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# Efficient transit network design and frequencies setting multi-objective optimization by alternating objective genetic algorithm

Renato Oliveira Arbex *, Claudio Barbieri da Cunha

*Av. Prof. Almeida Prado, trav.2 n°. 83, Edifício Paula Souza (Prédio da Engenharia Civil), Department of Transportation Engineering, University of São Paulo, Cidade Universitária, São Paulo, Brazil*

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The multi-objective transit network design and frequency setting problem (TNDFSP) involves finding a set of routes and their associated frequencies to operate in an urban area public transport system. The TNDFSP is a difficult combinatorial optimization problem, with a large search space and multiple constraints, leading to numerous infeasible solutions. We propose an Alternating Objective Genetic Algorithm (AOGA) to efficiently solve it, in which the objective to be searched is cyclically alternated along the generations. The two objectives are to minimize both passengers' and operators' costs. Transit users' costs are related to the total number of transfers, waiting and in-vehicle travel times, while operator's costs are related to the total required fleet to operate the set of routes. Our proposed GA also employs local search procedures to properly deal with infeasibility of newly generated individuals, as well as of those mutated. Extensive computational experiments results are reported using both Mandl's original benchmark set and instances with different demands and travel times as well, in order to determine Pareto Frontiers of optimal solutions, given that users' and operators' costs are conflicting objectives. The results evidence that the AOGA is very efficient, leading to improved solutions when compared to previously published results.



## 1. Introduction

The dramatic increasing use of private cars and motorcycles for daily commuting in urban and suburban areas, especially in developing and emerging countries, have led to several problems, including constant traffic jams, excessive and unreliable travel times, stress, GHG emissions and noise, more traffic accidents and energy consumption. Improving public transport systems and raising their attractiveness is a viable solution to mitigate these problems. The public transport barriers that need to be broken down in order to increase transit use are, among others: lack of direct transport, long travel times, need for multiple journeys, not frequent enough and poor information (Beirão and Cabral, 2007).

The transit network design and frequencies setting problem aims to find a set of routes and their associated frequencies, according to the users' travel demand in an urban area, to create an urban public transport system. The problem is applicable for public transportation networks of modes such as, for example, buses, bus rapid transit (BRT) systems and trams. The

* Corresponding author. Tel.: +55 (11) 3091 6092.
*E-mail addresses:* renatoarbex@gmail.com (R.O. Arbex), cbcunha@usp.br (C.B. da Cunha).

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transit planning process is a very complex one and consists of five main stages in the case of bus service planning (Ceder and Wilson, 1986): network design, frequencies setting, timetable development, bus scheduling and driver scheduling. The ideal scenario would be to optimize all five stages simultaneously; however, this is not viable due to the NP-hard nature of each stage on its own (Magnanti and Wong, 1984). Therefore, most of the previous works found in the literature have individually optimized one of the five stages, and none has optimized all stages simultaneously.

When designing a transit network, different and conflicting interests have to be addressed, thus, making the problem multi-objective. First, the total generalized cost for each user has to be as close as possible to a minimum; these costs comprise the sum of waiting times, in-vehicle travel time and transfer penalties due to the extra time and other inconveniences such as walking. In other words, passengers want to travel as fast as possible, waiting as little as necessary and making as few transfers as possible. Second, the operator costs, that is, the total required fleet, which is a function of the routes frequencies and roundtrip times, must be minimized; in other words, operators seek to minimize costs in order to achieve higher profits. Reducing users' costs usually leads to increase in operator costs, and vice versa. However, determining the most efficient operation for both users and operators, for given levels of service and cost requirements, is essential for the success of such systems.

In order to consider the multi-objective nature of the Transit Network Design and Frequencies Setting Problem (TNDFSP), this paper focus on solving the first two stages, given that frequencies setting stage is required to calculate fleet size. In our proposed approach, the optimal solution for this multi-objective problem is not a single solution, but a set of solutions defined as Pareto optimal solutions. A solution is Pareto optimal if it is not possible to improve a given objective without deteriorating at least another objective. This set of solutions represents the compromise solutions between different conflicting objectives (Talbi, 2009).

The TNDFSP is a NP-hard, multi-constrained combinatorial optimization problem with a vast search space for which the evaluation of candidate solutions route sets can be both challenging and time consuming, with many potential solutions rejected due to infeasibility (Fan et al., 2009). In this paper, the TNDFSP is addressed using a Genetic Algorithm (GA) approach, a search and optimization method based on natural selection and natural genetics. In the proposed algorithm, the GA is used for the optimal selection of routes from a larger candidate Route Database, which is created using Yen's *k*th shortest path algorithm (Yen, 1971). For a solution to be feasible, it must go through four feasibility criteria. The two genetic operators, namely crossover and mutation, are proposed for exchanging solutions' routes, with multiple feasibility local search procedures. The bi-objective nature of the problem is solved using an alternating objective function, which alternates from minimizing users' cost to minimizing fleet at each generation of our GA. A Pareto frontier of compromising solutions is generated by a sensitivity analysis approach.

We performed extensive computational experiments, involving different transfer penalties, as well as diverse input parameters, such as number of bus routes of the best solutions, in order to determine Pareto frontier sets of optimal solutions that are compared to published results, in a more far-reaching manner that takes into consideration both conflicting objectives and different demands. These results show that our GA approach is effective in generating better compromising solutions than previous results. Additionally, we propose a comprehensive visual dashboard to help better visualize and understand, in an overall and more intuitive manner, the solutions we have obtained, as well as the influence of the several input parameters, given the multi-objective nature of the TNDFSP.

This paper is organized as follows: the literature review is presented in Section 2, while the TNDFSP, with its objective functions and constraints, is detailed in Section 3. Section 4 describes our proposed GA heuristic, followed by computational experiments in Section 5. Lastly, concluding remarks are presented in Section 6.

## 2. Literature review

Traditionally, public transportation planners have developed bus route networks and schedules by leaning on past experience and knowledge, simple guidelines, ad hoc procedures and local community demands (Fan et al., 2009). Sometimes, route networks just evolve over time, with new routes being gradually added, as well as frequencies of existing routes increased, in order to cope with changing demand, without a major reviewing or restructuring to eliminate inefficiencies and inconsistencies. Recently, however, the computational power has risen greatly and planners are now able to use computer-based tools for designing public transport systems in order to optimize the use of available resources. Many public transit agencies are using route networks which have not been reviewed or revised for the last 20–50 years (Bagloee and Ceder, 2011).

An extensive review on the design, frequencies setting, timetabling of transit lines and their combinations can be found in the paper by Guihaire and Hao (2008), who classified 69 approaches dealing with concerning problem tackled and the solution method. We here adopt their terminology proposal to the sub-problems. Kepaptsoglou and Karlaftis (2009) present a comprehensive and systematic review of research on transit network design problems, based on three distinctive aspects: design objectives, operating environment parameters and solution approach. The authors also present flowcharts describing typical heuristic solutions and key characteristics of the models. More recently, Farahani et al. (2013) presented a comprehensive review of the definitions, classifications, objectives, constraints, network topology decision variables, and solution methods of the Urban Transportation Network Design Problem (UTNDP), which includes not only transit network design problem but also the road network design problem.

The first heuristic algorithm proposed to solve the transit network design problem was Lampkin and Saalmans (1967). In the first step, an initial skeleton route is produced; other nodes are then inserted, one by one, into the skeleton route in the following steps and frequencies are then assigned to the generated set of routes. Mandl (1980) developed a two-stage approach: first, a feasible set of routes was created; then, heuristics were applied to improve the quality of the initial set of route. Only in-vehicle travel costs were considered to evaluate route quality. In Mandl pioneering work a benchmark of 15 Swiss cities network was proposed, which has been extensively used for results comparisons.

Baaj and Mahmassani (1991) proposed an AI-based solution algorithm which consisted of three main components: a route generation algorithm (RGA), an analysis procedure named TRUST, a transit route analyst and a route improvement algorithm. The proposed solution combined designer's knowledge and experience to reduce search space and developed solutions search techniques using Artificial Intelligence tools. Ceder and Israeli (1998) proposed a nonlinear mixed integer programming that takes into account both passenger and operator costs. In the first step, a very large set of feasible routes connecting all nodes is generated; then a set covering problem is solved to find the minimal subsets of routes from which the most suitable subset is selected by applying a multi-objective analysis.

Pattnaik et al. (1998) proposed a two phase algorithm to solve the bus route network design problem; in the first phase a set of routes is generated; in the second phase a genetic algorithm is applied to determine the best set of routes that minimize the overall system cost. The algorithm was applied to a transportation network in part of Madras, India, with 25 nodes and 39 links.

Bielli et al. (2002) applied genetic algorithm for the bus network optimization problem. Their approach allowed computing the fitness function by means of a multicriteria analysis based on a series of the performance indicators and was applied to the case of city of Parma, Italy. Fusco et al. (2002) proposed a practical methodology to identify a transit network configuration consisting of a set of routes and associated frequencies that approaches the minimum overall system costs. Their approach, also based on genetic algorithm, combines transit network design methods proposed by Baaj and Mahmassani (1991) and Pattnaik et al. (1998). It considers a candidate route set selection and a network improvement, but also includes a different criteria for route generation. A hierarchical transit network is also proposed by suitably relaxing the set of constraints in order to take into account the town structure. This allows the designers' to include their current knowledge of the network.

Tom and Mohan (2003) followed the basic method structure by Pattnaik et al. (1998) and proposed a minimization of total system cost, including bus operating cost and passenger total travel time, using a two-phase solution process. First, a large set of candidate route is generated using a candidate route generation algorithm. A solution route set is then selected from the candidate route set using genetic algorithms, considering frequency of the route as a variable. The model is applied to the medium-sized network of Chennai, India, with 75 nodes and 125 links. Ngamchai and Lovell (2003) proposed seven genetic operators specifically for the route improvement part of their proposed algorithm to facilitate the search, including route-merge, route-break, route-sprout, add-link, remove-link, route-crossover and transfer-location genetic operators. The authors also applied headway coordination by ranking transfer demands at transfer terminals.

Chakroborty and Dwivedi (2002) described properties of an efficient route network and proposed a three step iterative process for the problem. First, various reasonable route sets for a given road network and demand matrix are determined. Second, the goodness of a route set as a whole is determined in the evaluation step. Third, the route sets are modified using proposed modification procedure. In a later work, Chakroborty (2003) considered both routing and scheduling, applying the method to Mandl's (1980) benchmark network.

Lee and Vuchic (2005) considered the transit network design problem in the case of transit variable demand (i.e., demand depends on the configuration of the transit network and frequencies of the routes) under a given fixed total demand. In their approach, a model split procedure is added to the basic model to generate the optimal transit network and to estimate transit demand simultaneously. Fan and Machemehl (2006) proposed a GA approach that also deals with variable transit demand, using a three main component solution framework: a candidate route set generation procedure, a network analysis procedure and a genetic algorithm procedure to combine the first two parts.

Zhao (2006) used simulated annealing global search scheme to minimize user cost and transfers; according to them, this approach is more suitable than GA approaches, commonly found in the literature, for large transit networks. Zhao and Zeng (2008) proposed a iteratively defined local solution search space combined with an integrated simulated annealing, tabu, and greedy search algorithm to optimize transit networks, including route network design, vehicle headway and timetable assignment. The methodology was applied to a large-scale realistic network optimization problem and showed to be capable of producing improved solutions to large-scale transit network design problems in reasonable amounts of time and computing resources.

More recently, Fan et al. (2009) proposed a simple evolutionary multi-objective optimization algorithm to solve the transit network design problem, aiming to minimize both passenger and operator costs. The algorithm relies on the make-small-change procedure, which modifies an existing route set to produce a new feasible route set, based on simple neighborhood moves: adding a node at the end of a route and removing a node at the start of a route. Fan and Mumford (2010) propose a basic metaheuristic framework for solving the urban transit routing problem, consisting of a representation scheme, an initialization procedure and a set of simple neighborhood moves. Their method improves upon published results for Mandl's benchmark problems and was tested against some larger instances, generated by them.

Bagloee and Ceder (2011) studied the design of a transit network of routes for handling actual road size networks. Their approach also considers important aspects including categorization of stops, multiclass of transit vehicles, hierarchy transportation mode planning and system capacity. First, the process starts with the construction of a set of potential stops using a clustering concept. Then a set of candidate routes is formed using Newton gravity theory and a special shortest-path procedure. Finally a metaheuristic search engine based on genetic algorithm and ant search is launched over the candidate routes until a good solution is found. The authors applied their method to Mandl's (1980) benchmark network and to the city of Winnipeg, Canada.

Szeto and Wu (2011) investigated a trunk bus network design problem for a suburban residential area in Hong Kong, aiming to reduce the number of transfers and the total travel time of the network. They integrated a specific genetic algorithm which aims to optimize the route design, and a neighborhood search heuristic which aims to optimize the frequency setting. To illustrate the robustness and quality of solutions obtained, computational experiments were performed based on 1000 perturbed demand matrices. The results show that the design obtained by the proposed solution method is robust under demand uncertainty, and the design is better than both the current design and the design obtained by solving the route design problem and the frequency-setting problem sequentially.

Cipriani et al. (2012) described a procedure and its application to TNDFSP in the city of Rome, considering a complex road network, multimodal public transport systems and many-to-many transit demand. The procedure includes a route set generation algorithm and a parallel genetic algorithm for finding the sub-optimal set of routes with the associated frequencies, reporting an improvement over existing network. Blum and Mathew (2012) applied an intelligent agent optimization system to optimize the transit route network redesign problem with the additional constraint that existing routes in the network remain, although perhaps serviced with lower frequency. Their approach yielded to significant improvement in the route network, when applied to the transit network in Mumbai, India.

Afandizadeh et al. (2013) applied genetic algorithms to solve the bus network design problem which also comprises frequency determination and assignment procedure and network evaluation procedure, using an objective function that also takes into consideration depot assignment, penalty for empty seats to optimize fleet capacity and penalty for unmet demand. The authors also uniquely considered a passenger assignment using a travel time utility when more than one transit option existed for the same origin and destination pair. The logit model was used to estimate a utility function instead of shortest path for transit demand allocation.

Nikolić and Teodorović (2013) developed a Swarm Intelligence model for the transit network design problem, based on the Bee Colony Optimization metaheuristics aiming to maximize the number of served passengers, to minimize the total in-vehicle time of all served passengers, and to minimize the total number of transfers in the network. Chew et al. (2013) proposed an approach based on genetic algorithm to minimize the passengers' and operators' costs. The authors conducted computational experiments on Mandl's (1980) benchmark network, reporting results that outperformed previous best published results in most cases. Kechagiopoulos and Beligiannis (2014) presented a competitive method compared to existing methods based on particle swarm optimization metaheuristic.

Szeto and Jiang (2014) dealt with a different version of the TNDFSP that takes into consideration in-vehicle congestion that leads to increased waiting and travel times, along with the comfort problem prompted by a lack of seats for passengers. The upper-level problem is formulated as a mixed integer non-linear program with the objective of minimizing the number of passenger transfers, and the lower-level problem is the transit assignment problem with capacity constraints. They propose a bi-level model for designing transit routes and their frequencies that explicitly minimizes the total number of passenger transfers in the objective function of the upper-level problem and incorporates strict capacity constraints to address the in-vehicle congestion in the lower-level problem. No other passengers' and operators' costs are explicitly considered. They propose a hybrid artificial bee colony (ABC) algorithm to determine route structures and a descent direction search method to determine an optimal frequency setting for a given route structure. Various experiments, including the area of Tin Shui Wai in Hong Kong and Winnipeg demonstrated that the proposed algorithm could find optimal solutions and that the descent search method saved considerable computation time.

Other related problems and approaches are also found in the literature. For instance, Ibarra-Hojas (2014) proposed a multi-objective optimization approach to jointly solve the timetabling and vehicle scheduling problems in transit systems, which allows studying the trade-off between the level of service (in terms of well-timed passenger transfers) and the operating costs (in terms of vehicle use). They defined a mathematical formulation and implemented an $\epsilon$-constraint algorithm to obtain the Pareto optimal solutions for scenarios of up to 50 lines and five depots, which are real sizes for agencies in their case study, in Monterrey, Mexico.

Hassold and Ceder (2014) addressed the single depot vehicle scheduling problem with multiple vehicle types that have different capacities and different fixed and variable costs as well. They propose a methodology that is based on a minimum-cost network flow model that utilizes sets of Pareto-optimal timetables for individual bus lines. Given a fixed fleet size, the proposed approach also allows a selection of the optimal timetable. The method allows determining the use of a particular vehicle type for a trip or its substitution by either a larger vehicle or a combination of smaller vehicles with the same or higher total capacity. Their methods have been applied to a real-life situation from Auckland, New Zealand. The data used is based on a small existing bus network in one section of Auckland that is comprised of two bi-directional bus routes starting at the same location, and covering two suburbs. The results show that vehicle substitution can improve the results by reducing the total scheduling cost by more than 35%. In addition the proposed methodology with the input of sets of Pareto efficient timetables attains total vehicle scheduling cost reductions greater than 12% in comparison with a standard optimization of vehicle scheduling.

In summary, the majority of the related articles found in the literature propose heuristics and metaheuristics to solve the transit network design and frequency setting problem, given the its difficulty to be solved to optimality. The most commonly input data comprises the network travel time matrix and a fixed transit demand origin–destination (OD) matrix. The decision variables are the node sequence for each route and the corresponding bus frequencies. The objective functions focus on minimizing both user and operator costs, many of them using weights to determine a single objective function; user costs generally corresponds to total travel time, including waiting time, in-vehicle travel time and transfer penalties, while operator costs may correspond to total bus routes length (in terms of distance) or fleet size. The constraints frequently considered are frequency feasibility, load factor constraint, fleet size restrictions, route size and network feasibility.

Other articles consider that these two problems are addressed separately. For instance, Ibarra-Rojas et al. (2015) provide a comprehensive and updated literature review on Transit Network Planning (TNP) problems and real-time control strategies suitable to bus transport systems. TNP problems are related to the planning process that spans every decision that should be taken before the operation of the bus system. The types of problems studied are divided according to the planning horizon into strategic, tactical, and operational levels. They comprise transit network design, frequency setting, transit network timetabling, vehicle scheduling and driver scheduling and rostering problems. The aim of the article is to complement previous reviews and to incorporate most relevant approaches to address these problems. The authors also present decision support techniques which are playing an important role on improving the efficiency of transit systems. According to them, the transit network design problem is a strategic planning (long-term) decision, whose aim is to determine the lines, types of vehicles, and stop spacing to meet population's movement requirements. The frequency setting sub-problem, along with the transit network timetabling sub-problem are both tactical decisions related to determine the frequency and fleet assigned to different lines in each period, the timetables and the design of operational strategies to be considered during the execution in such a way that the level of service is improved and the operational costs are reduced.

The network infeasibility issue was not broadly addressed in most papers. Fan et al. (2009) and Chew et al. (2013) highlighted this problem and its effect in their proposed method. With respect to demand allocation methods, most of the papers made simplified assumptions and relied on shortest path methods. Ideally, a Logit discrete choice model, based on generalized costs as utility, may yield to a more realistic representation to infer the probability of a passenger choosing a specific route option among competitive alternatives. Another specific aspect that yields to differences in quality of the final solutions is the exact point of transfer between routes, that is, to the best of our knowledge, not addressed in the previous works found in the literature. In other words, let us consider, for example, two distinct routes that share some segments; if the transfer is made at the first or at last node of the common segment, it affects both bus route loads, ultimately changing fleet requirements for both routes.

Some of the authors do not deal with the frequency setting part of the problem, considering that there are sufficient vehicles in the network. The required number of vehicles to operate the set of routes is essential to represent properly the overall operator costs. This fleet is calculated from the peak load of each route. In recent papers, solution quality parameters did not include fleet size, which is a much relevant parameter to achieve resources and budget optimization.

## 3. Problem description

### 3.1. The Transit Design and Frequencies Setting Problem (TNDFSP)

The TNDFSP, which corresponds to the first two stages of bus service planning (Ceder and Wilson, 1986), is related to design of a bus network and frequencies setting. The network design part consists of finding a set of transit routes that together forms an efficient transit network, serving users' travel demand represented by an origin–destination matrix with minimum total travel time, including transfer penalties. Transit routes take into consideration existing transportation network available and predefined stop points. The second phase, assigning frequencies to transit routes establishes waiting times and total capacity levels for routes, and makes possible the calculation of the total necessary fleet for network operation. Higher frequencies directly affect operator costs, once they directly influence the total fleet required; on the other hand, it also contributes to a better quality public transportation system, as passengers will wait less for their transport. These conflicting objectives of user and operator costs make the TNDFSP problem multi-objective in nature.

Therefore, there is not a single optimum solution, but a set of solutions known as Pareto optimal solutions which represent the compromise solutions between different conflicting objectives (Talbi, 2009). The main objective of the TNDFSP is finding a set of solutions that provides: (i) for the users view, fast travel times for every origin destination pair with existing demand, the lowest possible average waiting time and as few transfers as necessary in order to complete trips, maximizing direct trips and (ii) for the operators, fewer buses to operate the network and minimum total bus-kilometers.

The first objective function represents the generalized cost for users. For the second objective function, representing operator costs, total fleet is used as a proxy. As described by Wardman (2001), the value of time in each of the transit trip stages (waiting, in-vehicle and transfers) varies according to multiple parameters. Thus, in order to better represent the real perception of waiting time and transfer penalties values, a coefficient is applied to total waiting time and required transfers. The two objective functions are as follows:

$$\min f_1 = C_{wt} * TWT + TIVTT + C_{1t} * T1T + C_{2t} * T2T + C_{ud} * UD \quad \text{(User's costs)} \tag{1}$$

$$\min f_2 = TRF \quad \text{(Operator cost)} \tag{2}$$

where

$TWT$ total waiting time for all passengers,
$C_{wt}$ waiting time weight,
$TIVTT$ total in-vehicle travel time for all passengers,
$T1T$ total number of first transfers required by all passengers,
$T2T$ total number of second transfers required by all passengers,
$C_{1t}$ first transfer penalty,
$C_{2t}$ second transfer penalty,
$UD$ unsatisfied demand,
$C_{ud}$ time penalty for each unsatisfied demand,
$TRF$ total required fleet for network operation.

We assume that passengers will only use the transit system when they are able to reach their destination with up to two transfers. Trips that may require three or more transfers are declared as unsatisfied demand. As the number of transfers should also be minimized, it is incorporated into the user's costs objective function. For operators we adopt a simpler proxy, using solely required fleet only, as it not only represents the key operator costs but also it is affected by the route lengths.

The transportation network is represented by an undirected graph $G = (N, A)$ where $N$ denotes the set of nodes, each of them representing a bus stop or a centroid of bus stops within a zone, and $A$ is the set of links, which corresponds to street and roads segments that connect these nodes. A transit route is a fixed path defined by a sequence of nodes and the corresponding links that connect them. The TNDFSP solution is a route set $R$ with its associated frequency. Passengers may use any transit route available that includes their origin node to reach their destination. If a direct connection is not possible within available routes in origin node, the user will drop off in another node that has a transit service that connects to the final destination of interest, and this represents a transfer.

Chakroborty (2003) points out the difficulty for modeling the TNDFSP with a mathematical programming formulation, as the problem is discrete and involves difficult to represent concepts like transfers and continuity, along with an objective function that is difficult to calculate. Newell (1979) emphasizes that this problem is generally a non-convex optimization problem which is very difficult to solve. In addition, a solution is a single entity on its own, one cannot assess a single route quality without considering the whole network connectivity. Therefore, no such formulation is presented in this section. Most of methods proposed in the recent literature to solve the TNDFSP are based on heuristics.

### 3.2. Input data

The input data used by the method in the current paper uses the same input data as previous methods, and is described and commented as follows:

- Data describing the transportation network structure available for the bus routes to use; including location of each node in the network and the available links, that is, its connectivity with other nodes.
- Data outlining the travel times of each link in the network, namely, the fixed time necessary for a vehicle to travel between two nodes in the transportation network.
- Data describing travel demand by transit between every origin destination pair, known as the origin destination matrix.

To simplify model complexity, the travel demand matrix is considered symmetrical, and travel times between two nodes are the same regardless of direction. Transit services operate from an origin node to a destination node and from the destination node to its origin, back and forth during all period of study.

It should be noted that the optimized transit network is strongly based on the input data provided. In recent literature the TNDFSP, input data quality has not been subject to extensive analysis. Authors use existing or already available input data, as in Mandl's benchmark network. To apply the method to a larger real city, the input data quality should be assessed and validated extensively. Each of the three input data affects the final transit network quality, as is detailed subsequently.

Concerning the network structure, using very simplified models for the real road and avenues network structure reduces possible paths for bus routes and concentrates the transit lines in the modelled paths, reducing the capillarity of the 'optimized' transit network, thus making the services unattractive for some users, as they have to walk longer distances to reach transit stops. On the other hand, if the modelled network represents every single street, this implies an unnecessary computational burden to cope with the higher disaggregation level of the network. Therefore, one should aim to model the network as to make it possible for the transit services to reach a reasonable service coverage area, considering each stop's area of attractiveness.

Ultimately, travel times in the transportation network are not fixed. The variability in travel times in public transport services is a relevant topic and ought to be included in future models, as this implies a considerable effect that using free flow travel times the optimized network will not necessarily favor the faster paths during rush hour periods. Indeed, Mazloumi et al. (2009) describes high travel time variability in AM peak for public transportation services. The origin destination matrix also varies from period to period. Urban travel demands are so dynamic that AM peak transit demands differ greatly from mid-day demands. Late night and weekends travel needs are also considerably different from peak hour demands. Thereafter, it should be noted that creating an 'optimized' route network from one-hour transit demand using fixed travel time is a simplification to reduce model complexity.

### 3.3. Assumptions and constraints

The TNDRTP problem is multi-constrained and solutions' infeasibility is an important issue to be properly tackled. Inspired on Mumford (2013) and Chew et al. (2013), our problem is subject to the following constraints:

(1) The route network (i.e., a subgraph of the transportation network) is connected, that is, from each node it is possible to reach every other node using a set of routes of the network. This does not imply that all demand is necessarily satisfied, because even if the route set is connected there may be some OD pairs that require three or more transfers.
(2) Each route is free from repeated nodes in order to avoid backtracks and cycles in a route.
(3) All transportation network nodes must be present in at least one route in order to be able to serve all transit demand. This restriction guarantees service coverage area. The route network may not contain, however, all the original transportation network links.
(4) A single route must be unique within the route network. Two routes with the same path in fact correspond to a single route with a higher frequency.
(5) The route network consists of a fixed number of routes predefined by the transit operator. We perform a broad sensitivity analysis to evaluate the influence of the number of routes in the quality of the solutions found.
(6) Each routes frequency must lie within a given predefined interval. In a sense, very high frequencies may lead to bus crowding and bunching because of short headways and the interaction between them (Fattouche, 2011). Low frequency services, on the other hand, make passengers dependent on the service schedule, reducing trip departure time decision freedom and raising waiting times, strongly increasing total travel time if a service or connection is missed (Walker, 2011).
(7) Vehicle load factor (coefficient that represents crowing, explained in the next section) must be less than a predefined maximum value to provide service quality for the passengers by limiting crowding inside vehicles.
(8) Route length must be within predefined minimum and a maximum number of nodes, as to consider factors such as driver fatigue and difficulty of maintaining the schedule (Zhao and Gan, 2003).
(9) Total fleet required to operate the designated routes is subject to a predefined upper bound, in order to cope with resources and budget constraints.

## 4. Genetic algorithm for the TNDFSP

This section describes the proposed heuristic based on Genetic Algorithm (GA) to solve the TNDFSP. GAs are among the popular heuristic algorithms that represent a powerful and robust approach for developing heuristic for complex and large-scale combinatorial optimization problems. The GA works with a set of solutions known as population. Each solution is named an individual and is initially generated in the proposed algorithm by a specific problem heuristic to minimize infeasibility. At each generation, some individuals are selected to be parents with a bias towards those with better fitness, which represents their solution quality. Those selected individuals are subject to evolutionary operators, namely, crossover and mutation, to generate the offspring. Finally, a replacement strategy is used to select individuals for next generation based on their fitness function. The process continues until a stopping criteria is satisfied (Talbi, 2009). Our proposed GA heuristic, called Alternating Objective Genetic Algorithm (AOGA) incorporates a mechanism that allows the objective that is searched to be cyclically alternated along the search. Fig. 1 presents the flowchart of our AOGA heuristic for the TNDFSP. Each step is detailed in the subsections that follow.

### 4.1. Route Database Creation

The idea behind the algorithm of Route Database Creation is to form a wide set of bus routes that are attractive to users, in the sense that they offer travel times as close as possible to the fastest travel time to their specific origin–destination pair. Based on this assumption, any route that is used in a solution will be an efficient travel option for all the origin–destination pairs that it connects. This Route Database is used as a route repository for population initialization and mutation procedures.

The following heuristic procedure is used to generate the Route Database: for each origin–destination pair with nonzero travel demand, the associated OD path that corresponds to the ideal shortest direct travel time is added to the database as a bus route, along with those routes whose travel time does not exceed the fastest time by a maximum route detour factor. The planner specifies the route detour factor in order to allow a variety of choices for the possible bus routes connecting origin–destination pairs. However, this factor, expressed as a percentage of the shortest direct travel time, must be as low as possible in order to avoid routes with considerable deviation from their ideal path. As Beirão and Cabral (2007) state in their qualitative study, long travel time is perceived as a barrier for public transport use.

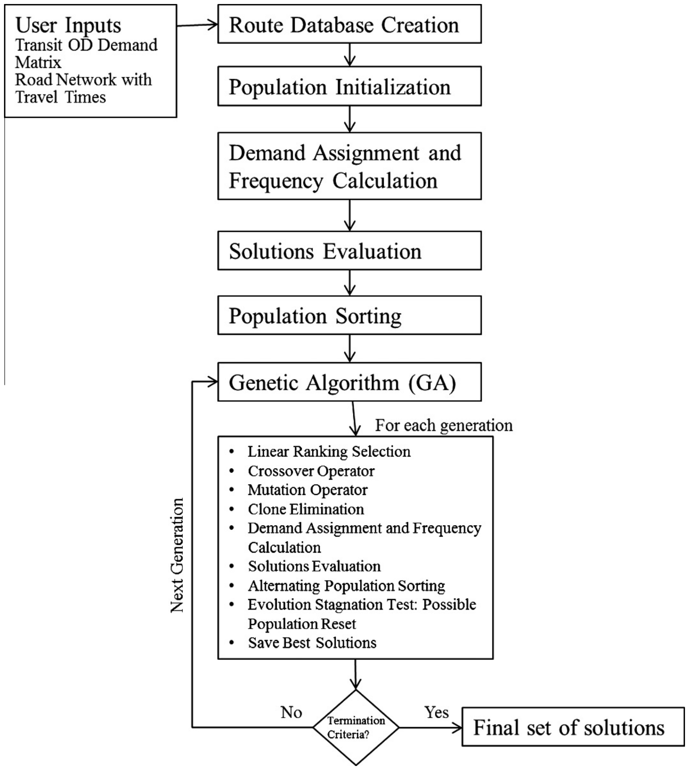


**Fig. 1.** Flowchart of the AOGA heuristic.

We employ Dijkstra's (1959) shortest path algorithm to determine the shortest travel time between each OD pair and the algorithm developed by Yen (1971) to find the other $k$-shortest paths. The value for $k$ cannot be established directly, as more paths are generated until the travel time corresponding to the limit of the route detour factor is reached. Two other Route Databases are also generated: one comprising all routes originating at a specific origin and one with all generated routes that directly link each OD pair.

### 4.2. Initial population

Our population initialization heuristic first randomly selects routes to serve each 'isolated node' (i.e. a node with only one incident arc) in the network; it should be noted that the selected route must necessarily start or end at this node. In the second step, we determine all nodes that are covered by the added routes. An OD pair is randomly selected among those nodes yet not serviced, and a route connecting them is randomly chosen from the Route Database among those that serve the OD pair. If a pair of unserved nodes does not have routes connecting them in the database (by being too close, for example), another pair of nodes is randomly selected until all pairs have been examined. Ultimately, if all remaining unserved pairs have been inspected and a route linking their OD pairs cannot be found, the individual is considered infeasible and thus discarded; the same applies if the number of generated routes is above pre-defined number of routes.

This procedures allows diverse solutions to be generated in order to prevent premature convergence of the GA (Talbi, 2009). The population initialization procedure runs until all initial population is formed only by feasible solutions.

### 4.3. Feasibility checks

Aside from the initialization aspects listed in the previous subsection, a complete network feasibility check is important in order to ensure that only valid solutions that are being generated are then submitted to the demand assignment and frequency calculation step. The proposed feasibility checks for network validation are:

(1) Check if route network is connected, that is, all nodes are reachable from all other nodes. This is accomplished using a breadth-first graph search approach. It acts on the sub graph formed by nodes and links of the solution's transit network. Firstly, the initial node from any route is chosen as the first node to be explored. Secondly, for each route that serves that node, every node is visited and it fills two lists: a found-nodes list and a nodes-yet-to-be-explored list. While there still exists nodes to be explored, that is, the number of found nodes is less than the ones to be explored,

then the algorithms goes back to searching for every node in each route that it serves and continues this process until that are none to be explored. If the number of found nodes is equal to the number of nodes in the network, then the solution's transit route network is considered connected. If not, the route set is disconnected and the individual is infeasible.

(2) Each node must be unique within its route, but may be repeated in the route network: this restriction is already met in Route Generation procedure, as only paths with unique nodes are added as possible routes in the Route Database.
(3) All transportation network nodes must be present in at least one route in order to be able to serve all transit demand: to check if this restriction is fulfilled, each node served by every route is added to a single data structure, and then are compared to the network nodes to check if every single node is served.
(4) A single route must be unique within the route network: the routes array that defines each solution is verified for equal or symmetrical routes and is considered infeasible if any coincident route is found.

Although the initial population procedure described above performs all these checks and guarantees only viable solutions, infeasible individuals might be generated as a result of the genetic operators. In this case, they are simply discarded and the same complete initialization procedure, including feasibility checks, is applied to generate a feasible solution to replace it.

### 4.4. Demand assignment and frequency calculation

Once an initial population with only valid solutions is formed, transit demand must be assigned to the route network. Demand assignment is a subproblem of the TNDFSP that requires some simplifying assumptions. Ideally, issues like risk-averse passenger behavior, demand and supply uncertainty, travel time variability and vehicle actual occupation should all be considered in transit assignment methods, as discussed by Szeto et al. (2011). However, in TNDFSP past literature, transit assignment issues may seem to have been overlooked, probably to reduce overall complexity. In this paper, we propose a further step in transit assignment method, in order to better represent passengers' transit option choice. It is based on the works of Baaj and Mahmassani (1991) and Afandizadeh et al. (2013).

For each OD pair with non-zero transit demand, we assume that the user initially searches for a direct route alternative, thus avoiding transfers. In the case that a direct route is not available, the user will then choose a path that requires only one transfer. If no single-transfer alternative is available, a two-transfer option is chosen. Ultimately, if there are none viable two-transfer options connecting the OD pair, it is then considered as unserved.

Whenever multiple direct options connecting an OD pair are available, each possible direct route has a chance to be selected that is directly proportional to its own frequency (frequency-based). In case no direct-route is available, but multiple one-transfer options do exist (or, similarly, only multiple two-transfer options exist), all alternatives are computed and the user chooses one of those with a probability that is calculated based on a multinomial logit model in which the utility function $U_i$ is given by the generalized cost of that transport alternative. Thus, for each non-direct transit option with the least number of transfers that serves an OD pair, the probability that a transit option will be chosen and the corresponding utility function are given by expressions (3) and (4), respectively.

$$P_{TransitOption} = \frac{e^{U_i}}{\sum_{i \in Options} e^{U_i}} \tag{3}$$

$$U_i = T_w \cdot C_{wt} + T_{iv} + x_{1t} \cdot C_{1t} + x_{2t} \cdot C_{2t} \tag{4}$$

where
$P_{TransitOption}$ probability to choose a specific transit option, consisting of a combination of multiple routes and the transfer point(s),
$U_i$ utility function,
$T_w$ waiting time,
$C_{wt}$ waiting time weight,
$T_{iv}$ in-vehicle travel time,
$x_{1t}$ 1st transfer,
$x_{2t}$ 2nd transfer,
$C_{1t}$ first transfer penalty,
$C_{2t}$ second transfer penalty.

The waiting time is assumed as half of the headway of a route. If there are multiple direct options, half of the total equivalent headway (that is, considering the sum of frequencies for all direct routes) is used. For the one-transfer and the two-transfer options, the waiting time is calculated as the sum of half the equivalent headway of the possible routes that connect the origin node to the transfer point; similarly, the waiting time at the transfer node is given by half the corresponding headway of all possible routes that link the transfer point to the final destination.

To calculate frequencies, we use the point-check period max load method from Ceder (1987). In this method, the frequency is calculated so that the desired occupancy for the buses is not exceeded even along the busiest segment of the route in terms of vehicle occupancy. The desired occupancy is modelled using a load factor that represents the proportion of standing passengers, calculated as total passengers divided by the number of seats. The expression that provides frequency calculation is given by expression (5).

$$f_r = \frac{Q_r^{max}}{LF_r \cdot CAP} \leqslant f_{r_{max}} \tag{5}$$

where
$f_r$ frequency of route $r$,
$f_{r_{\max}}$ maximum frequency of route $r$,
$Q_r^{max}$ peak load of route $r$,
$LF_r$ load factor of route $r$,
$CAP$ bus seating capacity.

We assume that the frequencies of all routes are initialized with the same value. After the first iteration, frequencies are updated to ensure that the maximum load factor is not exceeded along the busiest segments of all routes in terms of occupancy, thus affecting waiting times of all transit options and, as a result, changing overall generalized costs. Other iterations are performed until no considerable changes in frequencies exist. At the end of the assignment procedure, network parameters, user and operator costs are then calculated.

### 4.5. Solution evaluation

After the demand has been assigned and frequencies calculated, the next step is to evaluate each solution. As discussed before, we consider two distinct and conflicting objectives: total user generalized cost and fleet. The user cost is computed by summing the generalized costs for every OD pair. In order to determine the required fleet for each route, the roundtrip time for each bus route is divided by its respective headway and the value rounded up to the nearest integer. The total fleet is given by summing the number of buses of all routes.

### 4.6. Crossover operator

Individuals are selected for crossover based on linear ranking selection method, which associates to each individual a selection chance that is proportional to its position in the fitness ranking. A single point crossover is used as follows: a single point of crossover $k$ is chosen randomly, and two offspring are generated by interchanging routes at the $k$ position in the genes (routes vector) of the parents (that contains the id codes of the routes that form the Route Database).

Due to the high probability of generating infeasible individuals by using this operator, a feasibility search strategy is devised to increase the probability of obtaining a feasible offspring. It consists of the following steps: for each pair of parents mated using the Linear Ranking Method, two offspring are generated using the one-point crossover. In case one generated offspring is infeasible, every other potential point of crossover is randomly selected and a new crossover is performed. However, it is still possible that two valid offspring have not been generated. In this case, a new parent selection follows and, for each new pair of parents, all crossover points are tested until two valid individuals are generated. This search for new parents is performed until an upper bound limit that corresponds to 80% of all number of possible pairs of parents is reached. Although this strategy does not ensure viability by itself, it showed to increase considerably and consistently the proportion of generated feasible individuals. Also, since the number of routes in one solution is usually small, the time required to perform this feasibility search procedures does not influence the overall performance of our approach, as shown in the computational experiments.

### 4.7. Mutation operator

The mutation is a GA operator with the goal to prevent search stagnation in a local optimum by introducing new genes (bus lines) in the population. It affects a pre-defined, randomly selected small percentage of the population, switching a low number of routes with other routes from the Route Database. In order to preserve the original OD connectivity, the mutation operator swaps each of the selected routes for mutation with another in the Route Database that serves the same OD pair. In addition, a new route will only be chosen if there exist at least two routes in that OD Route Database.

Again, given the high likelihood of unfeasible mutated individuals, a viability search strategy is embedded into the mutation operator. It is a simple and straightforward procedure that works as follows: in case of infeasibility, another route from the same OD Route Database is randomly chosen and the network is tested for feasibility. In the worst-case scenario, every route in that specific OD Route Database is tested. If after that the mutated individual is not feasible, other routes to be mutated are chosen and the above steps are repeated. Other individual is then selected for mutation if no feasible mutation can be found. During computational experiments, we observed that we could always find mutated individuals that were viable. This feasibility is important for ensuring the continuity of the operators throughout the GA generations.

### 4.8. Alternating population sorting, replacement strategy, clones, stagnation test and stopping criteria

After all solutions have been evaluated, the parents and offspring are all ranked together based on their fitness functions. Since each individual has two conflicting fitness values (i.e. users and operator costs), we use each of two objectives in subsequent iterations; in other words, if user costs are used for ranking individuals after one generation, operator costs will be used in the subsequent iteration, thus interchanging repeatedly and regularly with one another. The initial population is ranked by user cost for the first crossover and mutation operators.

A clone verification procedure is applied to all individuals after ranking the current population and the generated offspring that is based on their fitness functions. If a clone is found, a new solution is generated using the initial population method in order to replace the clone. We adopt an elitist population replacement strategy, where the best solutions are transferred to the next generation, regardless of being a parent or offspring.

In order to avoid becoming prematurely trapped in a local optimum, an evolution stagnation test is applied after a given number of generations: whenever the lowest fleet and user cost is the same after more than 200 generations the entire population is renewed using the initial population procedure. This ensures that the search continues with an exploration concept until the maximum number of generations is reached.

## 5. Computational results and discussion

The AOGA heuristic that we propose was applied to Mandl's (1980) benchmark network, which has been used by several authors and is shown in Fig. 2. It contains 15 nodes and 21 bidirectional arcs, representing a group of cities in Switzerland. In Fig. 2, arc attributes denote travel times.

We assume the following parameters in our experiments:

- Number of seats on each bus: 40.
- Maximum load factor: 1.25.
- Waiting time penalty multiplier: 2.
- Minimum number of nodes per line: 3.
- Maximum number of generations: 4000.
- Maximum number of consecutive generations without improvement before restarting GA: 200.
- Crossover probability: 100%.
- Mutation probability: 10%.

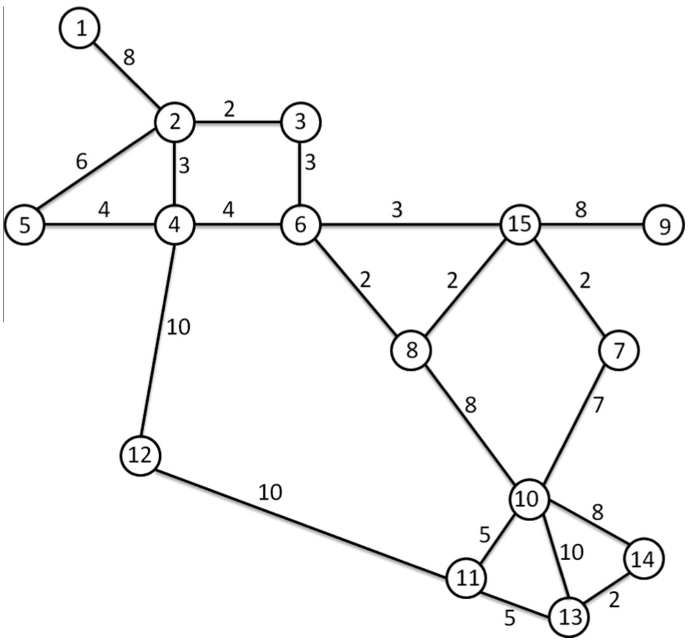


**Fig. 2.** Mandl's network.

In Section 5.1 we report the results considering Mandl's original demand matrix, while in Section 5.2 we evaluate the robustness of the best compromising solution we have found with respect to reductions in the OD matrix that reflect the case in which the symmetry of demand does not hold.

### 5.1. Results for Mandl's original symmetrical OD matrix

The transit demand (OD) matrix for transit services for the peak hour is given in Table 1, and the corresponding shortest paths flows intensity, (i.e., the number of passengers that would use each arc as part of its shortest path), also known as "desire lines", are represented in Fig. 3.

In order to represent the vast multitude of potential solutions alternatives, and also to better understand the effect of different parameters values in the results, we propose an extensive set of computational experiments considering each combination of following parameters, totaling 162 different possibilities:

- Population: 10, 14 and 18 individuals.
- Maximum route detour factor (with respect to minimum travel time for direct travel) in Route Database generation: 0%, 10%, 20%, 30%, 40% and 50%.
- Number of bus routes: 4–12.

Three different scenarios have been established regarding transfer penalties (Table 2): Scenario 1 and 2 seeks to better represent user behavior, as appears in Wardman (2001), who pointed to a value of 17 min with a 11 min standard deviation for perceived transfer penalty, while Scenario 3 aims to replicate transfer penalties commonly adopted in past literature. The aim is to investigate the influence of different transfer penalties in the quality of the solutions generated.

We performed 10 independent replications for each scenario and combination of input parameters, resulting in 4860 runs and running on multiple Intel Core i7 CPU 3770 computers, at an average execution processing time of 256 s per run. OpenMP library (Dagum and Menon, 1998) was used to leverage full processors performance. A total of 131,220 solutions were generated for each of the three above scenarios.

A visual dashboard depicted in Fig. 4 was developed using dc.js library (Zhu, 2013) to help better visualize and understand, in an overall and more intuitive manner, the solutions we have obtained, as well as the influence of the several input parameters, given the multi-objective nature of the TNDFSP. It allows filtering results based on different criteria; for instance, Fig. 4 shows solutions that satisfy the following criteria: 95% of demand serviced by direct routes and maximum headway of 20 min on all routes. The dashboard illustrates that results were generated from diverse input parameters. Our dashboard comprises (left-to-right, top-to-bottom):

- number of solutions found that satisfy the filtering criteria as a function of the number of iterations; it shows that the GA is converging, since the number of solutions that satisfy the filtering criteria increases as the generations progress; the oscillating behavior is due to the restart of the GA;

**Table 1**
Demand matrix.

| Origin/Destination | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | Sum by Destination |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 400 | 200 | 60 | 80 | 150 | 75 | 75 | 30 | 160 | 30 | 25 | 35 | 0 | 0 | 1320 |
| 2 | 400 | 0 | 50 | 120 | 20 | 180 | 90 | 90 | 15 | 130 | 20 | 10 | 10 | 5 | 0 | 1140 |
| 3 | 200 | 50 | 0 | 40 | 60 | 180 | 90 | 90 | 15 | 45 | 20 | 10 | 10 | 5 | 0 | 815 |
| 4 | 60 | 120 | 40 | 0 | 50 | 100 | 50 | 50 | 15 | 240 | 40 | 25 | 10 | 5 | 0 | 805 |
| 5 | 80 | 20 | 60 | 50 | 0 | 50 | 25 | 25 | 10 | 120 | 20 | 15 | 5 | 0 | 0 | 480 |
| 6 | 150 | 180 | 180 | 100 | 50 | 0 | 100 | 100 | 30 | 880 | 60 | 15 | 15 | 10 | 0 | 1870 |
| 7 | 75 | 90 | 90 | 50 | 25 | 100 | 0 | 50 | 15 | 440 | 35 | 10 | 10 | 5 | 0 | 995 |
| 8 | 75 | 90 | 90 | 50 | 25 | 100 | 50 | 0 | 15 | 440 | 35 | 10 | 10 | 5 | 0 | 995 |
| 9 | 30 | 15 | 15 | 15 | 10 | 30 | 15 | 15 | 0 | 140 | 20 | 5 | 0 | 0 | 0 | 310 |
| 10 | 160 | 130 | 45 | 240 | 120 | 880 | 440 | 440 | 140 | 0 | 600 | 250 | 500 | 200 | 0 | 4145 |
| 11 | 30 | 20 | 20 | 40 | 20 | 60 | 35 | 35 | 20 | 600 | 0 | 75 | 95 | 15 | 0 | 1065 |
| 12 | 25 | 10 | 10 | 25 | 15 | 15 | 10 | 10 | 5 | 250 | 75 | 0 | 70 | 0 | 0 | 520 |
| 13 | 35 | 10 | 10 | 10 | 5 | 15 | 10 | 10 | 0 | 500 | 95 | 70 | 0 | 45 | 0 | 815 |
| 14 | 0 | 5 | 5 | 5 | 0 | 10 | 5 | 5 | 0 | 200 | 15 | 0 | 45 | 0 | 0 | 295 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Sum by Origin | 1320 | 1140 | 815 | 805 | 480 | 1870 | 995 | 995 | 310 | 4145 | 1065 | 520 | 815 | 295 | 0 | **15570** |

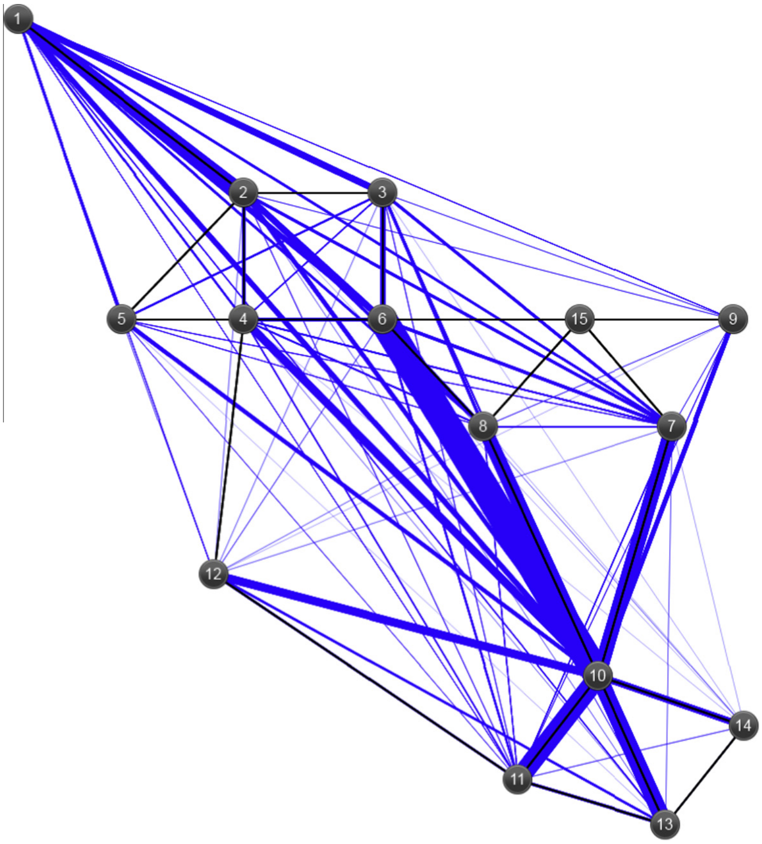


**Fig. 3.** Desire lines for the given OD matrix.

**Table 2**
Scenarios and transfer penalties.

| | Transfer penalty (min) | |
|---|---|---|
| Scenario | First | Second |
| 1 | 15 | 20 |
| 2 | 30 | 40 |
| 3 | 5 | 17 |

- number of solutions whose routes does not exceed route detour factors, evidencing the quality of found solutions in terms of direct trips; all except for 0% generated solutions that matched the specified criteria;
- histogram of number of generated solutions for different numbers of bus routes, evidencing the influence of the number of bus routes as an input in the number of solutions that satisfy the filtering criteria; solutions ranging from 4 to 12 routes were present in those that satisfied the filtering criteria;
- number of solutions found that satisfy each objective criteria;
- histogram of the number of solutions found as a function of the percentage of direct trips; this illustrates that the GA is generating high quality solutions from the user's perspective;
- histogram of the number of solutions found for different fleets (i.e., buses required);
- scatter plot of the solutions found for the two conflicting objectives;
- histogram of user's costs;
- histogram of the number of solutions found as a function of maximum and average route headways.

As mentioned before, the TNDFSP is a complex problem, given the two conflicting objectives that influence each other: higher frequencies improve user cost, since they wait less, but they also increase operator cost, once they directly impact the total number of buses required. We also observe that there is an opportunity to improve the way results have been presented in the literature. More specifically, it is not enough to report the best solution found in terms of percentage of direct trips and the one that minimizes the number of buses, as these two objectives are related. Since there is a set of solutions

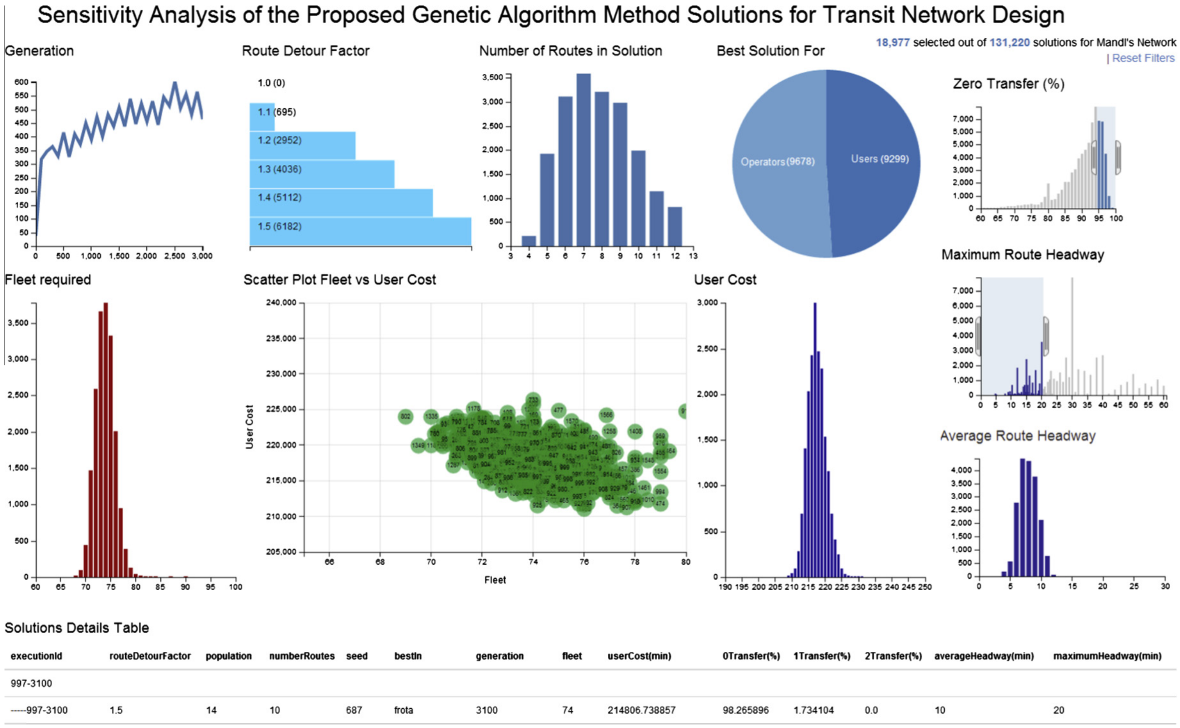


**Fig. 4.** Sensitivity analysis dashboard.

known as Pareto optimal solutions for the TNDFSP, which represent the compromise solutions between different conflicting objectives, we also determine the Pareto optimal subset of solution for each of the three different scenarios. A solution is Pareto optimal if it is not possible to improve a given objective without deteriorating at least another objective (Talbi, 2009). The set of Pareto Solutions is known as Pareto frontier. We initially compare the solutions we have obtained for the three proposed scenarios, whose results are presented in Tables 3–5.

As expected, total user costs increase as transfer penalty weights increase. The results also evidence that Scenario 2 yields to the overall best results considering both objectives. For instance, the direct trips percentage is 98.72% for 72 vehicles in the Scenario 2, followed by Scenarios 1 and 3 with 97.94% and 94.03%, respectively. The same applies for other values of required buses, which denote operators cost. This clearly demonstrates that higher transfer penalties than those that have been commonly used in the literature may yield to improved results with respect to both conflicting objectives.

With respect to other performance indicators, as average route headways, in-vehicle travel times and maximum route headways, they do not differ considerably among the three compared scenarios. Our solutions are robust and provide more

**Table 3**
Pareto frontier of Scenario 1.

| Pareto Frontier for Scenario 1 | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Solution | Route Detour Factor | Population | Routes | Seed | Generation Minimizing | Generation Obtained | Buses | User Cost (min) | Direct Trips (%) | 1 Transfer (%) | 2 Transfer (%) | Maximum Route Headway (min) | Average Route Headway (min) | Average In-vehicle Travel Time (min) | Average User Cost (min) | Average Waiting Time (min) |
| 1 | 1 | 18 | 8 | 700 | fleet | 2900 | **66** | **239130.49** | 88.05 | 11.88 | 0.06 | 10.00 | 5.55 | 11.13 | 15.36 | 1.31 |
| 2 | 1.2 | 10 | 8 | 956 | fleet | 3900 | **67** | **221631.50** | 95.18 | 4.82 | 0.00 | 19.33 | 7.50 | 11.21 | 14.23 | 1.31 |
| 3 | 1.2 | 18 | 9 | 811 | fleet | 700 | **68** | **220960.99** | 95.44 | 4.56 | 0.00 | 15.00 | 8.20 | 11.20 | 14.19 | 1.36 |
| 4 | 1.3 | 14 | 8 | 516 | user cost | 3800 | **69** | **217070.62** | 96.47 | 3.53 | 0.00 | 15.00 | 7.39 | 11.42 | 13.94 | 1.17 |
| 5 | 1.5 | 14 | 10 | 700 | fleet | 3900 | **70** | **214609.75** | 95.57 | 4.43 | 0.00 | 20.00 | 9.69 | 11.04 | 13.78 | 1.32 |
| 6 | 1.3 | 18 | 9 | 956 | fleet | 3300 | **71** | **214260.07** | 95.63 | 4.37 | 0.00 | 12.00 | 7.71 | 11.01 | 13.76 | 1.44 |
| 7 | 1.4 | 14 | 10 | 811 | fleet | 700 | **72** | **213308.76** | 97.94 | 2.06 | 0.00 | 17.00 | 9.90 | 10.86 | 13.70 | 1.43 |
| 8 | 1.5 | 14 | 8 | 516 | fleet | 2300 | **73** | **211822.93** | 97.50 | 2.50 | 0.00 | 18.00 | 8.43 | 11.29 | 13.60 | 1.42 |
| 9 | 1.5 | 14 | 8 | 516 | user cost | 2300 | **74** | **210487.89** | 97.56 | 2.44 | 0.00 | 18.00 | 8.39 | 11.29 | 13.52 | 1.42 |
| 10 | 1.4 | 14 | 10 | 811 | fleet | 2700 | **75** | **209060.47** | 97.94 | 2.06 | 0.00 | 20.00 | 10.53 | 10.86 | 13.43 | 1.43 |
| 11 | 1.4 | 14 | 10 | 811 | user cost | 3200 | **76** | **209004.11** | 97.94 | 2.06 | 0.00 | 20.00 | 9.93 | 10.86 | 13.42 | 1.43 |

**Table 4**
Pareto frontier for Scenario 2.

| Pareto Frontier for Scenario 2 | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Solution | Route Detour Factor | Population | Routes | Seed | Generation Minimizing | Generation Obtained | Buses | User Cost (min) | Direct Trips (%) | 1 Transfer (%) | 2 Transfer (%) | Maximum Route Headway (min) | Average Route Headway (min) | Average In-vehicle Travel Time (min) | Average User Cost (min) | Average Waiting Time (min) |
| 1 | 1.1 | 14 | 9 | 956 | fleet | 3700 | 66 | 244631.30 | 93.90 | 6.10 | 0.00 | 20.00 | 9.14 | 11.11 | 15.71 | 1.42 |
| 2 | 1.2 | 18 | 9 | 516 | fleet | 1500 | 67 | 233711.87 | 95.38 | 4.62 | 0.00 | 20.00 | 9.10 | 11.15 | 15.01 | 1.27 |
| 3 | 1.1 | 14 | 11 | 811 | user cost | 3300 | 68 | 231731.78 | 95.76 | 4.24 | 0.00 | 14.40 | 8.76 | 11.10 | 14.88 | 1.37 |
| 4 | 1.2 | 10 | 8 | 167 | user cost | 3000 | 69 | 227521.18 | 96.27 | 3.73 | 0.00 | 10.33 | 7.02 | 11.24 | 14.61 | 1.35 |
| 5 | 1.2 | 14 | 9 | 403 | fleet | 2400 | 70 | 221479.44 | 97.69 | 2.31 | 0.00 | 15.00 | 8.35 | 11.24 | 14.22 | 1.21 |
| 6 | 1.2 | 10 | 11 | 700 | fleet | 2400 | 71 | 221048.51 | 97.88 | 2.12 | 0.00 | 15.00 | 9.32 | 11.24 | 14.20 | 1.39 |
| 7 | 1.2 | 14 | 10 | 167 | user cost | 3800 | 72 | 217370.67 | 98.72 | 1.28 | 0.00 | 20.00 | 9.44 | 11.24 | 13.96 | 1.31 |
| 8 | 1.4 | 18 | 12 | 115 | fleet | 3600 | 73 | 214129.19 | 98.91 | 1.09 | 0.00 | 15.33 | 10.58 | 11.24 | 13.75 | 1.25 |
| 9 | 1.4 | 14 | 8 | 811 | fleet | 4000 | 74 | 213682.25 | 98.65 | 1.35 | 0.00 | 16.00 | 7.48 | 11.24 | 13.72 | 1.36 |
| 10 | 1.4 | 10 | 11 | 395 | fleet | 3200 | 75 | 213309.53 | 99.49 | 0.51 | 0.00 | 18.00 | 10.27 | 11.24 | 13.70 | 1.32 |
| 11 | 1.5 | 14 | 10 | 115 | fleet | 3400 | 76 | 211160.72 | 99.29 | 0.71 | 0.00 | 18.67 | 9.56 | 10.48 | 13.56 | 1.50 |
| 12 | 1.4 | 10 | 12 | 700 | fleet | 4000 | 77 | 210042.07 | 99.42 | 0.58 | 0.00 | 17.00 | 10.74 | 11.24 | 13.49 | 1.20 |

**Table 5**
Pareto front for Scenario 3.

| Pareto Frontier for Scenario 3 | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Solution | Route Detour Factor | Population | Routes | Seed | Generation Minimizing | Generation Obtained | Buses | User Cost (min) | Direct Trips (%) | 1 Transfer (%) | 2 Transfer (%) | Maximum Route Headway (min) | Average Route Headway (min) | Average In-vehicle Travel Time (min) | Average User Cost (min) | Average Waiting Time (min) |
| 1 | 1.1 | 18 | 6 | 687 | fleet | 3000 | 66 | 217582.67 | 86.64 | 13.36 | 0.00 | 8.33 | 5.13 | 10.65 | 13.97 | 1.57 |
| 2 | 1.2 | 14 | 8 | 167 | fleet | 3900 | 67 | 214327.90 | 89.72 | 10.28 | 0.00 | 19.00 | 8.72 | 11.05 | 13.77 | 1.36 |
| 3 | 1.2 | 14 | 8 | 395 | fleet | 4000 | 68 | 211647.05 | 93.13 | 6.87 | 0.00 | 19.33 | 8.91 | 10.75 | 13.59 | 1.32 |
| 4 | 1.5 | 10 | 9 | 956 | fleet | 4000 | 69 | 210204.12 | 93.64 | 6.36 | 0.00 | 15.33 | 8.78 | 11.05 | 13.50 | 1.27 |
| 5 | 1.4 | 14 | 10 | 403 | user cost | 2900 | 70 | 207275.47 | 93.96 | 6.04 | 0.00 | 17.00 | 9.31 | 11.06 | 13.31 | 1.27 |
| 6 | 1.5 | 14 | 10 | 278 | fleet | 3900 | 71 | 205962.61 | 95.12 | 4.88 | 0.00 | 16.50 | 10.01 | 11.62 | 13.23 | 1.16 |
| 7 | 1.4 | 14 | 8 | 115 | fleet | 2300 | 72 | 205863.08 | 94.03 | 5.97 | 0.00 | 17.00 | 8.81 | 11.65 | 13.22 | 1.16 |
| 8 | 1.4 | 10 | 8 | 687 | fleet | 4000 | 73 | 205340.69 | 93.00 | 7.00 | 0.00 | 20.00 | 8.77 | 11.24 | 13.19 | 1.36 |
| 9 | 1.4 | 14 | 8 | 115 | fleet | 3800 | 74 | 204457.02 | 94.80 | 5.20 | 0.00 | 20.00 | 9.82 | 11.65 | 13.13 | 1.16 |
| 10 | 1.5 | 14 | 11 | 956 | fleet | 3900 | 75 | 204497.27 | 94.61 | 5.39 | 0.00 | 20.00 | 10.19 | 11.38 | 13.13 | 1.30 |
| 11 | 1.4 | 10 | 9 | 403 | fleet | 3900 | 76 | 203859.55 | 93.96 | 6.04 | 0.00 | 16.67 | 9.28 | 11.92 | 13.09 | 1.22 |
| 12 | 1.4 | 14 | 11 | 516 | user cost | 3100 | 77 | 204232.58 | 93.83 | 6.17 | 0.00 | 16.00 | 10.64 | 11.11 | 13.12 | 1.45 |
| 13 | 1.5 | 18 | 7 | 700 | user cost | 1200 | 78 | 203956.02 | 93.58 | 6.42 | 0.00 | 16.00 | 7.51 | 11.18 | 13.10 | 1.12 |

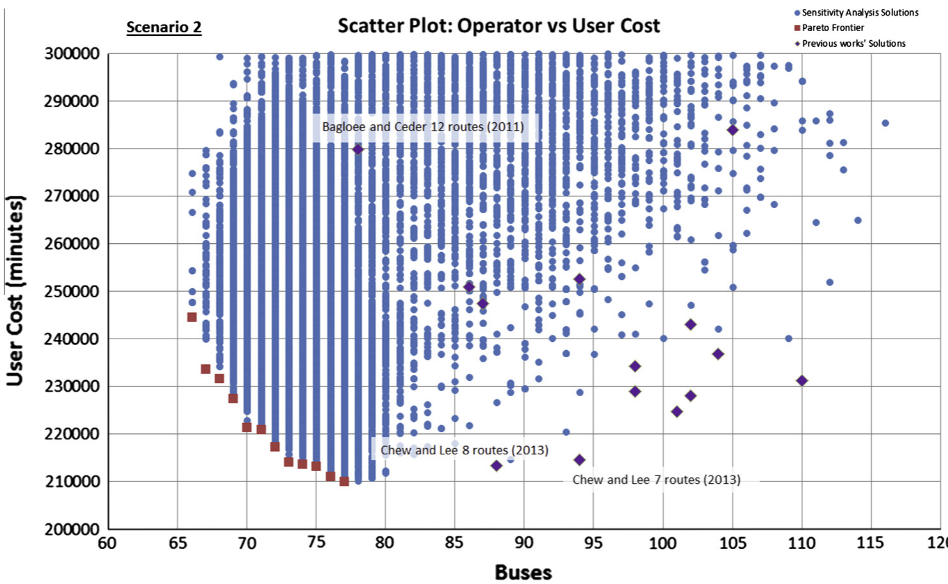


**Fig. 5.** Pareto frontier for Scenario 2.

**Table 6**
Pareto frontier solutions and previously published results.

**Previous Work and Scenario 2 Best Solutions**

| Routes | Source | Routes | Buses | User Cost (min) | Direct Trips (%) | 1-transfer (%) | 2-transfer (%) | Maximum Route Headway (min) | Average Route Headway (min) | Average in-vehicle travel time (AIVTT) (min) | Average User Cost (min) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | Mandl (1979) | 4 | **103** | **349230.26** | 69.94 | 29.93 | 0.13 | 6.67 | 3.48 | 11.40 | 22.43 |
| | Chakroborty (2003) | 4 | **105** | **284009.56** | 89.98 | 10.02 | 0.00 | 9.00 | 4.06 | 13.10 | 18.24 |
| | Mumford (2013) | 4 | **86** | **251015.35** | 91.14 | 8.86 | 0.00 | 9.33 | 4.43 | 10.79 | 16.12 |
| | Chew e Lee (2013) | 4 | **87** | **247498.31** | 92.74 | 7.26 | 0.00 | 5.11 | 3.64 | 11.47 | 15.90 |
| | Nikolic e Teodorovic (2013) | 4 | **94** | **252596.87** | 91.91 | 8.09 | 0.00 | 4.32 | 3.41 | 11.71 | 16.22 |
| | **AOGA (best 4 routes in Scenario 2)** | 4 | **79** | **223506.59** | 98.27 | 1.73 | 0.00 | 8.60 | 4.60 | 11.13 | 14.35 |
| 5 | **AOGA (best 5 routes in Scenario 2)** | 5 | **75** | **219257.75** | 98.20 | 1.80 | 0.00 | 9.56 | 5.39 | 11.08 | 14.08 |
| 6 | Baaj e Mahmassani (1991) | 6 | **87** | **311983.89** | 78.61 | 21.39 | 0.00 | 11.33 | 4.11 | 11.14 | 20.04 |
| | Mumford (2013) | 6 | **98** | **234358.26** | 96.08 | 3.92 | 0.00 | 8.00 | 5.06 | 11.77 | 15.05 |
| | Chew e Lee (2013) | 6 | **110** | **231258.66** | 98.14 | 1.86 | 0.00 | 8.80 | 4.86 | 12.18 | 14.85 |
| | Nikolic e Teodorovic (2013) | 6 | **102** | **228122.02** | 97.24 | 2.76 | 0.00 | 10.29 | 5.25 | 11.64 | 14.65 |
| | **AOGA (best 6 routes in Scenario 2)** | 6 | **77** | **215781.79** | 98.20 | 1.80 | 0.00 | 9.56 | 6.42 | 11.55 | 13.86 |
| 7 | Mumford (2013) | 7 | **102** | **243087.48** | 98.01 | 1.99 | 0.00 | 6.80 | 5.32 | 12.91 | 15.61 |
| | Chew e Lee (2013) | 7 | **94** | **214543.20** | 99.10 | 0.90 | 0.00 | 10.00 | 5.89 | 11.04 | 13.78 |
| | Nikolic e Teodorovic (2013) | 7 | **98** | **229039.11** | 98.84 | 1.16 | 0.00 | 17.50 | 7.00 | 11.99 | 14.71 |
| | **AOGA (best 7 routes in Scenario 2)** | 7 | **77** | **214989.45** | 98.52 | 1.48 | 0.00 | 12.80 | 7.58 | 11.91 | 13.81 |
| 8 | Baaj e Mahmassani (1991) | 8 | **78** | **311059.22** | 79.96 | 20.04 | 0.00 | 10.67 | 4.65 | 11.00 | 19.98 |
| | Mumford (2013) | 8 | **101** | **224805.54** | 99.10 | 0.90 | 0.00 | 15.00 | 6.91 | 11.95 | 14.44 |
| | Chew e Lee (2013) | 8 | **88** | **213344.37** | 99.04 | 0.96 | 0.00 | 31.00 | 9.67 | 10.72 | 13.70 |
| | Nikolic e Teodorovic (2013) | 8 | **104** | **236835.24** | 98.97 | 1.03 | 0.00 | 29.00 | 9.66 | 12.65 | 15.21 |
| | **AOGA (Scenario 2 Pareto Set)** | 8 | **69** | **227521.18** | 96.27 | 3.73 | 0.00 | 10.33 | 7.02 | 11.24 | 14.61 |
| | **AOGA (Scenario 2 Pareto Set)** | 8 | **74** | **213682.25** | 98.65 | 1.35 | 0.00 | 16.00 | 7.48 | 11.24 | 13.72 |
| 9 | **AOGA (Scenario 2 Pareto Set)** | 9 | **66** | **244631.30** | 93.90 | 6.10 | 0.00 | 20.00 | 9.14 | 11.11 | 15.71 |
| | **AOGA (Scenario 2 Pareto Set)** | 9 | **67** | **233711.87** | 95.38 | 4.62 | 0.00 | 20.00 | 9.10 | 11.15 | 15.01 |
| | **AOGA (Scenario 2 Pareto Set)** | 9 | **70** | **221479.44** | 97.69 | 2.31 | 0.00 | 15.00 | 8.35 | 11.24 | 14.22 |
| 10 | **AOGA (Scenario 2 Pareto Set)** | 10 | **72** | **217370.67** | 98.72 | 1.28 | 0.00 | 20.00 | 9.44 | 11.24 | 13.96 |
| | **AOGA (Scenario 2 Pareto Set)** | 10 | **76** | **211160.72** | 99.29 | 0.71 | 0.00 | 18.67 | 9.56 | 10.48 | 13.56 |
| 11 | **AOGA (Scenario 2 Pareto Set)** | 11 | **68** | **231731.78** | 95.76 | 4.24 | 0.00 | 14.40 | 8.76 | 11.10 | 14.88 |
| | **AOGA (Scenario 2 Pareto Set)** | 11 | **71** | **221048.51** | 97.88 | 2.12 | 0.00 | 15.00 | 9.32 | 11.24 | 14.20 |
| | **AOGA (Scenario 2 Pareto Set)** | 11 | **75** | **213309.53** | 99.49 | 0.51 | 0.00 | 18.00 | 10.27 | 11.24 | 13.70 |
| 12 | **AOGA (Scenario 2 Pareto Set)** | 12 | **73** | **214129.19** | 98.91 | 1.09 | 0.00 | 15.33 | 10.58 | 11.24 | 13.75 |
| | **AOGA (Scenario 2 Pareto Set)** | 12 | **77** | **210042.07** | 99.42 | 0.58 | 0.00 | 17.00 | 10.74 | 11.24 | 13.49 |
| | Bagloee e Ceder (2011) | 12 | **78** | **279862.79** | 86.90 | 13.10 | 0.00 | 11.33 | 7.23 | 11.00 | 17.97 |

than one route choice for most OD pairs, as can be observed by the low values for average waiting times, which are significantly less than average routes headways.

Fig. 5 maps the various solutions for Scenario 2 using a Pareto frontier with user cost on the vertical axis and number of required buses in the horizontal. Efficient frontier solutions are highlighted in red while past literature results are colored purple. One should note that the fleet requirements for the best compromising solutions are lower than all previously published results, evidencing the high quality of the solutions that we have obtained.

The corresponding details of Pareto frontier solutions and previous published results depicted in Fig. 5 are shown in Table 6. One should note that the solutions we have obtained clearly dominate other solutions in the Pareto sense, once fleet requirements are considerably lower and direct trips are higher than other results for all numbers of routes analyzed, which extends the usual range of number of routes proposed in the solutions for this benchmark. It is important to mention that, in order to compare results, the same assignment method and the same transfer penalties are used to estimate user costs taking into consideration the details of the solution routes found in each research paper. Therefore, only those papers that provided such details were considered for comparison.

For Scenario 1, where transfer penalties are 15 min and 20 min for the first and second required transfers, the best direct trips percentage (i.e. 0-transfer) obtained for all transit demand is 97.94% for the solution with 72 vehicles. The same value is only 95.12% using 71 vehicles in Scenario 3. When transfer penalties are raised to 30 and 40 min, as in Scenario 2, the best amount of direct trips raised to 99.49% with 75 vehicles (Table 4).

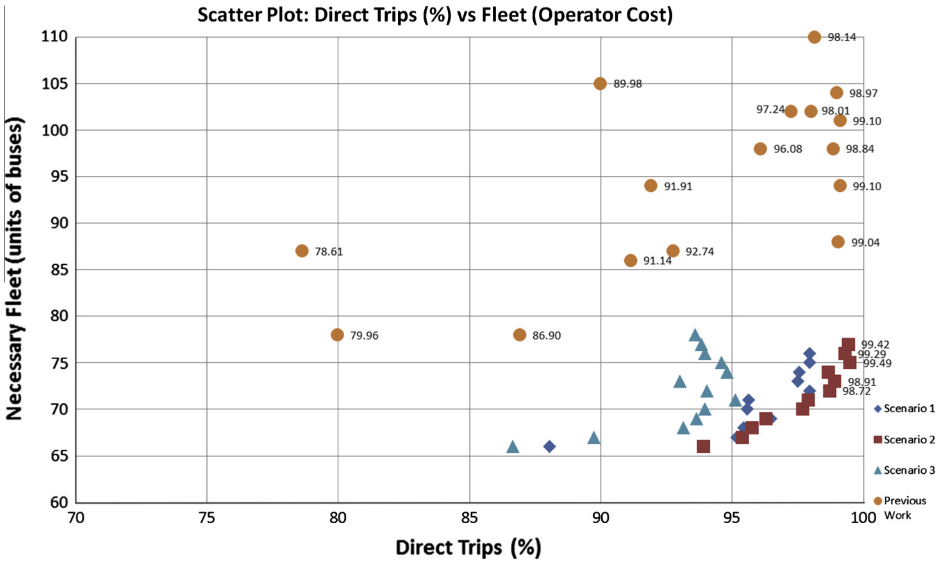


**Fig. 6.** Direct trips % vs fleet Pareto solutions scatter plot for every scenario.

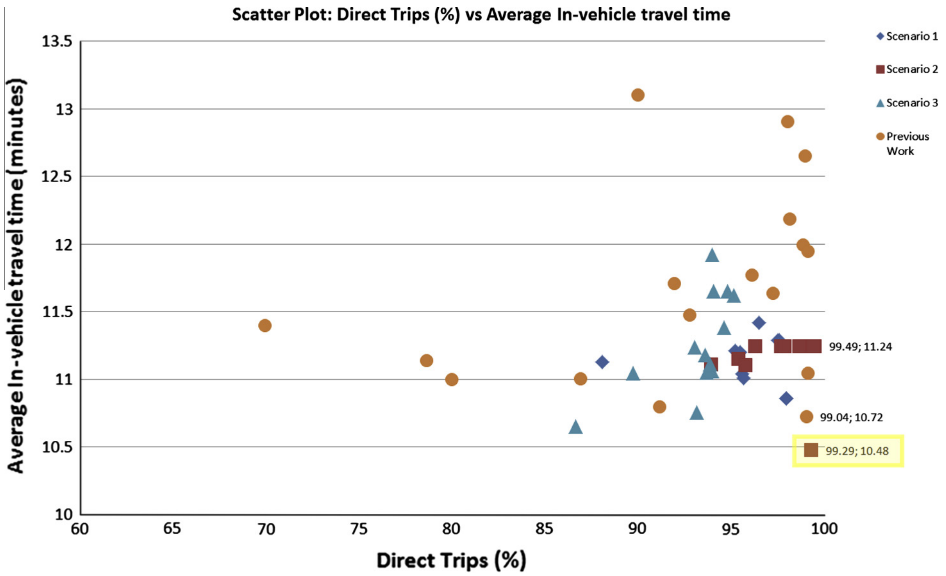


**Fig. 7.** Direct trips % vs average in-vehicle travel time for every scenario.

A Pareto set of optimal solutions in Scenario 2 generated 12 compromising solutions. In order to choose a single best solution, two other graphs are generated: the first one representing required buses and percentage of direct trips on vertical and horizontal axes, respectively (Fig. 6). The second one depicts average in-vehicle travel time and percentage of 0-transfer trips (Fig. 7).

As can be observed in Fig. 6, nearly all Pareto frontier solutions for the three scenarios clearly outperform previous solutions reported in the literature. In particular, Scenario 2 solutions (displayed in burgundy) yield to very high percentages of direct trips with much less buses required; the best previous result has a percentage of direct trips equal to 99.10% but requires 94 buses while our best solution yield to 99.49% of direct trips with a fleet of only 75 buses. Even if percentage of direct trips is solely analyzed, our proposed heuristic generates excellent solutions. The scatter plot graph displayed in Fig. 7 shows that our solutions are very competitive with published results in terms of both percentage of direct trips and average in-vehicle travel times. One of our solutions (lowest to the right and bottom, highlighted) results in 99.29% of direct trips and an average of 10.48 min of in-vehicle travel time, the lowest time among all those reported in the

**Table 7**
Route details for the best compromising solution selected.

| *Scenario 2 - Solution 11 - Best Compromising Solution Chosen* | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Route Number | Origin Node | Destination Node | Node Sequence | Number of Nodes | Travel Time (min) | Frequency (/h) | Fleet | Headway (min) | Peak Load | Peak Segment | Total Demand | Color in Map |
| 1 | 1 | 13 | 1-2-3-6-8-10-11-13 | 8 | 33 | 10.91 | 12 | 5.50 | 526 | 6-8 | 2784 | Red |
| 2 | 9 | 12 | 9-15-7-10-11-12 | 6 | 32 | 8.44 | 9 | 7.11 | 403 | 10-11 | 1646 | Light Green |
| 3 | 7 | 5 | 7-15-8-6-3-2-4-5 | 8 | 18 | 6.67 | 4 | 9.00 | 309 | 6-3 | 1096 | Dark Blue |
| 4 | 2 | 14 | 2-4-6-8-10-11-13-14 | 8 | 29 | 9.31 | 9 | 6.44 | 461 | 10-11 | 2114 | Brown |
| 5 | 13 | 4 | 13-14-10-8-6-3-2-4 | 8 | 28 | 8.57 | 8 | 7.00 | 406 | 10-8 | 1808 | Yellow |
| 6 | 1 | 12 | 1-2-5-4-12 | 5 | 28 | 3.21 | 3 | 18.67 | 131 | 1-2 | 346 | Pink |
| 7 | 11 | 1 | 11-10-7-15-6-3-2-1 | 8 | 30 | 13.00 | 13 | 4.62 | 649 | 10-7 | 2948 | Orange |
| 8 | 5 | 11 | 5-4-6-8-10-11 | 6 | 23 | 11.74 | 9 | 5.11 | 579 | 10-8 | 1674 | Light Blue |
| 9 | 13 | 1 | 13-11-12-4-5-2-1 | 7 | 43 | 3.49 | 5 | 17.20 | 167 | 1-2 | 708 | Purble |
| 10 | 9 | 12 | 9-15-8-6-3-2-4-12 | 8 | 30 | 4.00 | 4 | 15.00 | 158 | 6-3 | 546 | Dark Green |

literature (considering the demand allocation method used in this paper in order to equalize all methods' results). The details of the routes that form this solution are given in Table 7 and Fig. 8.

Finally, in Table 8 we summarize the comparison of our best solutions with previously published results for different number of routes in the route set. The authors' values are as obtained by applying the allocation method used in AOGA to the resulting routes sets reported by the authors. Transfer penalties correspond to Scenario 2. We have considered the following parameters to compare the resulting route sets:

- $d_0$ = the percentage of demand satisfied without any transfers;
- $d_1$ = the percentage of demand satisfied with one transfer;
- $d_2$ = the percentage of demand satisfied with two transfers;
- $d_{un}$ = the percentage of demand unsatisfied;
- AIVTT = average in-vehicle travel time in minutes per transit user (mpu);
- AUC = average user cost in minutes per transit user (mpu), comprising travel time and transfer penalties.

The results in Table 8 show that AOGA yield to improved solutions with lower travel times, lower user costs and higher demands satisfied without any transfer, and yet requiring significantly less vehicles.

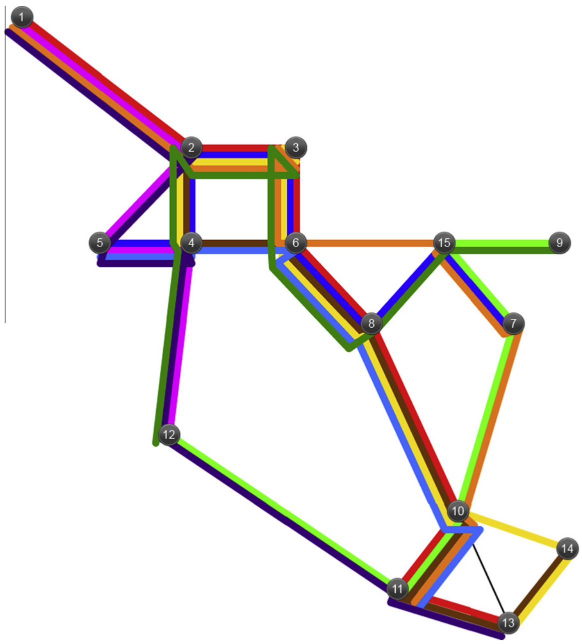


**Fig. 8.** Bus route network for the best compromising solution selected.

**Table 8**
Comparison among the final solutions generated by our AOGA heuristic and the previous approaches for Mandl's network and OD matrix.

| Number of routes | Parameters | Mandl (1980)* | Baaj and Mahmassani (1991) * | Chakroborty (2003)* | Bagloee and Ceder (2011)* | Mumford (2013)* | Chew et al. (2013)* | Nikolic and Teodorovic (2013)* | AOGA** |
|---|---|---|---|---|---|---|---|---|---|
| 4 | $d_0$ | 69.94 | – | 89.98 | – | 91.14 | 92.74 | – | 98.27 |
| | $d_1$ | 29.93 | – | 10.02 | – | 8.86 | 7.26 | – | 1.73 |
| | $d_2$ | 0.13 | – | 0.00 | – | 0.00 | 0.00 | – | 0.00 |
| | $d_{un}$ | 0 | – | 0 | – | 0 | 0 | – | 0 |
| | AIVTT | 11.40 | – | 13.10 | – | 10.79 | 11.47 | – | 11.13 |
| | AUC | 22.43 | – | 18.24 | – | 16.12 | 15.90 | – | 14.35 |
| | Fleet | 103 | – | 105 | – | 86 | 87 | – | 79 |
| 6 | $d_0$ | – | 78.61 | – | – | 96.08 | 98.14 | 97.24 | 98.20 |
| | $d_1$ | – | 21.39 | – | – | 3.92 | 1.86 | 2.76 | 1.80 |
| | $d_2$ | – | 0.00 | – | – | 0.00 | 0.00 | 0.00 | 0.00 |
| | $d_{un}$ | – | 0 | – | – | 0 | 0 | 0 | 0 |
| | AIVTT | – | 11.14 | – | – | 11.77 | 12.18 | 11.64 | 11.55 |
| | AUC | – | 20.04 | – | – | 15.05 | 14.85 | 14.65 | 13.86 |
| | Fleet | – | 87 | – | – | 98 | 110 | 102 | 77 |
| 7 | $d_0$ | – | – | – | –– | – | – | 98.84 | 98.52 |
| | $d_1$ | – | – | – | – | – | – | 1.16 | 1.48 |
| | $d_2$ | – | – | – | – | – | – | 0.00 | 0.00 |
| | $d_{un}$ | – | – | – | – | – | – | – | – |
| | AIVTT | – | – | – | – | – | – | 11.99 | 11.91 |
| | AUC | – | – | – | – | – | – | 14.71 | 13.81 |
| | Fleet | – | – | – | – | – | – | 98 | 77 |
| 8 | $d_0$ | – | 79.96 | – | – | 99.10 | – | – | 98.65 |
| | $d_1$ | – | 20.04 | – | – | 0.90 | – | – | 1.35 |
| | $d_2$ | – | 0.00 | – | – | 0.00 | – | – | 0.00 |
| | $d_{un}$ | – | 0 | – | – | 0 | – | – | – |
| | AIVTT | – | 11.00 | – | – | 11.95 | – | – | 11.24 |
| | AUC | – | 19.98 | – | – | 14.44 | – | – | 13.72 |
| | Fleet | – | 78 | – | – | 101 | – | – | 74 |
| 12 | $d_0$ | – | – | – | 86.90 | – | – | – | 99.42 |
| | $d_1$ | – | – | – | 13.10 | – | – | – | 0.58 |
| | $d_2$ | – | – | – | 0.00 | – | – | – | 0.00 |
| | $d_{un}$ | – | – | – | 0 | – | – | – | – |
| | AIVTT | – | – | – | 11.00 | – | – | – | 11.24 |
| | AUC | – | – | – | 17.97 | – | – | – | 13.49 |
| | Fleet | – | – | – | 78 | – | – | – | 77 |

* The author's values are as obtained by applying the allocation method used in AOGA to the resulting routes sets reported by the authors.
** When two solutions from the same Pareto set exist with the same number of lines, the one with a higher % of direct travel ($d_0$) is selected to be presented in this comparison table.

### 5.2. Non-symmetric and reduced demand

In order to evaluate the effectiveness of our proposed AOGA heuristic in determining high quality solutions in the cases in which the symmetry of demand does not hold and is reduced, as in off-peak periods, we have generated two new OD matrices for Mandl's network:

- Non-symmetrical demand: a disturbance factor $r$ is randomly (uniformly) generated in the interval [0–0.2] for each OD pair $(i, j)$ such that $i < j$ (i.e., those pairs located above the main diagonal of the demand matrix); demands are multiplied by $(1 + r)$ and $(1 - r)$ for each of the two symmetric OD pairs $(i, j)$ and $(j, i)$, $i < j$, respectively, in order to eliminate symmetry while keeping the total demand in the network unchanged.
- Reduced unsymmetrical demand: a reduction factor is randomly (uniformly) generated in the interval [0.3–0.5] and applied to each OD pair.

The aim is to evaluate the impact of these two different unsymmetrical demand matrices in the quality of the our best compromising solution, depicted in Table 7 and Fig. 8, which comprises 10 routes in the route set. In order to accomplish this, we have applied our demand allocation procedure to these two OD matrices for Mandl's network. Travel times are kept unchanged.

Similarly, in order to evaluate our best compromising solution with respect to non-symmetric travel times, we also generated a modified version of Mandl's network in which some travel times are unevenly increased by some minutes, in one or eventually both directions, mainly in those arcs towards and away from the central axis of demand defined by nodes 6, 8, 10 and 13. This modified network evidencing the new travel times is shown in Fig. 9.

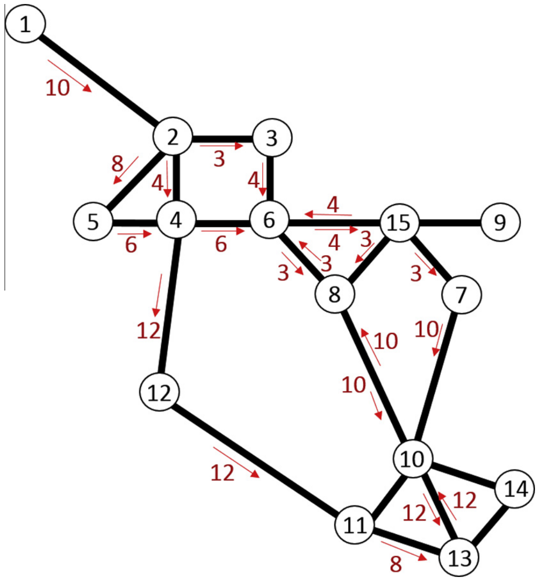


**Fig. 9.** Mandl's network with unevenly modified travel times.

**Table 9**
Results for non-symmetric and reduced demand matrices and modified travel times in Mandl's network.

| Parameters | Mandl's demand | | | |
|---|---|---|---|---|
| | Original | Original with modified travel times | Non-symmetric | Reduced |
| Total demand | 15,570 | 15,570 | 15,570 | 7283 |
| $d_0$ | 99.29 | 99.29 | 99.29 | 99.14 |
| $d_1$ | 0.71 | 0.71 | 0.71 | 0.85 |
| $d_2$ | 0.00 | 0.00 | 0.00 | 0.00 |
| $d_{un}$ | – | – | – | – |
| AIVTT | 10.48 | 12.49 | 10.54 | 10.71 |
| Average user cost (min) | 13.56 | 13.98 | 12.99 | 16.16 |
| Average waiting time (min) | 1.41 | 1.45 | 1.21 | 2.70 |
| Fleet | 76 | 91 | 92 | 44 |

The results for the modified demands and travel times are depicted in Table 9. They show that the set of ten routes of our best compromising solution still yield to high quality solutions in terms of travel times, user costs and demands satisfied without any transfer for the two distinct demand matrices. It should be noted that these results remain superior to previously published results depicted in Table 8 for both level of service indicators ($d_0$, AIVTT and AUC) and fleet.

## 6. Conclusions and future research

In this paper we propose a novel efficient GA heuristic in which the objective to be searched is cyclically alternated along the generations aiming to better tackle the multiobjective nature of the TNDFSP, a NP-hard, multiconstrained combinatorial optimization problem. Extensive computational results using Mandl's benchmark set show that our AOGA is very efficient, leading to improved solutions for both conflicting objectives when compared to previously reported results, regardless of the values adopted for transfer penalties, since solutions with very high percentage of direct trips, less buses and lower in-vehicle travel time could be found. Our best compromising solution was also evaluated considering two different non-symmetric demand matrices derived from Mandl's benchmark problem, as well as distinct travel times on selected arcs. The results show that the solution is insignificantly affected and remains very robust in terms of travel times, user costs and demands satisfied without any transfer.

For future research, we plan to apply our AOGA to determine an efficient late night bus service network for the city of São Paulo, Brazil. The results also evidence that our proposed heuristic can be helpful in establishing an intercity suburban bus network that connects the municipalities of metropolitan regions, as is the case of São Paulo.

On the other hand, designing efficient transit network for urban transport, especially in larger cities, requires considering many aspects that have not yet been fully addressed in the literature on the TNDFSP. Efficient bus networks for urban services in larger cities would require a different approach in terms of the network configuration. Instead of a direct-services configuration that allows reducing the number of transfers required, as is the case of Mandl's network, a trunk-feeder system is oftentimes deemed as the best alternative, as evidenced in several cities across the world such as Bogota (Colombia), Curitiba (Brazil), Guangzhou (China) and Jakarta (Indonesia). In a trunk-feeder system, smaller vehicles are utilized in lower-density areas and interconnect to the main corridors, operated with larger, trunk-line vehicles. When these larger vehicles operate mainly in dedicated central lanes (to avoid curb-side delays), bus stations are elevated and leveled with the bus floor and offer off-board fare collection (to reduce boarding delays), this system is also referred to as bus rapid transit or BRT.

Given that the TNDFSP is a difficult combinatorial optimization problem, a decomposition approach may be an alternative to be investigated in such circumstances that a hierarchy of services exists. In this case, our AOGA heuristic has shown to be an effective alternative to solve these sub-problems that may arise, for which the resulting number of nodes and links of the network can be reduced through some aggregation procedure. In addition, our Route Database Creation procedure can be adapted to generate routes that are adequate to each type of service being considered taking into other practical constraints that may arise such as maximum route length and some required topology of routes in terms of its desirable itinerary that would restrict the number of potential links.

In this sense, future work research direction may include considering existing metro lines, dedicated bus lanes and BRTs into the TNDFSP, which might also include aspects such specific bus services that would operate solely at peaks, such as express and limited-stop services, and also determining the most suitable vehicle size for each service. Expanding the methodology to include complete input data development process with smart card and bus GPS data analysis is also an interesting future research area for the TNDFSP. Exploring the concept of hyperpaths (Nguyen and Pallottino, 1986) in the process of improving the simulation of customer route choice behavior for transit network design is another potential research direction to be investigated.

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