13.2 SOLUÇÕES

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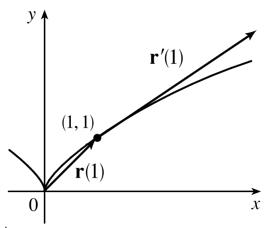
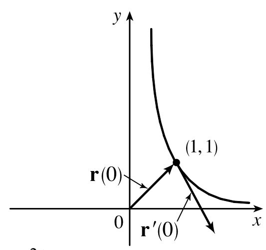
1. (a), (c)



(b) $\mathbf{r}'(t) = \langle 3t^2, 2t \rangle$

2. $x^{-2} = e^{-2t} = y$, então $y = 1/x^2, x > 0$.

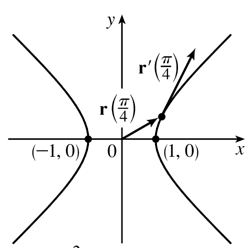
(a), (c)



(b) $\mathbf{r}'(t) = e^t\,\mathbf{i} - 2e^{-2t}\,\mathbf{j}$

3. $x^2 - y^2 = \sec^2 t - \operatorname{tg}^2 t = 1$, então a curva é uma hipérbole.

(a), (c)



(b) $\mathbf{r}'(t) = \sec t \operatorname{tg} t\,\mathbf{i} + \sec^2 t\,\mathbf{j}$

4. O domínio de $\mathbf{r}$ é $\mathbb{R}$ e $\mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle$.

5. O domínio de $\mathbf{r}$ é $\{t \mid t \geq 4 \text{ e } t \leq 6\}$ ou $\{t \mid 4 \leq t \leq 6\}$ e

$$\mathbf{r}'(t) = \left\langle 2t, \tfrac{1}{2}(t-4)^{-1/2}, \tfrac{1}{2}(6-t)^{-1/2}(-1) \right\rangle$$
$$= \left\langle 2t, \frac{1}{2\sqrt{t-4}}, -\frac{1}{2\sqrt{6-t}} \right\rangle$$

6. Uma vez que tg t e sec t não são definidos para múltiplos ímpares de $\frac{\pi}{2}$, o domínio de $\mathbf{r}$ é $\left\{t \mid t \neq (2n+1)\frac{\pi}{2}, n \text{ um inteiro}\right\}$.

$\mathbf{r}'(t) = \left(\sec^2 t\right)\mathbf{j} + (\sec t \operatorname{tg} t)\,\mathbf{k}$.

7. Uma vez que $\dfrac{t-1}{t+1}$ não é definido para $t = -1$ (e $\operatorname{tg}^{-1} t$ é definido para todo real t), o domínio é $\{t \mid t \neq -1\}$.

$$\mathbf{r}'(t) = (1+2t)\,e^{2t}\,\mathbf{i} + \frac{2}{(t+1)^2}\,\mathbf{j} + \frac{1}{1+t^2}\,\mathbf{k}.$$

8. $\mathbf{r}'(t) = -\dfrac{2t}{4-t^2}\,\mathbf{i} + \dfrac{1}{2\sqrt{1+t}}\,\mathbf{j} - 12e^{3t}\,\mathbf{k}$

9. $\mathbf{r}'(t) = -e^{-t}(\cos t + \operatorname{sen} t)\,\mathbf{i} + e^{-t}(\cos t - \operatorname{sen} t)\,\mathbf{j} + \dfrac{1}{t}\,\mathbf{k}$

10. $\mathbf{r}'(t) = \left\langle \dfrac{1}{2\sqrt{t}}, 1-2t, \dfrac{1}{1+t^2} \right\rangle \quad \Rightarrow$

$\mathbf{r}'(1) = \left\langle \frac{1}{2}, -1, \frac{1}{2} \right\rangle$. Então

$|\mathbf{r}'(1)| = \sqrt{\left(\frac{1}{2}\right)^2 + (-1)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{2}}$ e

$$\mathbf{T}(1) = \frac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \frac{1}{\sqrt{3/2}}\left\langle \tfrac{1}{2}, -1, \tfrac{1}{2} \right\rangle$$
$$= \left\langle \tfrac{1}{2}\sqrt{\tfrac{2}{3}}, -\sqrt{\tfrac{2}{3}}, \tfrac{1}{2}\sqrt{\tfrac{2}{3}} \right\rangle = \left\langle \tfrac{1}{\sqrt{6}}, -\sqrt{\tfrac{2}{3}}, \tfrac{1}{\sqrt{6}} \right\rangle$$

11. $\mathbf{r}'(t) = \mathbf{i} + 2\cos t\,\mathbf{j} - 3\operatorname{sen} t\,\mathbf{k}$, $\mathbf{r}'\left(\frac{\pi}{6}\right) = \mathbf{i} + \sqrt{3}\,\mathbf{j} - \frac{3}{2}\,\mathbf{k}$.

Logo,

$$\mathbf{T}\left(\tfrac{\pi}{6}\right) = \frac{1}{\sqrt{1^2 + \left(\sqrt{3}\right)^2 + (-3/2)^2}}\left(\mathbf{i} + \sqrt{3}\,\mathbf{j} - \tfrac{3}{2}\,\mathbf{k}\right)$$
$$= \frac{1}{5/2}\left(\mathbf{i} + \sqrt{3}\,\mathbf{j} - \tfrac{3}{2}\,\mathbf{k}\right) = \tfrac{2}{5}\,\mathbf{i} + \tfrac{2\sqrt{3}}{5}\,\mathbf{j} - \tfrac{3}{5}\,\mathbf{k}$$

12. $\mathbf{r}'(t) = 2e^{2t}(\cos t\,\mathbf{i} + \operatorname{sen} t\,\mathbf{j} + \mathbf{k}) + e^{2t}(-\operatorname{sen} t\,\mathbf{i} + \cos t\,\mathbf{j})$

$= e^{2t}\,[(2\cos t - \operatorname{sen} t)\,\mathbf{i} + (2\operatorname{sen} t + \cos t)\,\mathbf{j} + 2\,\mathbf{k}]$

$\mathbf{r}'\left(\frac{\pi}{2}\right) = e^{\pi}(-\mathbf{i} + 2\,\mathbf{j} + 2\,\mathbf{k})$

Logo, $\mathbf{T}\left(\frac{\pi}{2}\right) = \dfrac{e^{\pi}}{e^{\pi}\sqrt{9}}(-\mathbf{i} + 2\,\mathbf{j} + 2\,\mathbf{k}) = -\frac{1}{3}\,\mathbf{i} + \frac{2}{3}\,\mathbf{j} + \frac{2}{3}\,\mathbf{k}$.

13. $\mathbf{r}'(t) = \langle 2, 6t, 12t^2 \rangle$, $\mathbf{r}(1) = \langle 2, 3, 4 \rangle$, $\mathbf{r}'(1) = \langle 2, 6, 12 \rangle$.

Logo,

$$\mathbf{T}(1) = \frac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \frac{1}{\sqrt{188}}\langle 2, 6, 12 \rangle = \left\langle \tfrac{1}{\sqrt{46}}, \tfrac{3}{\sqrt{46}}, \tfrac{6}{\sqrt{46}} \right\rangle.$$

14. $\mathbf{r}'(t) = \langle 2e^{2t}, -2e^{-2t}, (1+2t)\,e^{2t} \rangle$, $\mathbf{r}'(0) = \langle 2, -2, 1 \rangle$.

Logo, $\mathbf{T}(0) = \frac{1}{\sqrt{9}}\langle 2, -2, 1 \rangle = \left\langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right\rangle$.

15. A equação vetorial da curva é $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$, então $\mathbf{r}'(t) = \mathbf{i} + 2t\,\mathbf{j} + 3t^2\,\mathbf{k}$. No ponto $(1, 1, 1)$, $t = 1$, então o vetor tangente é $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. A reta tangente passa através do ponto $(1, 1, 1)$ e tem um vetor diretor $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. Assim, as equações paramétricas são $x = 1 + t$, $y = 1 + 2t$, $z = 1 + 3t$.

16. $\mathbf{r}(t) = \langle 1 + 2t, 1 + t - t^2, 1 - t + t^2 - t^3 \rangle$,

$\mathbf{r}'(t) = \langle 2, 1 - 2t, -1 + 2t - 3t^2 \rangle$. Em $(1, 1, 1)$, $t = 0$ e $\mathbf{r}'(0) = \langle 2, 1, -1 \rangle$. Assim, as retas tangentes passam através do ponto $(1, 1, 1)$ e têm vetor diretor $\langle 2, 1, -1 \rangle$.

As equações paramétricas são $x = 1 + 2t$, $y = 1 + t$, $z = 1 - t$.

17. $\mathbf{r}(t) = \langle t\cos 2\pi t, t\operatorname{sen} 2\pi t, 4t \rangle$,

$\mathbf{r}'(t) = \langle \cos 2\pi t - 2\pi t \operatorname{sen} 2\pi t, \operatorname{sen} 2\pi t + 2\pi t \cos 2\pi t, 4 \rangle$.

Em $\left(0, \frac{1}{4}, 1\right)$, $t = \frac{1}{4}$ e

$\mathbf{r}'\left(\frac{1}{4}\right) = \left\langle 0 - \frac{\pi}{2}, 1 + 0, 4 \right\rangle = \left\langle -\frac{\pi}{2}, 1, 4 \right\rangle$. Assim, as equações paramétricas da reta tangente são $x = -\frac{\pi}{2}t$, $y = \frac{1}{4} + t$, $z = 1 + 4t$.

18. $\mathbf{r}(t) = \langle \operatorname{sen} \pi t, \sqrt{t}, \cos \pi t \rangle$, $\mathbf{r}'(t) = \langle \pi \cos \pi t, 1/(2\sqrt{t}), -\pi \operatorname{sen} \pi t \rangle$. Em $(0, 1, -1)$, $t = 1$ e $\mathbf{r}'(1) = \langle -\pi, \frac{1}{2}, 0 \rangle$. Assim, as equações paramétricas da reta tangente são $x = -\pi t$, $y = 1 + \frac{1}{2}t$, $z = -1$.

19. $\mathbf{r}(t) = \langle t, \sqrt{2}\cos t, \sqrt{2}\operatorname{sen} t \rangle$, $\mathbf{r}'(t) = \langle 1, -\sqrt{2}\operatorname{sen} t, \sqrt{2}\cos t \rangle$. Em $\left(\frac{\pi}{4}, 1, 1\right)$, $t = \frac{\pi}{4}$ e $\mathbf{r}'\left(\frac{\pi}{4}\right) = \langle 1, -1, 1 \rangle$. Assim, as equações paramétricas da reta tangente são $x = \frac{\pi}{4} + t$, $y = 1 - t$, $z = 1 + t$.

20. $\mathbf{r}(t) = \langle \cos t, 3e^{2t}, 3e^{-2t} \rangle$, $\mathbf{r}'(t) = \langle -\operatorname{sen} t, 6e^{2t}, -6e^{-2t} \rangle$. Em $(1, 3, 3)$, $t = 0$ e $\mathbf{r}'(0) = \langle 0, 6, -6 \rangle$. Assim, as equações paramétricas da reta tangente são $x = 1$, $y = 3 + 6t$, $z = 3 - 6t$.

21. $\int_0^1 \left(t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}\right) dt$

$$= \left(\int_0^1 t\,dt\right)\mathbf{i} + \left(\int_0^1 t^2\,dt\right)\mathbf{j} + \left(\int_0^1 t^3\,dt\right)\mathbf{k}$$

$$= \left[\frac{t^2}{2}\right]_0^1 \mathbf{i} + \left[\frac{t^3}{3}\right]_0^1 \mathbf{j} + \left[\frac{t^4}{4}\right]_0^1 \mathbf{k}$$

$$= \tfrac{1}{2}\,\mathbf{i} + \tfrac{1}{3}\,\mathbf{j} + \tfrac{1}{4}\,\mathbf{k}$$

22. $\int_1^2 \left[\left(1 + t^2\right)\mathbf{i} - 4t^4\,\mathbf{j} - \left(t^2 - 1\right)\mathbf{k}\right] dt$

$$= \left[\left(t + \tfrac{1}{3}t^3\right)\mathbf{i} - \tfrac{4}{5}t^5\,\mathbf{j} - \left(\tfrac{1}{3}t^3 - t\right)\mathbf{k}\right]_1^2$$

$$= \left[\left(2 + \tfrac{8}{3}\right)\mathbf{i} - \tfrac{128}{5}\,\mathbf{j} - \left(\tfrac{8}{3} - 2\right)\mathbf{k}\right] - \left[\left(1 + \tfrac{1}{3}\right)\mathbf{i} - \tfrac{4}{5}\,\mathbf{j} - \left(\tfrac{1}{3} - 1\right)\mathbf{k}\right]$$

$$= \tfrac{10}{3}\,\mathbf{i} - \tfrac{124}{5}\,\mathbf{j} - \tfrac{4}{3}\,\mathbf{k}$$

23. $\int_0^{\pi/4} (\cos 2t\,\mathbf{i} + \operatorname{sen} 2t\,\mathbf{j} + t \operatorname{sen} t\,\mathbf{k})\,dt$

$$= \left[\tfrac{1}{2}\operatorname{sen} 2t\,\mathbf{i} - \tfrac{1}{2}\cos 2t\,\mathbf{j}\right]_0^{\pi/4} + \left[\left[-t\cos t\right]_0^{\pi/4} + \int_0^{\pi/4}\cos t\,dt\right]\mathbf{k}$$

$$= \tfrac{1}{2}\,\mathbf{i} + \tfrac{1}{2}\,\mathbf{j} + \left[-\tfrac{\pi}{4}\cos\tfrac{\pi}{4} + \operatorname{sen}\tfrac{\pi}{4}\right]\mathbf{k}$$

$$= \tfrac{1}{2}\,\mathbf{i} + \tfrac{1}{2}\,\mathbf{j} + \tfrac{1}{\sqrt{2}}\left(1 - \tfrac{\pi}{4}\right)\mathbf{k}$$

$$= \tfrac{1}{2}\,\mathbf{i} + \tfrac{1}{2}\,\mathbf{j} + \tfrac{4-\pi}{4\sqrt{2}}\,\mathbf{k}$$