

**PEF3200**

**Aula 13**

**28 jun**

**PROF.**

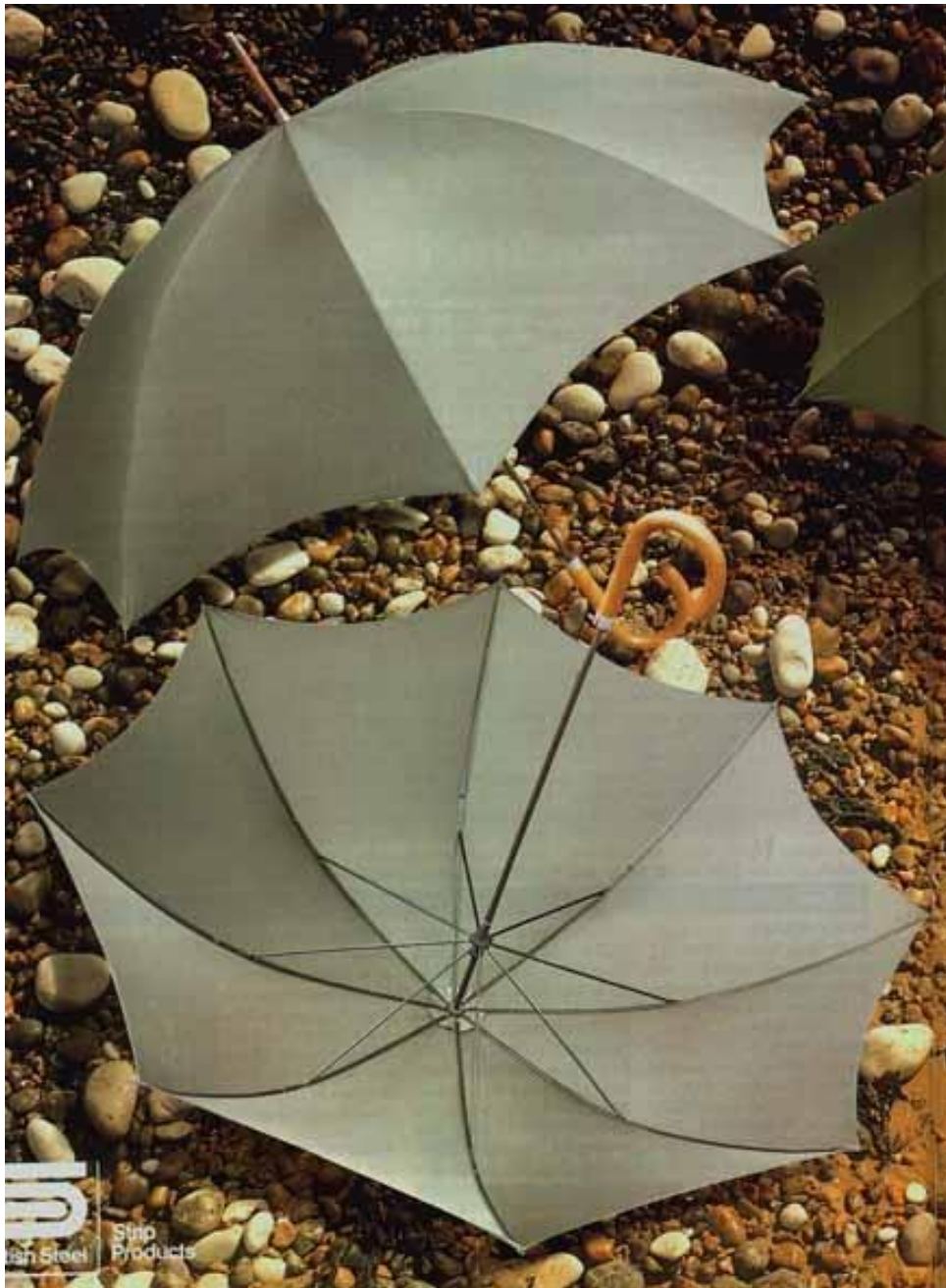
**NAKAO**

❖ **Estruturas associadas.**

# AGENDA

7	10 mai	Linhas de influência.
8	17 mai	<b>Prova P1</b>
9	24 mai	Linhas de influência. Diagramas de máximos e mínimos.
10	31 mai	Treliças.
11	07 jun	Pórticos triarticulados.
12	14 jun	Arcos triarticulados.
13	21 jun	Vigas Gerber.
14	28 jun	Estruturas associadas.
15	05 jul	<b>Prova P2</b>
16	12 jul	<b>Prova Substitutiva</b>
17	19 jul	<b>Prova de Recuperação</b>









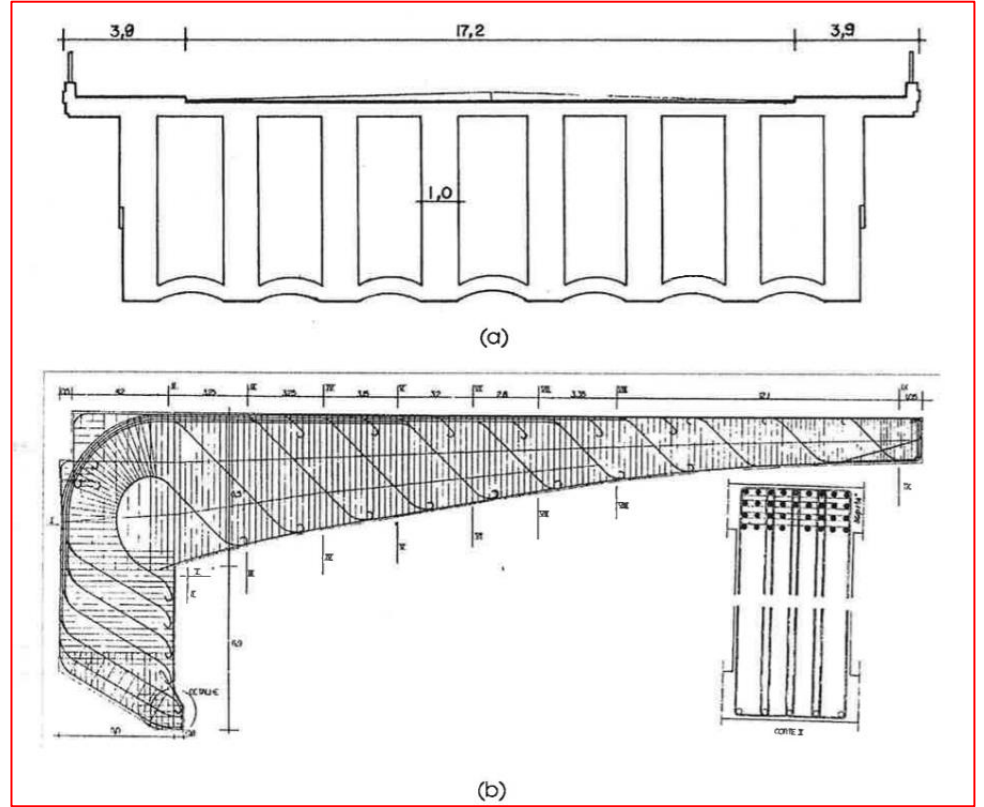
**Ponte estaiada sobre o rio Pinheiros**  
(fotografia de Anderson Glauco Benite)



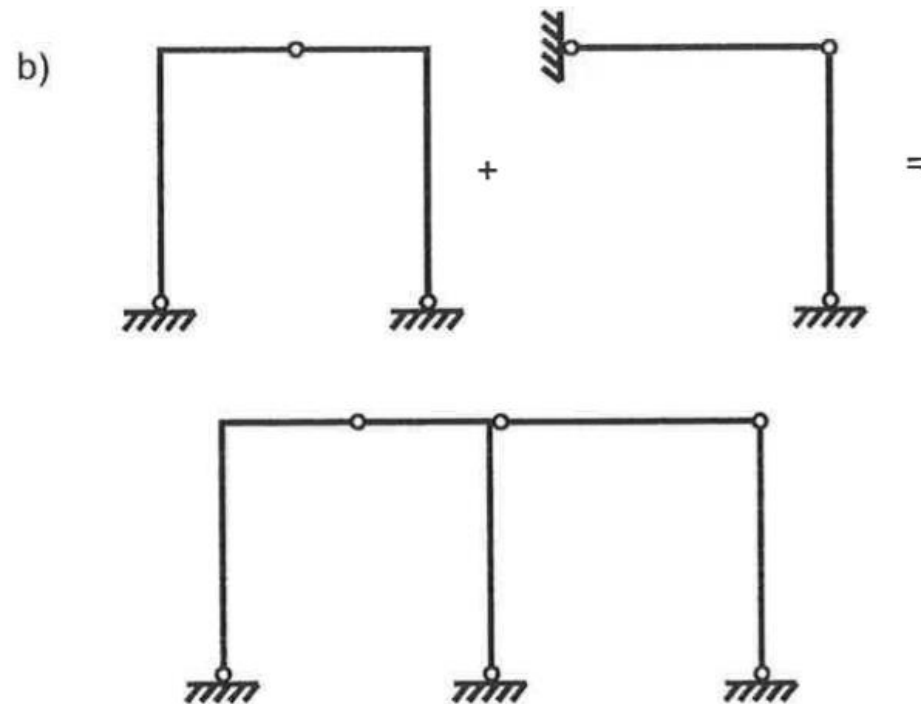
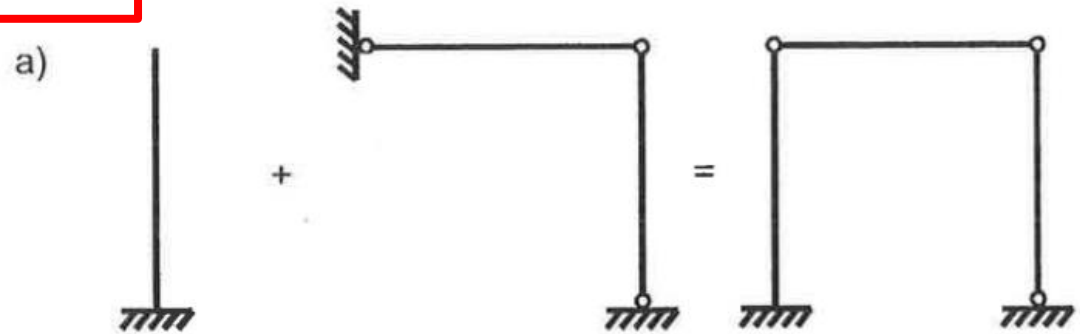
**Treliça de escoramento dos segmentos  
do tabuleiro da ponte**  
(fotografia de Anderson Glauco Benite)

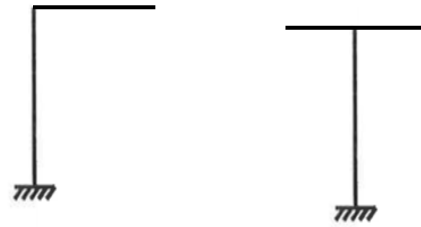
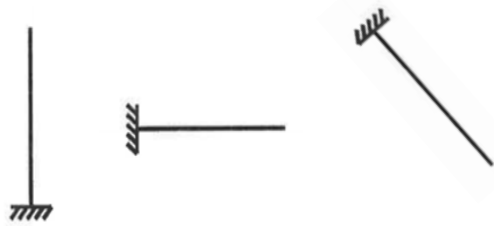


**VIADUTO DO CHÁ**









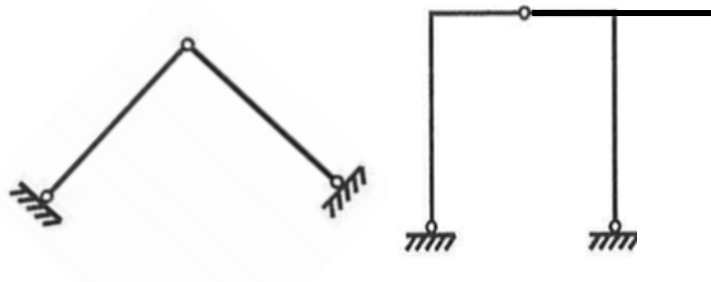
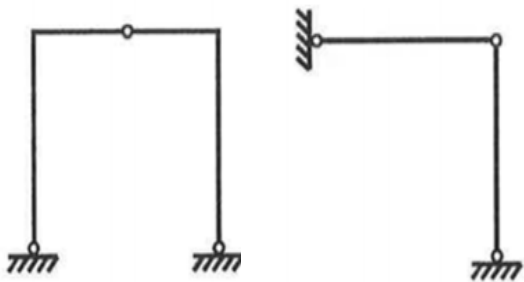
Viga em balanço ou  
Viga engastada



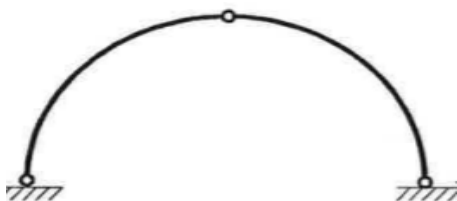
Viga simplesmente apoiada ou  
Viga biapoiada



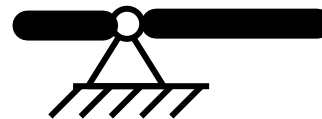
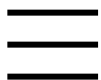
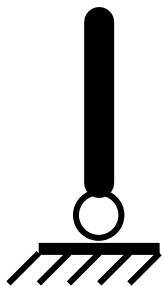
Viga simplesmente apoiada com balanço ou  
Viga biapoiada com balanço



Pórtico triarticulado



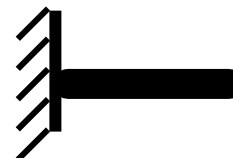
Arco triarticulado



Articulação  
fixa



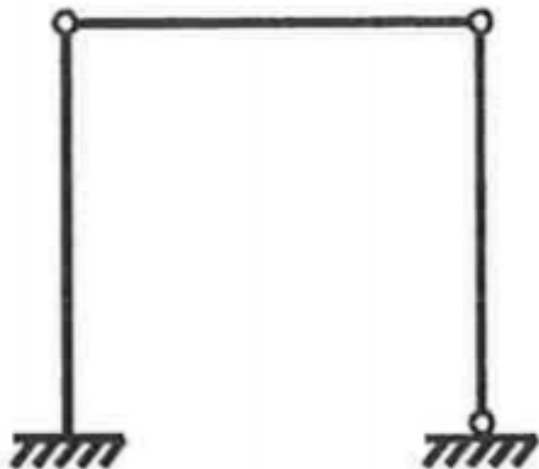
Articulação  
móvel



Engaste  
Engastamento

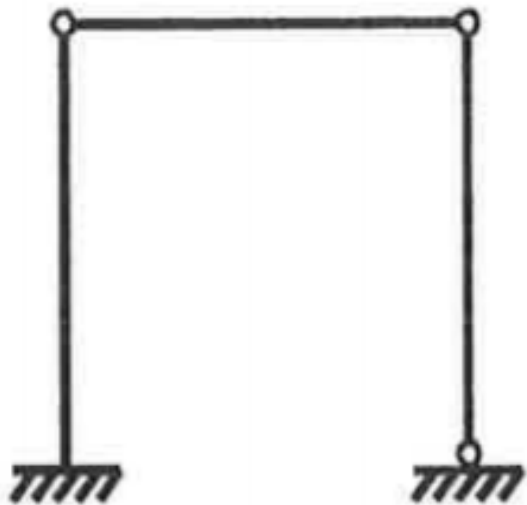
## Exercício 1.

**Decompor a estrutura e denominar as subestruturas**



### **COMO DECOMPOR?**

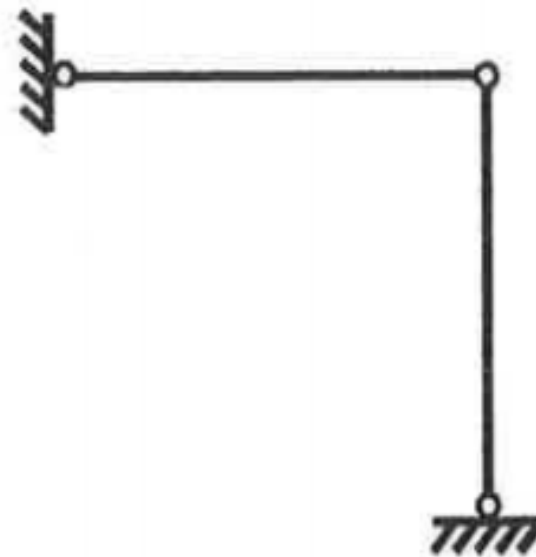
- **QUAL É A ESTRUTURA QUE SE APÓIA EM OUTRAS E NÃO DÁ APOIO A NENHUMA? COMEÇAR POR ELA.**
- **QUAL É A ESTRUTURA QUE NÃO SE APÓIA EM NENHUMA E DÁ APOIO ÀS OUTRAS? TERMINAR POR ELA.**



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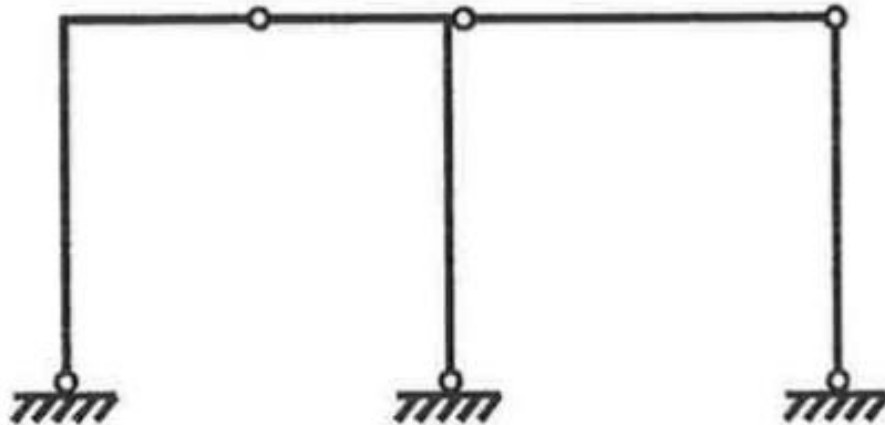


Viga em balanço ou  
Viga engastada

Pórtico triarticulado

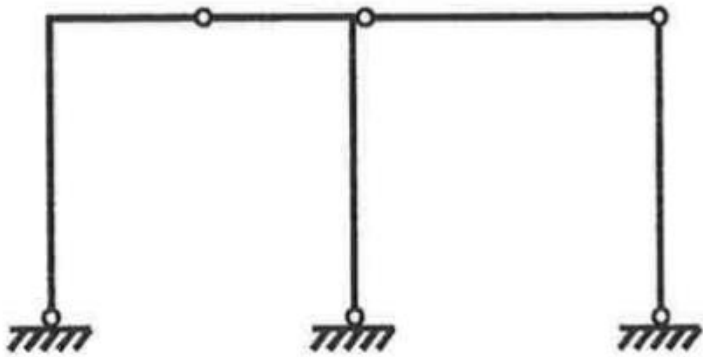
## Exercício 2.

**Decompor a estrutura e denominar as subestruturas**



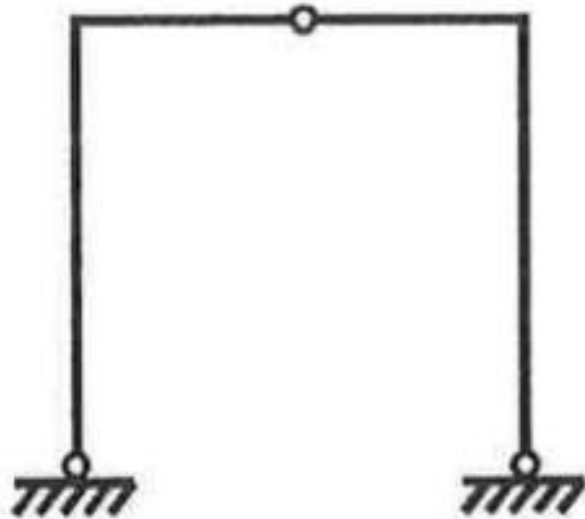
### **COMO DECOMPOR?**

- QUAL É A ESTRUTURA QUE SE APÓIA EM OUTRAS E NÃO DÁ APOIO A NENHUMA? COMEÇAR POR ELA.
- QUAL É A ESTRUTURA QUE NÃO SE APÓIA EM NENHUMA E DÁ APOIO ÀS OUTRAS? TERMINAR POR ELA.



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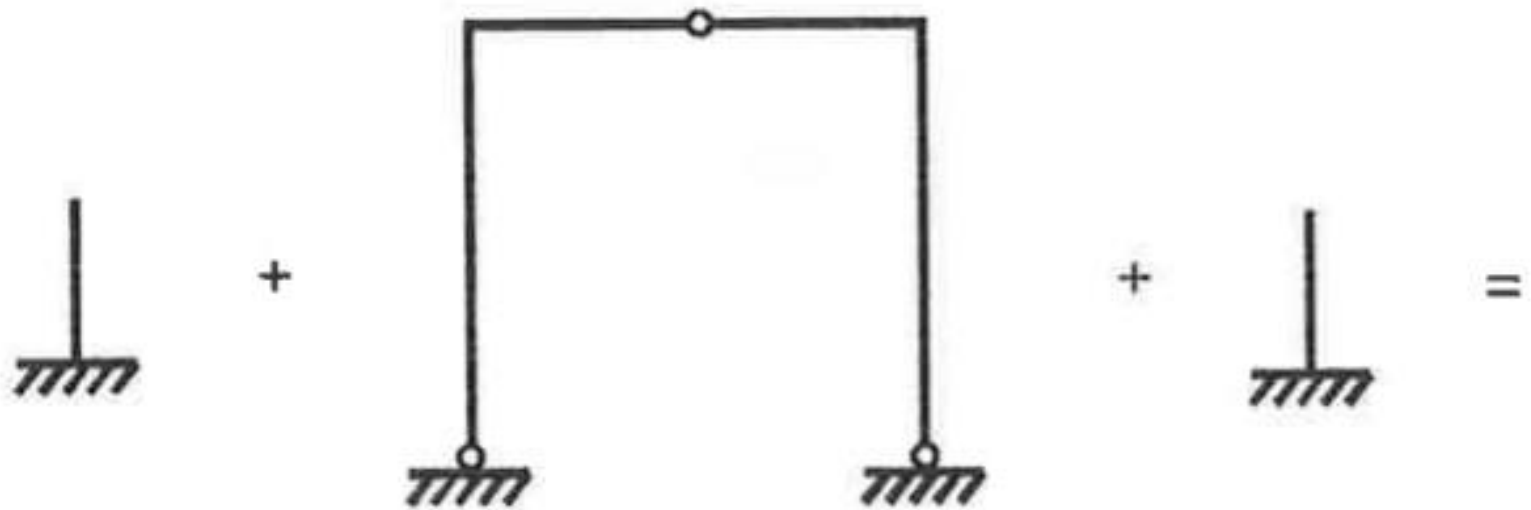


Pórtico triarticulado

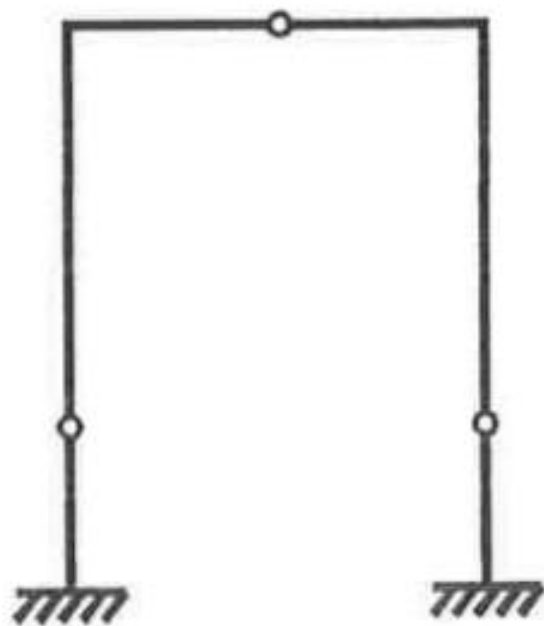
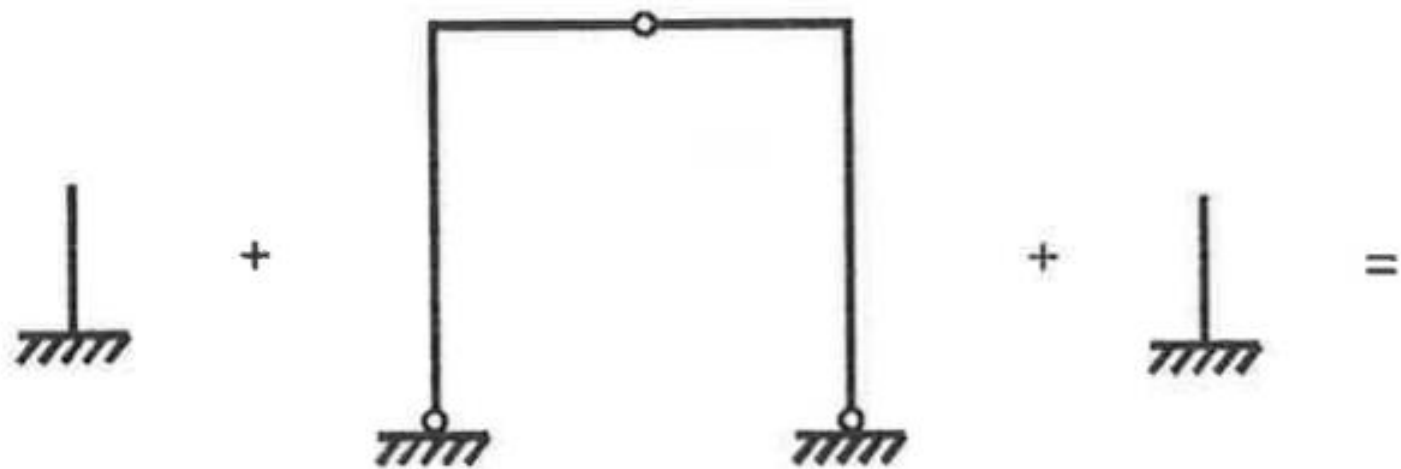
Pórtico triarticulado

### Exercício 3.

Como compor uma estrutura?

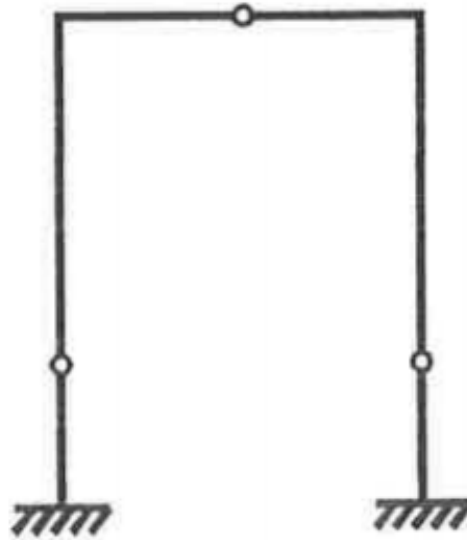






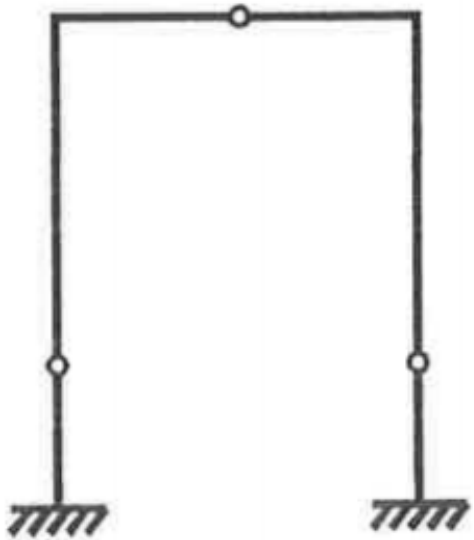
## Exercício 4.

**Decompor a estrutura e denominar as subestruturas**



### **COMO DECOMPOR?**

- **QUAL É A ESTRUTURA QUE SE APÓIA EM OUTRAS E NÃO DÁ APOIO A NENHUMA? COMEÇAR POR ELA.**
- **QUAL É A ESTRUTURA QUE NÃO SE APÓIA EM NENHUMA E DÁ APOIO ÀS OUTRAS? TERMINAR POR ELA.**

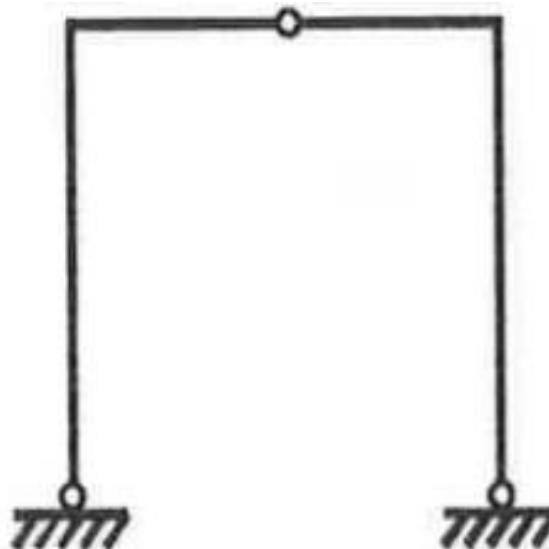


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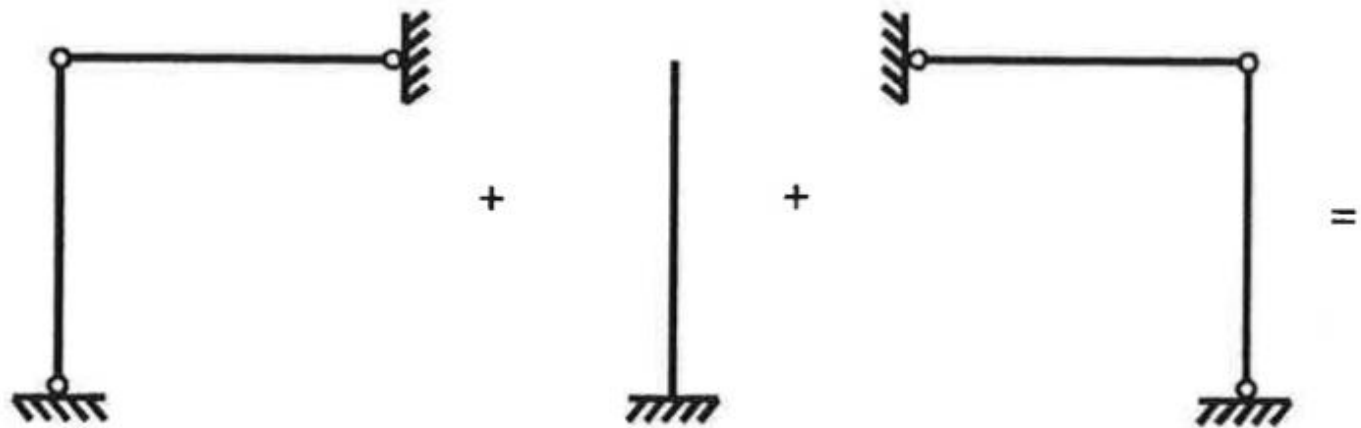
Viga em balanço ou  
Viga engastada

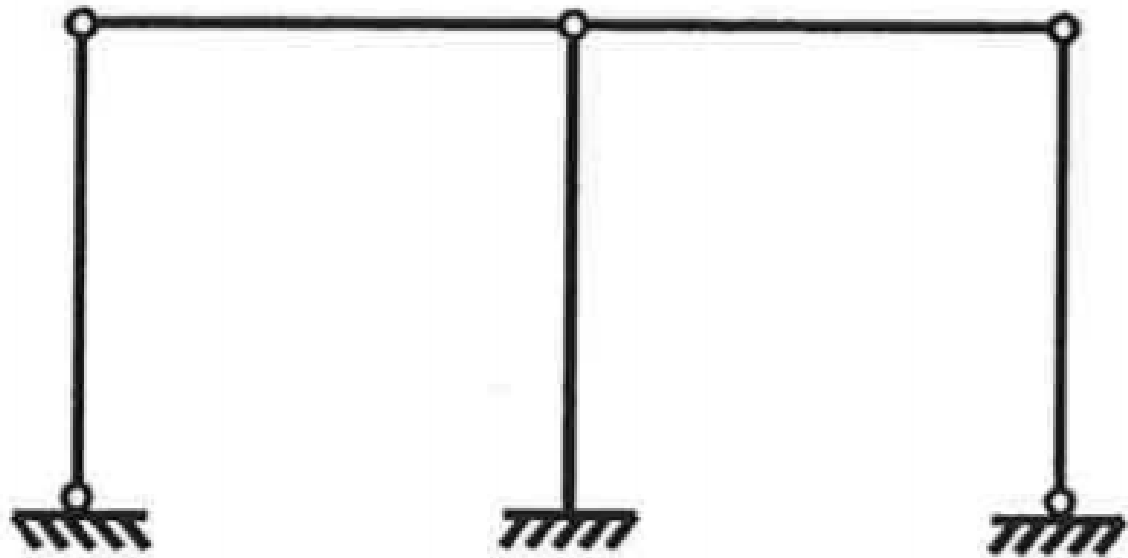
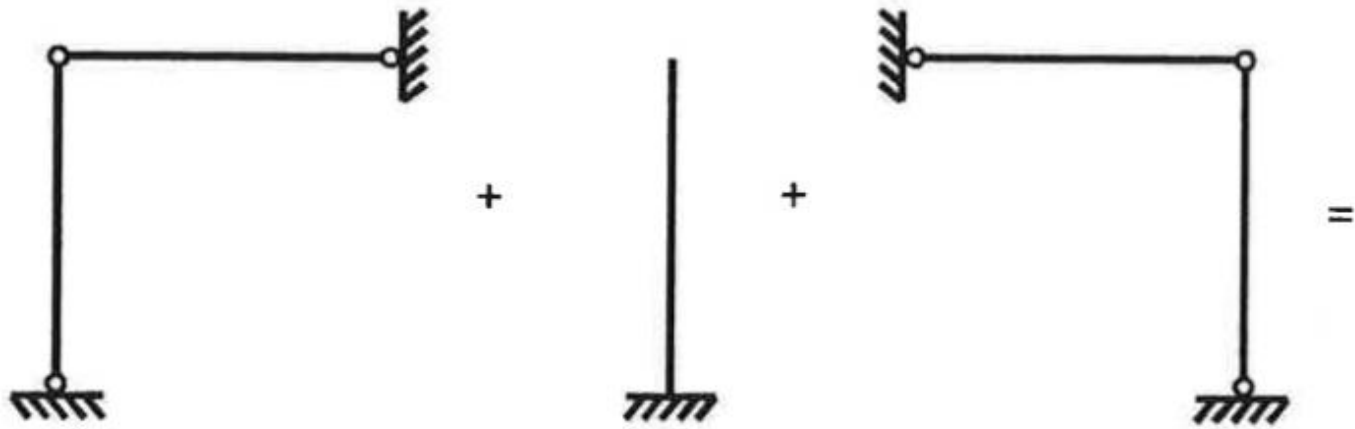
Pórtico triarticulado

Viga em balanço ou  
Viga engastada

## Exercício 5.

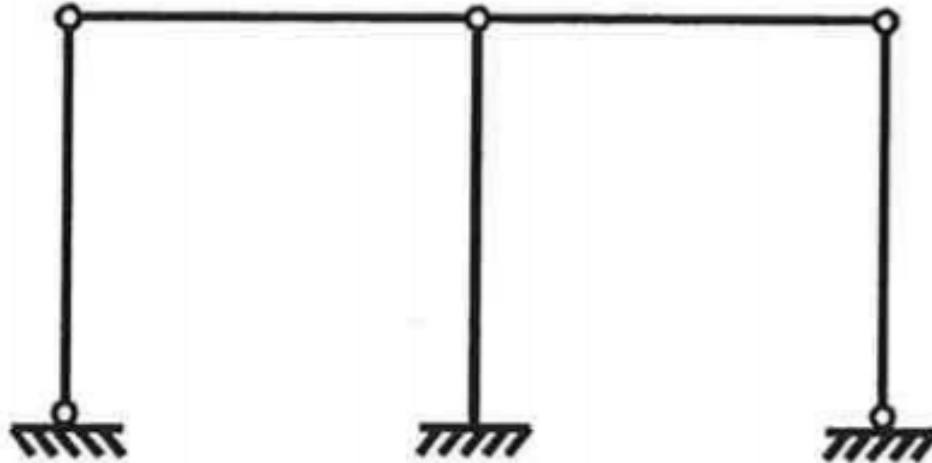
Como compor uma estrutura?





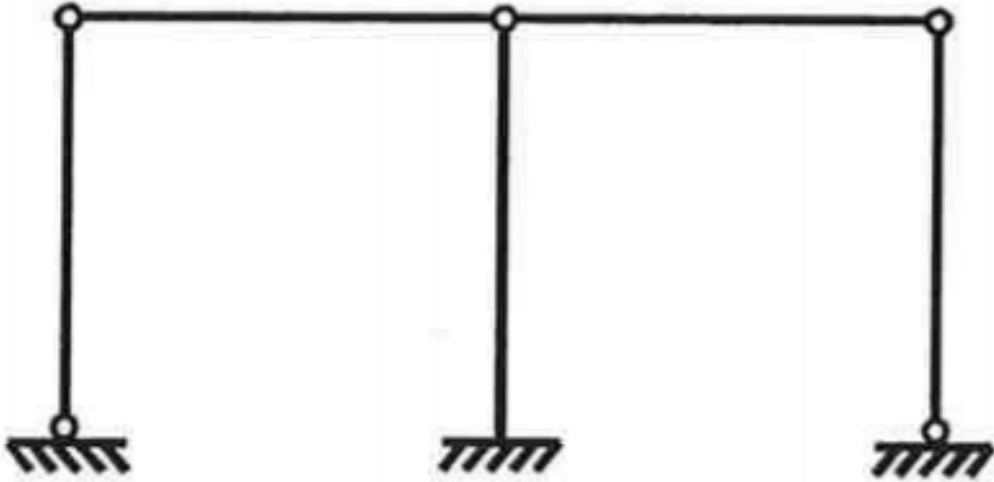
## Exercício 6.

**Decompor a estrutura e denominar as subestruturas**

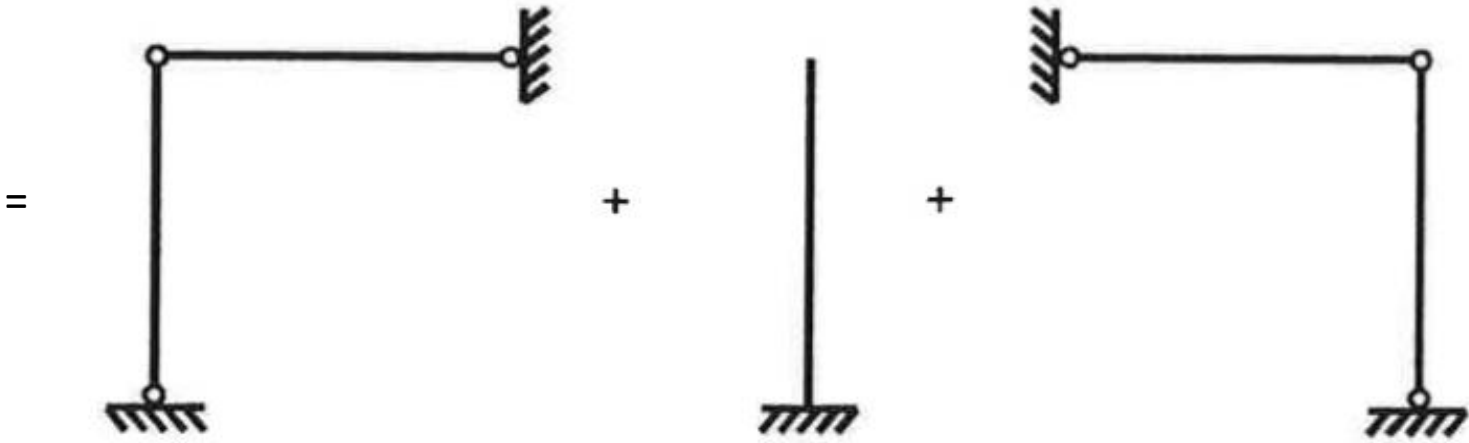


### **COMO DECOMPOR?**

- **QUAL É A ESTRUTURA QUE SE APÓIA EM OUTRAS E NÃO DÁ APOIO A NENHUMA? COMEÇAR POR ELA.**
- **QUAL É A ESTRUTURA QUE NÃO SE APÓIA EM NENHUMA E DÁ APOIO ÀS OUTRAS? TERMINAR POR ELA.**



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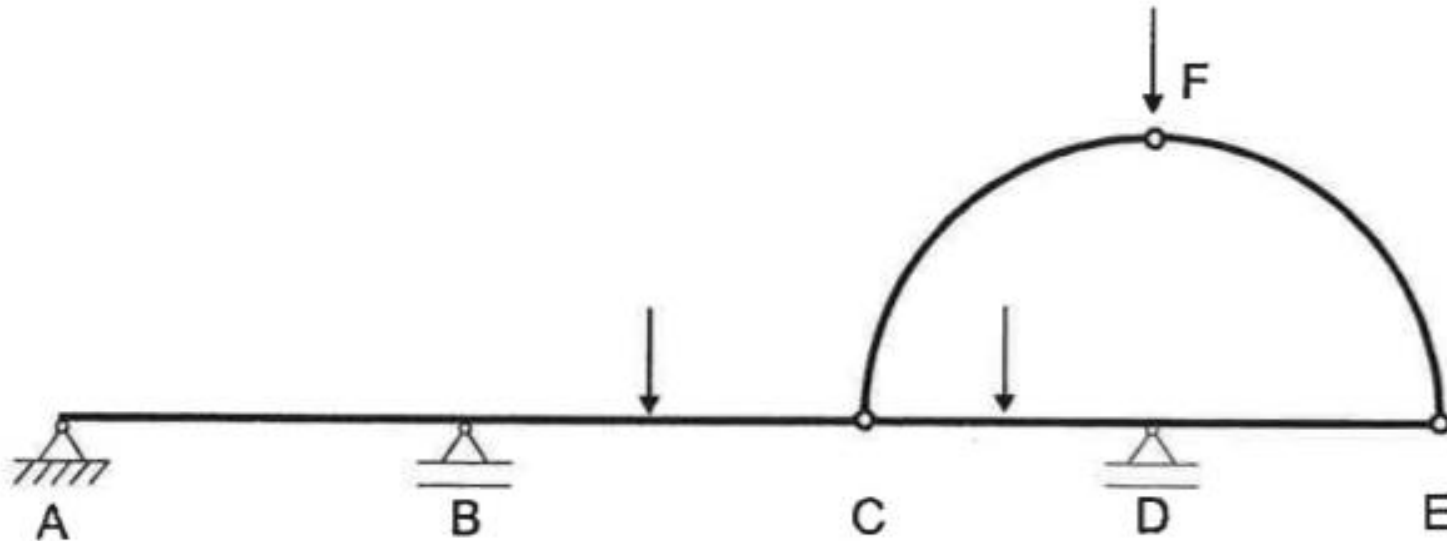
Pórtico triarticulado

Viga em balanço ou Viga engastada

Pórtico triarticulado

## Exercício 7.

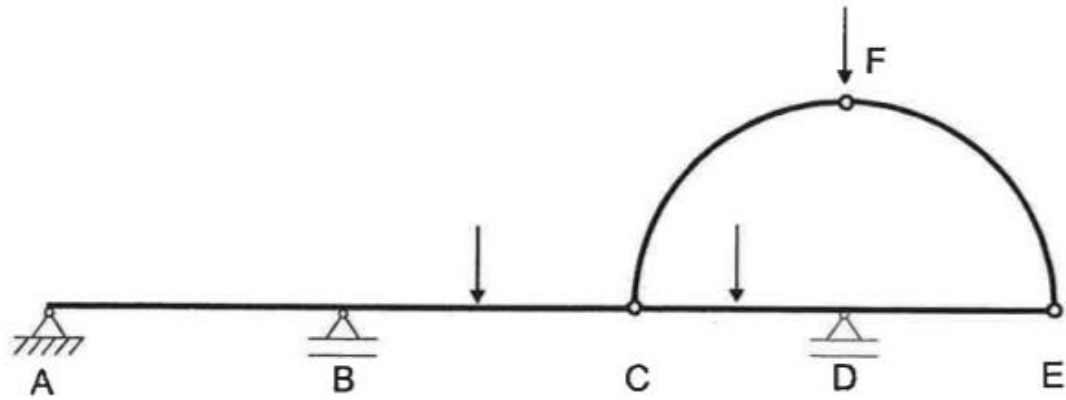
**Decompor a estrutura e denominar as subestruturas**



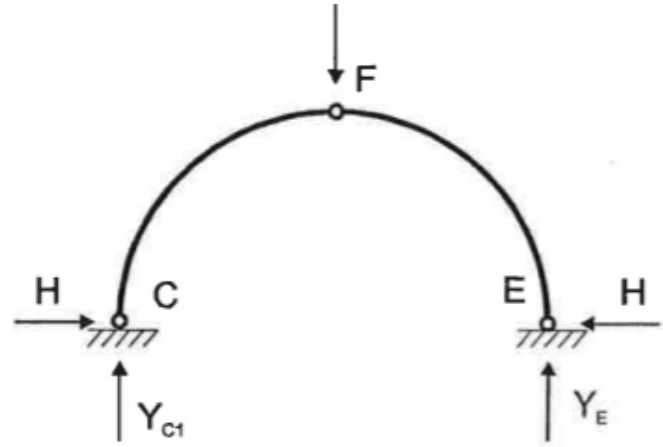
**COMO DECOMPOR?**

- QUAL É A ESTRUTURA QUE SE APÓIA EM OUTRAS E NÃO DÁ APOIO A NENHUMA? COMEÇAR POR ELA.
- QUAL É A ESTRUTURA QUE NÃO SE APÓIA EM NENHUMA E DÁ APOIO ÀS OUTRAS? TERMINAR POR ELA.

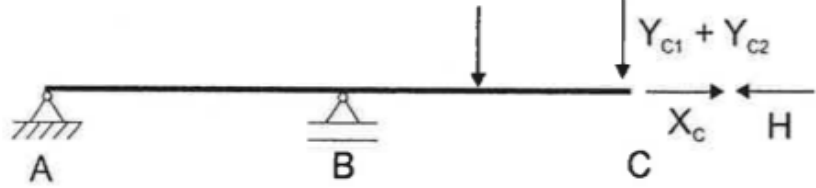
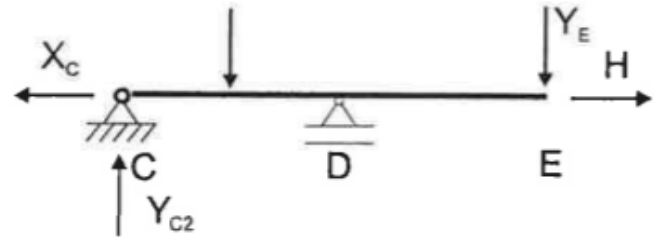




Arco triarticulado

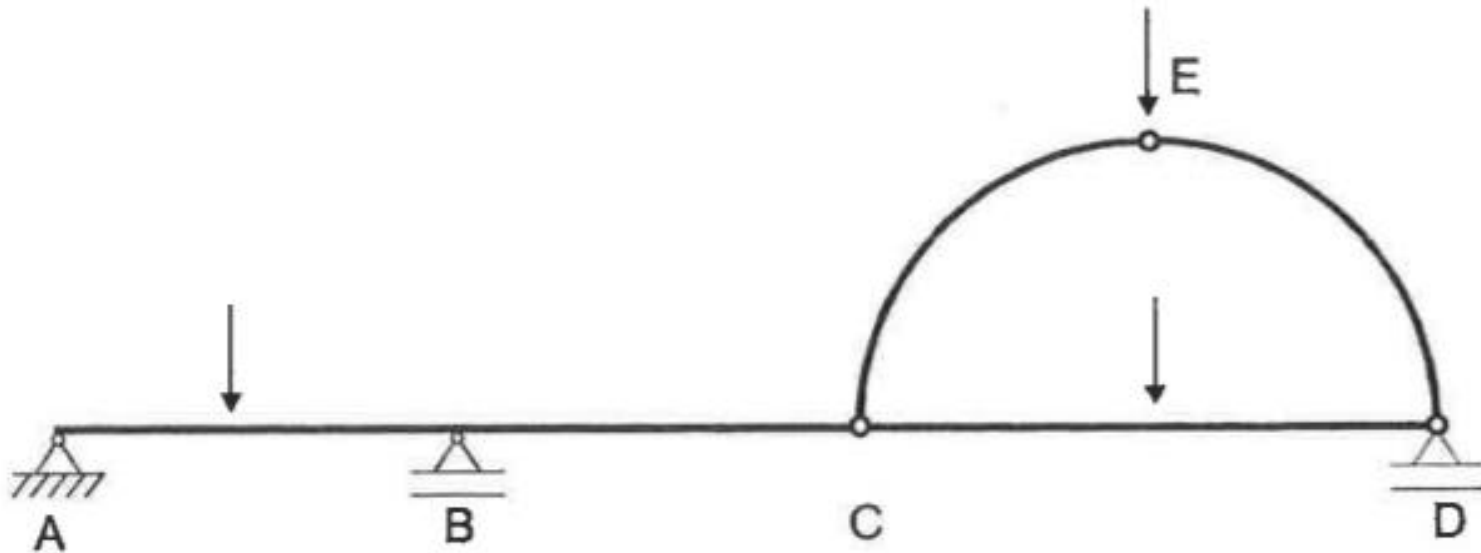


Viga simplesmente apoiada com balanço



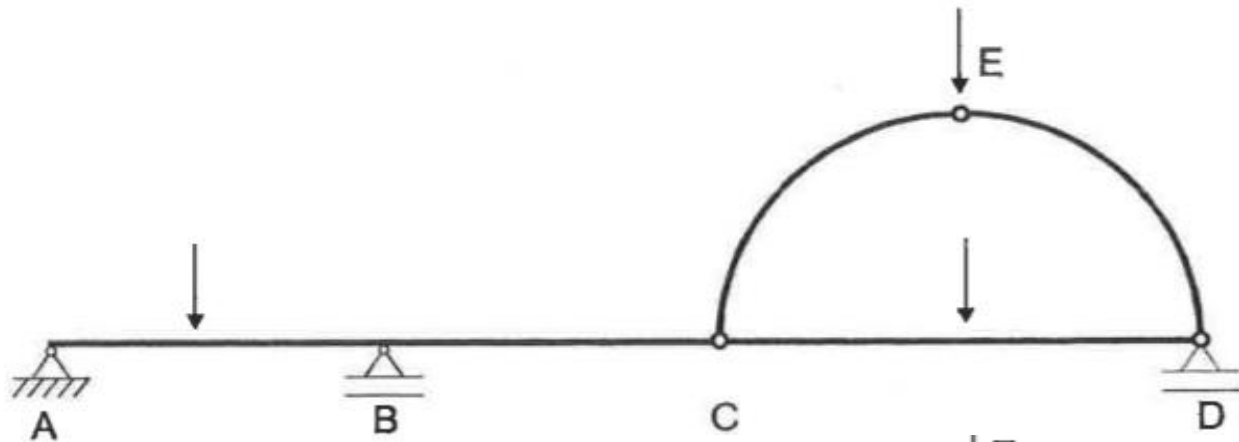
## Exercício 8.

**Decompor a estrutura e denominar as subestruturas**

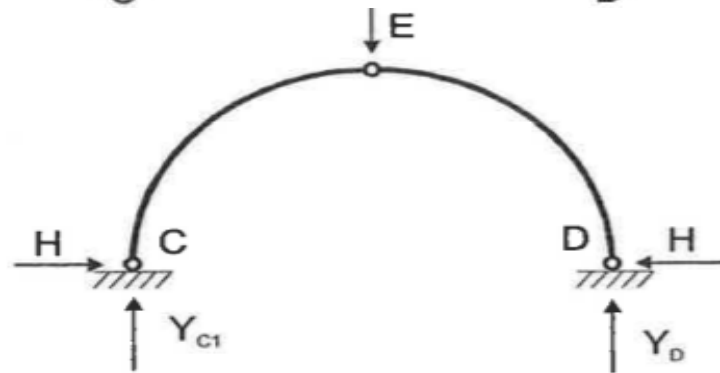


**COMO DECOMPOR?**

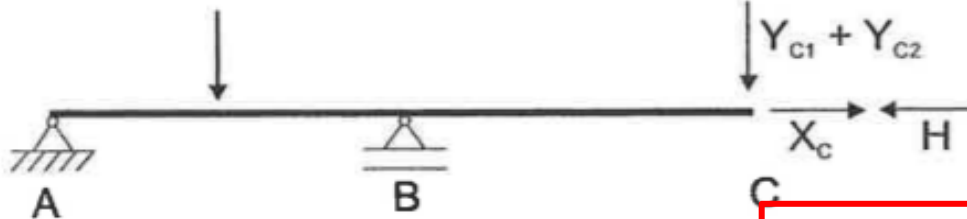
- QUAL É A ESTRUTURA QUE SE APÓIA EM OUTRAS E NÃO DÁ APOIO A NENHUMA? COMEÇAR POR ELA.
- QUAL É A ESTRUTURA QUE NÃO SE APÓIA EM NENHUMA E DÁ APOIO ÀS OUTRAS? TERMINAR POR ELA.



Arco triarticulado



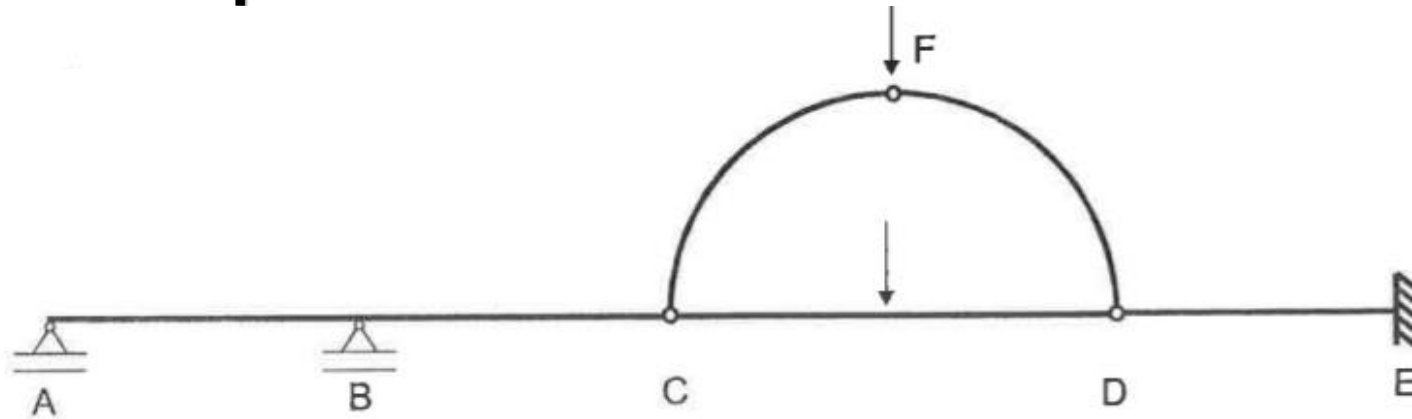
Viga simplesmente apoiada  
Viga biapoiada



Viga simplesmente apoiada com balanço

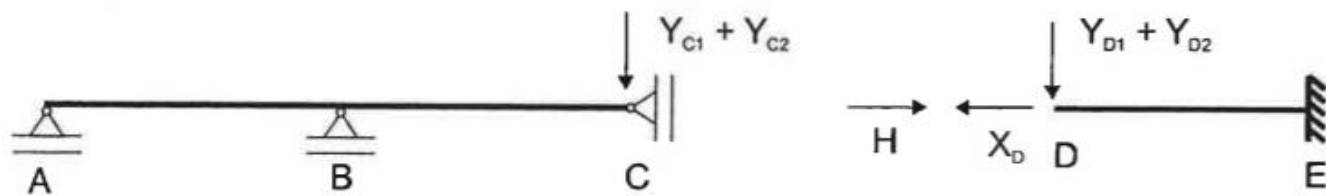
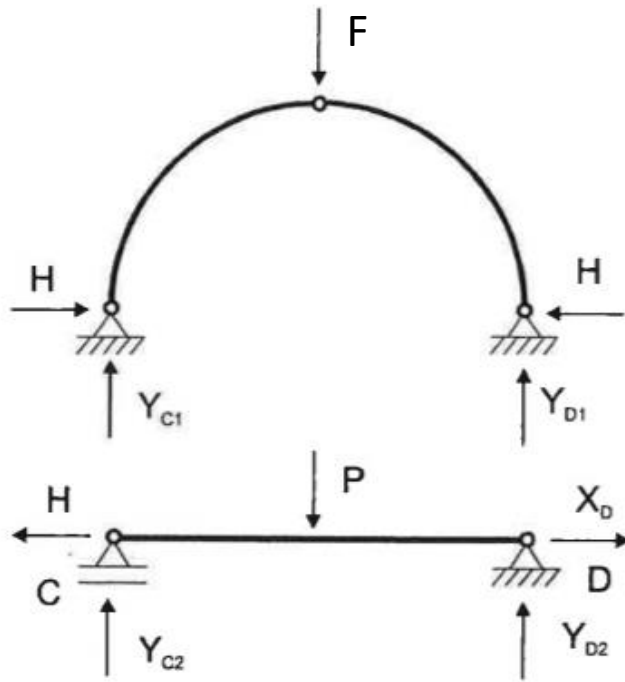
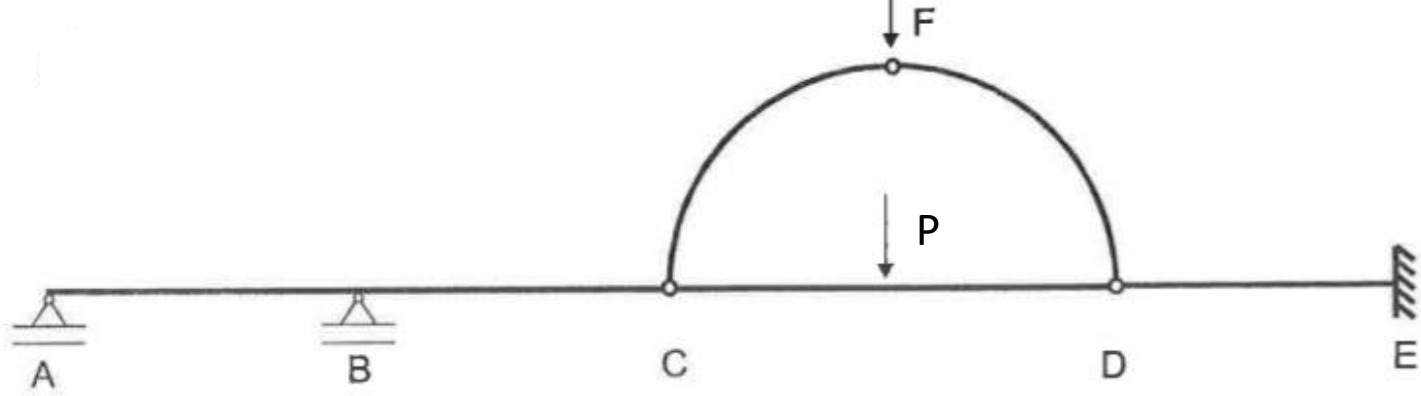
## Exercício 9.

### Decompor a estrutura



#### COMO DECOMPOR?

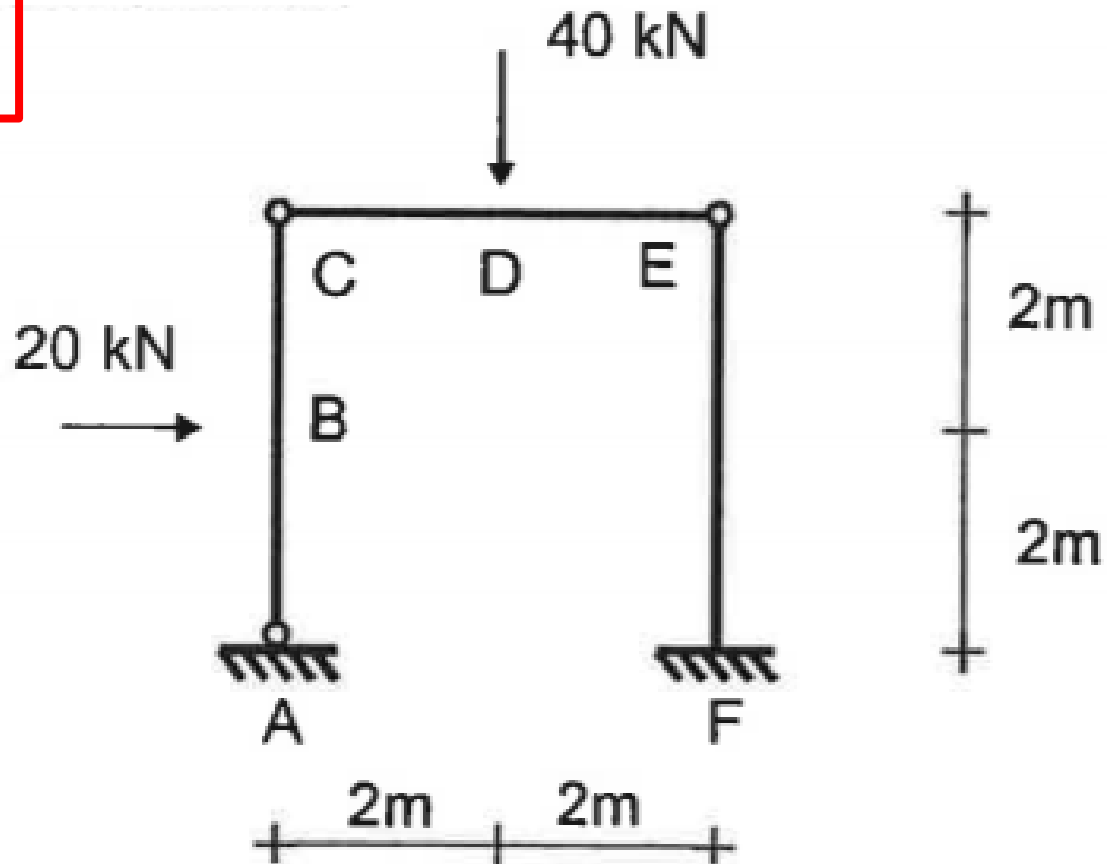
- QUAL É A ESTRUTURA QUE SE APÓIA EM OUTRAS E NÃO DÁ APOIO A NENHUMA? COMEÇAR POR ELA.
- QUAL É A ESTRUTURA QUE NÃO SE APÓIA EM NENHUMA E DÁ APOIO ÀS OUTRAS? TERMINAR POR ELA.

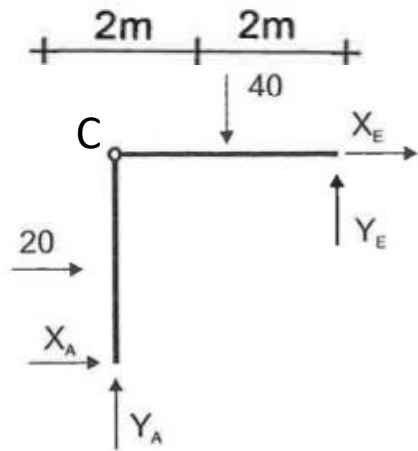
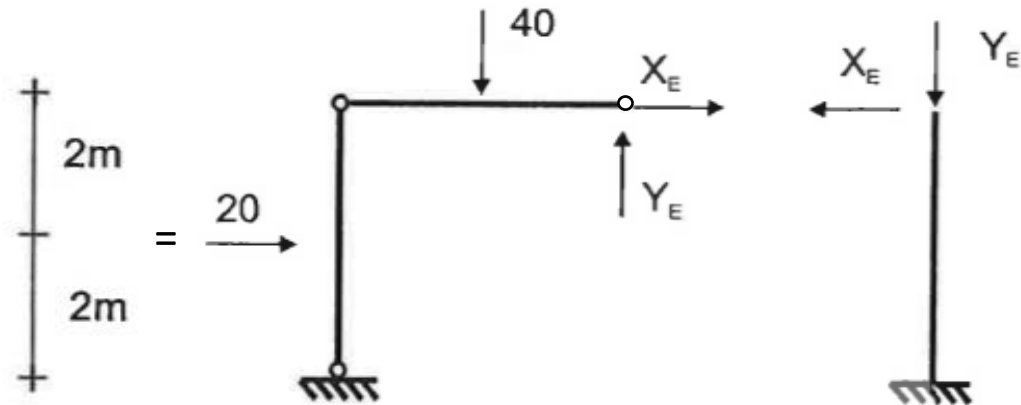
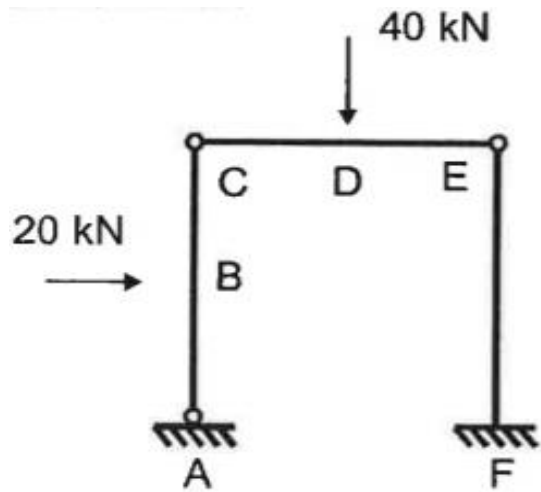


## Exercício 10.

Traçar os diagramas dos esforços solicitantes da estrutura da figura

APOSTILA  
ESTRUTURAS ASSOCIADAS  
PÁGINA 171



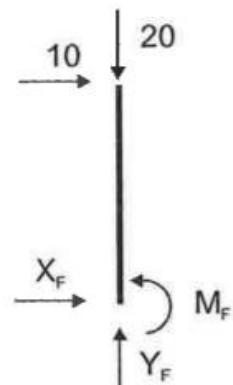


$$\sum M_{fletor}^{C, esquerda} = 0 = X_A * 4 + 20 * 2 \Rightarrow X_A = -10 \text{ kN}$$

$$\sum X = 0 = X_A + 20 + X_E \Rightarrow X_E = -10 \text{ kN}$$

$$\sum M_{(A)} = 0 = -20 * 2 - 40 * 2 - X_E * 4 + Y_E * 4 \Rightarrow Y_E = 20 \text{ kN}$$

$$\sum Y = 0 = Y_A - 40 + Y_E \Rightarrow Y_A = 20 \text{ kN}$$

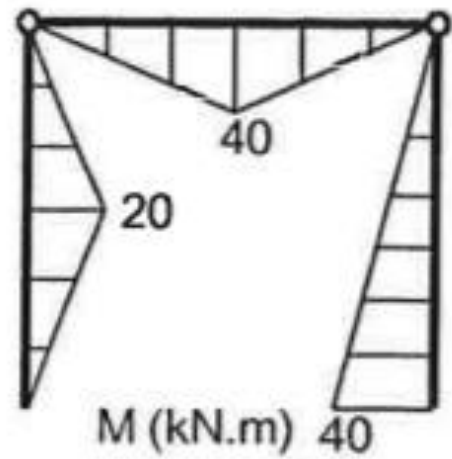
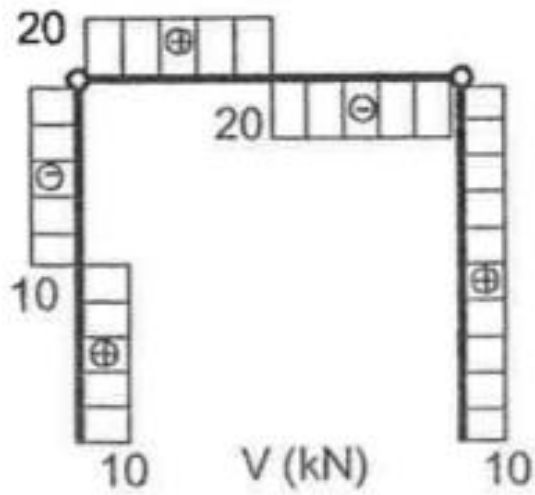
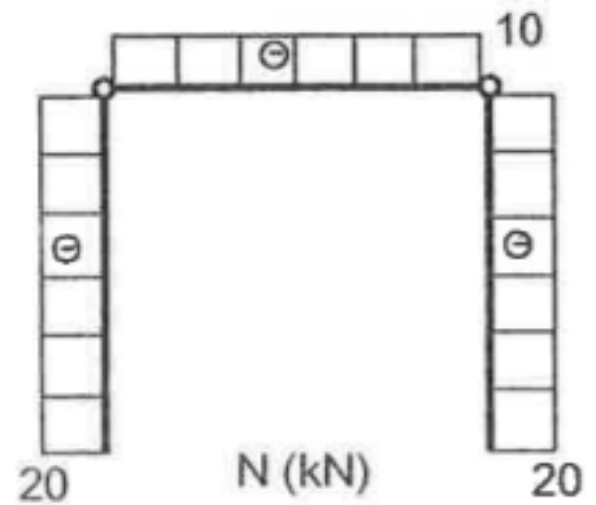
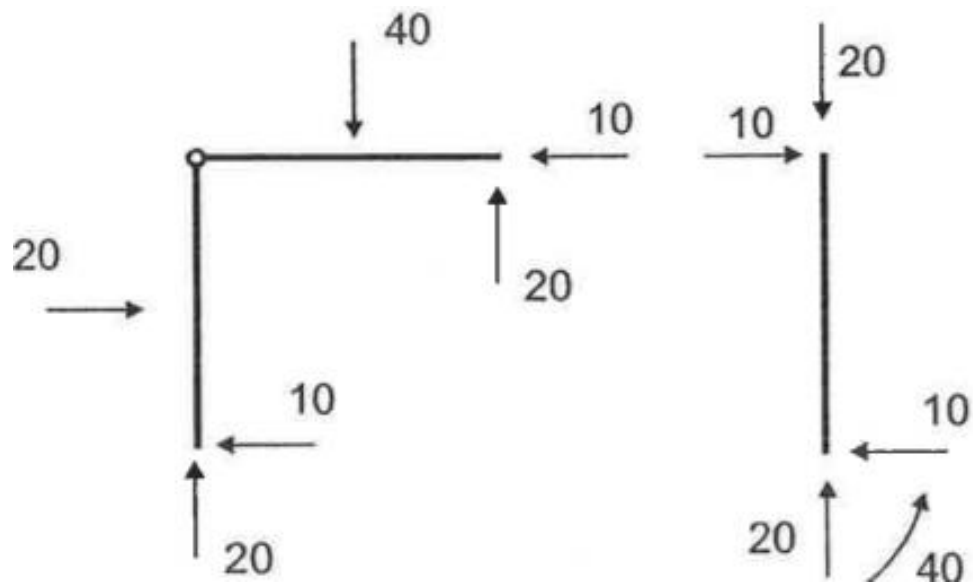


$$\sum M_{(F)} = 0 = M_F - 10 * 4 \Rightarrow M_F = 40 \text{ kNm}$$

$$\sum X = 0 = X_F + 10 \Rightarrow X_F = -10 \text{ kN}$$

$$\sum Y = 0 = Y_F - 20 \Rightarrow Y_F = 20 \text{ kN}$$

GRINTER

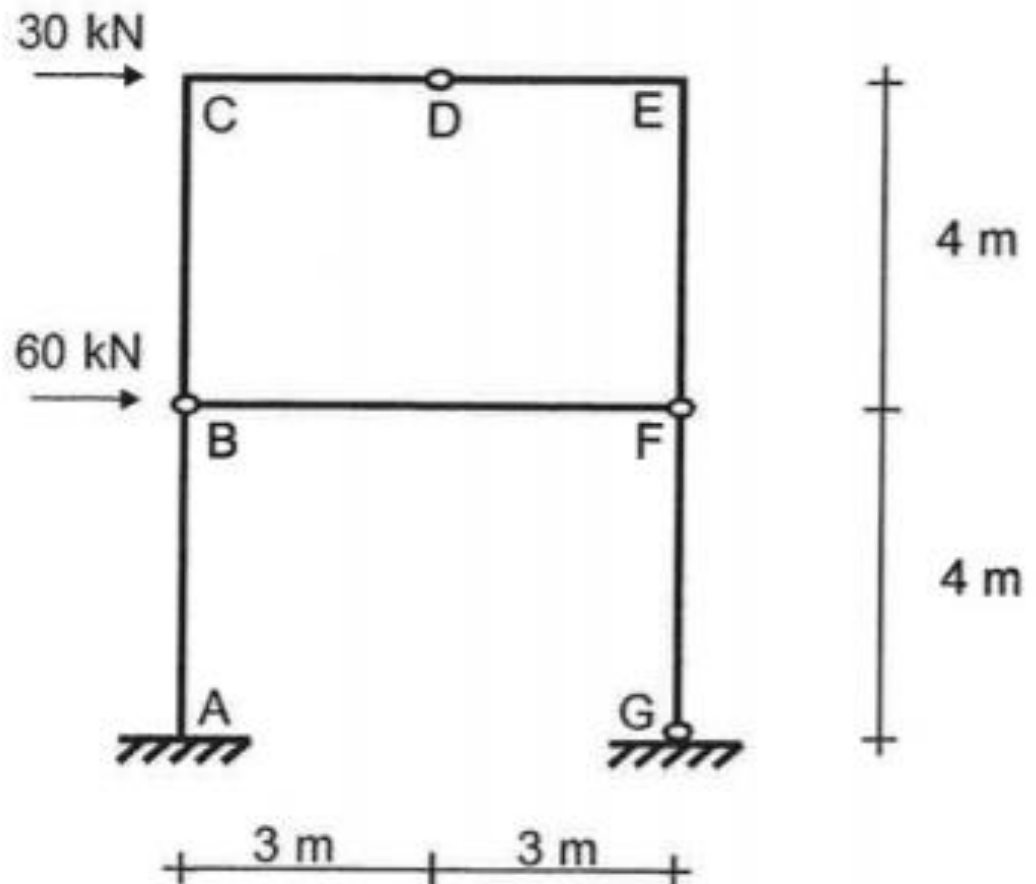


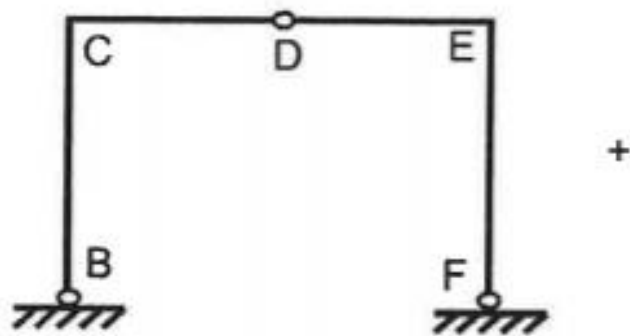
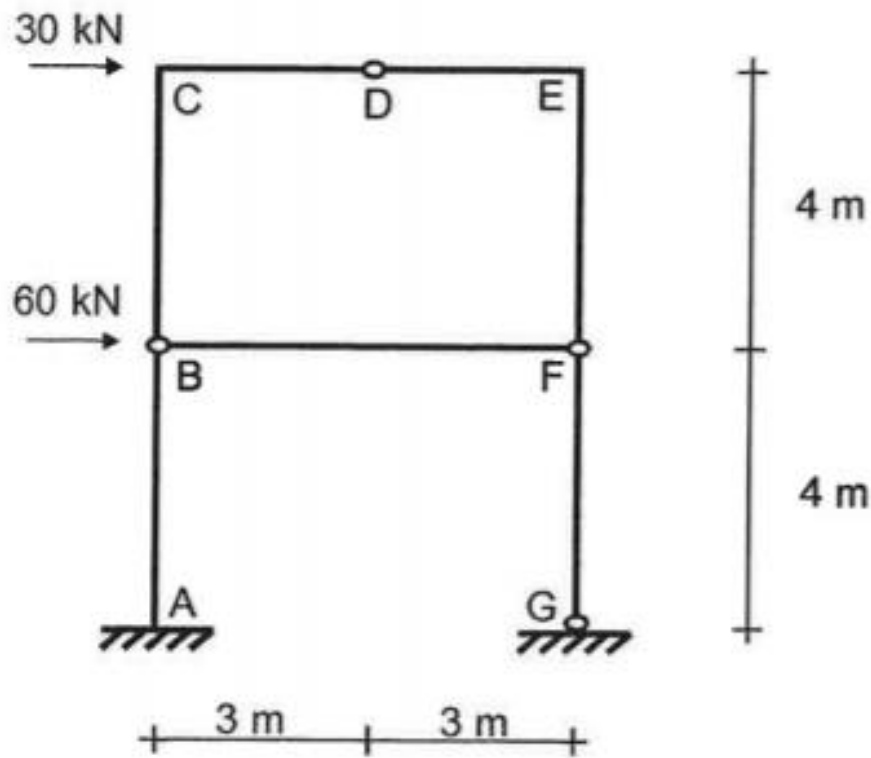


## Exercício 11.

Traçar os diagramas dos esforços solicitantes da estrutura da figura

APOSTILA  
ESTRUTURAS ASSOCIADAS  
PÁGINA 173

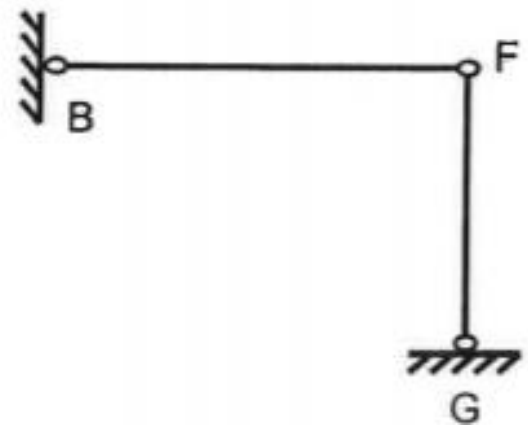


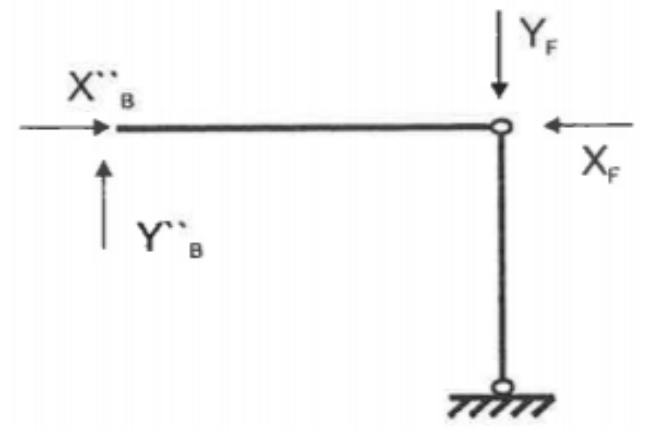
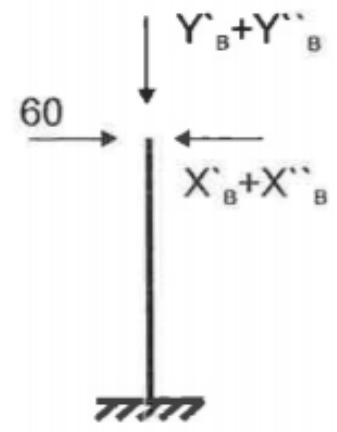
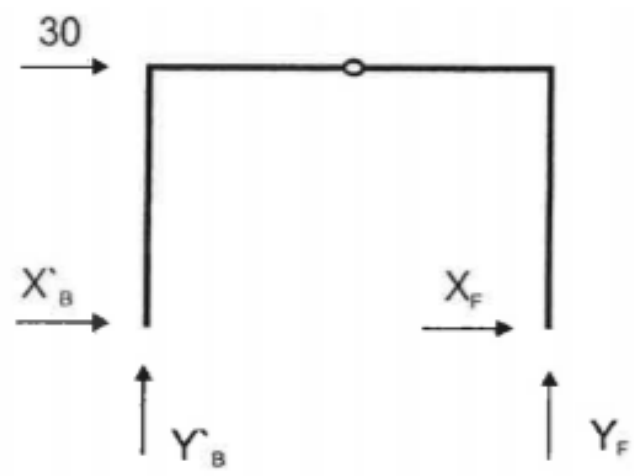
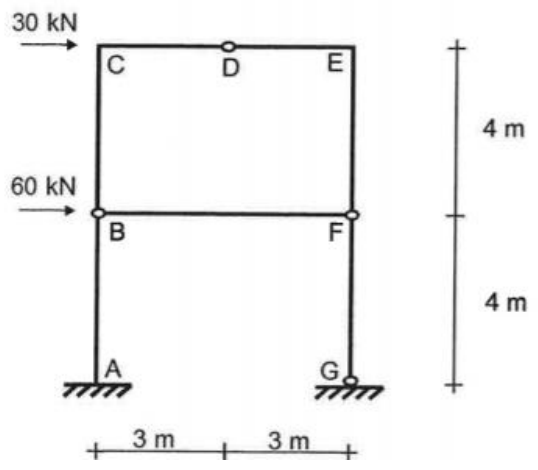


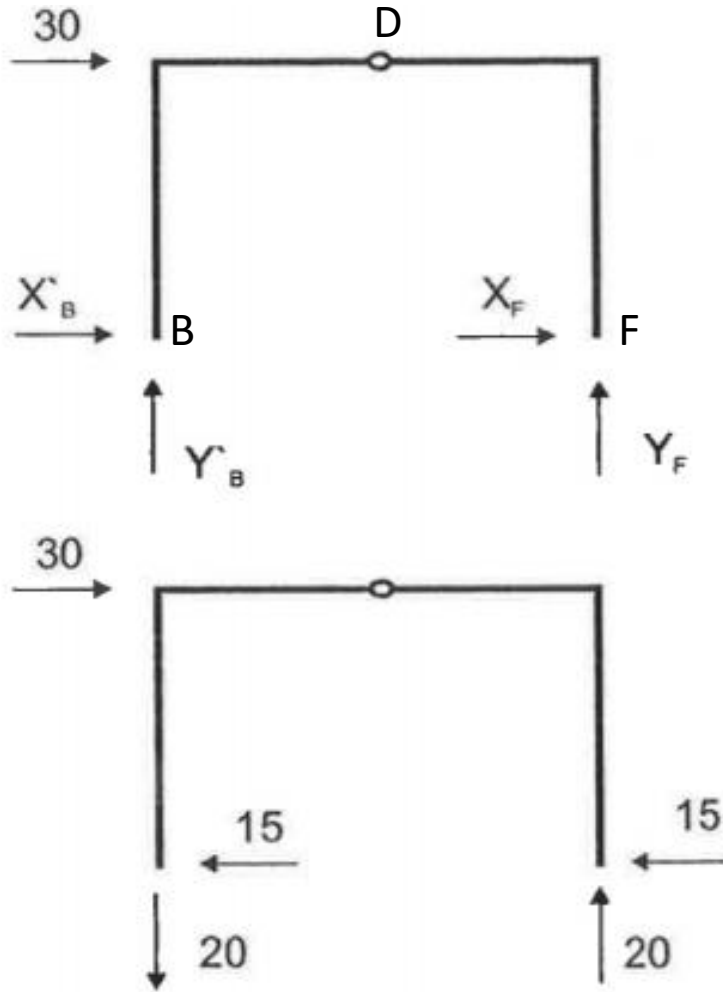
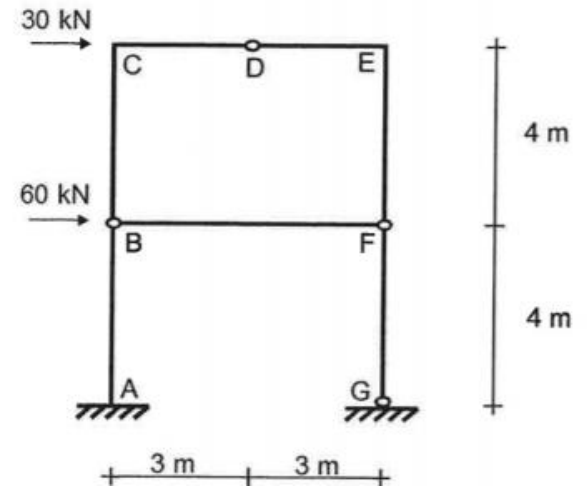
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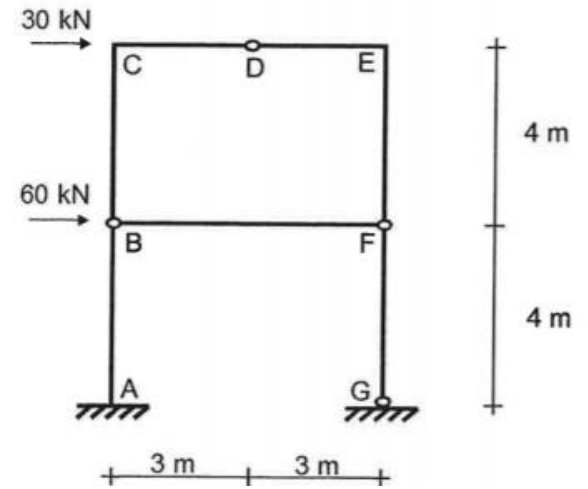
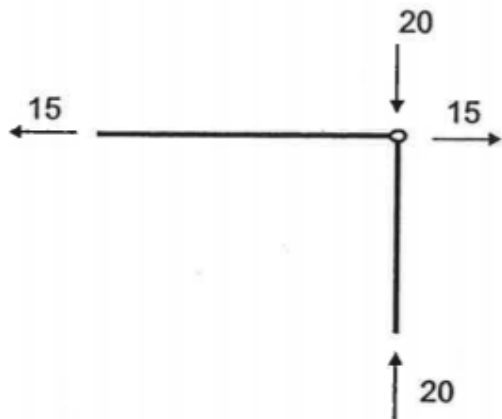
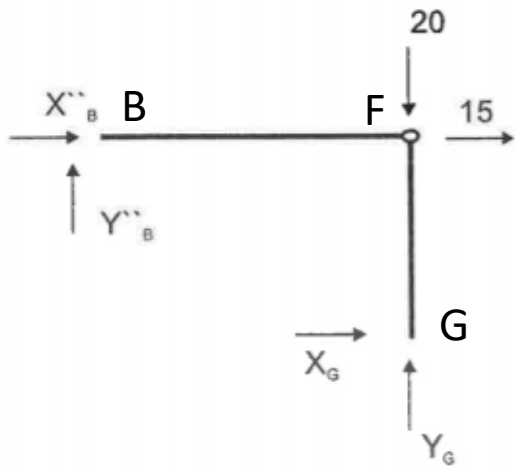


$$\sum M_{(B)} = 0 = -30 * 4 + Y_F * 6 \Rightarrow Y_F = 20 \text{ kN}$$

$$\sum M_{fletor}^{D,direita} = 0 = X_F * 4 + Y_F * 3 \Rightarrow X_F = -15 \text{ kN}$$

$$\sum X = 0 = X'_B + 30 + X_F \Rightarrow X'_B = -15 \text{ kN}$$

$$\sum Y = 0 = Y'_B + Y_F \Rightarrow Y'_B = -20 \text{ kN}$$

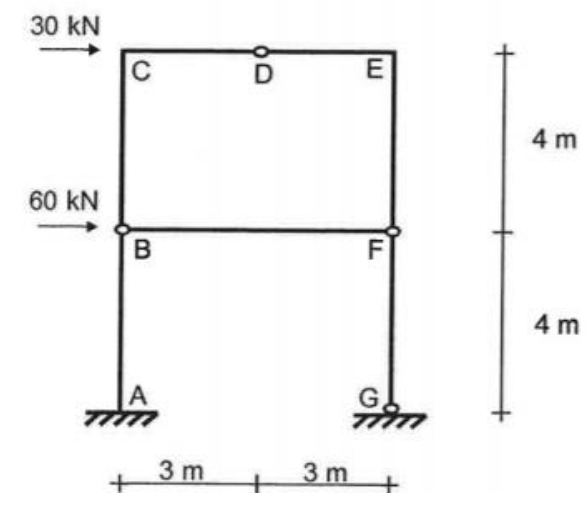
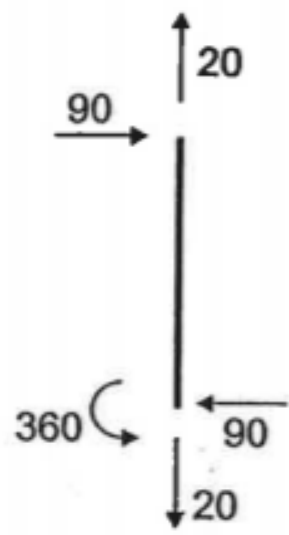
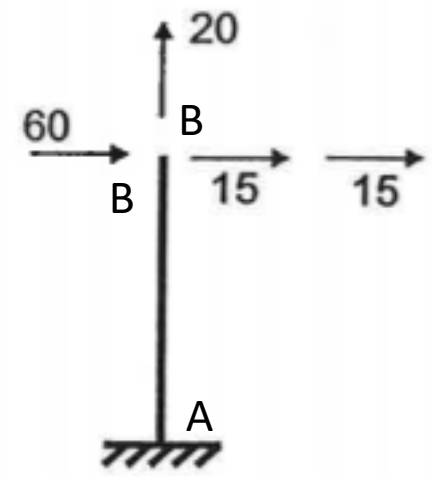
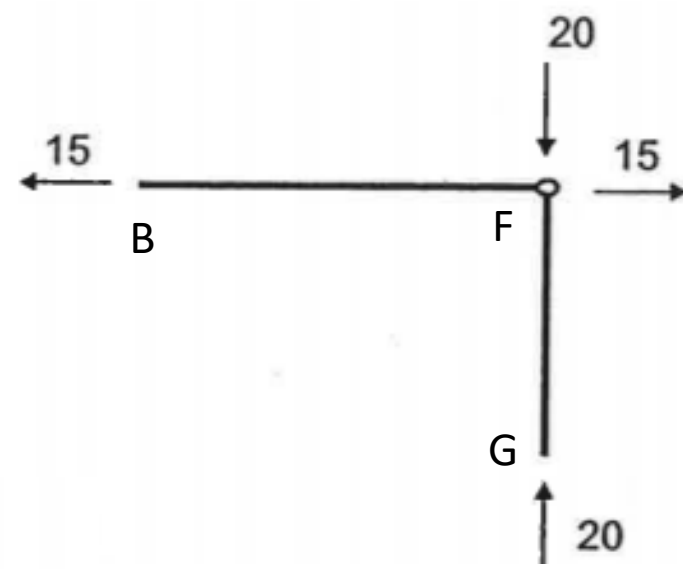
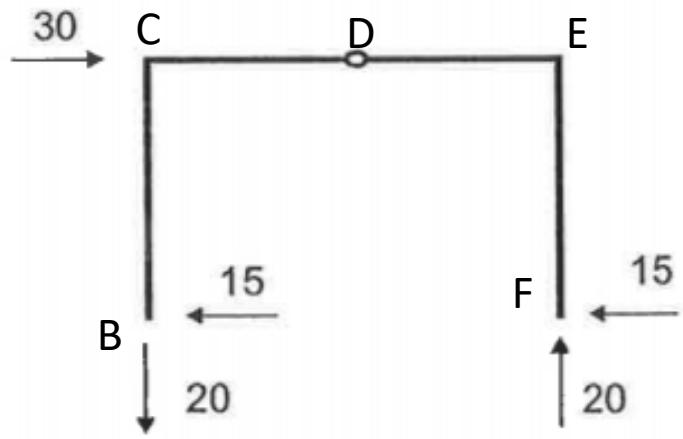


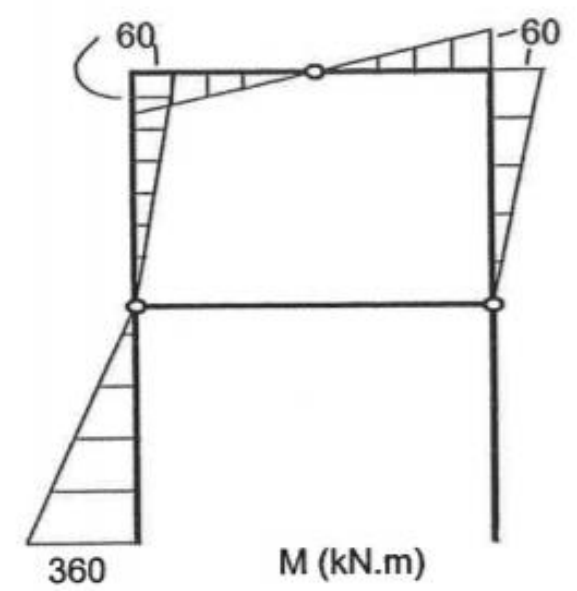
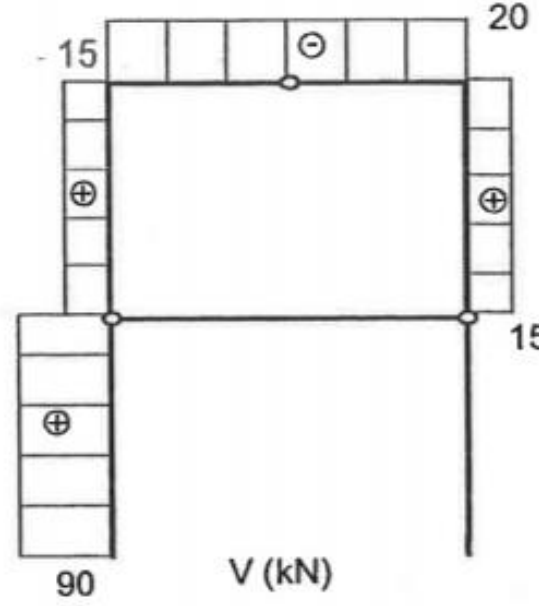
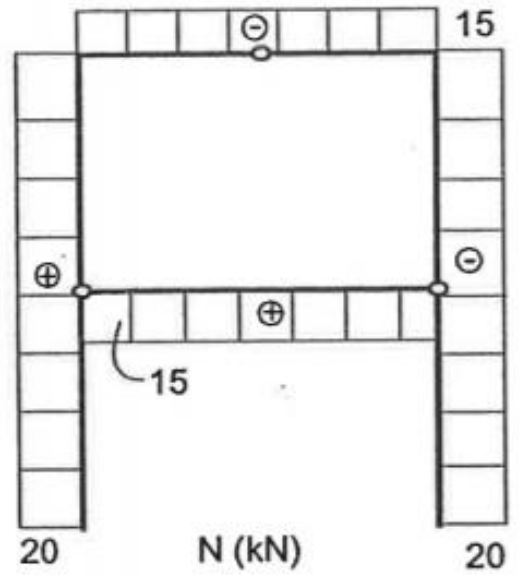
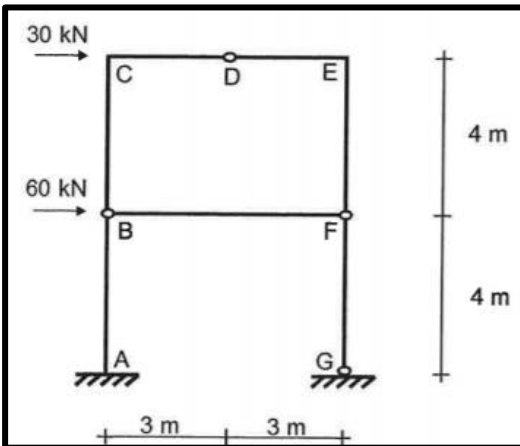
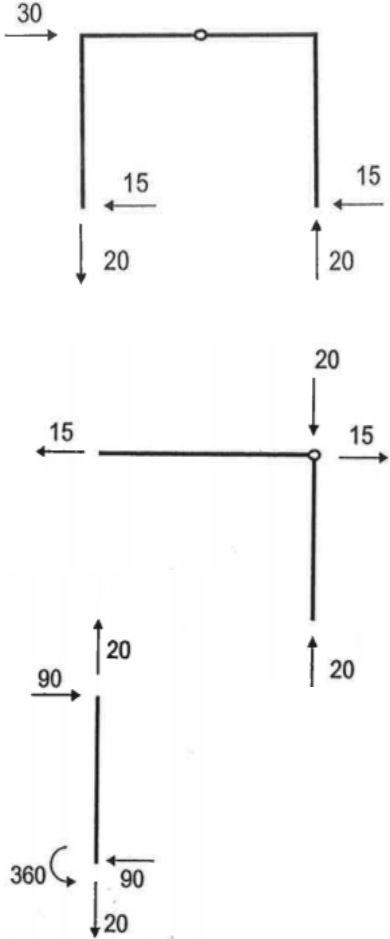
$$\sum M_{fletor}^{F,direita} = 0 = X_G * 4 \Rightarrow X_G = 0 \text{ kN}$$

$$\sum M_{(B)} = 0 = -20 * 6 + X_G * 4 + Y_G * 6 \Rightarrow Y_G = 20 \text{ kN}$$

$$\sum X = 0 = X''_B + 15 + X_G \Rightarrow X''_B = -15 \text{ kN}$$

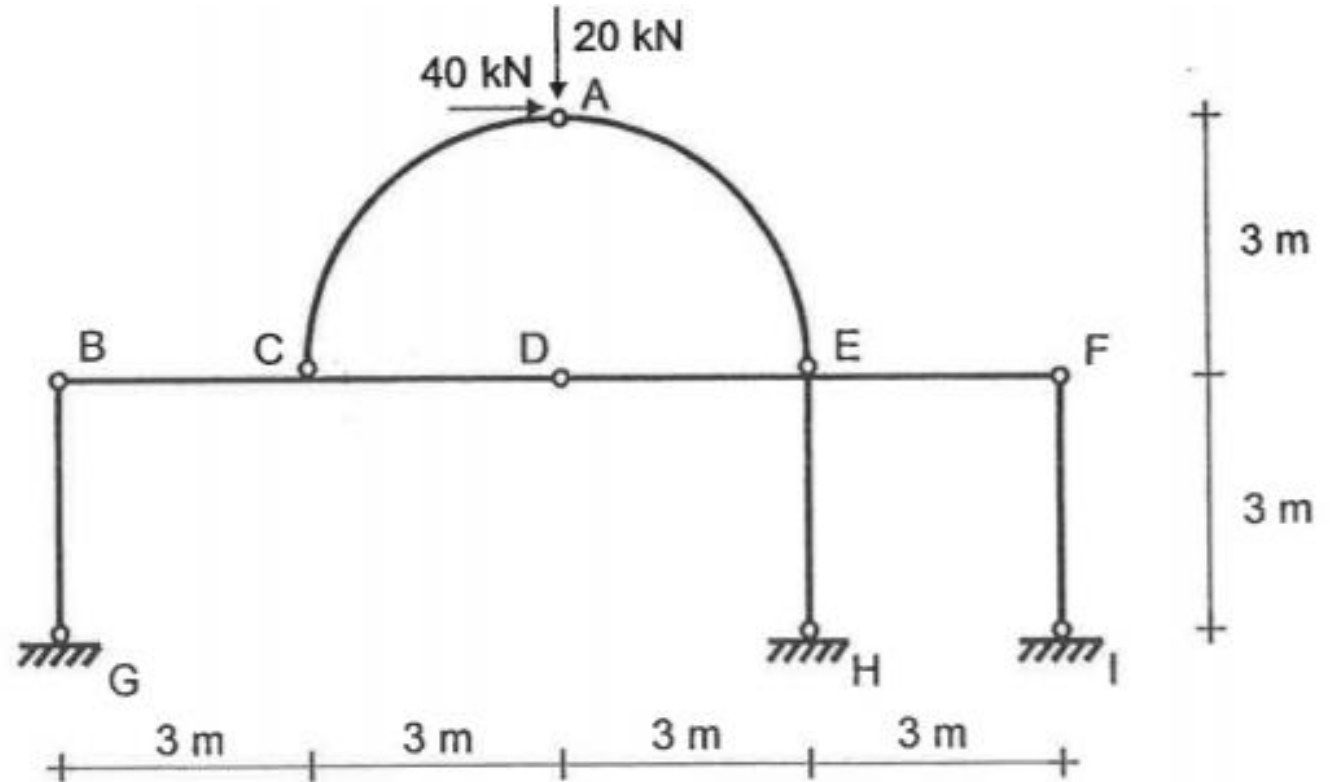
$$\sum Y = 0 = Y''_B - 20 + Y_G \Rightarrow Y''_B = 0 \text{ kN}$$





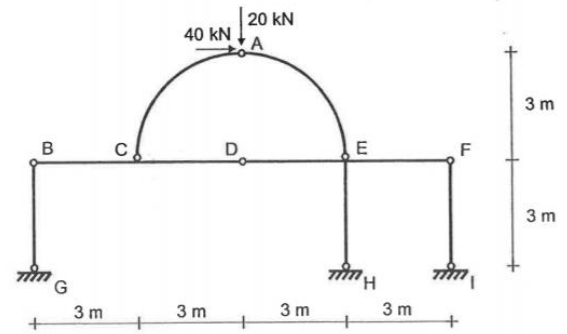
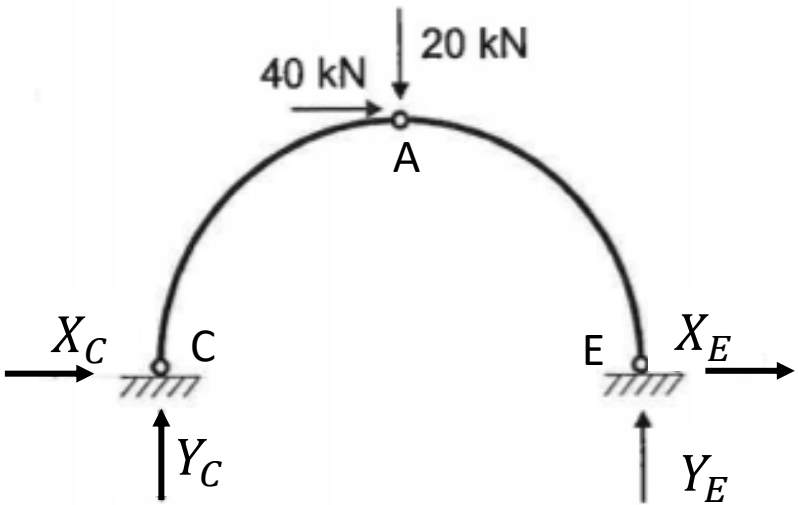
## Exercício 12.

Traçar o diagrama de momentos fletores dos trechos retos da estrutura da figura



**APOSTILA  
ESTRUTURAS ASSOCIADAS  
PÁGINA 176**



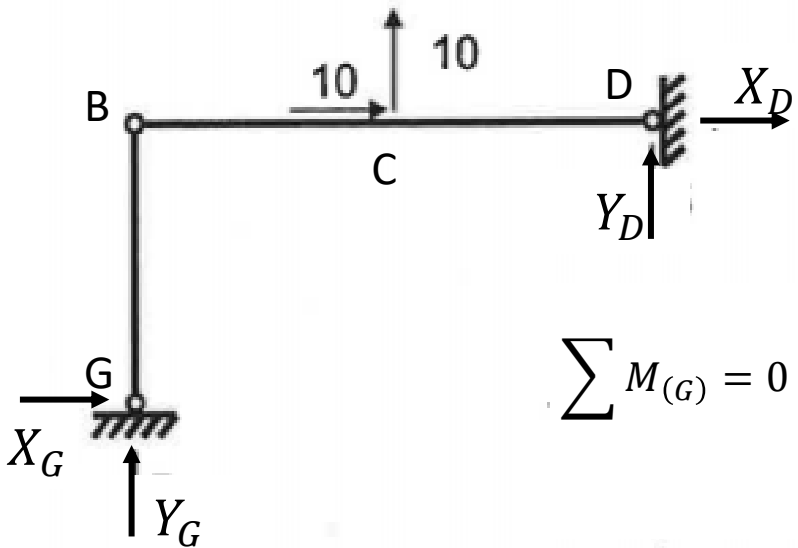


$$\sum M_{(C)} = 0 = -40 * 3 - 20 * 3 + Y_E * 6 \Rightarrow Y_E = 30 \text{ kN}$$

$$\sum M_{(E)} = 0 = -40 * 3 + 20 * 3 - Y_C * 6 \Rightarrow Y_C = -10 \text{ kN}$$

$$\sum M_{fletor}^{A,direita} = 0 = X_E * 3 + Y_E * 3 \Rightarrow X_E = -30 \text{ kN}$$

$$\sum M_{fletor}^{A,esquerda} = 0 = X_C * 3 - Y_C * 3 \Rightarrow X_C = -10 \text{ kN}$$

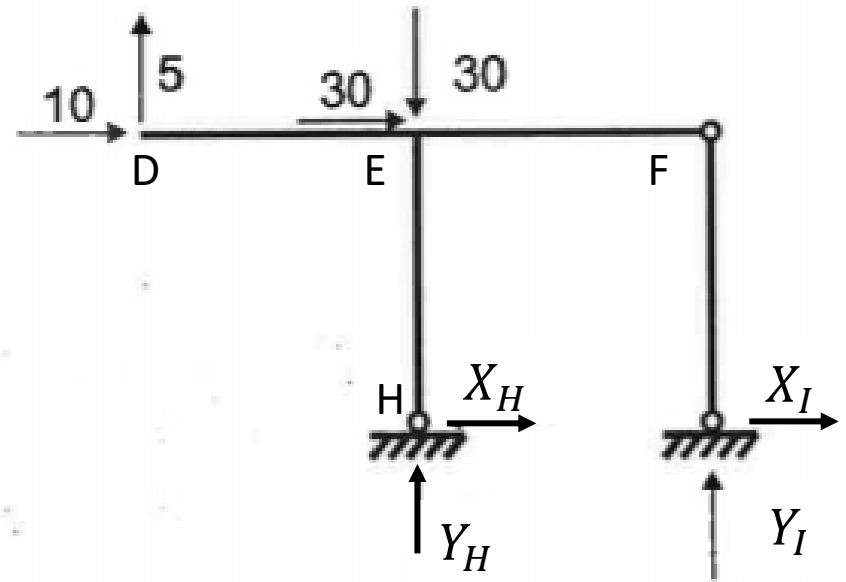
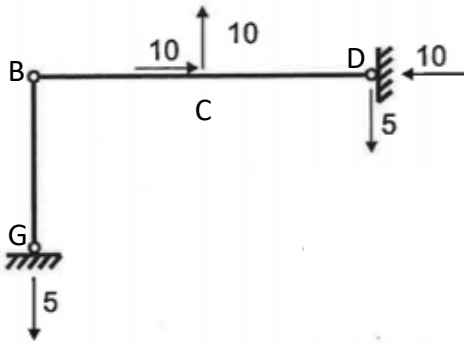
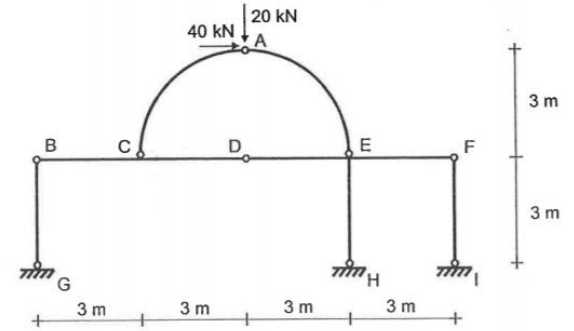
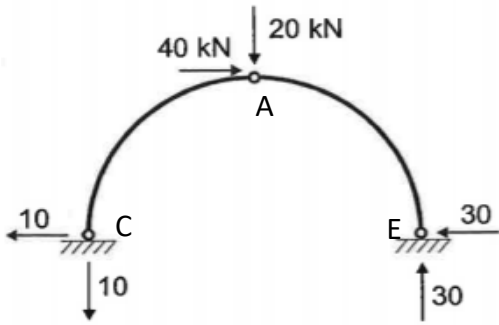


$$\sum M_{fletor}^{B,esquerda} = 0 = X_G * 3 \Rightarrow X_G = 0 \text{ kN}$$

$$\sum M_{fletor}^{B,direita} = 0 = 10 * 3 + Y_D * 6 \Rightarrow Y_D = -5 \text{ kN}$$

$$\sum M_{(G)} = 0 = -10 * 3 + 10 * 3 + Y_D * 6 + X_D * 3 \Rightarrow X_D = -10 \text{ kN}$$

$$\sum Y = 0 = Y_G + 10 + Y_D \Rightarrow Y_G = -5 \text{ kN}$$

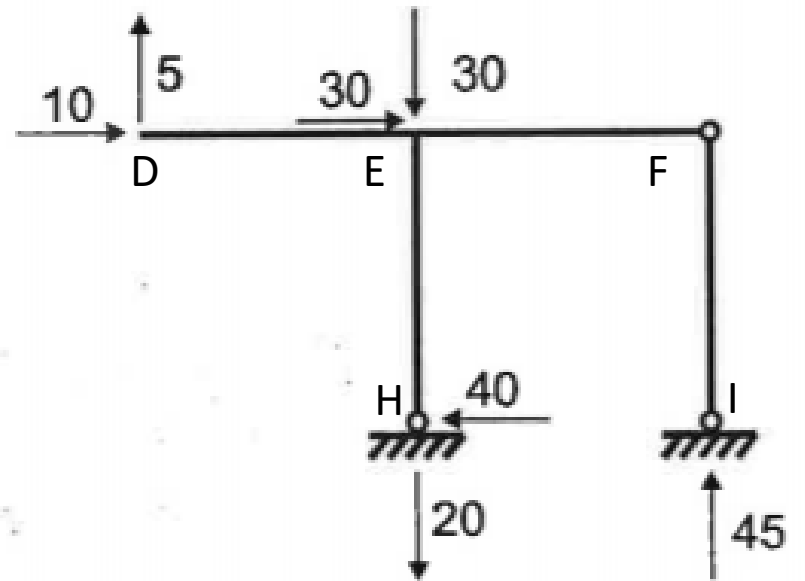
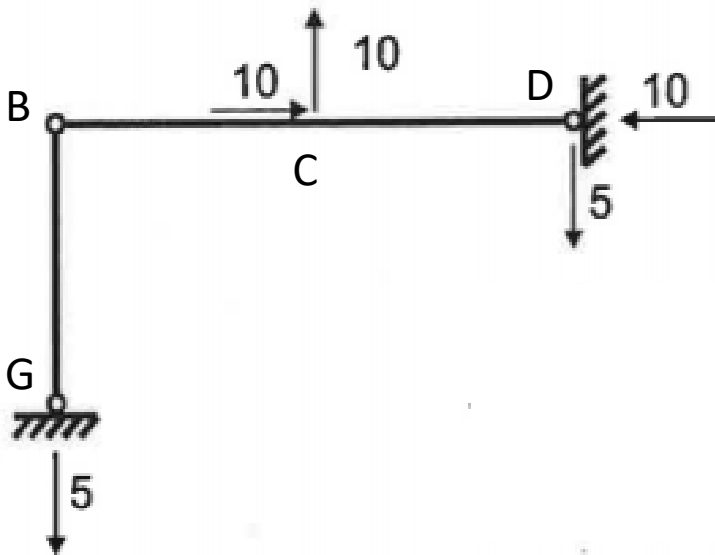
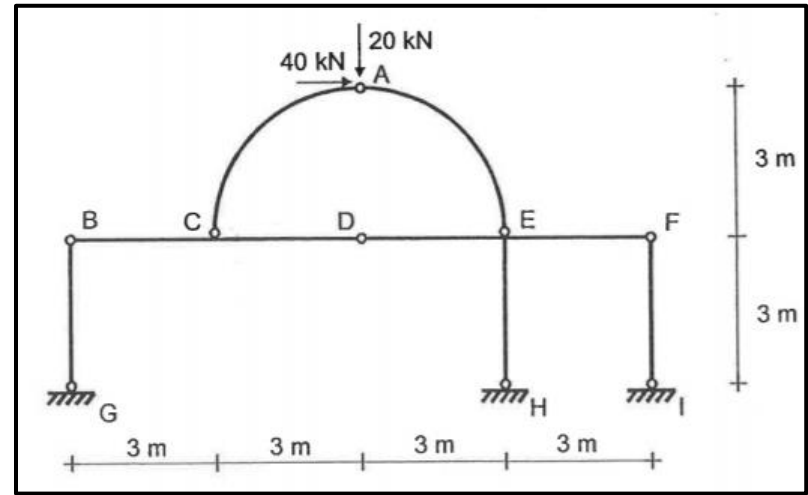
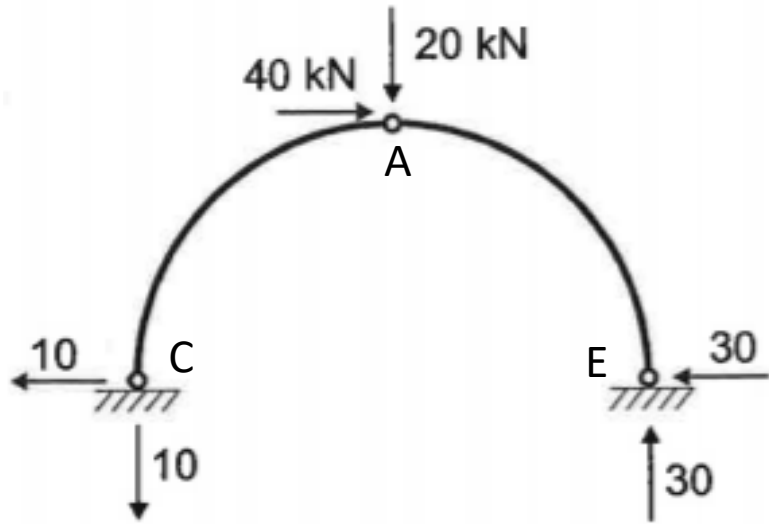


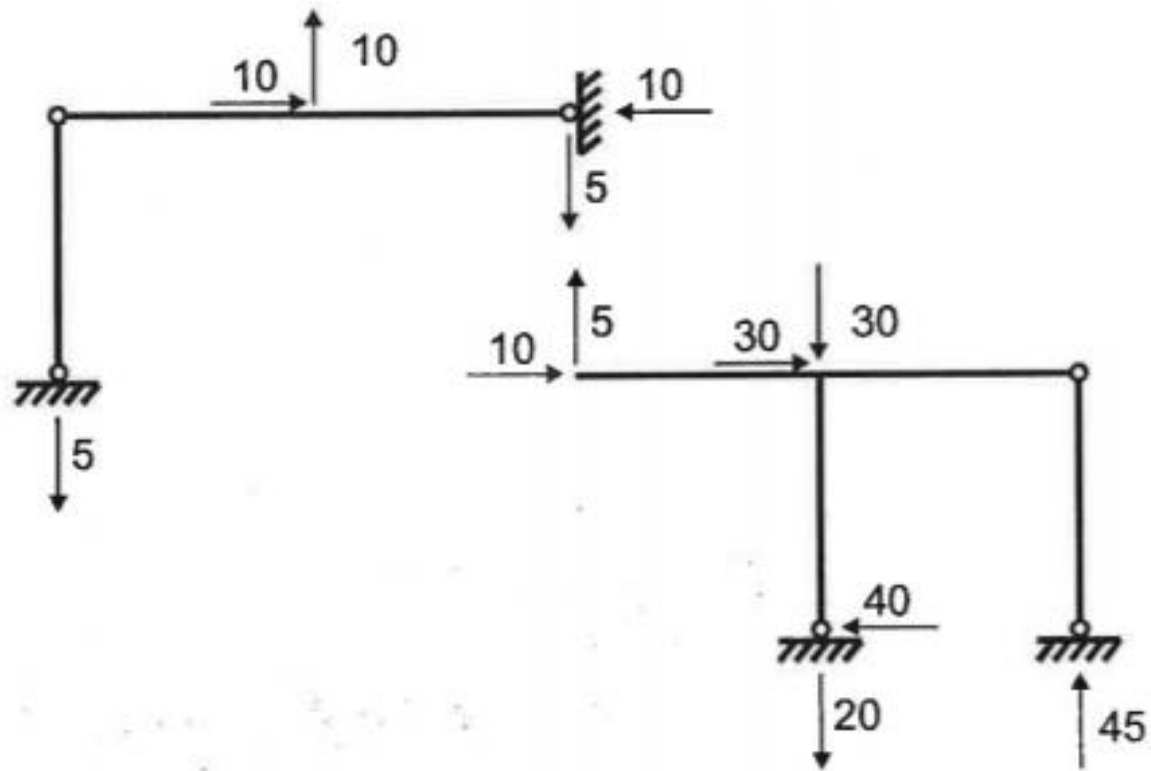
$$\sum M_{fletor}^{F,direita} = 0 = X_I * 3 \Rightarrow X_I = 0 \text{ kN}$$

$$\sum X = 0 = X_H + 10 + 30 + X_I \Rightarrow X_H = -40 \text{ kN}$$

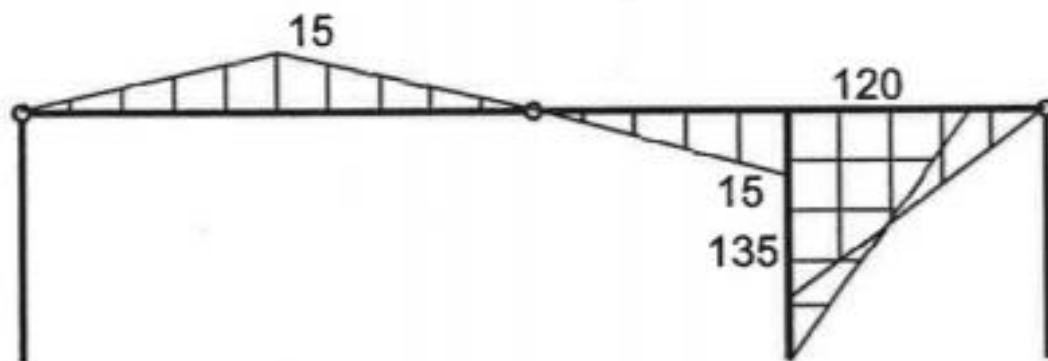
$$\sum M_{fletor}^{F,esquerda} = 0 = -5 * 6 + 30 * 3 - Y_H * 3 + X_H * 3 \Rightarrow Y_H = -20 \text{ kN}$$

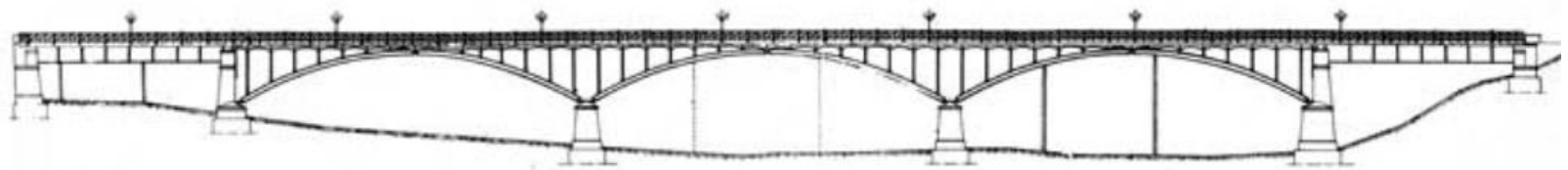
$$\sum Y = 0 = 5 - 30 + Y_H + Y_I \Rightarrow Y_I = 45 \text{ kN}$$





M (kNm)







**Ponte da Cidade Universitária**

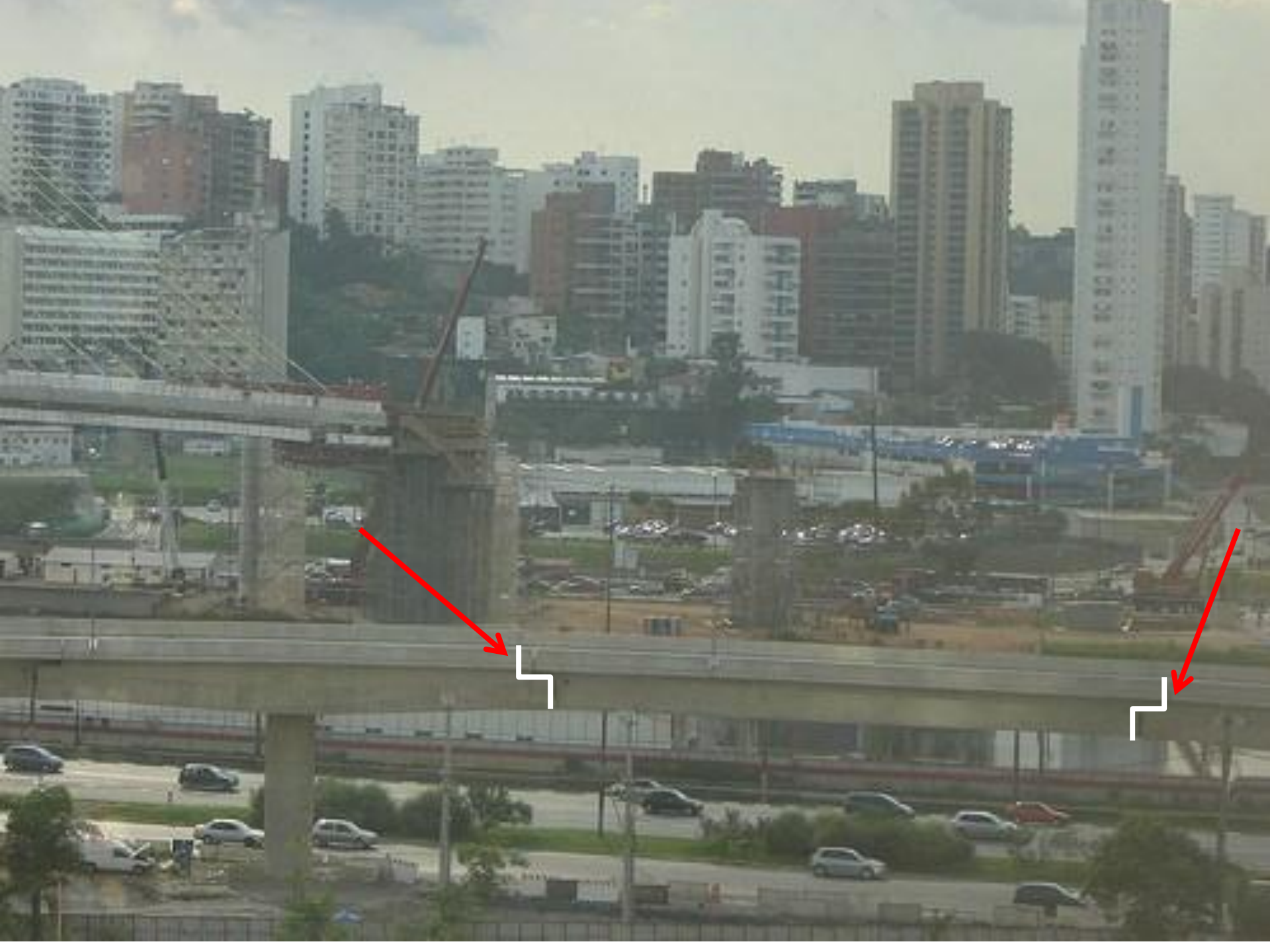




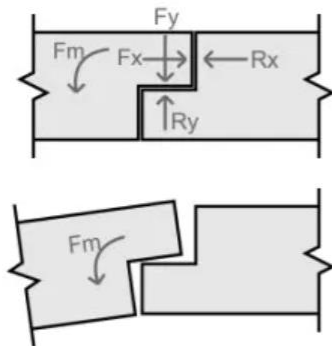
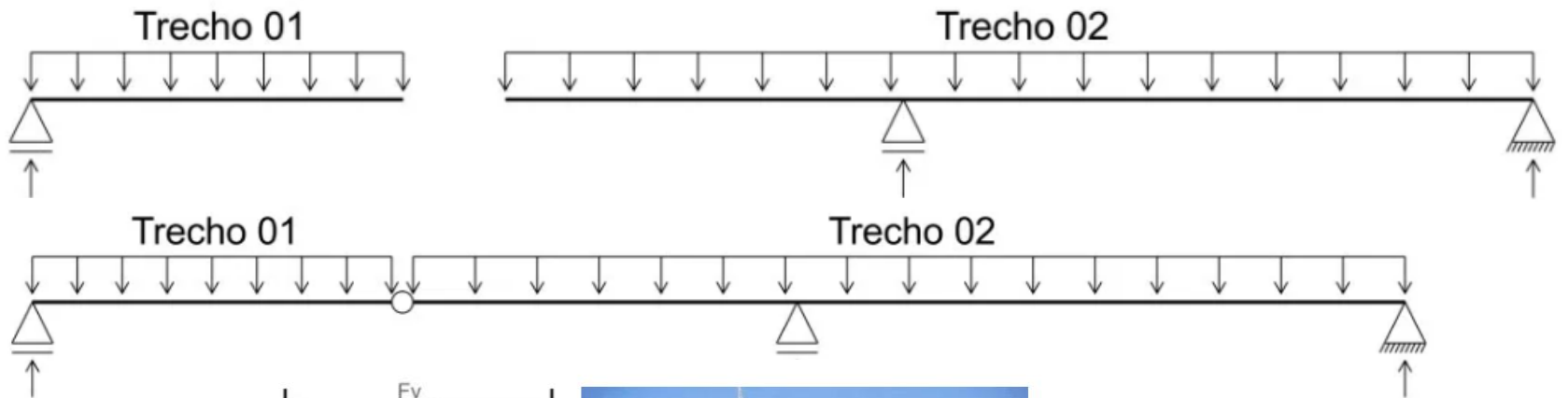












Ponte Rio-Niterói

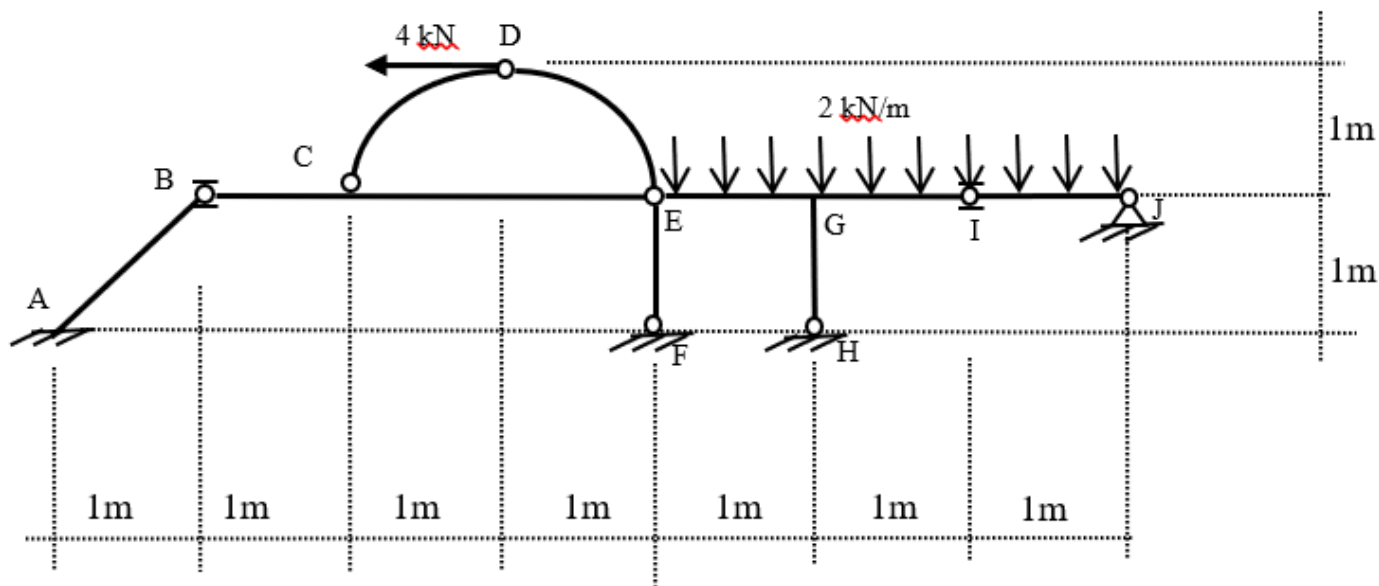
## Exercício 13.

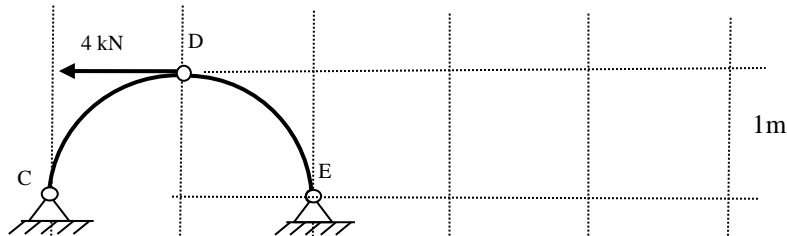
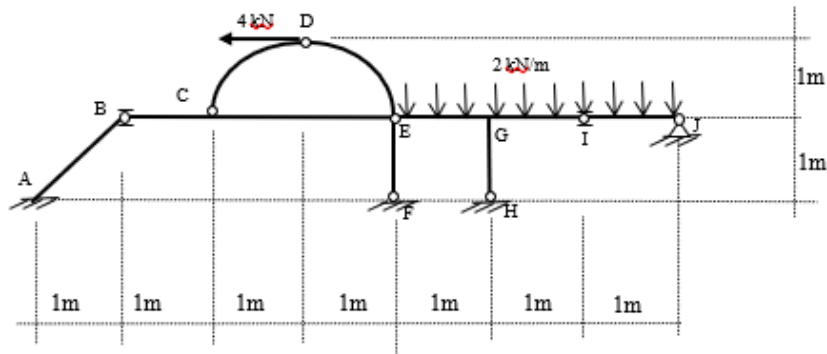
**PEF-3200 Introdução à Mecânica das Estruturas P3 27/6/2018**

nome.....no. USP.....

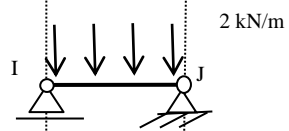
**Questão 3** Na estrutura associada ABCDEFGHIJ representada na figura:

- Identifique as subestruturas que a compõem (isolando e nomeando);
- Determine as reações nos apoios F e H.

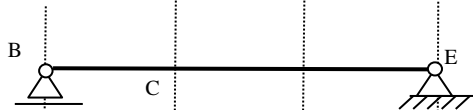




Arco triarticulado

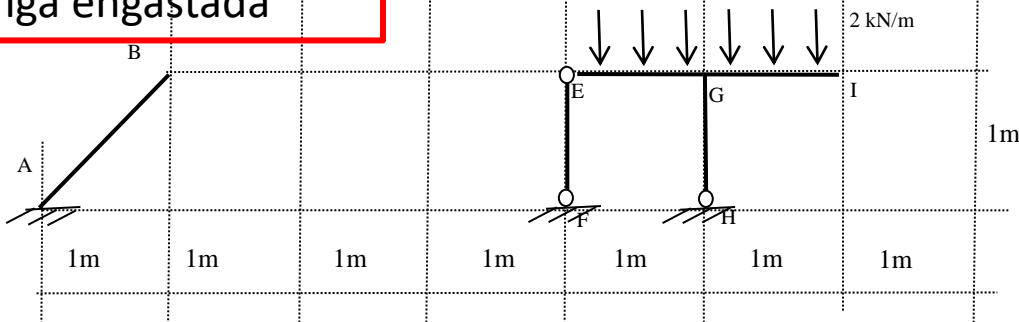


Viga simplesmente apoiada ou Viga biapoiada



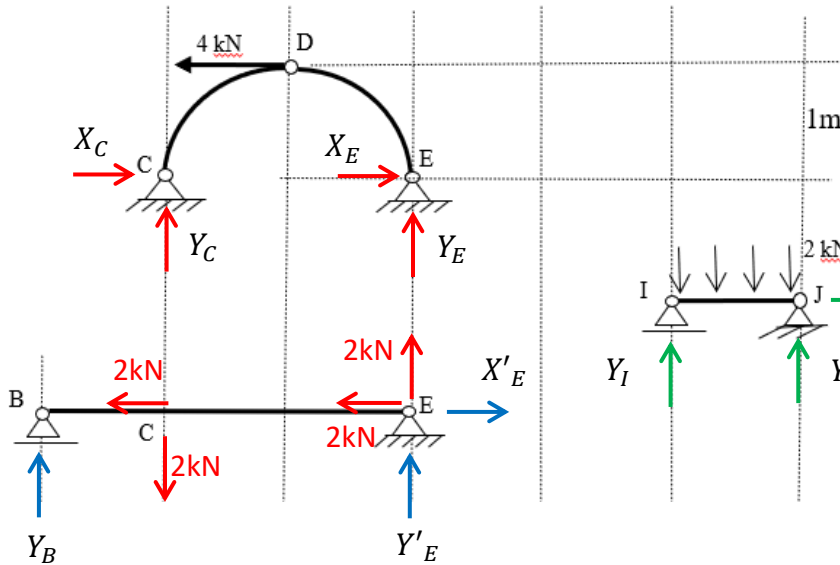
Viga simplesmente apoiada ou Viga biapoiada

Viga em balanço ou Viga engastada



Pórtico triarticulado

# 1ª OPÇÃO

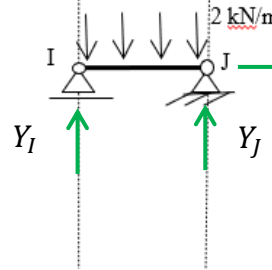


$$\sum M_{(C)} = 0 = 4 * 1 + Y_E * 2 \Rightarrow Y_E = -2 \text{ kN}$$

$$\sum Y = 0 = Y_C + Y_E \Rightarrow Y_C = 2 \text{ kN}$$

$$\sum M_{fletor}^{D, direita} = 0 = X_E * 1 + Y_E * 1 \Rightarrow X_E = 2 \text{ kN}$$

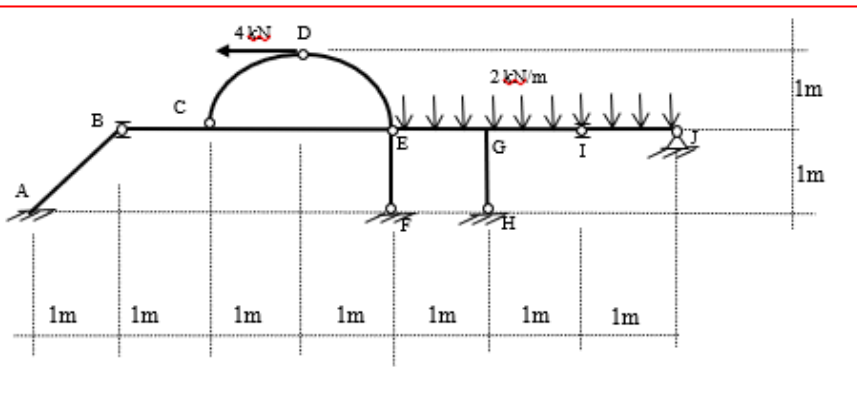
$$\sum X = 0 = X_C - 4 + X_E \Rightarrow X_C = 2 \text{ kN}$$



$$\sum X = 0 = X_J \Rightarrow X_J = 0 \text{ kN}$$

$$\sum M_{(I)} = 0 = -2 * 1 * 0,5 + Y_J * 1 \Rightarrow Y_J = 1 \text{ kN}$$

$$\sum Y = 0 = Y_I - 2 * 1 + Y_J \Rightarrow Y_I = 1 \text{ kN}$$

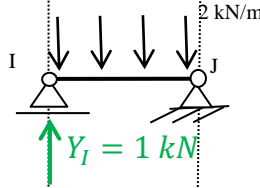
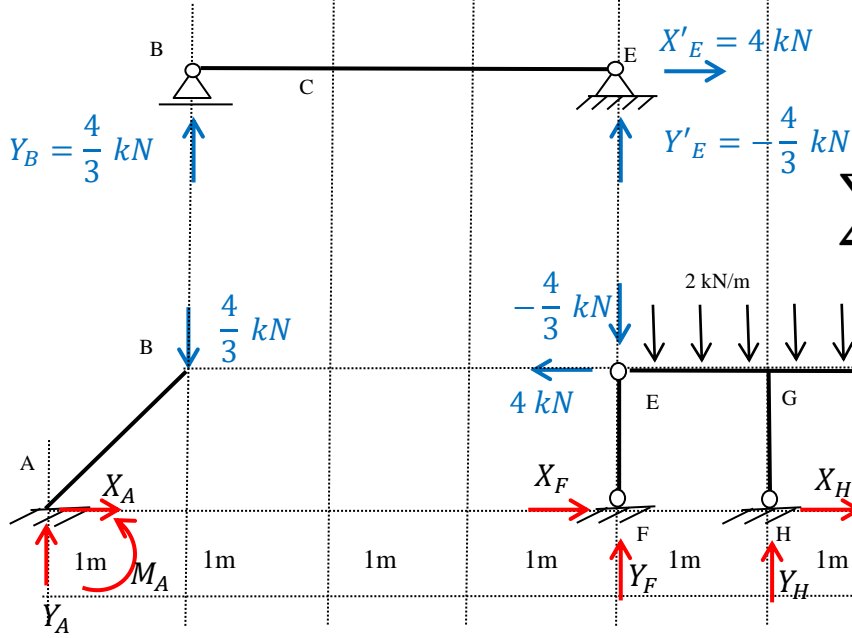
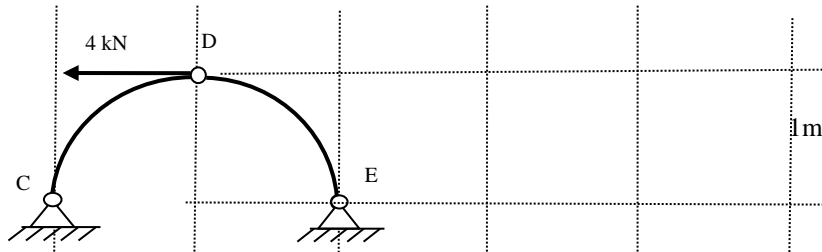


$$\sum X = 0 = -2 - 2 + X'_E \Rightarrow X'_E = 4 \text{ kN}$$

$$\sum M_{(B)} = 0 = -2 * 1 + 2 * 3 + Y'_E * 3 \Rightarrow Y'_E = -\frac{4}{3} \text{ kN}$$

$$\sum Y = 0 = Y_B - 2 + 2 + Y'_E \Rightarrow Y_B = \frac{4}{3} \text{ kN}$$





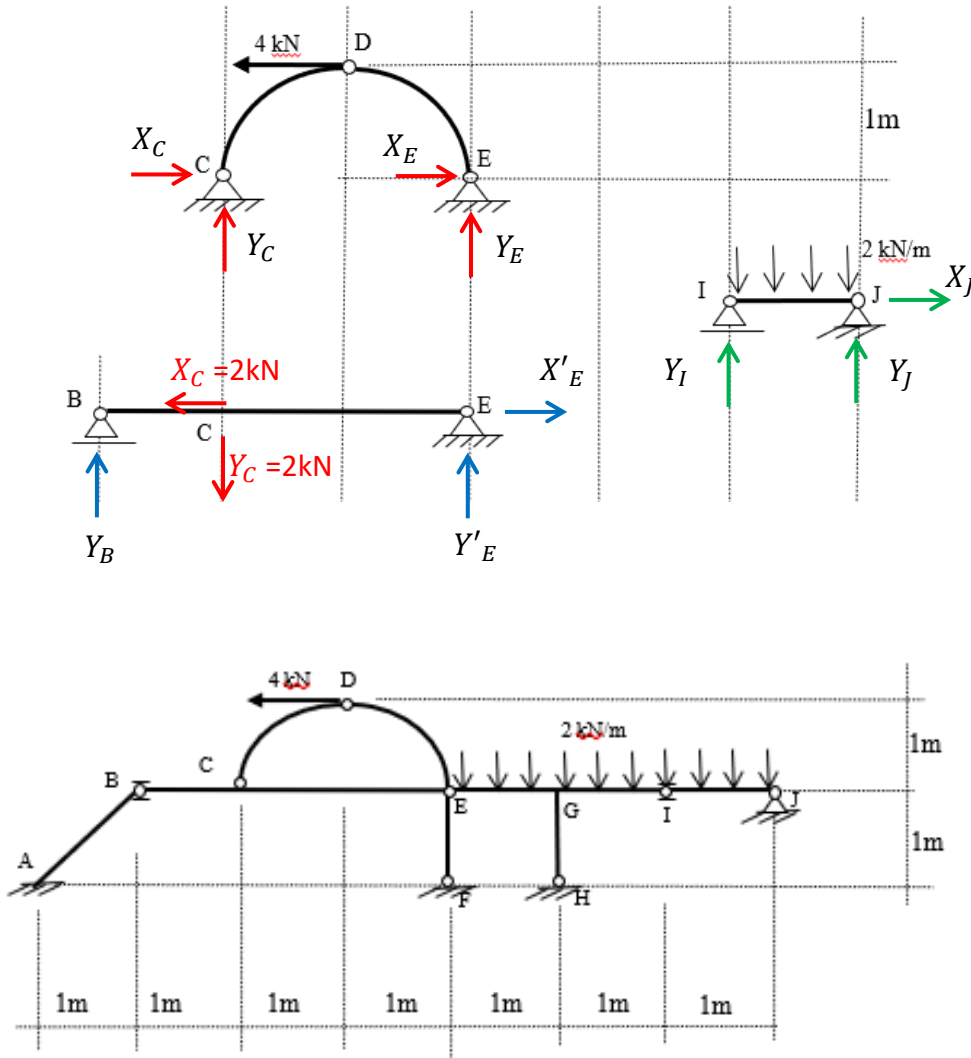
$$\sum M_{(F)} = 0 = 4 * 1 + Y_H * 1 - 2 * 2 * 1 - 1 * 2 \Rightarrow Y_H = 2 \text{ kN}$$

$$\sum Y = 0 = Y_F - (-\frac{4}{3}) - 2 * 2 - 1 + Y_H \Rightarrow Y_F = \frac{5}{3} \text{ kN}$$

$$\sum M_{flector}^{E, esquerda} = 0 = X_F * 1 \Rightarrow X_F = 0 \text{ kN}$$

$$\sum X = 0 = X_F - 4 + X_H \Rightarrow X_H = 4 \text{ kN}$$

## 2ª OPÇÃO



$$\sum M_{(C)} = 0 = 4 * 1 + Y_E * 2 \Rightarrow Y_E = -2 \text{ kN}$$

$$\sum Y = 0 = Y_C + Y_E \Rightarrow Y_C = 2 \text{ kN}$$

$$\sum M_{fletor}^{D, direita} = 0 = X_E * 1 + Y_E * 1 \Rightarrow X_E = 2 \text{ kN}$$

$$\sum X = 0 = X_C - 4 + X_E \Rightarrow X_C = 2 \text{ kN}$$

$$\sum X = 0 = X_J \Rightarrow X_J = 0 \text{ kN}$$

$$\sum M_{(I)} = 0 = -2 * 1 * 0,5 + Y_J * 1 \Rightarrow Y_J = 1 \text{ kN}$$

$$\sum Y = 0 = Y_I - 2 * 1 + Y_J \Rightarrow Y_I = 1 \text{ kN}$$

$$\sum X = 0 = -2 + X'_E \Rightarrow X'_E = 2 \text{ kN}$$

$$\sum M_{(B)} = 0 = -2 * 1 + Y'_E * 3 \Rightarrow Y'_E = \frac{2}{3} \text{ kN}$$

$$\sum Y = 0 = Y_B - 2 + Y'_E \Rightarrow Y_B = \frac{4}{3} \text{ kN}$$

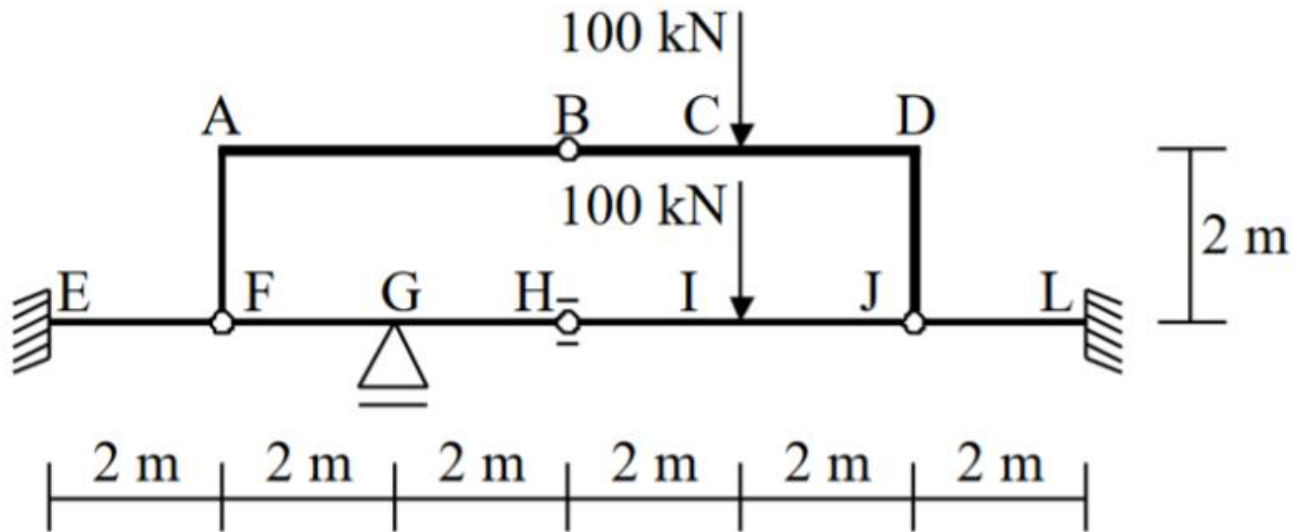


## EXERCÍCIO 14.

PEF-3200 – Prova de Recuperação – 24.7.2019

Nº USP: \_\_\_\_\_ Nome: \_\_\_\_\_

**3ª Questão ( xx pontos)** Para a estrutura da figura, obter os diagramas de esforços solicitantes do trecho EFGHIJL.



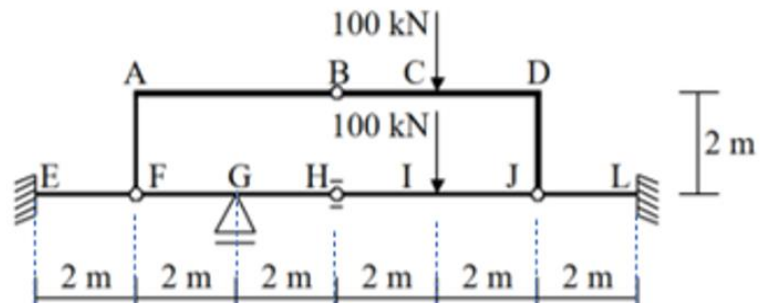


FIGURA 1

$$X_J = -50; Y_J = 75; X_F = 50; Y_F = 25$$

FIGURA 2

$$X'_J = 0; Y'_J = 50; Y'_H = 50$$

FIGURA 3

$$X''_F = 0; Y''_F = -50; Y''_G = 100$$

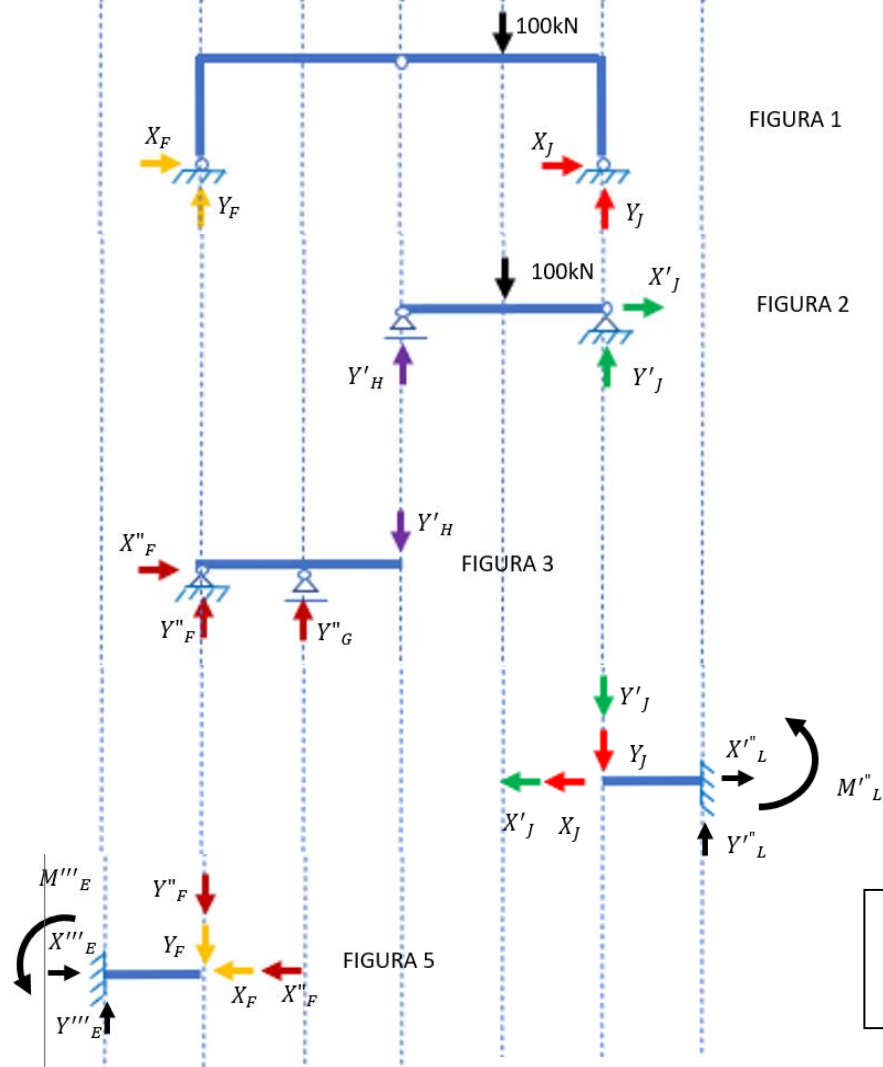
FIGURA 5

$$X'''_L = -50; Y'''_L = 125; M'''_L = -250;$$

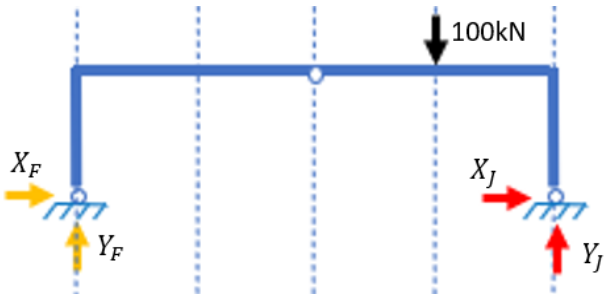
$$X_J = -50; Y_J = 75; X'_J = 0; Y'_J = 50$$

$$X'''_E = 50; Y'''_E = -25; M'''_E = 50;$$

$$X_F = 50; Y_F = 25; X''_F = 0; Y''_F = -50$$



**FIGURA 1**



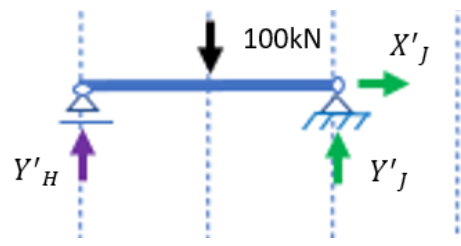
$$\sum M_{(F)} = 0 = -100 * 6 + Y_J * 8 \Rightarrow Y_J = 75 \text{ kN}$$

$$\sum Y = 0 = Y_F - 100 + Y_J \Rightarrow Y_F = 25 \text{ kN}$$

$$M_{fletor}^{B,esq} = 0 = X_F * 2 - Y_F * 4 \Rightarrow X_F = 50 \text{ kN}$$

$$\sum X = 0 = X_F + X_J \Rightarrow X_J = -50 \text{ kN}$$

**FIGURA 2**



$$\sum M_{(H)} = 0 = -100 * 2 + Y'_J * 4 \Rightarrow Y'_J = 50 \text{ kN}$$

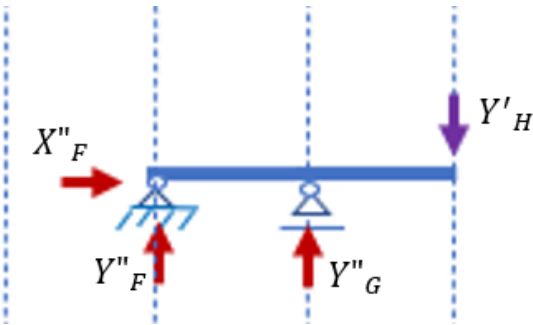
$$\sum Y = 0 = Y'_H + Y'_J - 100 \Rightarrow Y'_H = 50 \text{ kN}$$

$$\sum X = 0 = X'_J \Rightarrow X'_J = 0 \text{ kN}$$

$$X'_J = 0; Y'_J = 50; Y'_H = 50$$

$$X_J = -50; Y_J = 75; X'_J = 0; Y'_J = 50$$

**FIGURA 3**

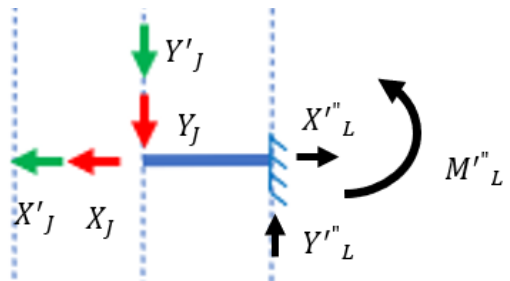


$$\sum M_{(F)} = 0 = Y''_G * 2 - Y'_H * 4 \Rightarrow Y''_G = 100 \text{ kN}$$

$$\sum Y = 0 = Y''_F + Y''_G - Y''_H \Rightarrow Y''_F = -50 \text{ kN}$$

$$\sum X = 0 = X''_F \Rightarrow X''_F = 0 \text{ kN}$$

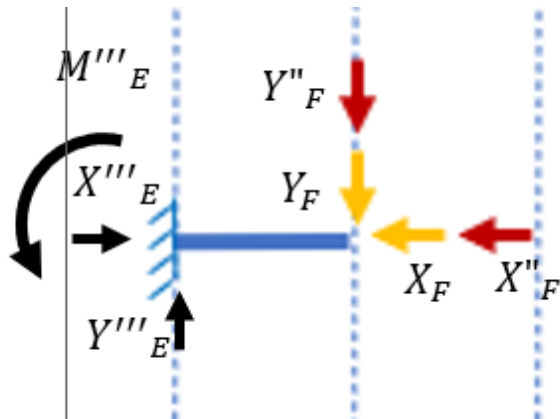
**FIGURA 4**



$$\sum M_{(L)} = 0 = Y_J * 2 + Y'_J * 2 + M'''_L \Rightarrow M'''_L = -250 \text{ kNm}$$

$$\sum Y = 0 = -Y'_J + Y_J + Y'''_L \Rightarrow Y'''_L = 125 \text{ kN}$$

$$\sum X = 0 = -X'_J - X_J + X'''_L \Rightarrow X'''_L = -50 \text{ kN}$$



$$X_F = 50; Y_F = 25; X''_F = 0; Y''_F = -50$$

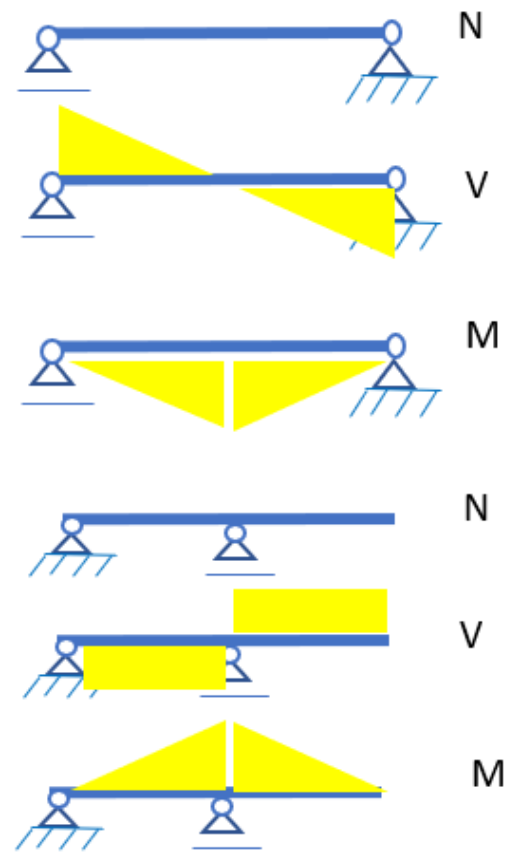
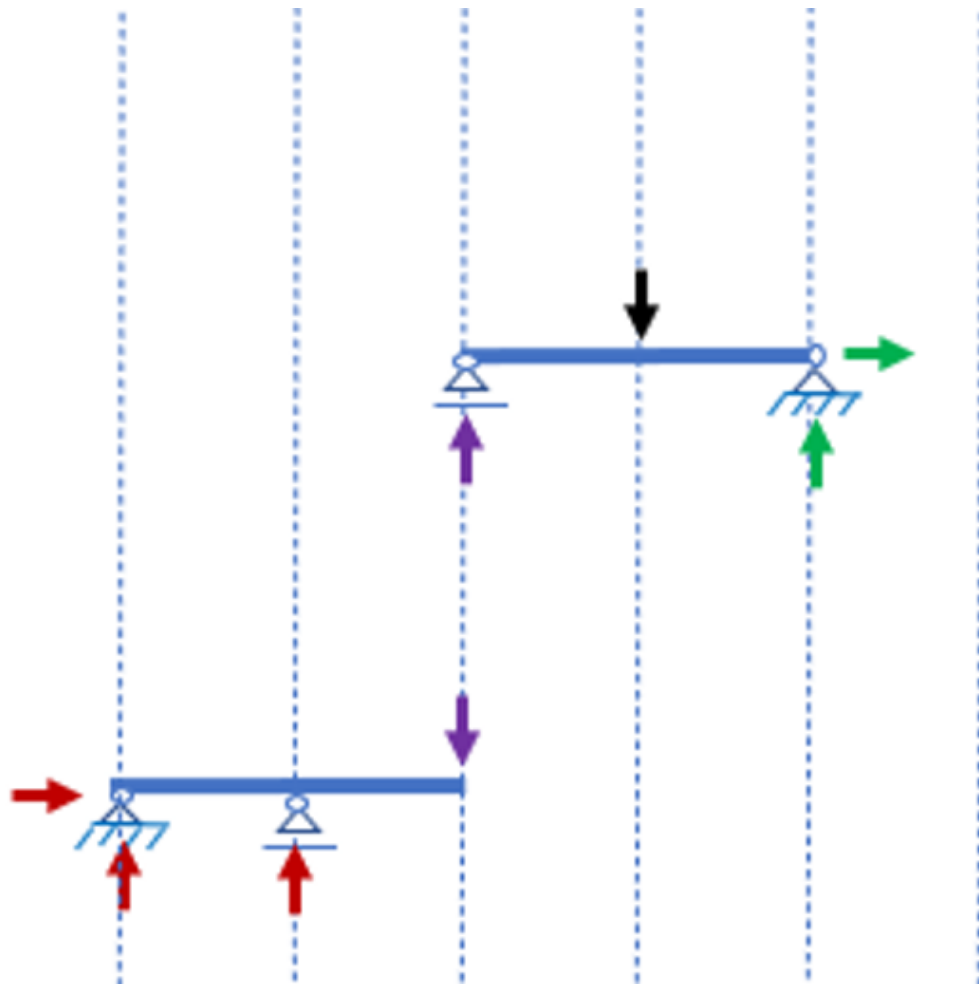
**FIGURA 5**

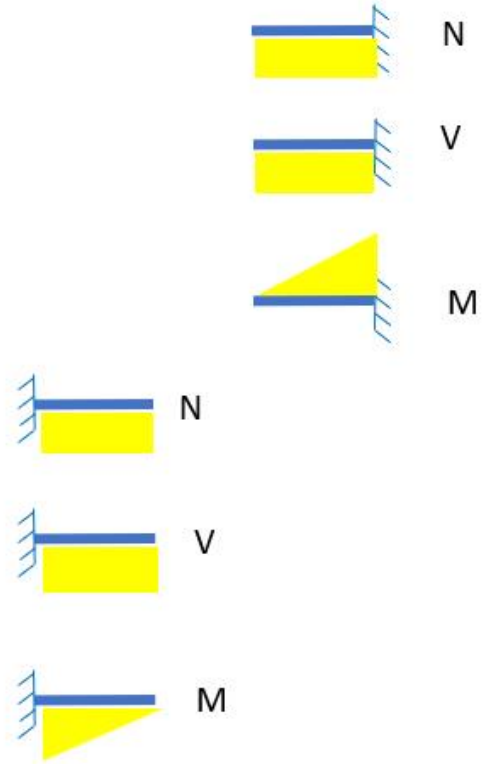
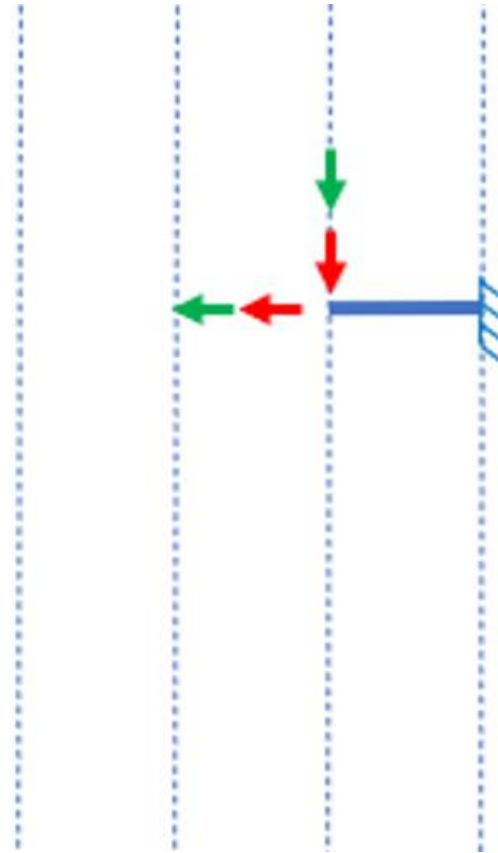
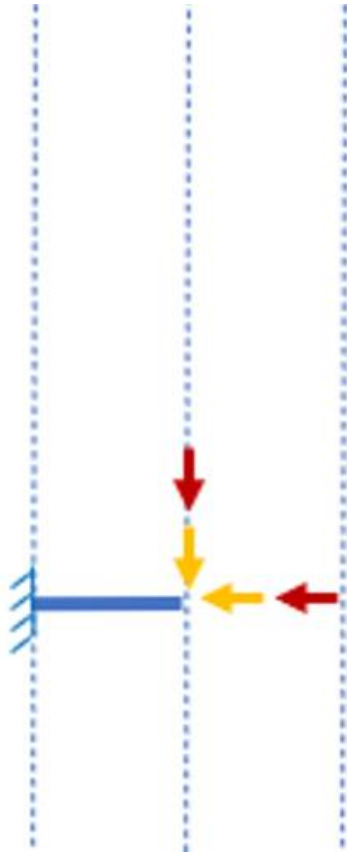
$$\sum M_{(E)} = 0 = -Y_F * 2 - Y''_F * 2 + M'''_E \Rightarrow M'''_E = -50 \text{ kNm}$$

$$\sum Y = 0 = -Y_F - Y''_F + Y'''_E \Rightarrow Y'''_E = -25 \text{ kN}$$

$$\sum X = 0 = -X''_F - X_F + X'''_E \Rightarrow X'''_E = 50 \text{ kN}$$





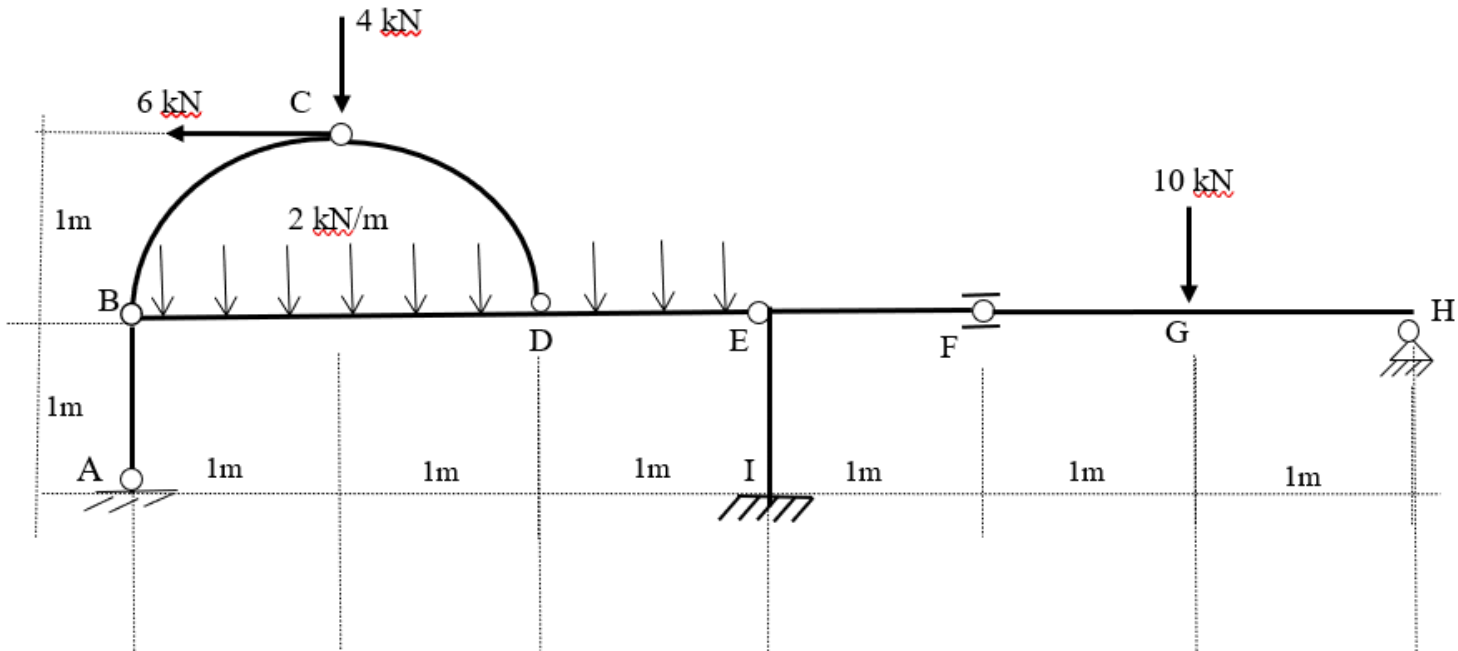


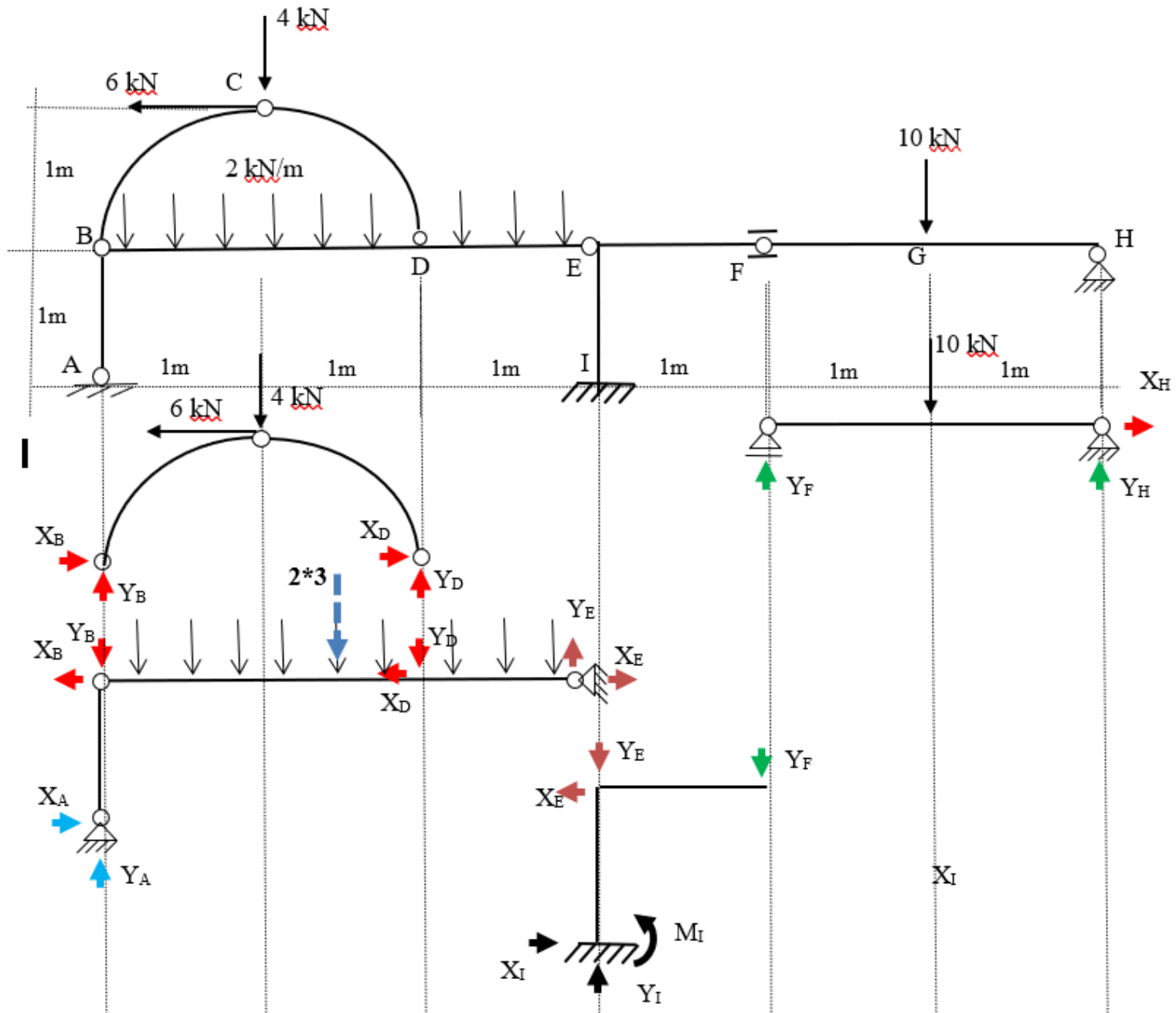
# EXERCÍCIO 15.

REC 2022

3ª Questão (pontos) Na estrutura associada ABCDEFGHI representada na figura:

- Identifique as subestruturas que a compõem (isolando e nomeando);
- Determine as reações nos apoios A, H e I;
- Esboce o diagrama dos momentos fletores no trecho reto BDEFGH.





- Arco triarticulado, pórtico triarticulado, viga bi apoiada, viga em balanço
- No arco:  $\Sigma M(B) = 0 = 6*1 - 4*1 + Y_D*2 \Rightarrow Y_D = -1 \text{ kN}$ ;  $\Sigma Y = 0 = Y_B + Y_D - 4 \Rightarrow Y_B = 5 \text{ kN}$ ;  
 $\Sigma M(C \text{ direita}) = 0 = X_D*1 + Y_D*1$  e  $Y_D = -1 \Rightarrow X_D = 1$ ;  $\Sigma X = 0 = X_B + X_D - 4 \Rightarrow X_B = 5 \text{ kN}$ .
- No pórtico:  $\Sigma M(B \text{ direita}) = 0 = -2*3*1,5 - Y_D*2 + Y_E*3$  e  $Y_D = -1 \Rightarrow Y_E = 7/3$ ;  
 $\Sigma Y = 0 = Y_A - Y_B - 6 - Y_D + Y_E$  e  $Y_B = 5, Y_D = -1, Y_E = 7/3 \Rightarrow Y_A = 23/3$ ;  $\Sigma M(B \text{ esquerda}) = 0 = X_A*1 \Rightarrow X_A = 0$ ;  
 $\Sigma X = 0 = X_A - X_B - X_D + X_E$  e  $X_B = 5, X_D = 1, X_A = 0, \Rightarrow X_E = 6 \text{ kN}$ .
- Na viga bi apoiada:  $\Sigma X = 0 = X_H \Rightarrow X_H = 0 \text{ kN}$ ;  $\Sigma M(F) = 0 = -10*1 + Y_H*2 \Rightarrow Y_H = 5 \text{ kN}$ ;  
 $\Sigma Y = 0 = Y_F - 10 + Y_H \Rightarrow Y_F = 5 \text{ kN}$
- Na viga em balanço:  $\Sigma X = 0 = X_I - X_E$  e  $X_E = 6 \Rightarrow X_I = 6 \text{ kN}$ ;  $\Sigma M(I) = 0 = M_I + X_E*1 - Y_F*1$  e  $X_E = 6, Y_F = 5 \Rightarrow M_I = -1 \text{ kN}$ ;  
 $\Sigma Y = 0 = Y_I - Y_E - Y_F$  e  $Y_E = 7/3, Y_F = 5 \Rightarrow Y_I = 22/3 \text{ kN}$
- Diagrama

