Noturno, Prova - 2, 2015, 10/12/2015, E. Alimentos

1) Calcule a integral $\int_0^{\pi/4} [\cos(2t)\vec{i} + \text{sen}(2t)\vec{j} + t\,\text{sen}(t)\vec{k}]\,dt.$

Solução: Para resolver o problema devemos calcular 3 integrais por separado

$$\int_a^b \vec{r}(t)dt = < \int_a^b x(t)dt \,,\, \int_a^b y(t)dt \,,\, \int_a^b z(t)dt >$$

$$\vec{r}(t) = < \cos(2t), \text{sen}(2t), t\,\text{sen}(t) >$$

$$- \int_a^b x(t)dt = \int_0^{\pi/4} \cos(2t)\,dt = \frac{1}{2}\text{sen}(2t)\Big|_0^{\pi/4} = \frac{1}{2}[\underbrace{\text{sen}(\pi/2)}_{1} - \underbrace{\text{sen}(0)}_{0}]$$

$$\int_a^b x(t)dt = \frac{1}{2}$$

$$- \int_a^b y(t)dt = \int_0^{\pi/4} \text{sen}(2t)\,dt = -\frac{1}{2}\cos(2t)\Big|_0^{\pi/4} = -\frac{1}{2}[\underbrace{\cos(\pi/2)}_{0} - \underbrace{\cos(0)}_{1}]$$

$$\int_0^{\pi/4} y(t)dt = \frac{1}{2}$$

$$- \int_a^b z(t)dt = \int_0^{\pi/4} t\,\text{sen}(t)\,dt \rightarrow$$

Integração por partes

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$u = t \qquad dv = \text{sen}(t)\,dt$

$du = dt \qquad v = -\cos(t)$

$$= -t\cos(t)\Big|_0^{\pi/4} - \int_0^{\pi/4} -\cos(t)dt$$

$$= -\frac{\pi}{4}\cos(\pi/4) + \int_0^{\pi/4} \cos(t)\,dt$$

$$= -\frac{\pi}{4}\frac{\sqrt{2}}{2} + \text{sen}(t)\Big|_0^{\pi/4} = -\frac{\sqrt{2}\,\pi}{8} + \text{sen}(\pi/4) - \overset{0}{\text{sen}(0)} = \frac{-\sqrt{2}\,\pi}{8} + \frac{\sqrt{2}}{2}$$

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$$\int_0^{\pi/4} z(t)\,dt = \frac{\sqrt{2}}{2}\left(\frac{-\pi}{4}+1\right)$$

$$\int_0^{\pi/4} \vec{r}(t)\,dt = \left\langle \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\left(-\frac{\pi}{4}+1\right) \right\rangle$$

2) Encontre a curvatura da função vetorial
$\vec{r}(t) = (1+t)\vec{i} + (1-t)\vec{j} + 3t^2\vec{k}$.

Solução: $K(t) = \dfrac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}$

$\vec{r}(t) = \langle 1+t, 1-t, 3t^2 \rangle$

$\vec{r}\,'(t) = \langle 1, -1, 6t \rangle \longrightarrow |\vec{r}\,'(t)| = \sqrt{2+36t^2}$

$\vec{r}\,''(t) = \langle 0, 0, 6 \rangle$

$$\vec{r}\,' \times \vec{r}\,'' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 6t \\ 0 & 0 & 6 \end{vmatrix} = \langle -6, -6, 0 \rangle$$

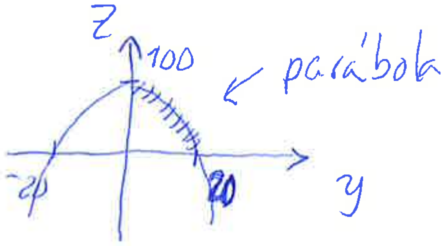
$$|\vec{r}\,' \times \vec{r}\,''| = \sqrt{2 \cdot 36} = 6\sqrt{2}$$

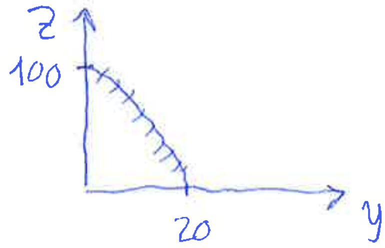
$$\boxed{K(t) = \frac{6\sqrt{2}}{(2+36t^2)^{3/2}}}$$

3) Esboce o gráfico da função $f(x,y) = 100 - x^2 - y^2/4$.

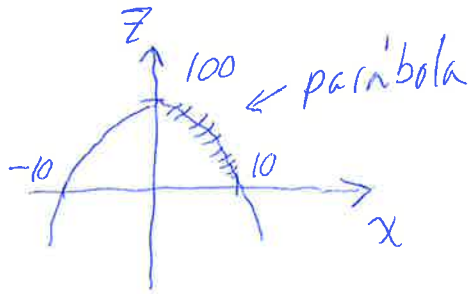
Solução: $z = 100 - x^2 - \frac{y^2}{4}$

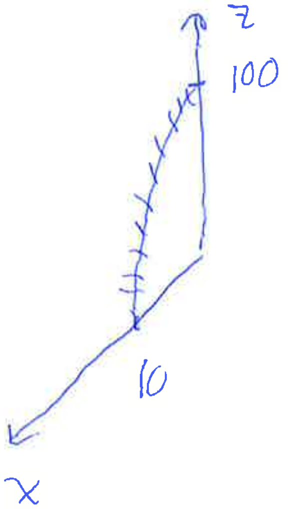
- Se $x = 0 \longrightarrow z = 100 - \frac{y^2}{4} \longrightarrow$





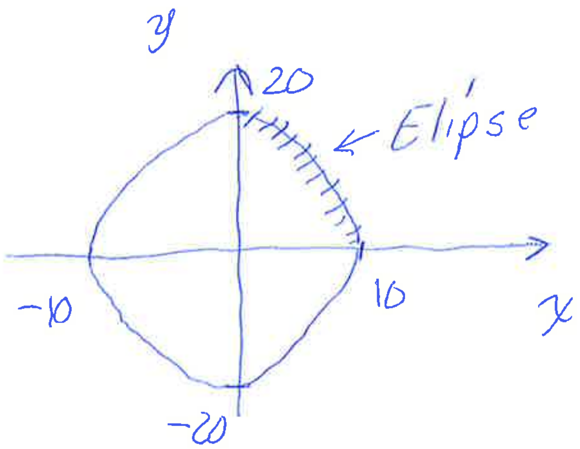
- Se $y = 0 \longrightarrow z = 100 - x^2 \longrightarrow$

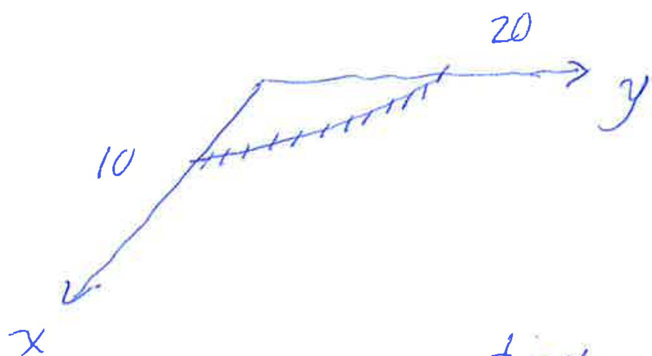




- Se $z = 0 \longrightarrow 0 = 100 - x^2 - \frac{y^2}{4} \longrightarrow x^2 + \frac{y^2}{4} = 100$ Elipse

$$\frac{x^2}{10^2} + \frac{y^2}{20^2} = 1$$





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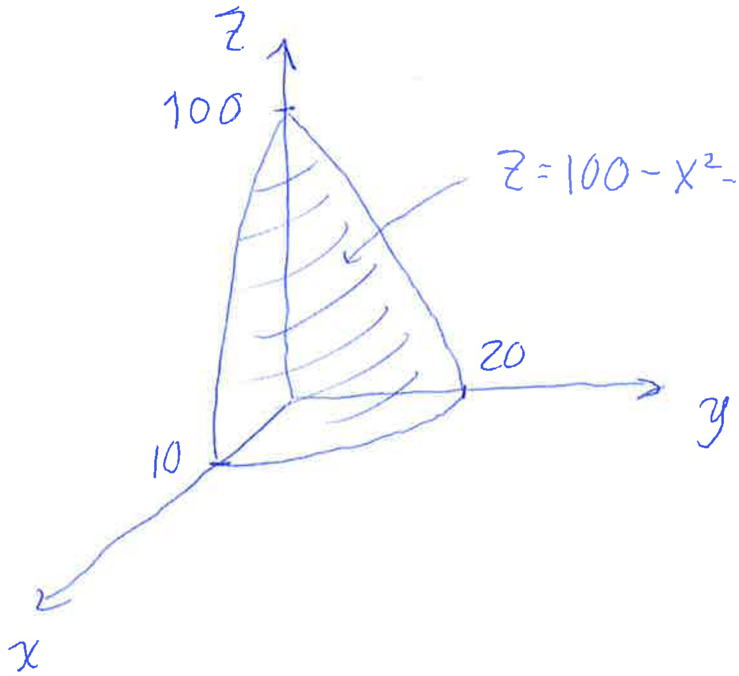
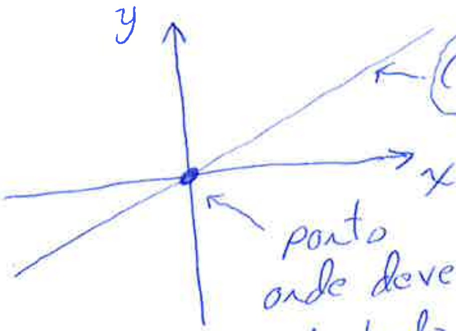


$z = 100 - x^2 - \frac{y^2}{4}$ superfície

Parabolóide Elíptico

4) Determine o limite, $\lim\limits_{(x,y)\to(0,0)} \left[\frac{2xy}{x^2+2y^2}\right]$, ou mostre que não existe.

<u>Solução:</u> Vou tentar mostrar que não existe.



Caminho de Teste
$y = mx$, $m \in \mathbb{R}$
m é uma constante qualquer

ponto onde deve ser calculado o limite

$$\lim_{\substack{(x,y)\to(0,0)\\ \boxed{y=mx}}} \left[\frac{2xy}{x^2+2y^2}\right] = \lim_{(x,mx)\to(0,0)} \left[\frac{2xmx}{x^2+2m^2x^2}\right] = \lim_{x\to 0} \left[\frac{2m}{1+2m^2}\right] = \frac{2m}{1+2m^2}$$

O resultado depende de m. Isso mostra que existem dois caminhos pelos quais o limite é diferente. Logo, o limite NÃO EXISTE.

5) Determine as derivadas parciais de primeira ordem da função $f(x,y) = x^2y^2(x^4+y^4)$. (5)

Solução: - Temos que usar a regra do produto para derivar

- Existe simetria na troca de $x \to y$. Logo,

$\frac{\partial f}{\partial x}$ coincide com $\frac{\partial f}{\partial y}$ se trocamos $x \to y$.

$$\frac{\partial f}{\partial x} = 2xy^2(x^4+y^4) + x^2y^2(4x^3)$$

$$\frac{\partial f}{\partial y} = 2yx^2(x^4+y^4) + x^2y^2(4y^3)$$

$\blacksquare$