Diurno, Alimentos, P2, 2016, Prof. Juan (1)

1) Reparametrize a curva $x = e^t \cos(t)$, $y = e^t \operatorname{sen}(t)$ em função do parâmetro comprimento de arco. Tome como ponto de referência $(0, e^{\pi/2})$.

Sol.: $\vec{r}(u) = e^u < \cos(u), \operatorname{sen}(u) >$

$\vec{r}\,'(u) = e^u < \cos(u), \operatorname{sen}(u) > + e^u < -\operatorname{sen}(u), \cos(u) >$

$\vec{r}\,'(u) = e^u < \cos(u) - \operatorname{sen}(u), \cos(u) + \operatorname{sen}(u) >$

$|\vec{r}\,'(u)| = |e^u| \cdot |< \cos(u) - \operatorname{sen}(u), \cos(u) + \operatorname{sen}(u) >|$

$|\vec{r}\,'(u)| = e^u \sqrt{[\cos(u) - \operatorname{sen}(u)]^2 + [\cos(u) + \operatorname{sen}(u)]^2}$

$|\vec{r}\,'(u)| = \sqrt{2}\, e^u$

$$\begin{cases} (I) \; 0 = e^{u_0} \cos(u_0) \longrightarrow e^{u_0} \neq 0 \longrightarrow \cos(u_0) = 0 \rightarrow u_0 = \pi/2 \text{ ou } u_0 = 3\pi/2 \\ (II) \; e^{\pi/2} = e^{u_0} \operatorname{sen}(u_0) \end{cases}$$

- Se $u_0 = \pi/2$ em $(II) \rightarrow 1 = \operatorname{sen}(\pi/2) \rightarrow$ verdadeiro
- Se $u_0 = 3\pi/2$ em $(II) \rightarrow e^{\pi/2} = e^{3\pi/2} \underbrace{\operatorname{sen}(3\pi/2)}_{-1} \Rightarrow e^{\pi/2} = -e^{3\pi/2}$ ABSURDO

Logo $u_0 = \pi/2 \longrightarrow \vec{r}(u_0 = \pi/2) = (0, e^{\pi/2})$.

Temos que encontrar a função comprimento de arco

$$s(t) = \int_{u_0}^{t} |\vec{r}\,'(u)|\, du$$

(2)

$$s(t) = \int_{\pi/2}^{t} \sqrt{2}\, e^{u}\, du = \sqrt{2}\, e^{u} \Big|_{\pi/2}^{t} = \sqrt{2}\left[e^{t} - e^{+\pi/2}\right]$$

$$s(t) = \sqrt{2}\left[e^{t} - e^{\pi/2}\right]$$

Agora encontramos a função inversa $(t(s))$

$$\frac{s}{\sqrt{2}} = e^{t} - e^{\pi/2}$$

$$\frac{s}{\sqrt{2}} + e^{\pi/2} = e^{t}$$

$$\ln\left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right] = \ln(e^{t}) = t$$

$$t(s) = \ln\left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]$$

Voltando nas eq. originais

$$\begin{cases} x(t) = e^{t}\cos(t) \\ y(t) = e^{t}\,\text{sen}(t) \end{cases} \Rightarrow \vec{r}(t) = e^{t}\langle \cos(t), \text{sen}(t) \rangle$$

e trocando t por $t(s)$

$$x(s) = \underbrace{e^{\ln\left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]}}_{\left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]} \cos\left(\ln\left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]\right)$$

$$\begin{cases} x(s) = \left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]\cos\left(\ln\left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]\right) \\ y(s) = \left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]\text{sen}\left(\ln\left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]\right) \end{cases}$$

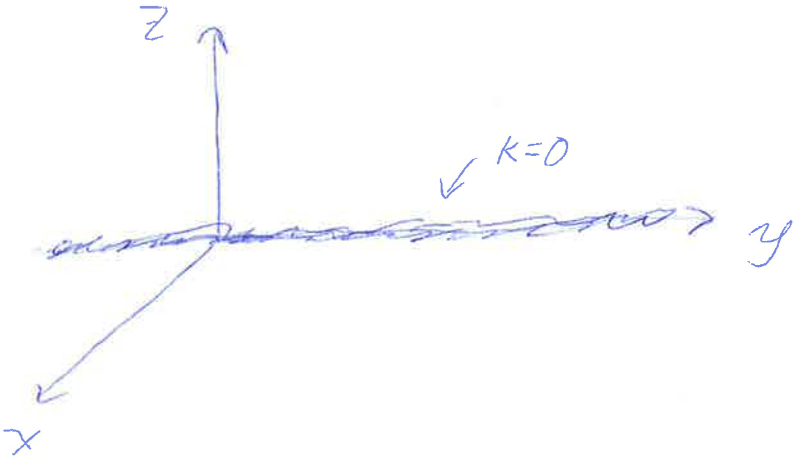
$$\vec{r}(s) = \left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]\left\langle \cos\left(\ln\left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]\right), \text{sen}\left(\ln\left[\frac{s}{\sqrt{2}} + e^{\pi/2}\right]\right)\right\rangle$$

2) Esboce três superfícies de nível da função $f(x,y,z) = x^2 + z^2$.

Sol.: $k = cte = f(x,y,z)$ ← Superfície de Nível

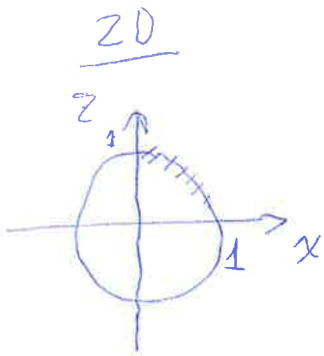
$\boxed{k = x^2 + z^2}$ ← Eq. das Superfícies de Nível de $f(x,y,z) = x^2 + z^2$.

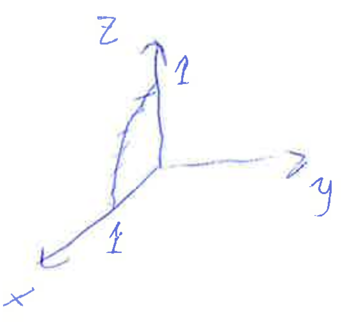
- Se $k = 0 \longrightarrow 0 = x^2 + z^2$
São todos os pontos do tipo $(0,y,0)$
Ou seja, sobre a reta do eixo y.



- Se $k = 1 \longrightarrow x^2 + z^2 = 1$

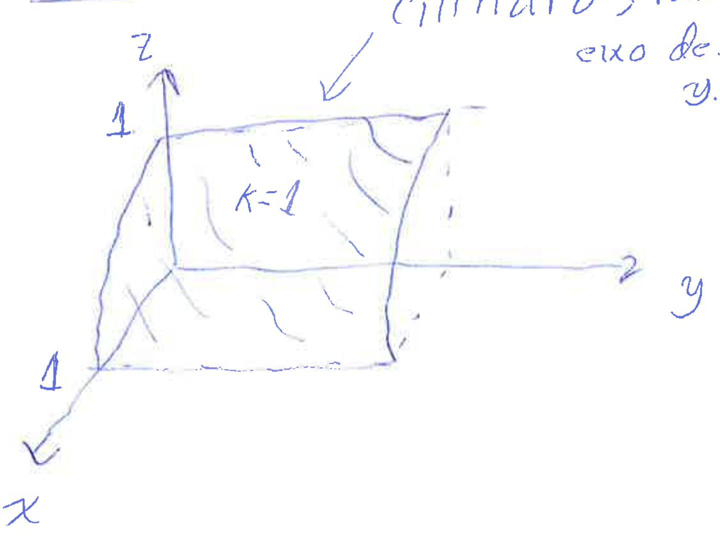
2D





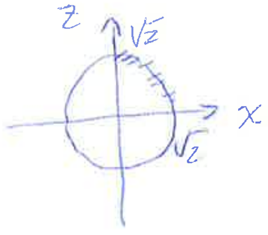
3D

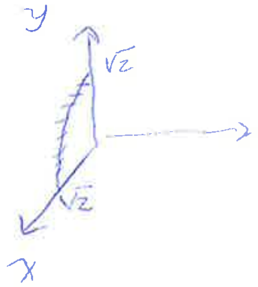
cilindro, raio 1, eixo de simetria y.



- Se $k=2 \Rightarrow x^2+z^2=2$

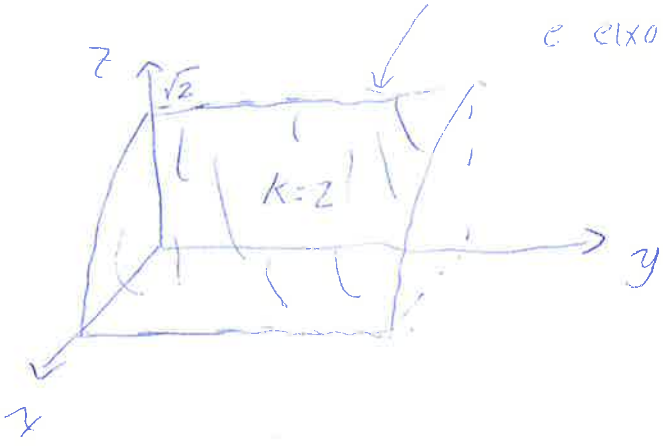
2D



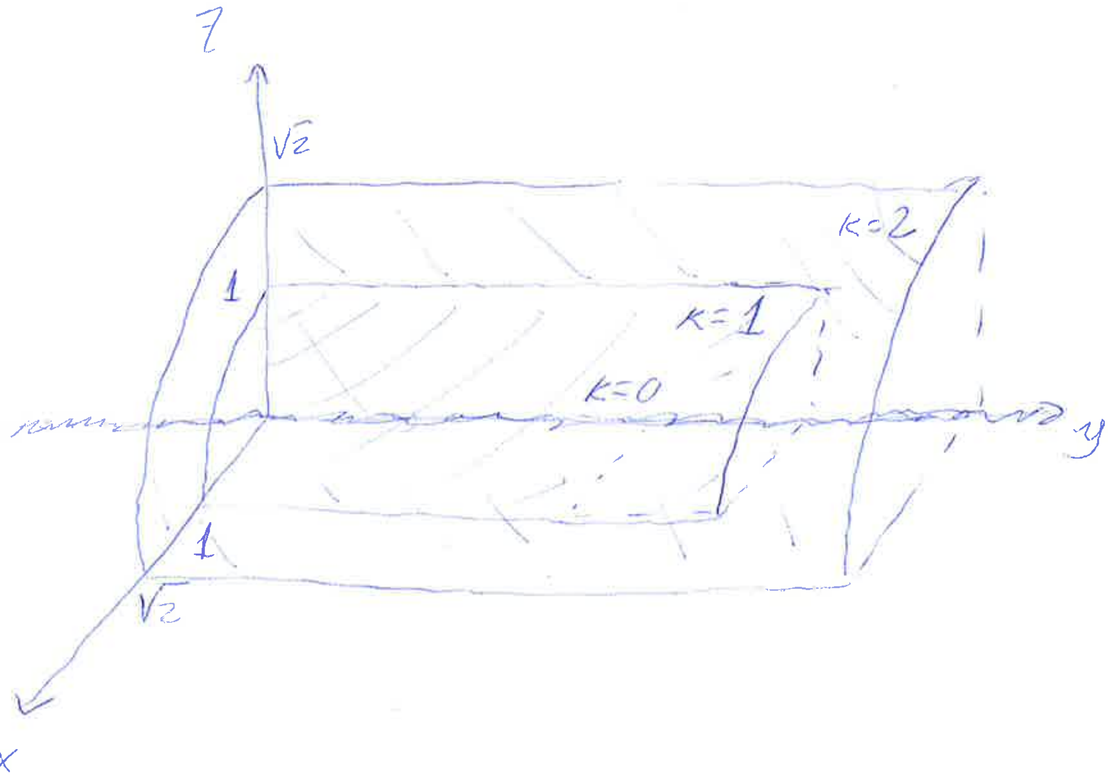


3D

cilindro de raio $\sqrt{2}$ e eixo de simetria y.



Todas juntas



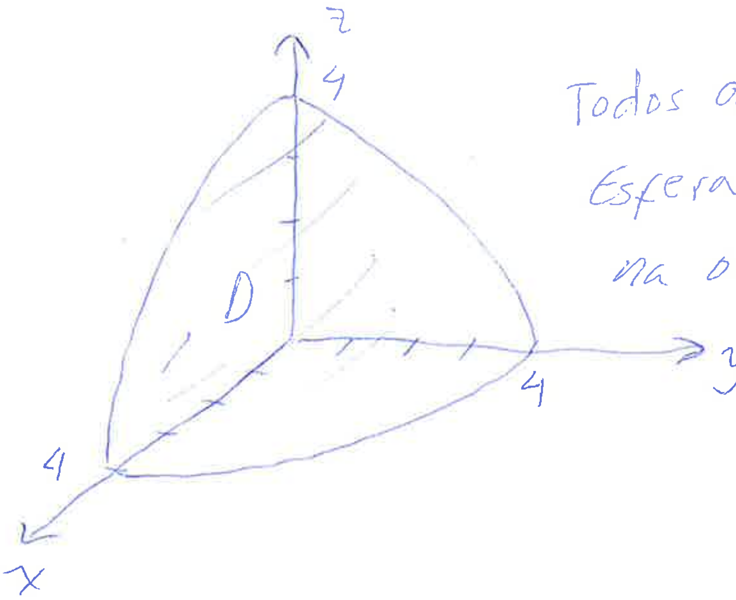
3) Determine o domínio da função.

$$f(x,y,z) = \frac{1}{\sqrt{16 - x^2 - y^2 - z^2}}$$

Sol.:

$$D = \{(x,y,z) \in \mathbb{R}^3 \mid 16 - x^2 - y^2 - z^2 > 0\}$$

$$x^2 + y^2 + z^2 < 4^2$$



Todos os pontos dentro da Esfera de raio 4 e centrada na origem. Excluídos os pontos da fronteira (da esfera).

4) Calcule $\frac{\partial z}{\partial y}$ se $zy^2x^3 + z^3y^2x = y + z$.

Sol.: $\boxed{\text{Caminho 1}}$

Vamos re-escrever a eq. dada na forma

$$F(x,y,z) = c \text{ (cte)}.$$

Isto é $F(x,y,z) = zy^2x^3 + z^3y^2x - y - z = 0$

$$\boxed{\frac{\partial z}{\partial y} = -\left(\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}\right)}$$

Fórmula para derivada implícita onde $F(x,y,z)$ é uma função com 3 variáveis independentes

$$\frac{\partial F}{\partial y} = 2x^3yz + 2xyz^3 - 1$$

$$\frac{\partial F}{\partial z} = x^3y^2 + 3xy^2z^2 - 1$$

$$\boxed{\frac{\partial z}{\partial y} = -\left(\frac{2x^3yz + 2xyz^3 - 1}{x^3y^2 + 3xy^2z^2 - 1}\right)} \quad \boxed{\text{se } x^3y^2 + 3xy^2z^2 - 1 \neq 0}$$

$\boxed{\text{Caminho - 2}}$

Considere que z depende implicitamente de x e y pela eq. dada.

$$z(x,y)\,y^2x^3 + [z(x,y)]^3y^2x = y + z(x,y)$$

$$\frac{\partial}{\partial y}\left[z(x,y)\,y^2x^3 + [z(x,y)]^3y^2x\right] = \frac{\partial}{\partial y}\left[y + z(x,y)\right]$$

$$x^3y^2\frac{\partial z}{\partial y} + 2x^3yz + 3xy^2z^2\frac{\partial z}{\partial y} + 2xyz^3 = 1 + \frac{\partial z}{\partial y}$$

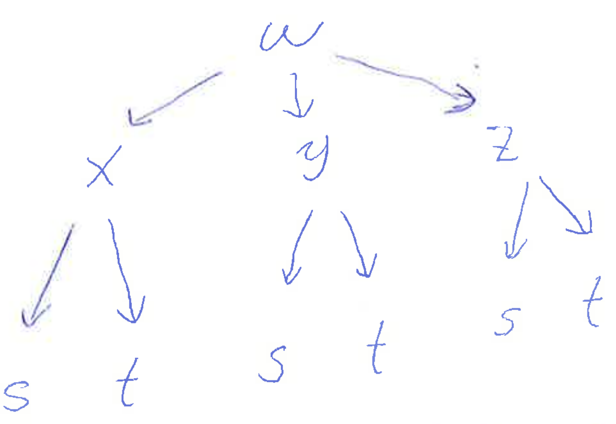
$$(x^3y^2 + 3xy^2z^2 - 1)\frac{\partial z}{\partial y} = 1 - 2x^3yz - 2xyz^3$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{1 - 2x^3yz - 2xyz^3}{x^3y^2 + 3xy^2z^2 - 1}} \quad \boxed{\text{Se } x^3y^2 + 3xy^2z^2 - 1 \neq 0}$$

5) Use a regra da cadeia para determinar as derivadas parciais $\frac{\partial w}{\partial s}$ e $\frac{\partial w}{\partial t}$ se $w(x,y,z) = x^2+y^2+z^2$; $x(s,t) = st$, $y(s,t) = s\cos(t)$ e $z(s,t) = s\,\mathrm{sen}(t)$.

Sol.:

$$w(x,y,z) = x^2+y^2+z^2$$
$$x(s,t) = st$$
$$y(s,t) = s\cos(t)$$
$$z(s,t) = s\,\mathrm{sen}(t)$$



$$\frac{\partial w}{\partial s} = \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial x}{\partial s}\right) + \left(\frac{\partial w}{\partial y}\right)\left(\frac{\partial y}{\partial s}\right) + \left(\frac{\partial w}{\partial z}\right)\left(\frac{\partial z}{\partial s}\right)$$
$$= 2x \cdot t + 2y\cos(t) + 2z\,\mathrm{sen}(t)$$

$$\frac{\partial w}{\partial s} = 2\cdot s\cdot t\cdot t + \underbrace{2s\cos^2(t) + 2s\,\mathrm{sen}^2(t)}_{2s\cdot 1}$$

$$\frac{\partial w}{\partial s} = 2st^2 + 2s$$

$$\boxed{\frac{\partial w}{\partial s} = 2s(t^2+1)}$$

$$\frac{\partial w}{\partial t} = \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial x}{\partial t}\right) + \left(\frac{\partial w}{\partial y}\right)\left(\frac{\partial y}{\partial t}\right) + \left(\frac{\partial w}{\partial z}\right)\left(\frac{\partial z}{\partial t}\right)$$

$$= 2x\cdot s + 2y(-1)s\,\mathrm{sen}(t) + 2z\,s\cos(t)$$

$$= 2s\cdot t\cdot s + 2s\cos(t)(-1)s\,\mathrm{sen}(t) + 2s\,\mathrm{sen}(t)\,s\cos(t)$$

$$= 2s^2\cdot t \cancel{- 2s^2\cos(t)\mathrm{sen}(t) + 2s^2\cos(t)\mathrm{sen}(t)}$$

$$\boxed{\frac{\partial w}{\partial t} = 2s^2t}$$