13.4 SOLUÇÕES

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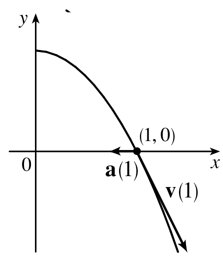
1. $\mathbf{r}(t) = \langle \sqrt{t}, 1 - t \rangle \Rightarrow \mathbf{v}(t) = \left\langle \frac{1}{2} t^{-1/2}, -1 \right\rangle,$

$\mathbf{a}(t) = \left\langle -\frac{1}{4} t^{-3/2}, 0 \right\rangle, |\mathbf{v}(t)| = \sqrt{\frac{1}{4} t^{-1} + 1}$

Em $t = 1$: $\mathbf{v}(1) = \left\langle \frac{1}{2}, -1 \right\rangle$, $\mathbf{a}(1) = \left\langle -\frac{1}{4}, 0 \right\rangle$

Uma vez que $x^2 = t$, $y = 1 - t = 1 - x^2$,

mas $x = \sqrt{t}$, então $x \geq 0$.



2. $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle \Rightarrow \mathbf{v}(t) = \mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle,$

$\mathbf{a}(t) = \mathbf{v}'(t) = \langle 0, 2, 6t \rangle,$

$|\mathbf{v}(t)| = \sqrt{1^2 + (2t)^2 + (3t^2)^2} = \sqrt{1 + 4t^2 + 9t^4}$

3. $\mathbf{r}(t) = \langle t^3, t^2 + 1, t^3 - 1 \rangle \Rightarrow \mathbf{v}(t) = \langle 3t^2, 2t, 3t^2 \rangle,$

$\mathbf{a}(t) = \langle 6t, 2, 6t \rangle,$

$|\mathbf{v}(t)| = \sqrt{9t^4 + 4t^2 + 9t^4} = \sqrt{18t^4 + 4t^2}$

$= |t| \sqrt{18t^2 + 4}$

4. $\mathbf{r}(t) = \langle \sqrt{t}, t, t\sqrt{t} \rangle \Rightarrow \mathbf{v}(t) = \left\langle \frac{1}{2} t^{-1/2}, 1, \frac{3}{2} t^{1/2} \right\rangle,$

$\mathbf{a}(t) = \left\langle -\frac{1}{4} t^{-3/2}, 0, \frac{3}{4} t^{-1/2} \right\rangle,$

$|\mathbf{v}(t)| = \sqrt{\frac{1}{4} t^{-1} + 1 + \frac{9}{4} t} = \frac{1}{2} \sqrt{\frac{1 + 4t + 9t^2}{t}}$

5. $\mathbf{r}(t) = \langle 1/t, 1, t^2 \rangle \Rightarrow \mathbf{v}(t) = \langle -t^{-2}, 0, 2t \rangle,$

$\mathbf{a}(t) = \langle 2t^{-3}, 0, 2 \rangle, |\mathbf{v}(t)| = \sqrt{t^{-4} + 4t^2} = \frac{1}{t^2} \sqrt{4t^6 + 1}$

6. $\mathbf{r}(t) = \langle e^t, 2t, e^{-t} \rangle \Rightarrow \mathbf{v}(t) = \langle e^t, 2, -e^{-t} \rangle,$

$\mathbf{a}(t) = \langle e^t, 0, e^{-t} \rangle, |\mathbf{v}(t)| = \sqrt{e^{2t} + 4 + e^{-2t}}$

7. $\mathbf{r}(t) = \langle \cosh t, \operatorname{senh} t, t \rangle \Rightarrow$

$\mathbf{v}(t) = \langle \operatorname{senh} t, \cosh t, 1 \rangle$, $\mathbf{a}(t) = \langle \cosh t, \operatorname{senh} t, 0 \rangle,$

$|\mathbf{v}(t)| = \sqrt{\operatorname{senh}^2 t + \cosh^2 t + 1} = \sqrt{\cosh 2t + 1}$

Lembre-se de que $\cosh^2 t - \operatorname{senh}^2 t = 1$.

8. Aqui a velocidade inicial $v_0 = 120$ m/s; seja α o ângulo de elevação. Assumindo que um objeto está no chão, o objeto será atingido no momento $t = \dfrac{240 \operatorname{sen} \alpha}{g}$ (novamente veja o Exemplo 5). Então $\dfrac{(120)^2 \operatorname{sen} 2\alpha}{g} = 500$ ou

$\operatorname{sen} 2\alpha = \dfrac{500g}{(120)^2} = \dfrac{5g}{144}$ e $2\alpha = \operatorname{sen}^{-1} \dfrac{5g}{144} \approx 19{,}9^\circ$, então

$\alpha \approx 9{,}9^\circ$.

9. $\mathbf{r}'(t) = 2t\,\mathbf{i} + 2\,\mathbf{j}$, $|\mathbf{r}'(t)| = 2\sqrt{t^2 + 1}$, $\mathbf{r}''(t) = 2\,\mathbf{i}$. Então,

$a_T = \dfrac{4t}{2\sqrt{t^2 + 1}} = \dfrac{2t}{\sqrt{t^2 + 1}}$ e

$a_N = \dfrac{|-4\mathbf{k}|}{2\sqrt{t^2 + 1}} = \dfrac{2}{\sqrt{t^2 + 1}}.$

10. $\mathbf{r}'(t) = (1 - \cos t)\,\mathbf{i} + (\operatorname{sen} t)\,\mathbf{j},$

$|\mathbf{r}'(t)| = \sqrt{1 - 2\cos t + 1} = \sqrt{2(1 - \cos t)},$

$\mathbf{r}''(t) = \operatorname{sen} t\,\mathbf{i} + \cos t\,\mathbf{j}$. Então $a_T = \dfrac{\operatorname{sen} t}{\sqrt{2(1 - \cos t)}}$ e

$a_N = \dfrac{\left|\left(\cos t - \cos^2 t - \operatorname{sen}^2 t\right)\mathbf{k}\right|}{\sqrt{2(1 - \cos t)}} = \dfrac{\sqrt{[(\cos t) - 1]^2}}{\sqrt{2}\sqrt{1 - \cos t}}$

$= \dfrac{1}{\sqrt{2}} \sqrt{\dfrac{(1 - \cos t)^2}{1 - \cos t}} = \dfrac{\sqrt{1 - \cos t}}{\sqrt{2}}$

11. $\mathbf{r}'(t) = \mathbf{i} + 4\cos t\,\mathbf{j} - 4\operatorname{sen} t\,\mathbf{k}$, $|r'(t)| = \sqrt{1 + 4} = \sqrt{5},$

$\mathbf{r}''(t) = -4\operatorname{sen} t\,\mathbf{j} - 4\cos t\,\mathbf{k}.$

$a_T = -16\cos t \operatorname{sen} t + 16\cos t \operatorname{sen} t = 0$ e

$a_N = \frac{1}{\sqrt{17}} |-16\,\mathbf{i} + 4\cos t\,\mathbf{j} - 4\operatorname{sen} t\,\mathbf{k}|$

$= \frac{1}{\sqrt{17}} \sqrt{256 + 16} = 4$

12. $\mathbf{r}'(t) = 3t^2\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k}$, $|\mathbf{r}'(t)| = \sqrt{9t^4 + 4t^2 + 1},$

$\mathbf{r}''(t) = 6t\,\mathbf{i} + 2\,\mathbf{j}$. Então $a_T = \dfrac{18t^3 + 4t}{\sqrt{9t^4 + 4t^2 + 1}}$ e

$a_N = \dfrac{\left|-2\,\mathbf{i} + 6t\,\mathbf{j} + \left(6t^2 - 12t^2\right)\mathbf{k}\right|}{\sqrt{9t^4 + 4t^2 + 1}}$

$= \dfrac{\sqrt{4 + 36t^2 + 36t^4}}{\sqrt{9t^4 + 4t^2 + 1}} = \dfrac{2\sqrt{9t^4 + 9t^2 + 1}}{\sqrt{9t^4 + 4t^2 + 1}}$