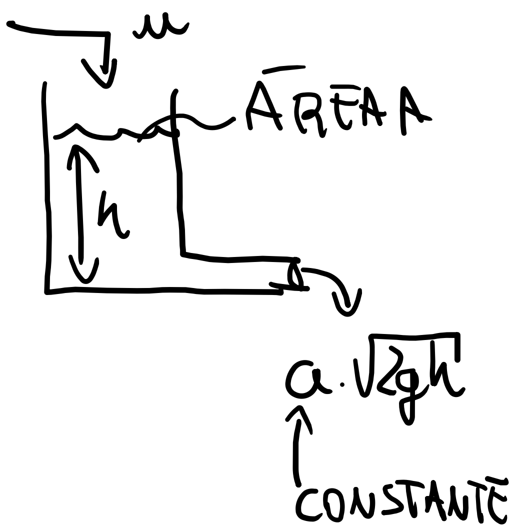
Aula 4 – Feedback Linearization

Prof. Eduardo A. Tannuri

PMR 5014

Controle Não Linear Aplicado a Sistemas Mecânicos e Mecatrônicos

CAP6 - FEEDBACK LINEARIZATION



$$\frac{dV}{dt} = u(t) - a\sqrt{2gh}$$

$$A\dot{h} = u(t) - a\sqrt{2gh}$$

SE $\underbrace{u(t)}_{\text{CONTROLE}} = \underbrace{a\sqrt{2gh}}_{\text{COMPENSAÇÃO NÃO LINEAR}} + \underbrace{A.v(t)}_{\text{CONTROLE LINEAR}}$

$\Rightarrow$ EM MALHA FECHADA

$$\cancel{A}.\dot{h} = \overbrace{\cancel{a\sqrt{2gh}} + \cancel{A}.v(t)}^{u(t)} - \cancel{a\sqrt{2gh}}$$

$\Rightarrow$ $\boxed{\dot{h} = v(t)}$ DINÂMICA EM MALHA FECHADA É LINEAR

↳ PODEMOS PROJETAR $v(t)$ POR TÉCNICAS DE CONTROLE LINEAR.

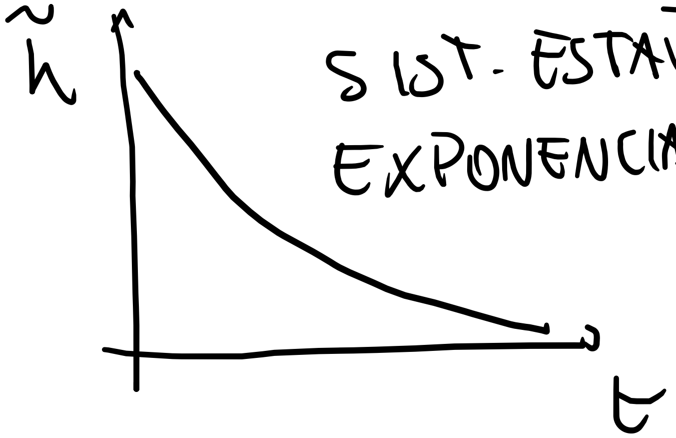
POR EXEMPLO

$v(t) = -\alpha \tilde{h}(t)$ COM $\tilde{h} = h(t) - h_d$

$\Rightarrow \boxed{\dot{h} = -\alpha . \tilde{h}(t)}$ $\rightarrow$ SE $h_d = cte \Rightarrow \dot{h}_d = 0 \Rightarrow$

$\dot{\tilde{h}} = \dot{h} - \cancel{\dot{h}_d} = \dot{h} \Rightarrow \boxed{\dot{\tilde{h}} = -\alpha \tilde{h}}$

$$\dot{\tilde{h}} = -\alpha \cdot \tilde{h}$$ → DINÂMICA DO ERRO



SIST. ESTÁVEL
EXPONENCIAL

$\dot{\tilde{h}} \to 0$ COM CONSTANTE DE TEMPO $1/\alpha$

$$\Rightarrow \text{logo} \quad \boxed{u(t) = \underbrace{a\sqrt{2gh}}_{\text{COMPENSAÇÃO DA NÃO LINEAR.}} - A \cdot \underbrace{\alpha \cdot \tilde{h}(t)}_{\text{CONTROLE LINEAR}}}$$

OBS) SE $h_d(t)$ NÃO FOR CONSTANTE

$$v(t) = \dot{h}_d - \alpha \cdot \tilde{h}(t)$$

POIS EM MF FICAMOS COM:

$$\dot{h} = v(t) \Rightarrow \dot{h} = \dot{h}_d - \alpha \cdot \tilde{h}(t)$$

$$\underbrace{\dot{h} - \dot{h}_d}_{\dot{\tilde{h}}} = -\alpha \cdot \tilde{h}(t)$$

$$\boxed{\dot{\tilde{h}} = -\alpha \cdot \tilde{h}} \rightarrow \text{DINÂMICA EM MF ESTÁVEL}$$

DE FORMA GERAL

SIST. FORMA CANÔNICA DE CONTROLABILIDADE $\Longrightarrow$

$$\underbrace{x^{(n)}}_{\text{SAÍDA } x} = f(\underset{\sim}{x}) + b(\underset{\sim}{x}) \cdot \overbrace{u}^{\text{ESCALAR}}$$

$$\underset{\sim}{x} = [x, \dot{x}, \ddot{x}, \dots x^{n-1}]$$

AÇÃO DE CONTROLE

$$\boxed{u = \frac{1}{b(\underset{\sim}{x})} \cdot \left[v(t) - f(\underset{\sim}{x})\right]}$$

$b(\underset{\sim}{x})$ E $f(\underset{\sim}{x})$ SÃO CONHECIDOS C/ EXATIDÃO

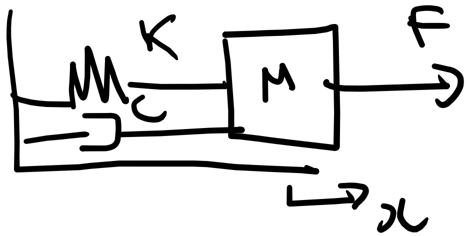
$\Downarrow$ EM MALHA FECHADA

$$x^{(n)} = \cancel{f(\underset{\sim}{x})} + \cancel{b(\underset{\sim}{x})} \cdot \frac{1}{\cancel{b(\underset{\sim}{x})}} \left[v(t) - \cancel{f(\underset{\sim}{x})}\right]$$

$$\boxed{x^{(n)} = v(t)} \rightarrow \text{SIST. LINEAR!}$$

EXEMPLO

$$\underbrace{\ddot{x}}_{x} = \overbrace{-\frac{K}{M} \cdot x - \frac{C}{M}\dot{x}}^{f(\underset{\sim}{x})} + \overbrace{\frac{1}{M}}^{b(\underset{\sim}{x})} \overbrace{F}^{u}$$



(1) $x^{(n)} = v(t)$ → DEVEMOS AGORA PROJETAR $v(t)$ POR TÉCNICAS DE CONTROLE LINEAR

se $v(t) = -K_0\tilde{x} - K_1\dot{\tilde{x}} - K_2\ddot{\tilde{x}} - \dots - K_{n-1}\tilde{x}^{(n-1)} + x_d^{(n)}$

COM $\tilde{x} = x - x_d$

$K_1, K_2, K_3 \dots K_{N-1}$ SÃO CONSTANTES OBTIDAS P/ QUE A MALHA FECHADA TENHA POLOS NAS POSIÇÕES DESEJADAS

⇒ VOLTANDO À (1) ⇒

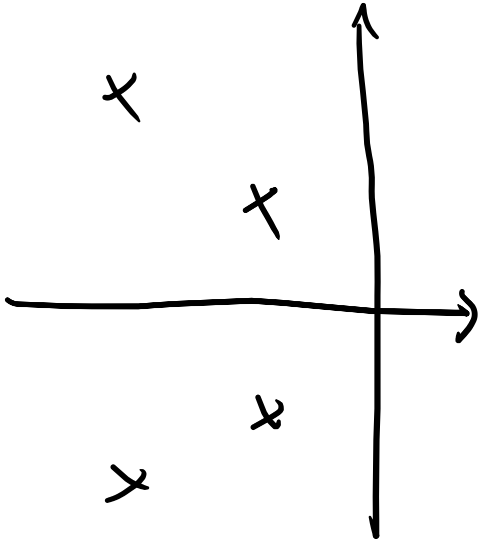
$x^{(n)} = -K_0\tilde{x} - K_1\dot{\tilde{x}} - \dots - K_{n-1}\tilde{x}^{(n-1)} + x_d^{(n)}$

$$\boxed{\tilde{x}^{(n)} + K_{n-1}\tilde{x}^{(n-1)} + \dots + K_1\dot{\tilde{x}} + K_0\tilde{x} = 0}$$

→ K_i POSICIONAM POLOS ONDE EU DESEJAR

$s^n + K_{n-1}s^{n-1} + \dots + K_1 s + K_0 = 0$

Ex)

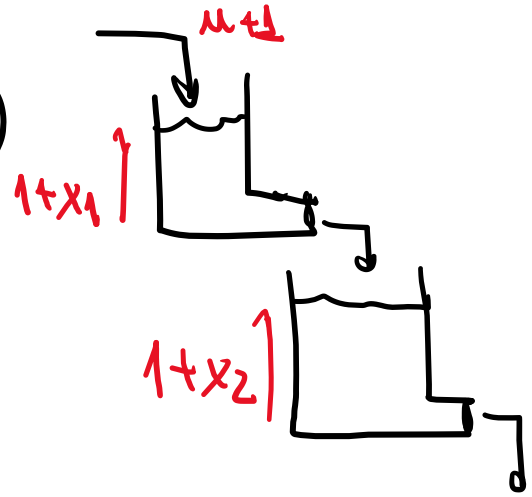


POLOS EM MF REQUERIDOS

$-1 \pm j$
(4º ORDEM)
$-5 \pm 5j$

$$(s+1+j)(s+1-j)(s+5+5j)(s+5-5j) = s^4 + k_3 s^3 + k_2 s^2 + k_1 s + k_0$$

Ex)



$$\begin{cases} \dot{x}_1 = 1 + u - \sqrt{1+x_1} \\ \dot{x}_2 = \sqrt{1+x_1} - \sqrt{1+x_2} \\ y = x_2 \end{cases}$$

$$y = x_2$$

$$\dot{y} = \dot{x}_2 = \sqrt{1+x_1} - \sqrt{1+x_2}$$

$$\ddot{y} = \frac{1}{2}(1+x_1)^{-\frac{1}{2}} \cdot \dot{x}_1 - \frac{1}{2} \cdot (1+x_2)^{-\frac{1}{2}} \cdot \dot{x}_2$$

$$\boxed{\ddot{y} = \underbrace{\frac{1}{2}\left(\frac{1}{\sqrt{1+x_1}} - \frac{\sqrt{1+x_1}}{\sqrt{1+x_2}}\right)}_{f} + \frac{u}{\underbrace{2\sqrt{1+x_1}}_{b}}}$$

$$f(\underset{\sim}{x}) = \frac{1}{2}\left(\frac{1}{\sqrt{1+x_1}} - \frac{\sqrt{1+x_1}}{\sqrt{1+x_2}}\right)$$

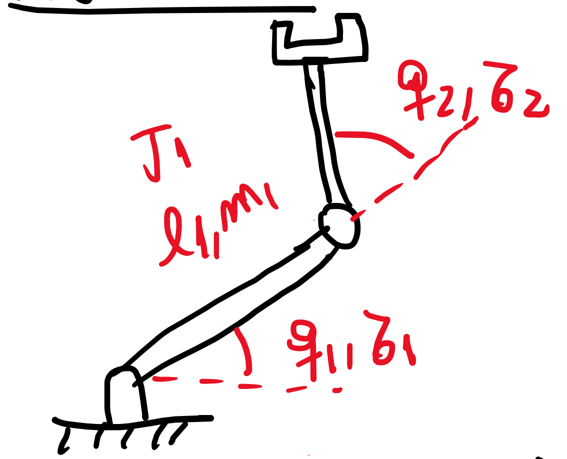
$$b(\underset{\sim}{x}) = \frac{1}{2\sqrt{1+x_1}}$$

$$\Rightarrow \ddot{y} = f(\underset{\sim}{x}) + b(\underset{\sim}{x}) \cdot u$$

$$\boxed{\begin{gathered} u = \frac{1}{b(\underset{\sim}{x})} \cdot \left[-f(\underset{\sim}{x}) + v(t)\right] \\ v(t) = \ddot{y}_d - \alpha_1 \dot{y} - \alpha_2 \tilde{y} \end{gathered}}$$

α_1 α_2 CALCULADOS P/ QUE A MF TENHA POLOS ONDE QUISER

ROBÓTICA



$$\underbrace{\underset{\sim}{H}(q)}_{\text{MATRIZ INERCIA}}\ddot{q} + \underbrace{\underset{\sim}{C}(q,\dot{q})\dot{q}}_{\text{CORIOLIS + CENTRIPETA}} + \underbrace{\underset{\sim}{g}(q)}_{\text{GRAVIDADE}} = \underset{\sim}{\tau} \rightarrow \text{TORQUES MOTORES} \quad (1)$$

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad \underset{\sim}{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

$$\ddot{q} = \underbrace{-\underset{\sim}{H}^{-1}\cdot\underset{\sim}{C}\cdot\dot{q} - \underset{\sim}{H}^{-1}\underset{\sim}{g}}_{\underset{\sim}{f}(q,\dot{q})} + \underbrace{H^{-1}}_{b(q,\dot{q})}\cdot\tau$$

$$\underset{\sim}{\tau}(t) = \underset{\sim}{H}\cdot\left[\underset{\sim}{H}^{-1}\cdot\underset{\sim}{C}\cdot\dot{q} + H^{-1}\cdot\underset{\sim}{g} + \underset{\sim}{v}\right] \Rightarrow \boxed{\underset{\sim}{\tau}(t) = \underset{\sim}{C}\cdot\dot{q} + g + \underset{\sim}{H}\cdot v(t)} \quad (2)$$

NÃO LINEARES

$$\boxed{\begin{gathered} u = \frac{1}{b(x)}\cdot\left[-f(x) + v(t)\right] \\ v(t) = \ddot{y}_d - \lambda_1\dot{y} - \lambda_2\tilde{y} \end{gathered}}$$

EM MALHA FECHADA

(2) EM (1) $\Rightarrow \boxed{\ddot{q} = v(t)} \Rightarrow v(t) = \ddot{q}_d - 2\lambda\cdot\dot{\tilde{q}} - \lambda^2\cdot\tilde{q}$

$$\underset{\sim}{\tau}(t) = \underset{\sim}{H} \cdot \left[\underset{\sim}{H}^{-1} \cdot \underset{\sim}{C} \cdot \dot{q} + \underset{\sim}{H}^{-1} \cdot g + \underset{\sim}{v}\right] \Rightarrow \boxed{\underset{\sim}{\tau}(t) = \underset{\sim}{C} \cdot \dot{q} + g + \underset{\sim}{H} \cdot \underline{v(t)}} \quad (2)$$

NÃO LINEARES

$$\boxed{\begin{aligned} u &= \frac{1}{b(\underset{\sim}{x})} \cdot \left[-f(\underset{\sim}{x}) + v(t)\right] \\ v(t) &= \ddot{y}_d - \lambda_1 \dot{\tilde{y}} - \lambda_2 \tilde{y} \end{aligned}}$$

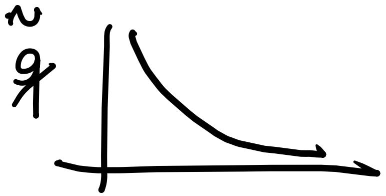
EM MALHA FECHADA

(2) EM (1) $\Rightarrow \boxed{\ddot{q} = v(t)} \Rightarrow v(t) = \ddot{q}_d - 2\lambda \cdot \dot{\tilde{q}} - \lambda^2 \cdot \tilde{q}$

$\Rightarrow$ EM MALHA FECHADA $\Rightarrow$

$$\ddot{q} = \ddot{q}_d - 2\lambda \dot{\tilde{q}} - \lambda^2 \tilde{q}$$

$$\ddot{\tilde{q}} + 2\lambda \dot{\tilde{q}} + \lambda^2 \tilde{q} = 0 \Rightarrow \text{2 POLOS EM } -\lambda$$



TIPO 1 - LINEARIZAÇÃO ENTRADA-ESTADO

↳ QUANDO SE QUER CONTROLAR ESTADO TODO

$$\boxed{\dot{x} = f(\underset{\sim}{x}, u)}$$

$$\begin{cases} \dot{x}_1 = -2x_1 + \overset{z_2}{a x_1 + \operatorname{sen} x_1} \\ \dot{x}_2 = -x_2 \cos x_2 + u \cos(2x_1) \end{cases}$$

PROJETAR $\underline{u}$ PARA GARANTIR QUE

$(x_1, x_2) \to (0,0)$ NUMA REGIÃO Ω

PASSO 1) TRANSFORMAR AS VARIÁVEIS P/ LINEARIZAR A DINÂMICA

$$\begin{cases} z_1 = x_1 \\ z_2 = a x_2 + \operatorname{sen} x_1 \end{cases}$$

$$\begin{cases} \dot{z}_1 = \dot{x}_1 \\ \dot{z}_2 = a \dot{x}_2 + \dot{x}_1 \cos x_1 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \dot{z}_1 = -2z_1 + z_2 \\ \dot{z}_2 = -2z_1 \cos z_1 + \cos z_1 \operatorname{sen} z_1 + a u \cos 2z_1 \end{cases}$$

$$\begin{cases} \dot{z}_1 = -2z_1 + z_2 \\ \dot{z}_2 = -2z_1\cos z_1 + \cos z_1 \operatorname{sen} z_1 + \underline{\underline{au}}\cos 2z_1 \end{cases}$$

$\longrightarrow$ LINEAR

$\longrightarrow$ NÃO LINEAR

PASSO 2) APLICAR FL. P/ z_2

$$u = \frac{1}{a\cos 2z_1} \cdot \left[2z_1\cos z_1 - \cos z_1 \operatorname{sen} z_1 + v(t) \right]$$

$\Downarrow$ em MF

$$\underbrace{\begin{cases} \dot{z}_1 = -2z_1 + z_2 \\ \dot{z}_2 = v(t) \end{cases}}$$

SIST. LINEAR E CONTROLÁVEL!

ooo

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} -2 & 1 \\ 0 & 0 \end{pmatrix}}_{A} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B} u$$

$\operatorname{rank}(\operatorname{ctrb}(A,B)) = 2$ ✓

$v(t) = -K \cdot [z_1 ; z_2] \rightarrow \underset{\sim}{K} = \operatorname{place}(A, B, \text{polos})$

PASSO 2) APLICAR F.L. P/ z_2

$$u = \frac{1}{a\cos 2z_1} \cdot \left[2z_1\cos z_1 - \cos z_1 \operatorname{sen} z_1 + v(t) \right]$$

$\Downarrow$ em MF

$$\underbrace{\begin{cases} \dot{z}_1 = -2z_1 + z_2 \\ \dot{z}_2 = v(t) \end{cases}}_{\text{SIST. LINEAR E CONTROLÁVEL!}}$$

$\circ\circ\circ$ $\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} -2 & 1 \\ 0 & 0 \end{pmatrix}}_{A} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B} u$

$\operatorname{rank}(\operatorname{ctrb}(A,B)) = 2$ ✓

$v(t) = -\underset{\sim}{K} \cdot [z_1 ; z_2] \rightarrow \underset{\sim}{K} = \operatorname{place}(A, B, \text{polos})$

SE POLOS $[-2, -2] \Rightarrow \underset{\sim}{K} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \Rightarrow v = -2z_2$

$$\Rightarrow \boxed{u = \frac{1}{a\cos 2z_1} \cdot \left[\underbrace{2z_1\cos z_1 - \cos z_1 \operatorname{sen} z_1}_{\text{F.L.}} \underbrace{- 2z_2}_{\text{CONT. LINEAR}} \right]}$$

$$u = \frac{1}{a\cos 2z_1} \cdot \left[\underbrace{2z_1\cos z_1 - \cos z_1 \operatorname{sen} z_1}_{\text{F.L.}} - 2z_2 \right]$$

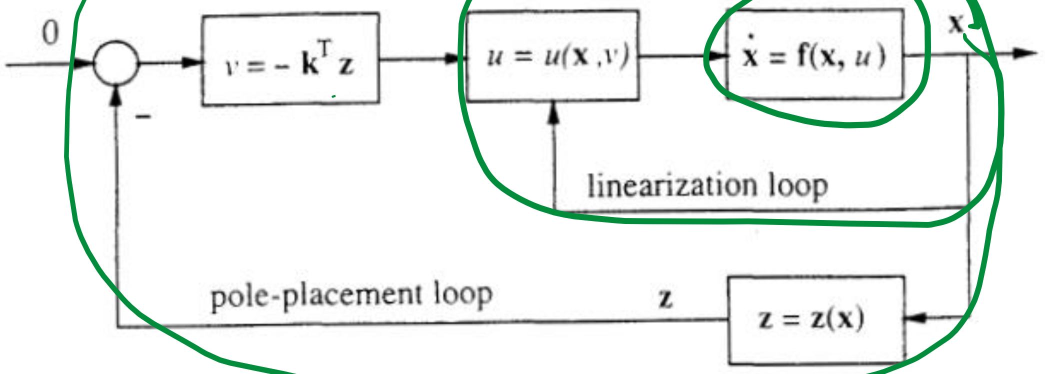
u

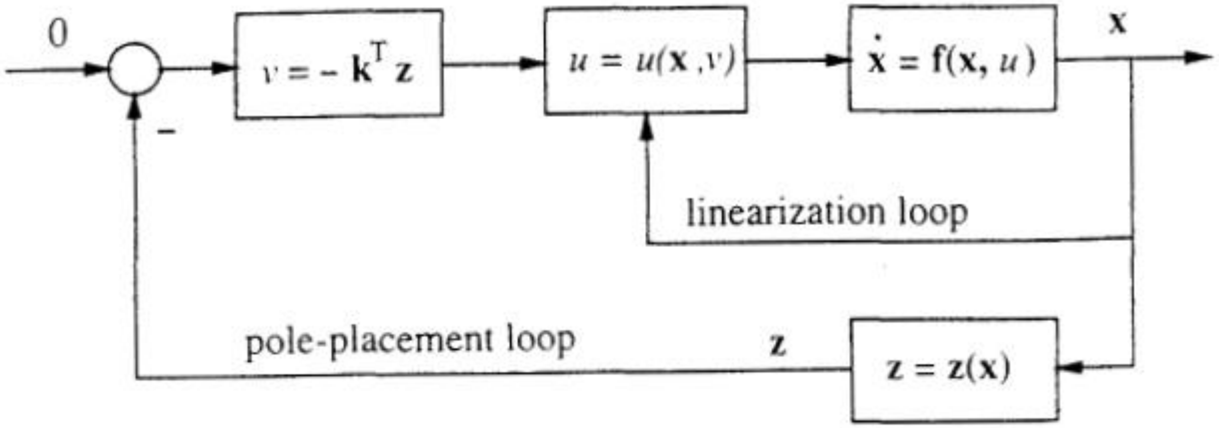
$$\begin{cases} \dot{x}_1 = -2x_1 + (a x_1 + \operatorname{sen} x_1) \\ \dot{x}_2 = -x_2 \cos x_2 + u\cos(2x_1) \end{cases}$$

x_1, x_2

TRANSF

z_1, z_2




0
$v = -\mathbf{k}^T\mathbf{z}$
$u = u(\mathbf{x}, v)$
$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u)$
x
–
linearization loop
pole-placement loop
z
$\mathbf{z} = \mathbf{z}(\mathbf{x})$