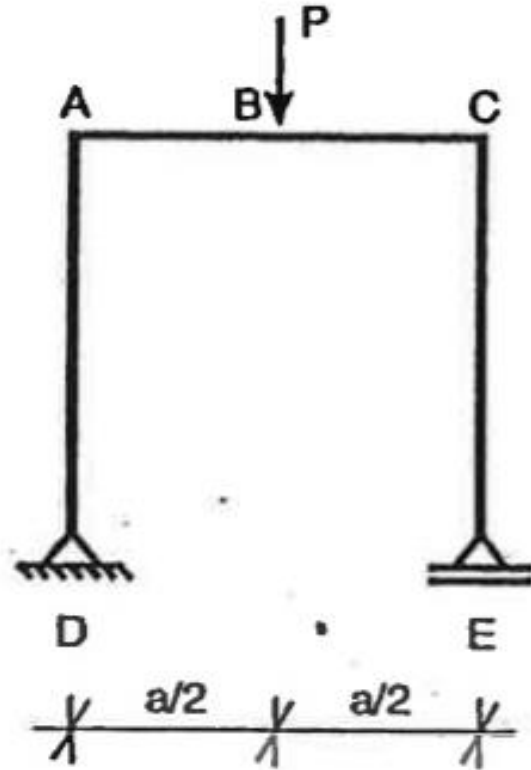


PEF3200
Aula 10
7 jun
prof. NAKAO

❖ **Pórticos triarticulados.**

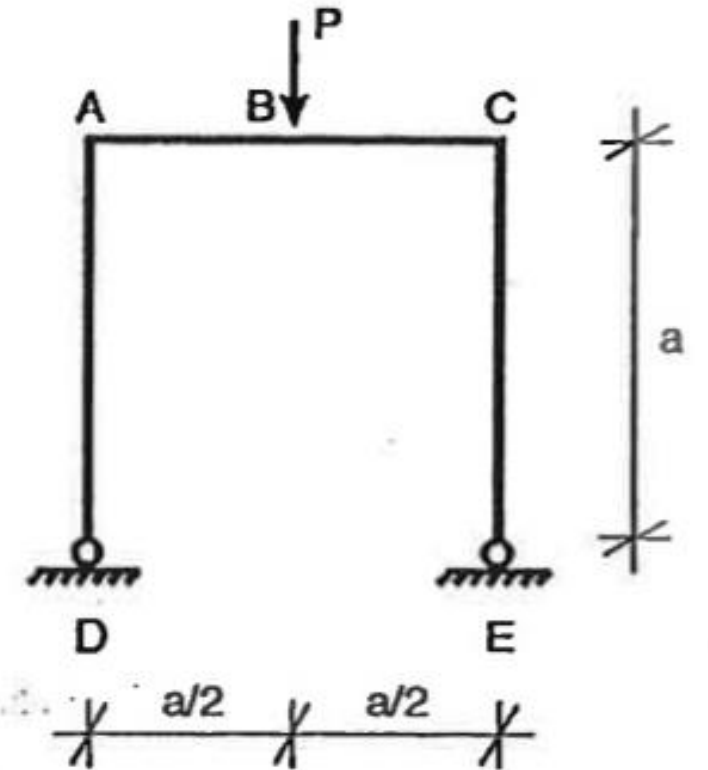
PÓRTICOS TRIARTICULADOS

APOSTILA
CAPÍTULO 7
PÁGINA 122



(a)

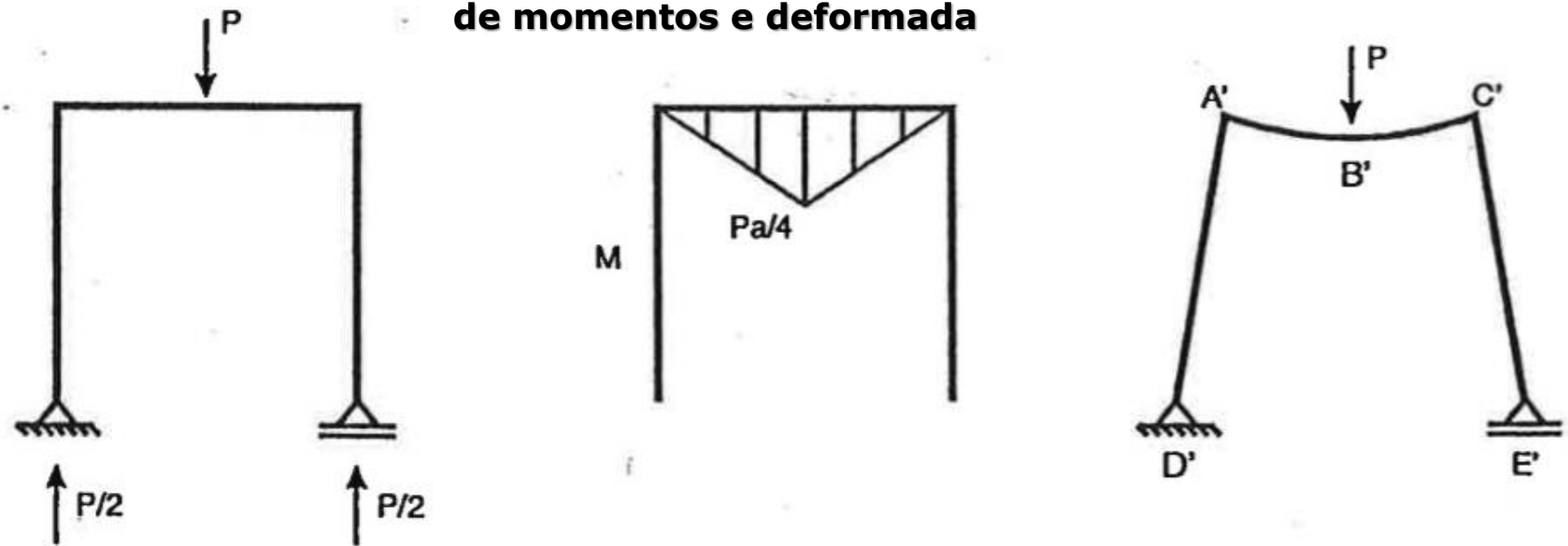
PÓRTICO SIMPLEMENTE APOIADO



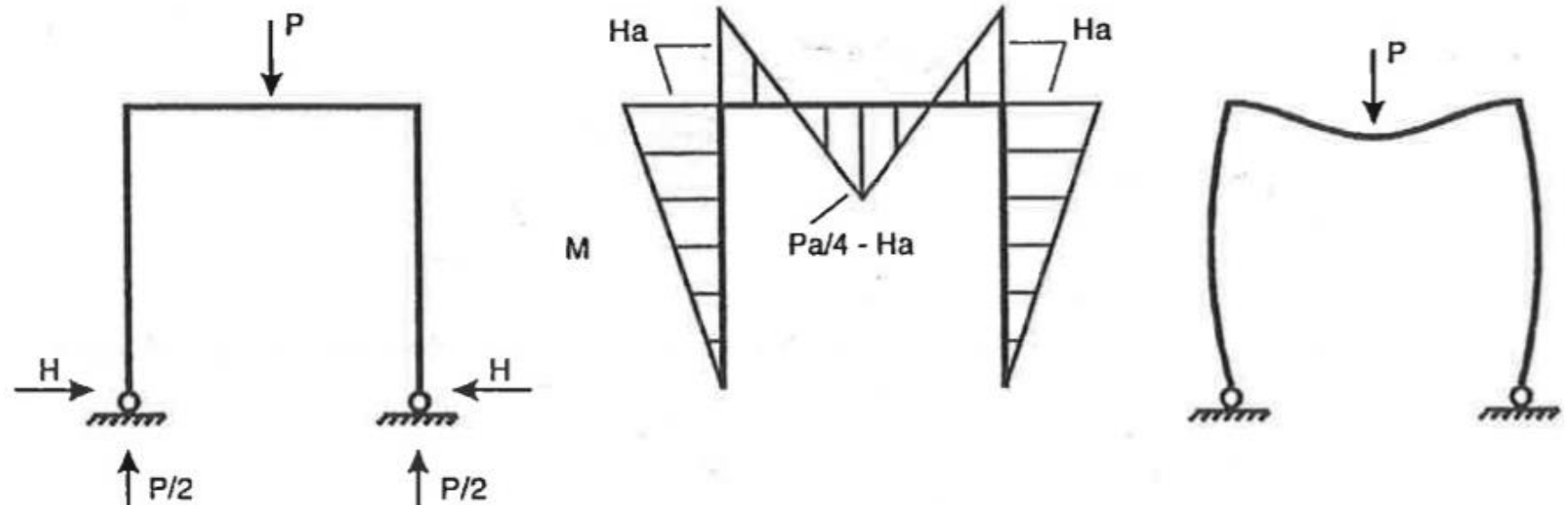
(b)

PÓRTICO BIARTICULADO

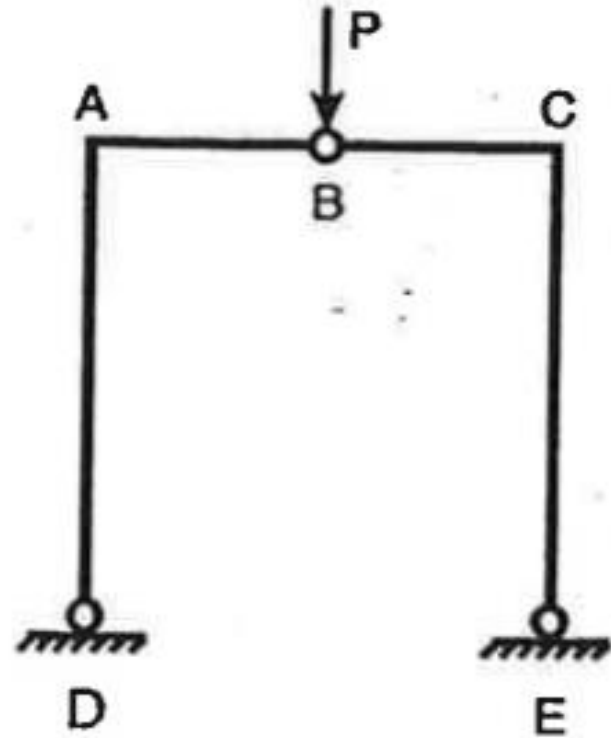
PÓRTICO SIMPLEMENTE APOIADO: diagrama de momentos e deformada



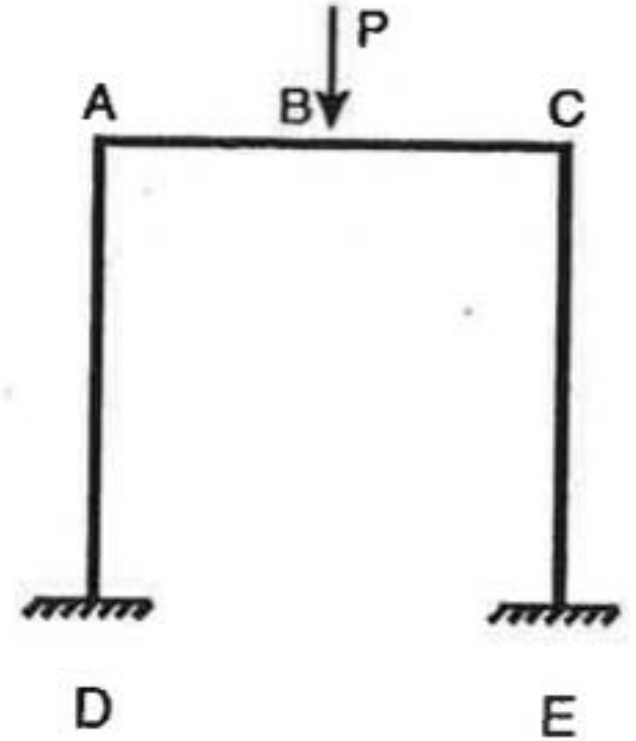
PÓRTICO BIARTICULADO: diagrama de momentos e deformada



APOSTILA
CAPÍTULO 7
PÁGINA 127



(a)

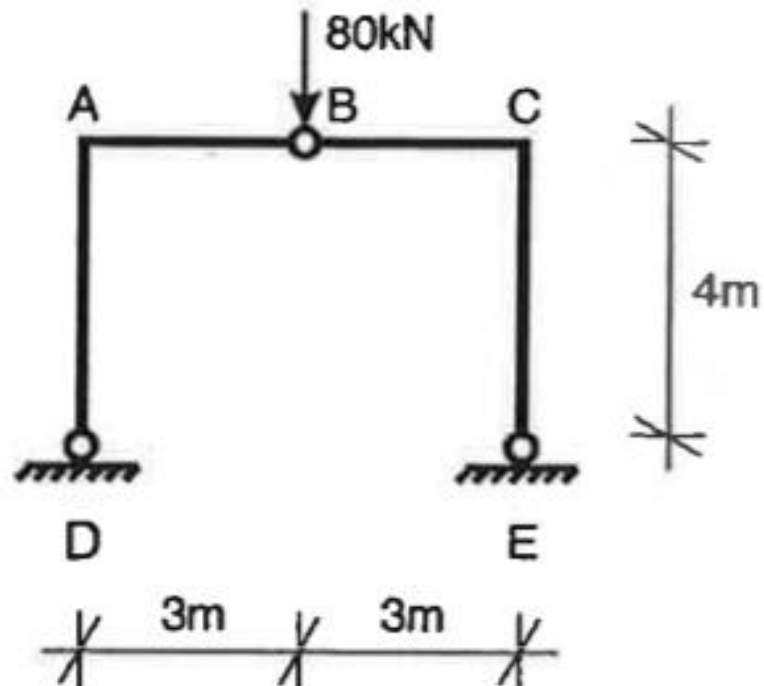


(b)

APOSTILA
CAPÍTULO 7
PÁGINA 137

Exemplo 8.1

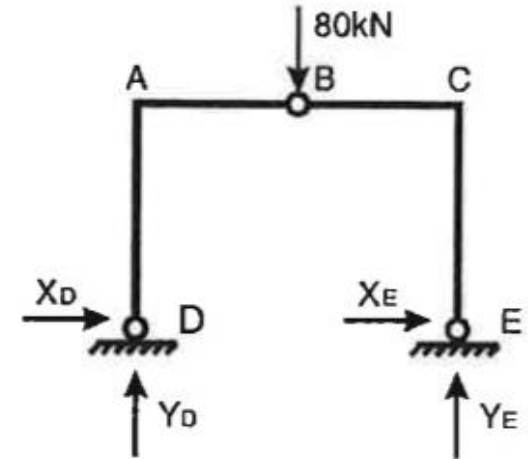
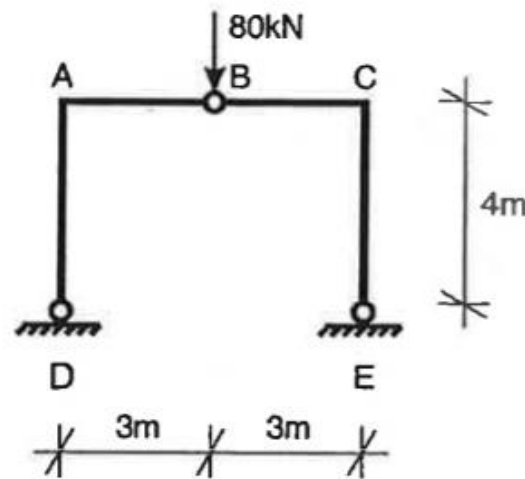
Traçar os diagramas de esforços solicitantes do pórtico triarticulado da Figura 8. 21.



Exemplo 8.1

Traçar os diagramas de esforços solicitantes do pórtico triarticulado da Figura 8. 21.

APOSTILA CAPÍTULO 7 PÁGINA 137

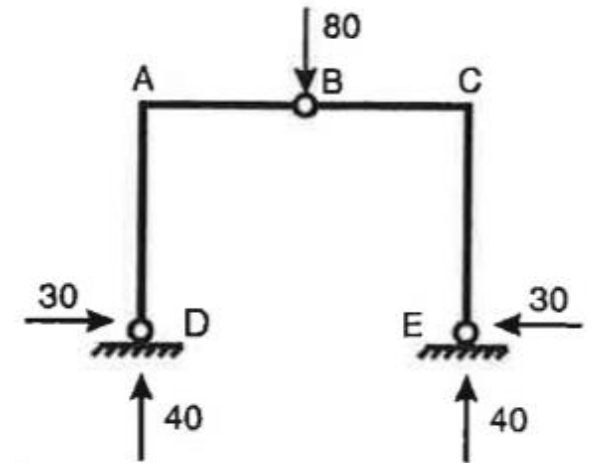


$$1. \sum M_D = 0 = -80 \cdot 3 + Y_E \cdot 6 \rightarrow Y_E = 40 \text{ kN}$$

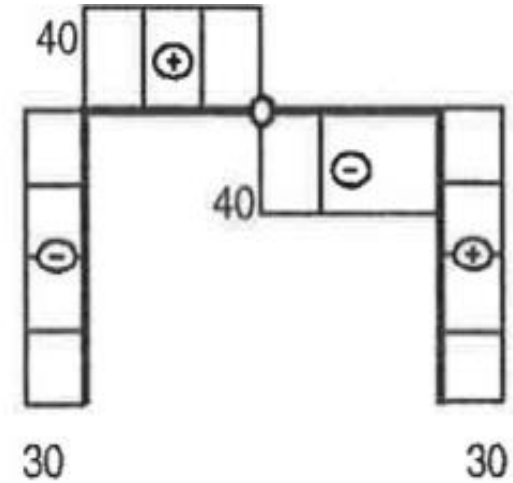
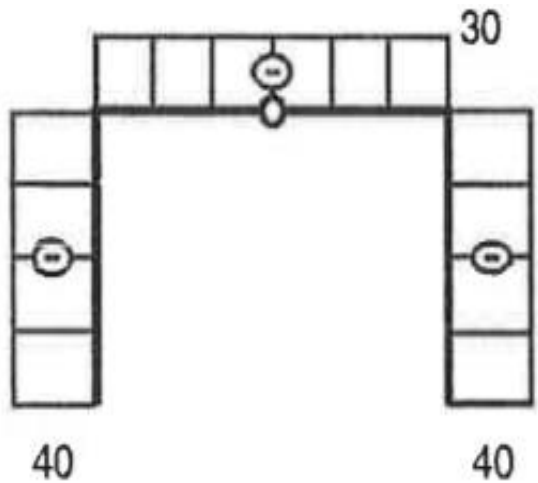
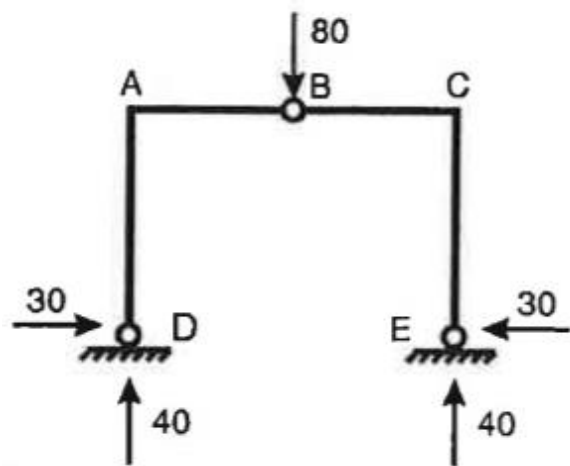
$$2. \sum Y = 0 = Y_D - 80 + Y_E \rightarrow Y_D = 40 \text{ kN}$$

$$3. \sum M_{fletor}^{B, esquerda} = 0 = +X_D \cdot 4 - Y_D \cdot 3 \text{ e } Y_D = 40 \text{ kN} \Rightarrow X_D = 30 \text{ kN}$$

$$4. \sum X = 0 = X_D + X_E \text{ e } X_D = 30 \text{ kN} \Rightarrow X_E = -30 \text{ kN}$$

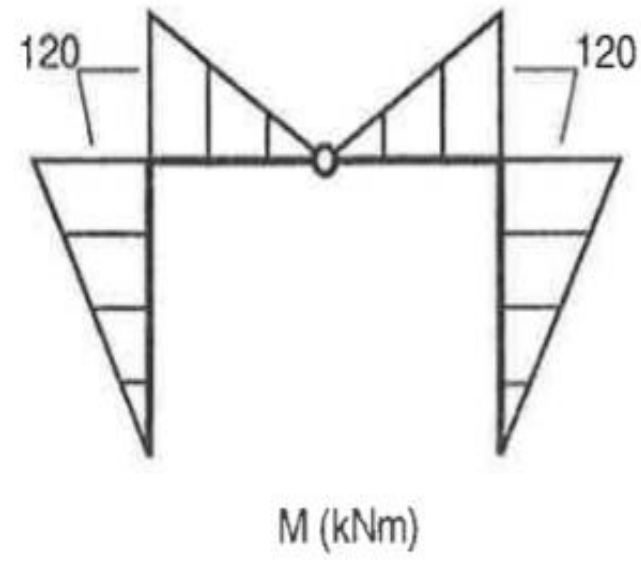


$$5. \text{verificação: } \sum M_A = X_D \cdot 4 - 80 \cdot 3 + X_E \cdot 4 + Y_E \cdot 6 \text{ e } X_D = 30 \text{ kN}, Y_E = 40 \text{ kN}, X_E = -30 \text{ kN} = 0 \text{ (OK!)}$$



N (kN)

V (kN)



**PÓRTICO TRIARTICULADO:
diagrama dos esforços solicitantes**

**APOSTILA
CAPÍTULO 7
PÁGINA 140**

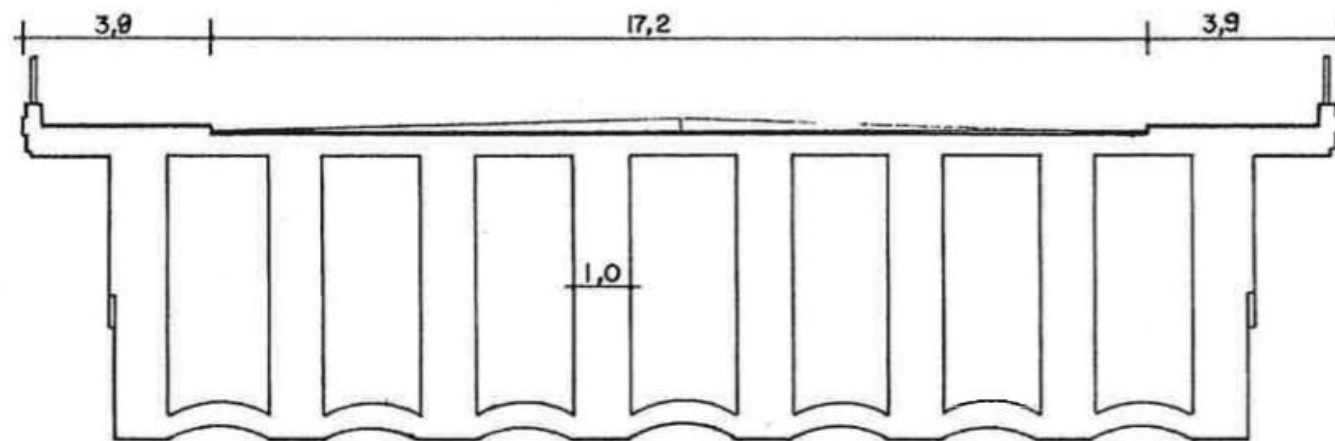


VIADUTO DO CHÁ

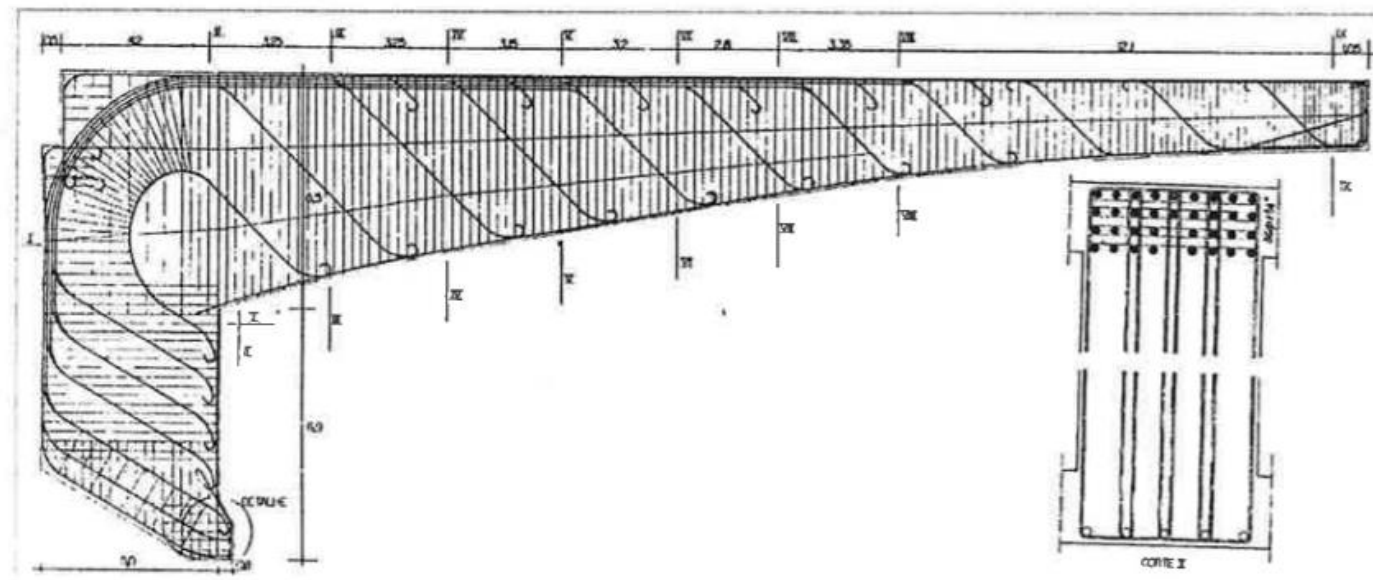


VIADUTO DO CHÁ

APOSTILA
CAPÍTULO 7
PÁGINA 133



(a)



(b)

VIADUTO DO CHÁ

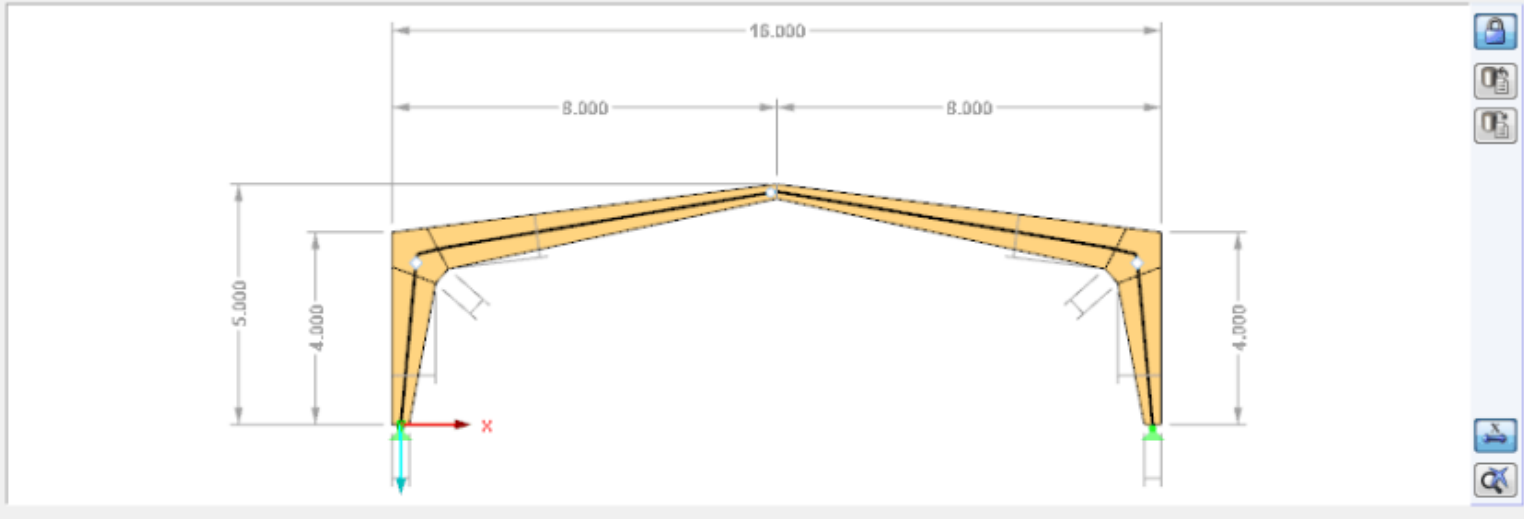
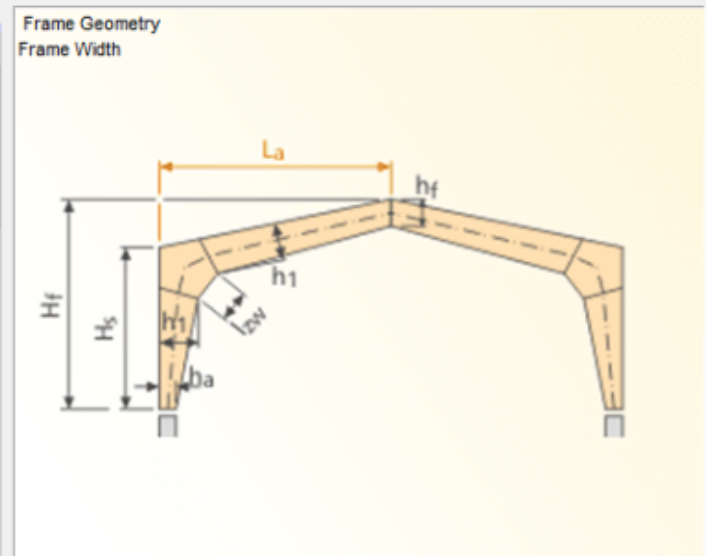


VIADUTO DO CHÁ

- Input Data
 - Frame Type and Material
 - Geometry**
 - Loads
 - Control Parameters
- Results
 - Result Combinations
 - Design - All
 - Design by Member
 - Design by Location X
 - Support Forces
 - Deformations

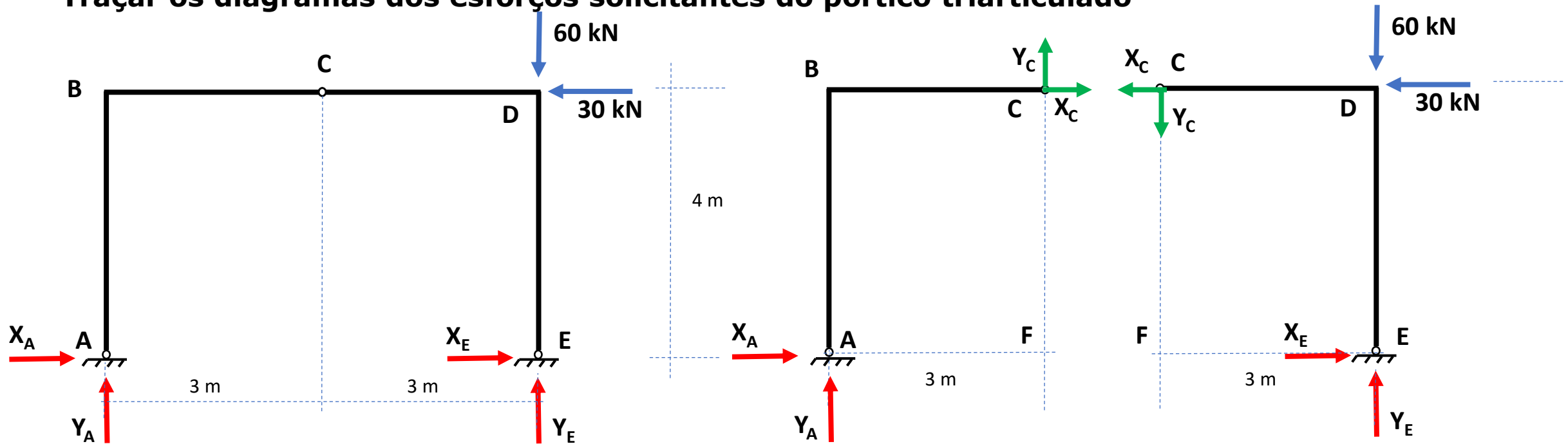
1.2 Geometry

Parameter	Description	Value	Unit
Building Dimensions			
<input type="checkbox"/>	Building Height	H	5.000 m
<input type="checkbox"/>	Building Depth	B	40.000 m
<input type="checkbox"/>	Truss Spacing	a	5.000 m
<input type="checkbox"/>	Distance to Roof Edge	u	0.000 m
<input type="checkbox"/>	Load coeff. for continuity	k	1.000 -
<input type="checkbox"/>	Edge Trusses		<input type="checkbox"/>
Frame Geometry			
<input checked="" type="checkbox"/>	Frame Width	L_a	8.000 m
<input type="checkbox"/>	Height on Edge	H_s	4.000 m
<input type="checkbox"/>	Height in the Middle	H_f	5.000 m
<input type="checkbox"/>	Cross-Section Height in Footing	h_a	35.00 cm
<input type="checkbox"/>	Cross-Section Height at Ridge	h_f	30.00 cm
<input type="checkbox"/>	Cross-Section Height at Frame Joint	h_1	90.00 cm
<input type="checkbox"/>	Length of Inserted Member	l_{zw}	40.00 cm
Releases			
<input type="checkbox"/>	Release Spring	$c_{y,f}$	0.000 kNm/
Cross-Section			



EXERCÍCIO 1

Traçar os diagramas dos esforços solicitantes do pórtico triarticulado



a) REAÇÕES DE APOIO

$$1. \sum M_A = 0 = -60 \cdot 6 + 30 \cdot 4 + Y_E \cdot 6 \rightarrow Y_E = 40 \text{ kN}$$

$$2. \sum M_E = 0 = -Y_A \cdot 6 + 30 \cdot 4 \rightarrow Y_A = 20 \text{ kN}$$

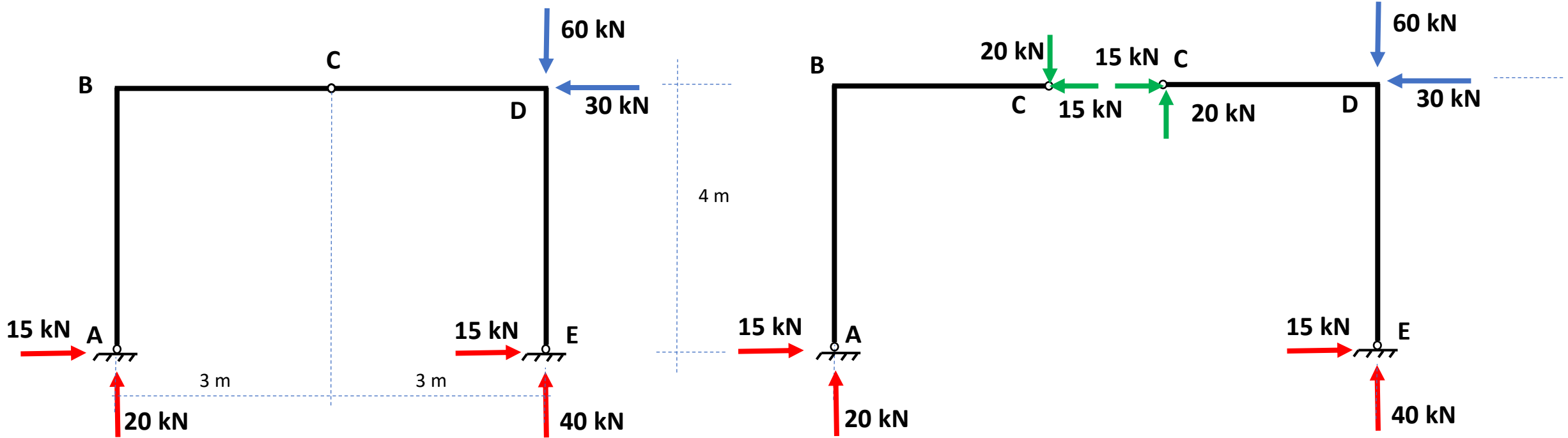
$$3. \sum M_{fletor}^{C,esq} = 0 = X_A \cdot 4 - Y_A \cdot 3 \text{ e } Y_A = 20 \text{ kN} \rightarrow X_A = 15 \text{ kN}$$

$$4. \sum M_{fletor}^{C,dir} = 0 = -60 \cdot 3 + X_E \cdot 4 + Y_E \cdot 3 \text{ e } Y_E = 40 \text{ kN} \rightarrow X_E = 15 \text{ kN}$$

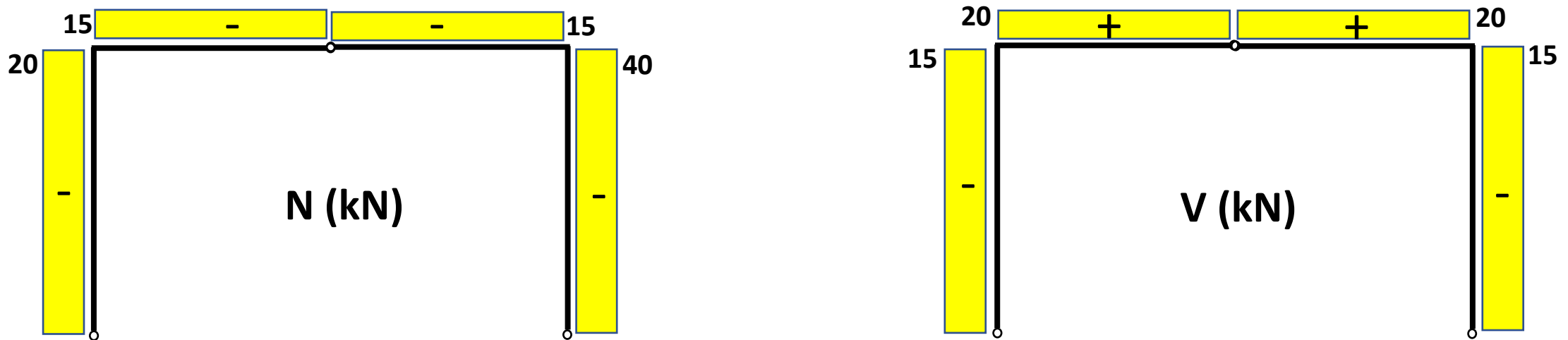
$$5. \sum M_F = 0 = -Y_A \cdot 3 - X_C \cdot 4 \text{ e } Y_A = 20 \text{ kN} \rightarrow X_C = -15 \text{ kN}$$

$$6. \sum Y = 0 = Y_A + Y_C \text{ e } Y_A = 20 \text{ kN} \rightarrow Y_C = -20 \text{ kN}$$

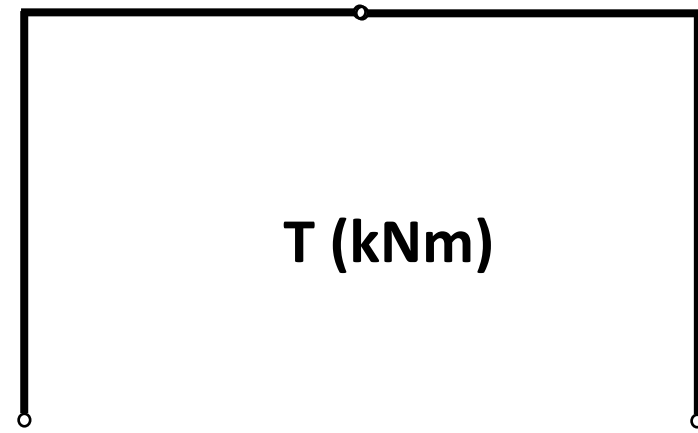
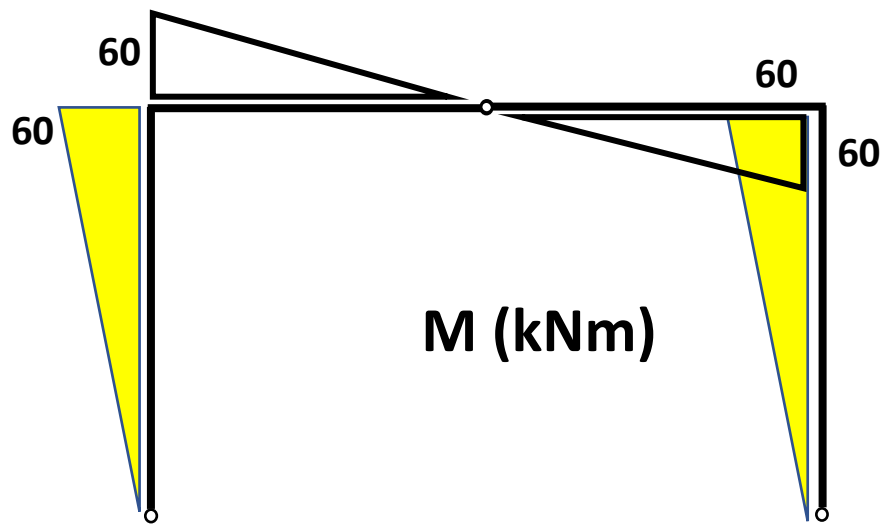
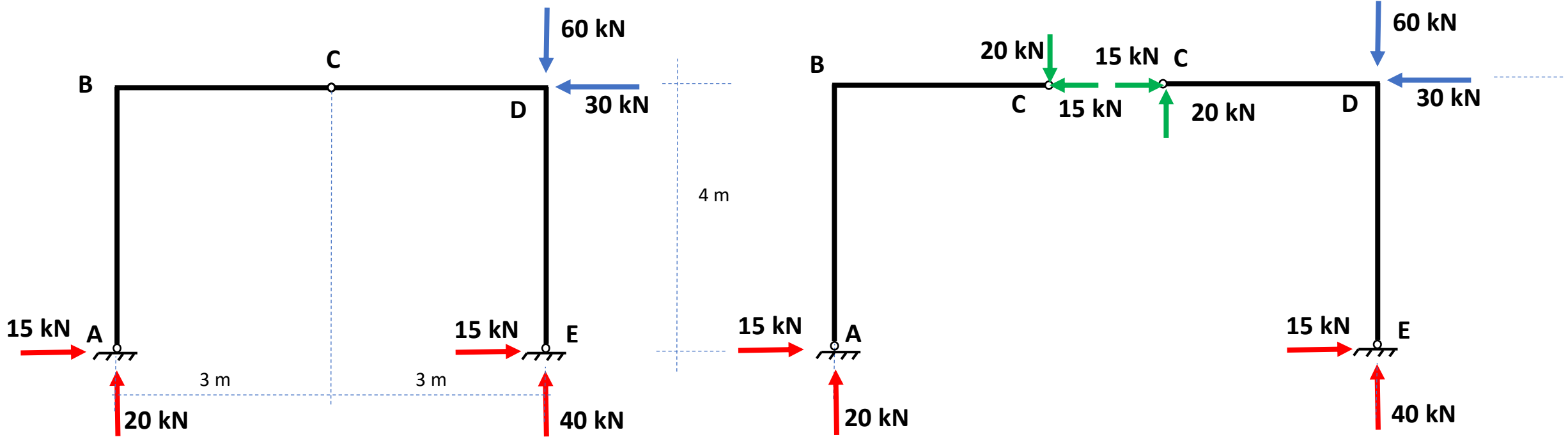
b) DIAGRAMA DO CORPO LIVRE



c) DIAGRAMAS DOS ESFORÇOS SOLICITANTES



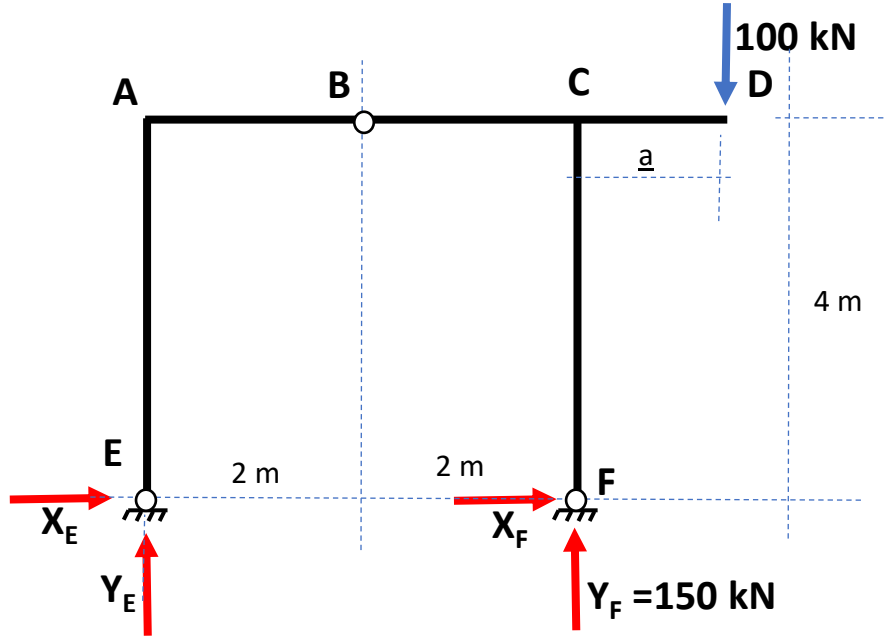
C) DIAGRAMAS DOS ESFORÇOS SOLICITANTES



EXERCÍCIO 2

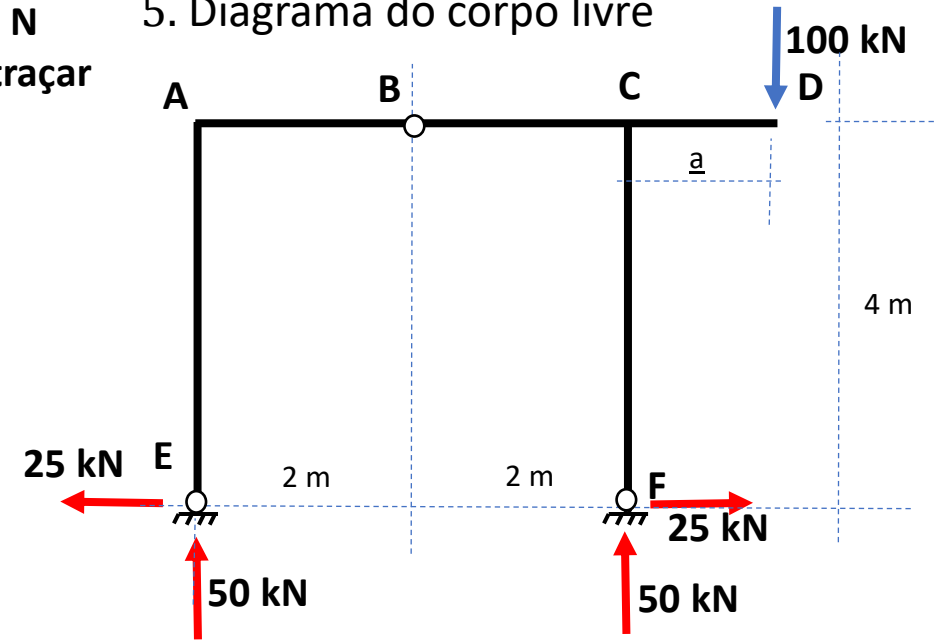
Para a estrutura:

- Determinar o comprimento a do balanço para que a reação vertical em F seja 150 N
- Com o valor de a encontrado no item anterior, determinar as reações de apoio e traçar o diagrama dos momentos fletores em todas as barras

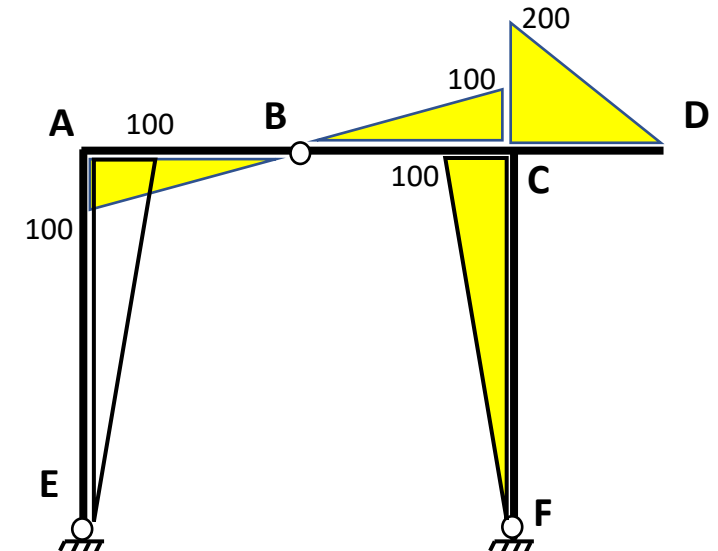


- $\sum M_{(E)} = 0 = -100 \cdot (4 + a) + Y_F \cdot 4 \rightarrow a = 2 \text{ m}$
- $\sum M_{(F)} = 0 = -Y_E \cdot 4 - 100 \cdot a \rightarrow Y_E = -50 \text{ kN}$
- $\sum M_{fletor}^{B,direita} = 0 = -100 \cdot (2 + a) + X_F \cdot 4 + Y_F \cdot 2 \rightarrow X_F = 25 \text{ kN}$
- $\sum M_{fletor}^{B,esquerda} = 0 = X_E \cdot 4 - Y_E \cdot 2 \rightarrow X_E = -25 \text{ kN}$

5. Diagrama do corpo livre



6. Diagrama do momento fletor (kNm)

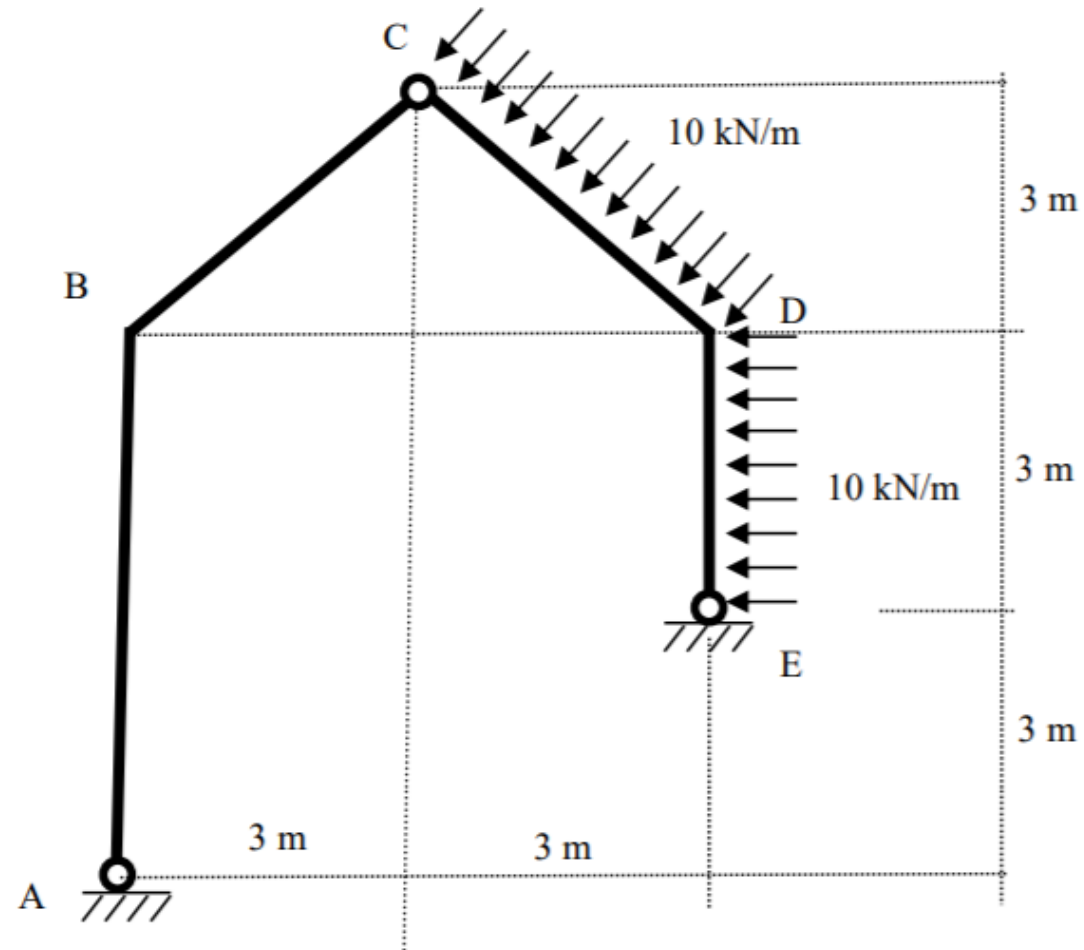


EXERCÍCIO 3

P3 2019

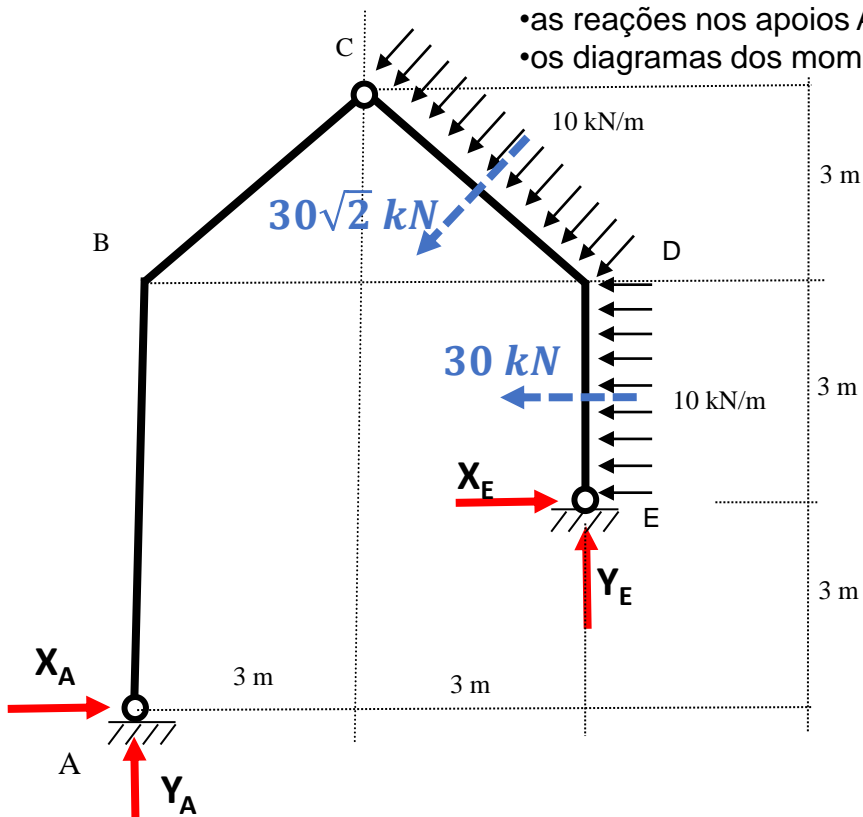
Para o pórtico triarticulado da figura representando parte da estrutura de um galpão recebendo a carga de um vento de 10 kN/m , determine:

- as reações nos apoios A e E;
- os diagramas dos momentos fletores na barra CDE indicando todos os valores relevantes.

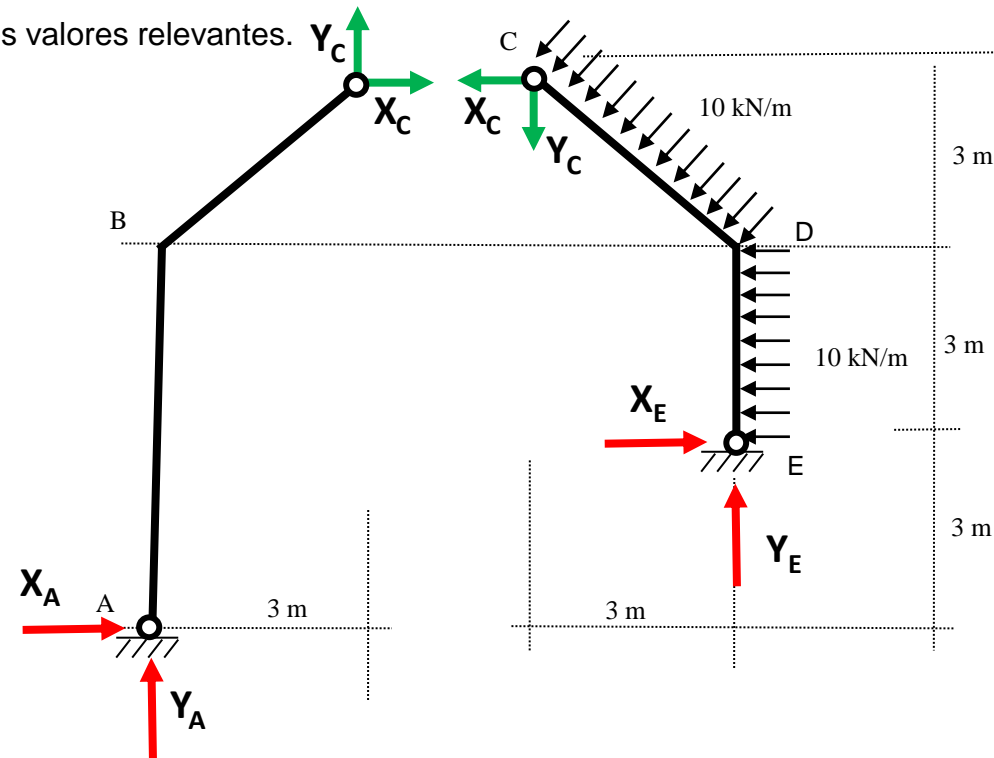


Para o pórtico triarticulado da figura representando parte da estrutura de um galpão recebendo a carga de um vento de 10 kN/m, determine:

- as reações nos apoios A e E;
- os diagramas dos momentos fletores na barra CDE indicando todos os valores relevantes.



a) REAÇÕES DE APOIO



$$1. \sum X = 0 = X_A + X_E - 30\sqrt{2} \cdot \frac{\sqrt{2}}{2} - 30 \rightarrow X_A + X_E = 60 \text{ kN}$$

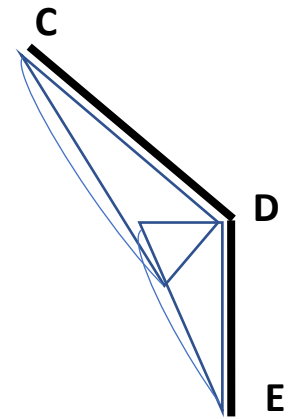
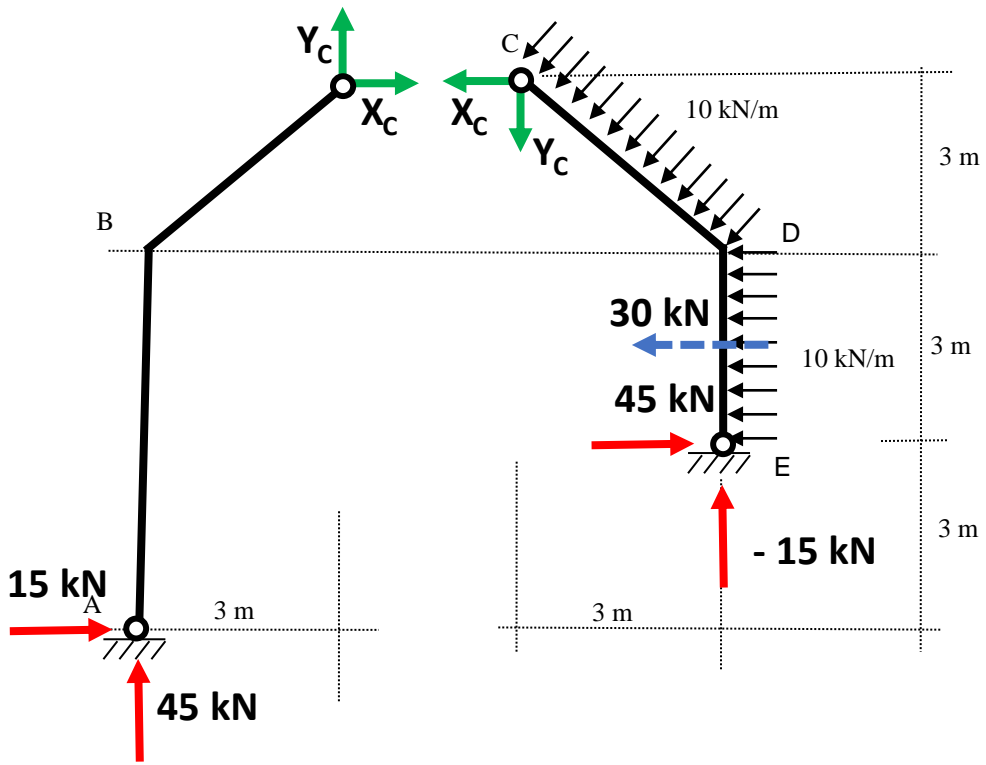
$$2. \sum Y = 0 = Y_A + Y_E - 30\sqrt{2} \cdot \frac{\sqrt{2}}{2} \rightarrow Y_A + Y_E = 30 \text{ kN}$$

$$3. \sum M_{fletor}^{C, esquerda} = 0 = X_A \cdot 9 - Y_A \cdot 3 \rightarrow 3X_A - Y_A = 0$$

$$4. \sum M_{fletor}^{C, direita} = 0 = -30\sqrt{2} \cdot \frac{3\sqrt{2}}{2} - 30 \cdot \left(3 + \frac{3}{2}\right) + X_E \cdot 6 + Y_E \cdot 3 \rightarrow 2X_E + Y_E = 75 \text{ kN}$$

$$5. \begin{cases} X_A + X_E = 60 \text{ kN} \\ Y_A + Y_E = 30 \text{ kN} \\ 3X_A - Y_A = 0 \text{ kN} \\ 2X_E + Y_E = 75 \text{ kN} \end{cases} \rightarrow \begin{cases} X_A = 15 \text{ kN} \\ X_E = 45 \text{ kN} \\ Y_A = 45 \text{ kN} \\ Y_E = -15 \text{ kN} \end{cases}$$

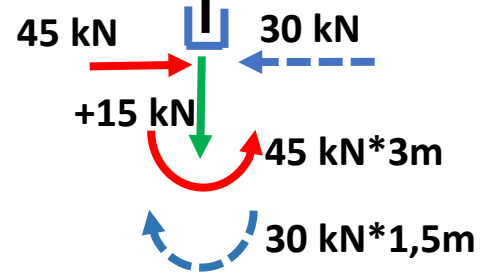




$$\sum M_{st} = 0 = M + 45x - 10x \cdot \frac{x}{2} \quad \text{GRINTER } \uparrow +$$

$$M = -45x + 5x^2 \quad (0 \leq x \leq 3)$$

6. seção D_-

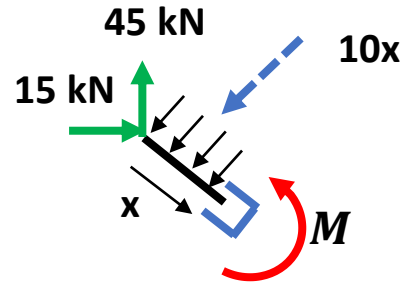
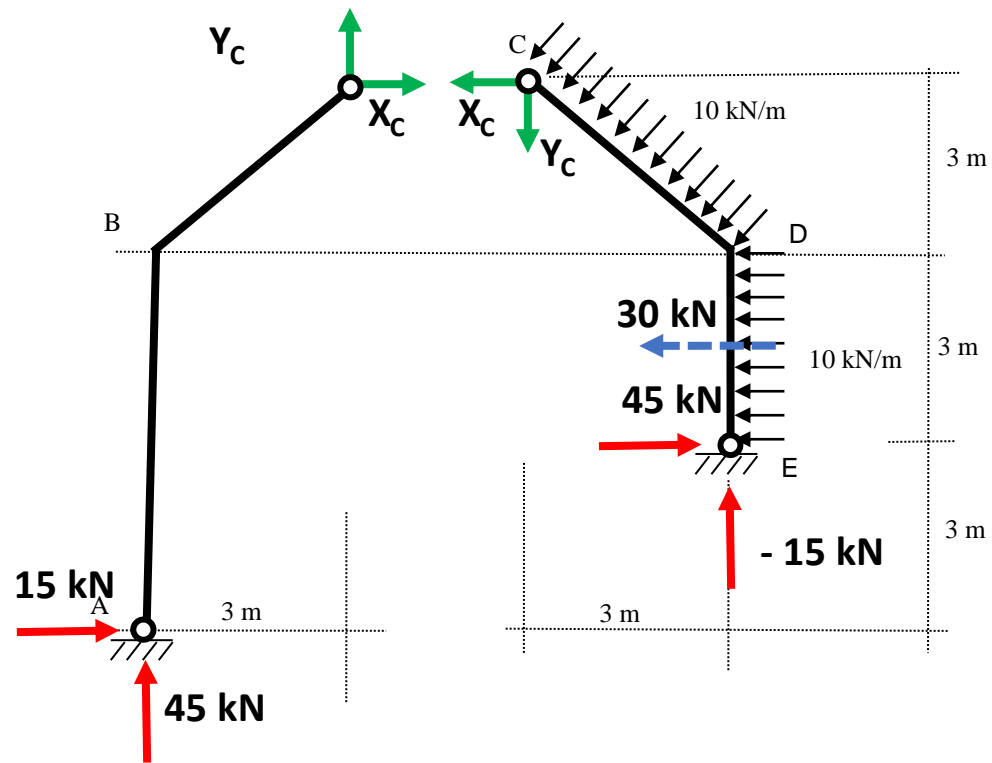


6. seção D_-



7. seção D_+





$$\sum M_{st} = 0 = M - 45x \frac{\sqrt{2}}{2} - 15x \frac{\sqrt{2}}{2} + 10x \cdot \frac{x}{2}$$

$$M = +30\sqrt{2}x - 5x^2 \quad (0 \leq x \leq 3\sqrt{2})$$

8. Parte da esquerda:

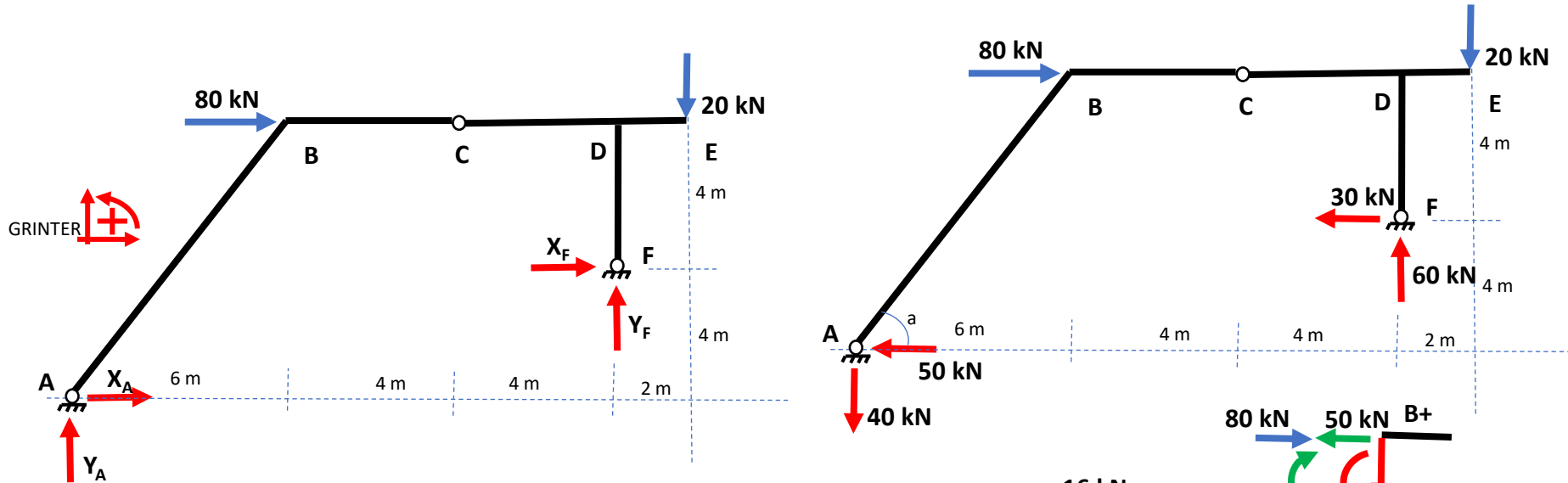
$$\sum X = 0 = 15 + X_c \Rightarrow X_c = -15 \text{ kN}$$

$$\sum Y = 0 = 45 + Y_c \Rightarrow Y_c = -45 \text{ kN}$$

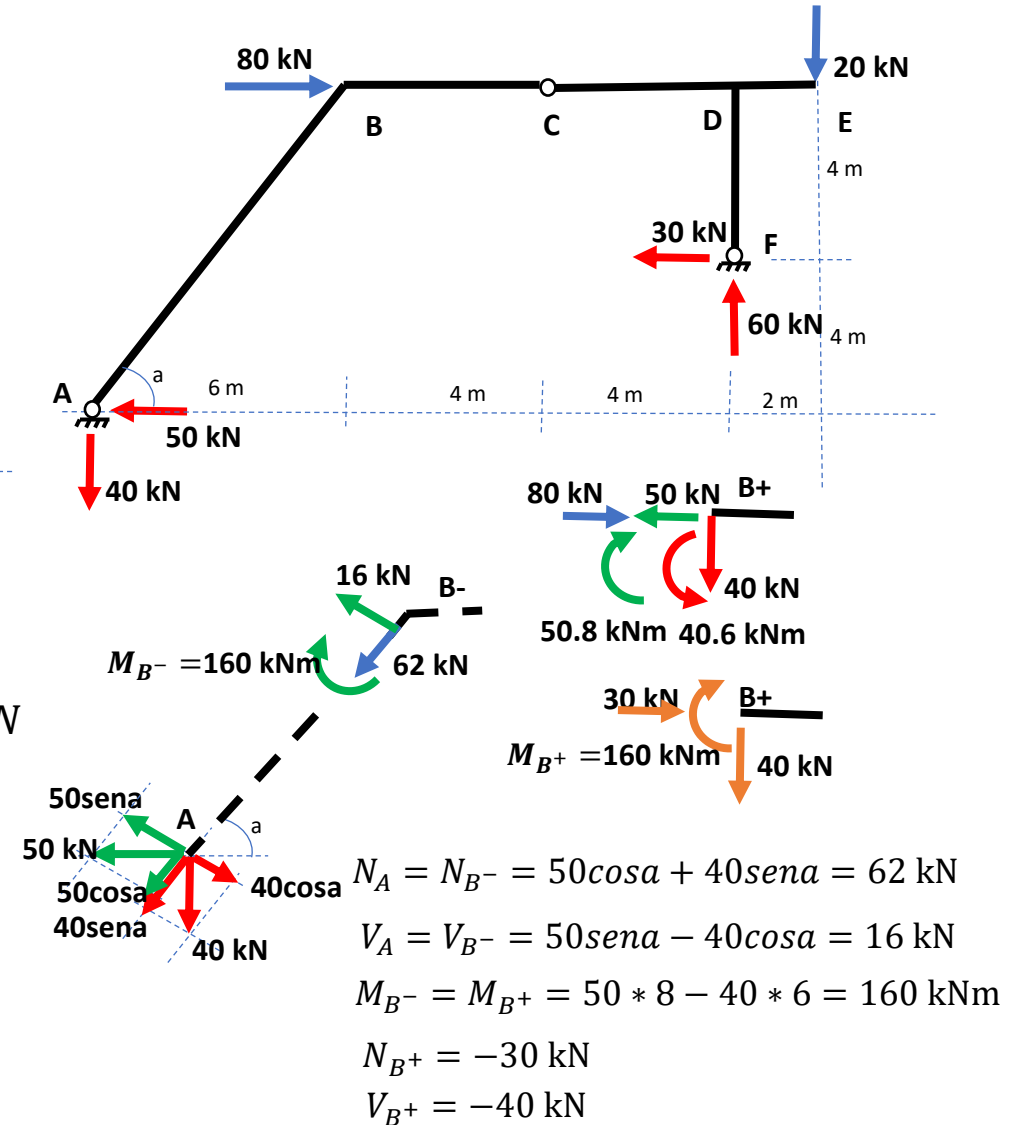
EXERCÍCIO 4

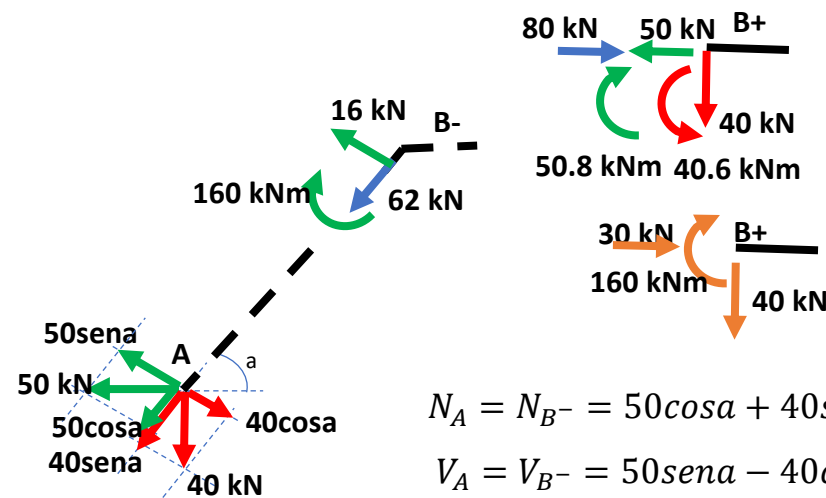
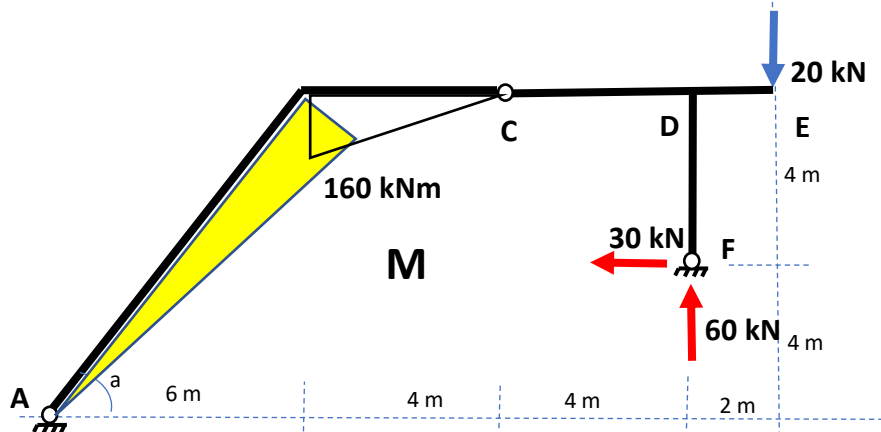
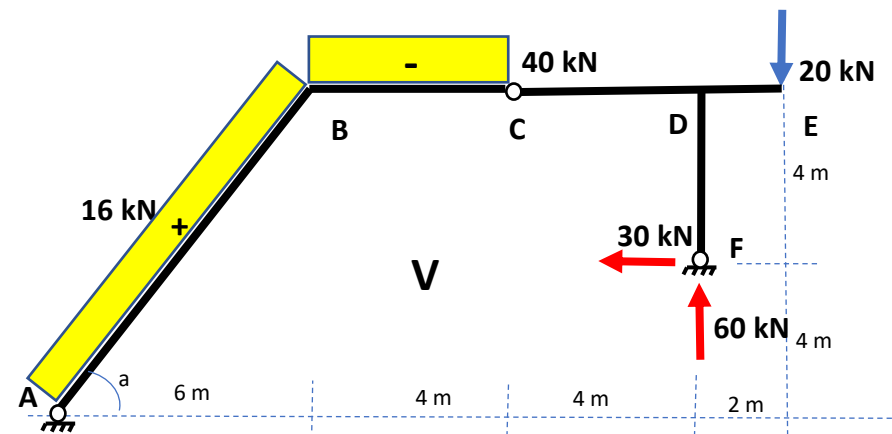
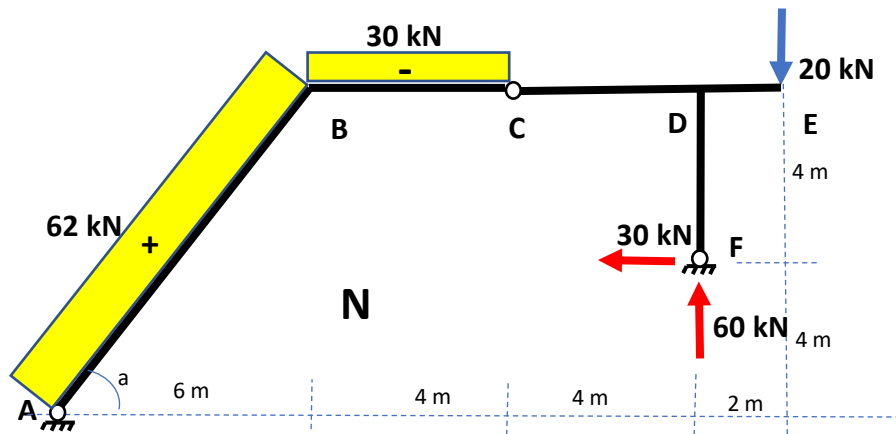
P3 2018

Traçar os diagramas de esforços solicitantes do pórtico triarticulado da figura:



1. $\sum X = 0 = X_A + 80 + X_F \rightarrow X_A + X_F = -80 \text{ kN}$
2. $\sum Y = 0 = Y_A + Y_F - 20 \text{ kN} \rightarrow Y_A + Y_F = 20 \text{ kN}$
3. $\sum M_{fletor}^{C,direita} = 0 = X_F * 4 + Y_F * 4 - 20 * 6 \rightarrow X_F + Y_F = 30 \text{ kN}$
4. $\sum M_{fletor}^{C,esquerda} = 0 = X_A * 8 - Y_A * 10 \rightarrow 4 * X_A - 5 * Y_A = 0$
5.
$$\begin{cases} X_A + X_F = -80 \text{ kN} \\ Y_A + Y_F = 20 \text{ kN} \\ X_F + Y_F = 30 \text{ kN} \\ 4 * X_A - 5 * Y_A = 0 \end{cases} \Rightarrow \begin{cases} X_A = -50 \text{ kN} \\ Y_A = -40 \text{ kN} \\ X_F = -30 \text{ kN} \\ Y_F = 60 \text{ kN} \end{cases}$$





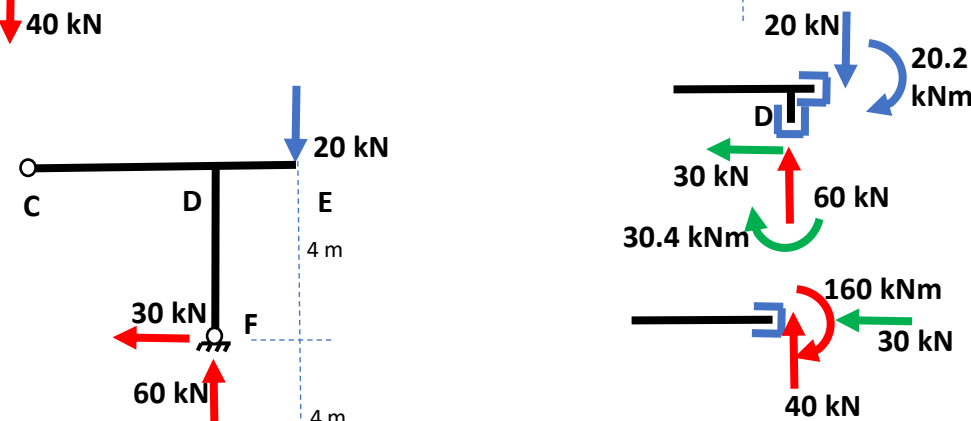
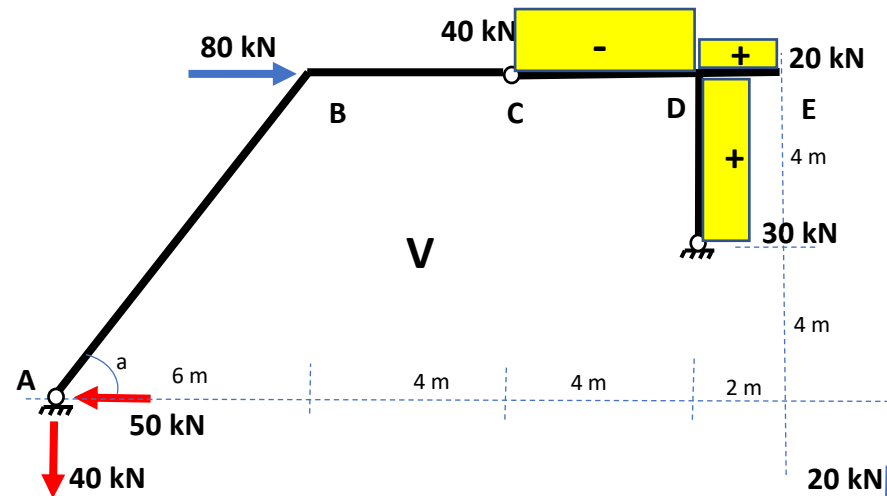
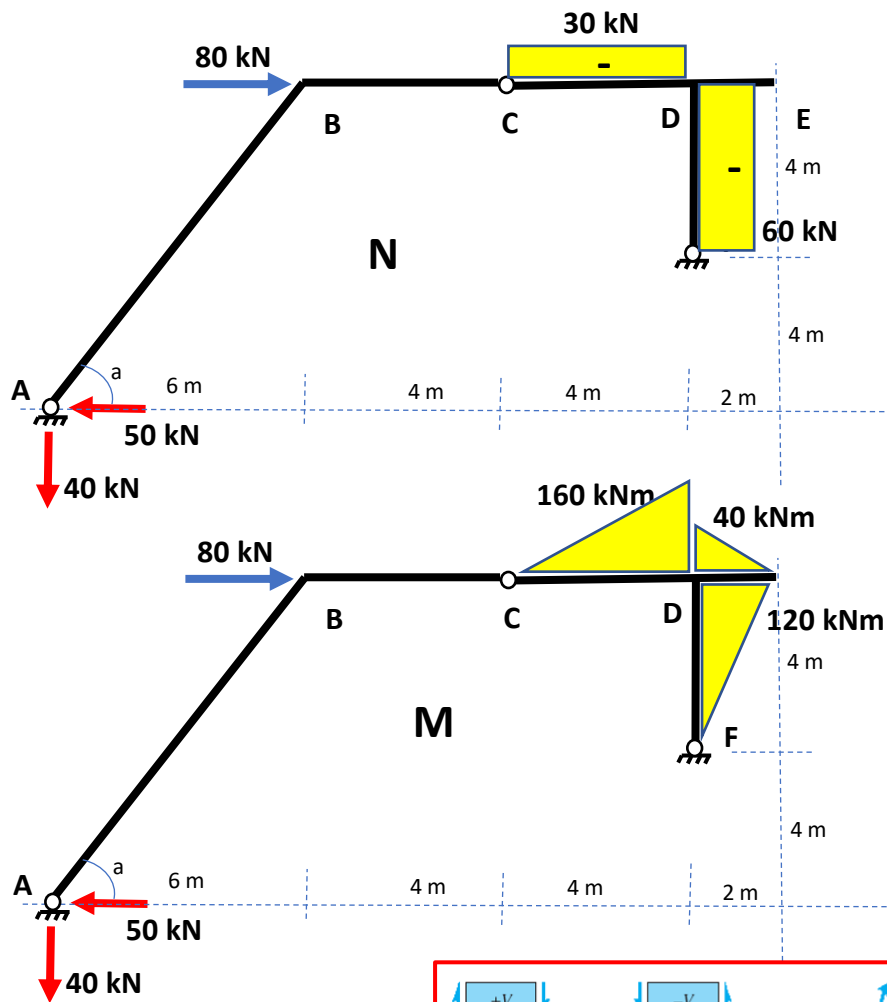
$$N_A = N_{B^-} = 50\cos a + 40\sin a = 62 \text{ kN}$$

$$V_A = V_{B^-} = 50\sin a - 40\cos a = 16 \text{ kN}$$

$$M_{B^-} = M_{B^+} = 50 \cdot 8 - 40 \cdot 6 = 160 \text{ kNm}$$

$$N_{B^+} = -30 \text{ kN}$$

$$V_{B^+} = -40 \text{ kN}$$



$$N_{Ddir} = 0; V_{Ddir} = +20kN; M_{Ddir} = -40kNm$$

$$N_{Dbaixo} = -60 kN; V_{Dbaixo} = +30kN; |M_{Dbaixo}| = 120kNm$$

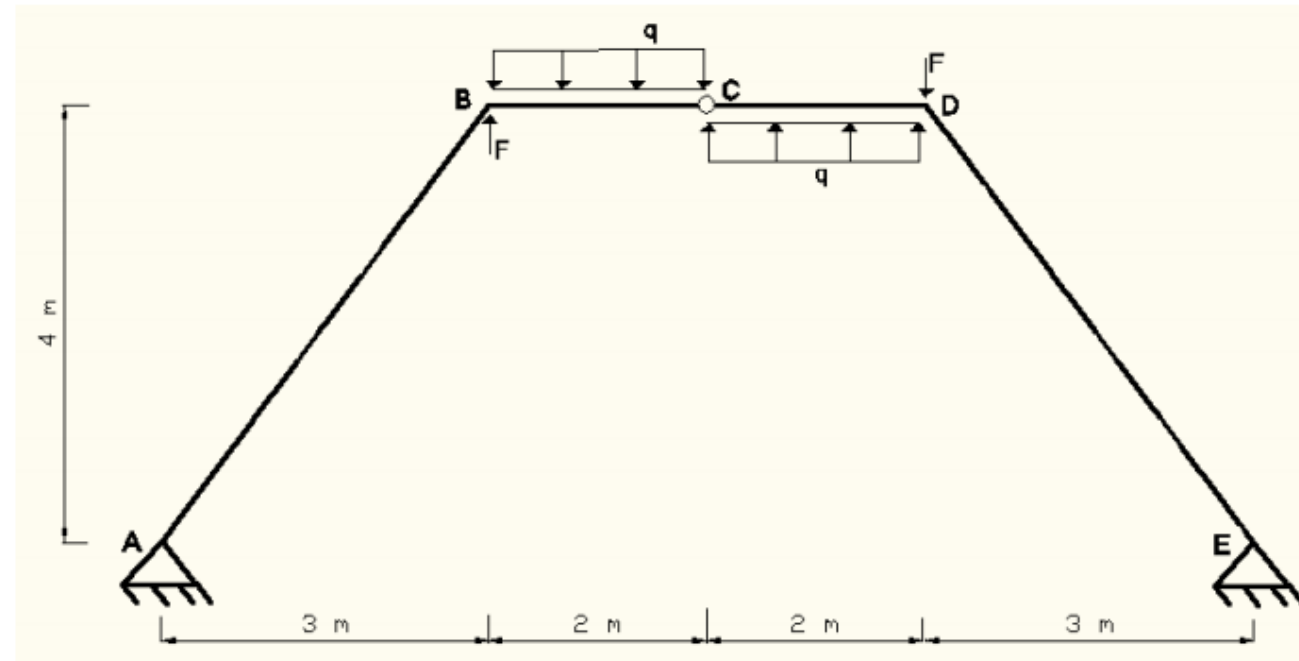
$$N_{Desq} = -30 kN; V_{Desq} = -40kN; M_{Desq} = -160kNm$$



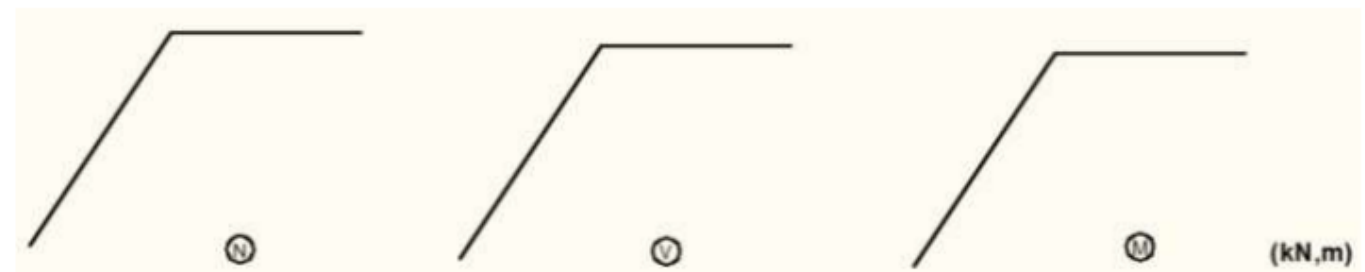
Nº USP: _____ Nome: _____

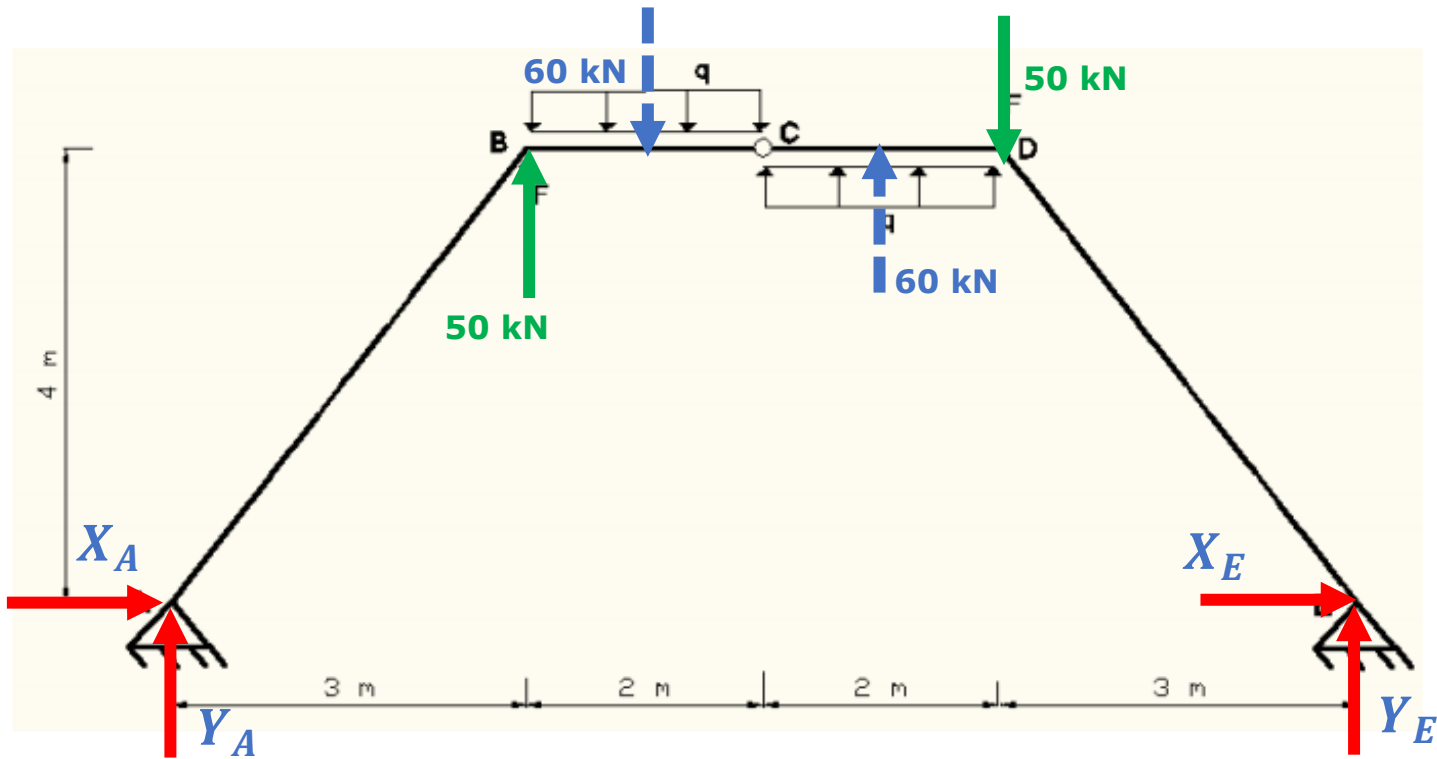
EXERCÍCIO 5

2ª Questão (4 pontos) Determinar os esforços solicitantes (M, V e N) no pórtico triarticulado **somente nos trechos ABCD**, sob a ações das cargas indicadas. Adote $q = 30 \text{ kN/m}$ e $F = 50 \text{ kN}$. Indique explicitamente os valores e os pontos de momentos extremos. Apresente os diagramas nos desenhos indicados na resposta.



Respostas





1. Reações nos apoios

$$\sum M_{(A)} = 0 = 50 \cdot 3 - 60 \cdot 4 + 60 \cdot 6 - 50 \cdot 7 + Y_E \cdot 10 \Rightarrow Y_E = 8 \text{ kN}$$

$$\sum M_{(E)} = 0 = -Y_A \cdot 10 - 50 \cdot 7 + 60 \cdot 6 - 60 \cdot 4 + 50 \cdot 3 \Rightarrow Y_A = -8 \text{ kN}$$

$$\sum M_{fletor}^{C,esq} = 0 = X_A \cdot 4 - Y_A \cdot 5 - 50 \cdot 2 + 60 \cdot 1 \text{ e } Y_A = -8 \text{ kN} \Rightarrow X_A = 0$$

$$\sum M_{fletor}^{C,dir} = 0 = X_E \cdot 4 + Y_E \cdot 5 - 50 \cdot 2 + 60 \cdot 1 \text{ e } Y_E = 8 \text{ kN} \Rightarrow X_E = 0$$

