

# ESCOAMENTOS E CONVECÇÃO DE CALOR

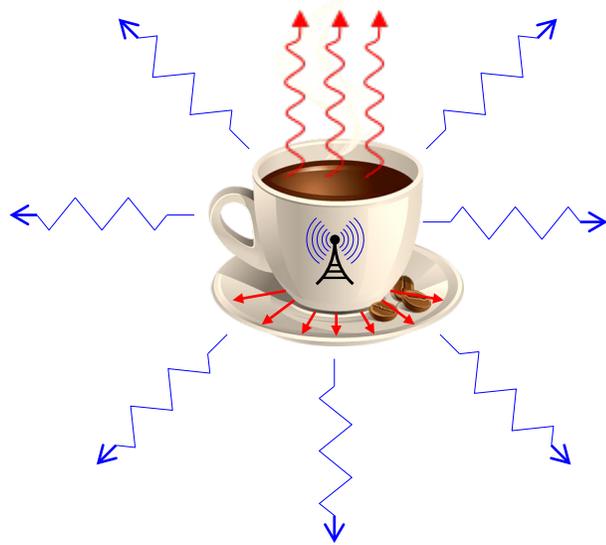
**Paulo Seleglim Jr.**  
**Universidade de São Paulo**



ANTES PORÉM:

RESISTÊNCIA TÉRMICA...

# DEFINIÇÃO DE UMA RESISTÊNCIA TÉRMICA → CIRCUITOS TÉRMICOS EQUIVALENTES



Condução de calor: lei de Fourier

$$Q = k \cdot A \cdot \Delta T / L$$

Convecção de calor: lei de Newton

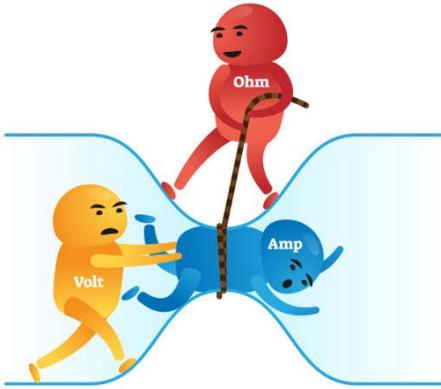
$$Q = h \cdot A \cdot \Delta T$$

Radiação de calor: lei de Stefan–Boltzmann

$$Q = \sigma \cdot A \cdot \Sigma(T^4 - T_{\infty}^4) \rightarrow Q = \bar{h}_{rad} \cdot A \cdot \Delta T$$



# DEFINIÇÃO DE UMA RESISTÊNCIA TÉRMICA → CIRCUITOS TÉRMICOS EQUIVALENTES



$$R_{elec} = \frac{\Delta V}{i}$$

Condução de calor: lei de Fourier

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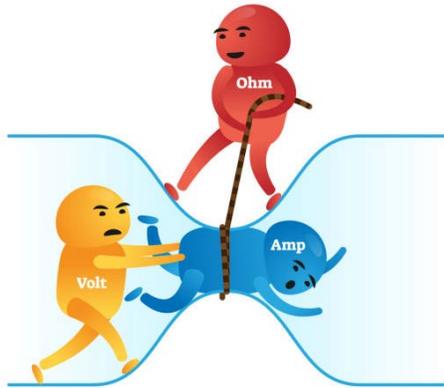
$$Q = \sigma \cdot A \cdot \Sigma(T^4 - T_{\infty}^4) \rightarrow Q = \bar{h}_{rad} \cdot A \cdot \Delta T$$

$$R_{term} = \frac{\Delta T}{Q}$$



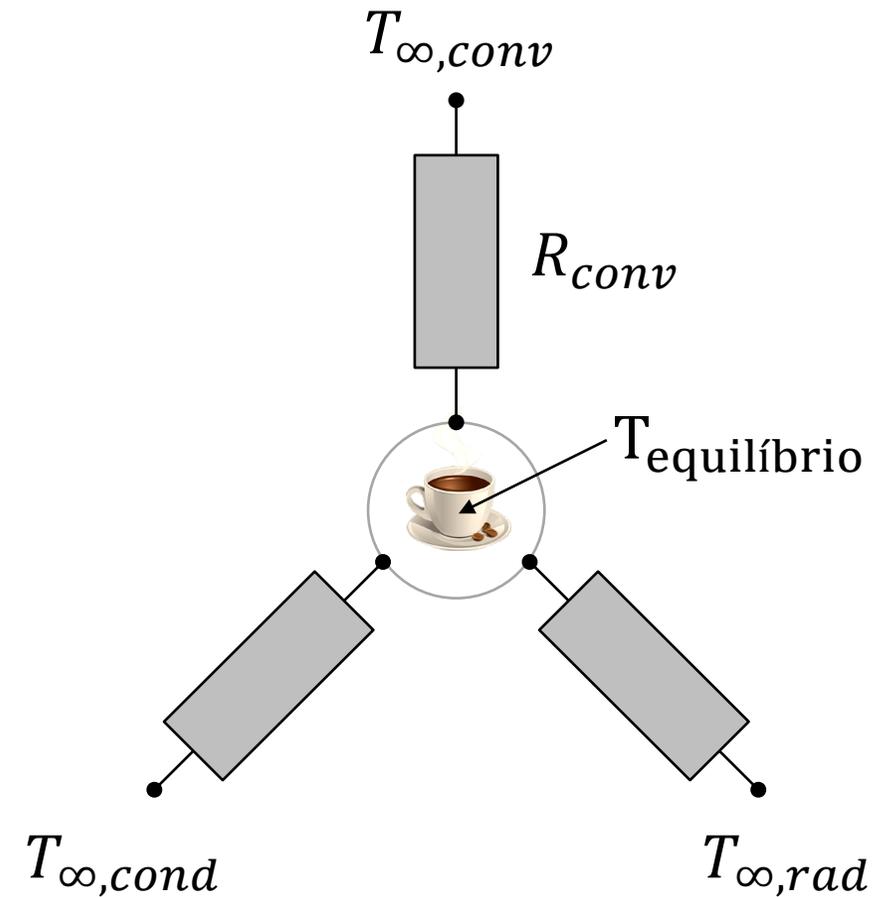
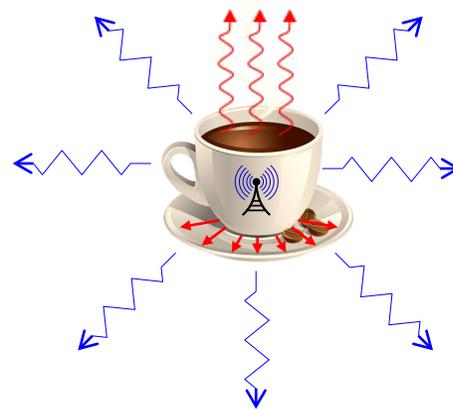
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# DEFINIÇÃO DE UMA RESISTÊNCIA TÉRMICA → CIRCUITOS TÉRMICOS EQUIVALENTES



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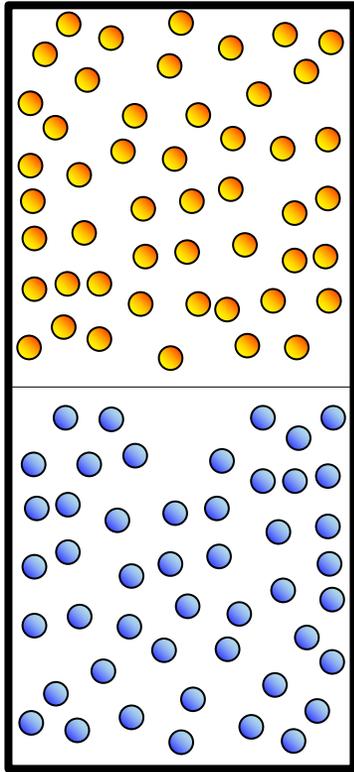


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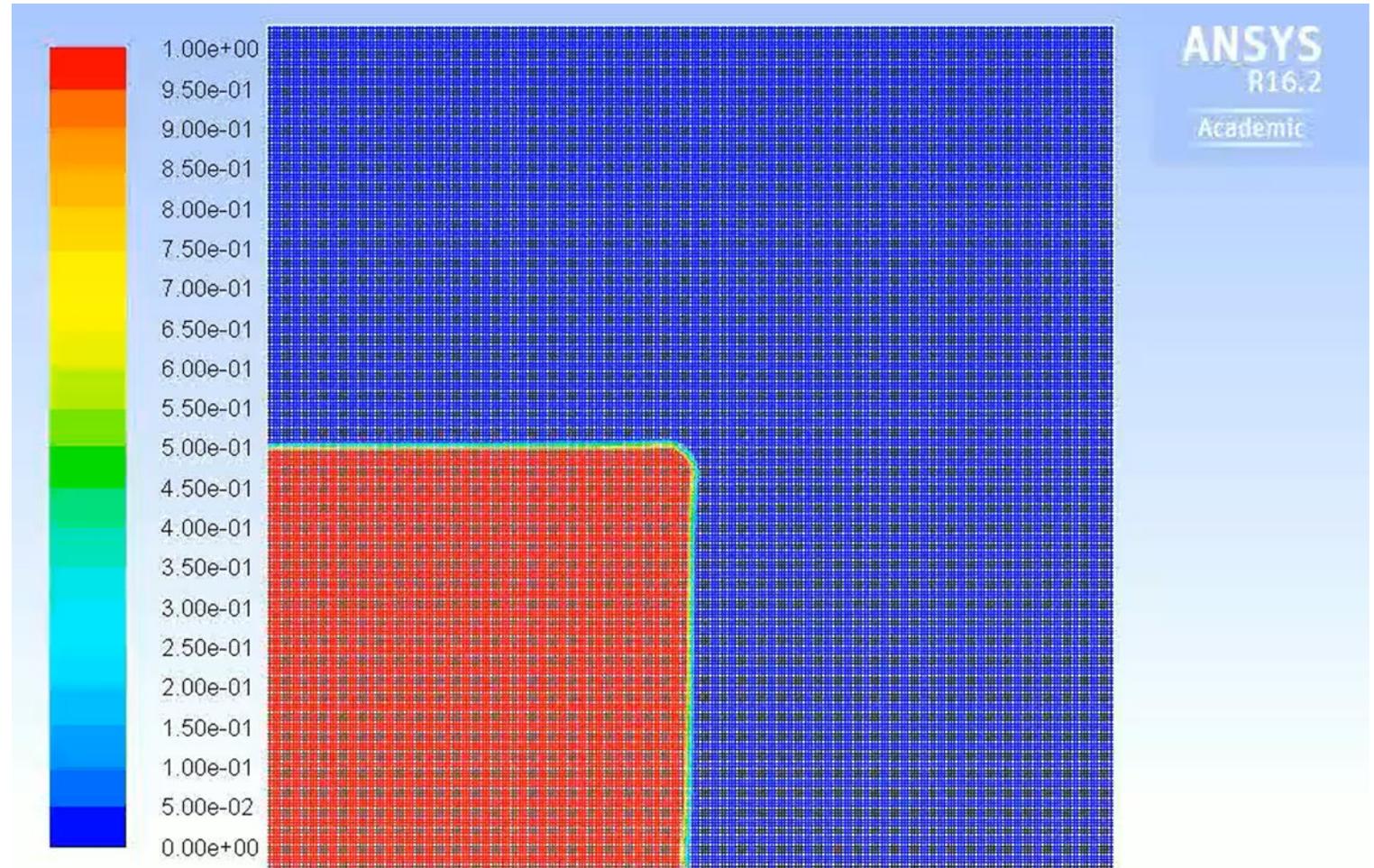
# CONVECÇÃO NATURAL × CONVECÇÃO FORÇADA...

movimentação do fluido ↔ transporte de energia térmica

# difusão



# convecção



Contours of Volume fraction (phase-2) (Time=5.0000e-02)

Mar 07, 2016

ANSYS Fluent Release 16.2 (2d, dp, pbns, vof, lam, transient)



Jingwei Zhu  
26 subscribers

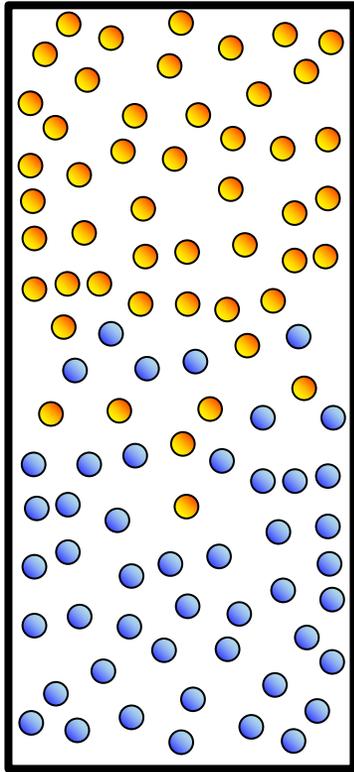
Oil-red; water-blue  
Software: ANSYS Fluent  
Oil density: 900kg/m<sup>3</sup>; viscosity: 0.1kg/m-s  
Water density: 1000kg/m<sup>3</sup>; viscosity: 0.005kg/m-s



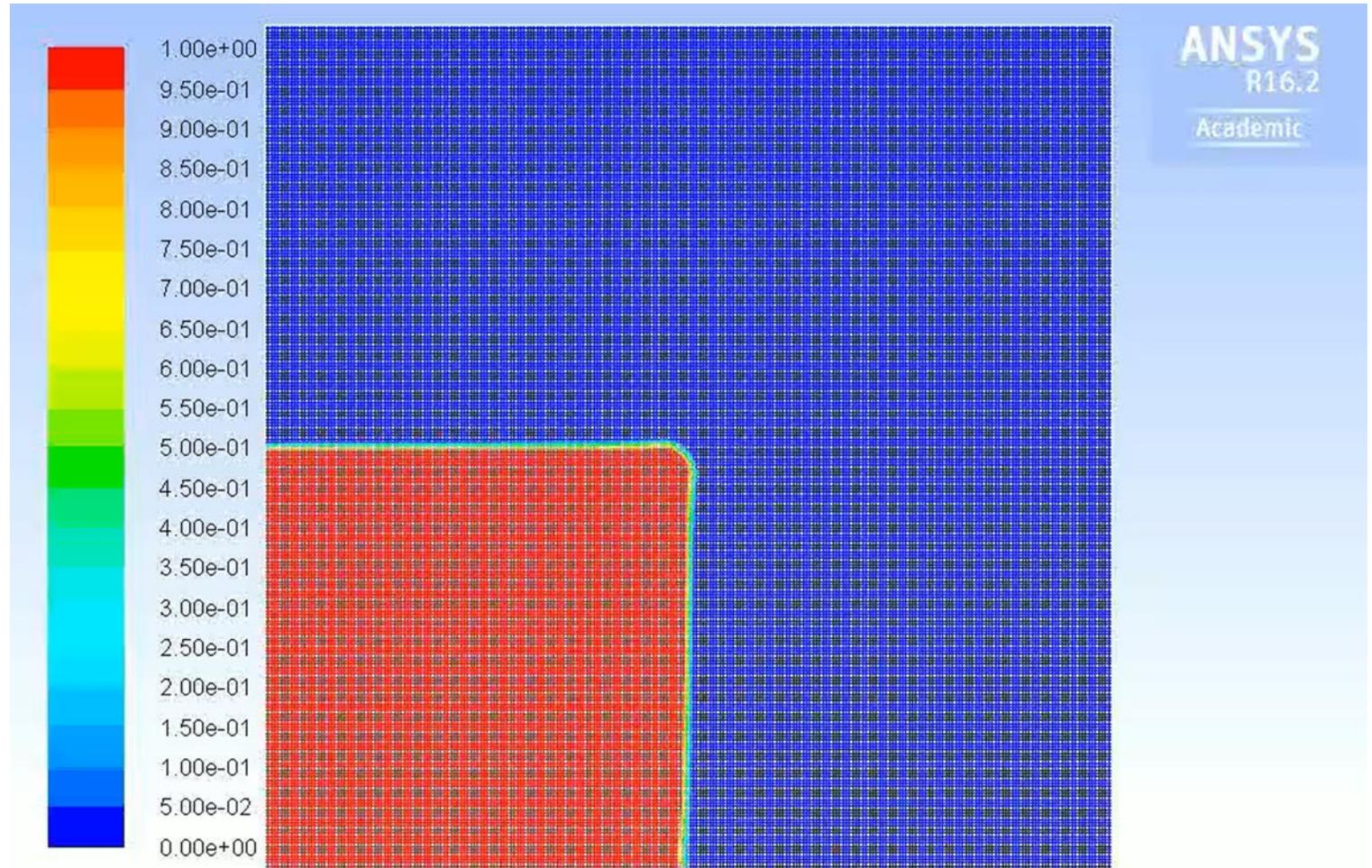
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<https://youtu.be/kNiUPuRLCXw>

# difusão



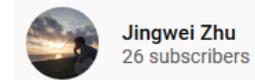
# convecção



Contours of Volume fraction (phase-2) (Time=5.0000e-02)

Mar 07, 2016

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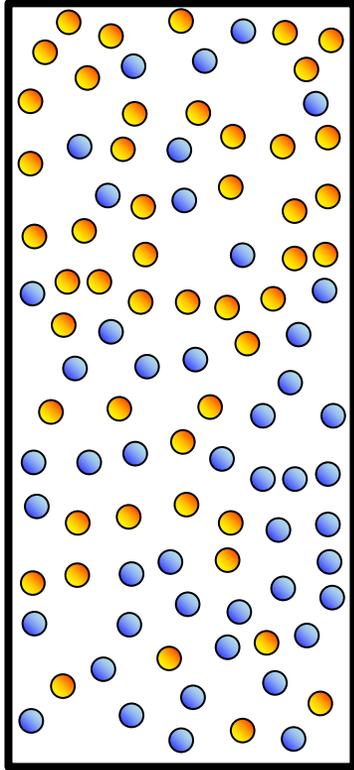
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difusão



convecção



$$F_M = -D \vec{\nabla} C$$

lei de Fick:



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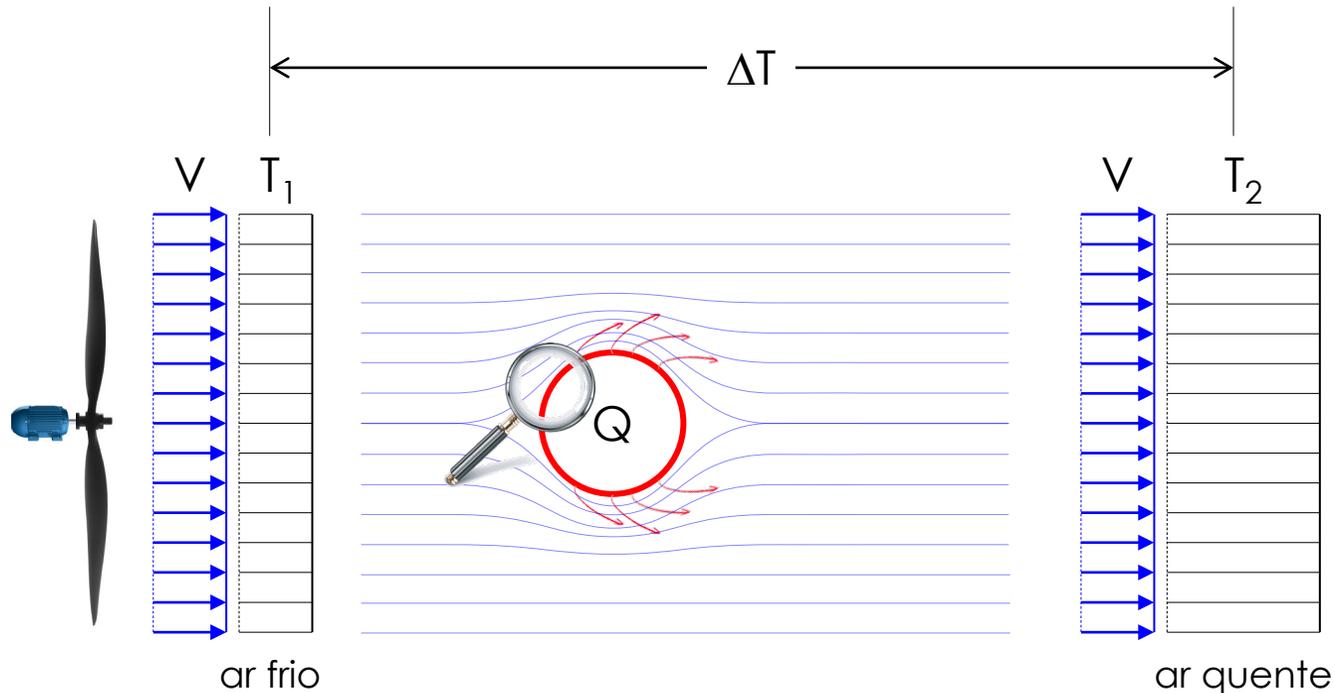
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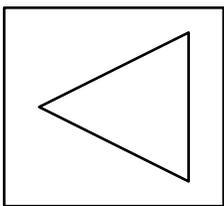
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# Convecção natural × convecção forçada...

**Forçada:** a movimentação do fluido é induzida por forças **externas**...



Estes são os efeitos globais.  
Qual é o mecanismo de transferência de calor da resistência para o ar ?  
(Efeitos locais)



$$\Delta E_{\text{Cin}} = 0$$

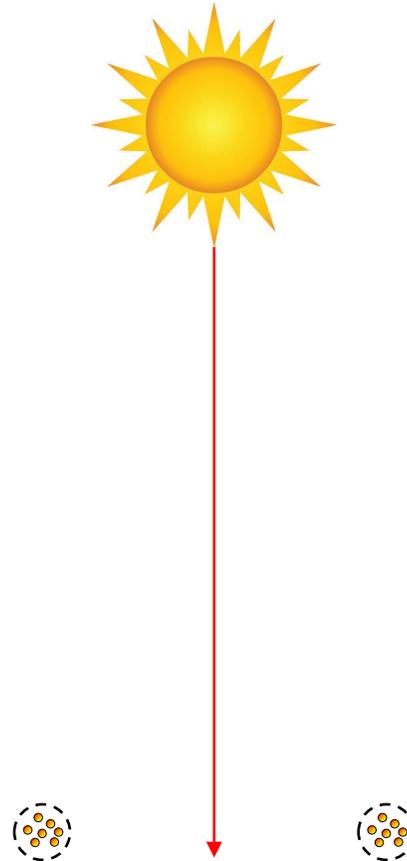
$$\Delta E_{\text{Ter}} = Q = \dot{m}_{\text{ar}} C_{P,\text{ar}} \Delta T$$



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# Convecção natural × convecção forçada...

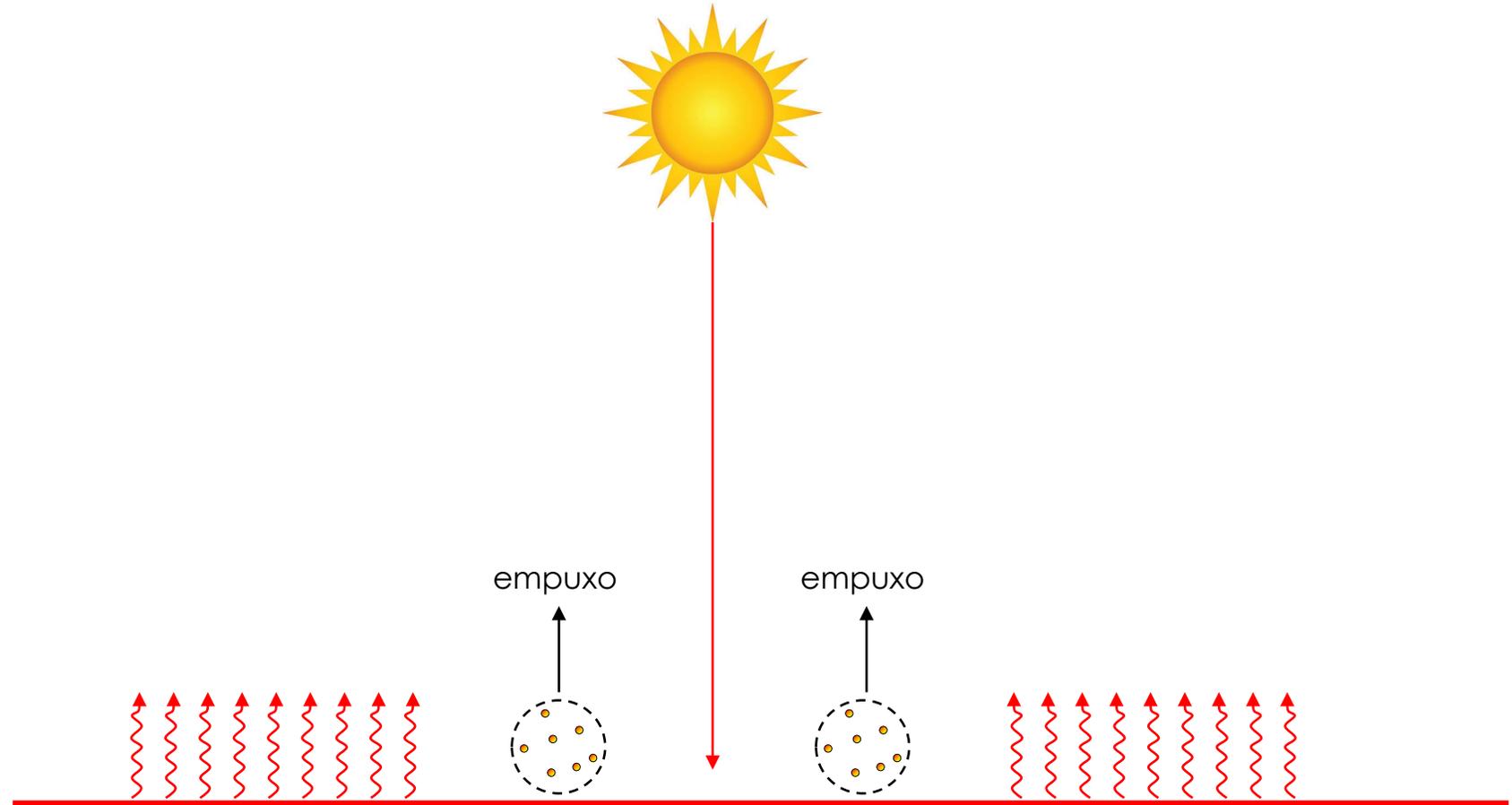
**Natural:** a movimentação do fluido é induzida por forças de **empuxo**...



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# Convecção natural × convecção forçada...

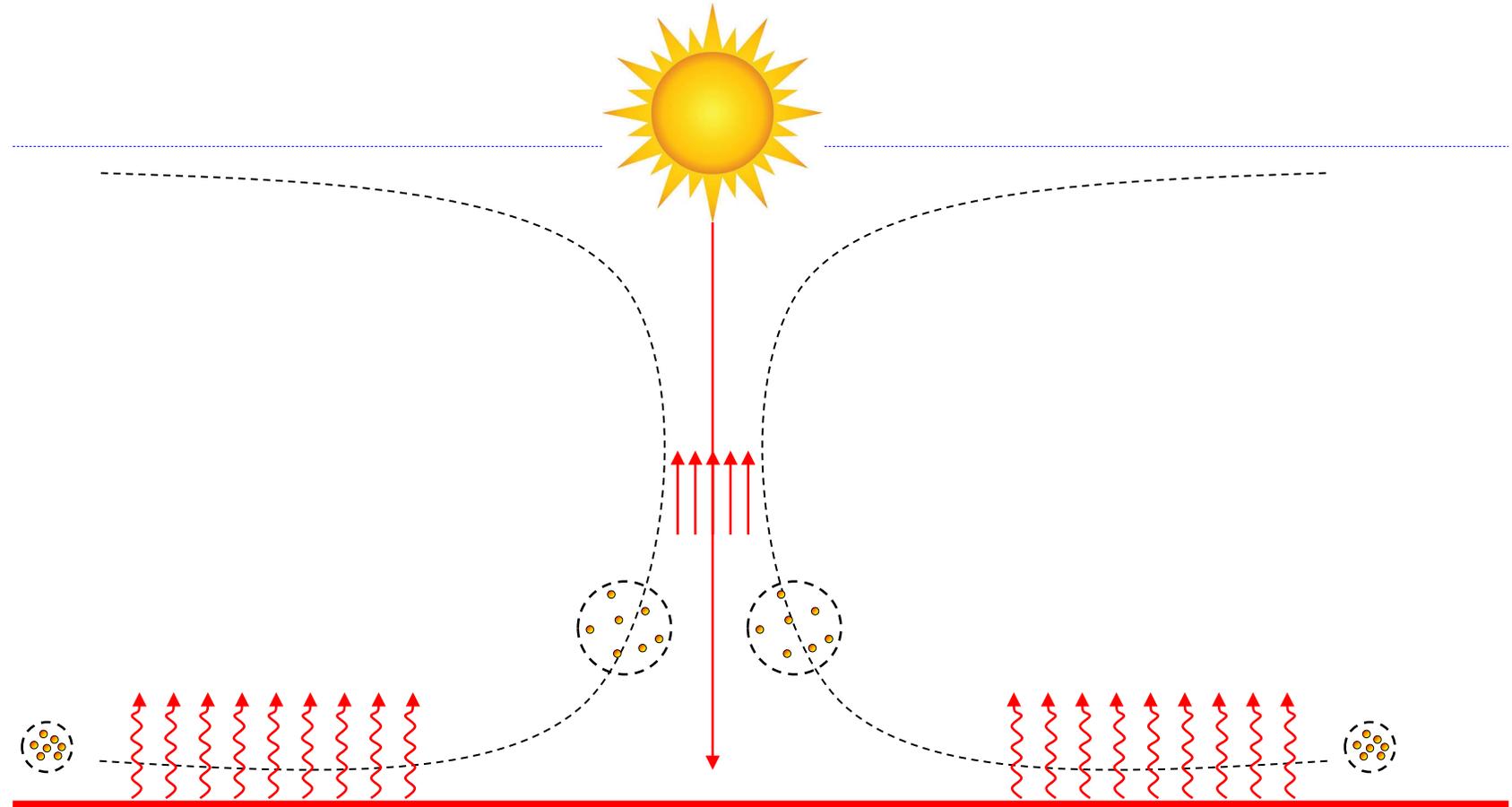
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# Convecção natural × convecção forçada...

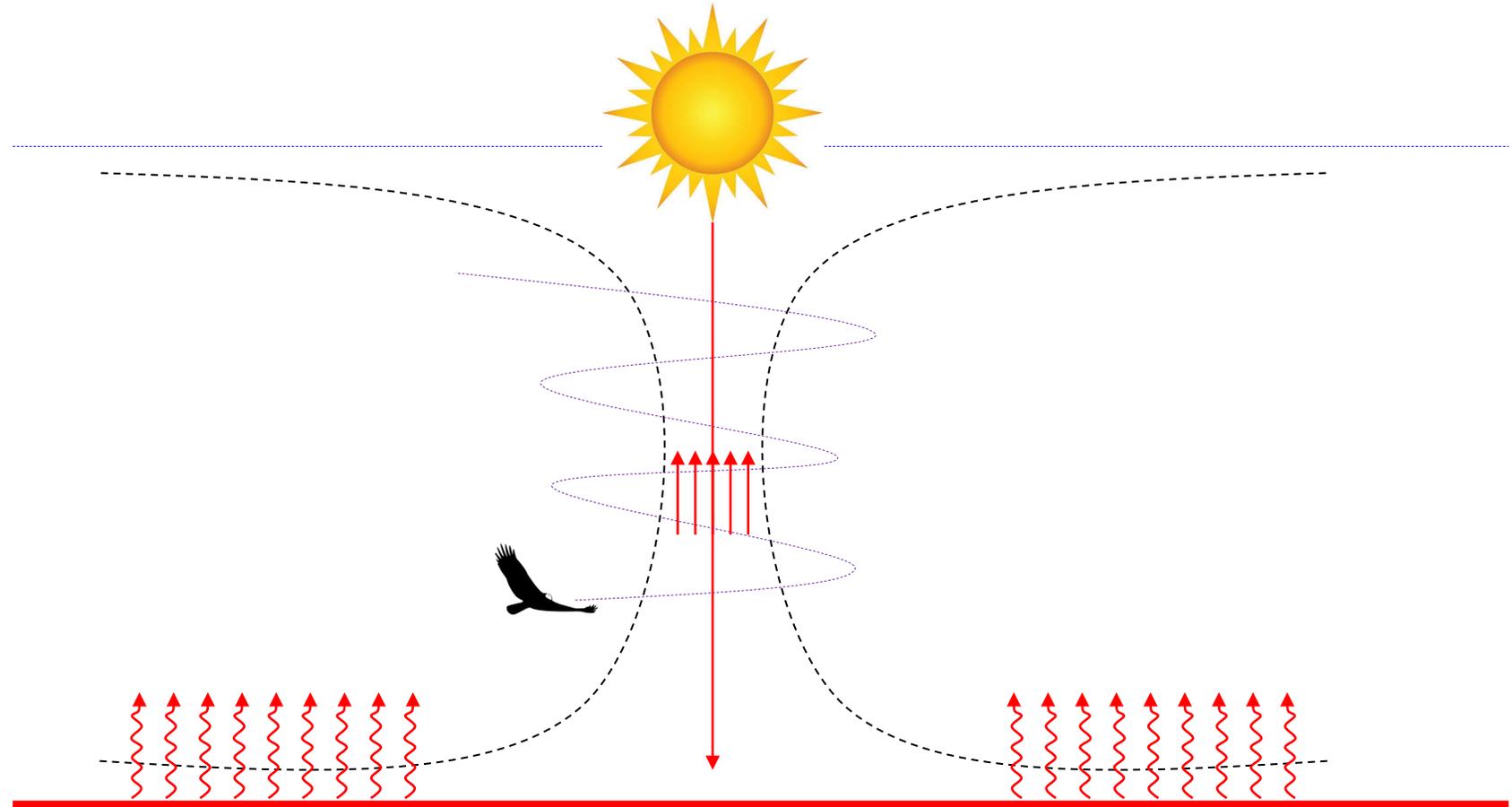
**Natural:** a movimentação do fluido é induzida por forças de **empuxo**...



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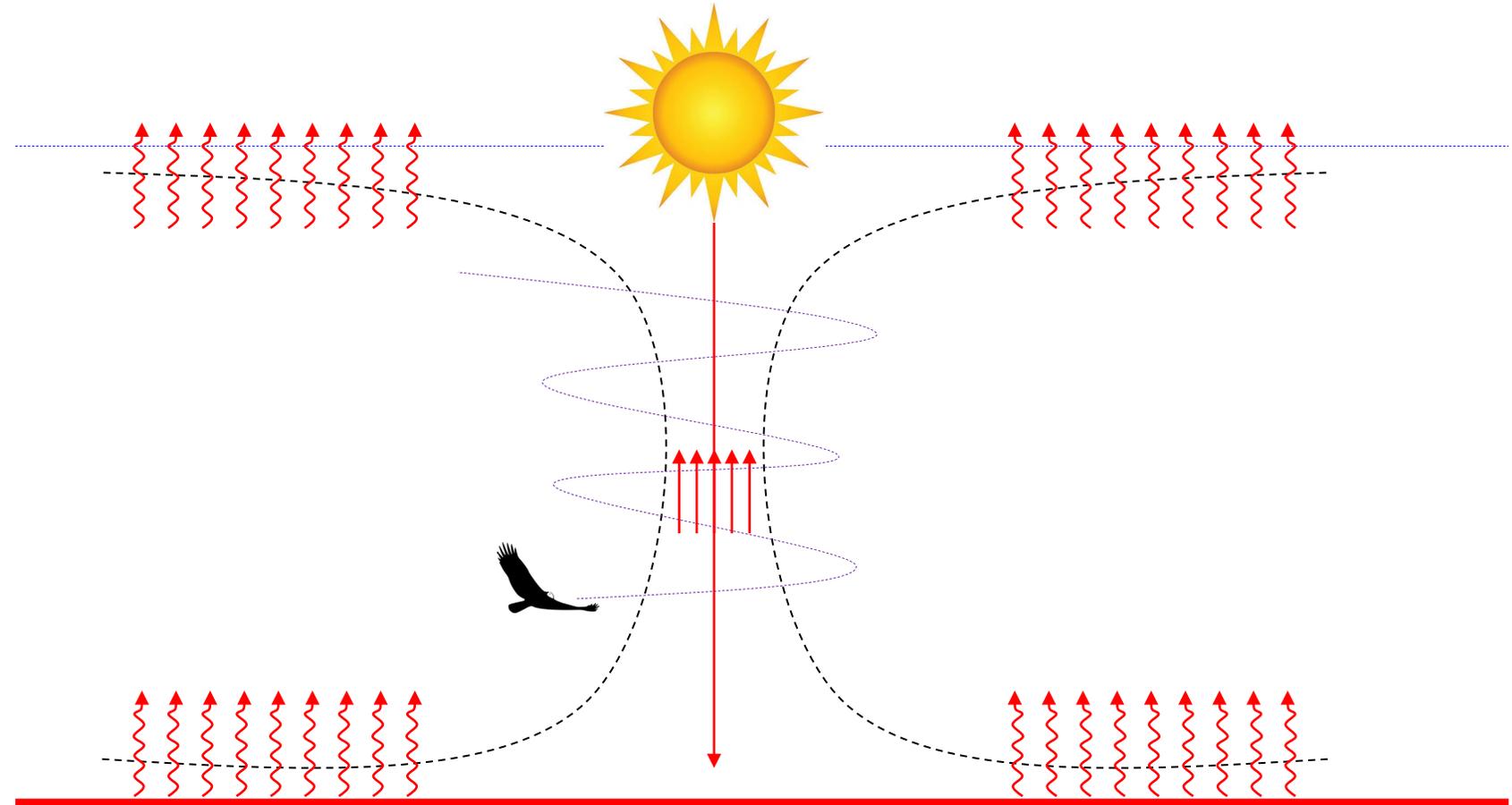
**Natural:** a movimentação do fluido é induzida por forças de **empuxo**...



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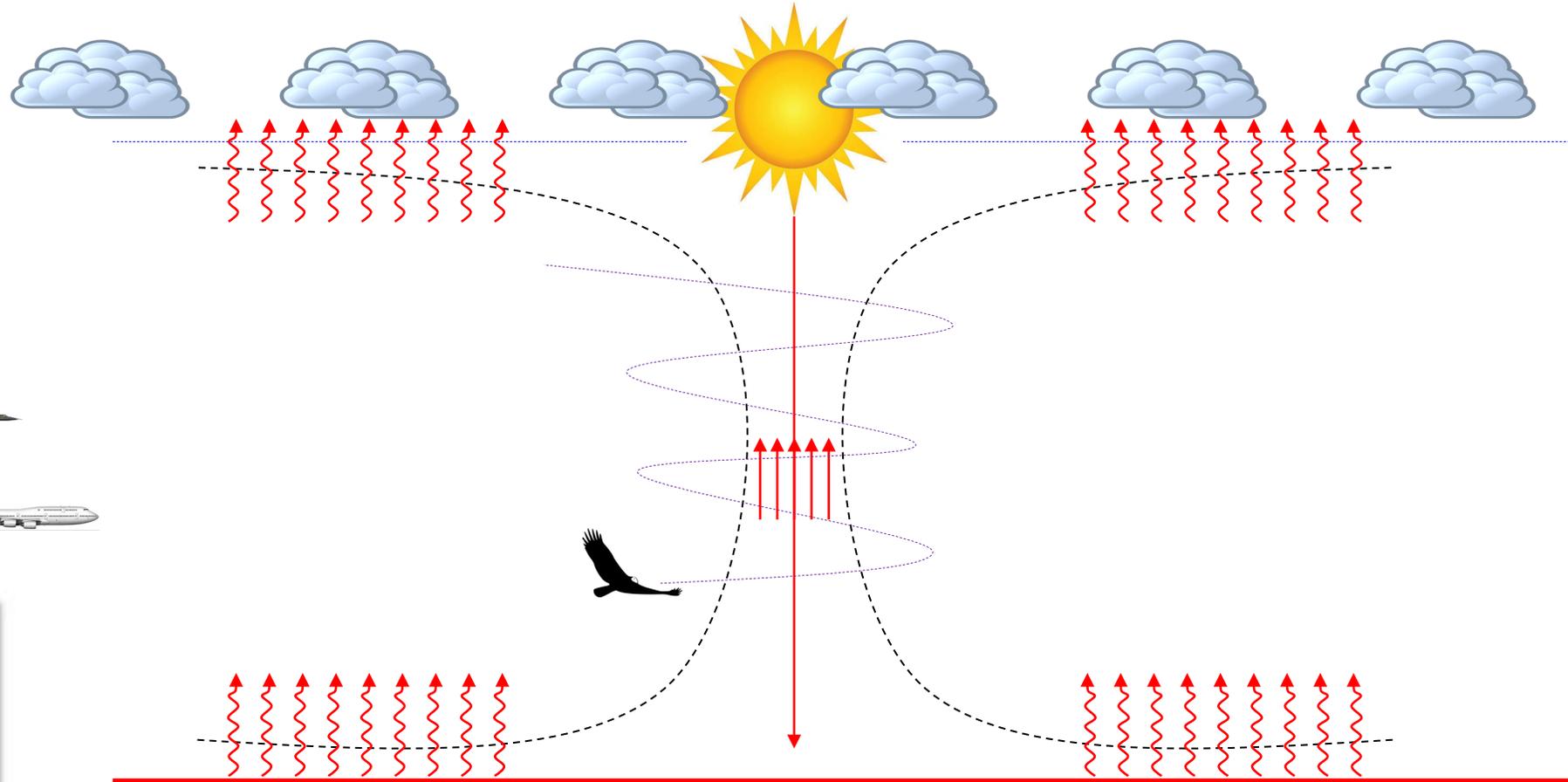
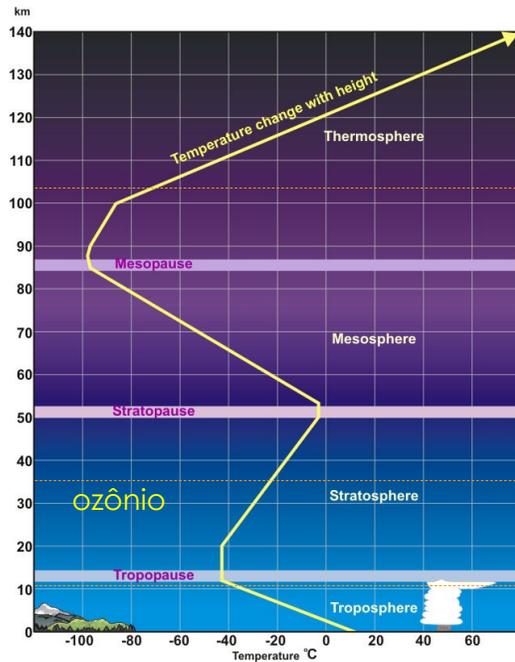
**Natural:** a movimentação do fluido é induzida por forças de **empuxo**...



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# Convecção natural × convecção forçada...

**Natural:** a movimentação do fluido é induzida por forças de **empuxo**...



**Cumulonimbus !**



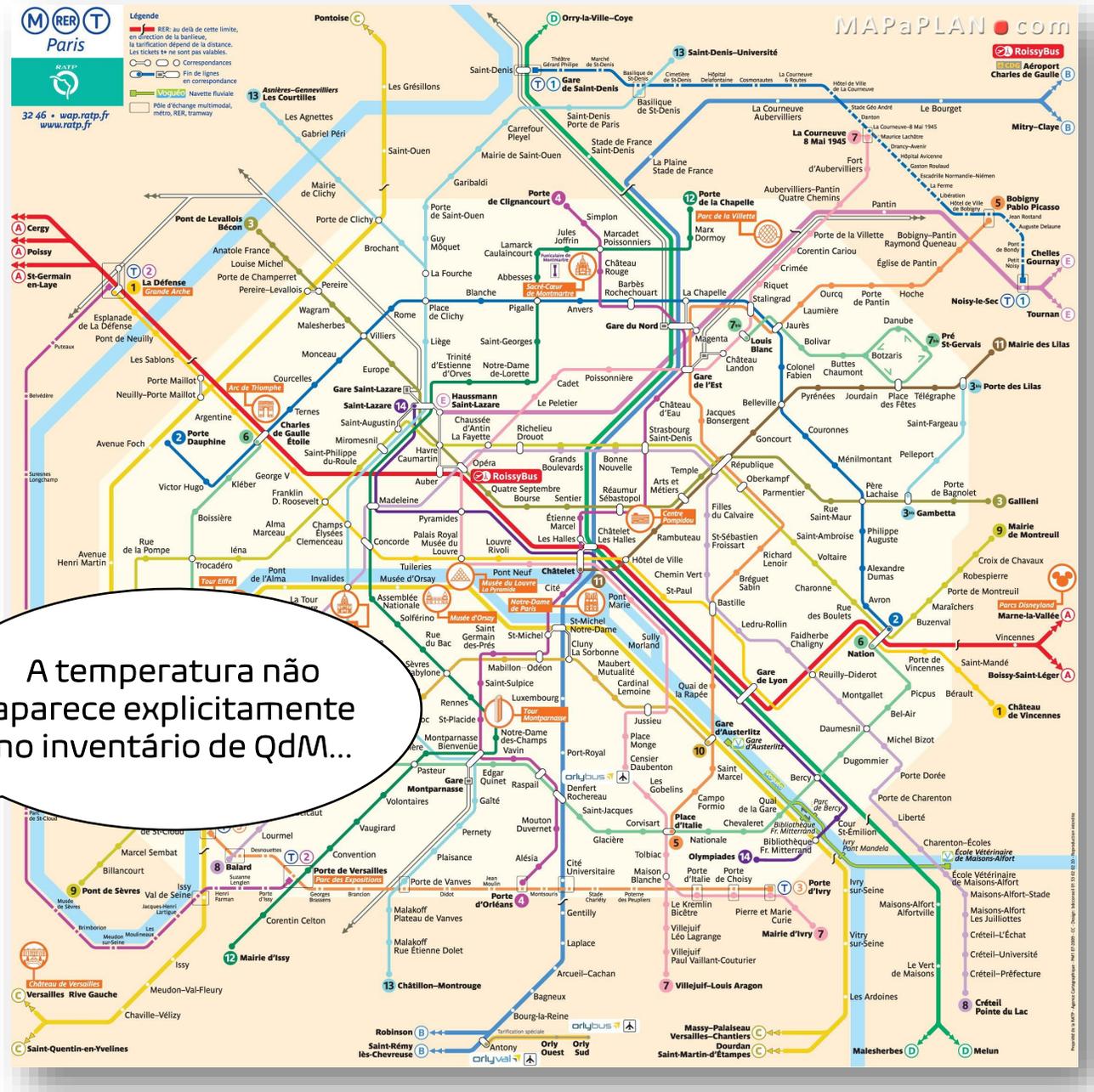
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A CONVECÇÃO DE CALOR É UM FENÔMENO ACOPLADO...

movimentação do fluido ↔ transporte de energia térmica



$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$\left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} \right) = -\vec{\nabla} P + \frac{1}{Re} \nabla^2 \vec{u} + \sum \frac{1}{R_k} F_k$$

$$\left( \frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T \right) = \frac{1}{Re Pr} \vec{\nabla} \cdot (k \vec{\nabla} T) + \frac{Ec}{Re} \Phi(\vec{u})$$

A temperatura não aparece explicitamente no inventário de QdM...



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# Escoamento incidindo sobre uma placa plana... CAMADA LIMITE



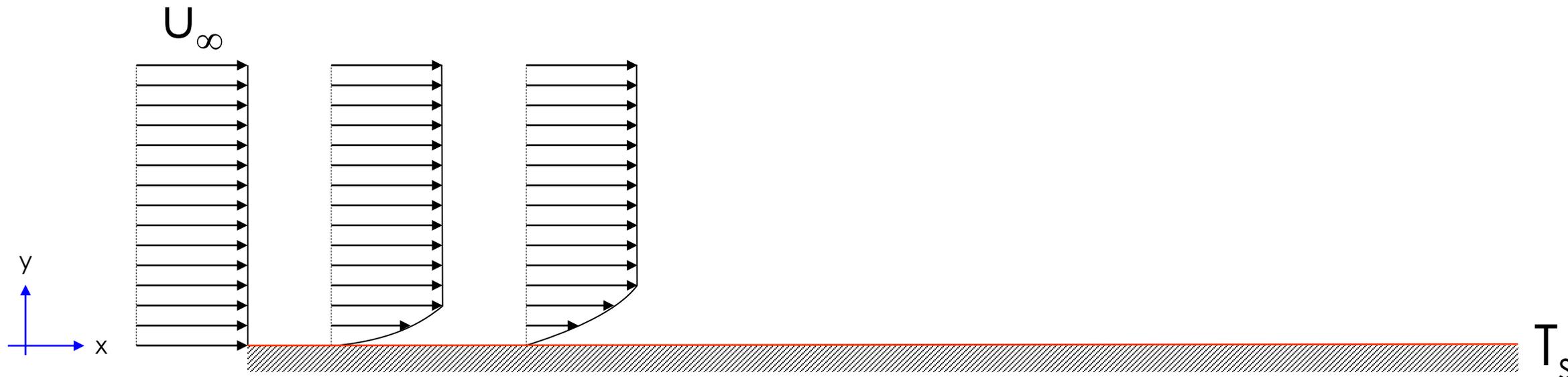
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# Escoamento incidindo sobre uma placa plana... CAMADA LIMITE



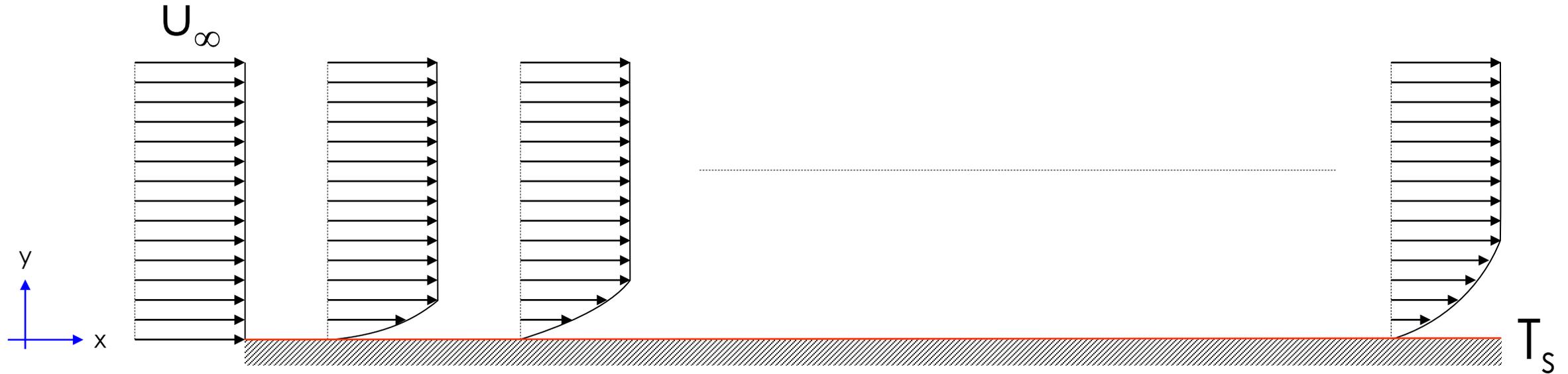
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# Escoamento incidindo sobre uma placa plana... CAMADA LIMITE



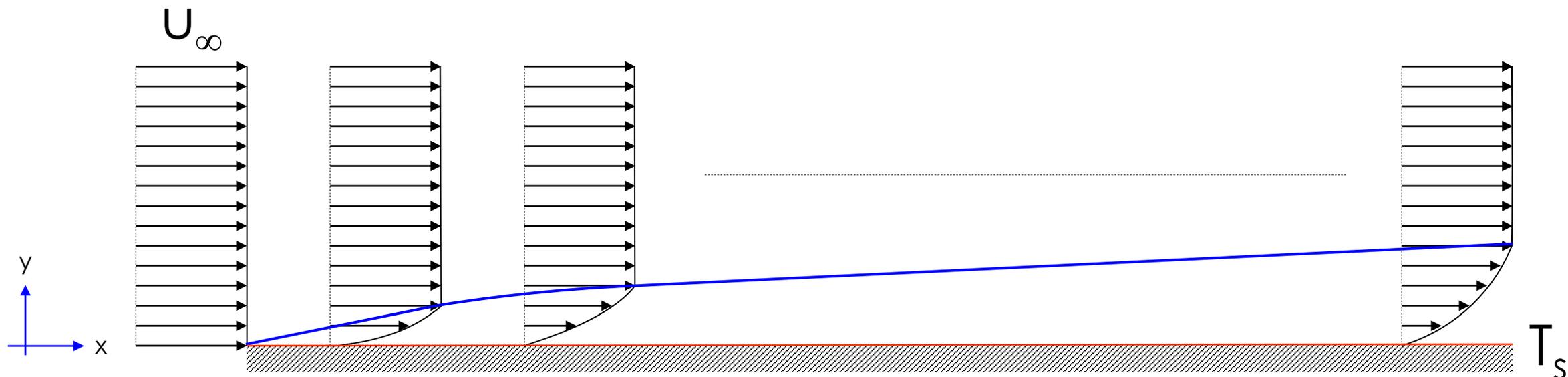
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# Escoamento incidindo sobre uma placa plana... CAMADA LIMITE



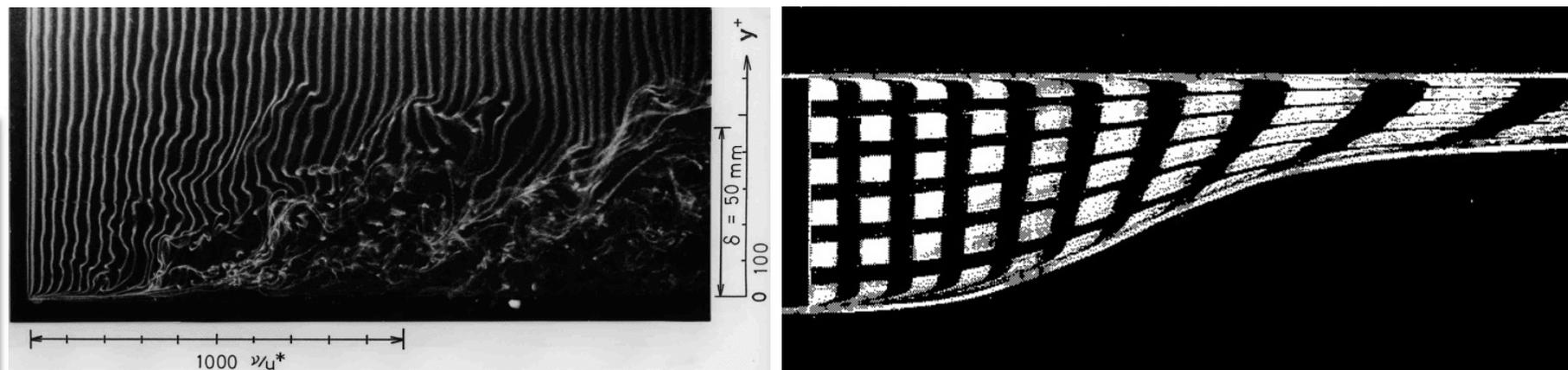
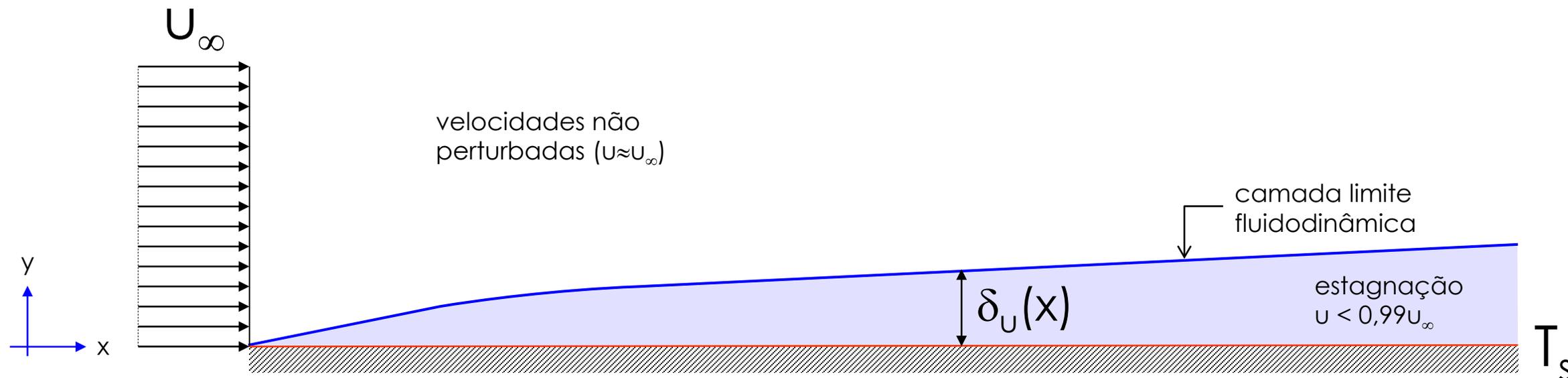
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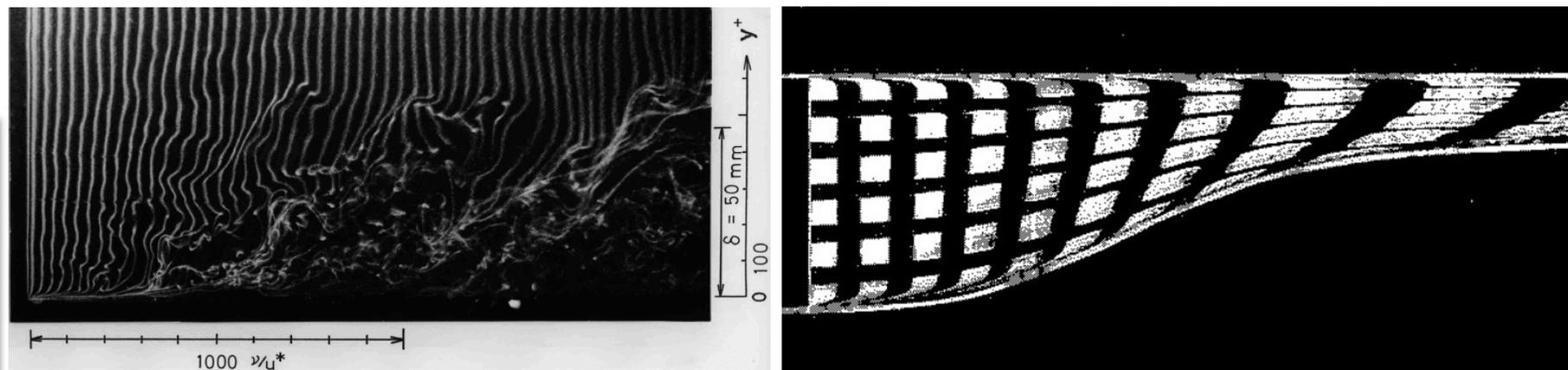
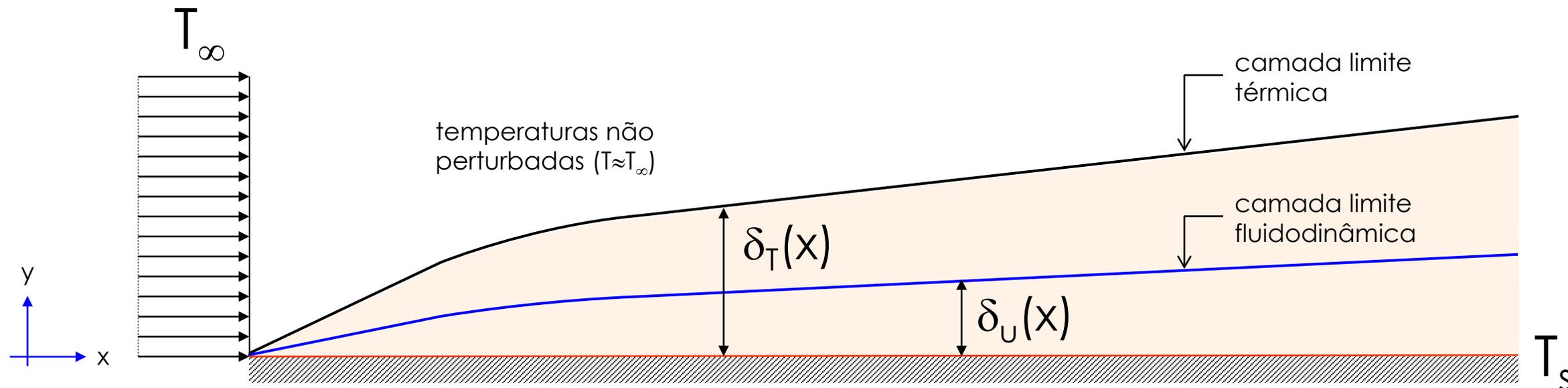
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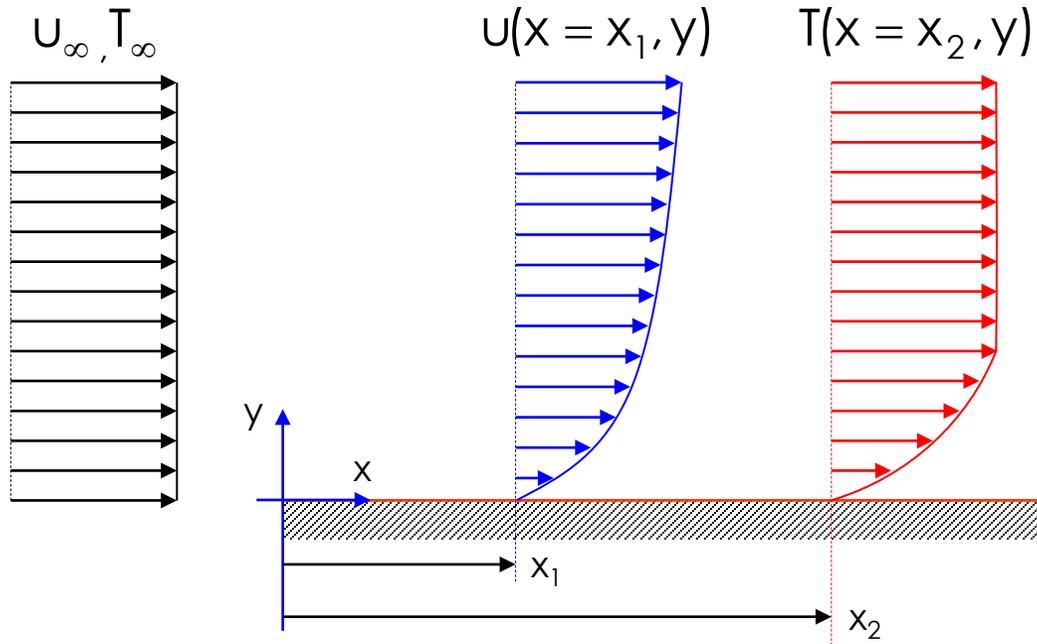
Y. IRITANI, N. KASAGI and M. HIRATA

# Escoamento incidindo sobre uma placa plana... CAMADA LIMITE



Y. IRITANI, N. KASAGI and M. HIRATA

# Cálculo das camadas limites hidrodinâmica e térmica...



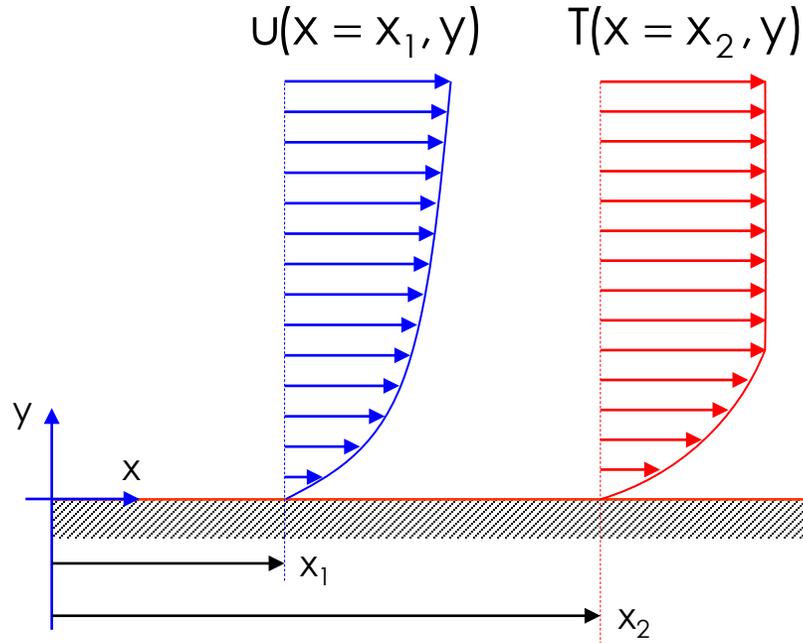
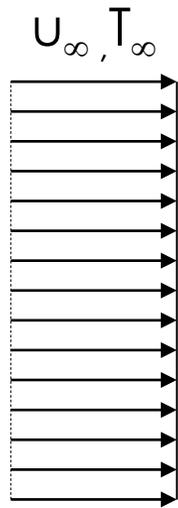
the following slides are  
**ANTI-EDUCATIONAL**

Mind the logical sequence not  
necessarily the mathematical  
manipulation...



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# Cálculo das camadas limites hidrodinâmica e térmica...



$$U \frac{\partial U}{\partial x} + v \frac{\partial U}{\partial y} = 0$$

← massa

$$U \frac{\partial U}{\partial x} + v \frac{\partial U}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 U}{\partial y^2}$$

← q. de movimento

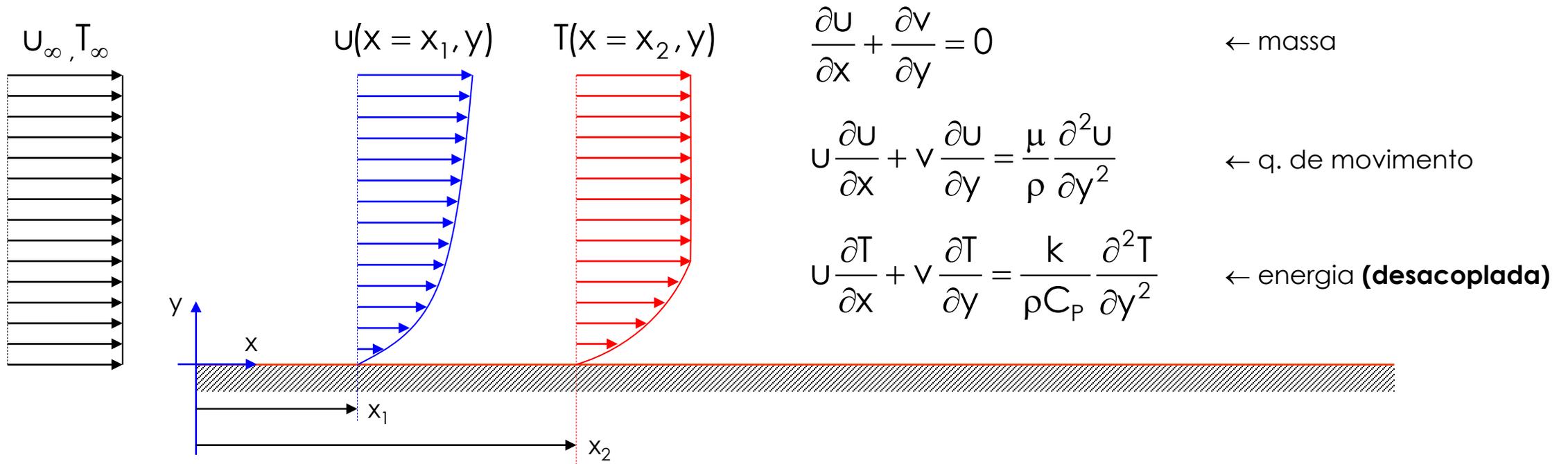
$$U \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$$

← energia



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# Cálculo das camadas limites hidrodinâmica e térmica...



$$p / x = 0 \rightarrow u(0, y) = u_\infty \text{ e } T(0, y) = T_\infty$$

$$p / y = 0 \rightarrow u(x, 0) = 0, v(x, 0) = 0 \text{ e } T(x, 0) = T_s$$

$$p / y \rightarrow \infty \rightarrow u(x, \infty) = u_\infty \text{ e } T(x, \infty) = T_\infty$$

# Solução de Blasius (1908)... velocidades

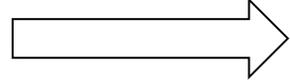


$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

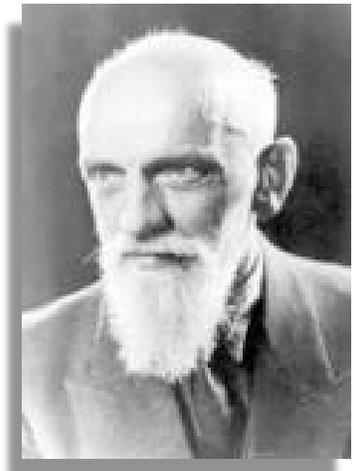


$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$

transformação



$$a_0 \cdot f(\eta) + a_1 \frac{f(\eta)}{d\eta} + a_2 \frac{f^2(\eta)}{d\eta^2} + a_3 \frac{f^3(\eta)}{d\eta^3} + \dots$$

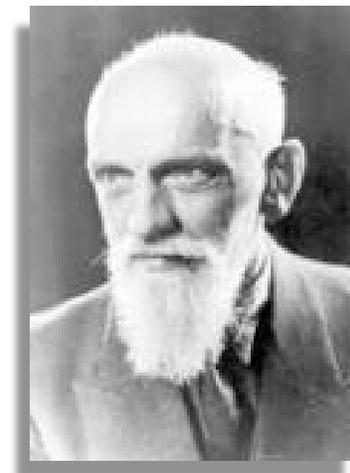


Paul R.H. Blasius  
(1883 – 1970)



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# Solução de Blasius (1908)... velocidades



Paul R.H. Blasius  
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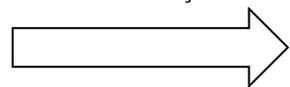


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$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$

transformação



$$a_0 \cdot f(\eta) + a_1 \frac{f(\eta)}{d\eta} + a_2 \frac{f^2(\eta)}{d\eta^2} + a_3 \frac{f^3(\eta)}{d\eta^3} + \dots$$



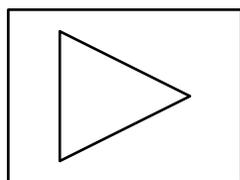
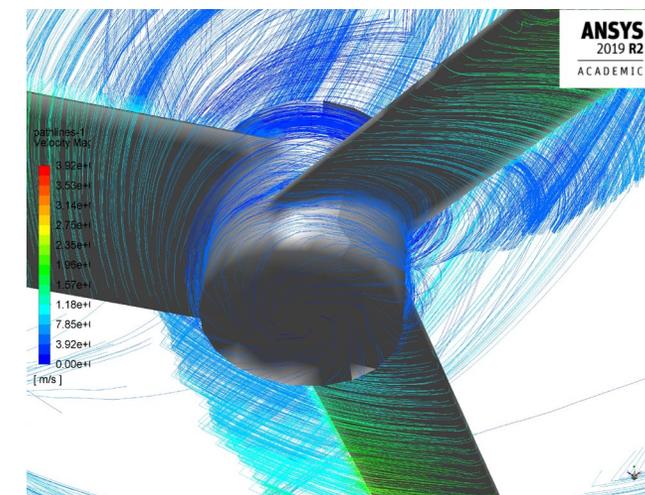
variável de similaridade



$$\eta \stackrel{\text{def}}{=} y \cdot \sqrt{\frac{U_\infty}{x \cdot \mu / \rho}}$$

$$f(\eta) \stackrel{\text{def}}{=} \frac{\psi}{U_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{U_\infty}}}$$

$\Psi$  = função de corrente



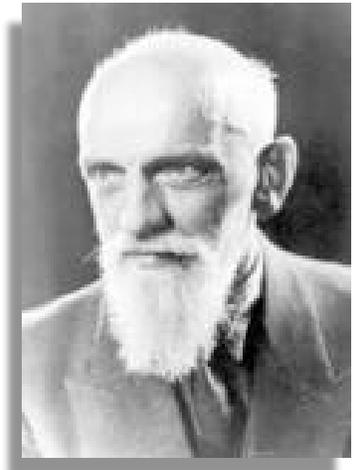
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# Solução de Blasius (1908)... **velocidades**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\psi(x, y) \stackrel{\text{def}}{=} \begin{cases} u = +\frac{\partial \psi}{\partial y} \\ v = -\frac{\partial \psi}{\partial x} \end{cases} \Rightarrow \text{🐒} \equiv 0$$

função de corrente

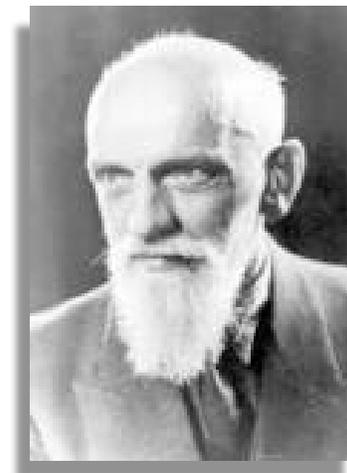


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Paul R.H. Blasius  
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função de corrente

As componentes da velocidade se escrevem então como:

$$\eta = y \cdot \sqrt{\frac{U_\infty}{x \cdot \rho / \mu}}$$

$$\psi = U_\infty \sqrt{x \cdot \frac{\mu / \rho}{U_\infty}} \cdot f(\eta)$$

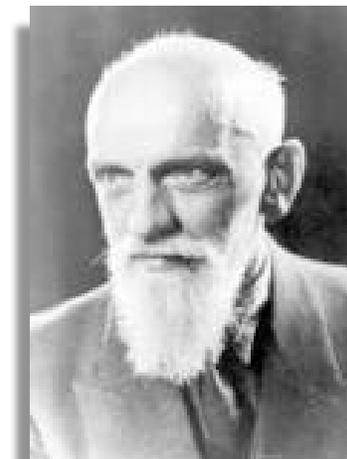
$$U = + \frac{\partial \psi}{\partial y} = + \left( \frac{\partial \psi}{\partial \eta} \right) \cdot \left( \frac{\partial \eta}{\partial y} \right) = \left( U_\infty \sqrt{\frac{xv}{U_\infty}} \cdot \frac{df}{d\eta} \right) \cdot \left( \sqrt{\frac{U_\infty}{xv}} \right) = U_\infty \frac{df}{d\eta}$$

$$v = \mu / \rho$$



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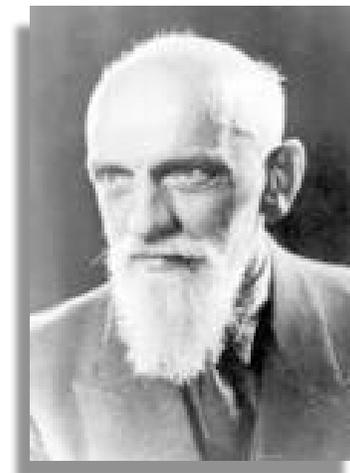
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$$v = - \frac{\partial \psi}{\partial x} = - U_\infty \sqrt{\frac{v}{U_\infty}} \cdot f(\eta) \cdot \left( \frac{x^{-1/2}}{2} \right) - \left( U_\infty \sqrt{x \cdot \frac{v}{U_\infty}} \cdot \frac{\partial f}{\partial x} \right) = \frac{1}{2} \sqrt{\frac{U_\infty v}{x}} \cdot \left( \eta \frac{df}{d\eta} - f \right)$$

$$v = \mu / \rho$$



# Solução de Blasius (1908)... **velocidades**



Paul R.H. Blasius  
(1883 – 1970)

$$\psi(x, y) \stackrel{\text{def}}{=} \begin{cases} U = + \frac{\partial \psi}{\partial y} \\ v = - \frac{\partial \psi}{\partial x} \end{cases} \Rightarrow \text{🙈} \equiv 0$$

função de corrente

As componentes da velocidade se escrevem então como:

$$\eta = y \cdot \sqrt{\frac{U_\infty}{x \cdot \rho / \mu}}$$

$$\psi = U_\infty \sqrt{x \cdot \frac{\mu / \rho}{U_\infty}} \cdot f(\eta)$$

$$U = + \frac{\partial \psi}{\partial y} = + \left( \frac{\partial \psi}{\partial \eta} \right) \cdot \left( \frac{\partial \eta}{\partial y} \right) = \left( U_\infty \sqrt{\frac{xv}{U_\infty}} \cdot \frac{df}{d\eta} \right) \cdot \left( \sqrt{\frac{U_\infty}{xv}} \right) = U_\infty \frac{df}{d\eta}$$

$$v = - \frac{\partial \psi}{\partial x} = - U_\infty \sqrt{\frac{v}{U_\infty}} \cdot f(\eta) \cdot \left( \frac{x^{-1/2}}{2} \right) - \left( U_\infty \sqrt{x \cdot \frac{v}{U_\infty}} \cdot \frac{\partial f}{\partial x} \right) = \frac{1}{2} \sqrt{\frac{U_\infty v}{x}} \cdot \left( \eta \frac{df}{d\eta} - f \right)$$

$$v = \mu / \rho$$

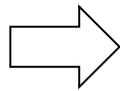
$$\begin{matrix} U = \dots \\ v = \dots \end{matrix} \Rightarrow U \frac{\partial U}{\partial x} + v \frac{\partial U}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 U}{\partial y^2} \text{ 🙈}$$

# Solução de Blasius (1908)... **velocidades**

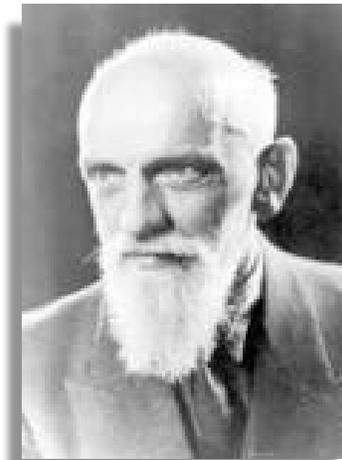
$$\frac{\partial u}{\partial x} = -\frac{u_{\infty}}{2x} \eta \frac{d^2 f}{d\eta^2}$$

$$\frac{\partial u}{\partial y} = u_{\infty} \sqrt{\frac{u_{\infty}}{\nu x}} \frac{d^2 f}{d\eta^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{\infty}^2}{\nu x} \frac{d^3 f}{d\eta^3}$$



$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$



Paul R.H. Blasius  
(1883 – 1970)



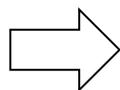
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# Solução de Blasius (1908)... **velocidades**

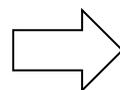
$$\frac{\partial u}{\partial x} = -\frac{u_\infty}{2x} \eta \frac{d^2 f}{d\eta^2}$$

$$\frac{\partial u}{\partial y} = u_\infty \sqrt{\frac{u_\infty}{\nu x}} \frac{d^2 f}{d\eta^2}$$

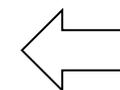
$$\frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^2}{\nu x} \frac{d^3 f}{d\eta^3}$$



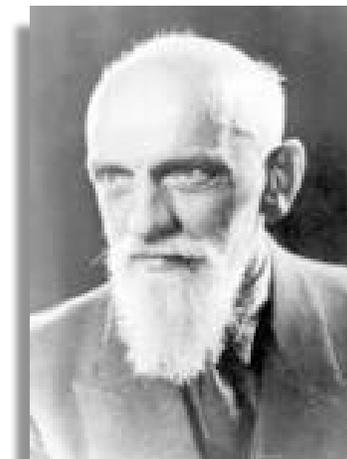
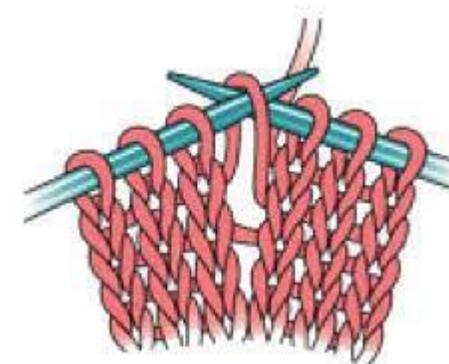
$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$



$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$



DIFERENÇAS FINITAS



Paul R.H. Blasius  
(1883 – 1970)



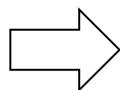
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# Solução de Blasius (1908)... **velocidades**

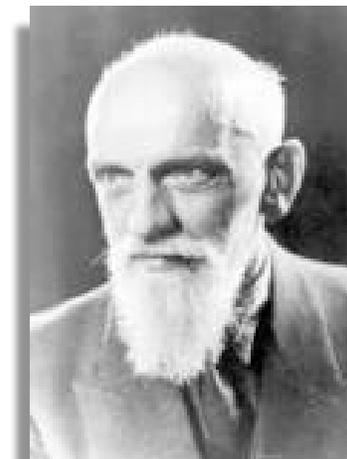
$$\frac{\partial u}{\partial x} = -\frac{u_{\infty}}{2x} \eta \frac{d^2 f}{d\eta^2}$$

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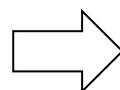
$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{\infty}^2}{\nu x} \frac{d^3 f}{d\eta^3}$$



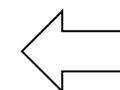
$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$



Paul R.H. Blasius  
(1883 – 1970)



$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$



DIFERENÇAS FINITAS

$$p/x = 0 \rightarrow u(0, y) = u_{\infty}$$

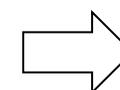
$$p/y = 0 \rightarrow u(x, 0) = 0, v(x, 0) = 0$$

$$p/y \rightarrow \infty \rightarrow u(x, \infty) = u_{\infty}$$

$$p/\eta = 0 \rightarrow f(\eta) = 0$$

$$p/\eta = 0 \rightarrow df/d\eta = 0$$

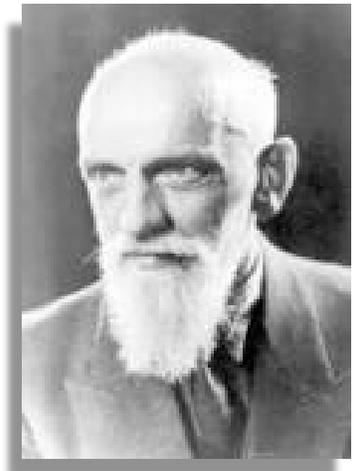
$$p/\eta = \infty \rightarrow df/d\eta = 1$$



# Solução de Blasius (1908)... **c. limite fluidodinâmica**

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$

$\eta$	$f$	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2 f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
2.0	0.650	0.630	0.267
2.5	0.996	0.751	0.217
3.0	1.397	0.846	0.161
3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
4.5	2.790	0.980	0.034
5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
$\infty$	$\infty$	1	0



Paul R.H. Blasius  
(1883 – 1970)



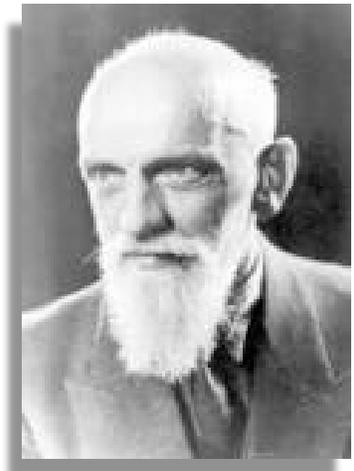
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# Solução de Blasius (1908)... **c. limite fluidodinâmica**

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$

$$\delta \Rightarrow u/u_\infty = 0.99 \Rightarrow \eta = 5.0$$

$\eta$	$f$	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2 f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
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$\infty$	$\infty$	1	0



Paul R.H. Blasius  
(1883 – 1970)



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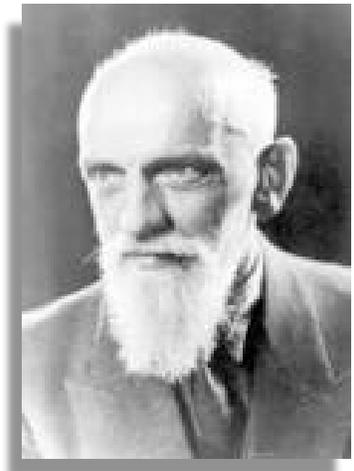
# Solução de Blasius (1908)... **c. limite fluidodinâmica**

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$

$$\delta \Rightarrow u/u_\infty = 0.99 \Rightarrow \eta = 5.0$$

$$\eta = y \cdot \sqrt{\frac{u_\infty}{xv}} \Rightarrow \delta = 5.0 \cdot \sqrt{\frac{xv}{u_\infty}}$$

$\eta$	$f$	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2 f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
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Paul R.H. Blasius  
(1883 – 1970)



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# Solução de Blasius (1908)... **c. limite fluidodinâmica**

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$

$$\delta \Rightarrow u/u_\infty = 0.99 \Rightarrow \eta = 5.0$$

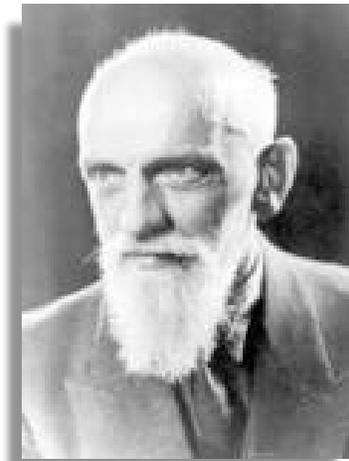
$$\eta = y \cdot \sqrt{\frac{u_\infty}{x\nu}} \Rightarrow \delta = 5.0 \cdot \sqrt{\frac{x\nu}{u_\infty}}$$



$$\delta = \frac{5.0 \cdot x}{\sqrt{Re_x}}$$

$$Re = \frac{\rho u_\infty x}{\mu}$$

$\eta$	$f$	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2 f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
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$\infty$	$\infty$	1	0



Paul R.H. Blasius  
(1883 – 1970)



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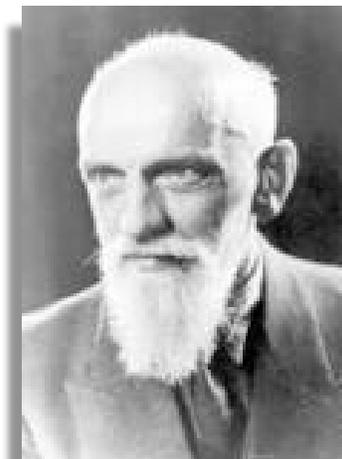
# Solução de Blasius (1908)... **tensões de cisalhamento**

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$

$$\tau_x = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \mu U_\infty \sqrt{\frac{U_\infty}{\nu x}} \left. \frac{d^2 f}{d\eta^2} \right|_{\eta=0}$$

$$\tau_x = 0.332 \frac{\rho U_\infty^2}{\sqrt{Re_x}}$$

$\eta$	$f$	$\frac{df}{d\eta} = \frac{u}{U_\infty}$	$\frac{d^2 f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
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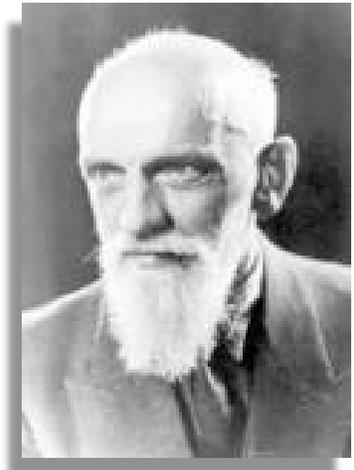
Paul R.H. Blasius  
(1883 – 1970)



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# Solução de Blasius (1908)... **c. limite térmica**

$$U \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$$



Paul R.H. Blasius  
(1883 – 1970)

$$\psi = U_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{U_\infty}} \cdot f(\eta)$$



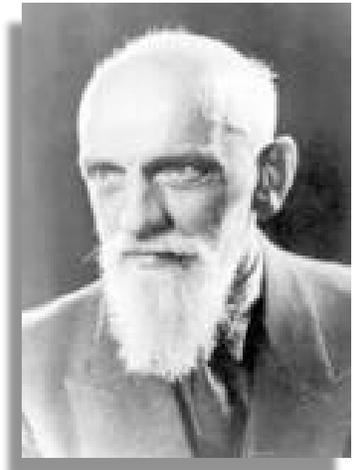
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# Solução de Blasius (1908)... **c. limite térmica**

$$\psi(x, y) \stackrel{\text{def}}{=} \begin{cases} u = +\frac{\partial\psi}{\partial y} \\ v = -\frac{\partial\psi}{\partial x} \end{cases}$$



$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$$



Paul R.H. Blasius  
(1883 – 1970)

$$\psi = U_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{U_\infty}} \cdot f(\eta)$$



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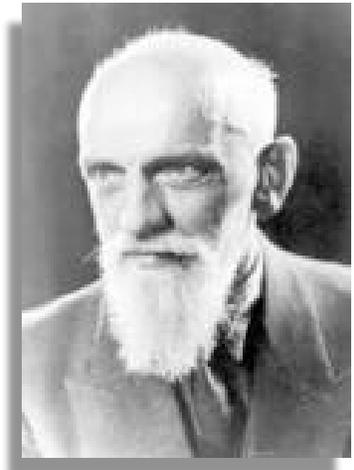
# Solução de Blasius (1908)... **c. limite térmica**

$$\psi(x, y) \stackrel{\text{def}}{=} \begin{cases} u = +\frac{\partial\psi}{\partial y} \\ v = -\frac{\partial\psi}{\partial x} \end{cases}$$



$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$$

$$\theta(x, y) \stackrel{\text{def}}{=} \frac{T(x, y) - T_s}{T_\infty - T_s} \leftarrow$$



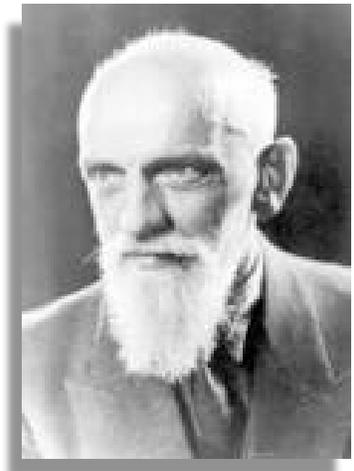
Paul R.H. Blasius  
(1883 – 1970)

$$\psi = U_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{U_\infty}} \cdot f(\eta)$$



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# Solução de Blasius (1908)... c. limite térmica



Paul R.H. Blasius  
(1883 – 1970)

$$\psi(x, y) \stackrel{\text{def}}{=} \begin{cases} u = + \frac{\partial \psi}{\partial y} \\ v = - \frac{\partial \psi}{\partial x} \end{cases}$$



$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$$

$$\theta(x, y) \stackrel{\text{def}}{=} \frac{T(x, y) - T_s}{T_\infty - T_s}$$



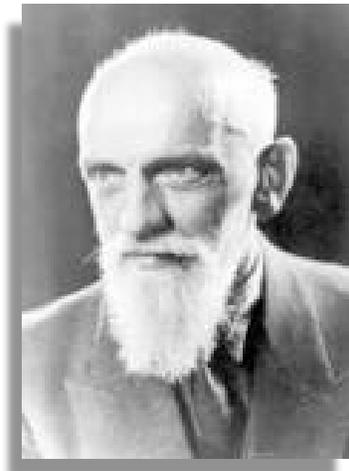
$$\psi = U_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{U_\infty}} \cdot f(\eta)$$

$$U_\infty \frac{df}{d\eta} \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{1}{2} \sqrt{\frac{U_\infty \nu}{x}} \left( \eta \frac{df}{d\eta} - f \right) \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial y} = \frac{k}{\rho C_p} \frac{d^2 \theta}{d\eta^2} \left( \frac{\partial \eta}{\partial y} \right)^2$$



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# Solução de Blasius (1908)... c. limite térmica



Paul R.H. Blasius  
(1883 – 1970)

$$u_{\infty} \frac{df}{d\eta} \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{1}{2} \sqrt{\frac{u_{\infty} \nu}{x}} \left( \eta \frac{df}{d\eta} - f \right) \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial y} = \frac{k}{\rho C_p} \frac{d^2 \theta}{d\eta^2} \left( \frac{\partial \eta}{\partial y} \right)^2$$

⋮

$$Pr = \frac{\mu}{k/C_p} \Rightarrow$$

$$2 \frac{d^2 \theta}{d\eta^2} + Pr \cdot f \frac{d\theta}{d\eta} = 0$$

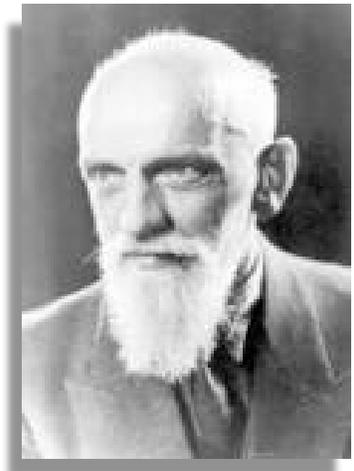
$$\theta(0) = 0 \quad e \quad \theta(\infty) = 1$$

$\eta$	$f$	$\frac{df}{d\eta} = \frac{u}{u_{\infty}}$	$\frac{d^2 f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
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4.0	2.306	0.956	0.064
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5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
$\infty$	$\infty$	1	0

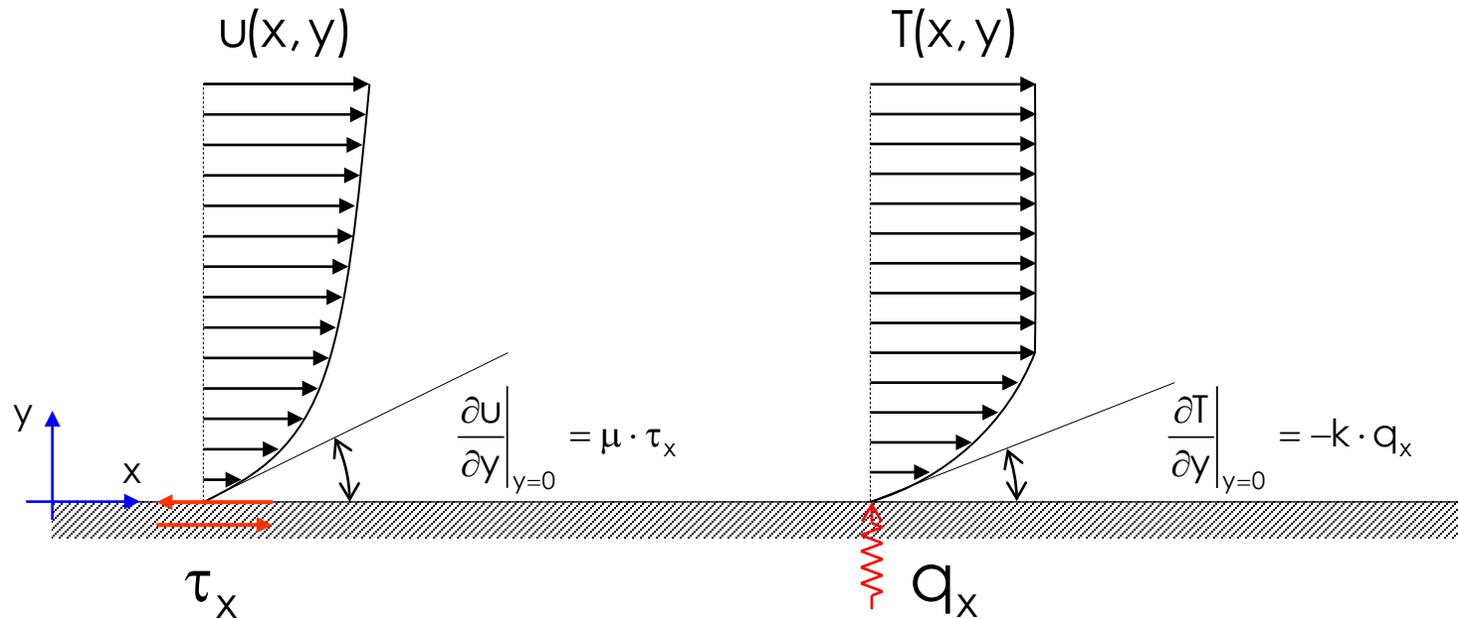


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# Solução de Blasius (1908)... c. limite térmica



Paul R.H. Blasius  
(1883 – 1970)



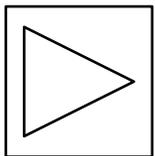
$$\tau_x = 0.332 \frac{\rho U_\infty^2}{\sqrt{Re_x}}$$

$$-\frac{q_x}{k} = 0.332 \cdot Pr^{1/3} (T_\infty - T_s) \sqrt{\frac{U_\infty}{\nu x}}$$

$$\delta_{FD} = \frac{5.0 \cdot x}{\sqrt{Re_x}}$$

$$\delta_{TM} = \frac{5.0 \cdot x}{Pr^{1/3} \sqrt{Re_x}}$$

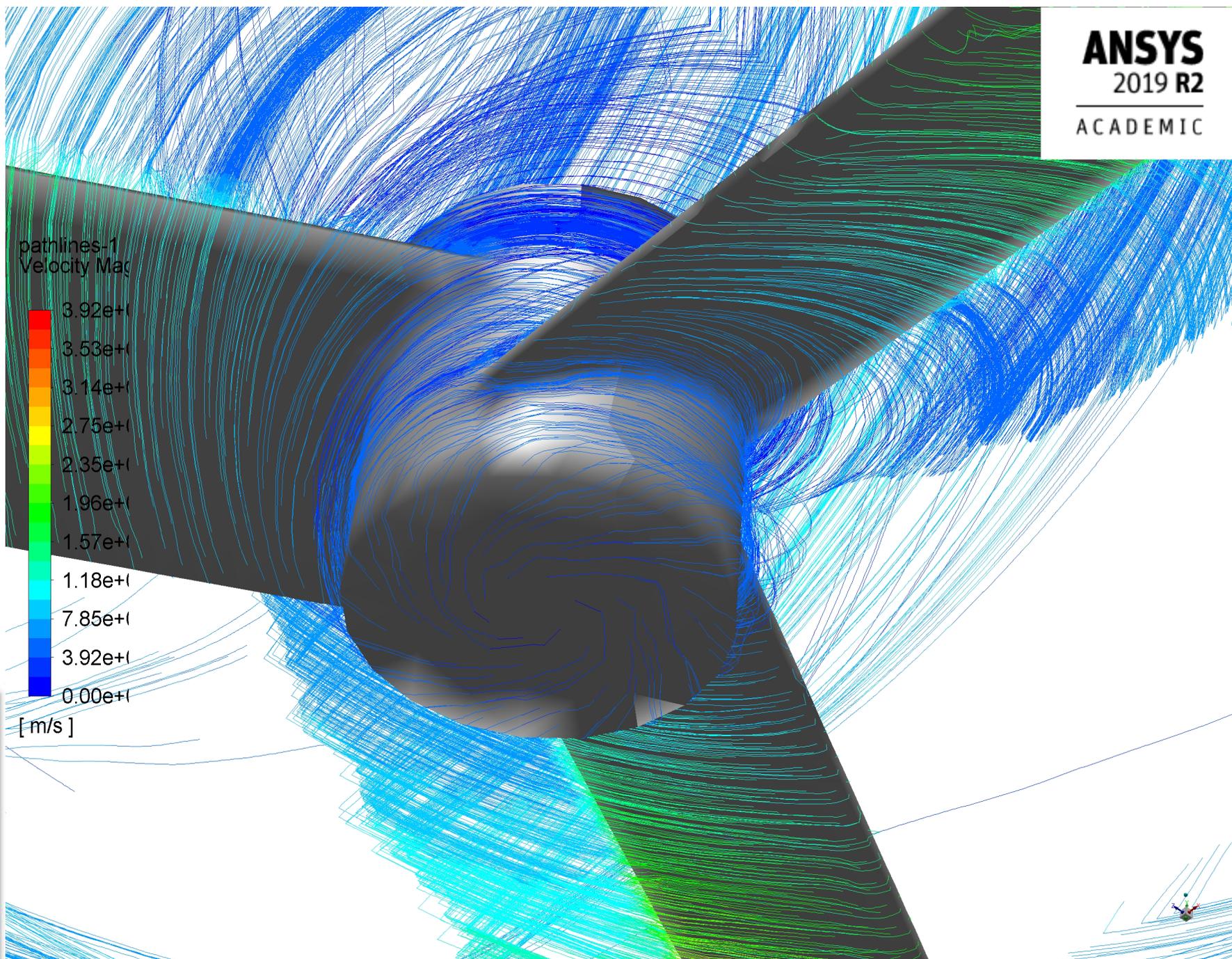
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# MÉTODOS DE SOLUÇÃO

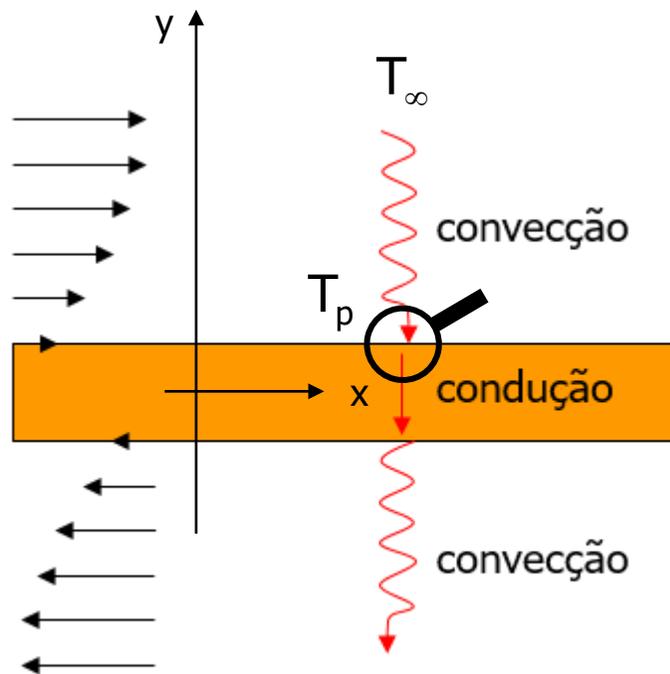
↓  
empíricos

↓  
computacionais

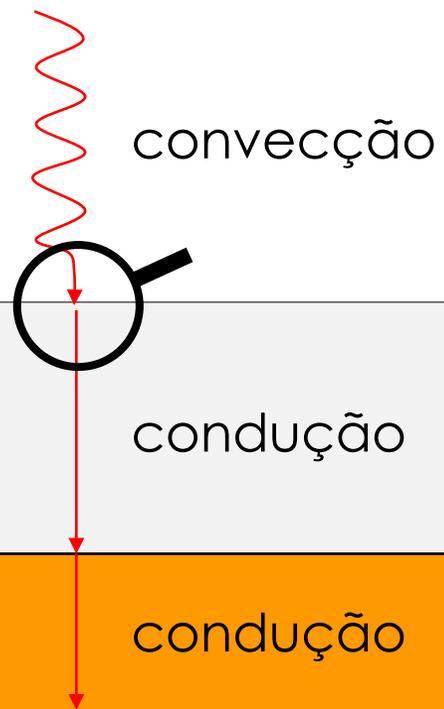


# A CONVECÇÃO DE CALOR – ABORDAGEM EMPÍRICA...

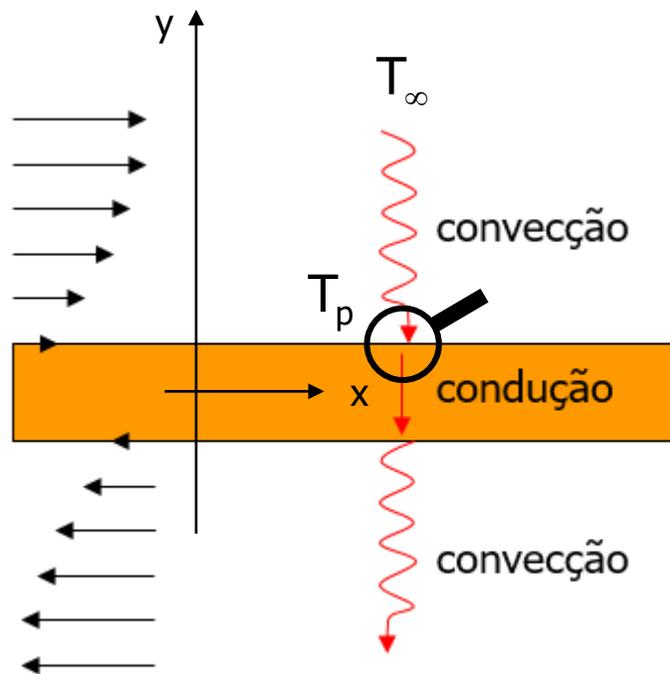
movimentação do fluido ↔ transporte de energia térmica



camada em movimento



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$$q = h_q \cdot (T_\infty - T_p) = -k \cdot \frac{dT}{dy} \Big|_{\text{camada limite}}$$

$$y^* \leftarrow y/D \quad T^* \leftarrow \frac{T - T_p}{T_\infty - T_p}$$

$$h_q \cdot (T_\infty - T_p) = -k \cdot \frac{d}{dy} (T_p + (T_\infty - T_p) \cdot T^*)$$

$$h_q \cdot (T_\infty - T_p) = -k(T_\infty - T_p) \cdot \frac{dT^*}{dy}$$

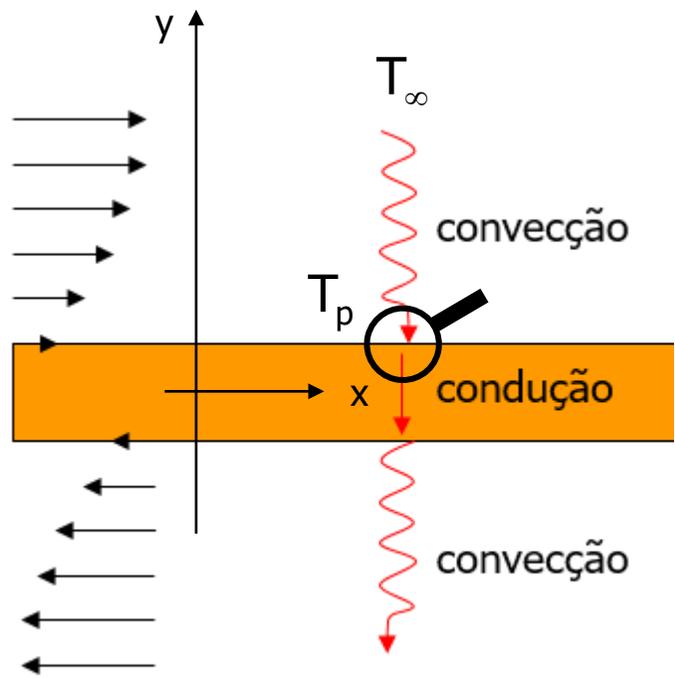
$$h_q = -k \cdot \frac{dT^*}{dy^*} \frac{dy^*}{dy}$$

$$\text{Nr. de Nusselt} \rightarrow \frac{h_q}{k/D} = -\frac{dT^*}{dy^*} \leftarrow \frac{\text{convecção}}{\text{condução}}$$



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$$Nu \stackrel{\text{def}}{=} \frac{h_q}{k/D}$$



As equações diferenciais de balanço indicam que:

Solução de Blasius (1908)... c. limite térmica

Paul R.H. Blasius (1883–1970)

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3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
4.5	2.790	0.980	0.034
5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
$\infty$	$\infty$	1	0

$$\tau_x = 0.332 \frac{\rho u_\infty^2}{\sqrt{Re_x}} \quad -q_x = 0.332 \cdot Pr^{1/3} (T_\infty - T_p) \sqrt{\frac{u_\infty}{\nu x}}$$

$$\delta_{FD} = \frac{5.0 \cdot x}{\sqrt{Re_x}} \quad \delta_{TM} = \frac{5.0 \cdot x}{Pr^{1/3} \sqrt{Re_x}}$$

aula MF2

Adimensionalização das equações governantes...

$$\frac{\partial \bar{u}}{\partial x} = \frac{u_\infty}{L} \frac{\partial \bar{u}^*}{\partial x^*} \quad \frac{\partial \bar{v}}{\partial y} = \frac{u_\infty}{L} \frac{\partial \bar{v}^*}{\partial y^*} \quad \frac{\partial \bar{P}}{\partial x} = \frac{\rho u_\infty^2}{L} \frac{\partial \bar{P}^*}{\partial x^*} \quad \frac{\partial \bar{P}}{\partial y} = \frac{\rho u_\infty^2}{L} \frac{\partial \bar{P}^*}{\partial y^*}$$

$$\frac{\partial \bar{\rho}^*}{\partial t^*} + \bar{\nabla} \cdot (\bar{\rho}^* \bar{U}^*) = 0$$

$$\bar{\rho}^* \left( \frac{\partial \bar{U}^*}{\partial t^*} + \bar{U}^* \cdot \bar{\nabla} \bar{U}^* \right) = -\bar{\nabla} \bar{P}^* + \frac{1}{Re} \bar{\nabla}^2 \bar{U}^* + \sum \frac{1}{R_k} \bar{F}_k^*$$

$$\left( \frac{\partial \bar{T}^*}{\partial t^*} + \bar{U}^* \cdot \bar{\nabla} \bar{T}^* \right) = \frac{1}{Re \cdot Pr} \bar{\nabla} \cdot \bar{\nabla} \bar{T}^* \quad \text{Forças externas}$$



$$Re = \frac{\rho u_0 D}{\mu} \rightarrow \frac{\text{inércia}}{\text{d.viscosa}}$$

$$Pr = \frac{\mu}{k/C_p} \rightarrow \frac{\text{d.viscosa}}{\text{d.térmica}}$$

As equações governam o fenômeno via leis de conservação. Os números adimensionais definem o comportamento na escala do problema modelado...



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$$Nu \stackrel{\text{def}}{=} \frac{h_q}{k/D}$$

As equações diferenciais de balanço indicam que:

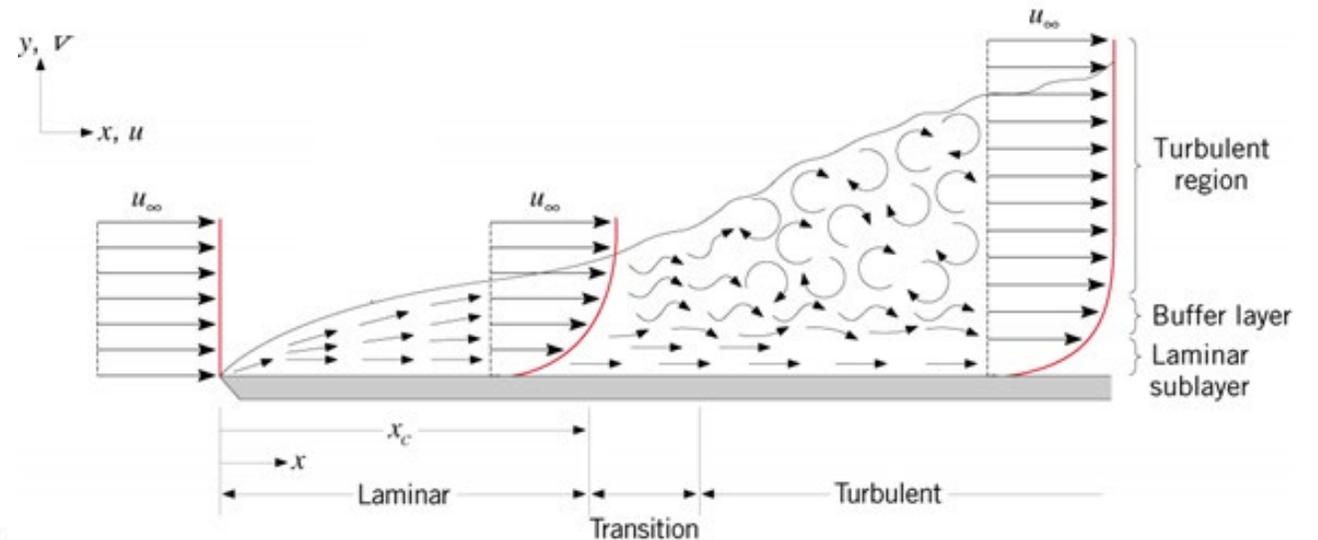
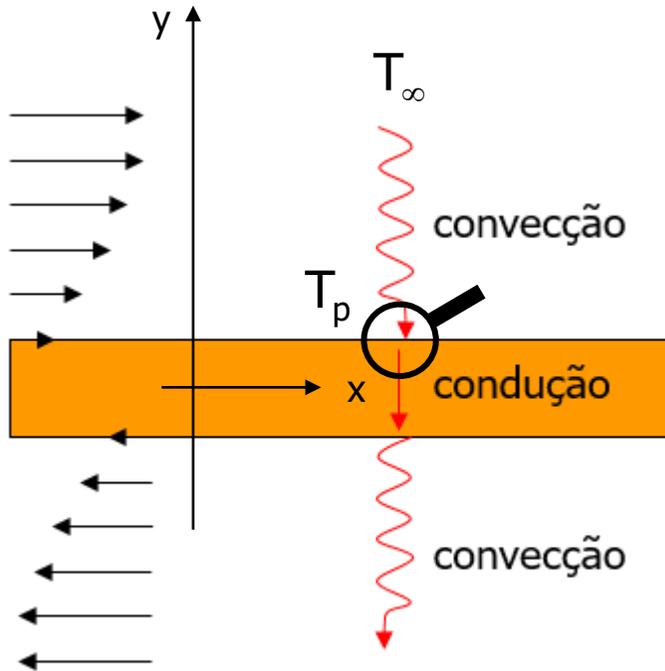
$$Nu = f(Re, Pr)$$

$$Re = \frac{\rho u_0 D}{\mu}$$

caracteriza o escoamento

caracteriza o fluido

$$Pr = \frac{C_p \mu}{k}$$



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$$Nu \stackrel{\text{def}}{=} \frac{h_q}{k/D}$$

As equações diferenciais de balanço indicam que:

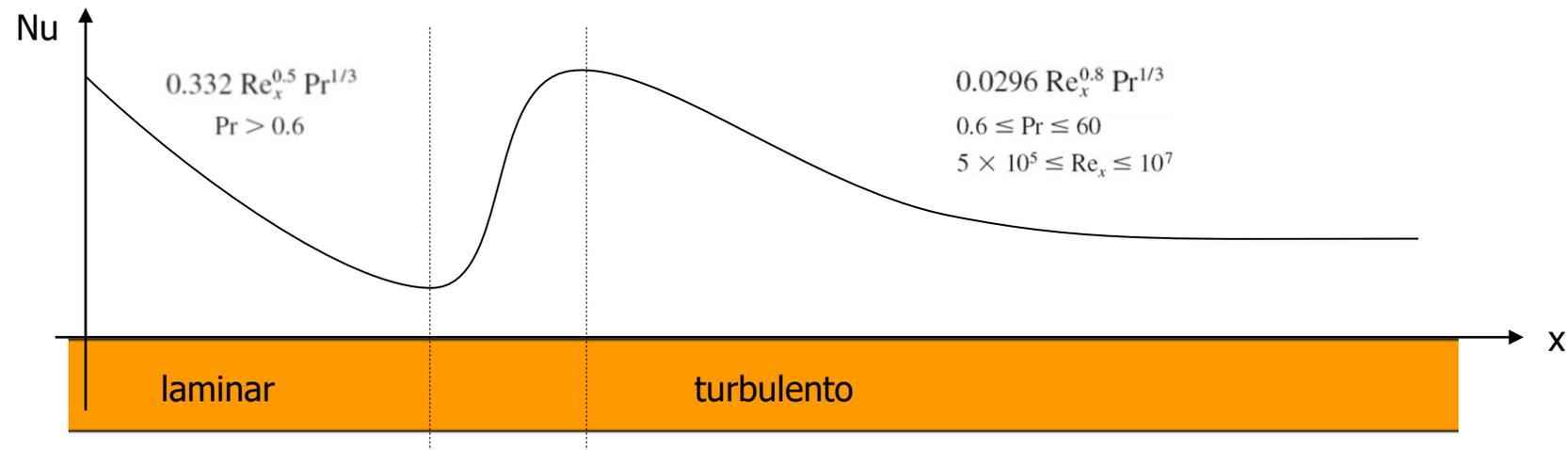
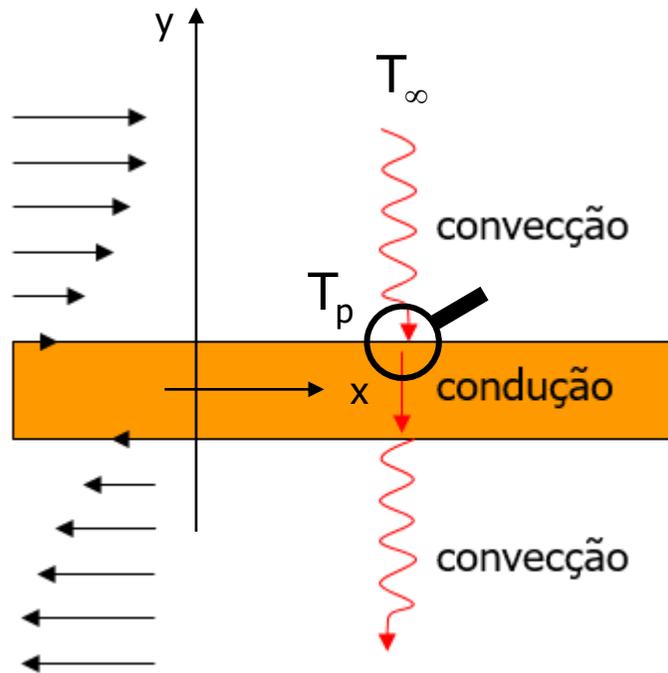
$$Nu = f(Re, Pr)$$

$$Re = \frac{\rho u_0 D}{\mu}$$

caracteriza o escoamento

caracteriza o fluido

$$Pr = \frac{C_p \mu}{k}$$



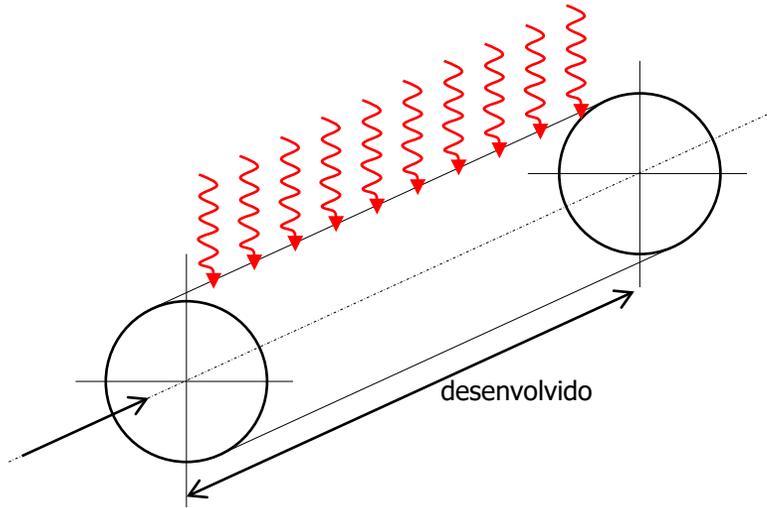
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# Correlações para escoamentos internos...

## Escoamento laminar:

Fluxo de calor constante:  $Nu = 4.36$

Temperatura constante:  $Nu = 3.66$



## Escoamento turbulento desenvolvido ( $Re > 10^4$ ):

Superfície lisa:

$$Nu = 0.023 \cdot Re^{4/5} \cdot Pr^n$$

(Dittus-Boelter)

$$\left[ \begin{array}{l} n = 0.3 \quad p / T_s < T_m \\ n = 0.4 \quad p / T_s > T_m \end{array} \right] \left[ \begin{array}{l} 0.7 \leq Pr \leq 160 \\ Re_D \geq 10,000 \\ \frac{L}{D} \geq 10 \end{array} \right]$$

$$Re = \frac{\rho u_0 D}{\mu} \quad Pr = \frac{C_p \mu}{k}$$



HOMEM DE FERRO

AVALIAÇÃO P2



Superfície rugosa:

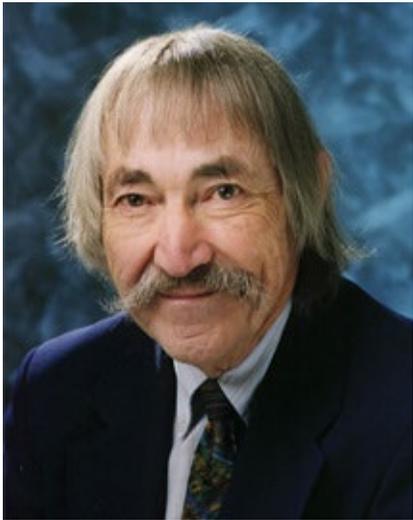
$$Nu = \frac{f}{8} \cdot \frac{(Re - 1000) \cdot Pr}{1 + 12.7(f/8)^{1/2} (Pr^{2/3} - 1)}$$

$f$  = friction factor

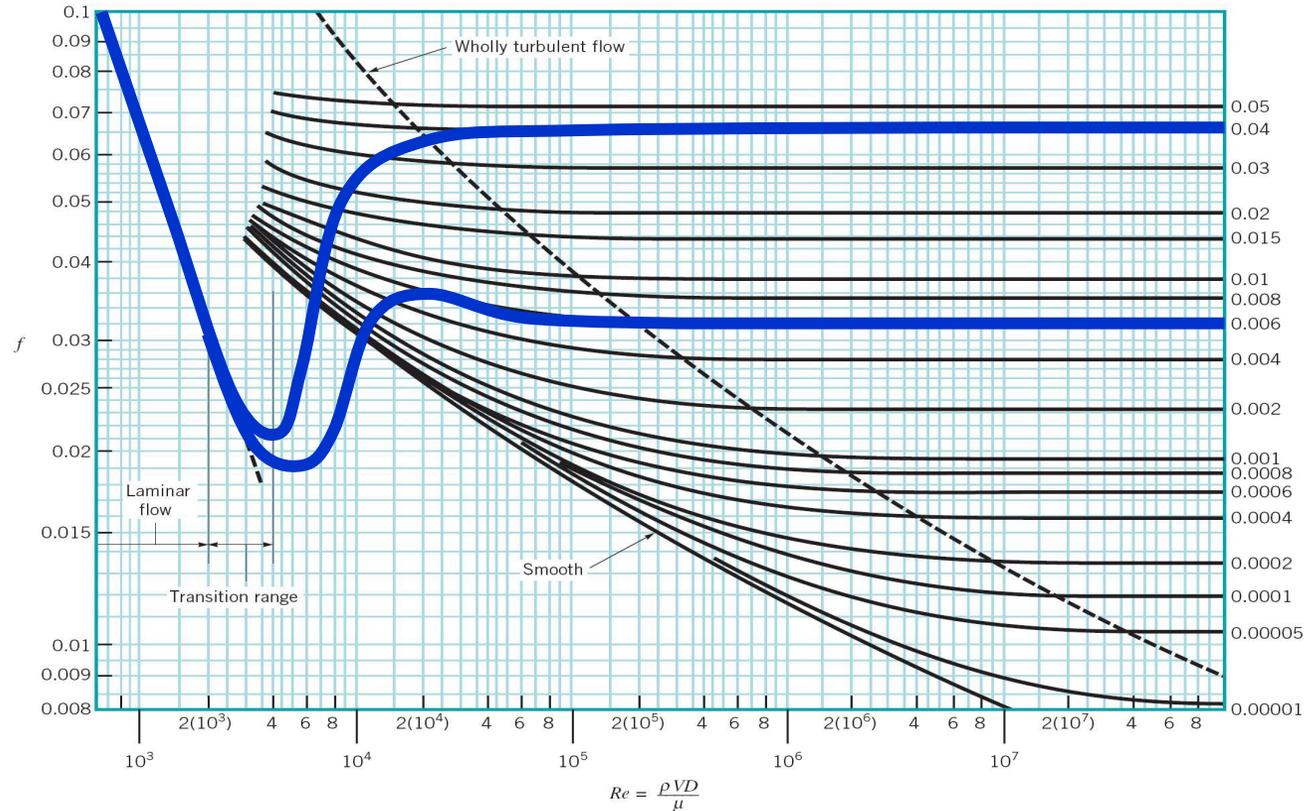


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# A equação de Darcy e cálculo do fator de atrito...



Stuart W. Churchill



**Laminar (Re < 2500)**

$$f = \frac{64}{Re}$$



HOMEM ARANHA

$\frac{\epsilon}{D}$  Turbulento (Re > 4000)

**Colebrook-White**

$$\frac{1}{\sqrt{f}} = -2 \log \left( 3.7 \frac{\epsilon}{D} + \frac{2.51}{Re \sqrt{f}} \right)$$

$$f = 8 \cdot \left[ \left( \frac{8}{Re} \right)^{12} + (A + B)^{-1,5} \right]^{1/12}$$

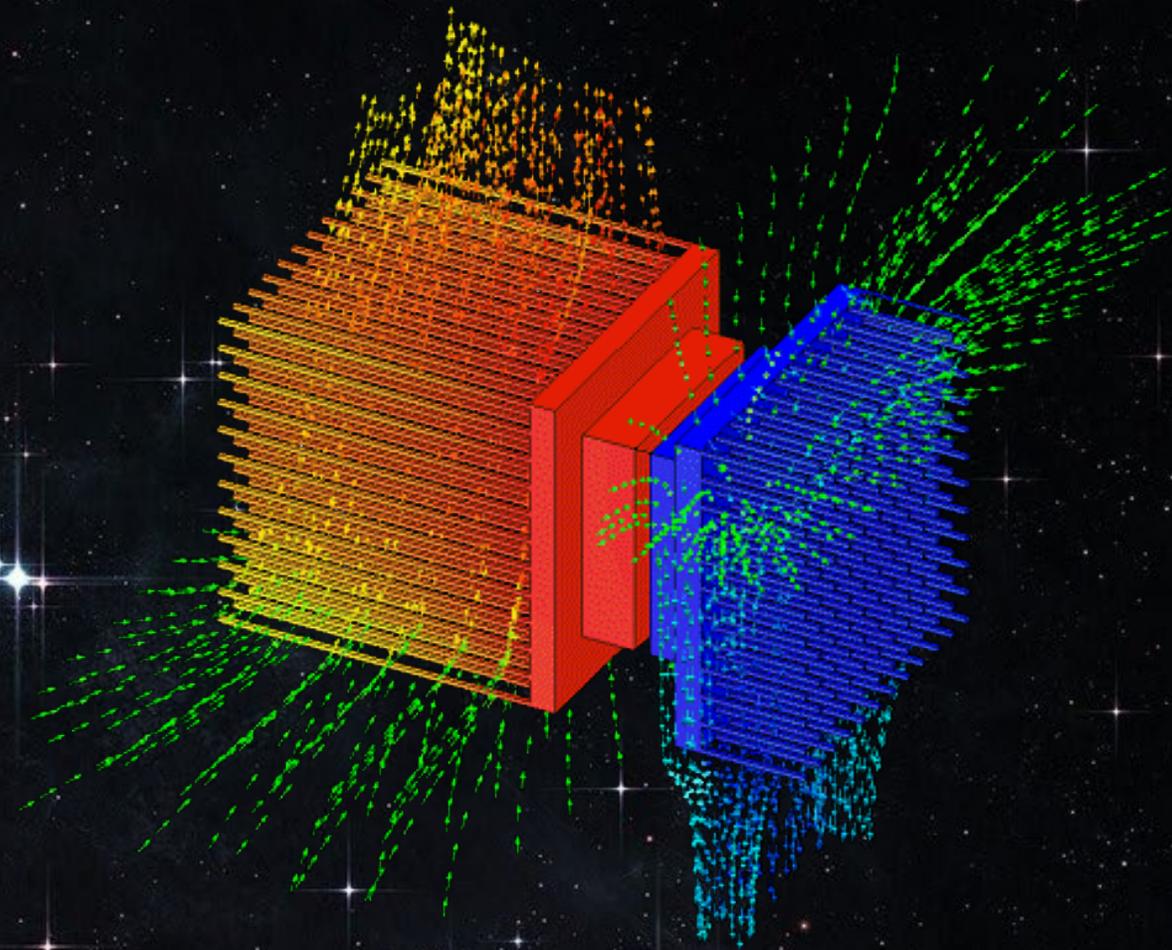
$$A = \left\{ 2,457 \ln \left( \left( \left( \frac{7}{Re} \right)^{0,9} + 0,27 \cdot \frac{\epsilon}{D} \right)^{-1} \right) \right\}^{16}$$

$$B = \left( \frac{37530}{Re} \right)^{16} \quad Re = \frac{4 \cdot m}{\mu \pi D}$$

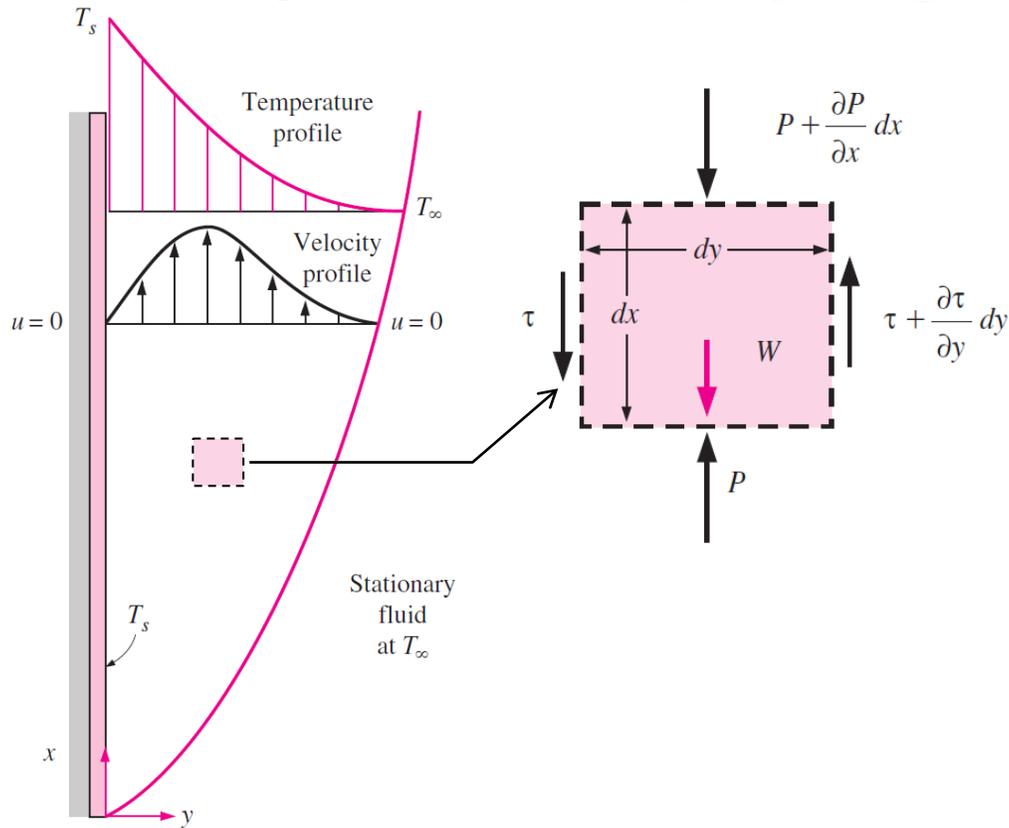


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# Convecção natural / escoamentos externos



# Convecção natural: equações governantes e o número de Grashof...



$$F_x = a_x \cdot dm$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$\frac{\partial P}{\partial y} = 0 \rightarrow \downarrow \leftarrow \frac{\partial P}{\partial x} = -\rho_\infty g$$

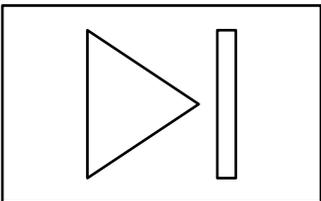
$$\rightarrow \rho \cdot \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g$$

$$\downarrow \leftarrow \Delta \rho = \rho \cdot \beta \cdot \Delta T \quad \text{Coef. expansão volumétrica}$$

$$\rightarrow \rho \cdot \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + g\beta \cdot (T - T_\infty)$$

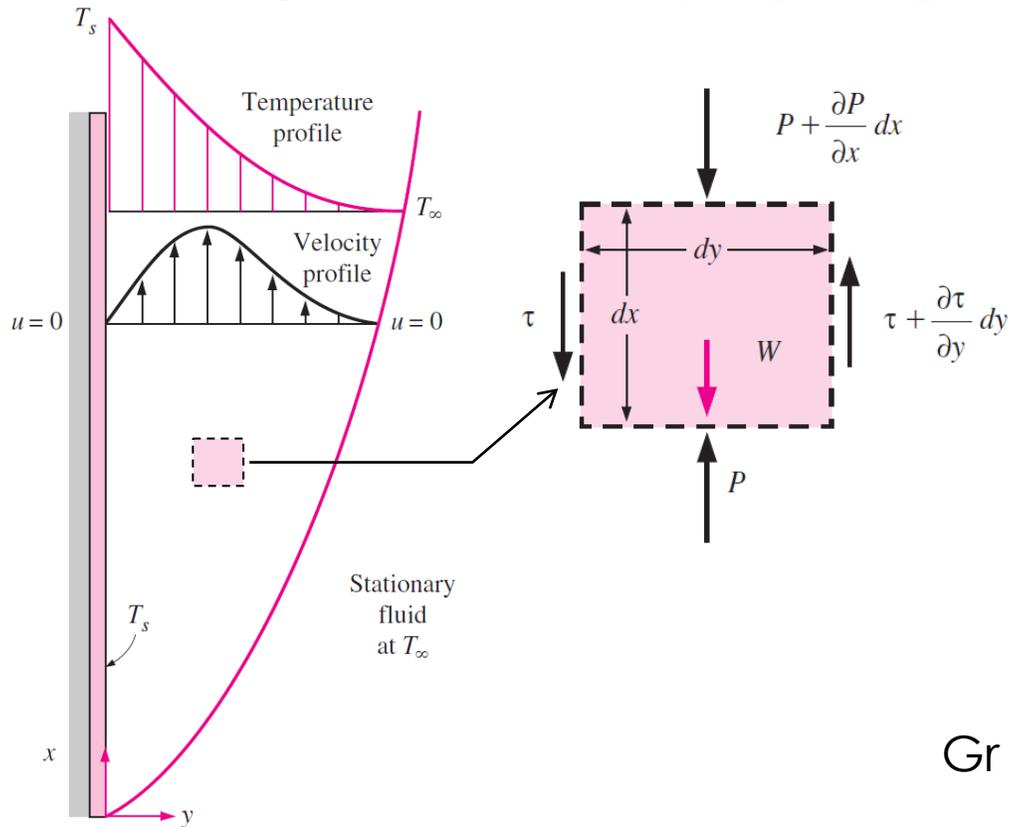
Força de empuxo dependente da temperatura

p/ eq. convecção forçada



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# Convecção natural: equações governantes e o número de Grashof...



$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{U}, \quad v^* = \frac{v}{U}, \quad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

adimensionalização

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}} + \left( \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \right) \cdot \frac{1}{Re} T^*$$

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{\text{força de empuxo}}{\text{dissipação viscosa}}$$

Análogo de Reynolds para convecção natural...

$$Ra = Gr \cdot Pr$$

Forma genérica para o Nr. Nusselt:

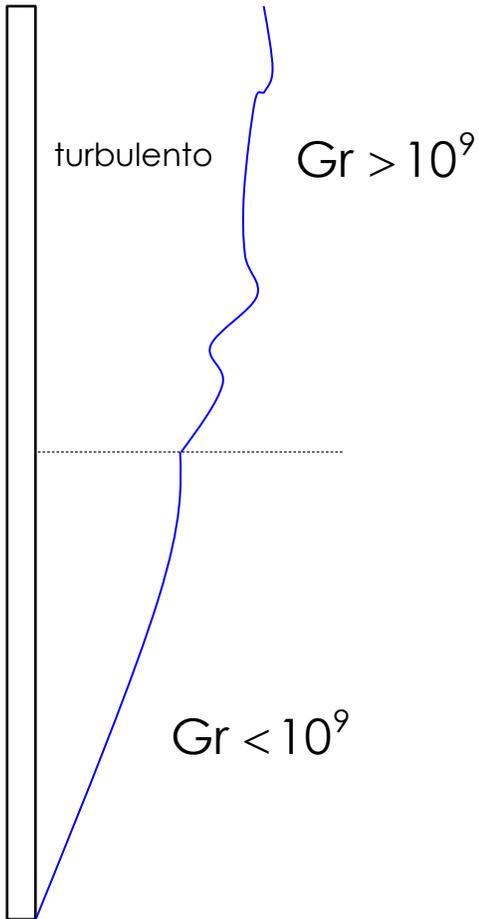
$$Nu = C \cdot (Gr \cdot Pr)^n = C \cdot Ra^n$$



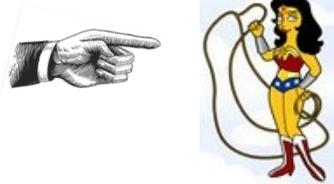
HE-MAN



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## Avaliação P2



MULHER-MARAVILHA

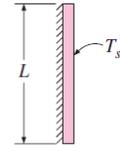
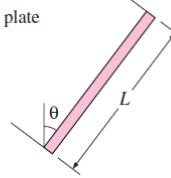
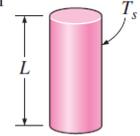
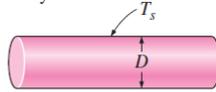
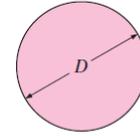
$$Q = hA_{\text{lateral}} \cdot (T - T_{\infty})$$

$$Nu = C \cdot Ra^n$$

$$\text{properties @ } T_{\text{fluid}} = \left( \frac{T_s + T_{\infty}}{2} \right)$$



### Empirical correlations for the average Nusselt number for natural convection over surfaces

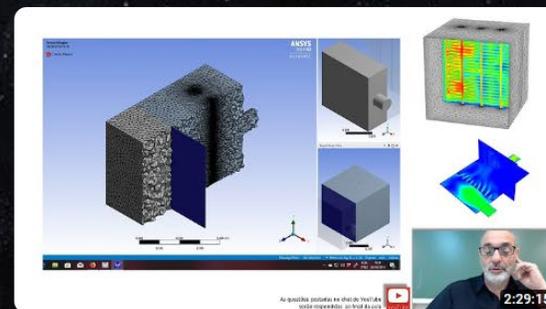
Geometry	Characteristic length $L_c$	Range of Ra	Nu
Vertical plate 	$L$	$10^4 - 10^9$ $10^9 - 10^{13}$	$Nu = 0.59Ra_l^{1/4}$ (9-19) $Nu = 0.1Ra_l^{1/3}$ (9-20)
Inclined plate 	$L$		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace $g$ by $g \cos \theta$ for $Ra < 10^9$
Horizontal plate (Surface area $A$ and perimeter $p$ ) (a) Upper surface of a hot plate (or lower surface of a cold plate) 	$A_s/p$	$10^4 - 10^7$ $10^7 - 10^{11}$	$Nu = 0.54Ra_l^{1/4}$ (9-22) $Nu = 0.15Ra_l^{1/3}$ (9-23)
(b) Lower surface of a hot plate (or upper surface of a cold plate) 		$10^5 - 10^{11}$	$Nu = 0.27Ra_l^{1/4}$ (9-24)
Vertical cylinder 	$L$		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_l^{1/4}}$
Horizontal cylinder 	$D$	$Ra_D \leq 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$ (9-25)
Sphere 	$D$	$Ra_D \leq 10^{11}$ ( $Pr \geq 0.7$ )	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (9-26)



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# A CONVECÇÃO DE CALOR – ABORDAGEM NUMÉRICA...

movimentação do fluido  $\leftrightarrow$  transporte de energia térmica



TUTORIAL CFD ANSYS/FLUENT: PROJETO DE UM AQUECEDOR DE AR

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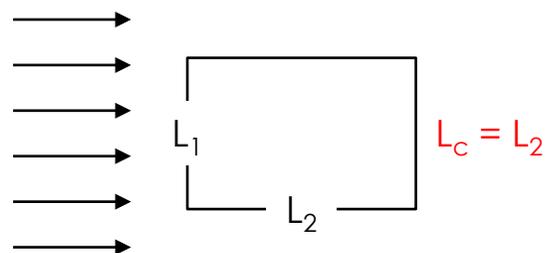
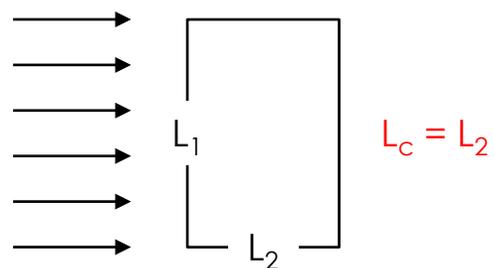
Prof. P. Seleghim

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<https://youtube.com/live/wgQCpJSJz0Y>



HULK



**Table 2.** Empirical correlations for the average Nusselt number for forced convection over isothermal surfaces. The characteristic length,  $L_c$ , is defined in Section 3.1.3.

Geometry	$L_c$	Range of Validity	$Nu$
<b>Parallel to a plate</b>			
Laminar flow	L	$Re \leq 5 \times 10^5$ $0.6 \leq Pr \leq 60$	$Nu = 0.664Re^{1/2}Pr^{1/3}$ [33]
Turbulent flow	L	$5 \times 10^5 \leq Re \leq \times 10^7$ $0.6 \leq Pr \leq 60$	$Nu = 0.037Re^{4/5}Pr^{1/3}$ [33]
Combined flow	L	$5 \times 10^5 \leq Re \leq \times 10^7$ $0.6 \leq Pr \leq 60$	$Nu = (0.037Re^{4/5} - 871)Pr^{1/3}$ [33]
<b>Around a sphere</b>			
	D	$0 \leq Re < 200$ $0 \leq Pr \leq 250$	$Nu = 2 + 0.6Re^{1/2}Pr^{1/3}$ [35]

[33] Welty, J.; Rorrer, G.L.; Foster, D.G. Fundamentals of Momentum, Heat and Mass Transfer; John Wiley & Sons: New York, NY, USA, 2014.

[35] Ranz, W.E.; Marshall, W.R. Evaporation from Drops. Chem. Eng. Prog. 1952, 48, 141–146.

Maragkos, Georgios, and Tarek Beji. "Review of convective heat transfer modelling in CFD simulations of fire-Driven Flows." *Applied Sciences* 11.11 (2021): 5240.

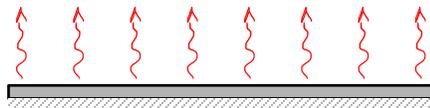
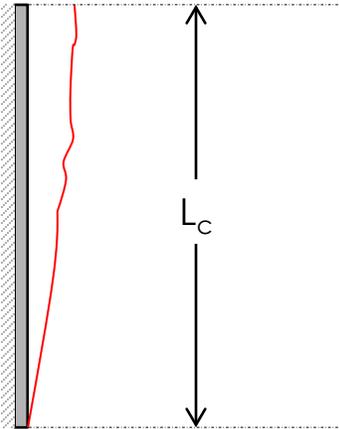
**TUTORIAL CFD ANSYS/FLUENT: PROJETO DE UM AQUECEDOR DE AR**

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$L_c = \text{Area}/\text{perímetro}$

**Table 3.** Empirical correlations for the average Nusselt number for natural convection over isothermal surfaces. The characteristic length,  $L_c$ , is defined in Section 3.1.3.

Geometry	$L_c$	Range of Validity	$Nu$
<b>Vertical plate</b>			
Laminar flow	L	$10^4 \leq Ra \leq 10^9$	$Nu = 0.59Ra^{1/4}$ [32]
Turbulent flow	L	$10^9 \leq Ra \leq 10^{13}$	$Nu = 0.1Ra^{1/3}$ [32]
Any type of flow	L	$Ra \leq 10^{12}$	$Nu = \left(0.825 + \frac{0.387Ra^{1/6}}{(1+(0.492/Pr)^{9/16})^{8/27}}\right)^2$ [36]
<b>Horizontal plate</b>			
Laminar flow	$A_s/p$	$10^4 \leq Ra \leq 10^7$	$Nu = 0.54Ra^{1/4}$ [32]
Turbulent flow	$A_s/p$	$10^7 \leq Ra \leq 10^{11}$	$Nu = 0.15Ra^{1/3}$ [32]
<b>Sphere</b>			
	D	$Ra \leq 10^{11}$ $Pr \geq 0.7$	$Nu = 2 + \frac{0.589Ra^{1/4}}{(1+(0.469/Pr)^{9/16})^{4/9}}$ [37]

[32] McAdams, W.H. Heat Transmission; McGraw-Hill Book Company: New York, NY, USA, 1957.

[36] Churchill, S.W.; Chu, H.H.S. Correlating Equations for Laminar and Turbulent Free Convection from a Vertical Plate. Int. J. Heat Mass Transf. 1975, 18, 1323–1329. [CrossRef]

[37] Churchill, S.W. Free Convection around Immersed Bodies. In Heat Exchanger Design Handbook; Hemisphere Publishing: New York, NY, USA, 1983.

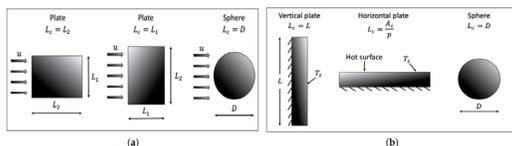
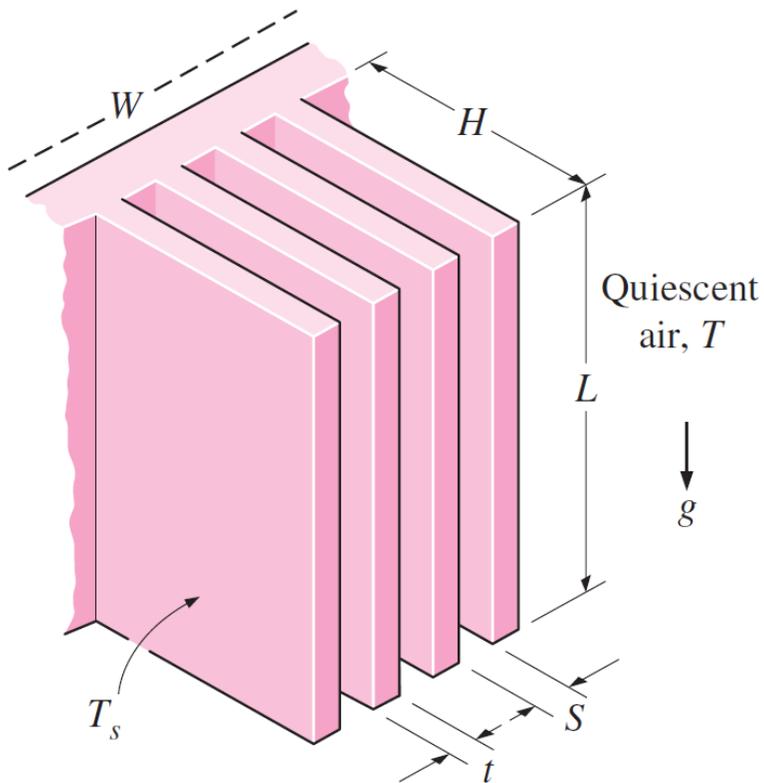


Figure 1. Characteristic length,  $L_c$ , for (a) forced and (b) natural convection scenarios.  $A_s$  and  $p$  are the surface area and perimeter of the plate, respectively.



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# Convecção natural em superfícies aletadas @ $T_s = \text{cte} \dots$



$$Ra_s = \frac{g\beta(T_s - T_\infty) \cdot S^3}{\nu^2} Pr \quad \text{ou} \quad Ra_L = \frac{g\beta(T_s - T_\infty) \cdot L^3}{\nu^2} Pr$$

$$Nu = \frac{hS_{opt}}{k} = \left[ \frac{576}{(Ra_s \cdot S/L)^2} + \frac{2.873}{(Ra_s \cdot S/L)^{0.5}} \right]^{-0.5}$$

**AVALIAÇÃO P2**  
Bar-Cohen and Rohsenow

Espaçamento ótimo (trade-off área x vazão):

$$@ T_s = \text{cte} \rightarrow S_{opt} = 2.714 \frac{L}{Ra_L^{0.25}} \rightarrow Nu = 1.307$$

$$Q = h \cdot (2nLH) \cdot (T_s - T_\infty) \quad \leftarrow t \ll S$$

$$\text{properties @ } T_{avg} = (T_s + T_\infty) / 2$$

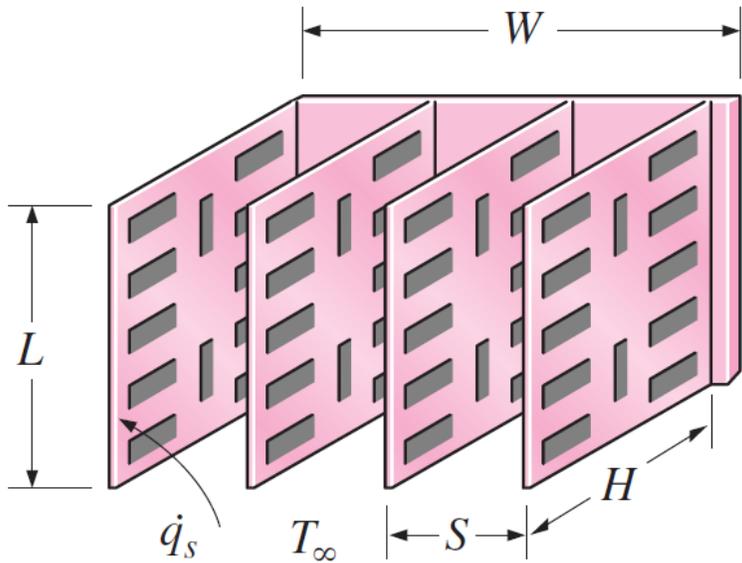


AQUAMAN



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# Convecção natural em superfícies aletadas @ $q_s = \text{cte} \dots$



$$Ra_s = \frac{g\beta q_s \cdot S^4}{k\nu^2} Pr$$

$$Nu = \frac{hL}{k} = \left[ \frac{48}{Ra_s \cdot S/L} + \frac{2.51}{(Ra_s \cdot S/L)^{0.4}} \right]^{-0.5}$$

Espaçamento ótimo (trade-off área x vazão):

$$@ q_s = \text{cte} \rightarrow S_{\text{opt}} = 2.12 \cdot \left( \frac{S^4 L}{Ra_s} \right)^{0.2}$$

$$Q = q_s \cdot (2nLH) \quad \leftarrow t \ll S$$

$$\text{properties @ } T_{\text{avg}} = (T_L + T_\infty) / 2 \quad \leftarrow T_L = T_\infty + q_s / h$$

temperatura crítica  
ocorrendo na borda  
superior



**AVALIAÇÃO P2**  
Bar-Cohen and Rohsenow



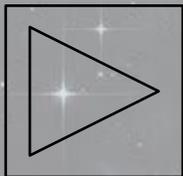
TARTARUGA  
NINJA



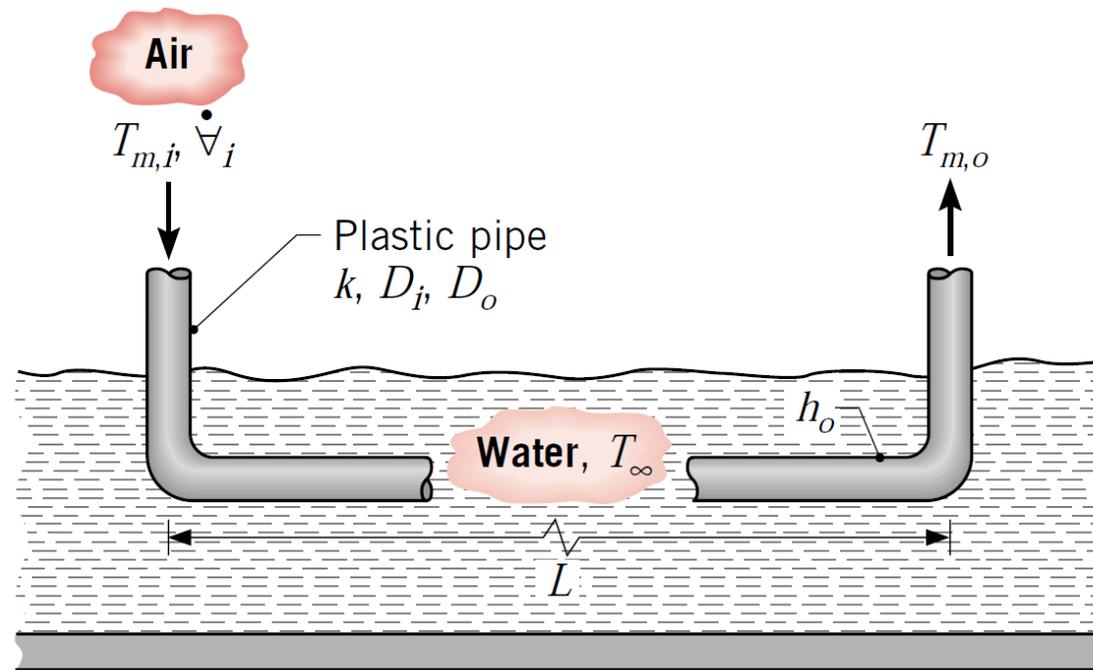
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# A CONVECÇÃO DE CALOR – ABORDAGEM EMPÍRICA...

exercício resolvido – tutorial excel



**8.31:** Para resfriar uma casa de verão sem uso de um ciclo frigorífico, ar é encaminhado através de uma tubulação de plástico ( $k=0.15\text{W/m/K}$ ,  $D_i=0.15\text{m}$ ,  $D_o=0.17\text{m}$ ) submersa em um corpo d'água adjacente. A temperatura da água é normalmente de  $T_\infty=17^\circ\text{C}$ , e o coeficiente de convecção é mantido em  $h_o=1500\text{ W/m}^2/\text{K}$  na superfície externa da tubulação. Se ar proveniente da casa entra no tubo a uma temperatura de  $T_{m,i}=29^\circ\text{C}$  e uma vazão volumétrica de  $V_i=0.025\text{m}^3/\text{s}$ , qual extensão  $L$  é necessária para que a temperatura na saída seja de  $T_{m,o}=21^\circ\text{C}$  ?



Próxima aula ?



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

**8.31:** Para resfriar uma casa de verão sem uso de um ciclo frigorífico, ar é encaminhado através de uma tubulação de plástico ( $k=0.15\text{W/m/K}$ ,  $D_i=0.15\text{m}$ ,  $D_o=0.17\text{m}$ ) submersa em um corpo d'água adjacente. A temperatura da água é normalmente de  $T_\infty=17^\circ\text{C}$ , e o coeficiente de convecção é mantido em  $h_o=1500\text{ W/m}^2/\text{K}$  na superfície externa da tubulação. Se ar proveniente da casa entra no tubo a uma temperatura de  $T_{m,i}=29^\circ\text{C}$  e uma vazão volumétrica de  $V_i=0.025\text{m}^3/\text{s}$ , qual extensão  $L$  é necessária para que a temperatura na saída seja de  $T_{m,o}=21^\circ\text{C}$  ?



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REFPROP (air (dry)) - NIST Reference Fluid Properties (DLL version 9,1)

File Edit Options Substance Calculate Plot Window Help Cautions

2: air (dry): Specified state points

	Temperature (°C)	Pressure (bar)	Density (kg/m³)	Enthalpy (kJ/kg)	Cp (kJ/kg-K)	Therm. Cond. (mW/m-K)	Viscosity (μPa-s)	Prandtl
1	29,000	1,0000	1,1533	428,46	1,0064	26,544	18,641	0,70678
2	25,000	1,0000	1,1688	424,44	1,0063	26,247	18,448	0,70729
3	21,000	1,0000	1,1848	420,41	1,0062	25,948	18,254	0,70781
4								

$$Q = \dot{m} \cdot (h_i - h_o) = \rho_i V_i \cdot (h_i - h_o)$$



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$$\dot{m} = 0.02883 \text{kg/s}$$



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$$Q = 1.1533 \frac{\text{kg}}{\text{m}^3} 0.025 \frac{\text{m}^3}{\text{s}} \cdot (428.46 - 420.41) \frac{\text{kJ}}{\text{kg}}$$

$$\dot{m} = 0.02883 \text{ kg/s}$$

$$Q = 0.2321 \text{ kW}$$



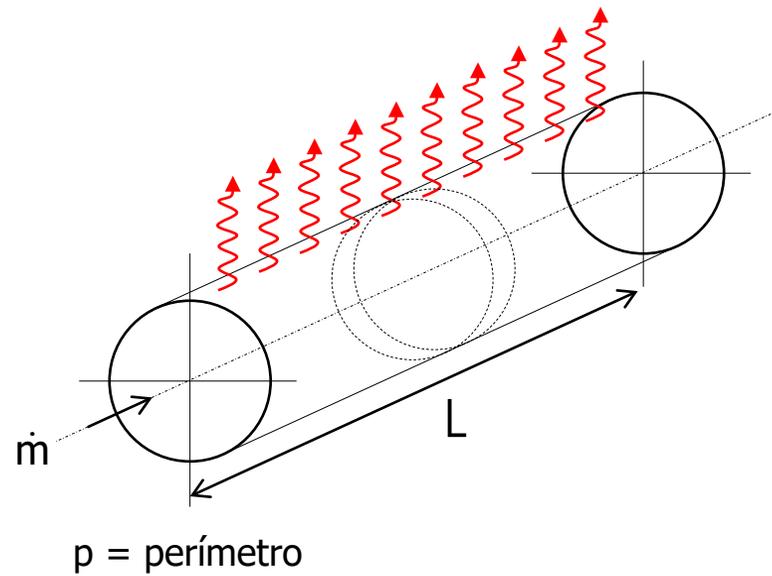
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# Equação do decaimento da temperatura ao longo da tubulação



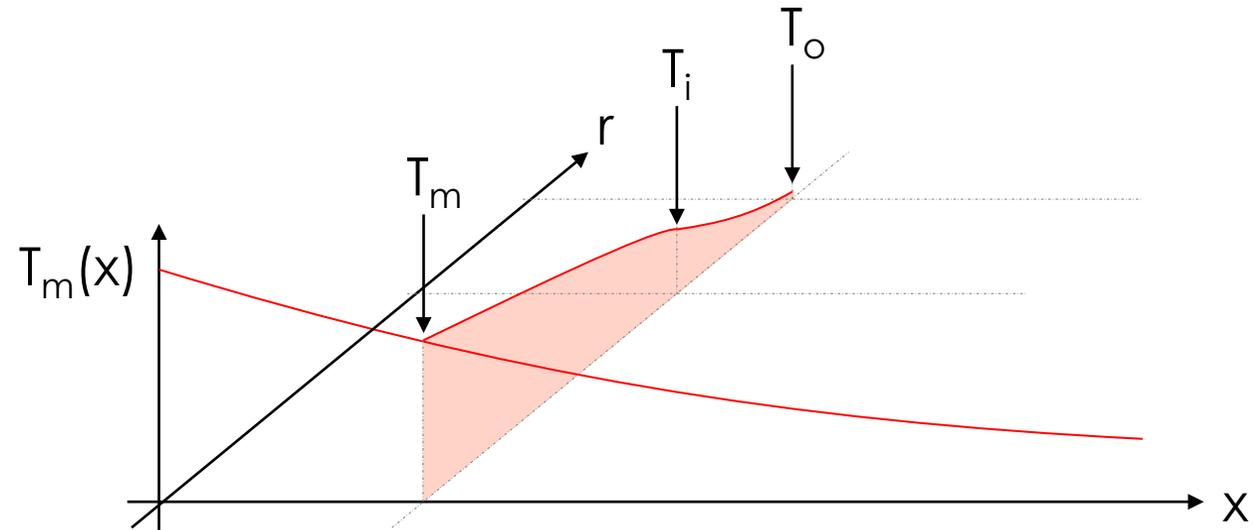
As questões postadas no Chat do YouTube  
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# Balço global de energia...

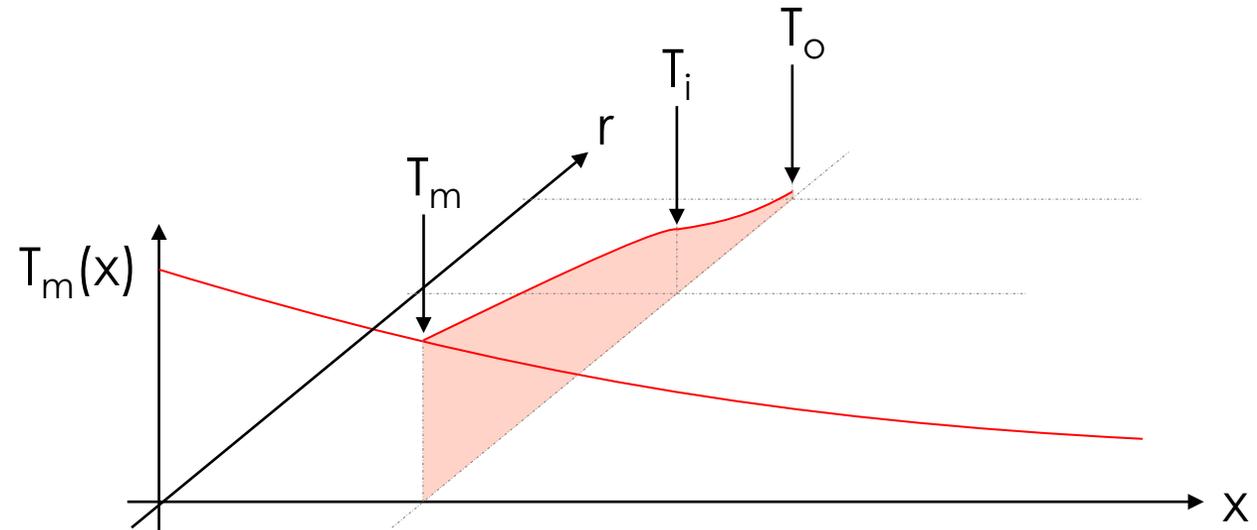
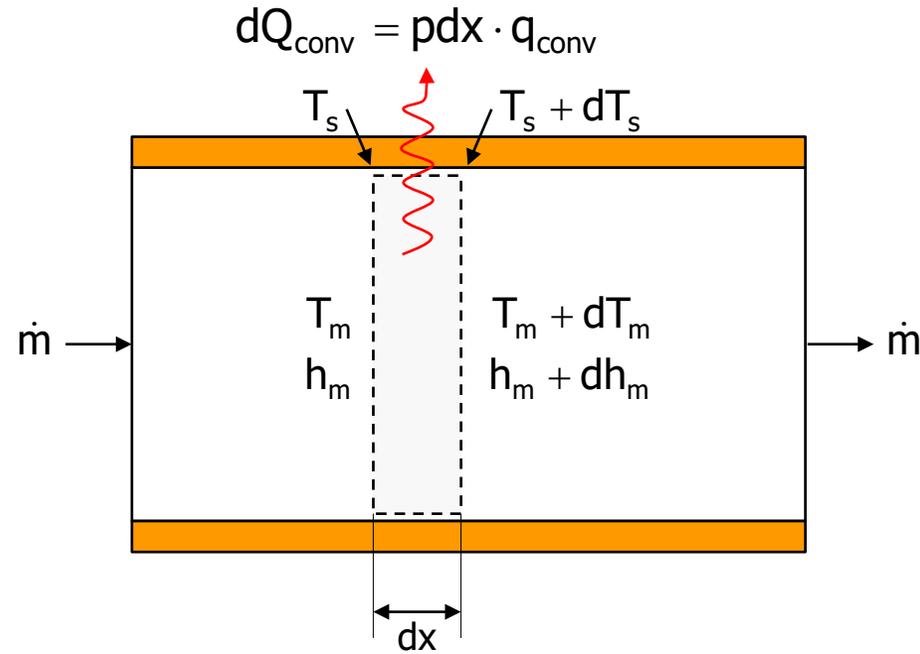
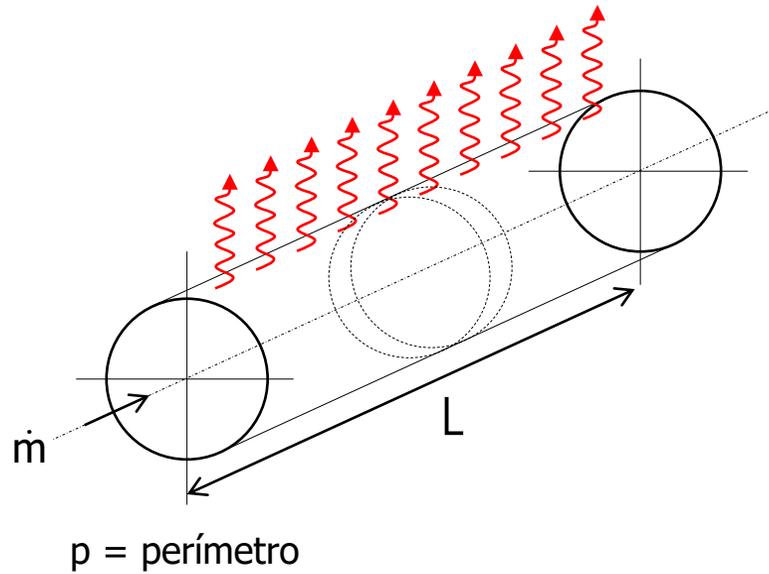


$T_m(x) \rightarrow$  perfil axial de temperaturas

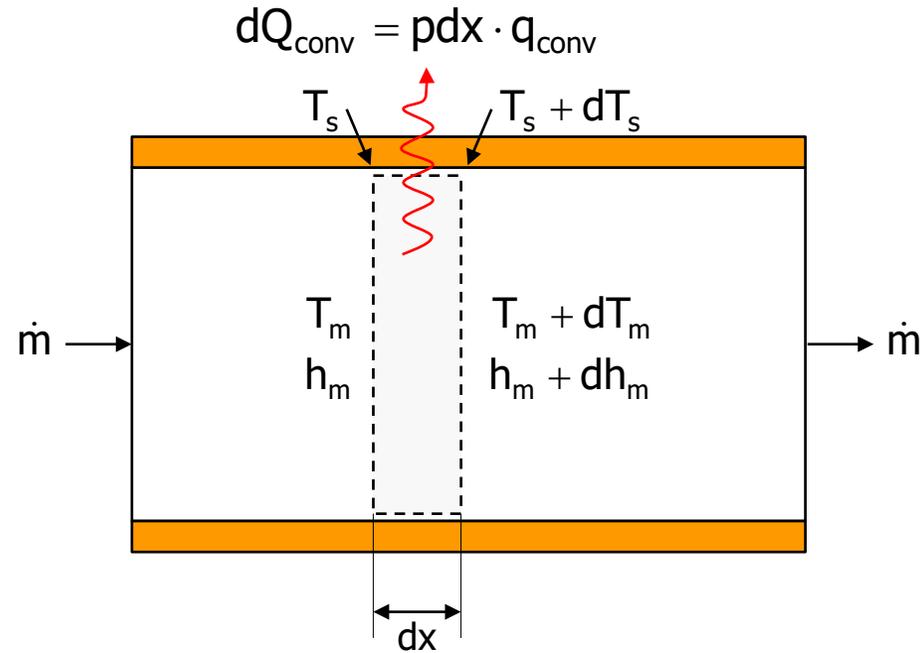
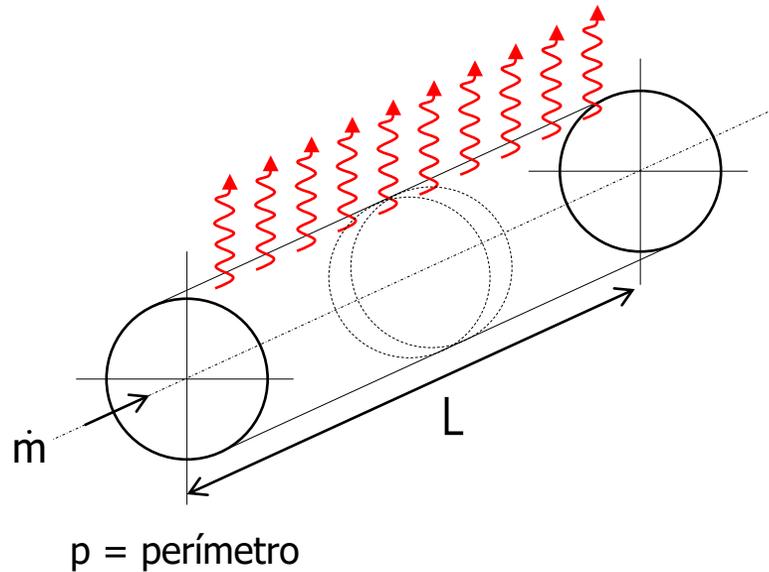
$T_r(x) \rightarrow$  perfil radial de temperaturas



# Balço global de energia...



# Balço global de energia...



$$\dot{Q} - \dot{W} = \sum_{\text{sai}} \dot{m}_k \cdot \left( h_k + gz_k + \frac{v_k^2}{2} \right) - \sum_{\text{entra}} \dot{m}_k \cdot \left( h_k + gz_k + \frac{v_k^2}{2} \right)$$

$$p q_{\text{conv}} dx = \dot{m} \cdot (h_m + dh_m) - \dot{m} \cdot h_m$$

$$q_{\text{conv}} = h_{\text{conv}} \cdot (T_s - T_m) \rightarrow$$

$$\dot{m} \cdot \frac{dh_m}{dx} - p h_{\text{conv}} \cdot (T_s - T_m) = 0$$



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## Balanco global de energia...

$$\dot{m} \cdot \frac{dh_m}{dx} - ph_{\text{conv}} \cdot (T_s - T_m) = 0$$

Solução direta via método numérico de  
solução de equação diferencial

$$h = h(P, T) \downarrow$$

Implementação numérica...



As questões postadas no Chat do YouTube  
serão respondidas ao final da aula.

# Balço global de energia...



$$\dot{m} \cdot \frac{dh_m}{dx} - ph_{conv} \cdot (T_s - T_m) = 0$$

Solução direta via método numérico de solução de equação diferencial

$$h = h(P, T) \downarrow$$

Implementação numérica...

Solução analítica a partir de hipóteses simplificadoras

$$h = C_p \cdot T \downarrow \begin{array}{l} \text{gases perfeitos} \\ \text{"fluido incompressível"} (C_p = C_v) \end{array}$$

$$\dot{m} \cdot \frac{dT_m}{dx} - \frac{ph_{conv}}{C_p} \cdot (T_s - T_m) = 0$$



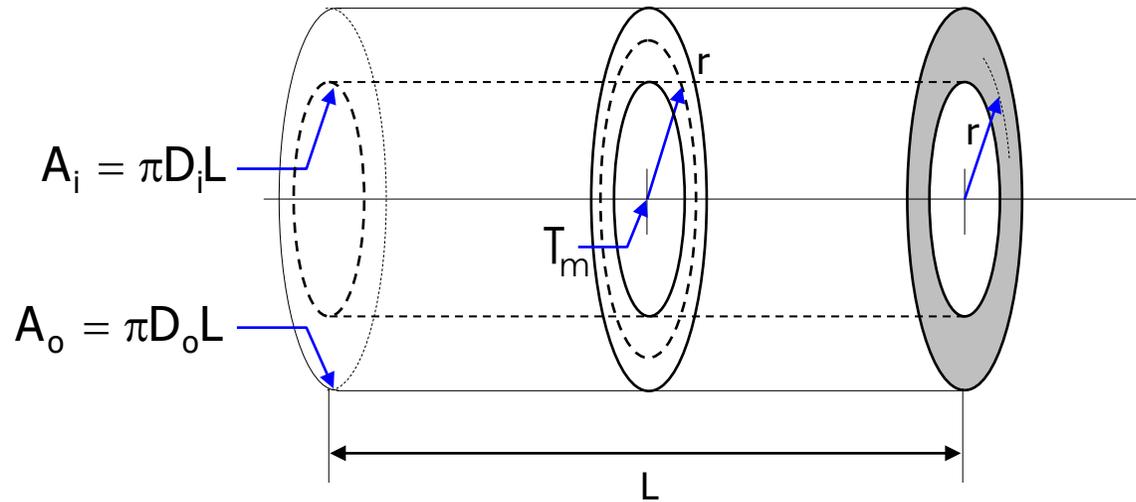
Fluxo de calor constante →

$$T_m(x) = T_{m,i} + \frac{pq_{conv}}{\dot{m}C_p} \cdot x$$

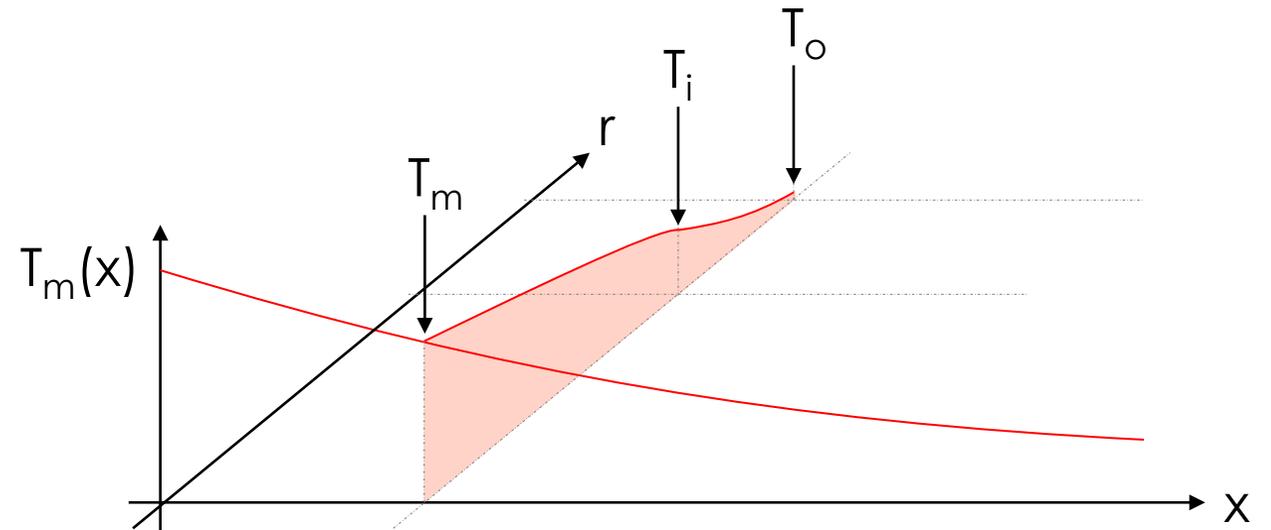
Temperatura superficial constante →

$$T_m(x) = T_s - (T_s - T_{m,i}) \cdot \exp\left(\frac{p\bar{h}_{conv}}{\dot{m}C_p} \cdot x\right)$$

# Geometria cilíndrica... **variação radial da temperatura**

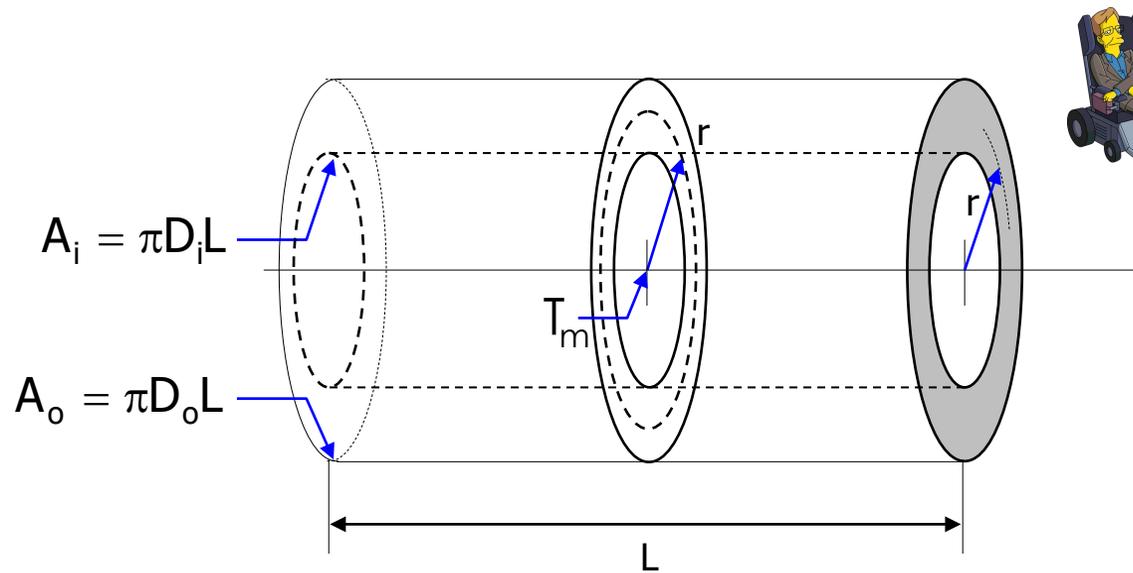


Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k \vec{\nabla} T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

$$\rightarrow \frac{dT}{dr} = C_1 + \frac{C_2}{r}$$

Obs.:  $\lim_{r \rightarrow \infty} \frac{dT}{dr} = 0$

$$\rightarrow dT = \frac{C_2}{r} dr$$

$$\rightarrow \int_{T_i}^{T(r)} dT = \int_{r_i}^r \frac{C_2}{r} dr$$

$$\rightarrow T(r) - T_i = \ln\left(\frac{r}{r_i}\right) \cdot C_2 \quad \leftarrow T(r_o) = T_o$$

Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$

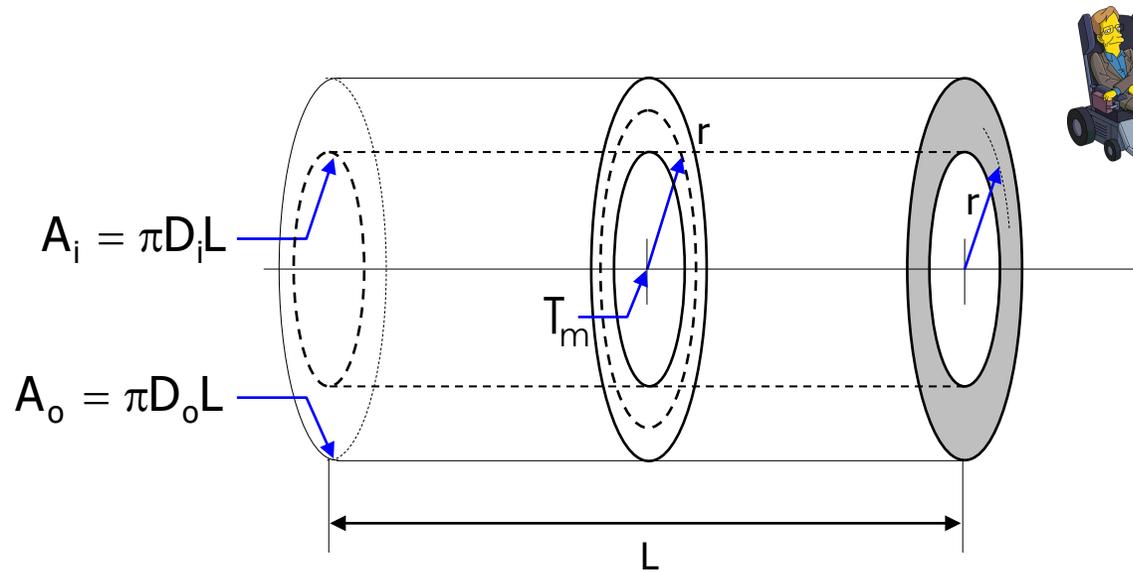
Equação do decaimento radial da temperatura

$$\rightarrow T(r) = T_i + \frac{T_o - T_i}{\ln(r_o / r_i)} \cdot \ln\left(\frac{r}{r_i}\right)$$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k \vec{\nabla} T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

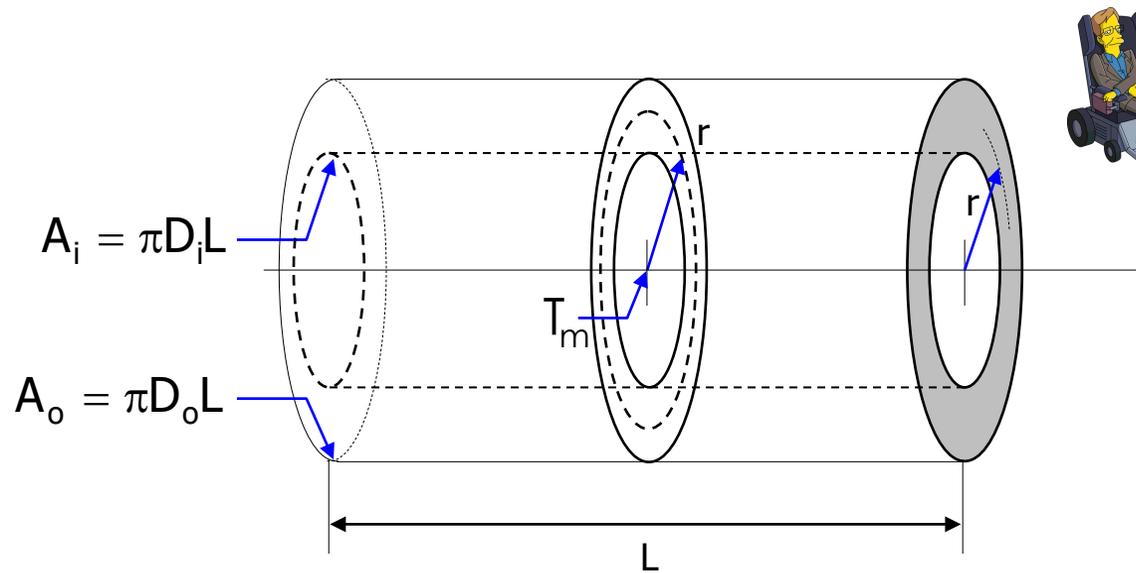
$$T(r) = T_i + \frac{T_o - T_i}{\ln(r_o / r_i)} \cdot \ln\left(\frac{r}{r_i}\right)$$

Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k \vec{\nabla} T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

$$T(r) = T_i + \frac{T_o - T_i}{\ln(r_o / r_i)} \cdot \ln\left(\frac{r}{r_i}\right)$$

$$Q = -kA(r) \frac{dT}{dr}(r) = \dots = \frac{2\pi Lk}{\ln(r_o / r_i)} \cdot (T_i - T_o)$$

Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$

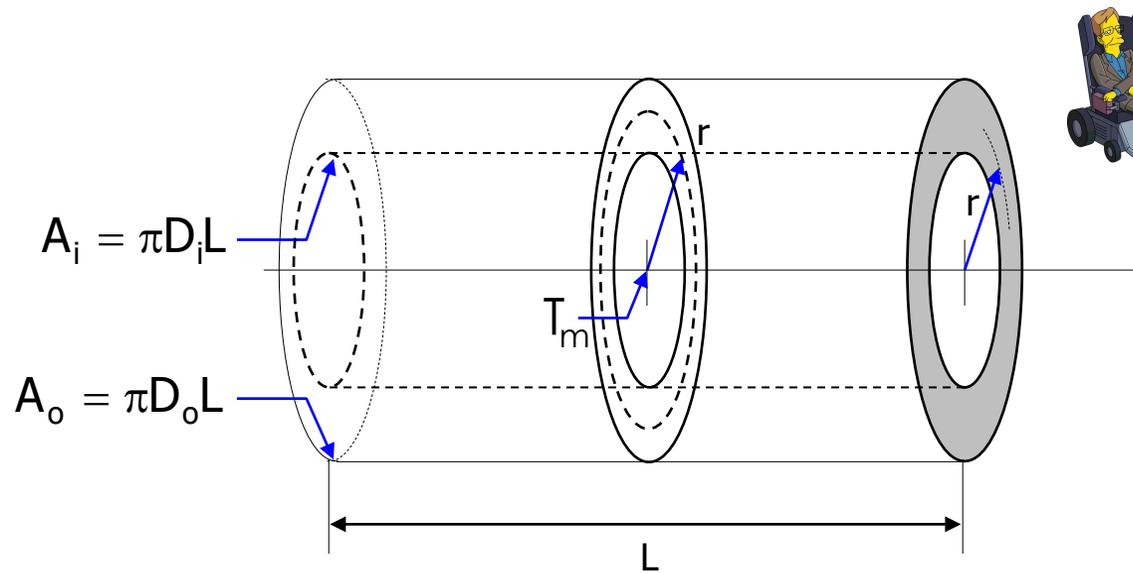
$$Q = h_i A_i (T_i - T_m) \rightarrow T_i = \frac{Q}{h_i A_i} + T_m$$

$$Q = h_o A_o (T_o - T_\infty) \rightarrow T_o = \frac{Q}{h_o A_o} - T_\infty$$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k \vec{\nabla} T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

$$T(r) = T_i + \frac{T_o - T_i}{\ln(r_o / r_i)} \cdot \ln\left(\frac{r}{r_i}\right)$$

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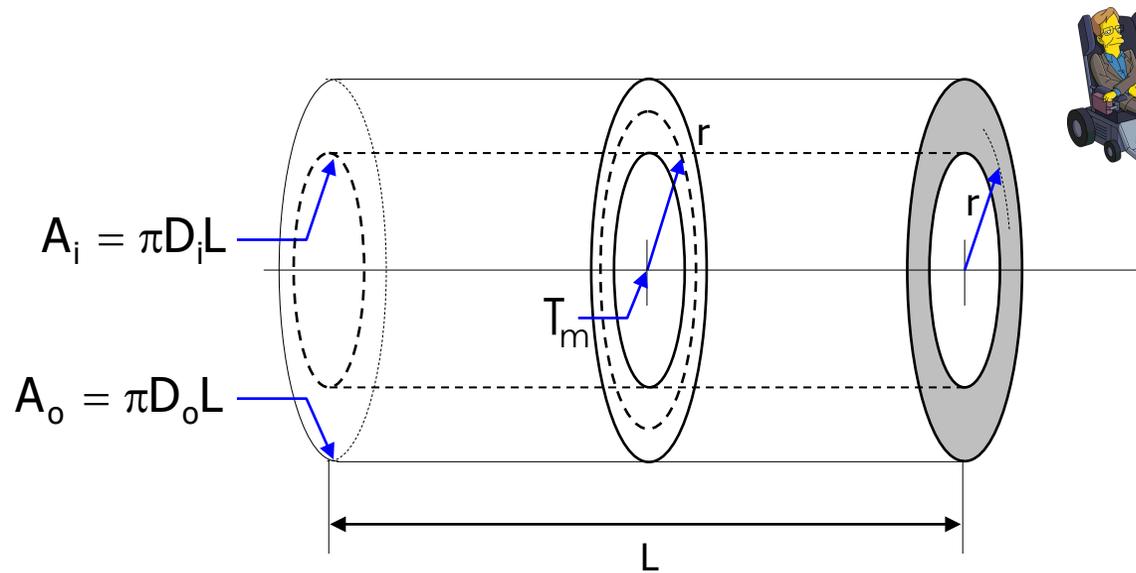
Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$

$$Q = \frac{T_m - T_\infty}{\frac{1}{2\pi r_i L h_i} + \frac{\ln(r_o / r_i)}{2\pi L k} + \frac{1}{2\pi r_o L h_o}}$$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k \vec{\nabla} T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

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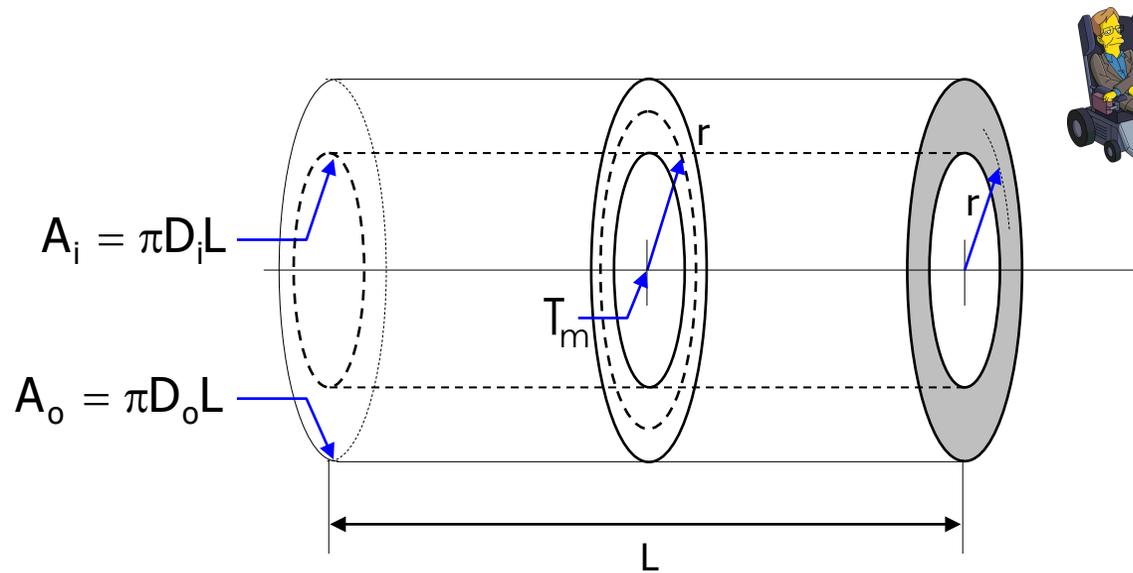
$$Q = \frac{T_m - T_\infty}{\frac{1}{2\pi r_i L h_i} + \frac{\ln(r_o / r_i)}{2\pi L k} + \frac{1}{2\pi r_o L h_o}}$$

$$\dots = \frac{T_m - T_\infty}{R_{\text{total}}} = UA \cdot (T_m - T_\infty)$$



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# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k\vec{\nabla}T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

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$$Q = -kA(r) \frac{dT}{dr}(r) = \dots = \frac{2\pi Lk}{\ln(r_o/r_i)} \cdot (T_i - T_o)$$

Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$

$$Q = \frac{T_m - T_\infty}{\frac{1}{2\pi r_i L h_i} + \frac{\ln(r_o/r_i)}{2\pi L k} + \frac{1}{2\pi r_o L h_o}}$$

$$\dots = \frac{T_m - T_\infty}{R_{\text{total}}} = UA \cdot (T_m - T_\infty)$$

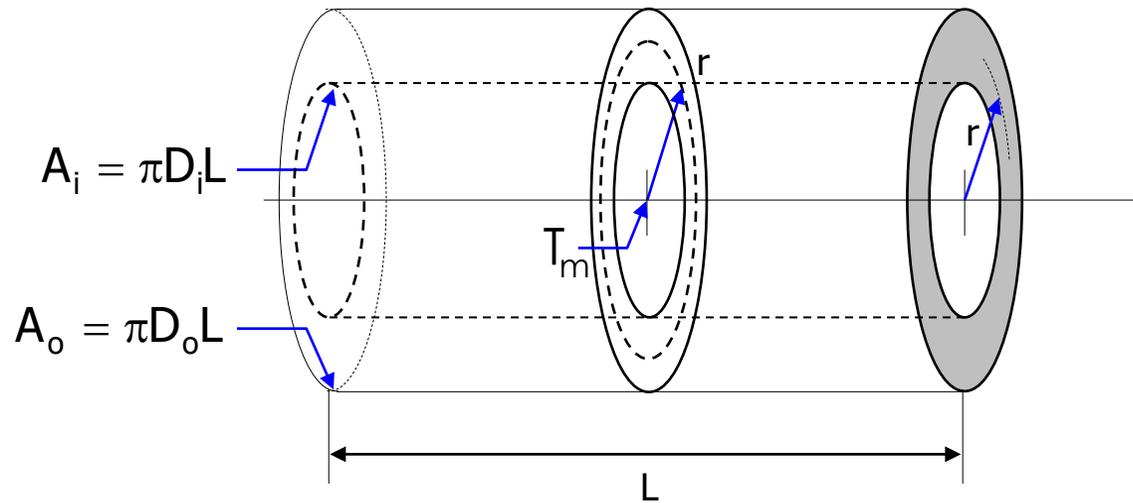
$$A = \pi D_i L \xrightarrow{\text{def}} U = \left[ \frac{1}{h_i} + \frac{r_i}{k} \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o} \cdot \frac{r_i}{r_o} \right]^{-1}$$

coeficiente global de transferência de calor (condutância)



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# Geometria cilíndrica... variação radial da temperatura



Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$

TABLE 13-1

Representative values of the overall heat transfer coefficients in heat exchangers

Type of heat exchanger	$U$ , $W/m^2 \cdot ^\circ C^*$
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Water-to-air in finned tubes (water in tubes)	30–60 <sup>†</sup>
	400–850 <sup>†</sup>
Steam-to-air in finned tubes (steam in tubes)	30–300 <sup>†</sup>
	400–4000 <sup>‡</sup>

\*Multiply the listed values by 0.176 to convert them to  $Btu/h \cdot ft^2 \cdot ^\circ F$ .

<sup>†</sup>Based on air-side surface area.

<sup>‡</sup>Based on water- or steam-side surface area.

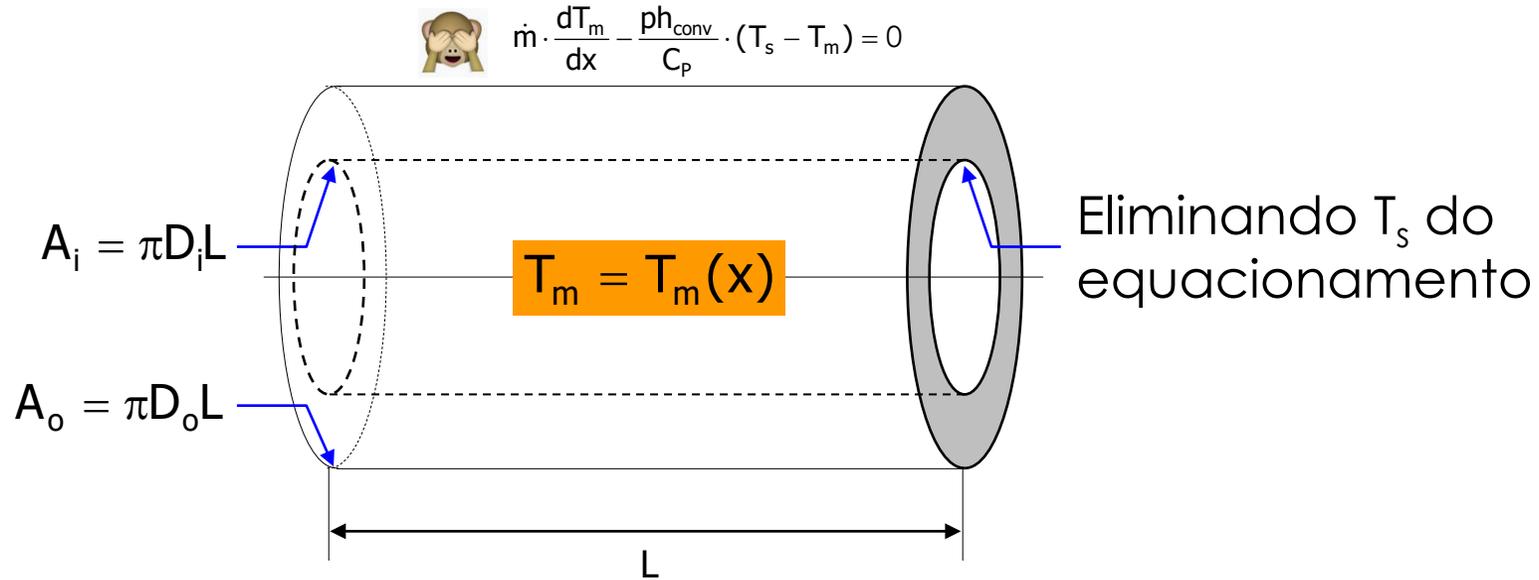
$$A = \pi D_i L \xrightarrow{\text{def}} U = \left[ \frac{1}{h_i} + \frac{r_i}{k} \ln \left( \frac{r_o}{r_i} \right) + \frac{1}{h_o} \cdot \frac{r_i}{r_o} \right]^{-1}$$

coeficiente global de transferência de calor (condutância)

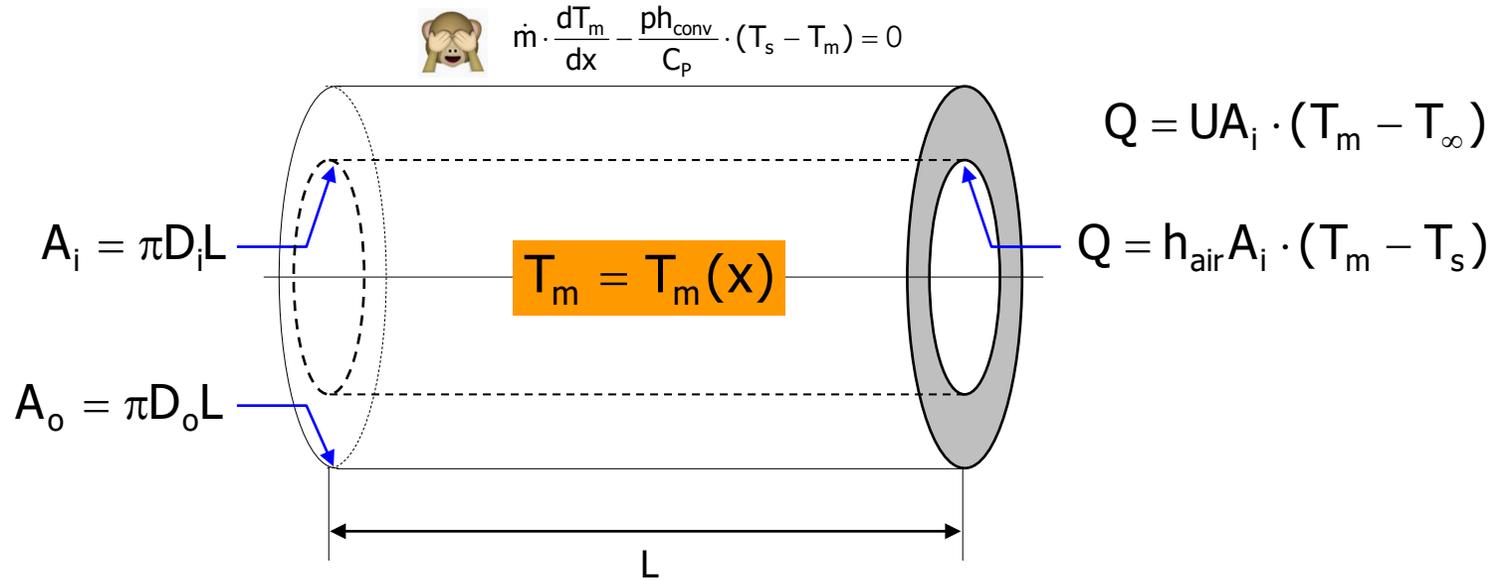


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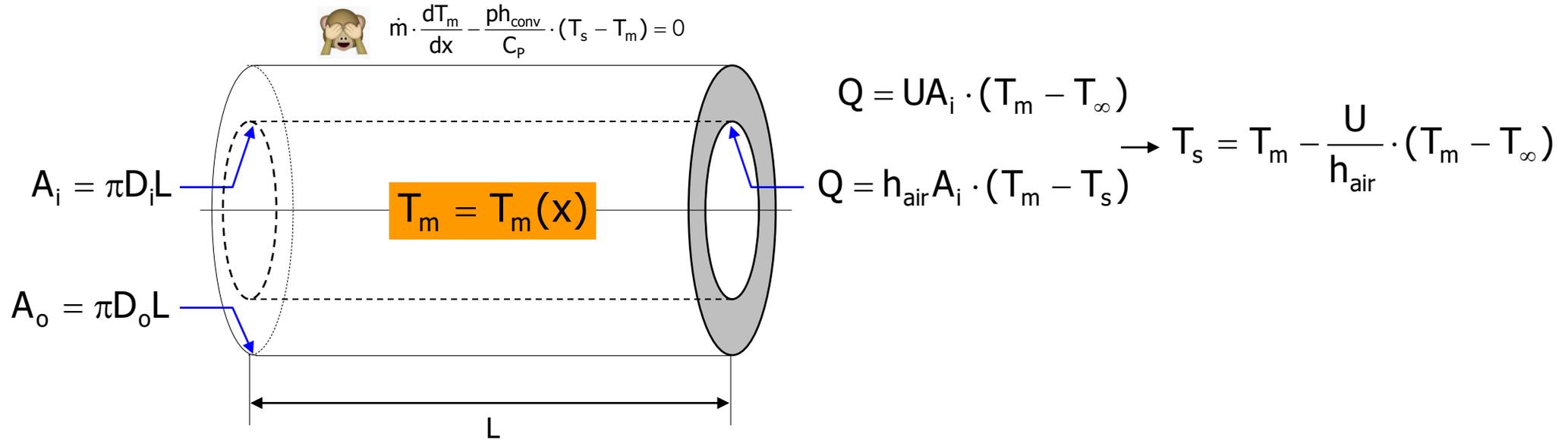
# Geometria cilíndrica... **variação axial da temperatura**



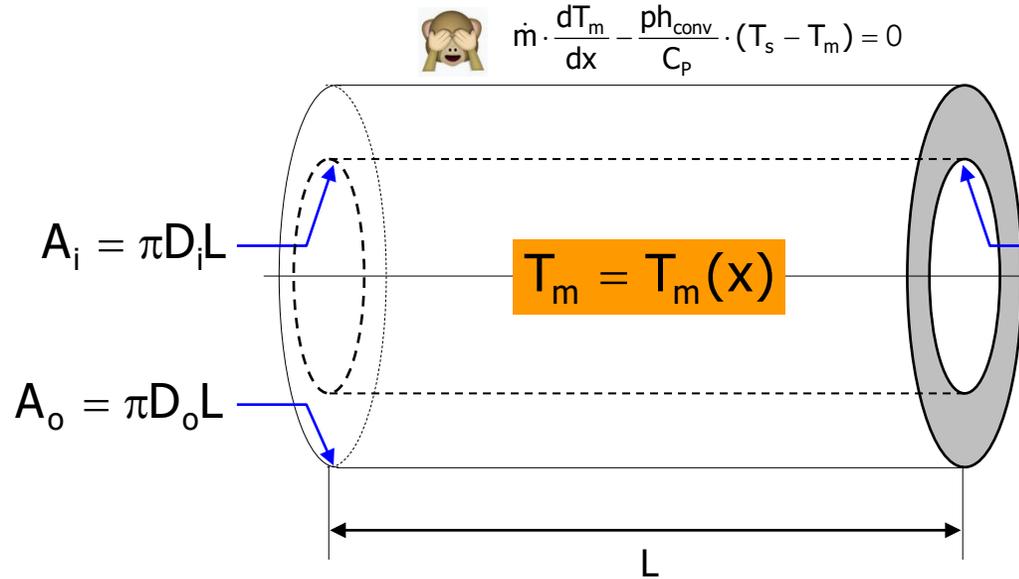
# Geometria cilíndrica... variação axial da temperatura



# Geometria cilíndrica... variação axial da temperatura



# Geometria cilíndrica... variação axial da temperatura



$$Q = UA_i \cdot (T_m - T_\infty)$$

$$Q = h_{air} A_i \cdot (T_m - T_s)$$

$$\rightarrow T_s = T_m - \frac{U}{h_{air}} \cdot (T_m - T_\infty)$$

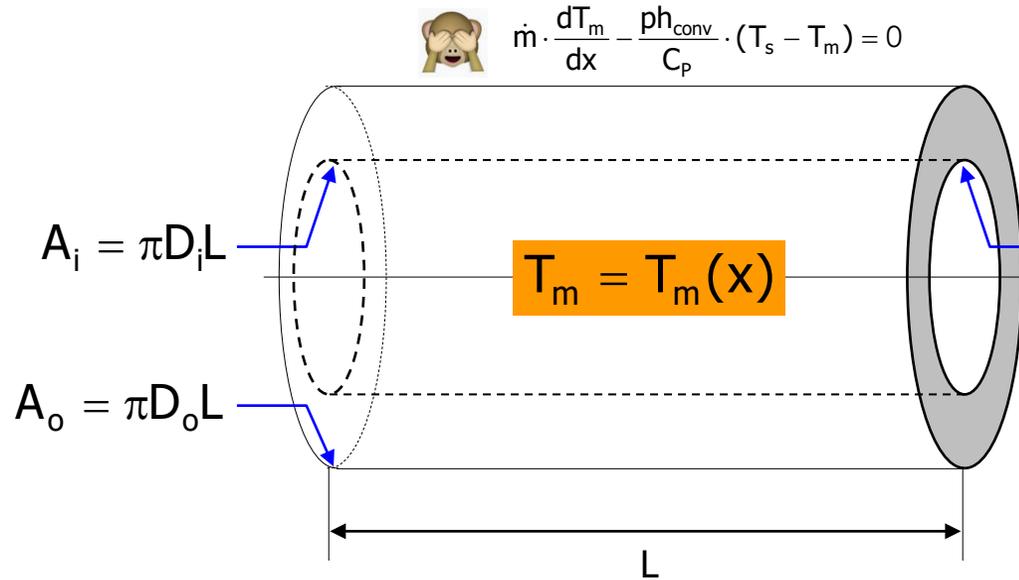
equação do balanço global de energia

$$\dot{m}C_p \cdot \frac{dT_m}{dx} - ph_{air} \cdot (T_s - T_m) = 0$$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Geometria cilíndrica... variação axial da temperatura



$$Q = UA_i \cdot (T_m - T_\infty)$$

$$Q = h_{air} A_i \cdot (T_m - T_s)$$

$$\rightarrow T_s = T_m - \frac{U}{h_{air}} \cdot (T_m - T_\infty)$$

equação do balanço global de energia

$$\dot{m}C_p \cdot \frac{dT_m}{dx} - ph_{air} \cdot (T_s - T_m) = 0$$

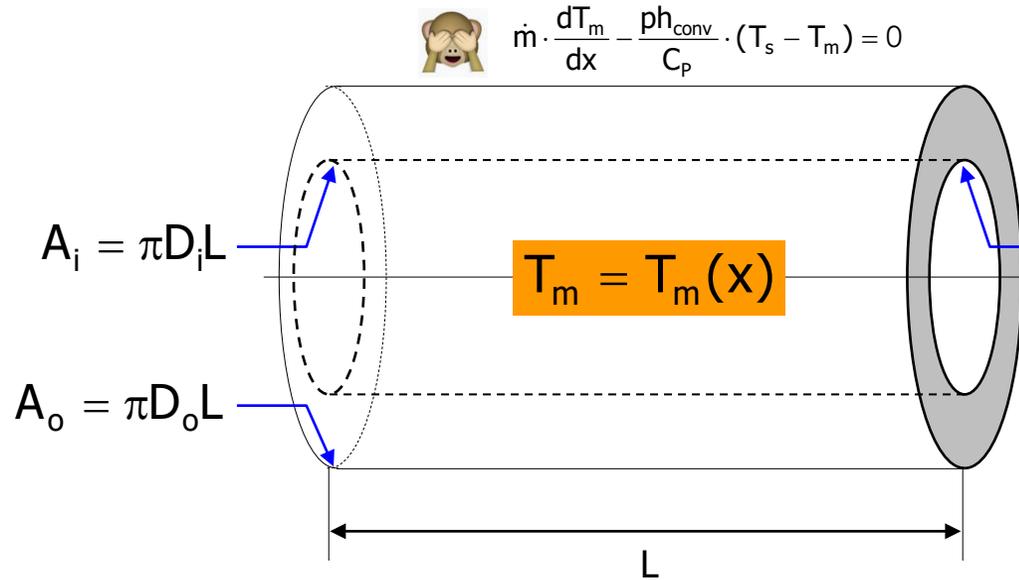


$$\dot{m}C_p \cdot \frac{dT_m}{dx} - ph_{air} \cdot \left[ \left( T_m - \frac{U}{h_{air}} (T_m - T_\infty) \right) - T_m \right] = 0 \rightarrow$$



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# Geometria cilíndrica... variação axial da temperatura



$$Q = UA_i \cdot (T_m - T_\infty)$$

$$Q = h_{air} A_i \cdot (T_m - T_s)$$

$$\rightarrow T_s = T_m - \frac{U}{h_{air}} \cdot (T_m - T_\infty)$$

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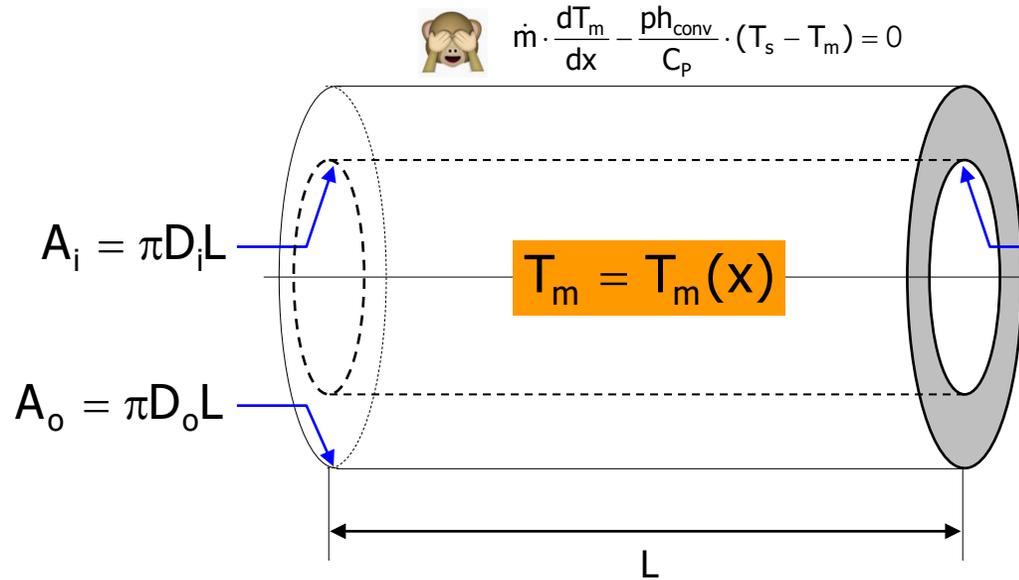
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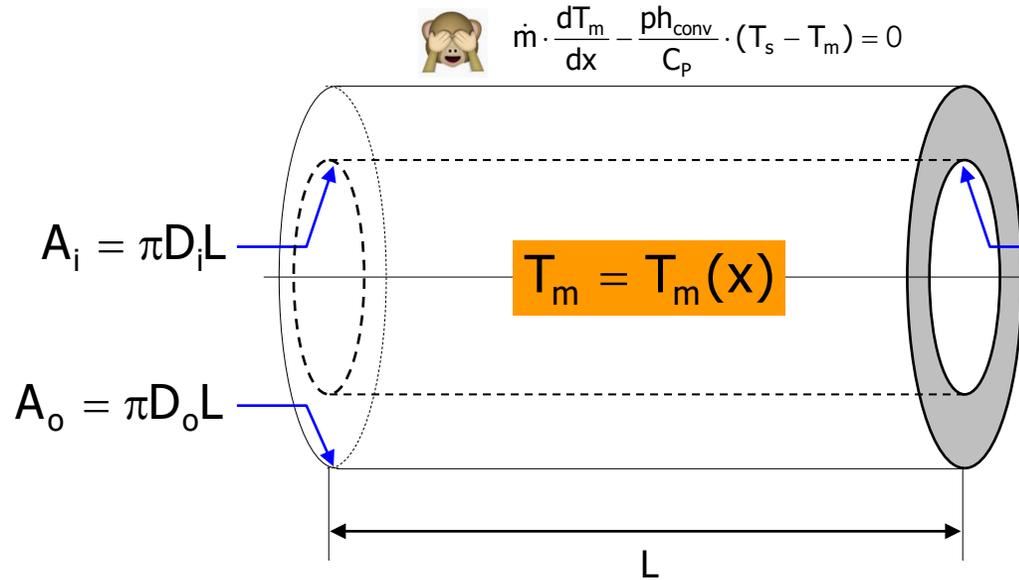
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$$\frac{d(T_m - T_\infty)}{T_m - T_\infty} = -\frac{pU}{\dot{m}C_p} \cdot dx \rightarrow$$



# Geometria cilíndrica... variação axial da temperatura



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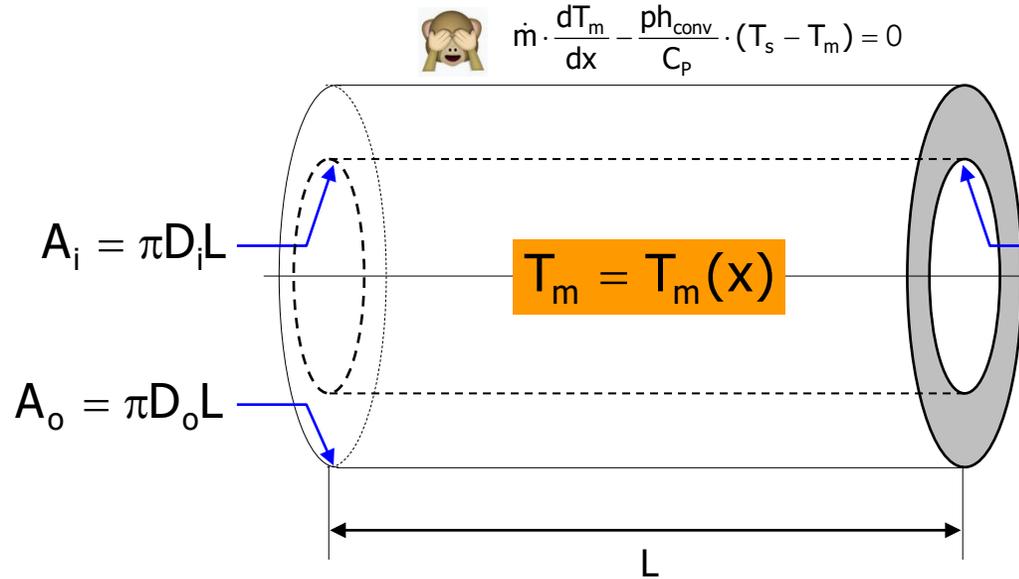


$$\int_{T_{m,e}}^{T_{m,s}} \frac{d(T_m - T_\infty)}{T_m - T_\infty} = -\frac{pL}{\dot{m}C_p} \cdot \frac{1}{L} \int_0^L U dx$$



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# Geometria cilíndrica... variação axial da temperatura



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$$\frac{T_{m,s} - T_\infty}{T_{m,e} - T_\infty} = \exp\left(-\frac{\bar{U}A}{\dot{m}C_p}\right)$$

... calculado @ temperatura axial média

$$\frac{T_{m,s} - T_{\infty}}{T_{m,e} - T_{\infty}} = \exp\left(-\frac{\bar{U}A}{\dot{m}C_p}\right) \rightarrow$$

$$(\bar{U}A_i)^{-1} = \frac{1}{2\pi r_i L h_i} + \frac{\ln(r_o / r_i)}{2\pi L k} + \frac{1}{2\pi r_o L h_o}$$

Dittus-Boelter ?

=cte

=cte



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Dittus-Boelter ?

=cte

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Escoamento turbulento desenvolvido ( $Re > 10^4$ ):

Superfície lisa:

$$Nu = 0.023 \cdot Re^{4/5} \cdot Pr^n$$

(Dittus-Boelter)

$$n = 0.3 \quad p / T_s < T_m$$

$$n = 0.4 \quad p / T_s > T_m$$

$$\left[ \begin{array}{l} 0.7 \leq Pr \leq 160 \\ Re_D \geq 10,000 \\ \frac{L}{D} \geq 10 \end{array} \right]$$



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Dittus-Boelter ?      =cte      =cte

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$$\bar{T} = 25^{\circ}C \rightarrow \rho = 1.1688 \text{ kg/m}^3 \quad \mu = 18.448 \text{ } \mu\text{Pa} \cdot \text{s} \quad C_p = 1.0063 \text{ kJ/kg/K}$$

$$k = 26.247 \text{ mW/m/K} \quad Pr = 0.70729$$



$$\left[ \begin{array}{l} 0.7 \leq Pr \leq 160 \\ Re_D \geq 10,000 \\ \frac{L}{D} \geq 10 \end{array} \right]$$



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$$Re = \frac{\rho U D}{\mu} = \frac{\rho D}{\mu} \frac{V_i}{\pi D_i^2 / 4} = \frac{1.1688 \cdot 0.15}{18.448 \cdot 10^{-6}} \cdot \frac{0.025}{\pi \cdot 0.15^2 / 4} = 1.344 \cdot 10^4$$



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$$Nu = 0.023 \cdot Re^{4/5} \cdot Pr^{0.3} = 0.023 \cdot (1.344 \cdot 10^4)^{4/5} \cdot (0.70729)^{0.4} = 40.206$$



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$$\frac{T_{m,s} - T_{\infty}}{T_{m,e} - T_{\infty}} = \exp\left(-\frac{\bar{U}A}{\dot{m}C_p}\right) \rightarrow$$

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$$Nu = \frac{h_i D_i}{k} \rightarrow h_i = \frac{26.247 \cdot 10^{-3} \cdot 40.206}{0.15} = 7.035 \frac{\text{W}}{\text{m}^2 \text{K}}$$

$$\frac{T_{m,s} - T_{\infty}}{T_{m,e} - T_{\infty}} = \exp\left(-\frac{\bar{U}A}{\dot{m}C_p}\right) \rightarrow$$

$$(\bar{U}A_i)^{-1} = \frac{1}{2\pi r_i L h_i} + \frac{\ln(r_o / r_i)}{2\pi L k} + \frac{1}{2\pi r_o L h_o}$$

$$7.035 \frac{W}{m^2 K}$$

$$0.15 \frac{W}{mK}$$

$$1500 \frac{W}{m^2 K}$$



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$$0.15 \frac{\text{W}}{\text{mK}}$$

$$1500 \frac{\text{W}}{\text{m}^2 \text{K}}$$

$$(\bar{U}A_i)^{-1} = \frac{1}{\pi L} \left( \frac{1}{7.035 \cdot 0.15} + \frac{1}{2 \cdot 0.15} \ln\left(\frac{0.17}{0.15}\right) + \frac{1}{1500 \cdot 0.17} \right) \rightarrow \bar{U}A_i = 2.295 \cdot L$$



$$\frac{T_{m,s} - T_{\infty}}{T_{m,e} - T_{\infty}} = \exp\left(-\frac{\bar{U}A}{\dot{m}C_p}\right) \rightarrow$$

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$$\frac{21-17}{29-17} = \exp\left(-\frac{2.295 \cdot L}{0.02883 \cdot 1006.3}\right)$$



$$\frac{T_{m,s} - T_{\infty}}{T_{m,e} - T_{\infty}} = \exp\left(-\frac{\bar{U}A}{\dot{m}C_p}\right) \rightarrow$$

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$$L = 13.88 \text{ m}$$



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Considerando propriedades termofísicas variáveis...



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$$Re = \frac{\rho u_0 D}{\mu}$$

$$Pr = \frac{C_p \mu}{k}$$

# Balanco global de energia...

$$\dot{m} \cdot \frac{dh_m}{dx} - ph_{conv} \cdot (T_s - T_m) = 0$$

Solução direta via método numérico de solução de equação diferencial

$$h = h(P, T) \downarrow$$

Implementação numérica...

Solução analítica a partir de hipóteses simplificadoras

$$h = C_p \cdot T \downarrow \begin{array}{l} \text{gases perfeitos} \\ \text{"fluido incompressível" } (C_p=C_v) \end{array}$$

$$\dot{m} \cdot \frac{dT_m}{dx} - \frac{ph_{conv}}{C_p} \cdot (T_s - T_m) = 0$$

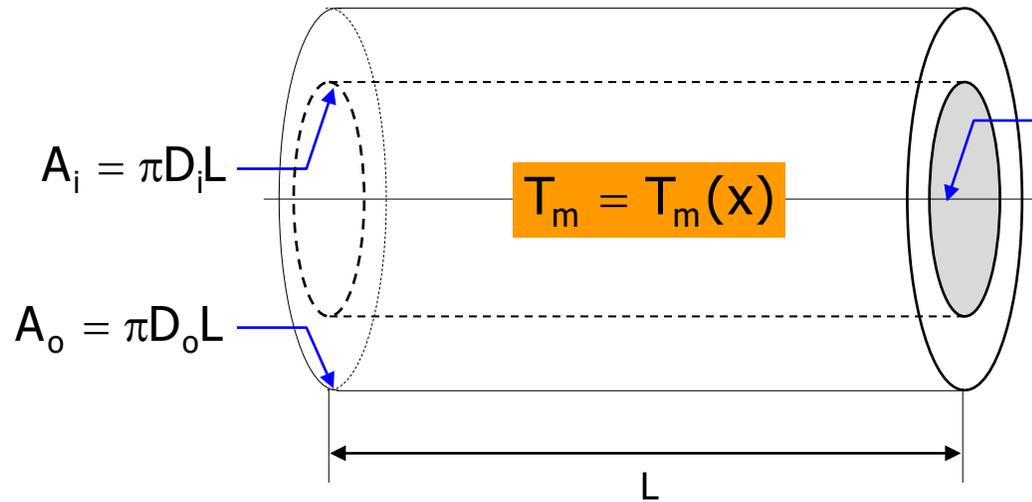


$$\mu = \mu(P, T), \dots \downarrow$$

Implementação numérica...



# Geometria cilíndrica... variação axial da temperatura



$$\dot{m}C_p \cdot \frac{dT_m}{dx} + pU \cdot (T_m - T_\infty) = 0$$

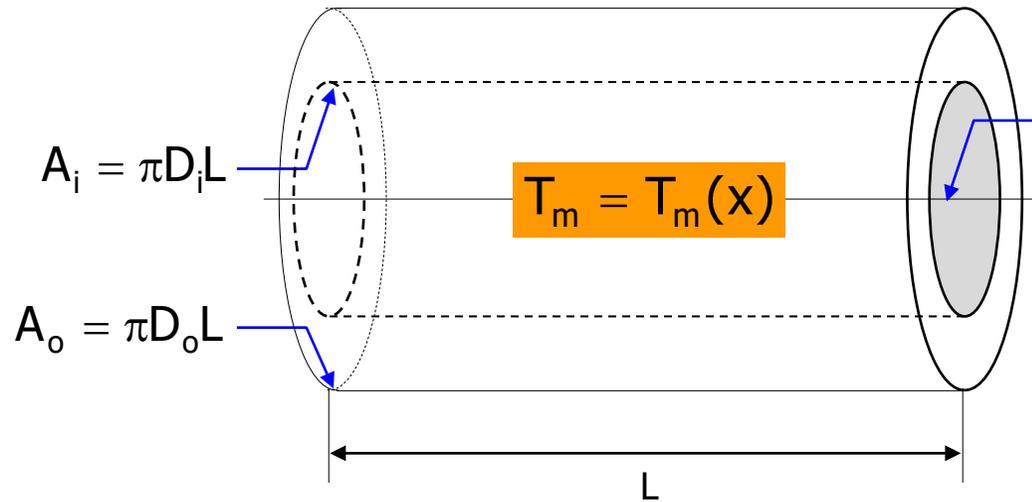


Eliminada  $T_s$  da eq.



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# Geometria cilíndrica... variação axial da temperatura

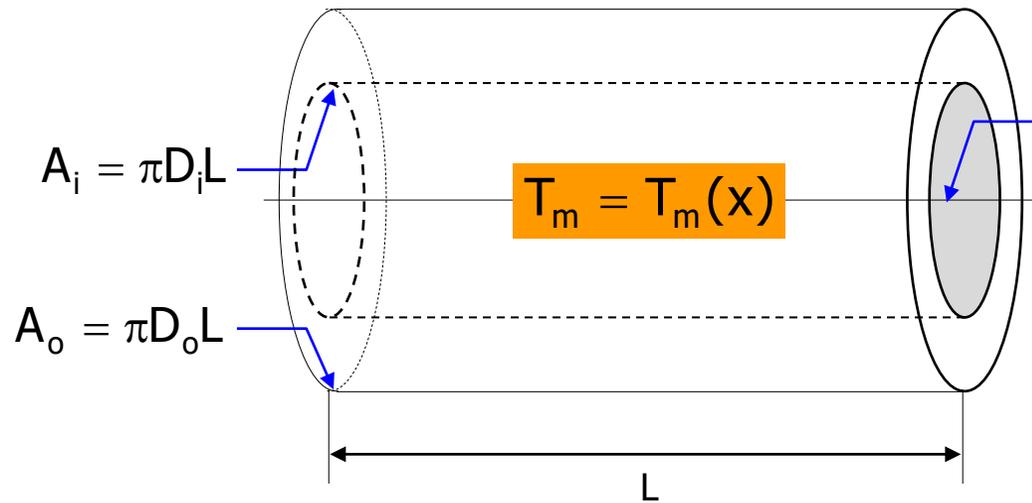


$$\dot{m}C_p \cdot \frac{dT_m}{dx} + pU \cdot (T_m - T_\infty) = 0$$

$$U = \left[ \frac{1}{h_i} + \frac{r_i}{k} \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o} \cdot \frac{r_i}{r_o} \right]^{-1}$$



# Geometria cilíndrica... variação axial da temperatura



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Dittus-Boelter

=cte

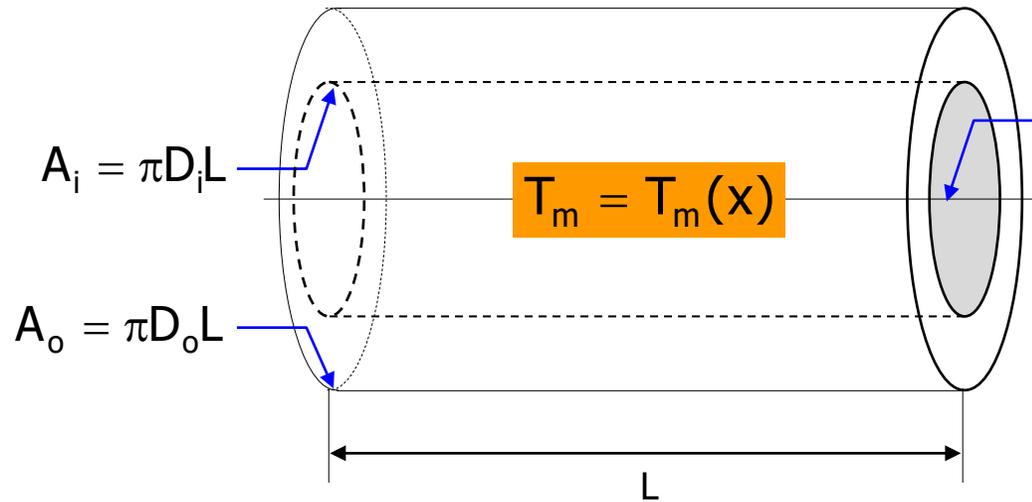
=cte

$$Nu = 0.023 \cdot Re^{4/5} \cdot Pr^n$$



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# Geometria cilíndrica... variação axial da temperatura



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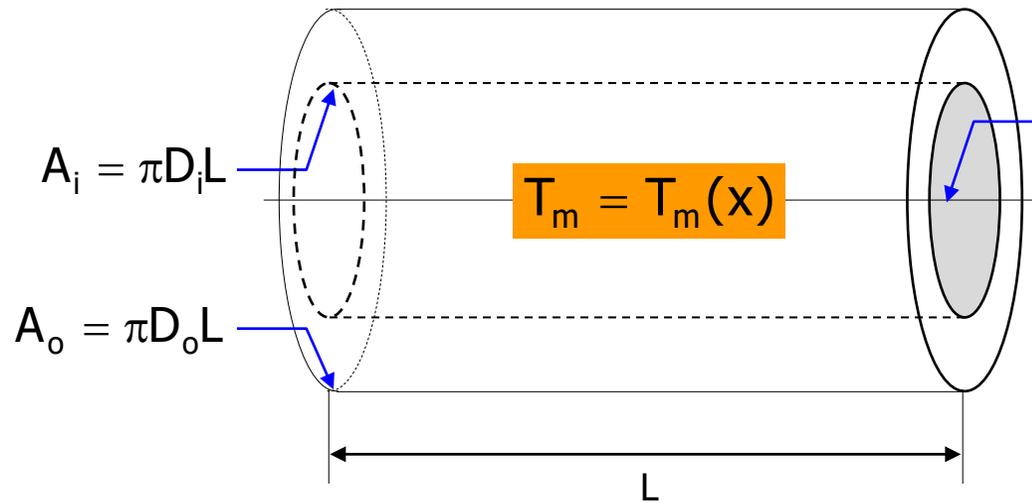
Dittus-Boelter  $\frac{r_i}{k} \ln\left(\frac{r_o}{r_i}\right) = \text{cte}$   $\frac{1}{h_o} \cdot \frac{r_i}{r_o} = \text{cte}$

$$Nu = 0.023 \cdot Re^{4/5} \cdot Pr^n$$

$$Re = \frac{\rho U_0 D}{\mu} \Big|_{T_m} \quad Pr = \frac{C_p \mu}{k} \Big|_{T_m}$$



# Geometria cilíndrica... variação axial da temperatura



$$\dot{m}C_p \cdot \frac{dT_m}{dx} + pU \cdot (T_m - T_\infty) = 0$$

$$\frac{dT_m}{dx} + \alpha(T_m) \cdot (T_m - T_\infty) = 0 \quad \alpha(T_m) = \frac{\rho}{\dot{m}} \frac{U(T_m)}{C_p(T_m)}$$


$$\frac{T_{m,j+1} - T_{m,j}}{dx} + \alpha_j \cdot (T_{m,j} - T_\infty) = 0$$

$$T_{m,j+1} = T_{m,j} - \alpha_j \cdot (T_{m,j} - T_\infty) \cdot dx$$

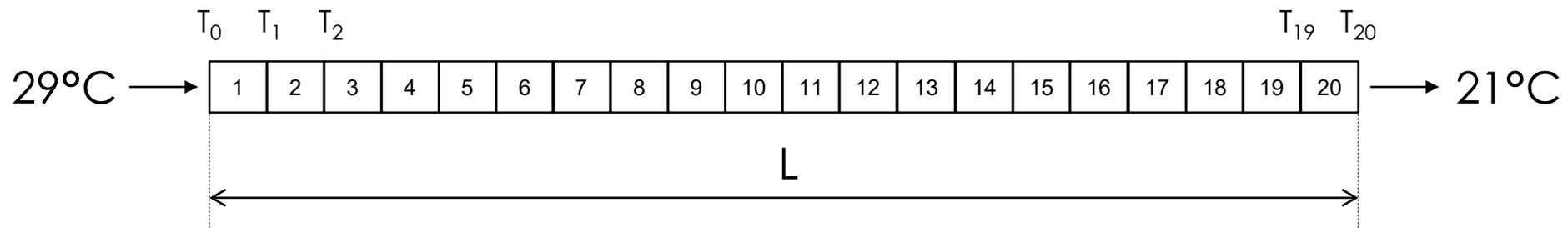

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$$T_{m,j+1} = T_{m,j} - \alpha_j \cdot (T_{m,j} - T_\infty) \cdot dx$$



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**8.31:** Para resfriar uma casa de verão sem uso de um ciclo frigorífico, ar é encaminhado através de uma tubulação de plástico ( $k=0.15\text{W/m/K}$ ,  $D_i=0.15\text{m}$ ,  $D_o=0.17\text{m}$ ) submersa em um corpo d'água adjacente. A temperatura da água é normalmente de  $T_\infty=17^\circ\text{C}$ , e o coeficiente de convecção é mantido em  $h_o=1500\text{ W/m}^2/\text{K}$  na superfície externa da tubulação. Se ar proveniente da casa entra no tubo a uma temperatura de  $T_{m,i}=29^\circ\text{C}$  e uma vazão volumétrica de  $V_i=0.025\text{m}^3/\text{s}$ , qual extensão  $L$  é necessária para que a temperatura na saída seja de  $T_{m,o}=21^\circ\text{C}$ ?

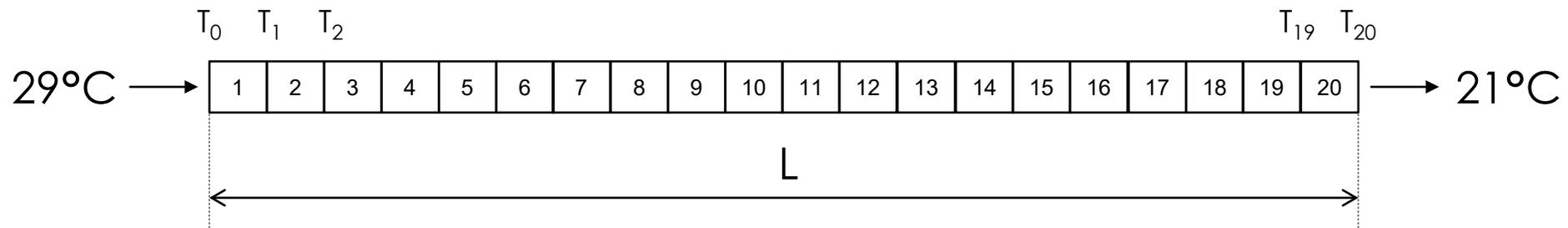


$$T_{m,j+1} = T_{m,j} - \alpha_j \cdot (T_{m,j} - T_\infty) \cdot dx \rightarrow$$



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$$T_{m,j+1} = T_{m,j} - \alpha_j \cdot (T_{m,j} - T_\infty) \cdot dx \rightarrow \begin{cases} T_{m,1} = T_{m,0} - \alpha_0 \cdot (T_{m,0} - T_\infty) \cdot dx \\ \rightarrow T_{m,2} = T_{m,1} - \alpha_1 \cdot (T_{m,1} - T_\infty) \cdot dx \\ \rightarrow T_{m,3} = T_{m,2} - \alpha_2 \cdot (T_{m,2} - T_\infty) \cdot dx \\ \vdots \\ \rightarrow T_{m,20} = T_{m,19} - \alpha_{19} \cdot (T_{m,19} - T_\infty) \cdot dx \end{cases}$$



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Salvamento Automático | exemplo aula TC3.xlsm - Excel | Entrar | Compartilhar

Arquivo | Página Inicial | Inserir | Layout da Página | Fórmulas | Dados | Revisão | Exibir | Diga-me o que você deseja fazer

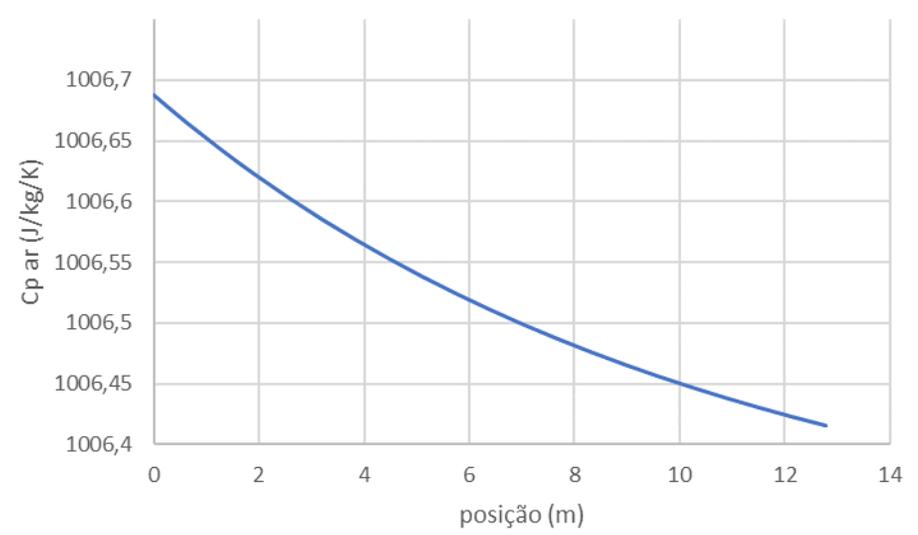
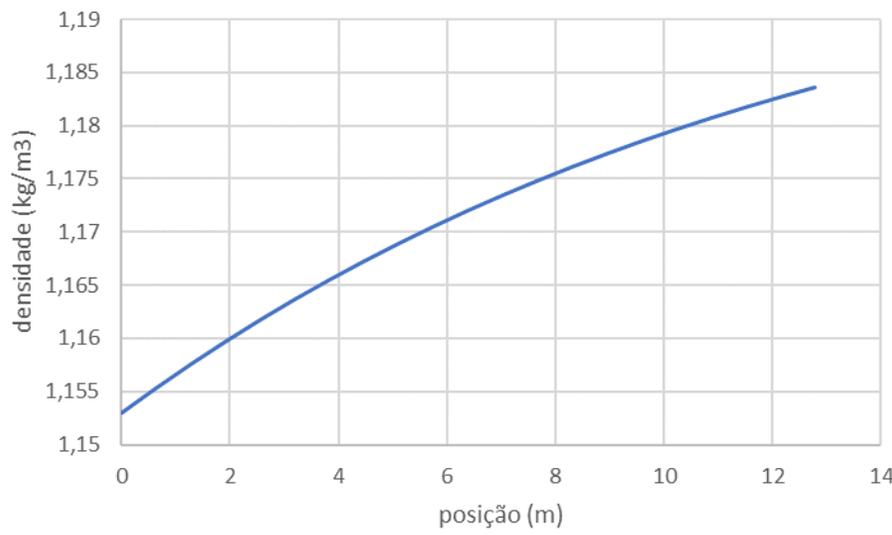
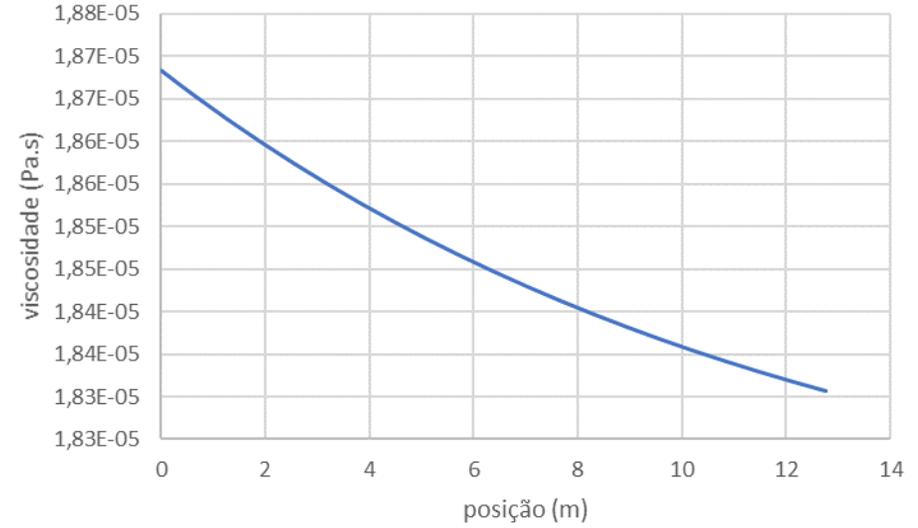
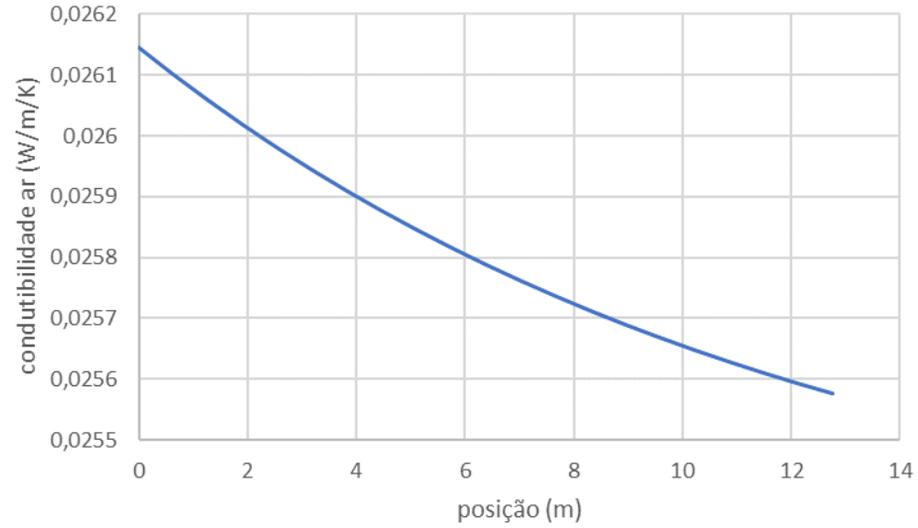
Recortar | Copiar | Pincel de Formatação | Área de Transferência

Calibri 11 | Quebrar Texto Automaticamente | Geral | Formatação Condicional | Formatar como Tabela

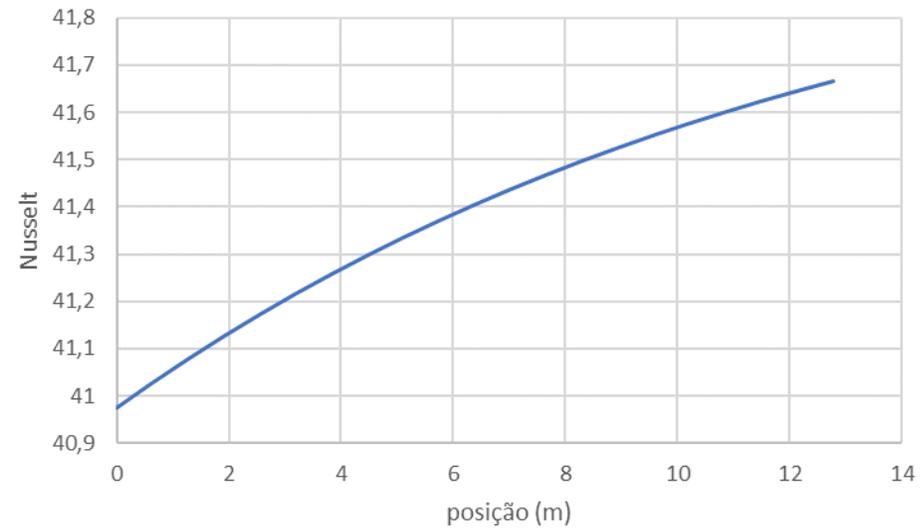
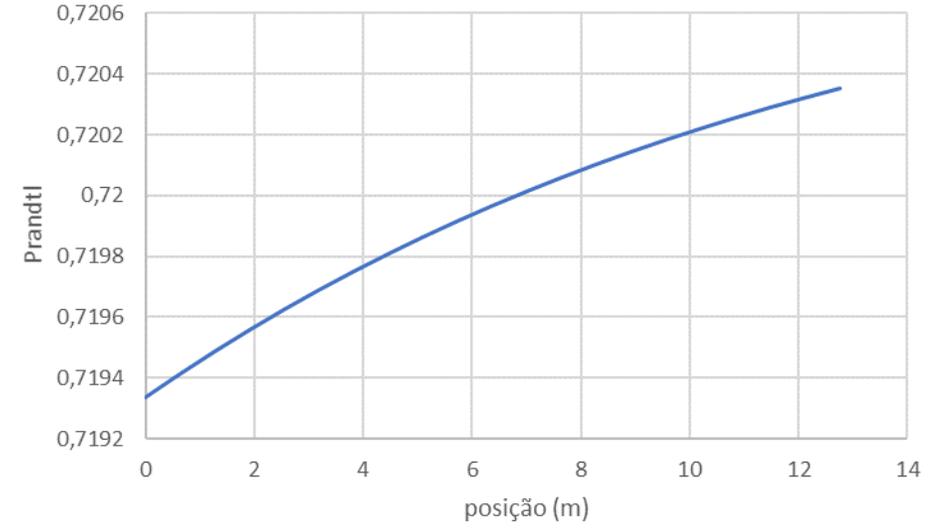
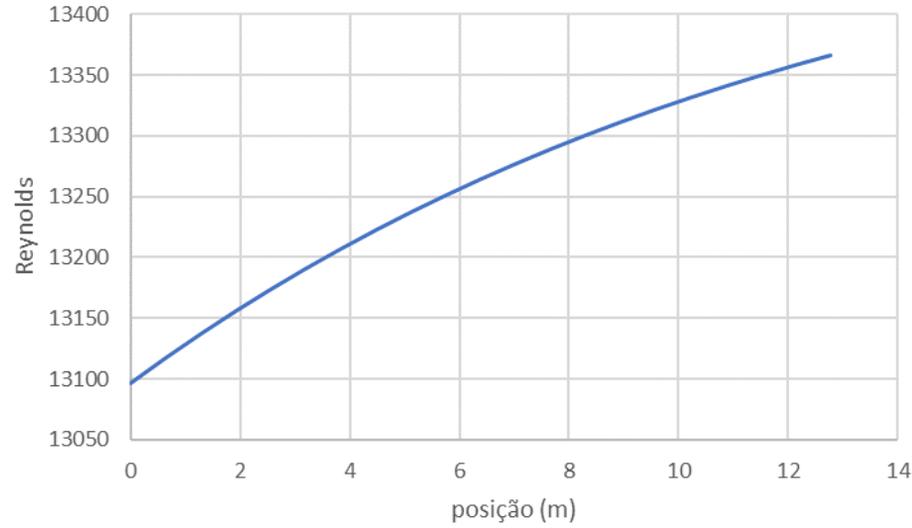
Normal | Bom | Neutro | Ruim | Cálculo | Célula de Ve... | Célula Vincul... | Entrada

Inserir | Excluir | Formatar | AutoSoma | Preencher | Limpar | Classificar e Filtrar | Localizar e Selecionar | Edição

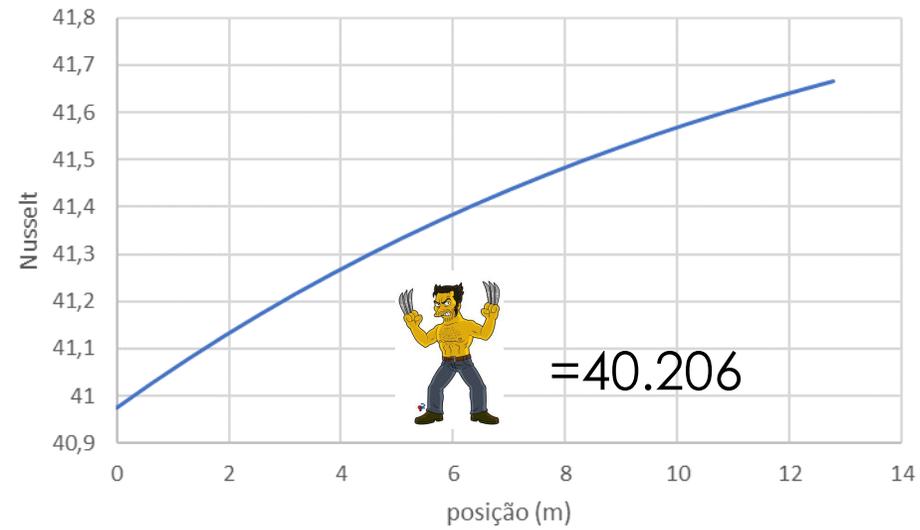
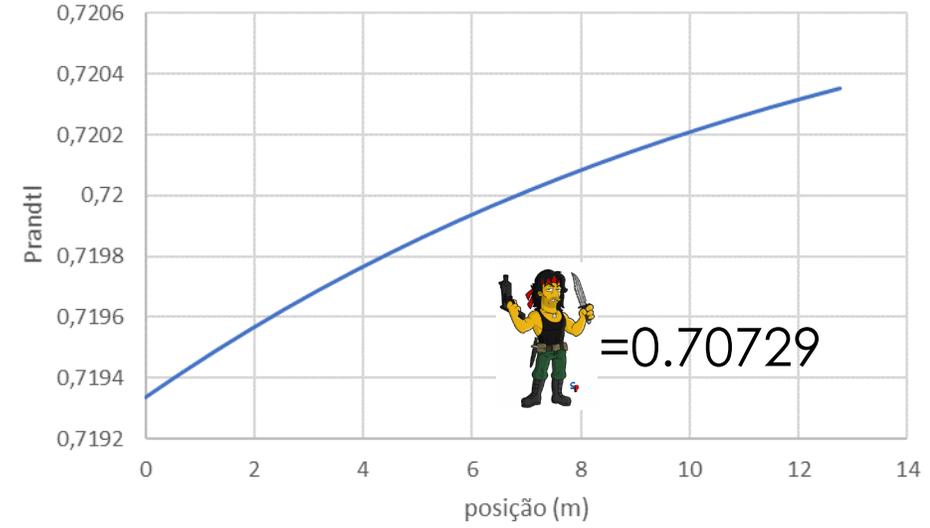
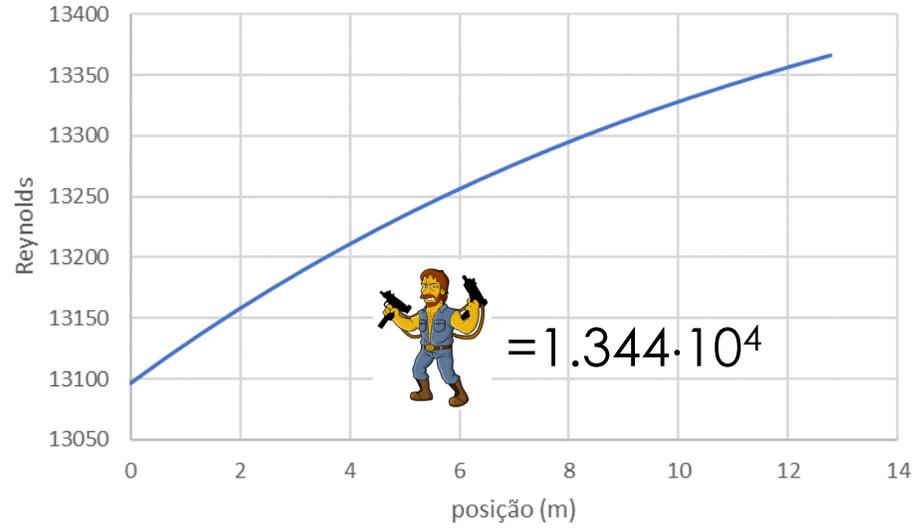
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	P	1 bar			posição	x	Tj	$\rho$	U (m/s)	$\mu$ (Pa.s)	kar	Cp	Re	Pr	Nu	hi	U	$\alpha(T)$	Tj+1	
2	ho	1500 W/m2/K			0	0	29	1,15301	1,41471	1,8683E-05	0,02615	1006,69	13096,2	0,71934	40,9754	7,14239	4,92177	0,07993	28,3574	
3	k	0,15 W/m/K			1	0,67	28,3574	1,15548	1,41169	1,8652E-05	0,0261	1006,66	13117,9	0,71942	41,0312	7,13928	4,92029	0,0799	27,7494	
4	Di	0,15 m			2	1,34	27,7494	1,15782	1,40884	1,8623E-05	0,02606	1006,64	13138,6	0,71949	41,0842	7,13634	4,9189	0,07988	27,174	
5	Do	0,17 m			3	2,01	27,174	1,16004	1,40614	1,8595E-05	0,02601	1006,62	13158,2	0,71957	41,1345	7,13355	4,91757	0,07986	26,6296	
6	Tinf	17 oC			4	2,68	26,6296	1,16215	1,40358	1,8569E-05	0,02597	1006,6	13176,8	0,71964	41,1823	7,13091	4,91631	0,07985	26,1145	
7	Tmi	29 oC			5	3,35	26,1145	1,16416	1,40117	1,8544E-05	0,02594	1006,58	13194,5	0,7197	41,2276	7,1284	4,91512	0,07983	25,627	
8	Vz @ i	0,025 m3/s			6	4,02	25,627	1,16606	1,39888	1,852E-05	0,0259	1006,56	13211,3	0,71977	41,2707	7,12603	4,91399	0,07981	25,1657	
9	VzM @ i	0,02883 kg/s			7	4,69	25,1657	1,16787	1,39671	1,8498E-05	0,02587	1006,55	13227,2	0,71983	41,3116	7,12378	4,91292	0,07979	24,7291	
10					8	5,36	24,7291	1,16959	1,39466	1,8477E-05	0,02583	1006,53	13242,4	0,71989	41,3504	7,12164	4,91191	0,07978	24,316	
11					9	6,03	24,316	1,17122	1,39272	1,8457E-05	0,0258	1006,52	13256,7	0,71994	41,3872	7,11962	4,91095	0,07976	23,925	
12	Tmo	21 oC			10	6,7	23,925	1,17276	1,39089	1,8438E-05	0,02578	1006,5	13270,4	0,71999	41,4221	7,11771	4,91004	0,07975	23,555	
13	L	13,4 m			11	7,37	23,555	1,17423	1,38915	1,842E-05	0,02575	1006,49	13283,3	0,72004	41,4553	7,1159	4,90917	0,07974	23,2048	
14	N	20 nd			12	8,04	23,2048	1,17562	1,38751	1,8403E-05	0,02572	1006,48	13295,6	0,72009	41,4867	7,11418	4,90836	0,07973	22,8733	
15	dx	0,67 m			13	8,71	22,8733	1,17694	1,38595	1,8387E-05	0,0257	1006,47	13307,2	0,72013	41,5165	7,11255	4,90758	0,07971	22,5597	
16					14	9,38	22,5597	1,17819	1,38448	1,8372E-05	0,02567	1006,46	13318,2	0,72017	41,5448	7,11101	4,90685	0,0797	22,2628	
17					15	10,05	22,2628	1,17938	1,38308	1,8357E-05	0,02565	1006,45	13328,7	0,72021	41,5716	7,10955	4,90615	0,07969	21,9818	
18					16	10,72	21,9818	1,18051	1,38176	1,8343E-05	0,02563	1006,44	13338,6	0,72025	41,597	7,10817	4,9055	0,07968	21,7158	
19					17	11,39	21,7158	1,18157	1,38051	1,8331E-05	0,02561	1006,43	13348,1	0,72029	41,6211	7,10686	4,90487	0,07967	21,4641	
20					18	12,06	21,4641	1,18259	1,37933	1,8318E-05	0,02559	1006,42	13357	0,72032	41,644	7,10562	4,90428	0,07966	21,2258	
21					19	12,73	21,2258	1,18355	1,37821	1,8307E-05	0,02558	1006,42	13365,4	0,72035	41,6656	7,10444	4,90372	0,07966	21,0003	
22					20	13,4	21,0003	1,18446	1,37716	1,8296E-05	0,02556	1006,41	13373,5	0,72038	41,6861	7,10333	4,90319	0,07965	20,7868	
23																				
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As questões postadas no Chat do YouTube serão respondidas ao final da aula.

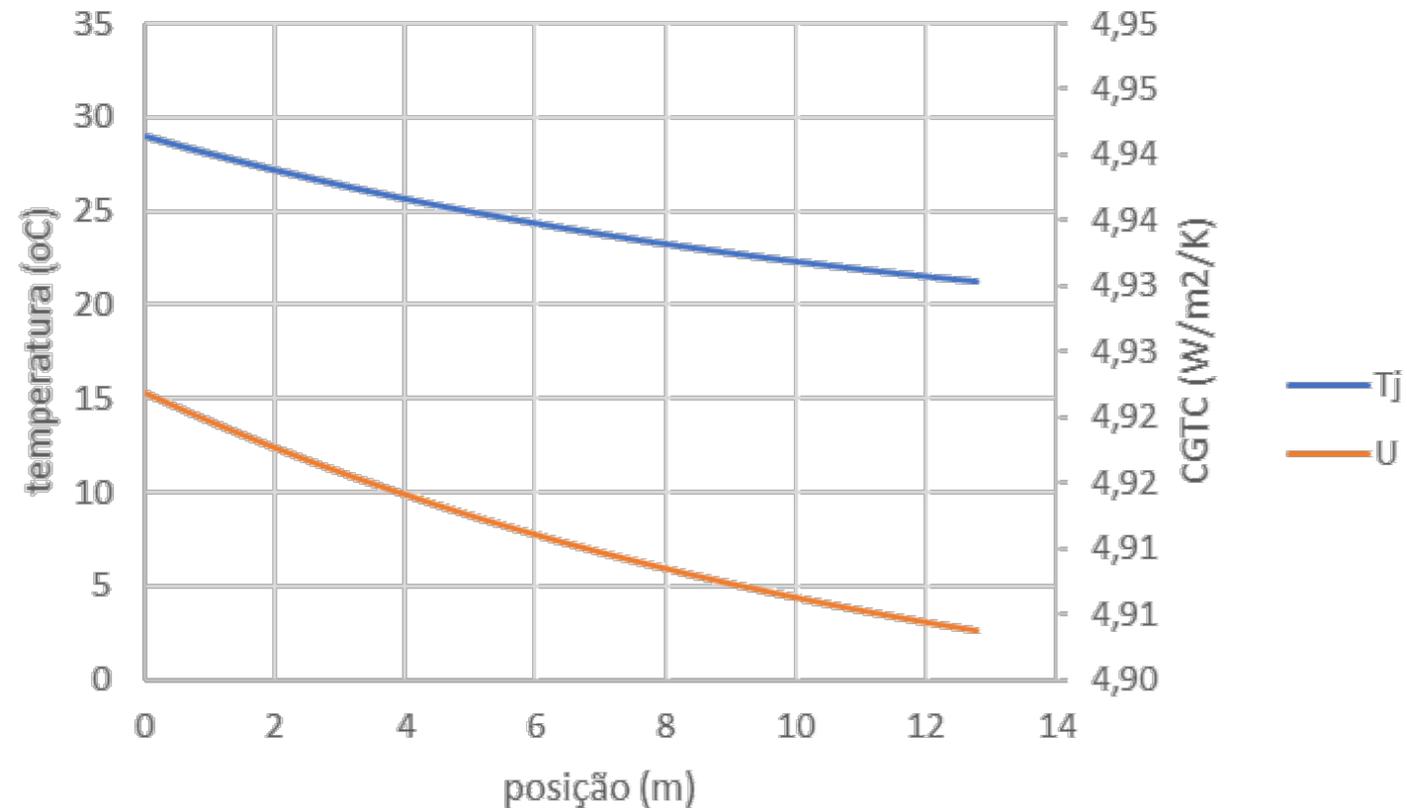


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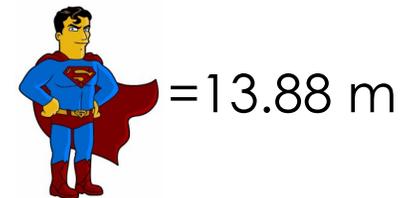
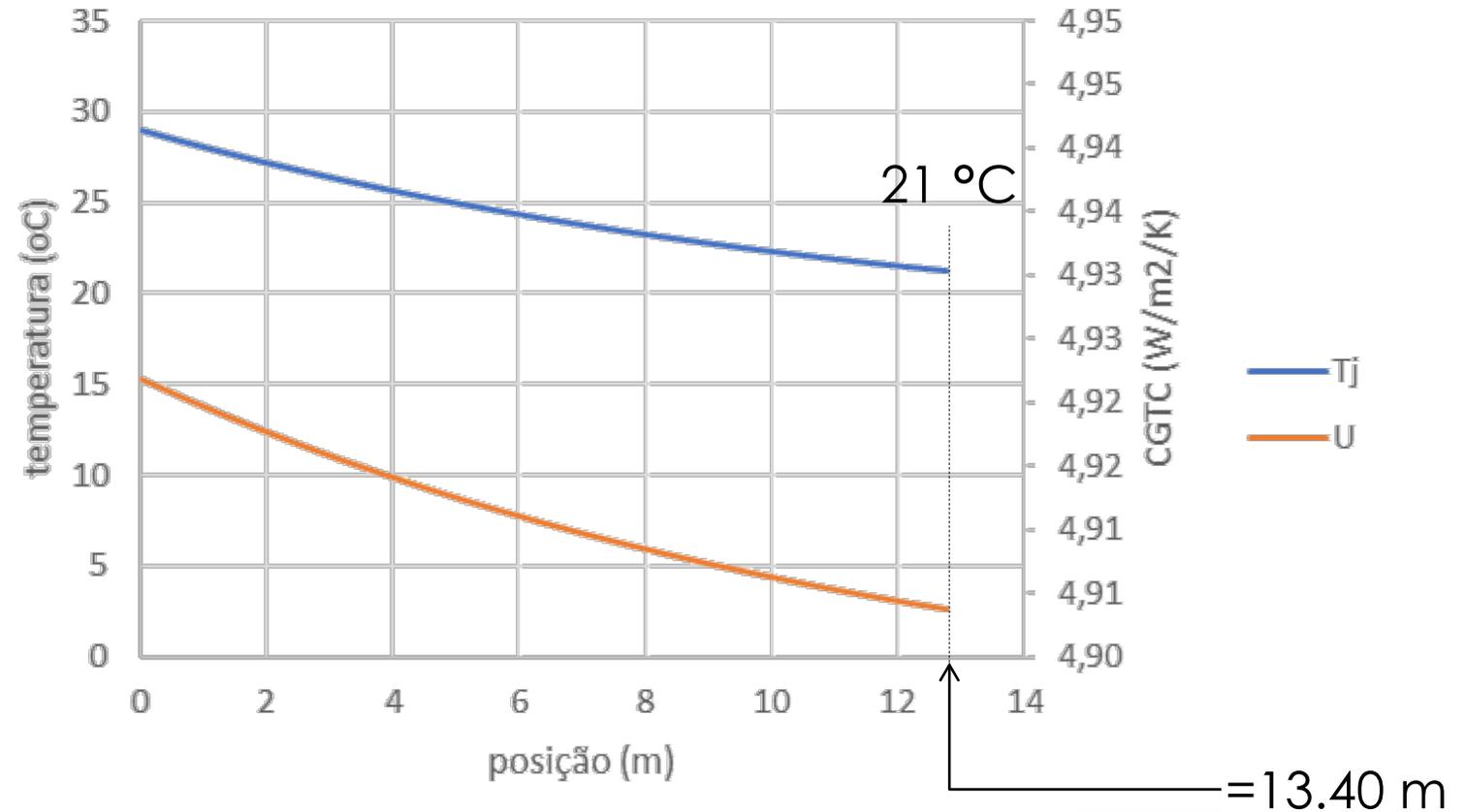
As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Tutorial: montagem da planilha de simulação...



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Tutorial: montagem da planilha de simulação...



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Dúvidas ?



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# CONVECÇÃO DE CALOR

