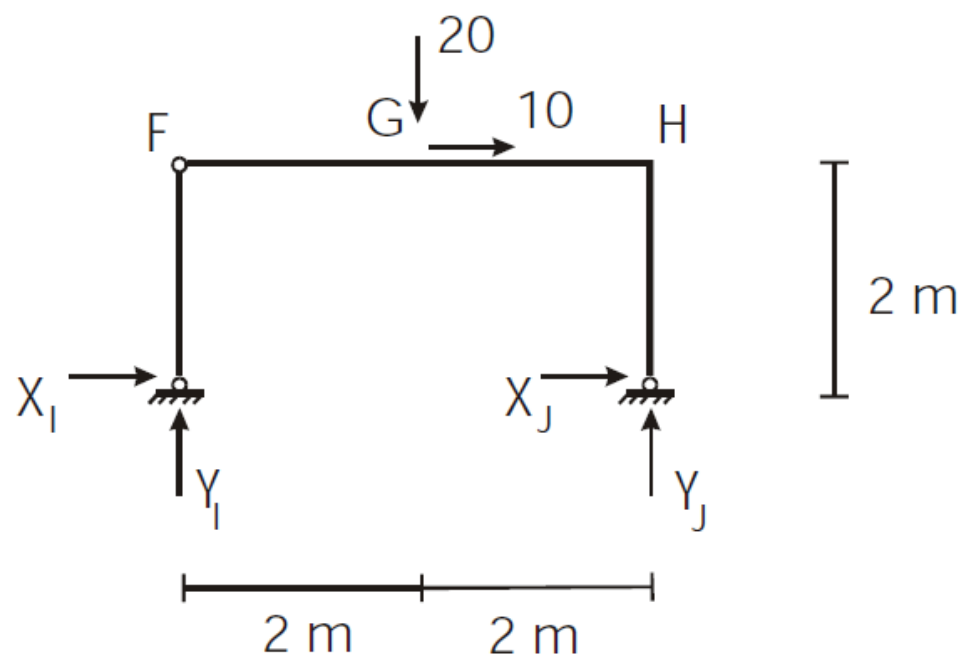


$$\Sigma X = 0 \quad X_I + X_J + 10 = 0$$

$$\Sigma Y = 0 \quad Y_I + Y_J - 20 = 0$$

$$\Sigma M_G = 0 \quad -20 \cdot 2 - 10 \cdot 2 + Y_J \cdot 4 = 0$$

$$M_{\text{fleitor em F}} = 0 \quad -X_I \cdot 2 = 0$$



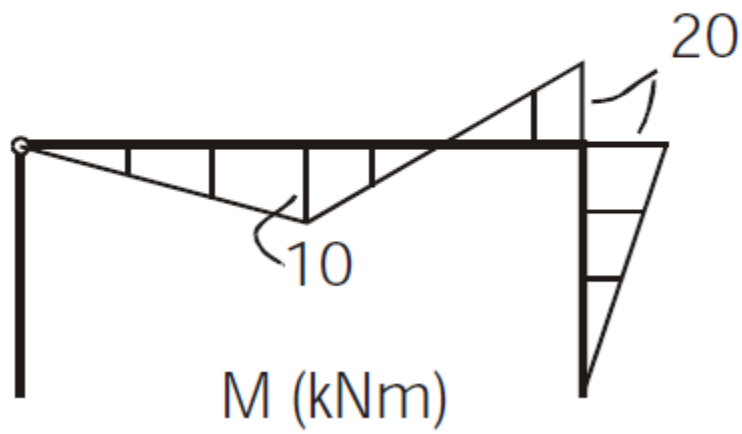
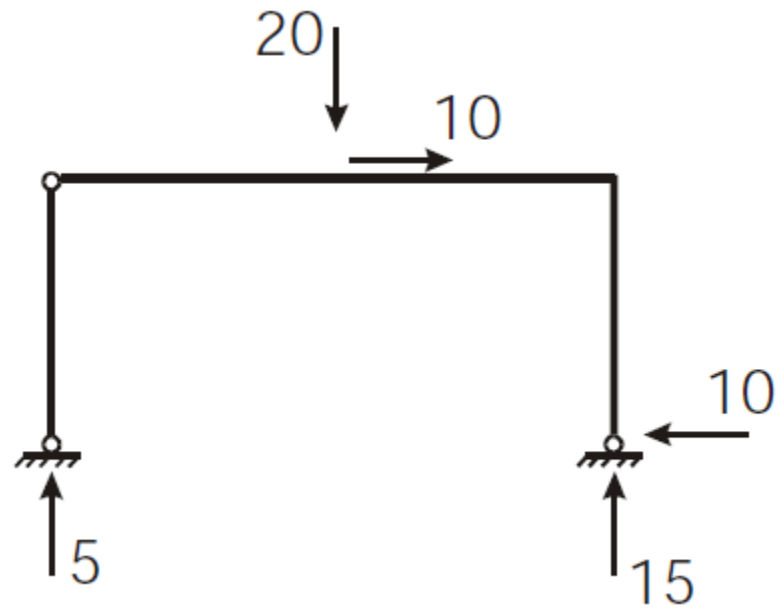
$$X_I = 0$$

$$Y_J = 15$$

$$Y_I = 5$$

$$X_G = -10$$

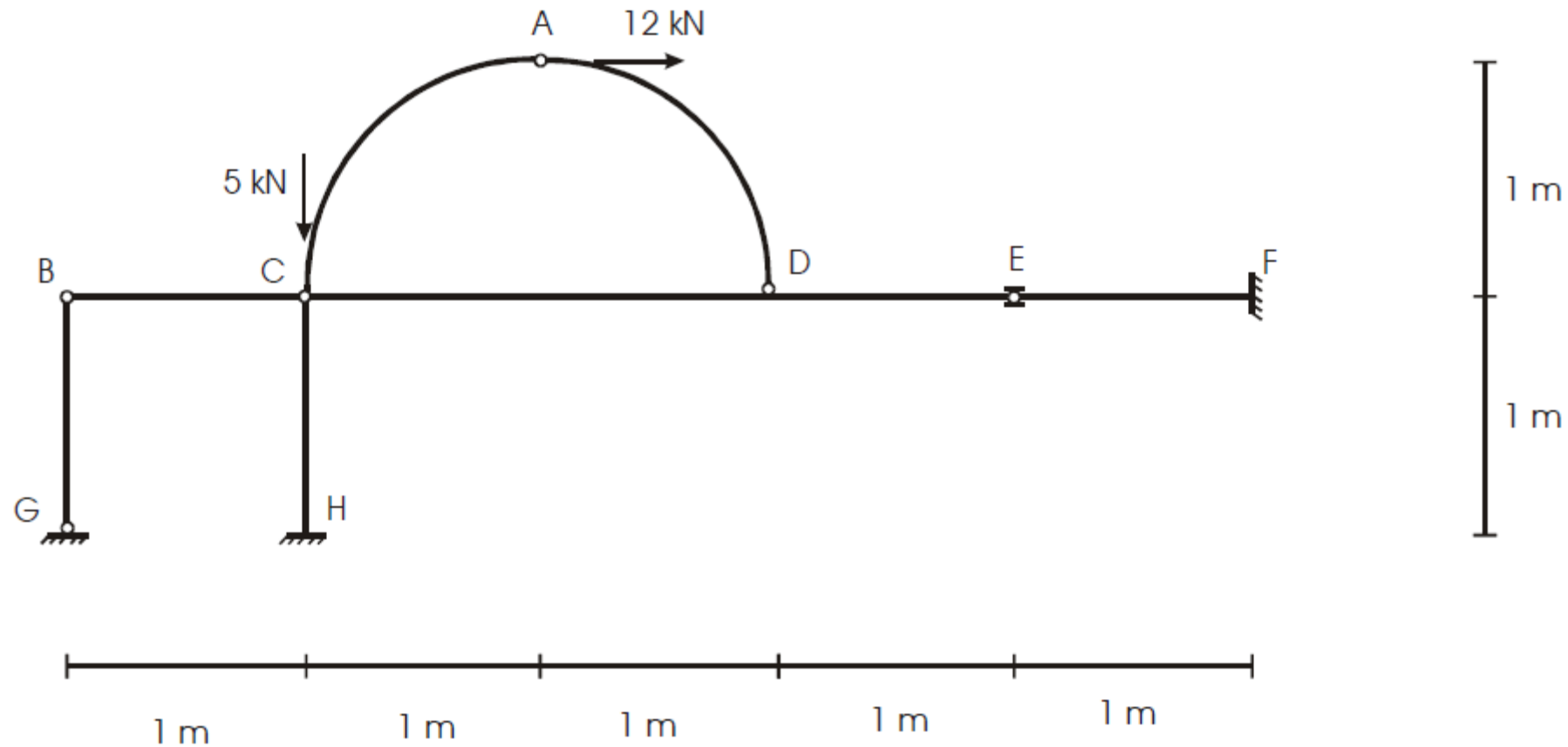




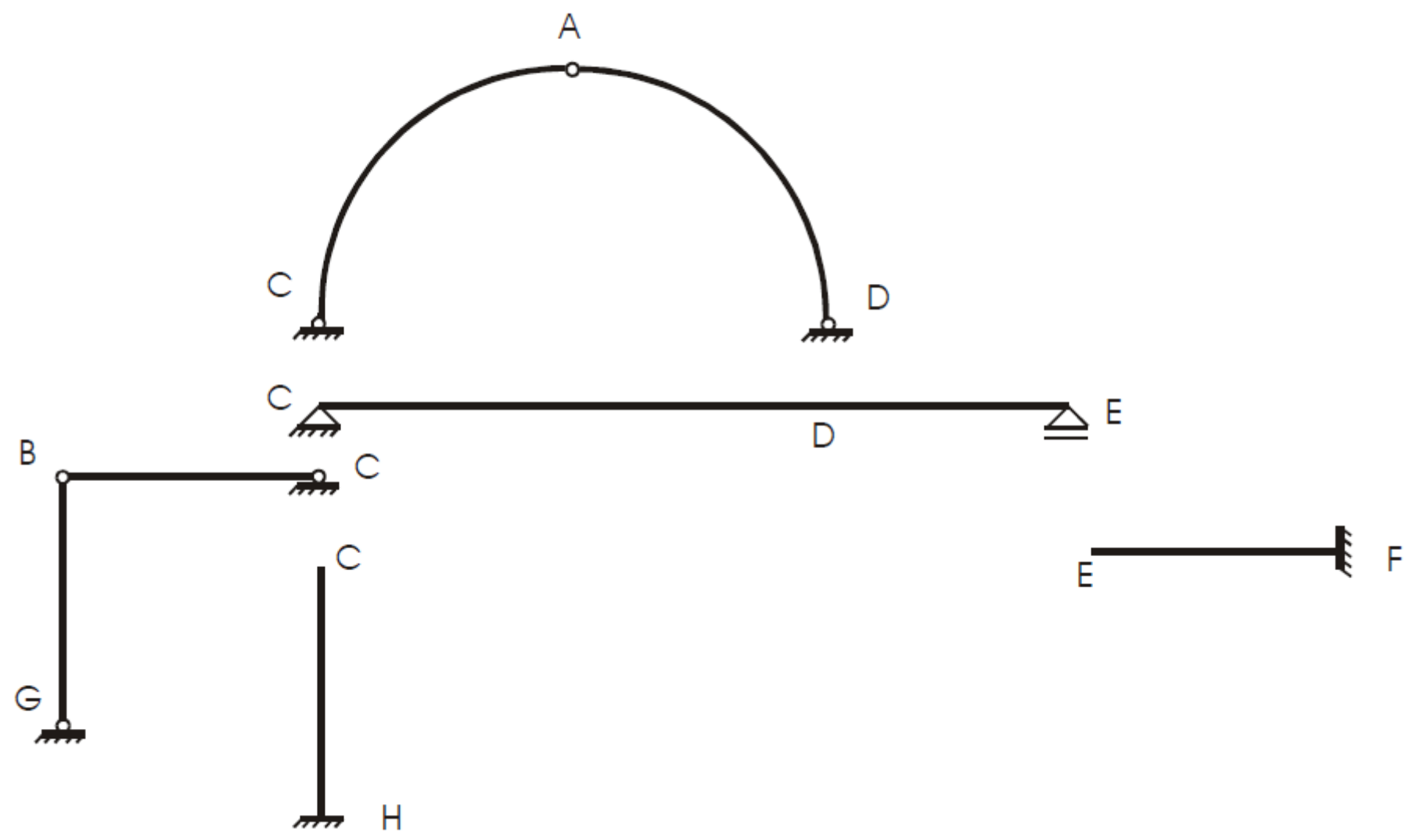
Para a estrutura associada da figura, determinar:

# Exemplo 11: P3 2004

- a) As subestruturas que a compõem;
- b) As reações de cada uma destas subestruturas.

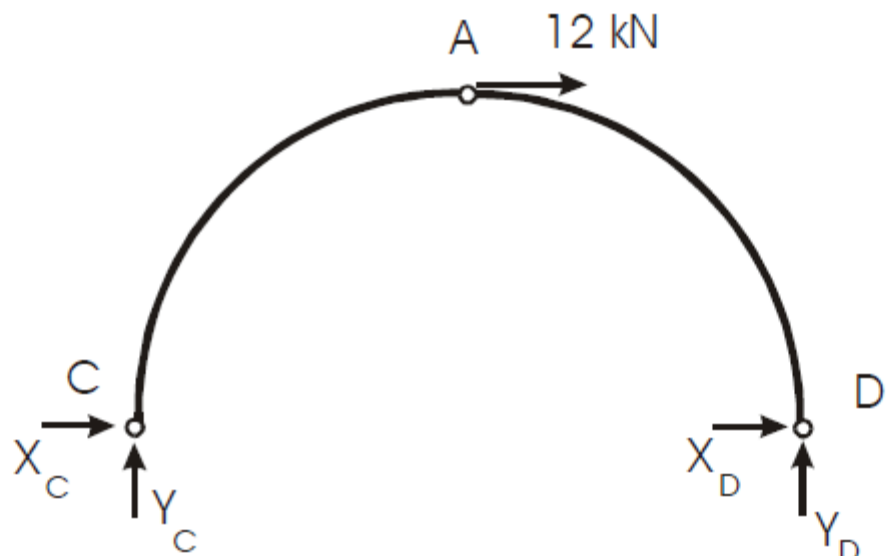


a)



b)

1)

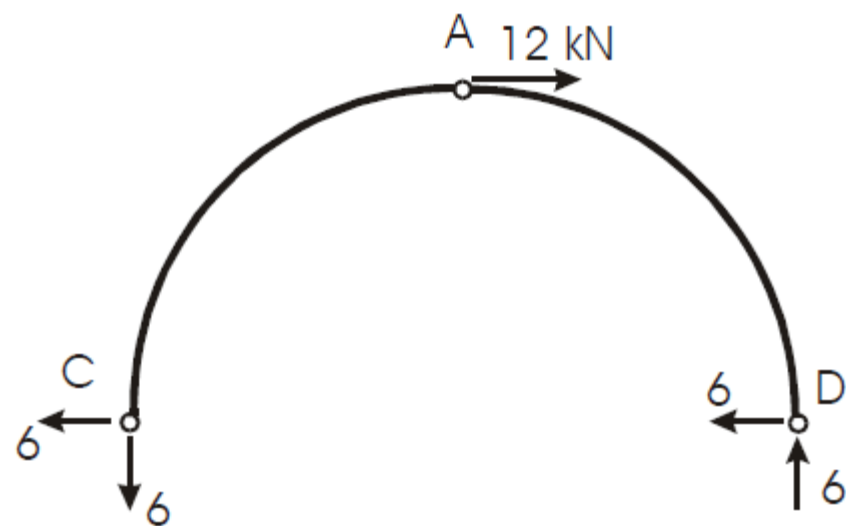


$$\sum M_{(C)} = 0 \quad -12 \cdot 1 + Y_D \cdot 2 = 0 \quad Y_D = 6 \text{ kN}$$

$$\sum Y = 0 \quad Y_C + Y_D = 0 \quad Y_C = -6 \text{ kN}$$

$$M_{\text{fleitor em A}} = 0 \quad X_D \cdot 1 + Y_D \cdot 1 = 0 \quad X_D = -6 \text{ kN}$$

$$\sum X = 0 \quad X_C + X_D + 12 = 0 \quad X_C = -6 \text{ kN}$$



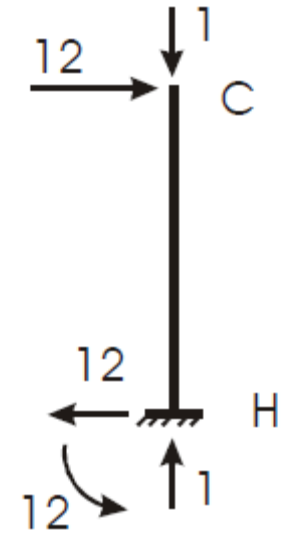
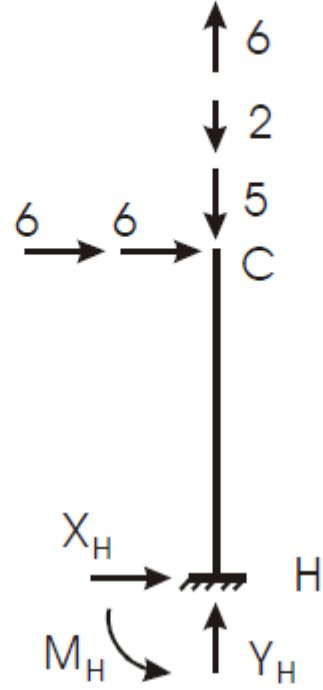
2)



$$\begin{array}{lll} \Sigma M_{(E)} = 0 & -Y'_C \cdot 3 + 6 \cdot 1 = 0 & Y'_C = 2 \text{ kN} \\ \Sigma X = 0 & X'_C + 6 = 0 & X'_C = -6 \text{ kN} \\ \Sigma Y = 0 & Y'_C - 6 + Y_E = 0 & Y_E = 4 \text{ kN} \end{array}$$



3)



$$\sum X = 0$$

$$X_H + 6 + 6 = 0$$

$$X_H = -12 \text{ kN}$$

$$\sum Y = 0$$

$$Y_H - 5 - 2 + 6 = 0$$

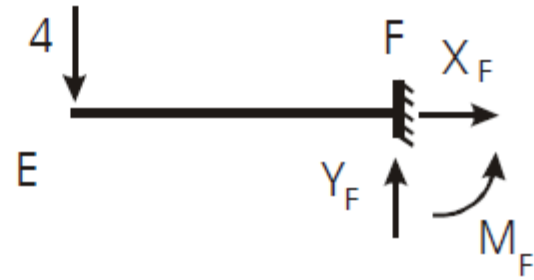
$$Y_H = 1 \text{ kN}$$

$$\sum M_{(H)} = 0$$

$$M_H - 6 \cdot 1 \cdot 2 = 0$$

$$M_H = 12 \text{ kNm}$$

4)



$$\sum X = 0$$

$$X_F = 0$$

$$\sum Y = 0$$

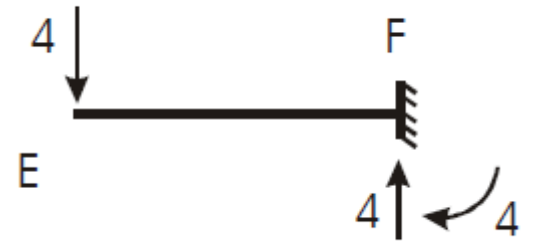
$$-4 + Y_F = 0$$

$$Y_F = 4 \text{ kN}$$

$$\sum M_{(F)} = 0$$

$$4 \cdot 1 + M_F = 0$$

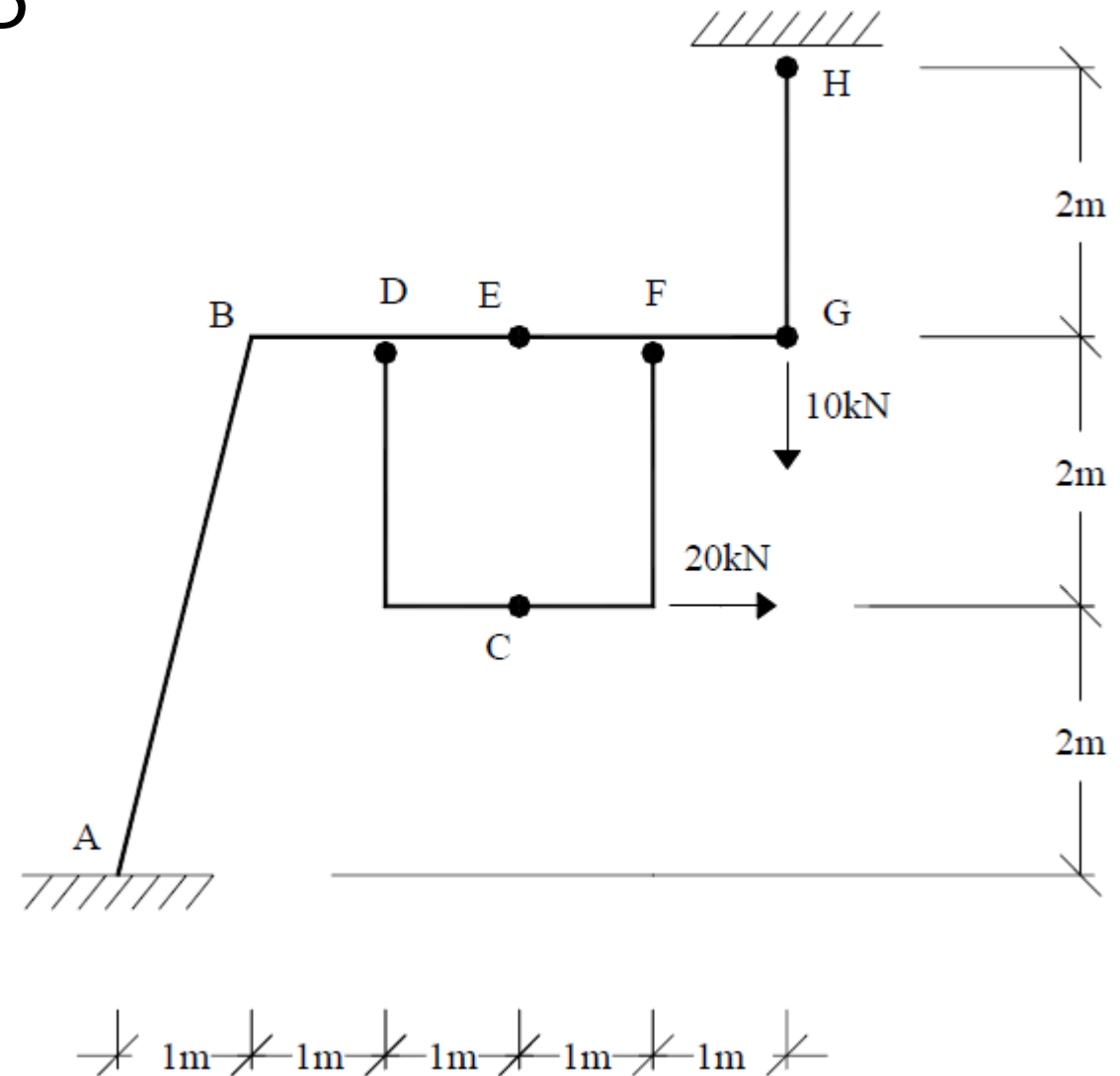
$$M_F = -4 \text{ kNm}$$



# Exemplo 12: SUB 2003

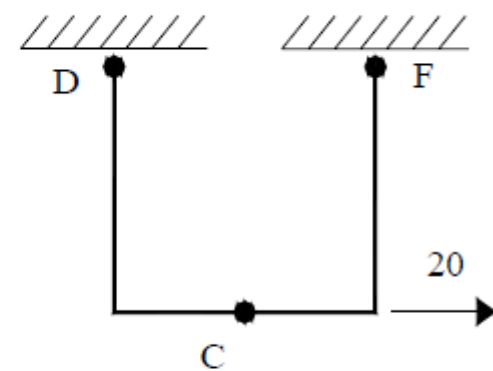
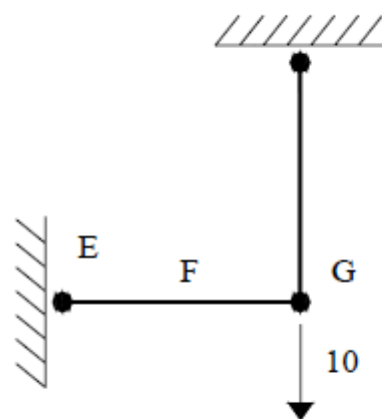
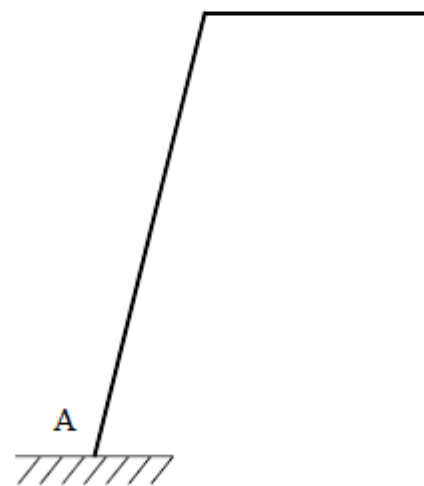
Na estrutura associada da figura, determine:

- as subestruturas;
- as reações em  $A$ ;
- o diagrama do momento fletor no trecho  $EFGH$ .



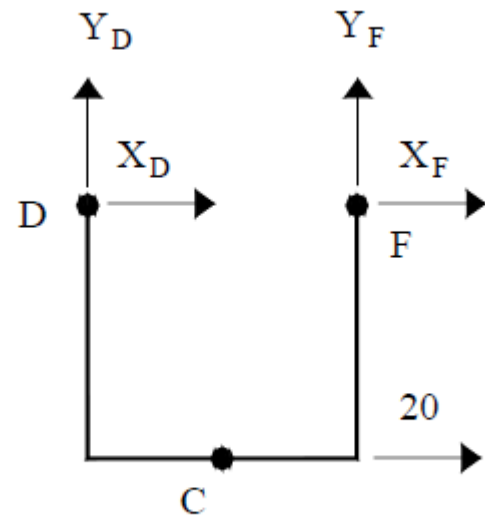


a) (1,0)



b)

1)

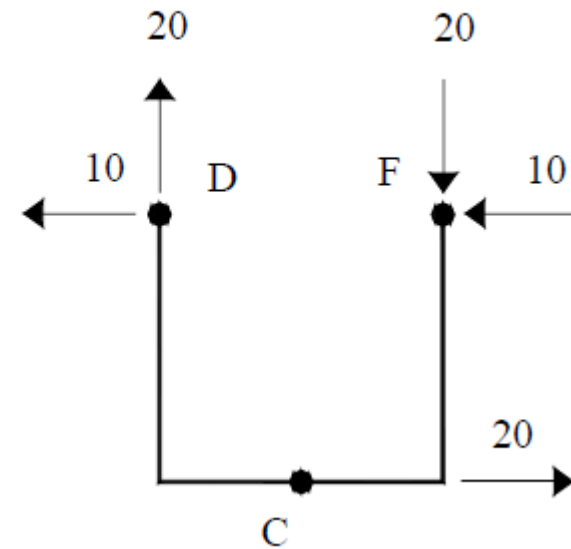


$$\sum M_D = 0 = Y_F \cdot 2 + 20 \cdot 2 \Rightarrow Y_F = -20 \text{ kN}$$

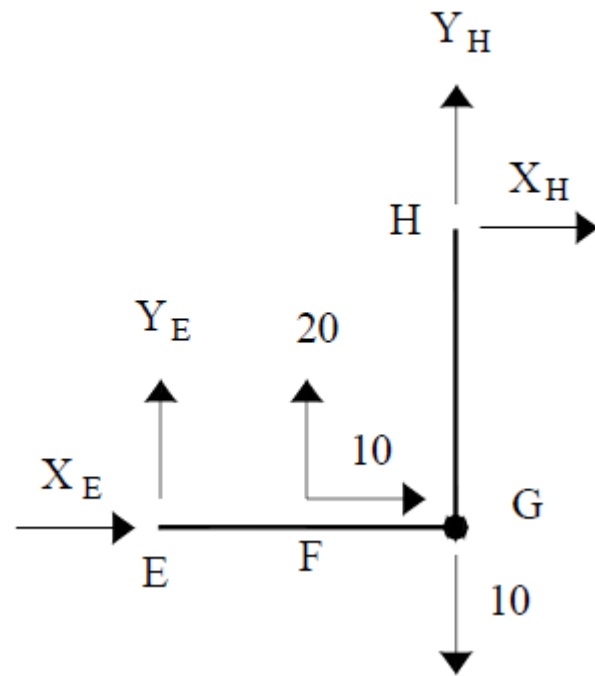
$$\sum Y = 0 = Y_D + Y_F \Rightarrow Y_D = 20 \text{ kN}$$

$$M_C = 0 = Y_F \cdot 1 - X_F \cdot 2 \Rightarrow X_F = -10 \text{ kN}$$

$$\sum X = 0 = X_D + X_F + 20 \Rightarrow X_D = -10 \text{ kN}$$



2)

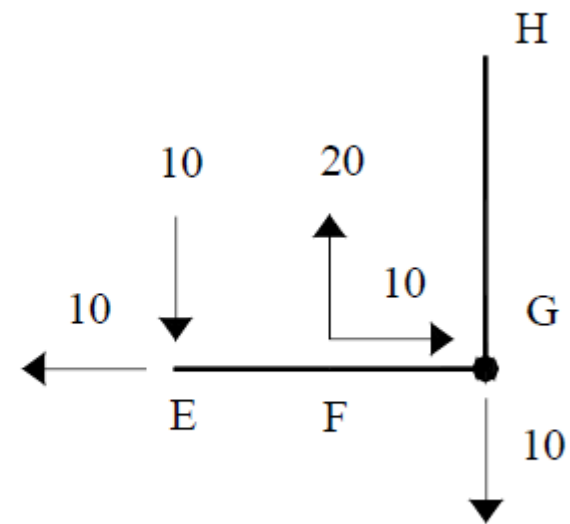


$$M_G = 0 = X_H$$

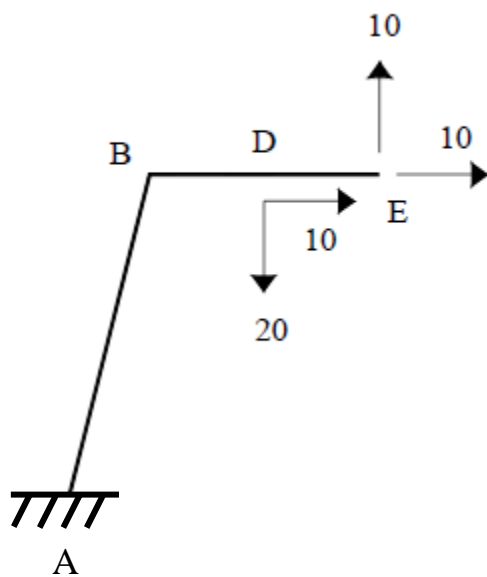
$$\sum X = 0 = X_E + 10 + X_H \Rightarrow X_E = -10 \text{ kN}$$

$$\sum M_E = 0 = 20 \cdot 1 - 10 \cdot 2 + Y_H \cdot 2 - X_H \cdot 2 \Rightarrow Y_H = 0$$

$$\sum Y = 0 = Y_E + 20 - 10 + Y_H \Rightarrow Y_E = -10 \text{ kN}$$



3)

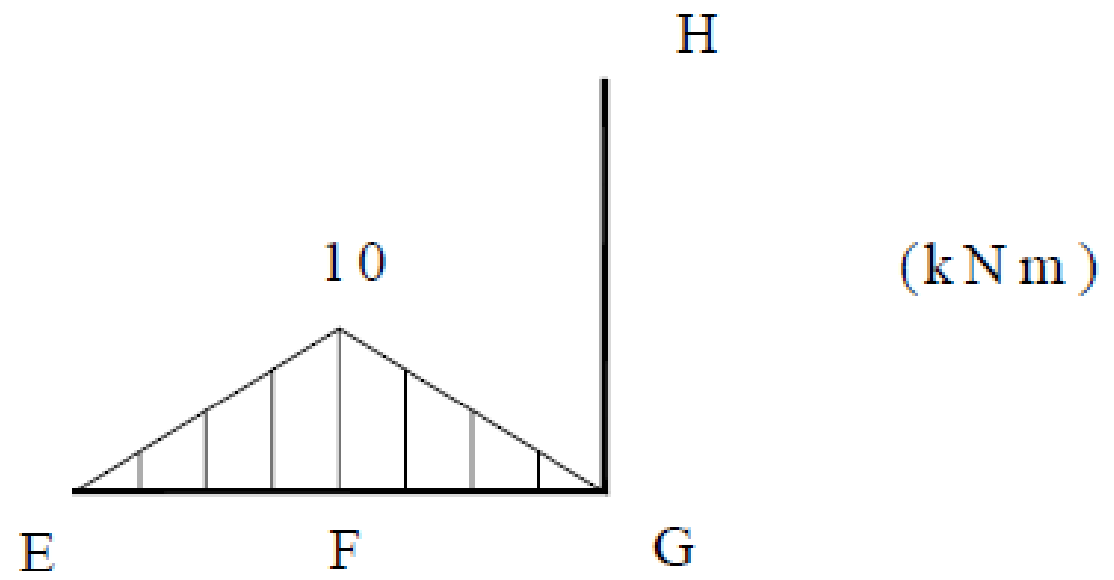


$$\sum X = 0 = Y_A + 10 + 10 \Rightarrow X_A = -20kN$$

$$\sum Y = 0 = Y_A + 10 - 20 \Rightarrow Y_A = 10kN$$

$$\sum M_A = 0 = -20 \cdot 2 - 10 \cdot 4 - 10 \cdot 4 + 10 \cdot 3 + M_A \Rightarrow M_A = 90kN$$

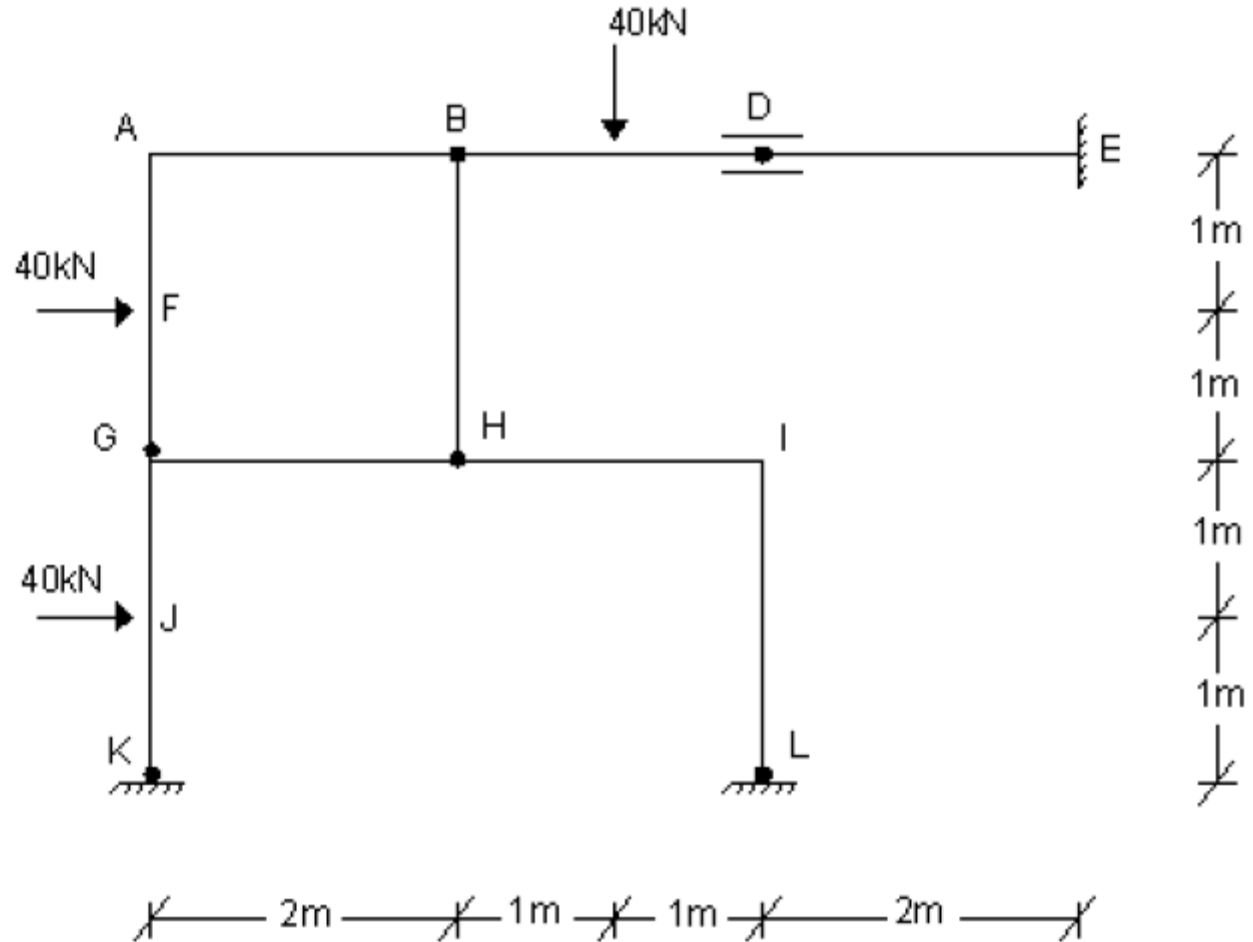
c)

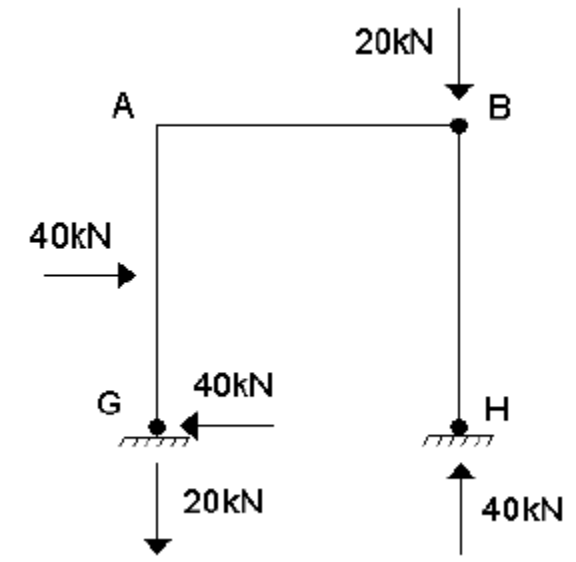
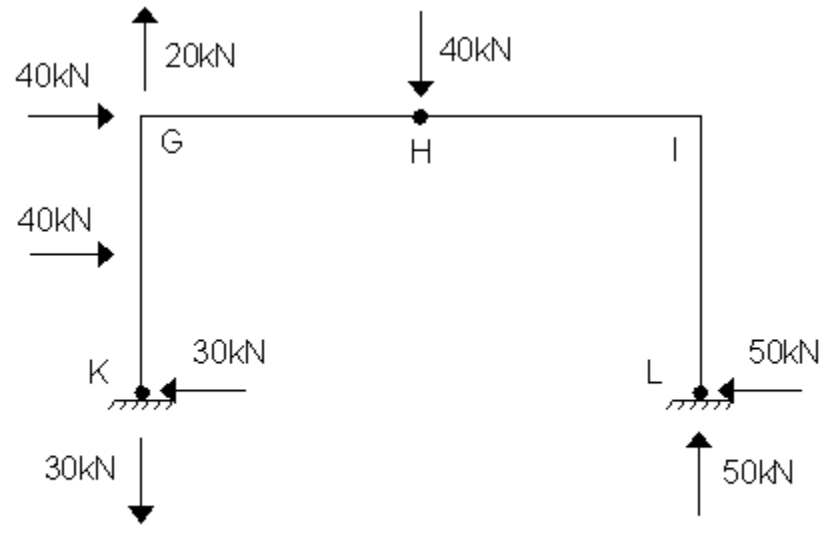
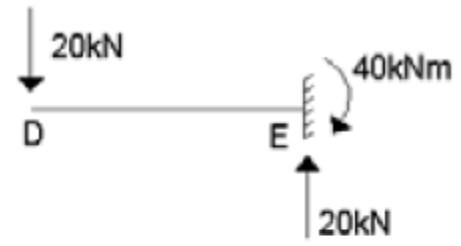
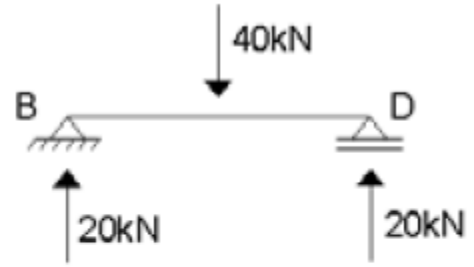


# Exemplo 13: Q3 2003

Para a estrutura associada da figura:

- Decompô-la nas subestruturas que a formam;
- Determinar as reações de apoio de cada uma destas subestruturas.



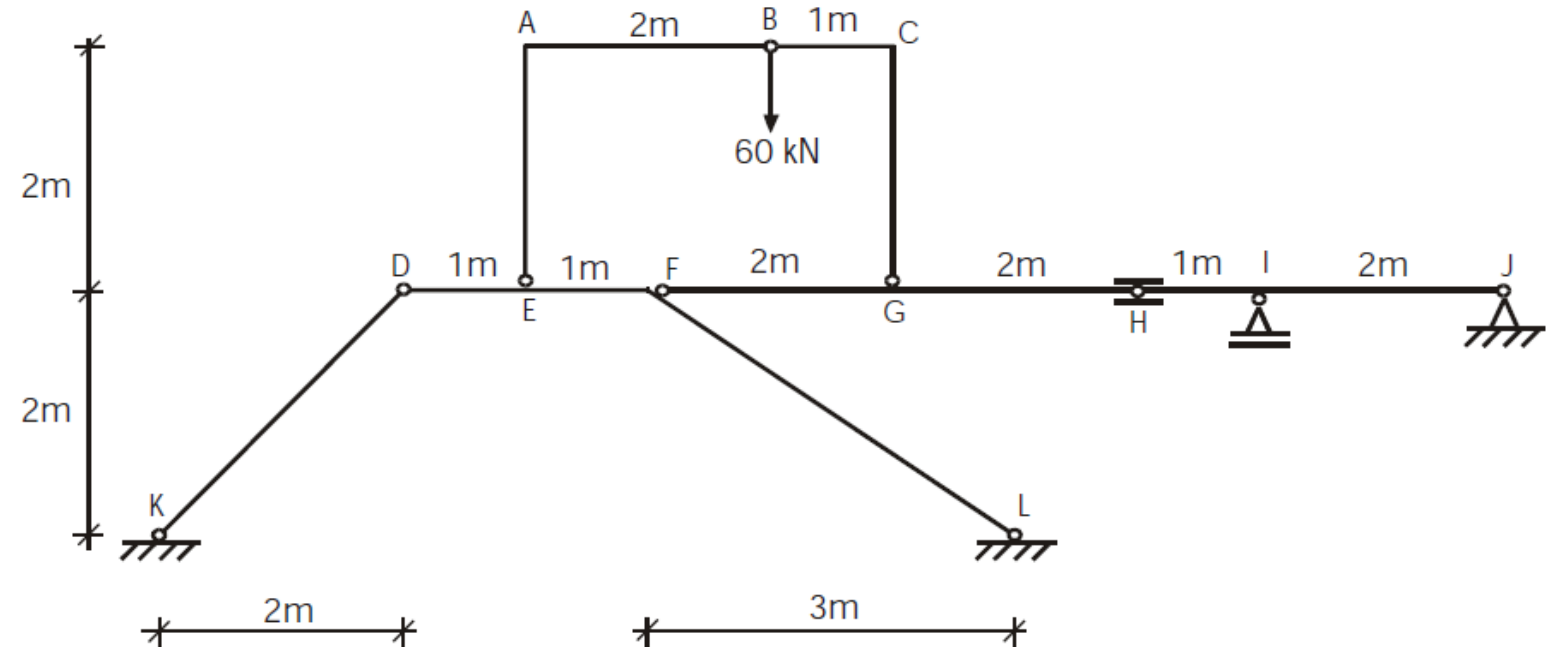
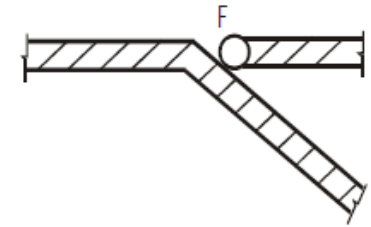


# Exemplo 14: P3 2002

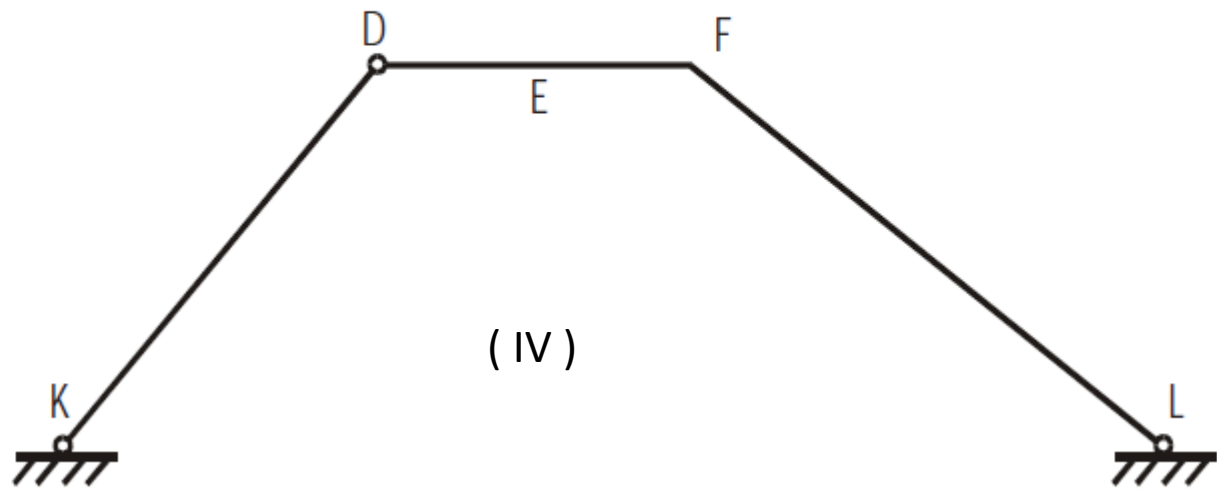
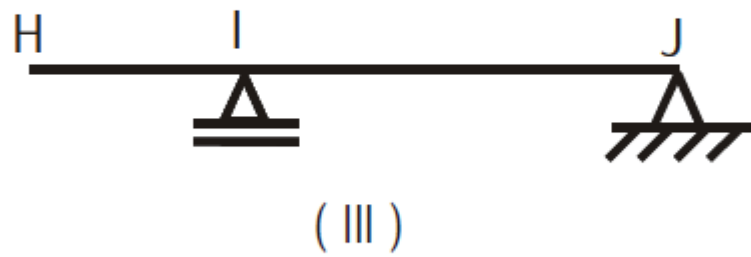
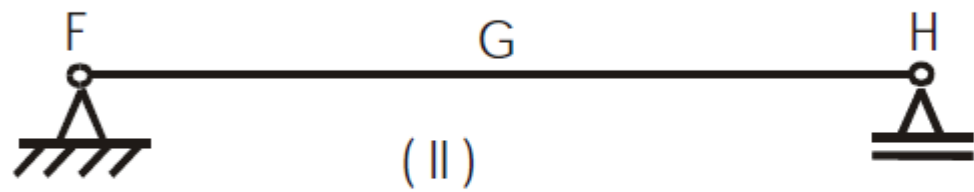
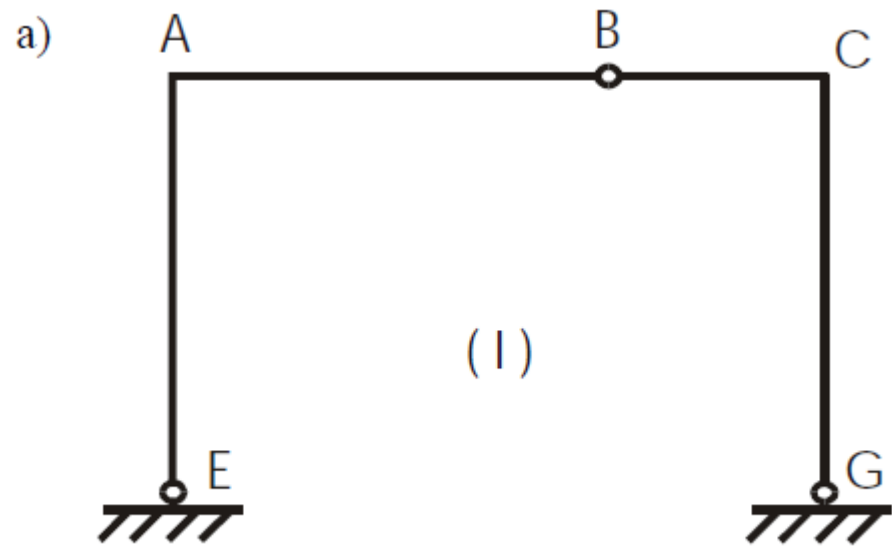
Para a estrutura da figura, solicitada por uma carga vertical de 60 kN em B, obter:

- as subestruturas das quais ela se compõe;
- as reações de apoio das subestruturas e da estrutura como um todo;
- os diagramas de esforços solicitantes do trecho DEF.

Obs: a articulação em F é externa ao trecho DEFL, conforme o detalhe.

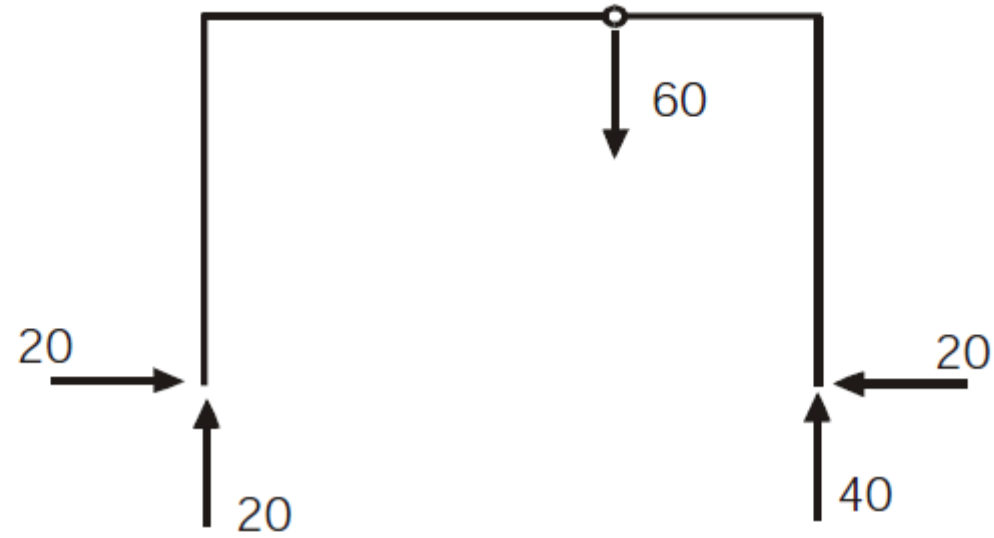




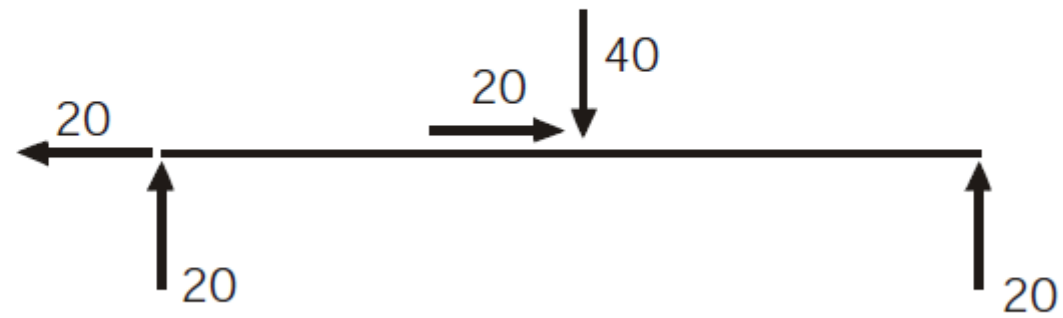


b)

$$(I) \quad \sum M_E = 0$$
$$\sum V = 0$$
$$M_{Ee} = 0$$
$$\sum H = 0$$



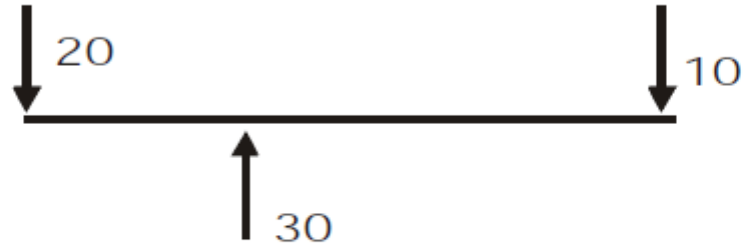
$$(II) \quad \sum H = 0$$
$$\sum M_F = 0$$
$$\sum V = 0$$



$$(III) \quad \sum H = 0$$

$$\sum M_I = 0$$

$$\sum V = 0$$

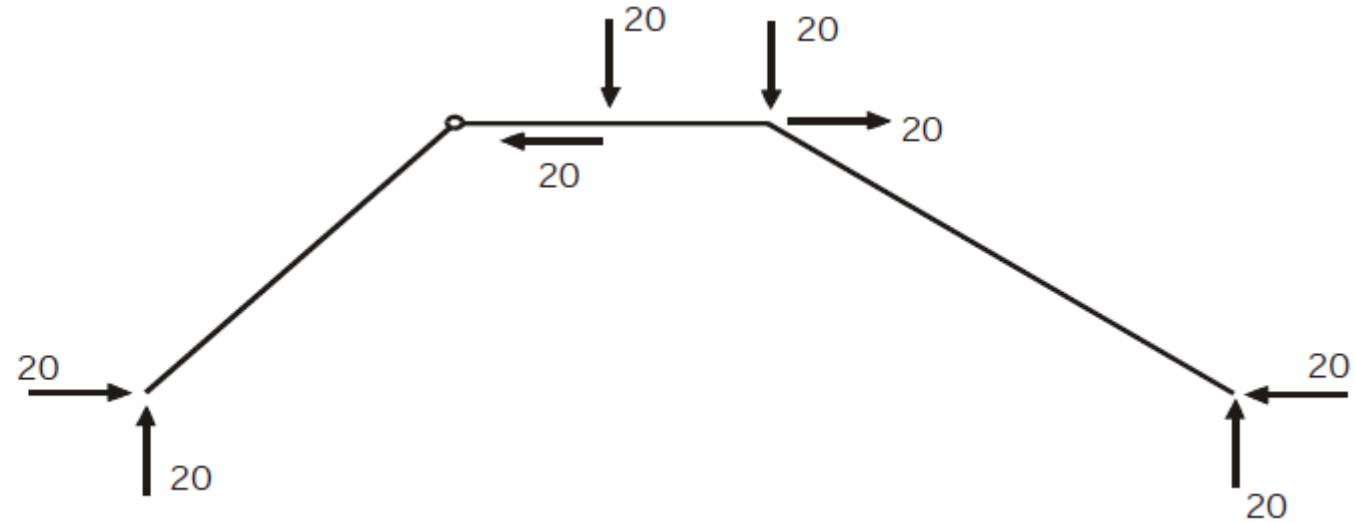


$$(IV) \quad \sum M_K = 0$$

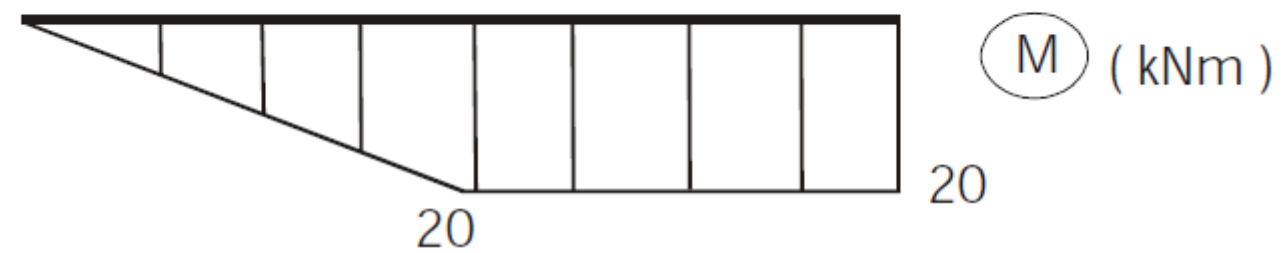
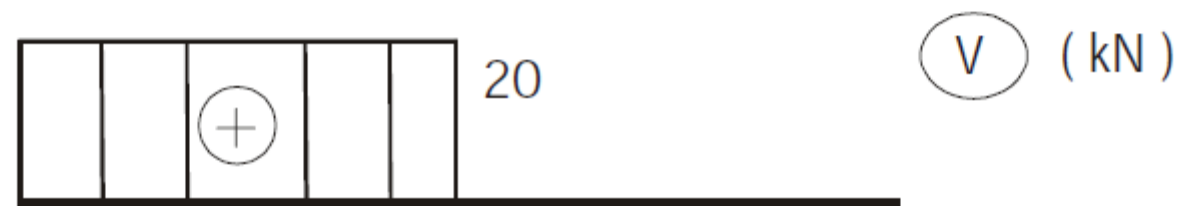
$$\sum V = 0$$

$$M_{De} = 0$$

$$\sum H = 0$$

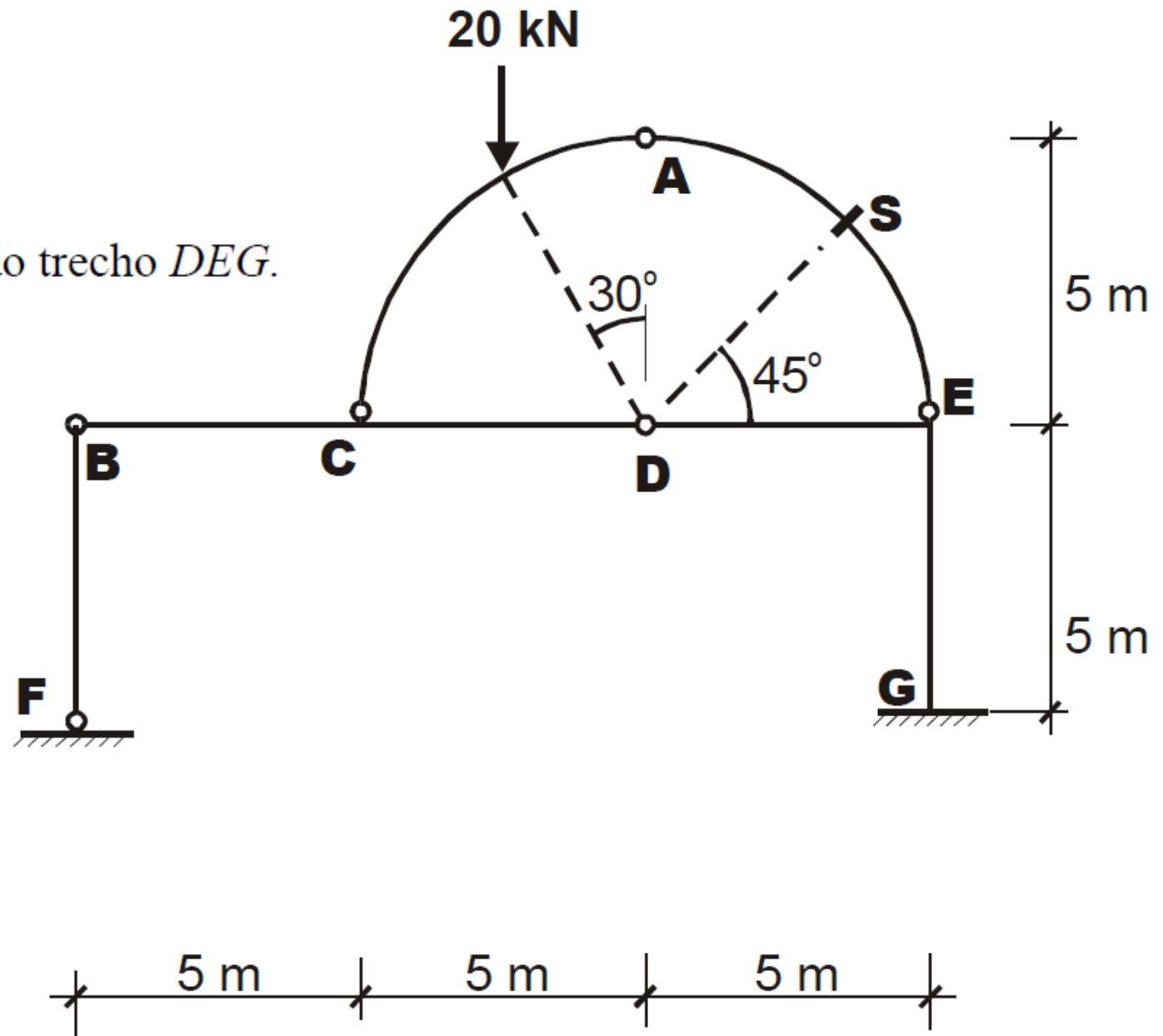


c)

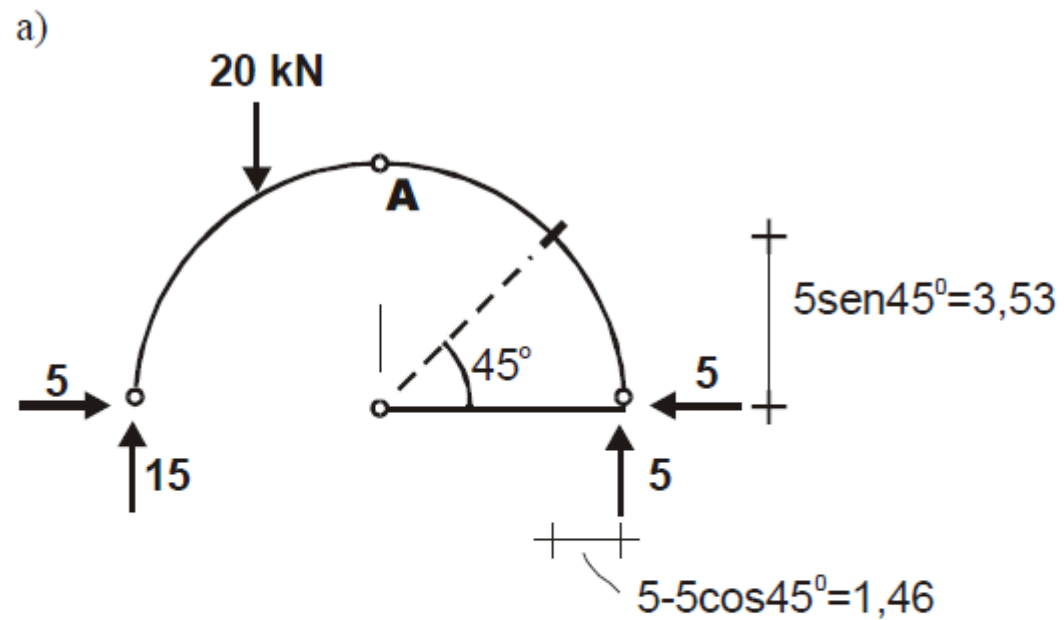


Para a estrutura da figura, determinar:

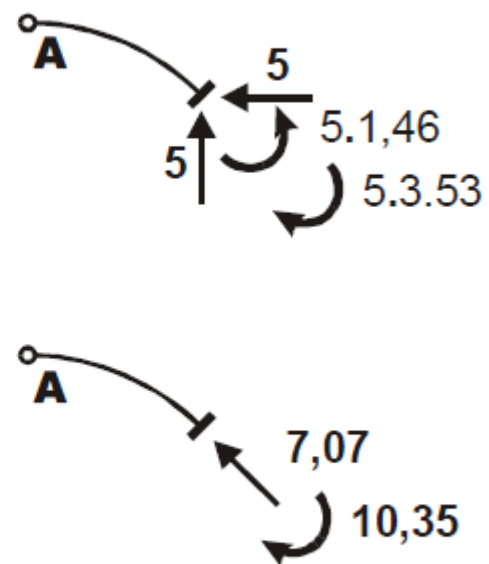
- os esforços solicitantes em  $S$ ;
- o diagrama de momentos fletores do trecho  $DEG$ .



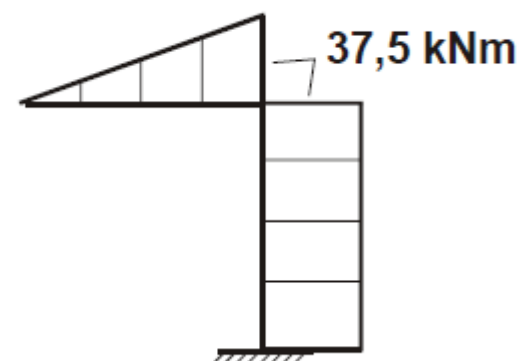
Exemplo 15  
P3 2001



$N = -7,07 \text{ kN}$   
 $V = 0$   
 $M = -10,35 \text{ kNm}$

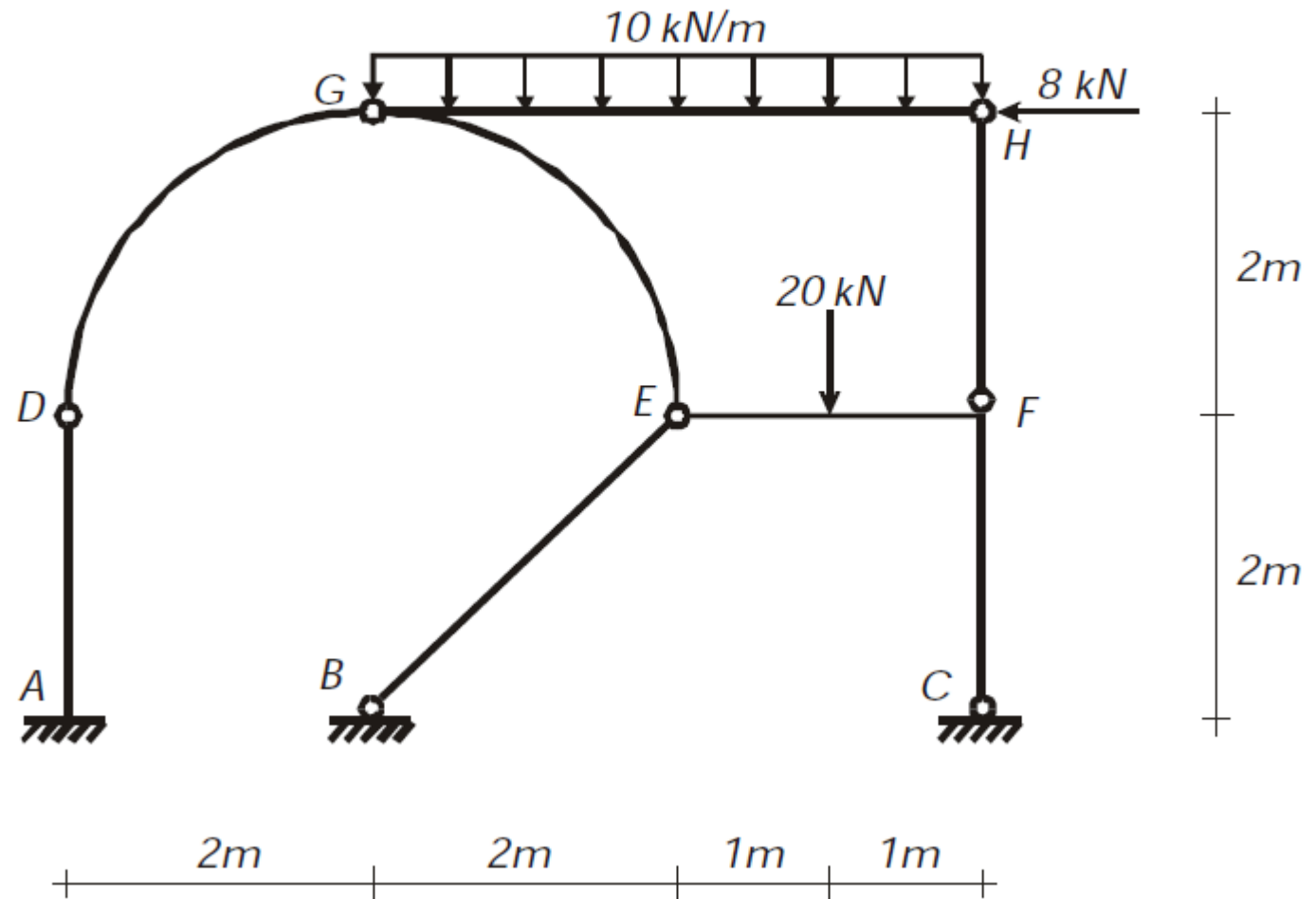


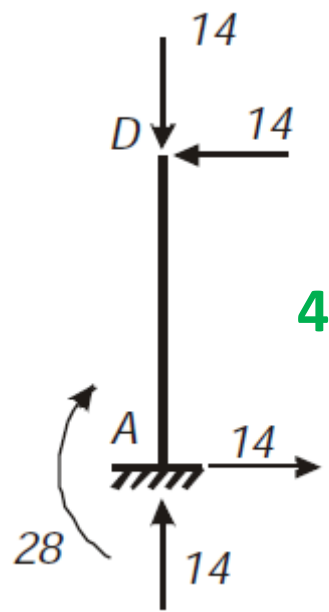
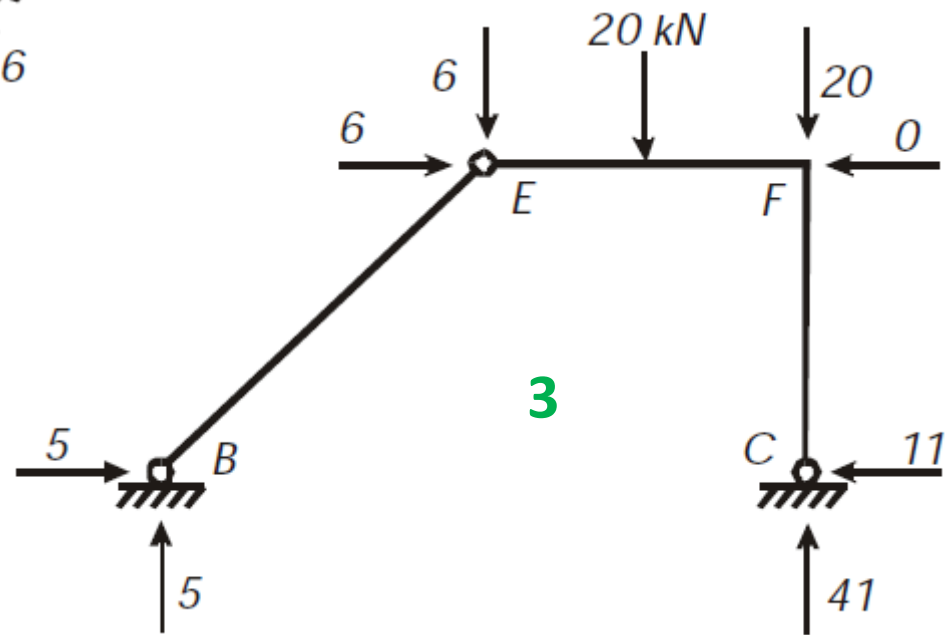
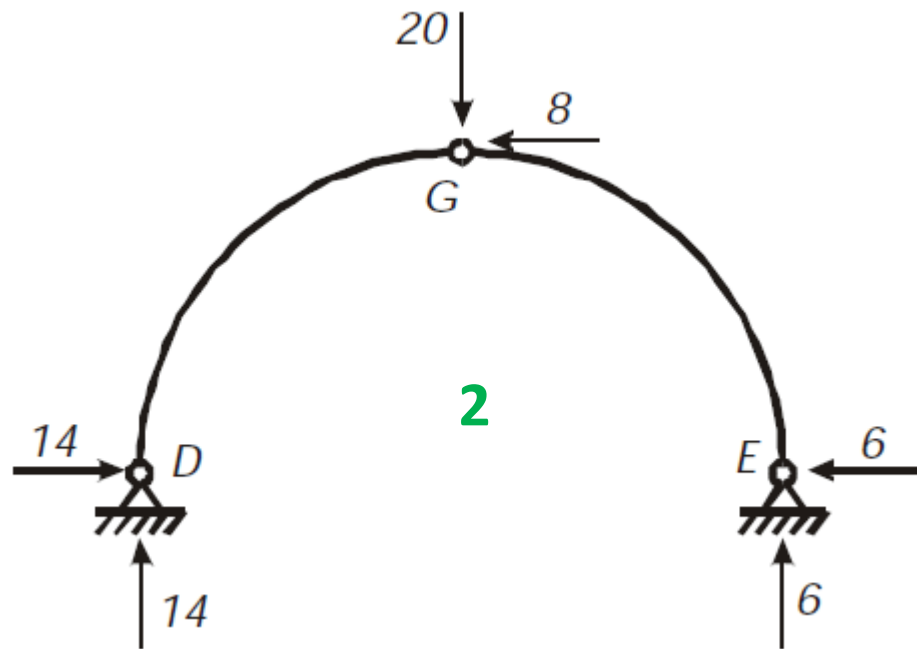
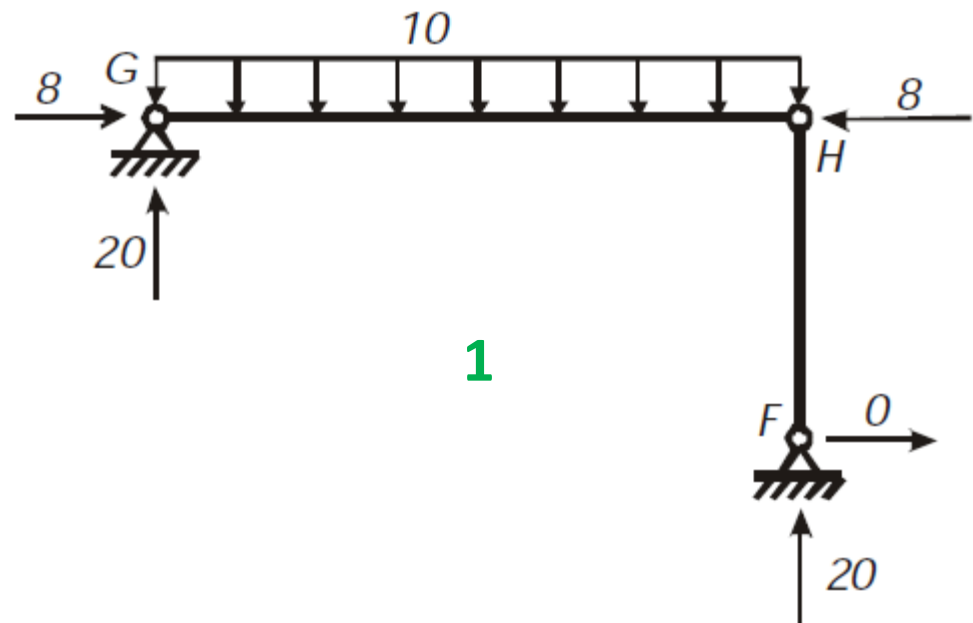
b)



# Exemplo 16: P3 1999

Determinar as reações vinculares da estrutura associada da figura abaixo.







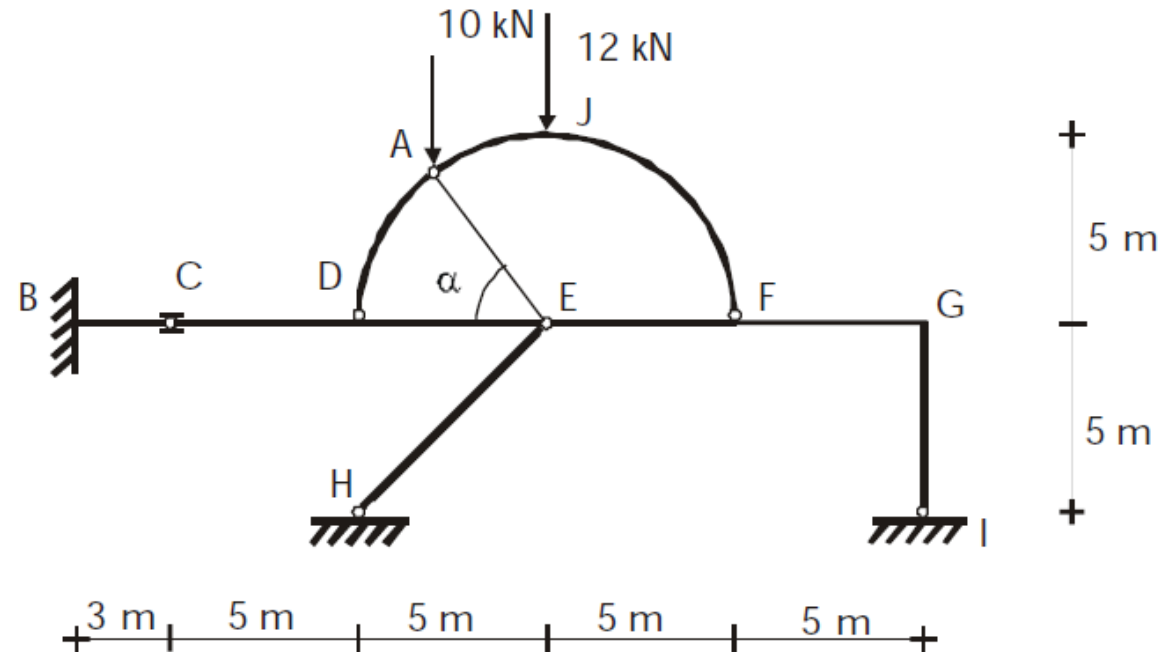
# Exemplo 17: P3 1998

Para a estrutura da figura abaixo:

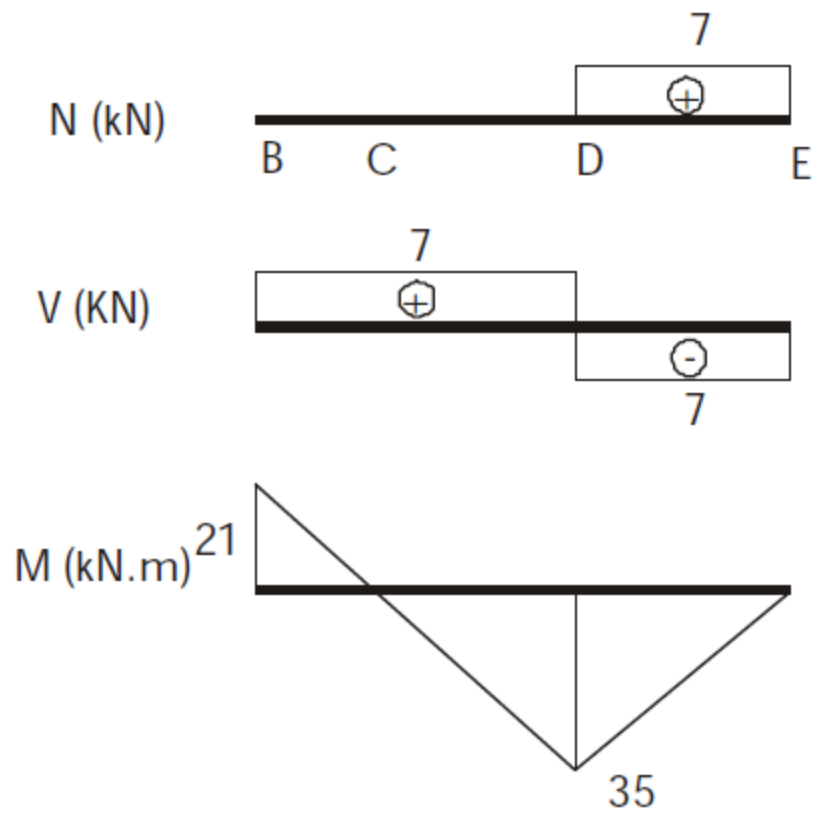
- Determinar as reações vinculares;
- Traçar os diagramas de esforços solicitantes **do trecho *BCDE***.

O trecho *DAJF* corresponde a uma semicircunferência de raio igual a 5 m e centro em *E*.

São dados:  $\text{sen}\alpha = 0,8$  e  $\text{cos}\alpha = 0,6$ .





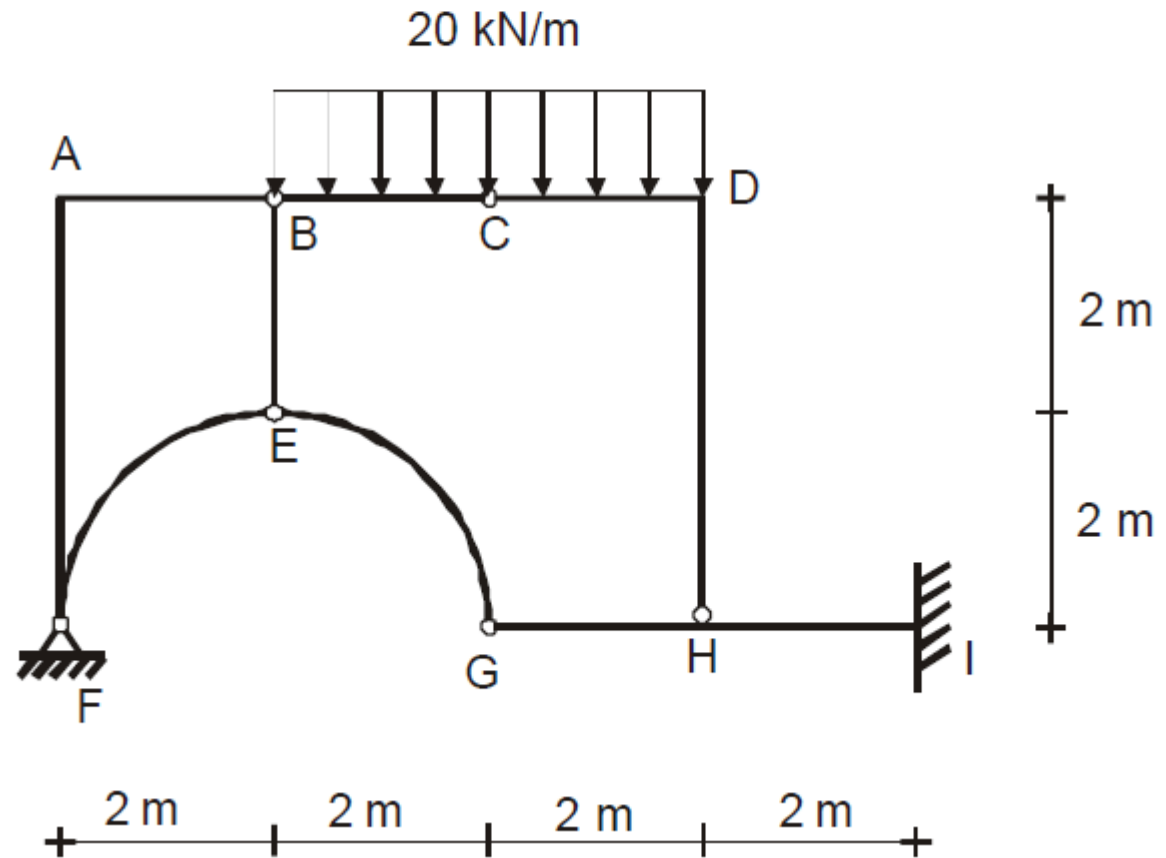


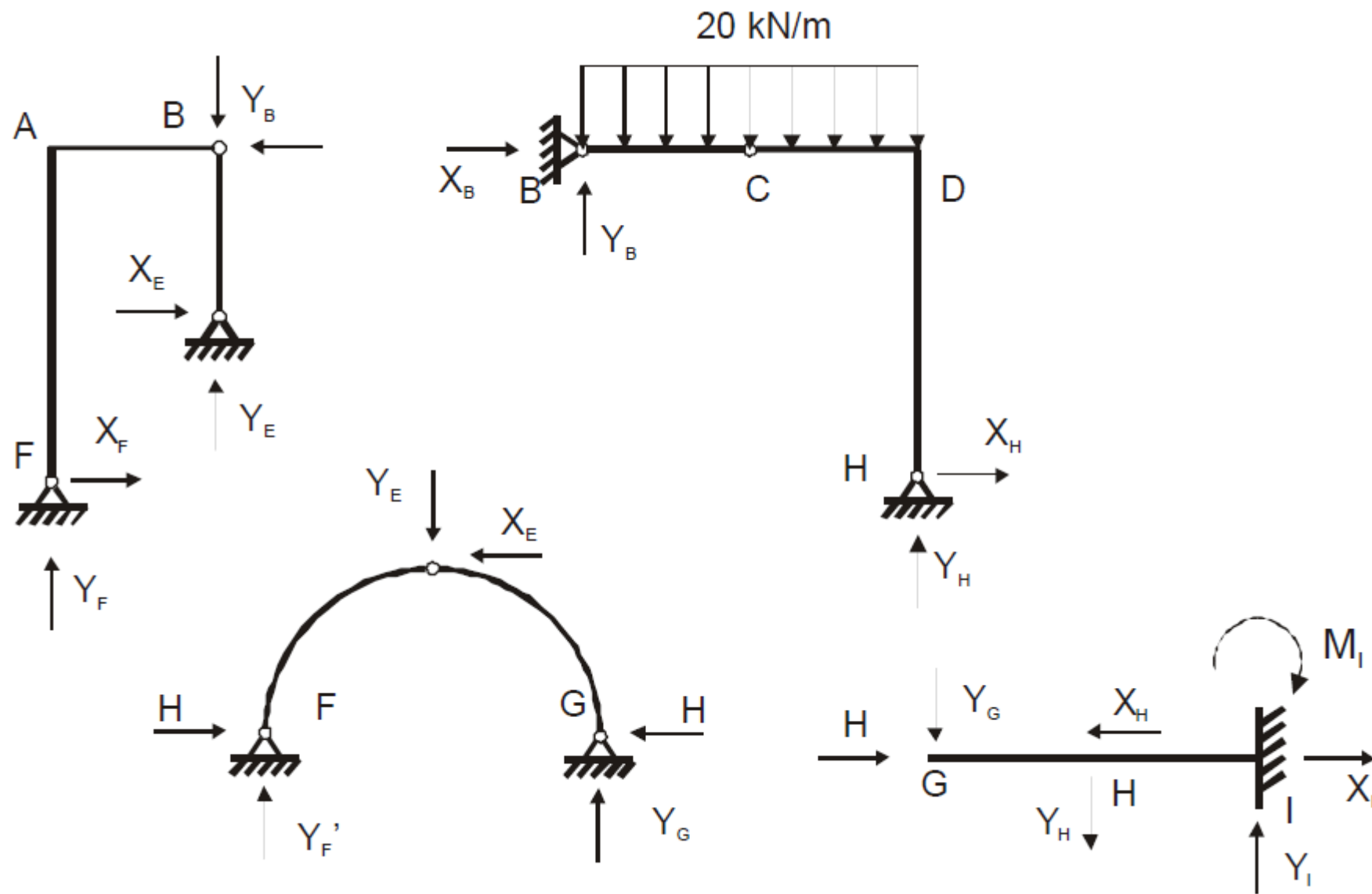
# Exemplo 18: REC 1998

Para a estrutura da figura:

a) Determinar as reações de apoio em  $F$  e  $I$ .

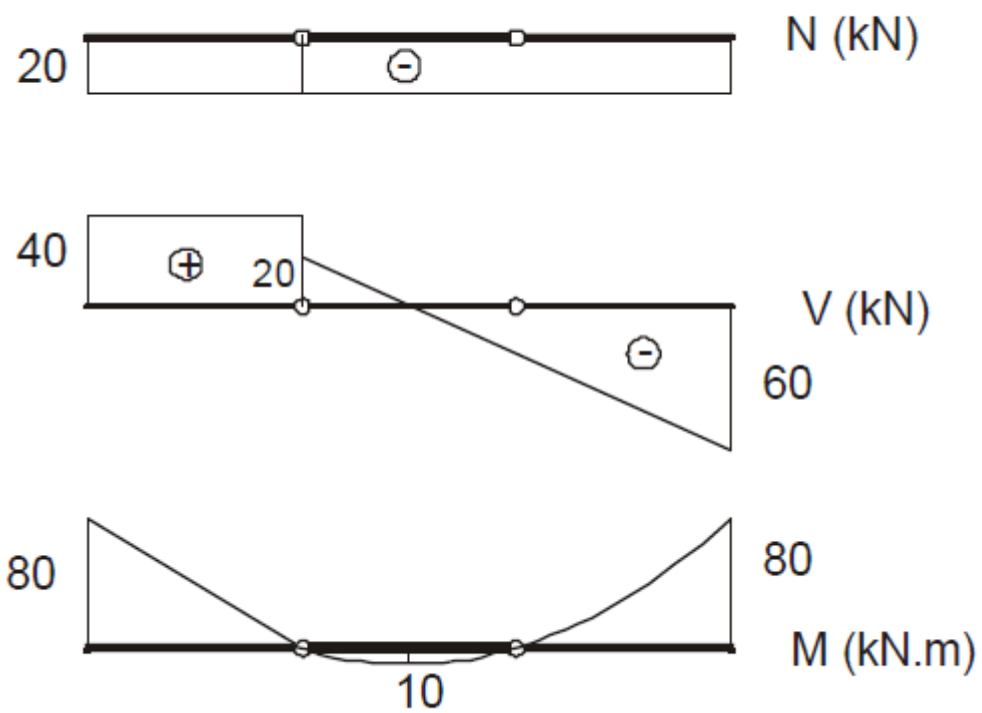
b) Traçar os diagramas de esforços solicitantes dos trechos  $ABCD$  e  $GHI$ .



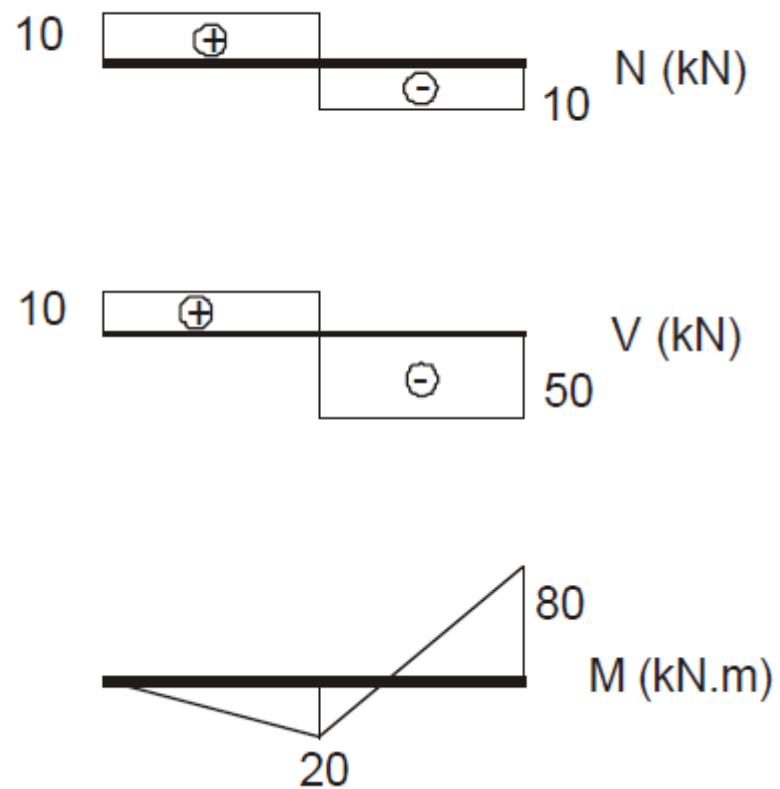


1.  $X_B = 20$
2.  $Y_B = 20$
3.  $X_H = -20$
4.  $Y_H = 60$
5.  $X_E = 0$
6.  $Y_E = -20$
7.  $X_F = 20$
8.  $Y_F = 40$
9.  $Y_F' = -10$
10.  $Y_G = -10$
11.  $H = -10$
12.  $X_I = -10$
13.  $Y_I = 50$
14.  $M_I = 80$

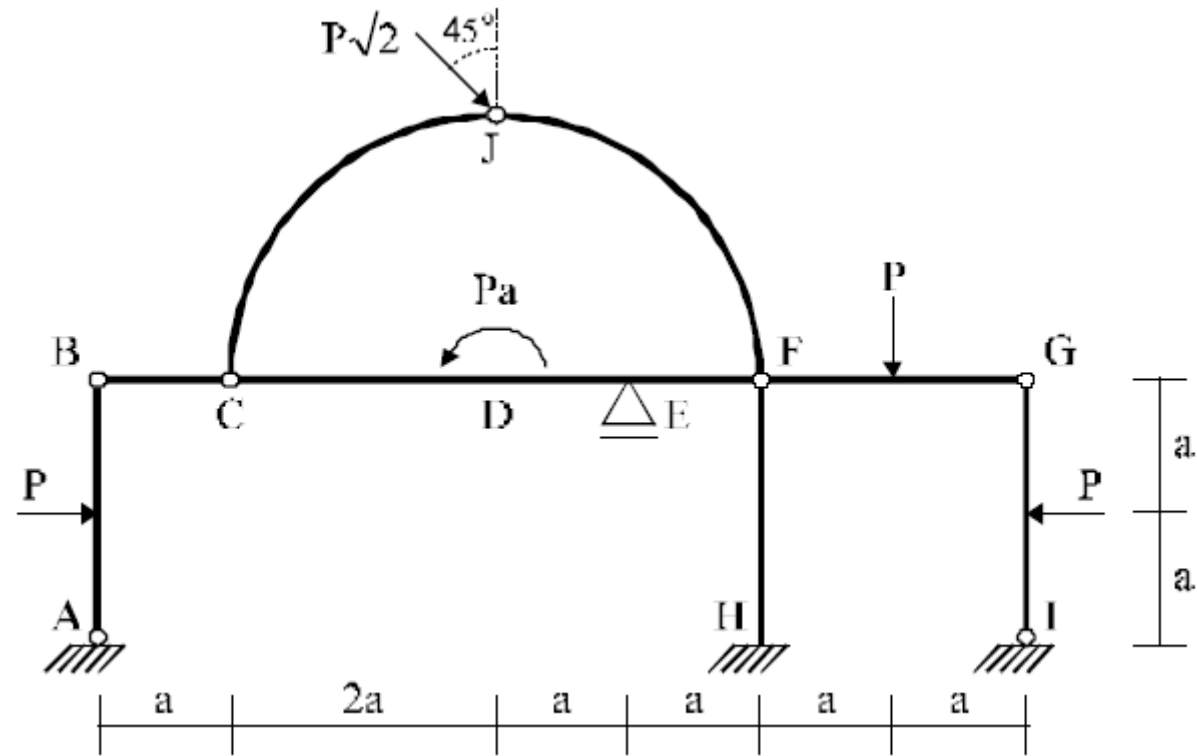
Trecho ABCD



Trecho GHI



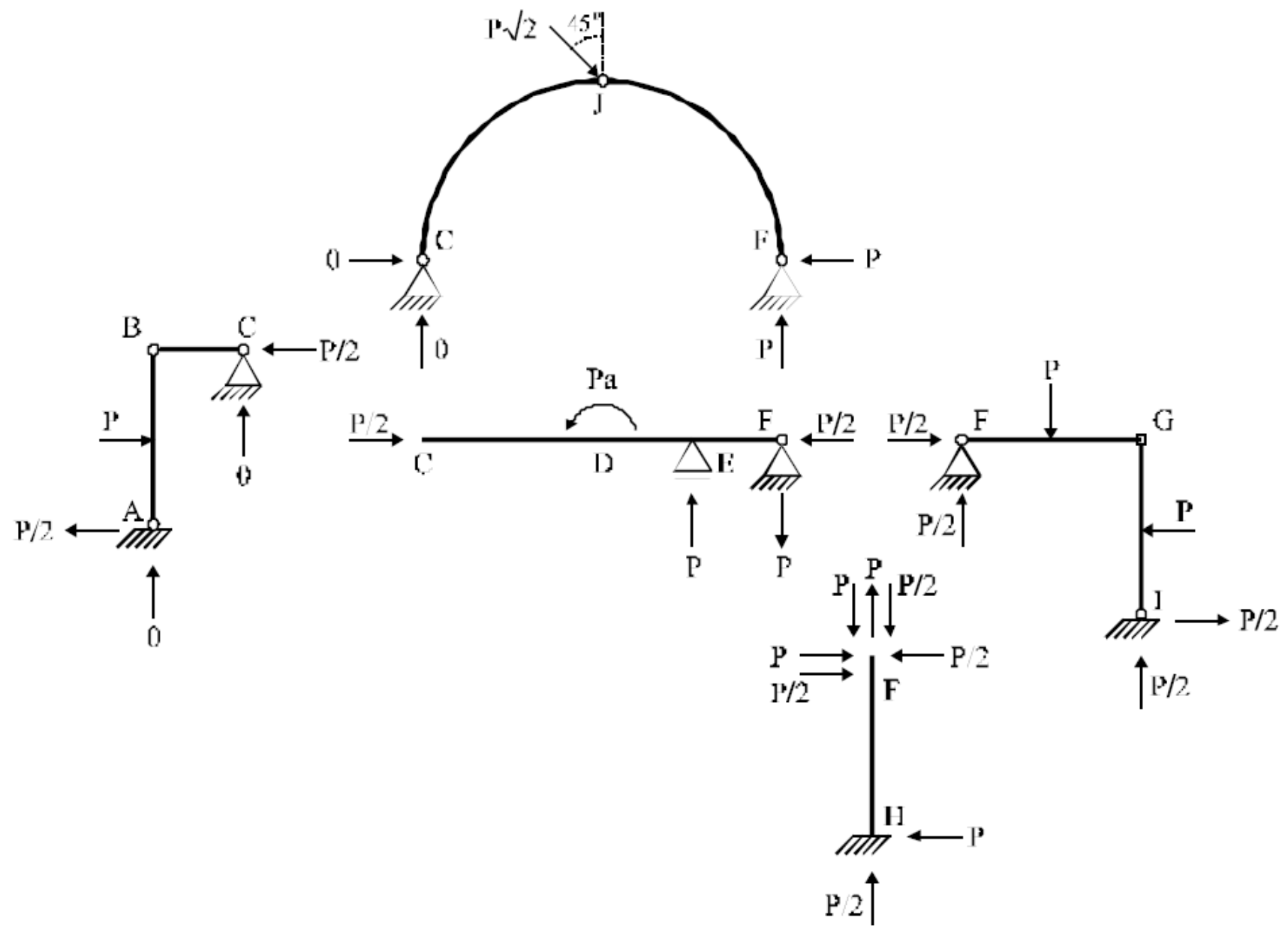
# Exemplo 19: P2 1996



Determinar as reações vinculares para a estrutura abaixo. Traçar os diagramas de esforços solicitantes para o trecho BCDEF.

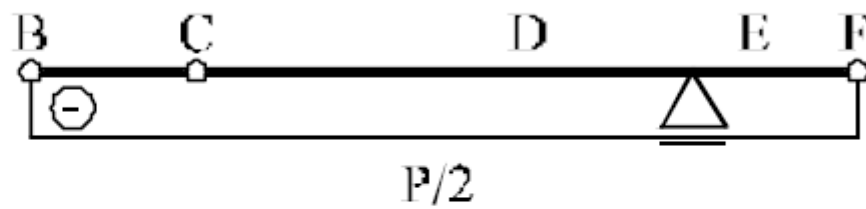
CJF corresponde a uma semi-circunferência de raio  $2a$  e centro D.

O momento concentrado  $Pa$  está aplicado em D.

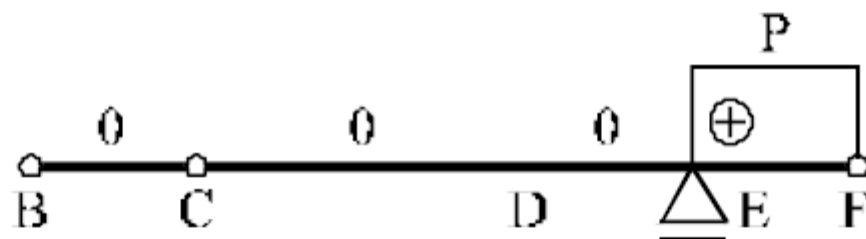




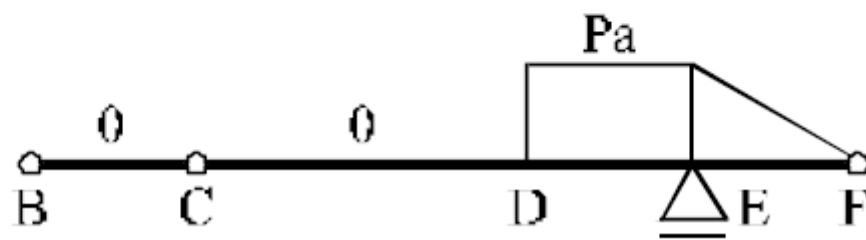
(N)



(V)

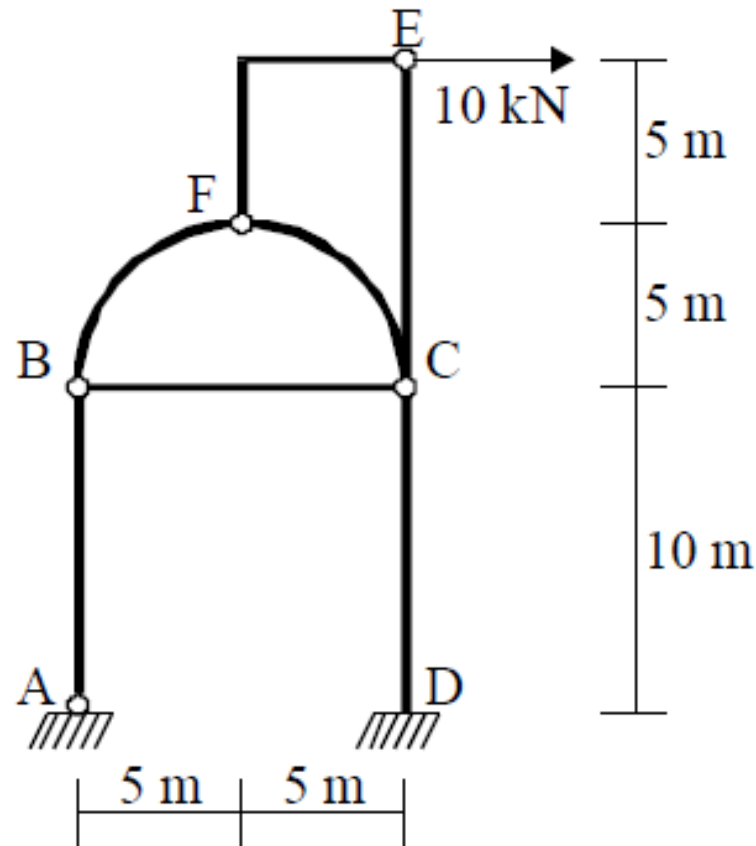


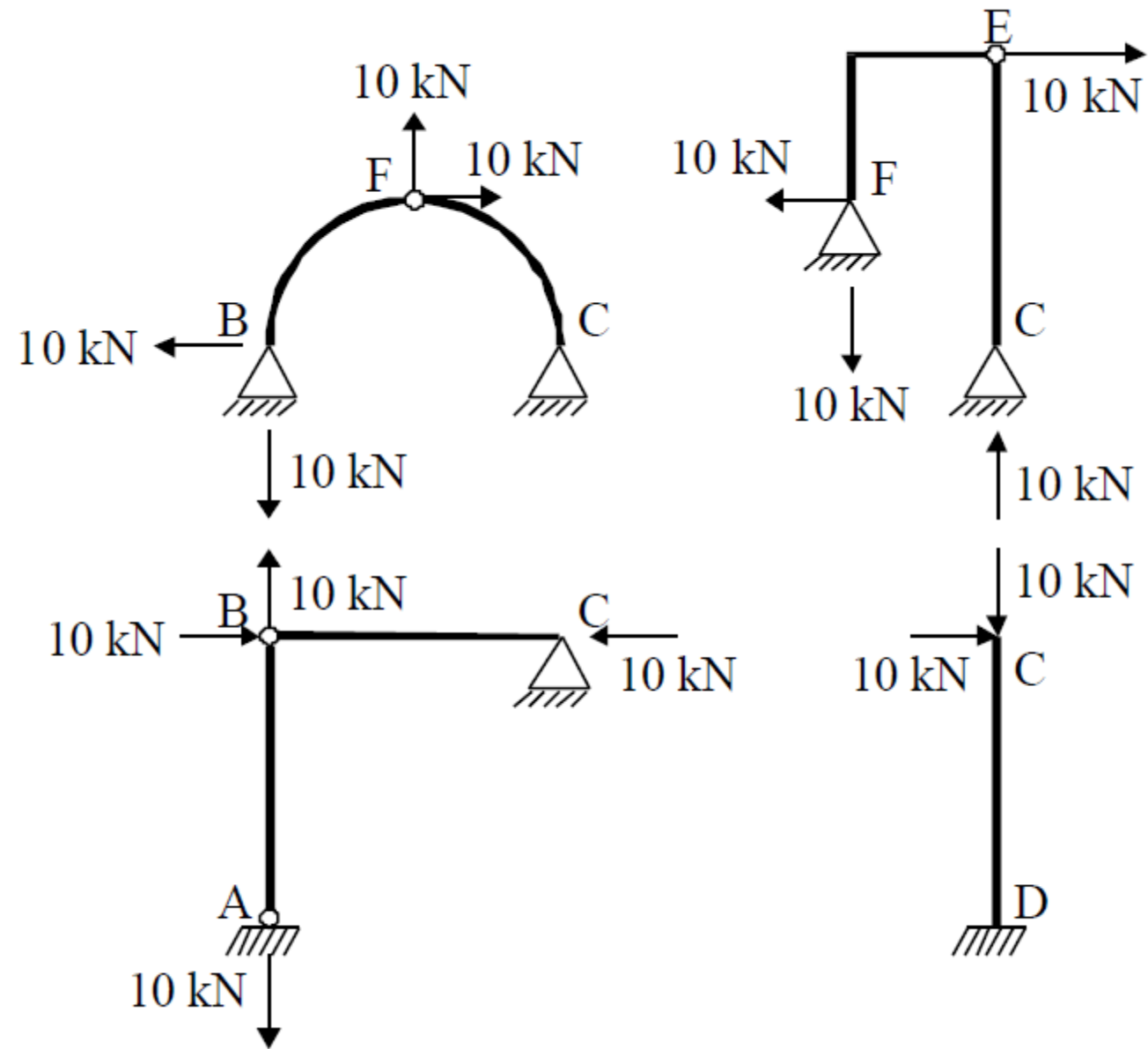
(M)

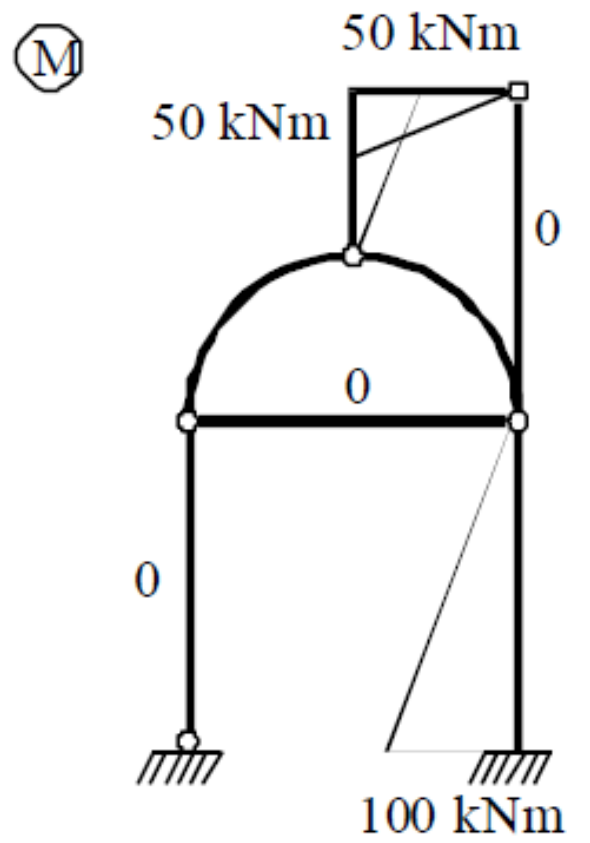
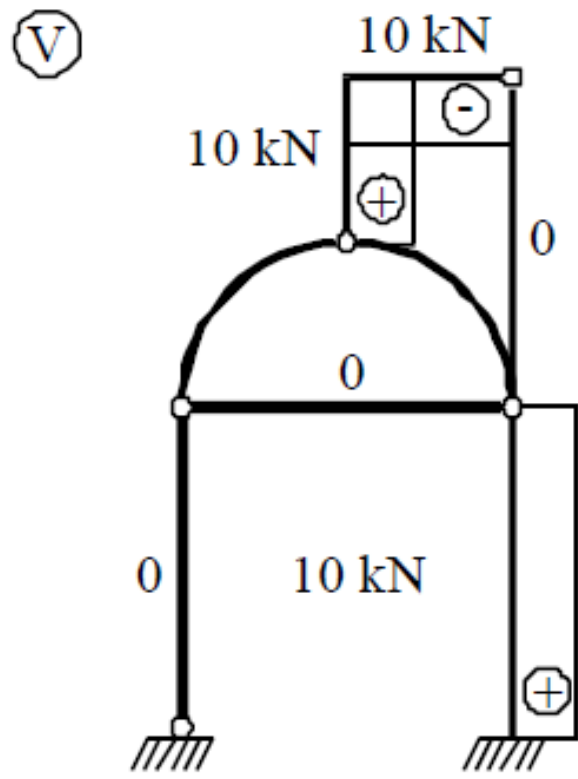
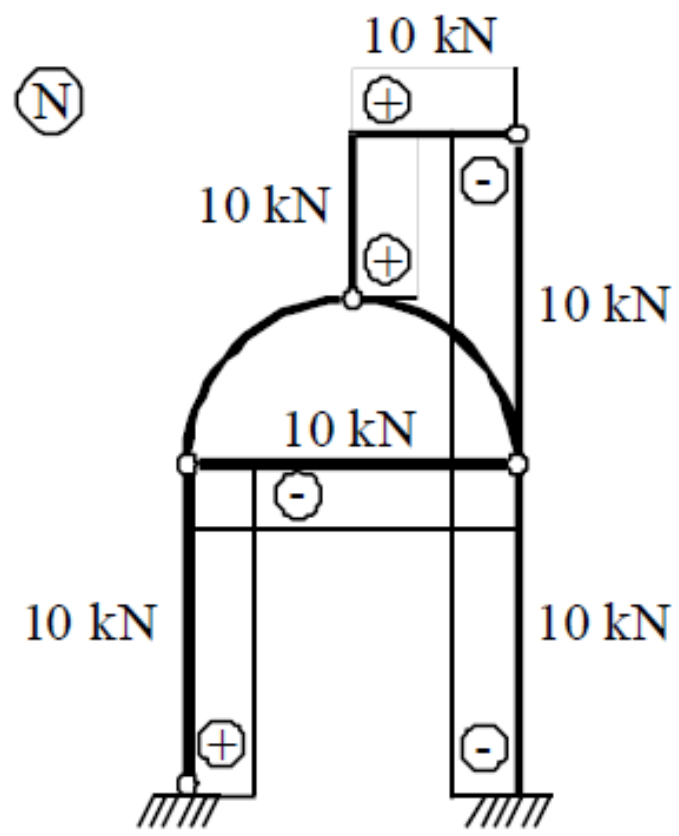


# Exemplo 20: P2 1995

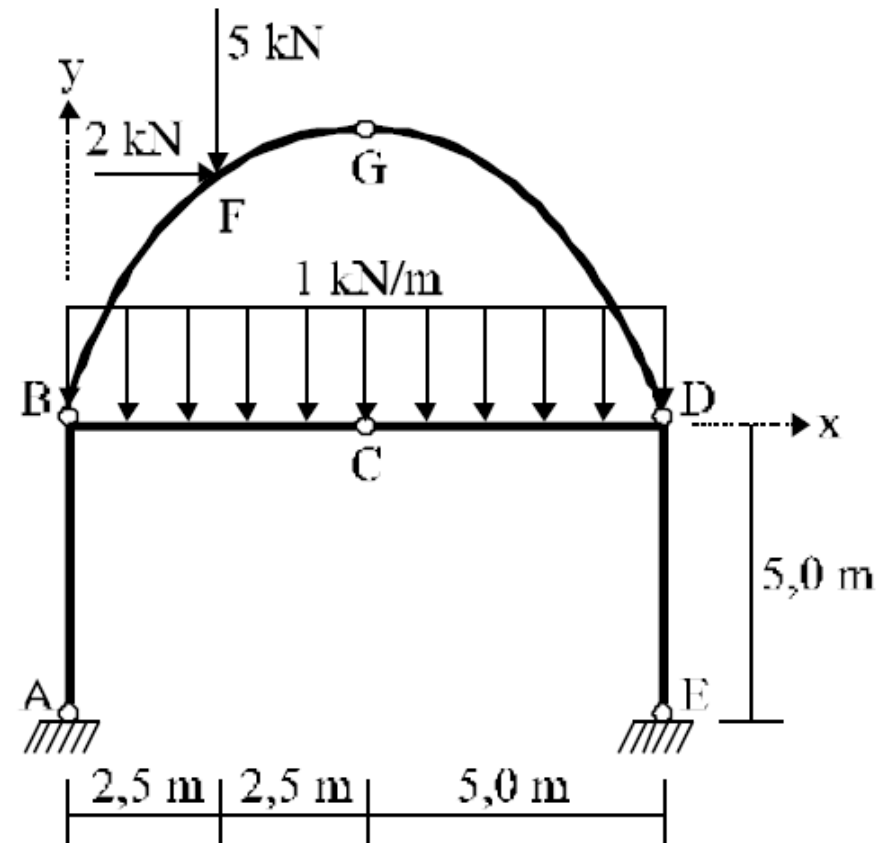
Traçar os diagramas de esforços solicitantes dos trechos retilíneos da estrutura abaixo esquematizada.







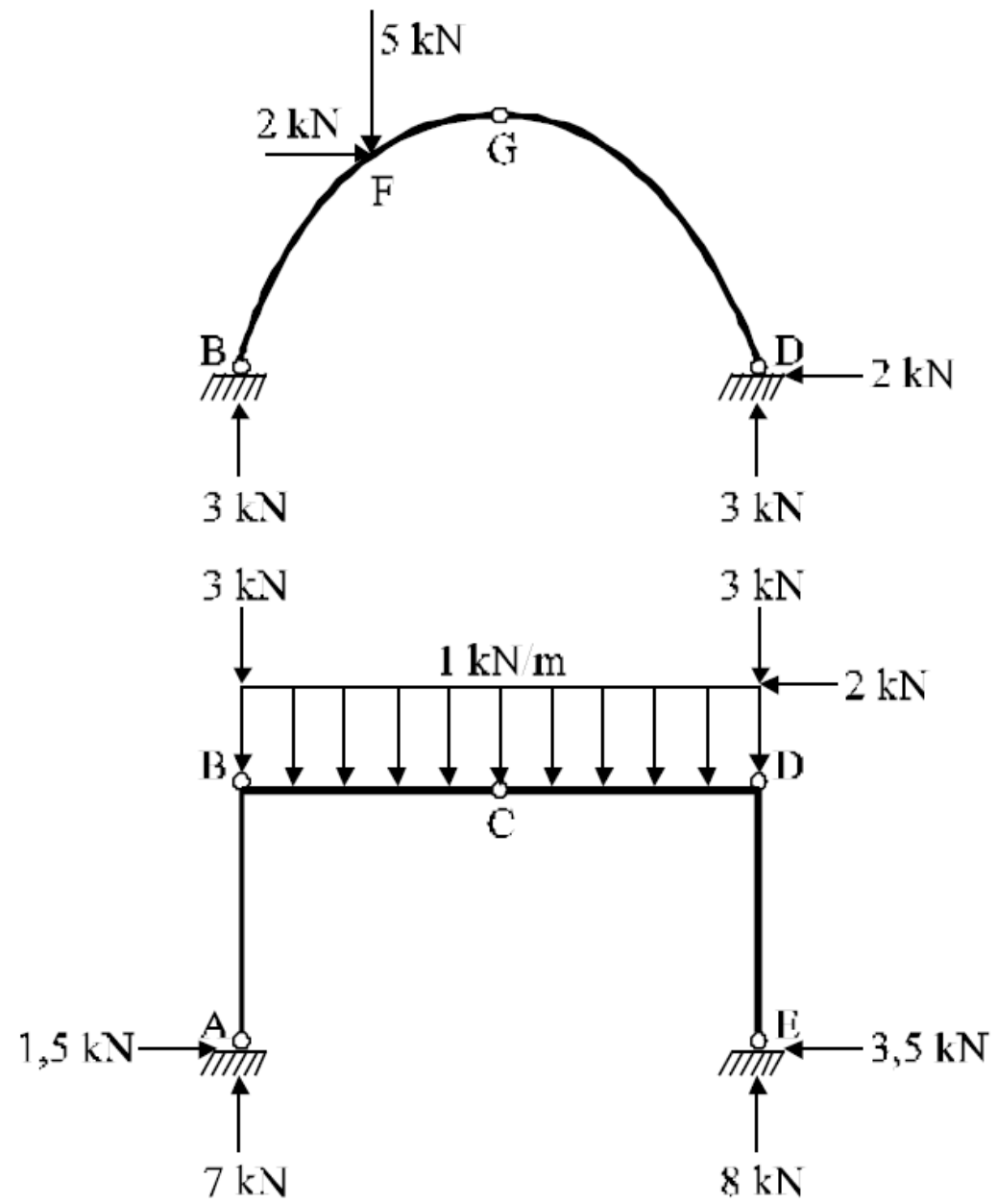
# Exemplo 21: P2 1994

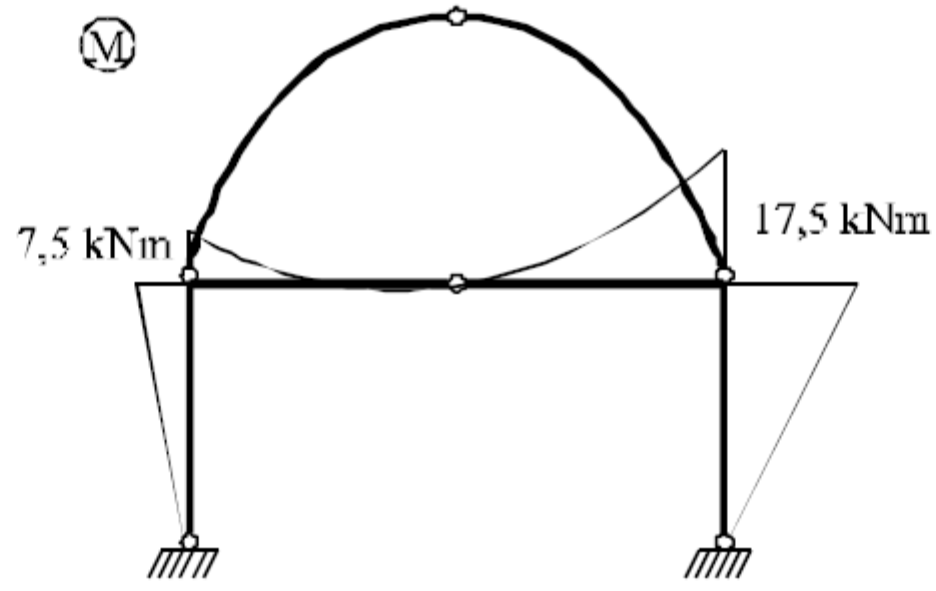
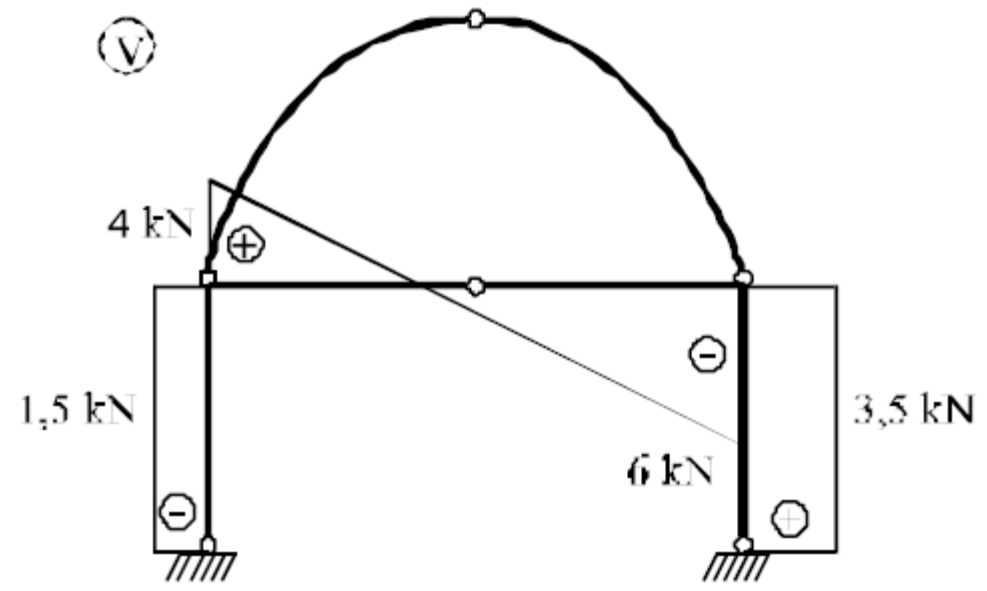
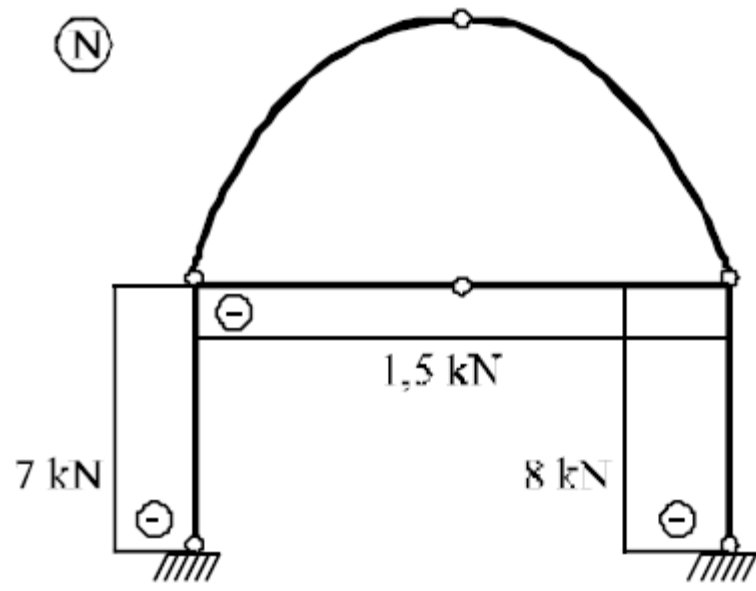


Desenhar os diagramas de esforços solicitantes do trecho ABCDE da estrutura esquematizada e calcular os esforços solicitantes na seção F ( seção imediatamente abaixo daquela onde são aplicados os esforços concentrados ). O trecho BFGD da estrutura é descrito pela função

$$y = -\frac{x^2}{5} + 2x$$

conforme o sistema de referência indicado.





*A tangente à curva no ponto F é dada pela derivada da função:*

$$y = -\frac{x^2}{5} + 2x$$

$$y' = -\frac{2x}{5} + 2$$

*- para  $x = 2,5 \implies y' = 1 \implies \theta = 45^\circ$ .*

*$\therefore$  no ponto F:  $N = -3 * \sqrt{2} / 2 \text{ kN}$  (de compressão)*

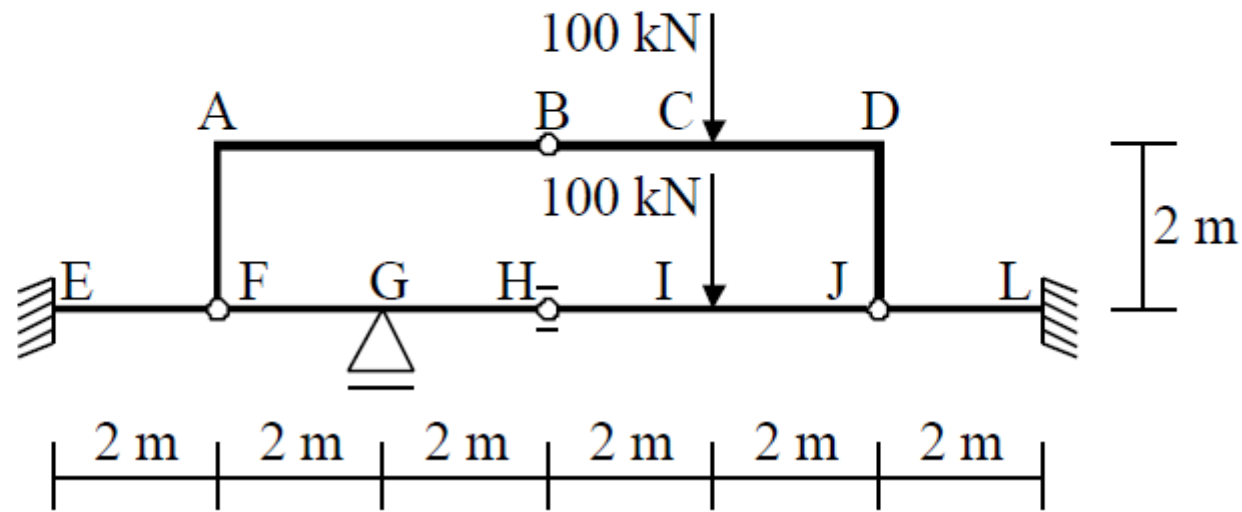
$$V = 3 * \sqrt{2} / 2 \text{ kN}$$

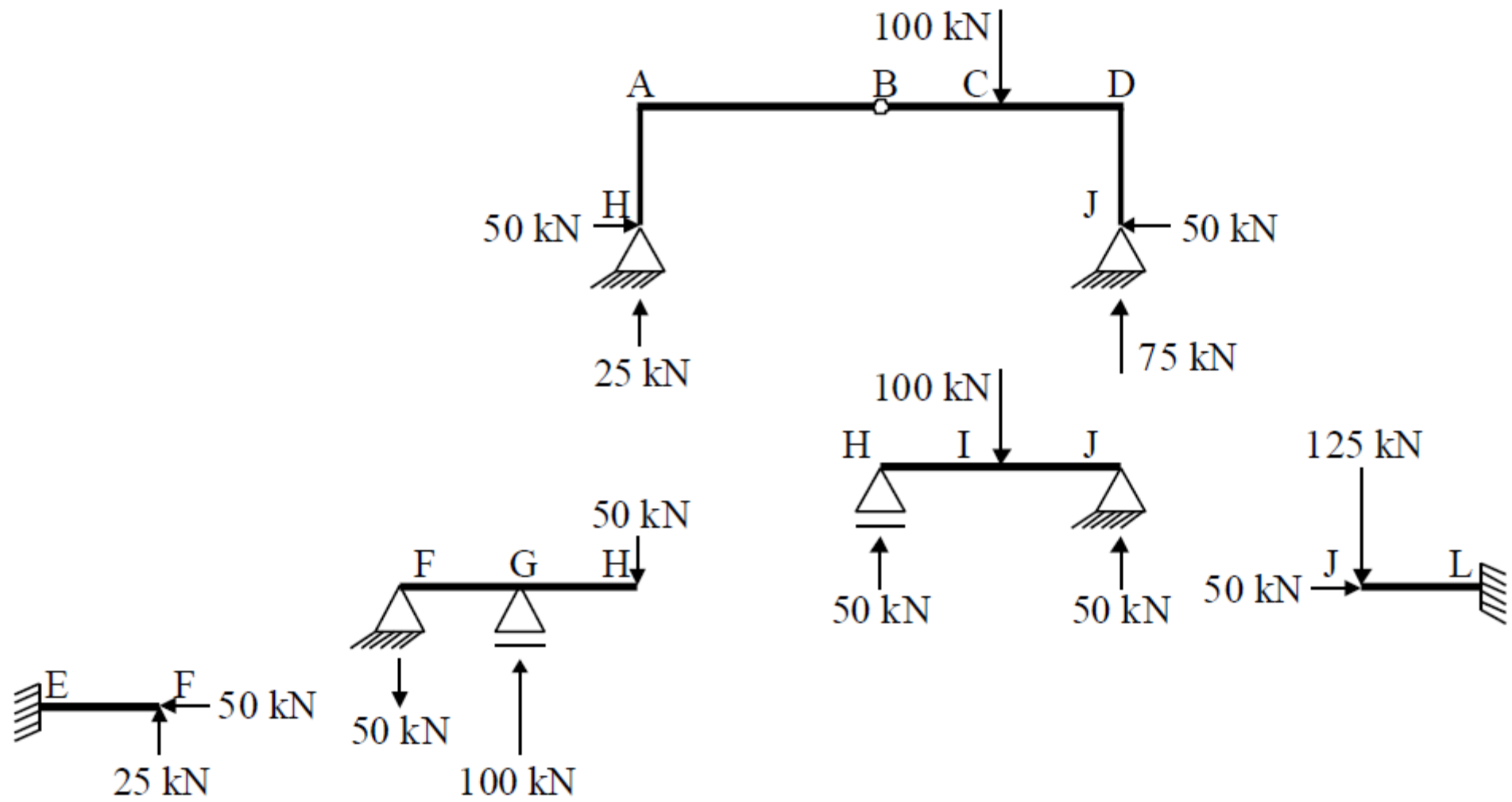


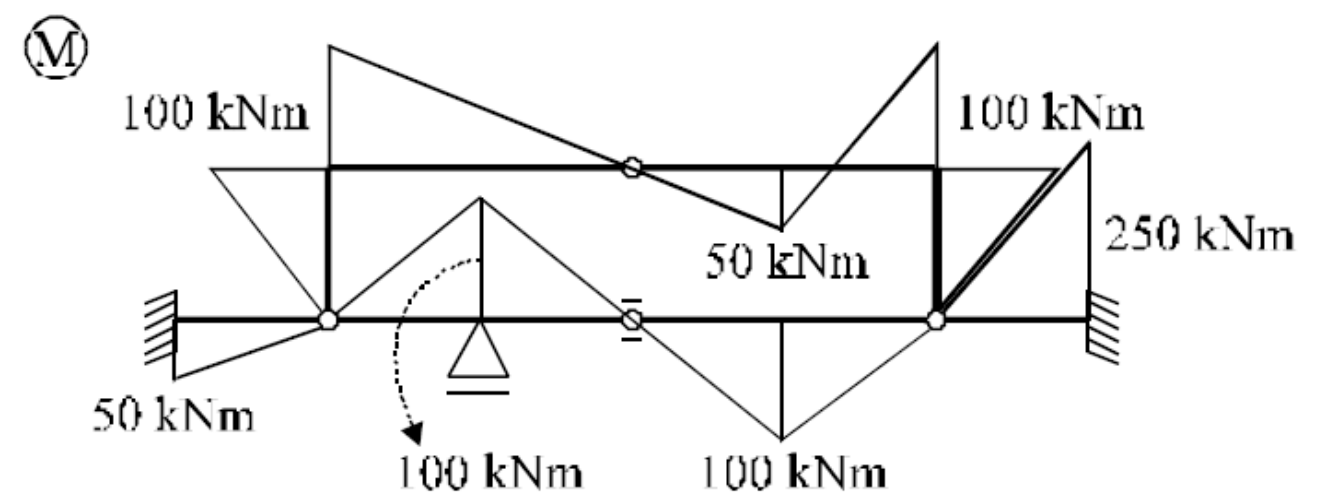
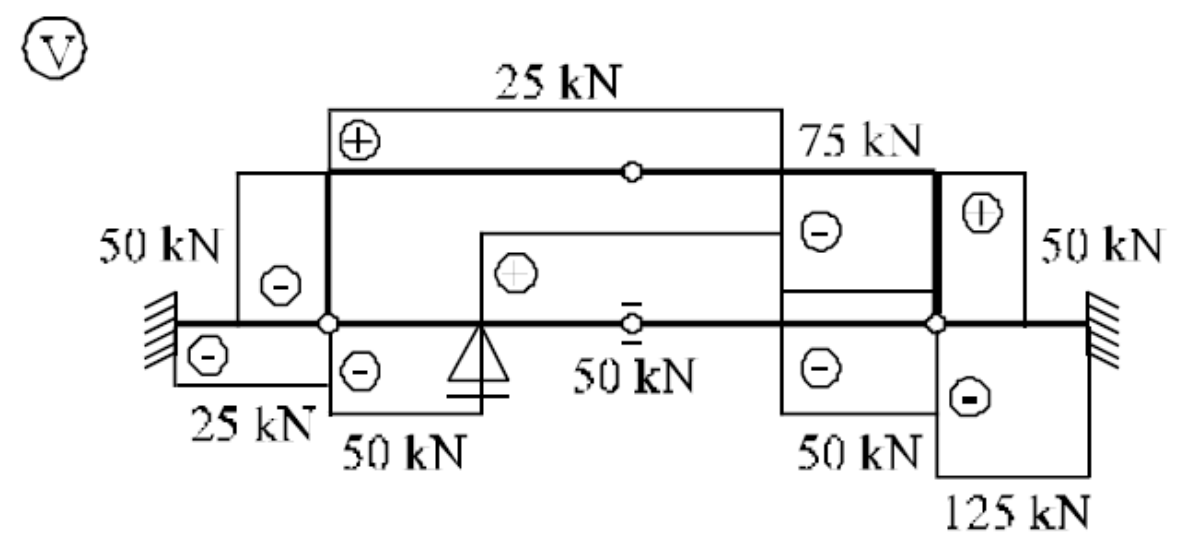
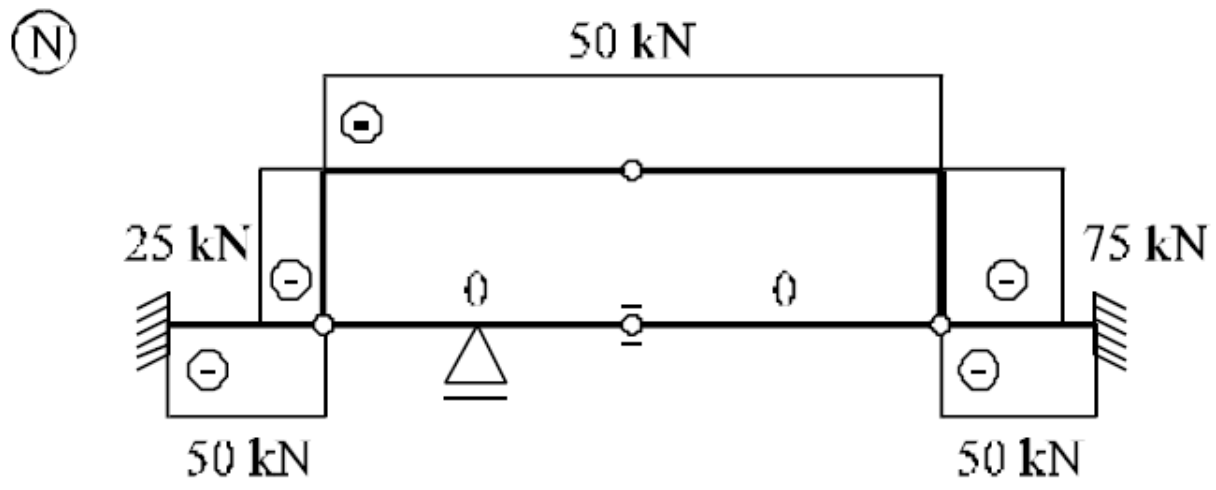
# Exemplo 22: SUB 1993

Para a estrutura da figura pede-se:

- Decompor a estrutura;
- Traçar os diagramas de esforços solicitantes.







# Exemplo 23: P3 1993

Considere a estrutura abaixo

- Decomponha a estrutura;
- Trace os diagramas de esforços solicitantes. Não é preciso traçar os diagramas no trecho CHE.

