10.2 SOLUÇÕES

Revisão técnica: Ricardo Miranda Martins – IMECC – Unicamp

1. $x = \sqrt{t} - t,\ y = t^3 - t \quad \Rightarrow \quad \dfrac{dy}{dt} = 3t^2 - 1,$

$\dfrac{dx}{dt} = \dfrac{1}{2\sqrt{t}} - 1$, e

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 1}{1/\left(2\sqrt{t}\right) - 1} = \frac{\left(3t^2 - 1\right)\left(2\sqrt{t}\right)}{1 - 2\sqrt{t}}$$

2. $x = t \ln t,\ y = \operatorname{sen}^2 t \quad \Rightarrow \quad \dfrac{dy}{dt} = 2 \operatorname{sen} t \cos t,$

$\dfrac{dx}{dt} = t\left(\dfrac{1}{t}\right) + (\ln t) \cdot 1 = 1 + \ln t$, e

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2 \operatorname{sen} t \cos t}{1 + \ln t}$$

3. $x = t^2 + t,\ y = t^2 - t;\ t = 0.\ \dfrac{dy}{dt} = 2t - 1,\ \dfrac{dx}{dt} = 2t + 1,$

logo $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t - 1}{2t + 1}$. Quando $t = 0,\ x = y = 0$

e $\dfrac{dy}{dx} = -1$. Uma equação da tangente é

$y - 0 = (-1)(x - 0)$ ou $y = -x$.

4. $x = t \operatorname{sen} t,\ y = t \cos t;\ t = \pi.\ \dfrac{dy}{dt} = \cos t - t \operatorname{sen} t,$

$\dfrac{dx}{dt} = \operatorname{sen} t + t \cos t$, e $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\cos t - t \operatorname{sen} t}{\operatorname{sen} t + t \cos t}$.

Quando $t = \pi,\ (x, y) = (0, -\pi)$ e $\dfrac{dy}{dx} = \dfrac{-1}{-\pi} = \dfrac{1}{\pi}$, então

a equação da tangente é $y + \pi = \frac{1}{\pi}(x - 0)$ ou

$y = \frac{1}{\pi}x - \pi$.

5. $x = t^2 + t,\ y = \sqrt{t};\ t = 4.\ \dfrac{dy}{dt} = \dfrac{1}{2\sqrt{t}},\ \dfrac{dx}{dt} = 2t + 1$, logo

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{1}{2\sqrt{t}\,(2t + 1)}$. Quando $t = 4$,

$(x, y) = (20, 2)$ e $\dfrac{dy}{dx} = \dfrac{1}{36}$, então a equação da tangente

é $y - 2 = \frac{1}{36}(x - 20)$ ou $y = \frac{1}{36}x + \frac{13}{9}$.

6. $x = 2 \operatorname{sen} \theta,\ y = 3 \cos \theta;\ \theta = \frac{\pi}{4}.\ \dfrac{dx}{d\theta} = 2 \cos \theta,$

$\dfrac{dy}{d\theta} = -3 \operatorname{sen} \theta,\ \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = -\frac{3}{2} \operatorname{tg} \theta$. Quando $\theta = \frac{\pi}{4}$,

$(x, y) = \left(\sqrt{2}, \frac{3\sqrt{2}}{2}\right)$, e $dy/dx = -\frac{3}{2}$, então a equação da

tangente é $y - \frac{3\sqrt{2}}{2} = -\frac{3}{2}\left(x - \sqrt{2}\right)$ ou $y = -\frac{3}{2}x + 3\sqrt{2}$.

7. (a) $x = 2t + 3,\ y = t^2 + 2t;\ (5, 3).\ \dfrac{dy}{dt} = 2t + 2,\ \dfrac{dx}{dt} = 2,$

e $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = t + 1$. Em $(5, 3),\ t = 1$ e $\dfrac{dy}{dx} = 2$,

então a tangente $y - 3 = 2(x - 5)$ ou $y = 2x - 7$.

(b) $y = t^2 + 2t = \left(\dfrac{x - 3}{2}\right)^2 + 2\left(\dfrac{x - 3}{2}\right)$

$= \dfrac{(x - 3)^2}{4} + x - 3$

logo $\dfrac{dy}{dx} = \dfrac{x - 3}{2} + 1$. Quando $x = 5,\ \dfrac{dy}{dx} = 2$, então

a equação da tangente é $y = 2x - 7$, como anteriormente.

8. (a) $x = 5 \cos t,\ y = 5 \operatorname{sen} t;\ (3, 4).\ \dfrac{dy}{dt} = 5 \cos t,$

$\dfrac{dx}{dt} = -5 \operatorname{sen} t,\ \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = -\operatorname{cotg} t$. Em $(3, 4)$,

$t = \operatorname{tg}^{-1} \dfrac{y}{x} = \operatorname{tg}^{-1} \frac{4}{3}$, logo $\dfrac{dy}{dx} = -\dfrac{3}{4}$, então a equação

da tangente é $y - 4 = -\frac{3}{4}(x - 3)$ ou $y = -\frac{3}{4}x + \frac{25}{4}$.

(b) $x^2 + y^2 = 25$, logo $2x + 2y\dfrac{dy}{dx} = 0$, ou $\dfrac{dy}{dx} = -\dfrac{x}{y}$. Em

$(3, 4),\ \dfrac{dy}{dx} = -\dfrac{3}{4}$, e como na parte (a), uma equação da

tangente é $y = -\frac{3}{4}x + \frac{25}{4}$.

9. (a) $x = 1. - t,\ y = 1 - t^2;\ (1, 1).\ \dfrac{dy}{dt} = -2t,\ \dfrac{dx}{dt} = -1,$

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = 2t$. Em $(1, 1),\ t = 0$, logo $\dfrac{dy}{dx} = 0$,

e a tangente é $y - 1 = 0(x - 1)$ ou $y = 1$.

(b) $y = 1 - t^2 = 1 - \left(1 - x^2\right) = 2x - x^2$, logo

$[dy/dx]_{x=1} = [2 - 2x]_{x-1} = 0$, e como na parte (a)

a tangente é $y = 1$.

10. (a) $x = t^3,\ y = t^2;\ (1, 1).\ \dfrac{dy}{dt} = 2t,\ \dfrac{dx}{dt} = 3t^2$, e

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t}{3t^2} = \dfrac{2}{3t}$ (para $t \neq 0$). Em $(1, 1)$,

temos $t = 1$ e $dy/dx = \frac{2}{3}$, então a equação da tangente

é $y - 1 = \frac{2}{3}(x - 1)$ ou $y = \frac{2}{3}x + \frac{1}{3}$.

(b) $y = x^{2/3}$, logo $dy/dx = \frac{2}{3}x^{-1/3}$. Quando $x = 1$,

$dy/dx = \frac{2}{3}$, então a tangente é $y = \frac{2}{3}x + \frac{1}{3}$ como

anteriormente.

11. $x = t^2 + t,\ y = t^2 + 1.$

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t}{2t + 1} = 1 - \dfrac{1}{2t + 1};$

$\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) = \dfrac{2}{(2t + 1)^2};$

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d\,(dy/dx)\,/dt}{dx/dt} = \dfrac{2}{(2t + 1)^3}$

12. $x = t + 2\cos t,\ y = \text{sen}\ 2t.\ \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2\cos 2t}{1 - 2\,\text{sen}\ t};$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{(1-2\,\text{sen}\ t)(-4\,\text{sen}\ 2t) - 2\cos 2t\,(-2\cos t)}{(1-2\,\text{sen}\ t)^2}$$

$$= \frac{4(\cos t - \text{sen}\ 2t + \text{sen}\ t\ \text{sen}\ 2t)}{(1-2\,\text{sen}\ t)^2};$$

$$\frac{d^2y}{dx^2} = \frac{d(dy/dx)/dt}{dx/dt} = \frac{4(\cos t - \text{sen}\ 2t + \text{sen}\ t\ \text{sen}\ 2t)}{(1-2\,\text{sen}\ t)^3}$$

13. $x = t^4 - 1,\ y = t - t^2 \quad \Rightarrow \quad \dfrac{dy}{dt} = 1 - 2t,\ \dfrac{dx}{dt} = 4t^3,$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-2t}{4t^3} = \tfrac{1}{4}t^{-3} - \tfrac{1}{2}t^{-2};$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = -\tfrac{3}{4}t^{-4} + t^{-3},$$

$$\frac{d^2y}{dx^2} = \frac{d(dy/dx)/dt}{dx/dt} = \frac{-\tfrac{3}{4}t^{-4} + t^{-3}}{4t^3} \cdot \frac{4t^4}{4t^4} = \frac{-3+4t}{16t^7}.$$

14. $x = t^3 + t^2 + 1,\ y = 1 - t^2.\ \dfrac{dy}{dt} = -2t,$

$$\frac{dx}{dt} = 3t^2 + 2t;\ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2t}{3t^2 + 2t} = -\frac{2}{3t+2};$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{6}{(3t+2)^2};$$

$$\frac{d^2y}{dx^2} = \frac{d(dy/dx)/dt}{dx/dt} = \frac{6}{t(3t+2)^3}.$$

15. $x = \text{sen}\ \pi t,\ y = \cos \pi t.$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\pi\ \text{sen}\ \pi t}{\pi \cos \pi t} = -\text{tg}\ \pi t;$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d(dy/dx)/dt}{dx/dt} = \frac{-\pi \sec^2 \pi t}{\pi \cos \pi t}$$

$$= -\sec^3 \pi t.$$

16. $x = 1 + \text{tg}\ t,\ y = \cos 2t \quad \Rightarrow \quad \dfrac{dy}{dt} = -2\,\text{sen}\ 2t,$

$$\frac{dx}{dt} = \sec^2 t,$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\,\text{sen}\ 2t}{\sec^2 t} = -4\,\text{sen}\ t\cos t \cdot \cos^2 t$$

$$= -4\,\text{sen}\ t\cos^3 t;$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = -4\,\text{sen}\ t\,\left(3\cos^2 t\right)(-\text{sen}\ t) - 4\cos^4 t$$

$$= 12\,\text{sen}^2 t\cos^2 t - 4\cos^4 t,$$

$$\frac{d^2y}{dx^2} = \frac{d(dy/dx)/dt}{dx/dt} = \frac{4\cos^2 t\left(3\,\text{sen}^2 t - \cos^2 t\right)}{\sec^2 t}$$

$$= 4\cos^4 t\left(3\,\text{sen}^2 t - \cos^2 t\right).$$

17. $x = e^{-t},\ y = te^{2t}.$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{(2t+1)e^{2t}}{-e^{-t}} = -(2t+1)e^{3t};$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = -3(2t+1)e^{3t} - 2e^{3t} = -(6t+5)e^{3t};$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d(dy/dx)/dt}{dx/dt} = \frac{-(6t+5)e^{3t}}{-e^{-t}}$$

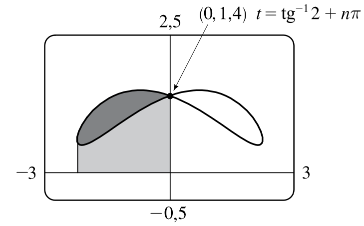
$$= (6t+5)e^{4t}.$$

18. $x = 1 + t^2,\ y = t\ln t.\ \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{1+\ln t}{2t};$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{2t(1/t) - (1+\ln t)2}{(2t)^2} = -\frac{\ln t}{2t^2};$$

$$\frac{d^2y}{dx^2} = \frac{d(dy/dx)/dt}{dx/dt} = -\frac{\ln t}{4t^3}.$$

19.



O gráfico de $x = \text{sen}\ t - 2\cos t,\ y = 1 + \text{sen}\ t\cos t$ é simétrico em torno do eixo y. O gráfico intersecta o eixo y quando $x = 0 \Rightarrow \text{sen}\ t - 2\cos t = 0 \Rightarrow \text{sen}\ t = 2\cos t \Rightarrow \text{tg}\ t = 2 \Rightarrow t = \text{tg}^{-1} 2 + n\pi$. O ciclo esquerdo é traçado no sentido horário de $t = \text{tg}^{-1} 2 - \pi$ para $t = \text{tg}^{-1} 2$, então a área do ciclo é dada (como no Exemplo 4) por

$$A = \int_{\text{tg}^{-1} 2 - \pi}^{\text{tg}^{-1} 2} y\,dx$$

$$\approx \int_{-2{,}0344}^{1{,}1071} (1 + \text{sen}\ t\cos t)(\cos t + 2\,\text{sen}\ t)\,dt$$

$$\approx 0{,}8944$$

Esta integral pode ser avaliada exatamente, seu valor é $\frac{2}{5}\sqrt{5}$.

20. $L = \int_0^1 \sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt$

e $dx/dt = 3t^2,\ dy/dt = 4t^3 \quad \Rightarrow$

$$L = \int_0^1 \sqrt{9t^4 + 16t^6}\,dt = \int_0^1 t^2\sqrt{9 + 16t^2}\,dt$$

21. $L = \int_0^2 \sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt$ e $dx/dt = 2t,$

$dy/dt = 4$, então $L = \int_0^2 \sqrt{4t^2 + 16}\,dt = 2\int_0^2 \sqrt{t^2 + 4}\,dt.$

22. $\dfrac{dx}{dt} = \text{sen}\ t + t\cos t$ e $\dfrac{dy}{dt} = \cos t - t\,\text{sen}\ t \quad \Rightarrow$

$$L = \int_0^{\pi/2} \sqrt{(\text{sen}\ t + t\cos t)^2 + (\cos t - t\,\text{sen}\ t)^2}\,dt$$

$$= \int_0^{\pi/2} \sqrt{1 + t^2}\,dt$$

23. $dx/dt = -e^{-t}$ e $dy/dt = e^{2t} + 2te^{2t} = e^{2t}(1 + 2t)$, então

$$L = \int_{-1}^{1} \sqrt{e^{-2t} + e^{4t}(1+2t)^2}\,dt.$$

24. $x = t^3,\ y = t^2,\ 0 \le t \le 4.$

$$(dx/dt)^2 + (dy/dt)^2 = \left(3t^2\right)^2 + (2t)^2 = 9t^4 + 4t^2.$$

$$L = \int_0^4 \sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt$$

$$= \int_0^4 \sqrt{9t^4 + 4t^2}\,dt = \int_0^4 t\sqrt{9t^2 + 4}\,dt$$

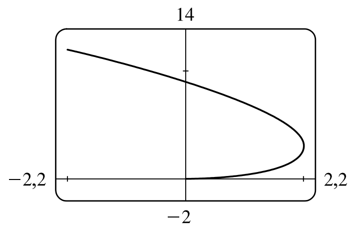
$$= \tfrac{1}{18}\int_4^{148} \sqrt{u}\,du \quad (\text{onde } u = 9t^2 + 4)$$

$$= \tfrac{1}{18}\left(\tfrac{2}{3}\right)\left[u^{3/2}\right]_4^{148} = \tfrac{1}{27}\left(148^{3/2} - 4^{3/2}\right)$$

$$= \tfrac{8}{27}\left(37^{3/2} - 1\right)$$

25. $x = 3t - t^3, y = 3t^2, 0 \le t \le 2$.

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(3 - 3t^2\right)^2 + (6t)^2$$
$$= 9\left(1 + 2t^2 + t^4\right) = \left[3\left(1 + t^2\right)\right]^2$$
$$L = \int_0^2 3\left(1 + t^2\right) dt = \left[3t + t^3\right]_0^2 = 14$$



26. $x = 2 - 3\,\text{sen}^2\,\theta, y = \cos 2\theta, 0 \le \theta \le \frac{\pi}{2}$.

$$(dx/d\theta)^2 + (dy/d\theta)^2 = (-6\,\text{sen}\,\theta\cos\theta)^2 + (-2\,\text{sen}\,2\theta)^2$$
$$= (-3\,\text{sen}\,2\theta)^2 + (-2\,\text{sen}\,2\theta)^2$$
$$= 13\,\text{sen}^2\,2\theta$$
$$\Rightarrow$$
$$L = \int_0^{\pi/2} \sqrt{13}\,\text{sen}\,2\theta d\theta = \left[-\frac{\sqrt{13}}{2}\cos 2\theta\right]_0^{\pi/2}$$
$$= -\frac{\sqrt{13}}{2}(-1-1) = \sqrt{13}$$

27. $x = 1 + 2\,\text{sen}\,\pi t, y = 3 - 2\cos\pi t, 0 \le t \le 1$.

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (2\pi\cos\pi t)^2 + (2\pi\,\text{sen}\,\pi t)^2 = 4\pi^2$$
$$\Rightarrow \quad L = \int_0^1 \sqrt{(dx/dt)^2 + (dy/dt^2)}\,dt = \int_0^1 2\pi\,dt = 2\pi$$

28. $x = 5t^2 + 1, y = 4 - 3t^2, 0 \le t \le 2$.

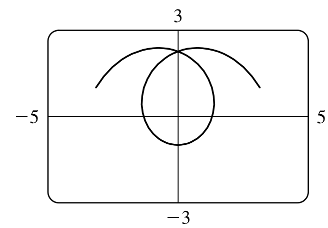
$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (10t)^2 + (-6t)^2 = 136t^2 \Rightarrow$$
$$L = \int_0^2 \sqrt{136t^2}\,dt = \int_0^2 \sqrt{136}\,t\,dt$$
$$= \left[\tfrac{1}{2}\cdot 2\sqrt{34}\,t^2\right]_0^2 = 4\sqrt{34}$$

29. $x = e^t\cos t, y = e^t\,\text{sen}\,t, 0 \le t \le \pi$.

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left[e^t(\cos t - \text{sen}\,t)\right]^2$$
$$+ \left[e^t(\cos t + \text{sen}\,t)\right]^2$$
$$= e^{2t}\left(2\cos^2 t + 2\,\text{sen}^2\,t\right) = 2e^{2t}$$
$$\Rightarrow \quad L = \int_0^\pi \sqrt{2}\,e^t\,dt = \sqrt{2}\left(e^\pi - 1\right)$$

30. $x = t\cos t + \text{sen}\,t, y = t\,\text{sen}\,t - \cos t, -\pi \le t \le \pi$.
$dx/dt = -t\,\text{sen}\,t + 2\cos t$ e $dy/dt = t\cos t + 2\,\text{sen}\,t$, assim
$(dx/dt)^2 + (dy/dt)^2 = t^2\,\text{sen}^2\,t - 4t\,\text{sen}\,t\cos t +$
$4\cos^2 t + t^2\cos^2 t + 4t\,\text{sen}\,t\cos t + 4\,\text{sen}^2\,t = t^2 + 4$ e

$$L = \int_{-\pi}^{\pi} \sqrt{t^2 + 4}\,dt = 2\int_0^\pi \sqrt{t^2 + 4}\,dt$$
$$\overset{21}{=} 2\left[\tfrac{1}{2}t\sqrt{t^2 + 4} + 2\ln\left(t + \sqrt{t^2 + 4}\right)\right]_0^\pi$$
$$= 2\left[\tfrac{\pi}{2}\sqrt{\pi^2 + 4} + 2\ln\left(\pi + \sqrt{\pi^2 + 4}\right) - 2\ln 2\right]$$
$$= \pi\sqrt{\pi^2 + 4} + 4\ln\left(\pi + \sqrt{\pi^2 + 4}\right) - 4\ln 2$$
$$\approx 16{,}633506$$



31. $x = \ln t$ e $y = e^{-t} \quad \Rightarrow \quad \dfrac{dx}{dt} = \dfrac{1}{t}$ e $\dfrac{dy}{dt} = -e^{-t} \quad \Rightarrow$
$L = \int_1^2 \sqrt{t^{-2} + e^{-2t}}\,dt$. Utilizando a Regra de Simpson com
$n = 10, \Delta x = (2-1)/10 = 0{,}1$ e $f(t) = \sqrt{t^{-2} + e^{-2t}}$
obtemos $L \approx \frac{0{,}1}{3}\,[f(1{,}0) + 4f(1{,}1) + 2f(1{,}2) +$
$\cdots + 2f(1{,}8) + 4f(1{,}9) + f(2{,}0)] \approx 0{,}7314$

32. $x = t^3$ e $y = t^4 \quad \Rightarrow \quad dx/dt = 3t^2$ e $dy/dt = 4t^3$.
Logo $S = \int_0^1 2\pi t^4\sqrt{9t^4 + 16t^6}\,dt = \int_0^1 2\pi t^6\sqrt{9 + 16t^2}\,dt$.

33. $$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(2t - \frac{2}{t^2}\right)^2 + \left(\frac{4}{\sqrt{t}}\right)^2$$
$$= 4t^2 + \frac{8}{t} + \frac{4}{t^4} = 4\left(t + \frac{1}{t^2}\right)^2$$
$$S = \int_1^9 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$
$$= 2\pi\int_1^9 \left(8\sqrt{t}\right)2\left(t + \frac{1}{t^2}\right)dt$$
$$= 32\pi\int_1^9 \left(t^{3/2} + t^{-3/2}\right)dt = 32\pi\left[\tfrac{2}{5}t^{5/2} - 2t^{-1/2}\right]_1^9$$
$$= 32\pi\left\{\left[\tfrac{2}{5}(243) - 2\left(\tfrac{1}{3}\right)\right] - \left[\tfrac{2}{5}(1) - 2(1)\right]\right\}$$
$$= \frac{47\,104}{15}\pi$$

34. $x = e^t\cos t, y = e^t\,\text{sen}\,t, 0 \le t \le \frac{\pi}{2}$.

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left[e^t(\cos t - \text{sen}\,t)\right]^2$$
$$+ \left[e^t(\cos t + \text{sen}\,t)\right]^2$$
$$= e^{2t}\left(2\cos^2 t + 2\,\text{sen}^2\,t\right) = 2e^{2t}\text{, assim}$$
$$S = \int_0^{\pi/2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$
$$= \int_0^{\pi/2} 2\pi e^t\,\text{sen}\,t\sqrt{2}e^t\,dt \overset{98}{=} 2\sqrt{2}\pi\int_0^{\pi/2} e^{2t}\,\text{sen}\,t\,dt$$
$$= \left[2\sqrt{2}\pi\frac{e^{2t}}{5}(2\,\text{sen}\,t - \cos t)\right]_0^{\pi/2}$$
$$= \frac{2\sqrt{2}}{5}\pi\left[2e^\pi - (-1)\right] = \frac{2\sqrt{2}\pi}{5}\left(2e^\pi - 1\right)$$

35. $\left(\dfrac{dx}{d\theta}\right)^2 + \left(\dfrac{dy}{d\theta}\right)^2$

$$= (-2\,\text{sen}\,\theta + 2\,\text{sen}\,2\theta)^2 + (2\cos\theta - 2\cos 2\theta)^2$$
$$= 4\left[\left(\text{sen}^2\,\theta - 2\,\text{sen}\,\theta\,\text{sen}\,2\theta + \text{sen}^2\,2\theta\right) + \left(\cos^2\theta - 2\cos\theta\cos 2\theta + \cos^2 2\theta\right)\right]$$
$$= 4\left[1 + 1 - 2\left(\cos 2\theta\cos\theta + \text{sen}\,2\theta\,\text{sen}\,\theta\right)\right]$$
$$= 8\left[1 - \cos(2\theta - \theta)\right] = 8(1 - \cos\theta)$$

Observe que $x(2\pi - \theta) = x(\theta)$ e $y(2\pi - \theta) = -y(\theta)$, logo a parte da curva de $\theta = 0$ a $\theta = \pi$ gera a mesma superfície que a parte de $\theta = \pi$ a $\theta = 2\pi$.

Além disto, $y = 2\,\text{sen}\,\theta - \text{sen}\,2\theta = 2\,\text{sen}\,\theta\,(1 - \cos\theta)$. Logo

$$S = \int_0^\pi 2\pi \cdot 2\,\text{sen}\,\theta\,(1 - \cos\theta)\,2\sqrt{2}\sqrt{1 - \cos\theta}\,d\theta$$
$$= 8\sqrt{2}\pi\int_0^\pi (1 - \cos\theta)^{3/2}\,\text{sen}\,\theta\,d\theta$$
$$= 8\sqrt{2}\pi\int_0^2 \sqrt{u^3}\,du \quad (u = 1 - \cos\theta,\ du = \text{sen}\,\theta\,d\theta)$$
$$= \left[8\sqrt{2}\pi\left(\tfrac{2}{5}u^{5/2}\right)\right]_0^2$$
$$= \frac{128\pi}{5}$$