• DISTRIBUIÇÃO GAUSSIANA

(i) IDENTIDADE FUNDAMENTAL

$$\boxed{\int_{-\infty}^{+\infty} e^{-\alpha x^2}\, dx = \sqrt{\frac{\pi}{\alpha}}}$$

$\alpha > 0$

• $I \equiv \int_{-\infty}^{+\infty} e^{-x^2} dx = \int_{-\infty}^{+\infty} e^{-y^2} dy$

$$I^2 = \left[\int e^{-x^2} dx\right] . \left[\int e^{-y^2} dy\right]$$

$$= \iint e^{-(x^2+y^2)} dx\,dy$$

$$\begin{cases} x = r.\cos\varphi \\ y = r.\sin\varphi \end{cases}$$

$$= \iint e^{-r^2} r.\,dr\,d\varphi$$

$$= \left[\int_0^{2\pi} d\varphi\right] . \left[\int_0^{\infty} \frac{1}{2} e^{-r^2} (2r\,dr)\right]$$

$$= 2\pi . \frac{1}{2} \int_0^{\infty} e^{-u} du$$

$$= \pi \left[-e^{-u}\right]_0^{\infty}$$

$$= \pi$$

$$\therefore \boxed{I = \sqrt{\pi}}$$

GAUSS

01

• $\int_{-\infty}^{+\infty} e^{-\alpha x^2}dx = \int_{-\infty}^{+\infty} e^{-u^2}\left(\frac{1}{\sqrt{\alpha}}du\right) = \frac{I}{\sqrt{\alpha}}$

(ii) NORMALIZAÇÃO, $X \sim N(\mu,\sigma^2)$

$$\int \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}dx =$$

$$z = \frac{x-\mu}{\sigma}$$

$$= \int \frac{1}{\sqrt{2\pi\cancel{\sigma^2}}} e^{-z^2/2}(\cancel{\sigma}\, dz) =$$

$$= \frac{1}{\sqrt{2\pi}}\left[\int e^{-z^2/2}dz\right] = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{\pi}{1/2}} = 1$$

$$X \sim N(\mu,\sigma^2) \Rightarrow Z \sim N(0,1)$$

$$Z \equiv \frac{X-\mu}{\sigma}$$

TRANSLAÇÃO POR μ

ESCALA POR $1/\sigma$

$$\rho_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

$$X = \sigma Z + \mu \Rightarrow \begin{cases} \langle X \rangle = \sigma\langle Z\rangle + \mu \\ \text{var}\, X = \sigma^2 \cdot \text{var}\, Z \end{cases}$$

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$\boxed{02}$

VEREMOS QUE $\langle Z \rangle = 0$ E

$\text{var}\, Z = 1$, DE MODO QUE

$$\boxed{\langle X \rangle = \mu} \quad \text{E} \quad \boxed{\text{var}\, X = \sigma^2},$$

SENDO OS PARÂMETROS μ E σ^2 A MÉDIA E A VARIÂNCIA DE X, RESPECTIVAMENTE.

$$\langle Z \rangle = \int_{-\infty}^{+\infty} z \cdot \rho_Z(z)\, dz$$

$$= \underbrace{\int_{-\infty}^{+\infty} \underbrace{z}_{\text{ÍMPAR}} \underbrace{\left[\frac{1}{\sqrt{2\pi}} e^{-z^2/2}\right]}_{\text{PAR}}}_{\text{ÍMPAR}} dz = 0$$

$$\text{var}\, Z = \langle Z^2 \rangle - \overset{0}{\overset{\shortparallel}{\langle Z \rangle}}{}^2 = \langle Z^2 \rangle =$$

$$= \int_{-\infty}^{+\infty} z^2 \left[\frac{1}{\sqrt{2\pi}} e^{-z^2/2}\right] dz =$$

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$$= \frac{1}{\sqrt{2\pi}} \left\{ \int_{-\infty}^{+\infty} (-1) \frac{\partial}{\partial \alpha} \left(e^{-\alpha z^2}\right) dz \right\}_{\alpha = 1/2} = \boxed{03}$$

$$= (-1)\frac{1}{\sqrt{2\pi}}\left\{\frac{d}{d\alpha}\left[\int e^{-\alpha z^2}dz\right]\right\}_{\alpha=\frac{1}{2}} =$$

$$= (-1)\frac{1}{\sqrt{2\pi}}\left\{\frac{d}{d\alpha}\left(\frac{\sqrt{\pi}}{\sqrt{\alpha}}\right)\right\}_{\alpha=1/2} =$$

$$= (-1)\frac{1}{\sqrt{2\pi}}\sqrt{\pi}\left\{\left(-\frac{1}{2}\right)\alpha^{-3/2}\right\}_{\alpha=1/2} =$$

$$= \frac{1}{2\sqrt{2}}\left(1/2\right)^{-3/2} = 1$$