

PNV3412- Mecânica de Estruturas Navais e Oceânicas II

Introdução ao método de elementos finitos:
Elementos barra (Truss)

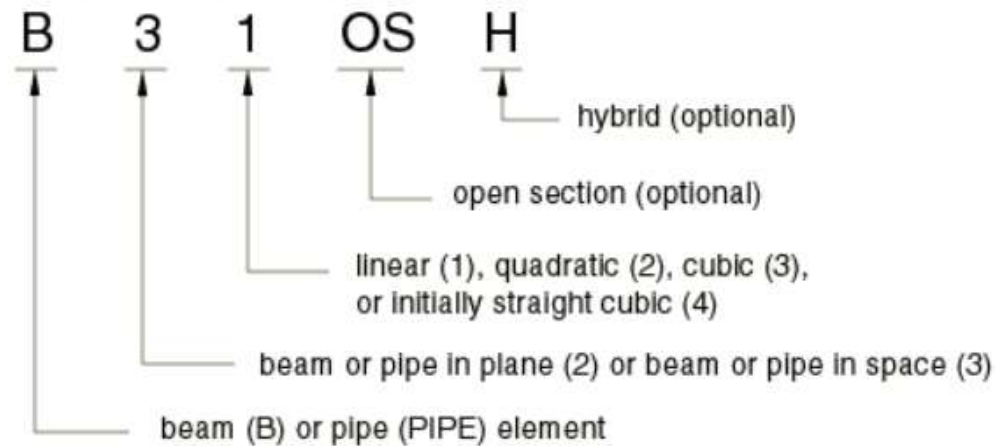
2023

Agenda

1. Viga com seção constante e carga variável
2. Viga com suporte

Naming convention

Beam elements in Abaqus are named as follows:



For example, B21H is a planar beam that uses linear interpolation and a hybrid formulation.

Agenda

1. Viga com seção constante e carga variável
2. Pórtico

Euler-Bernoulli (slender) beams

Euler-Bernoulli beams (B23, B23H, B33, and B33H) are available only in Abaqus/Standard. These elements do not allow for transverse shear deformation; plane sections initially normal to the beam's axis remain plane (if there is no warping) and normal to the beam axis. They should be used only to model slender beams: the beam's cross-sectional dimensions should be small compared to typical distances along its axis (such as the distance between support points or the wavelength of the highest mode that participates in a dynamic response). For beams made of uniform material, typical dimensions in the cross-section should be less than about 1/15 of typical axial distances for transverse shear flexibility to be negligible. (The ratio of cross-section dimension to typical axial distance is called the slenderness ratio.)

Load stiffness for pressure loads is not included for these elements.

Interpolation

The Euler-Bernoulli beam elements use cubic interpolation functions, which makes them reasonably accurate for cases involving distributed loading along the beam. Therefore, they are well suited for dynamic vibration studies, where the d'Alembert (inertia) forces provide such distributed loading.

The cubic beam elements are written for small-strain, large-rotation analysis. They may not be appropriate for torsional stability problems due to the approximations in the underlying formulation and cannot be used in analyses involving very large rotations (of the order 180°); quadratic or linear beam elements should be used instead.

Mass formulation

The Euler-Bernoulli beam elements use a consistent mass formulation. Rotary inertia for twist around the beam axis is the same as for Timoshenko beams. For details, see [Mass and inertia for Timoshenko beams](#). Any additional inertia defined for these elements (see [Adding inertia to the beam section behavior for Timoshenko beams](#)) is ignored.



Viga com seção constante

- Determinar campo de deslocamentos e tensões da viga



Specifying line loads on beam elements

You can specify line loads on beam elements in the global X-, Y-, or Z-direction. In addition, you can specify line loads on beam elements in the beam local 1- or 2-direction.

Input File Usage:

Use the following option to define a force per unit length in the global X-, Y-, or Z-direction on beam elements:

```
*DLOAD  
element number or element set, load type label, magnitude
```

where *load type label* is PX, PY, PZ, PXNU, PYNU, or PZNU.

Use the following option to define a force per unit length in the beam local 1- or 2-direction:

```
*DLOAD  
element number or element set, load type label, magnitude
```

where *load type label* is P1, P2, P1NU, or P2NU.

Abaqus/CAE Usage:

Load module: **Create Load**: choose **Mechanical** for the **Category** and **Line load** for the **Types for Selected Step**

Viga com seção constante

- Determinar campo de deslocamentos e tensões da viga

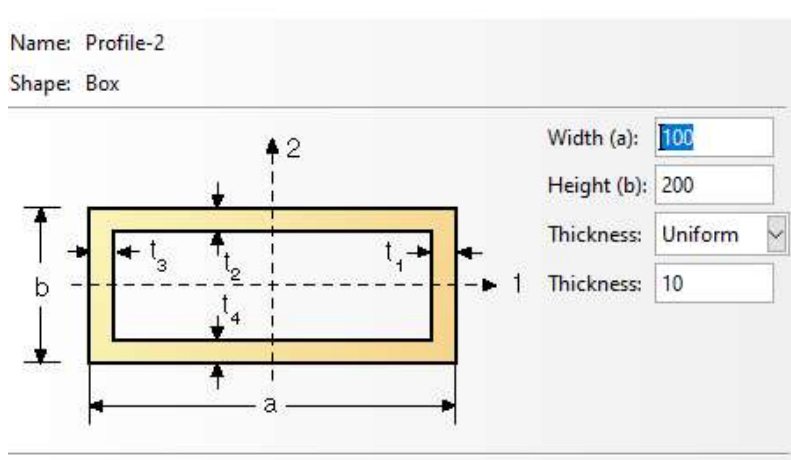
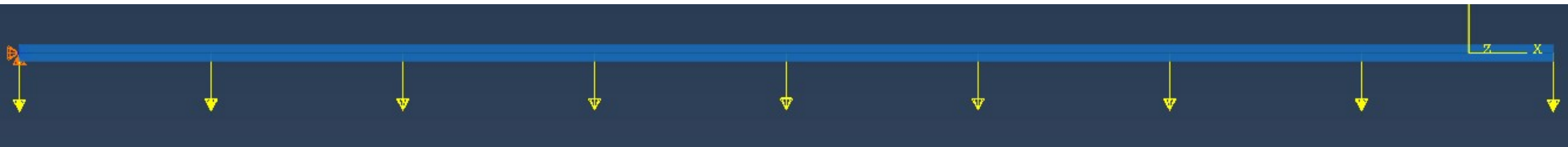
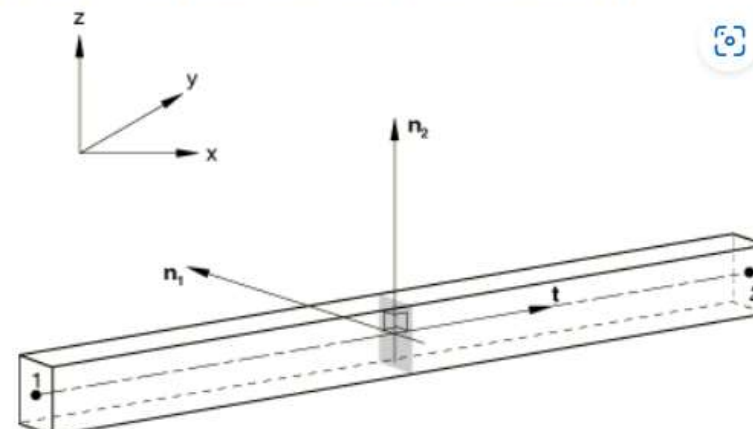
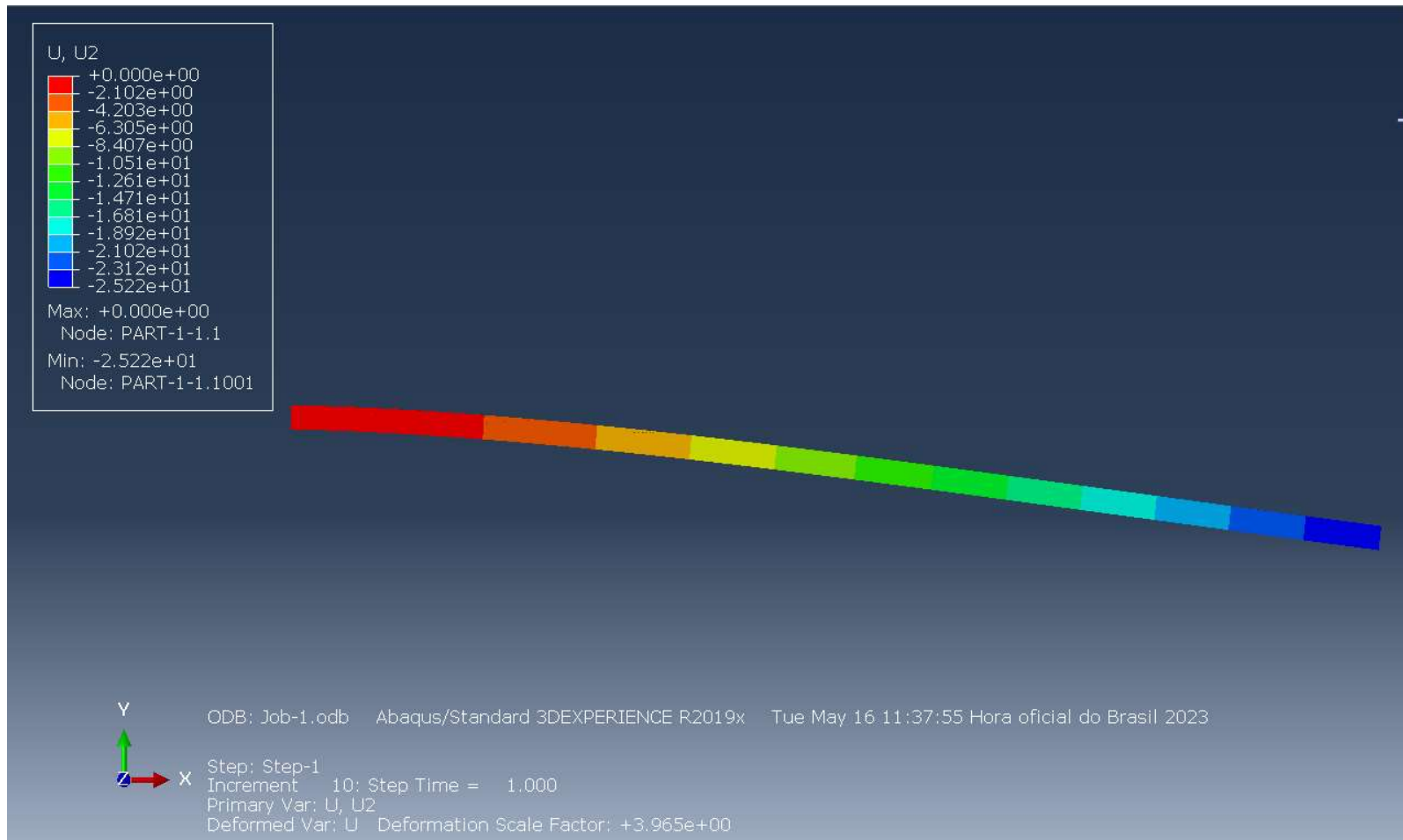


Figure 1. Local axis definition for beam-type elements.



Viga com seção constante

- Determinar campo de deslocamentos e tensões da viga

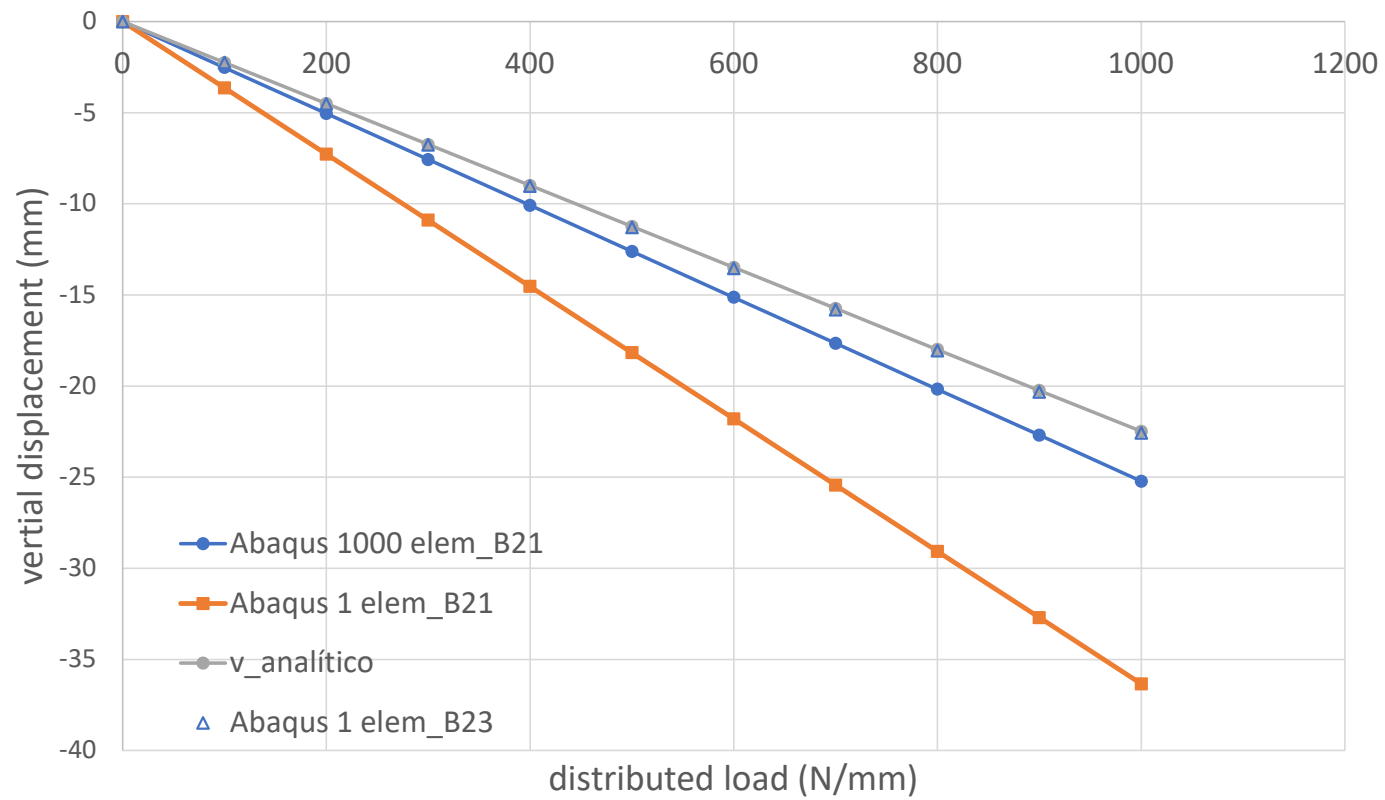


Viga com seção constante

TABELA 3.1a
DESLOCAMENTOS ELÁSTICOS EM VIGAS

CASO	VINCULAÇÃO E CARREGAMENTO	FLECHA		EQUAÇÃO DA ELÁSTICA
		w_{\max}	x	
1		$\frac{1}{8} \frac{p\ell^4}{EI}$	0	$\frac{p\ell^4}{24EI} (\alpha^4 - 4\alpha + 3)$
2		$\frac{1}{30} \frac{p\ell^4}{EI}$	0	$\frac{p\ell^4}{120EI} (\alpha^5 - 5\alpha + 4)$
3		$\frac{11}{120} \frac{p\ell^4}{EI}$	0	$\frac{p\ell^4}{120EI} (-\alpha^5 + 5\alpha^4 - 15\alpha + 11)$
4		$\frac{1}{3} \frac{P\ell^3}{EI}$	0	$\frac{P\ell^3}{6EI} (\alpha^3 - 3\alpha + 2)$
5		$\frac{1}{2} \frac{M\ell^2}{EI}$	0	$\frac{M\ell^2}{2EI} (1 - \alpha)^2$
6		$\frac{5}{384} \frac{p\ell^4}{EI}$	0,5l	$\frac{p\ell^4\alpha}{24EI} (\alpha^3 - 2\alpha^2 + 1)$
7		$\frac{3}{460} \frac{p\ell^4}{EI}^{(*)}$	0,519l	$\frac{p\ell^4\alpha}{360EI} (3\alpha^4 - 10\alpha^2 + 7)$
8		$\frac{1}{120} \frac{p\ell^4}{EI}$	0,5l	$\frac{p\ell^4\alpha}{960EI} (16\alpha^4 - 40\alpha^2 + 25)^{(**)}$
9		$\frac{1}{48} \frac{P\ell^3}{EI}$	0,5l	$\frac{P\ell^3\alpha}{48EI} (-4\alpha^2 + 3)^{(**)}$

Viga com seção constante



Viga com seção constante

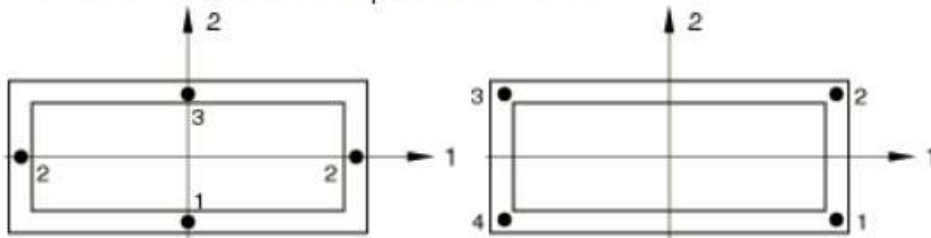
Default stress output points if a beam section integrated during the analysis is used

Beam in a plane: Bottom and top (points 1 and 5 above for default integration).

Beam in space: 4 corners (points 1, 5, 9, and 13 above for default integration).

Temperature and field variable input at specific points for beam sections integrated during the analysis

Give the value at each of the points shown below.



Beam in a plane

Beam in space

Viga com seção constante

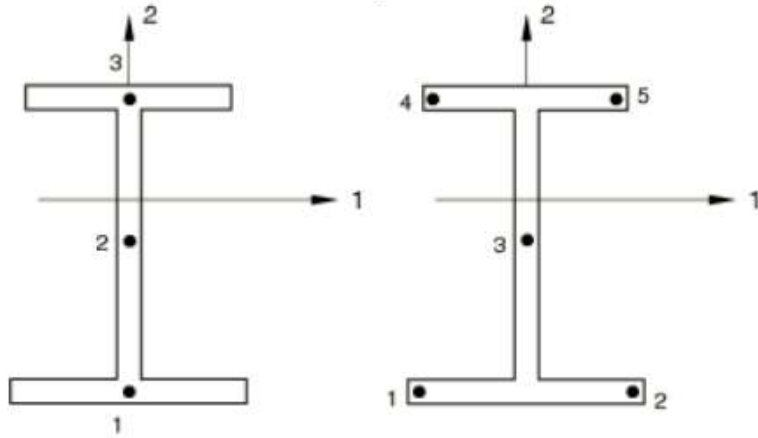
Default stress output points if a beam section integrated during the analysis is used

Beam in a plane: Flanges (points 1 and 5 above for default integration).

Beam in space: Ends of flanges (points 1, 5, 9, and 13 above for default integration).

Temperature and field variable input at specific points for beam sections integrated during the analysis

Give the value at each of the points shown below.



Beam in a plane

Beam in space

Viga com seção constante

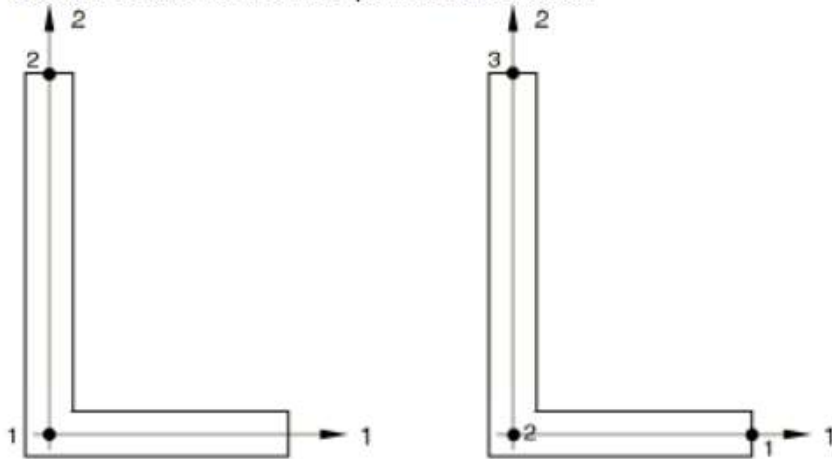
Default stress output points if a beam section integrated during the analysis is used

Beam in a plane: Bottom and top (points 1 and 5 above for default integration).

Beam in space: End of flange along positive local 1-axis; section corner; end of flange along positive local 2-axis (points 1, 5, and 9 above for default integration).

Temperature and field variable input at specific points for beam sections integrated during the analysis

Give the value at each of the points shown below.

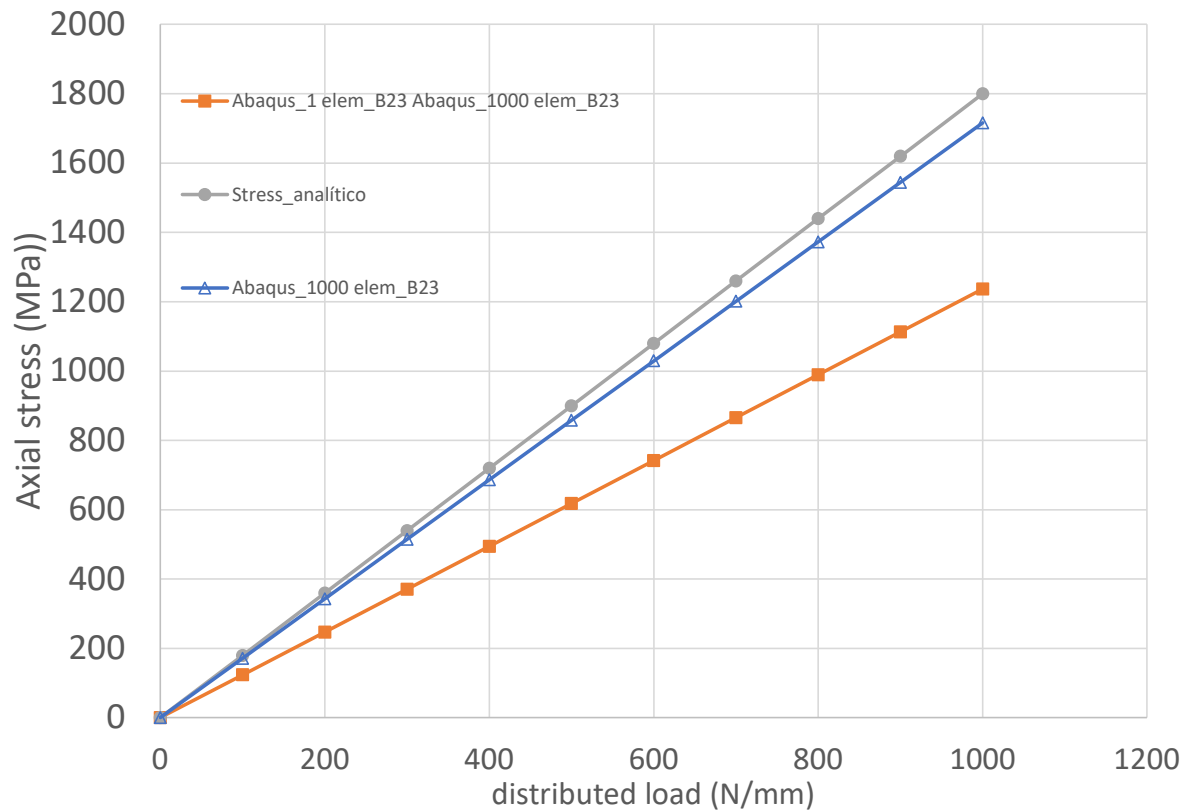


Beam in a plane

Beam in space



Viga com seção constante



$$\sigma = -\frac{Ey}{\rho}$$

$$\frac{1}{\rho} = \frac{\frac{d^2v(x)}{dx^2}}{\left[1 + \left(\frac{dv(x)}{dx}\right)^2\right]^{3/2}}$$

$$v(x) = \frac{p_0}{24EI} (x^4 - 4L^3x + 3L^4)$$

$$\frac{d^2v(x)}{dx^2} = \frac{12p_0}{24EI} x^2$$

No engaste, $x = L$

$$\frac{d^2v(L)}{dx^2} = \frac{p_0}{2EI} L^2$$

$$\sigma = -Ey \frac{p_0}{2EI} L^2$$

$$\sigma = -y \frac{p_0}{2I} L^2$$