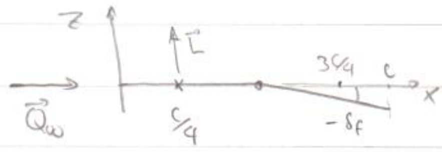
P5.3 Find the Hinge moment of the flapped airfoil of Eq. (5.90)

$$\frac{dy_c}{dx} = \begin{cases} 0 & \text{for } 0 < x < kc \\ -\delta_f & \text{for } kc < x < c \end{cases}$$



$\alpha = 0 \Rightarrow$ from eqs. (5.94) and (5.96)

$$\begin{cases} C_l = \left[2(\pi - \theta_k) + 2\sin\theta_k\right]\delta_f \\ C_{m_{c/4}} = \left[\frac{1}{4}\sin(2\theta_k) - \frac{1}{2}\sin\theta_k\right]\delta_f \end{cases}$$

where $kc = \frac{c}{2}(1 - \cos\theta_k) \Rightarrow \cos\theta_k = 1 - 2k$

The hinge moment: ($C_{m_{kc}}$) should be equal to the moment about the aerodinamic center, which does not depend on α and where the Lift force acts, for this torque is transferred to the hinge point (kc) without any change — plus the moment of the lift force with respect to the hinge point:

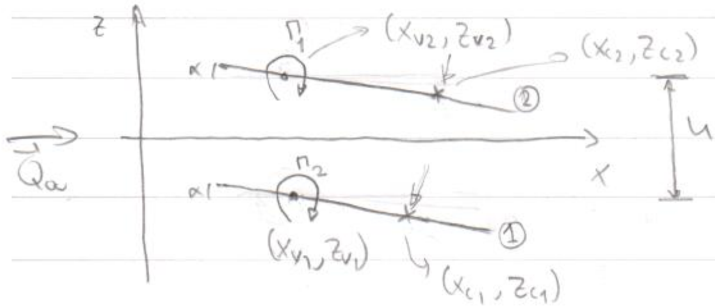
$$M_{kc} = M_{c/4} + L \cdot \left(kc - \frac{c}{4}\right)$$

$$C_{m_{kc}} = C_{m_{c/4}} + \frac{C_l}{c}\left(kc - \frac{c}{4}\right)$$

$$C_{m_{kc}} = C_{m_{c/4}} + C_l\left(k - 1/4\right)$$

$$C_{m_{kc}} = \left\{\frac{1}{4}\sin(2\theta_k) - \frac{1}{2}\sin\theta_k + \left(k - \frac{1}{4}\right)\left[2(\pi - \theta_k) + 2\sin\theta_k\right]\right\}\delta_f$$

P.5.5. $Q_\infty, \alpha \Rightarrow$ Biplane



$$u\Big|_{(x,z)} = \frac{\Gamma}{2\pi}\frac{(z-z_0)}{\left[(z-z_0)^2+(x-x_0)^2\right]}$$

$$w\Big|_{(x,z)} = -\frac{\Gamma}{2\pi}\frac{(x-x_0)}{\left[(z-z_0)^2+(x-x_0)^2\right]}$$

Vortex points: $\begin{cases} (1) \Rightarrow (x_{v_1}, z_{v_1}) = (0, -h/2) \\ (2) \Rightarrow (x_{v_2}, z_{v_2}) = (0, +h/2) \end{cases}$

Collocation Points: $\begin{cases} (1) \Rightarrow (x_{c_1}, z_{c_1}) = \left(\frac{c}{2}\cos\alpha, -\frac{(h+c\sin\alpha)}{2}\right) \\ (2) \Rightarrow (x_{c_2}, z_{c_2}) = \left(\frac{c}{2}\cos\alpha, \frac{(h-c\sin\alpha)}{2}\right) \end{cases}$

$\hat{n}_1 \parallel \hat{n}_2 = \sin\alpha\,\hat{\imath} + \cos\alpha\,\hat{k}$

Self-induced velocities at the collocation points are normal to each chord:

$$\begin{cases} V_{n_{11}} = -\frac{\Gamma_1}{\pi c} \\ V_{n_{22}} = -\frac{\Gamma_2}{\pi c} \end{cases}$$

Mutually induced velocities are given by (at the collocation points)

$$u_{12} = \frac{\Gamma_1}{2\pi}\frac{\left(h-\frac{c\sin\alpha}{2}\right)}{\left[\left(h-\frac{c\sin\alpha}{2}\right)^2+\frac{c^2}{4}\cos^2\alpha\right]} = \frac{\Gamma_1}{2\pi}\frac{2h-c\sin\alpha}{2\left[h^2-ch\sin\alpha+c^2/4\right]}$$

$$w_{12} = -\frac{\Gamma_1}{2\pi}\frac{c\cos\alpha}{2\left[h^2-ch\sin\alpha+c^2/4\right]}$$

$$v_{n12} = u_{12} \cdot \sin\alpha + w_{12} \cos\alpha$$

$$v_{n12} = \frac{\Gamma_1}{4\pi}\left[\frac{(2h - c\sin\alpha)\sin\alpha}{(h^2 - ch\sin\alpha + c^2/4)} - \frac{c\cos^2\alpha}{(h^2 - ch\sin\alpha + c^2/4)}\right]$$

$$v_{n12} = \frac{\Gamma_1}{4\pi} \frac{(2h\sin\alpha - c)}{(h^2 - ch\sin\alpha + c^2/4)} \checkmark$$

Similarly, we can compute v_{n21}

$$u_{21} = \frac{\Gamma_2}{2\pi} \frac{-\left(h + \frac{c\sin\alpha}{2}\right)}{\left[\left(h + \frac{c\sin\alpha}{2}\right)^2 + \frac{c^2}{4}\cos^2\alpha\right]} = -\frac{\Gamma_2}{4\pi} \frac{(2h + c\sin\alpha)}{(h^2 + ch\sin\alpha + c^2/4)} \checkmark$$

$$w_{21} = -\frac{\Gamma_2}{4\pi} \frac{c\cos\alpha}{(h^2 + ch\sin\alpha + c^2/4)} \checkmark$$

$$v_{n21} = u_{21}\sin\alpha + w_{21}\cos\alpha$$

$$v_{n21} = \frac{\Gamma_2}{4\pi}\left[\frac{-(2h + c\sin\alpha)\sin\alpha - c\cos\alpha\cos\alpha}{(h^2 + ch\sin\alpha + c^2/4)}\right]$$

$$v_{n21} = +\frac{\Gamma_2}{4\pi} \frac{(-2h\sin\alpha - c)}{(h^2 + ch\sin\alpha + c^2/4)} = -\frac{\Gamma_2}{4\pi} \frac{(2h\sin\alpha + c)}{(h^2 + ch\sin\alpha + c^2/4)} \checkmark$$

The boundary conditions at the collocation points comprise the set:

$$\begin{cases} v_{n11} + v_{n21} + Q_\infty \sin\alpha = 0 \\ v_{n22} + v_{n12} + Q_\infty \sin\alpha = 0 \end{cases}$$

$$\begin{cases} -\dfrac{\Gamma_1}{\pi c} - \dfrac{(c+2h\sin\alpha)}{\pi(4h^2+4ch\sin\alpha+c^2)}\Gamma_2 = -Q_\infty \sin\alpha \\ \dfrac{(2h\sin\alpha-c)\,\Gamma_1}{\pi(4h^2-4ch\sin\alpha+c^2)} - \dfrac{\Gamma_2}{\pi c} = -Q_\infty\sin\alpha \end{cases}$$

On having "Mathematica" solve this set, we get for the circulations: (+)

$$\Gamma_1 = \pi Q_\infty \sin(\alpha)\, c\, \frac{[8h^3+2c^2h\cos(2\alpha)+c(c^2-4h^2)\sin(\alpha)]}{h[c^2+8h^2+3c^2\cos(2\alpha)]}$$

$$\Gamma_2 = -\pi Q_\infty \sin(\alpha)\, c\, \frac{[8h^3+2c^2h\cos(2\alpha)-c(c^2-4h^2)\sin(\alpha)]}{h[c^2+8h^2+3c^2\cos(2\alpha)]}$$

And, on accounting for the fact that $\alpha \ll 1$, it implies the following assumptions:

$$\left.\begin{array}{l}\sin\alpha \simeq \alpha \\ \cos\alpha \simeq 1\end{array}\right\} \Rightarrow \begin{array}{l}\sin^2\alpha \simeq \alpha^2 \to 0, \quad \sin^3\alpha \simeq 0 \\ \cos(2\alpha) = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1 \simeq 1\end{array}$$

which lead to the following small disturbance expressions for the circulations:

$$\Gamma_1 \simeq \pi\alpha Q_\infty c \frac{(c^2+4h^2)}{2(c^2+2h^2)}$$

$$\Gamma_2 \simeq \pi\alpha Q_\infty c \frac{(c^2+4h^2)}{2(c^2+2h^2)}$$

Having found the sdt values for the circulations, it is easy to get the velocities each vortex induces on the other; in the normal direction, which in this case is the x direction:

$$u_{1v2v} = \frac{\Gamma_1}{2\pi h} \cong Q_\infty \alpha c \frac{(c^2+4h^2)}{4h(c^2+2h^2)}$$

$$u_{2v1v} = \frac{-\Gamma_2}{2\pi h} \cong -Q_\infty \alpha c \frac{(c^2+4h^2)}{4h(c^2+2h^2)}$$

On substituting these results into the generalized form of the Kutta-Joukowski theorem, we get:

$$L = \rho Q_\infty \Gamma \left(1 + \frac{\vec{Q}_\infty \cdot \vec{q}_I}{Q_\infty^2} \right) \quad , \text{where } \vec{Q}_\infty \cdot \vec{q}_I = u_{1v2v}$$

$$L_1 = \pi \rho Q_\infty \alpha c \frac{(c^2+4h^2)}{(c^2+2h^2)} \left[-\frac{\alpha c (c^2+4h^2)}{8h(c^2+2h^2)} + \frac{Q_\infty}{2} \right]$$

$$L_2 = \pi \rho Q_\infty \alpha c \frac{(c^2+4h^2)}{(c^2+2h^2)} \left[+\frac{\alpha c (c^2+4h^2)}{8h(c^2+2h^2)} + \frac{Q_\infty}{2} \right]$$

$$L_T = L_1 + L_2 = \pi \rho Q_\infty^2 \alpha c \frac{(c^2+4h^2)}{(c^2+2h^2)}$$

It is worth noting that the lift for a single flat plate is given by: $L_{fp} = \pi \rho Q_\infty^2 \alpha c$; and the ratio L_T / L_{fp} yields

$$\frac{L_T}{L_{fp}} = 2 - \frac{c^2}{c^2+2h^2}$$

which implies that the influence each plate exerts on one another lessens the total lift from what it could be, provided that the plates are farther appart: $h \to \infty$