

# ESCOAMENTOS E CONVECÇÃO DE CALOR

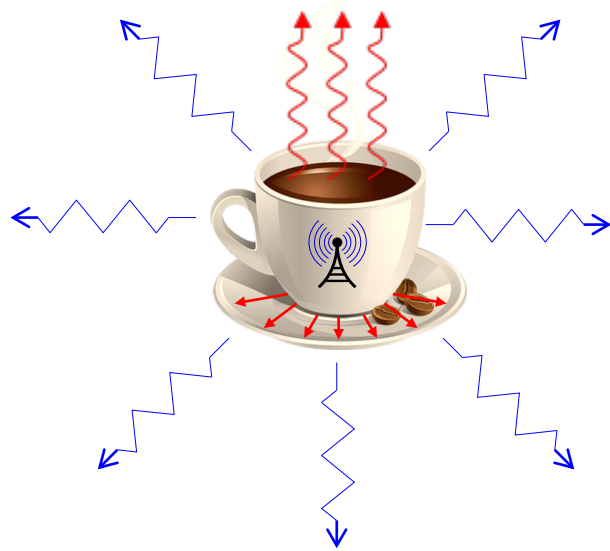
**Paulo Seleglim Jr.**  
**Universidade de São Paulo**



ANTES PORÉM:

RESISTÊNCIA TÉRMICA...

# DEFINIÇÃO DE UMA RESISTÊNCIA TÉRMICA → CIRCUITOS TÉRMICOS EQUIVALENTES



Condução de calor: lei de Fourier

$$Q = k \cdot A \cdot \Delta T / L$$

Convecção de calor: lei de Newton

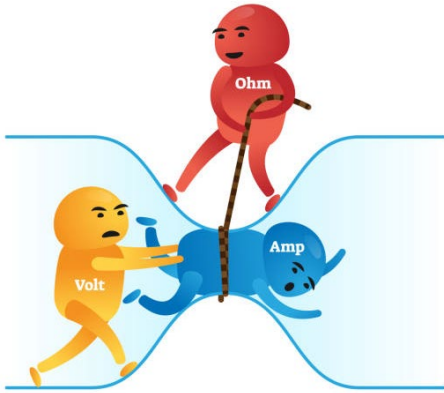
$$Q = h \cdot A \cdot \Delta T$$

Radiação de calor: lei de Stefan–Boltzmann

$$Q = \sigma \cdot A \cdot \Sigma(T^4 - T_{\infty}^4) \rightarrow Q = \bar{h}_{rad} \cdot A \cdot \Delta T$$



# DEFINIÇÃO DE UMA RESISTÊNCIA TÉRMICA → CIRCUITOS TÉRMICOS EQUIVALENTES



$$R_{elec} = \frac{\Delta V}{i}$$

Condução de calor: lei de Fourier

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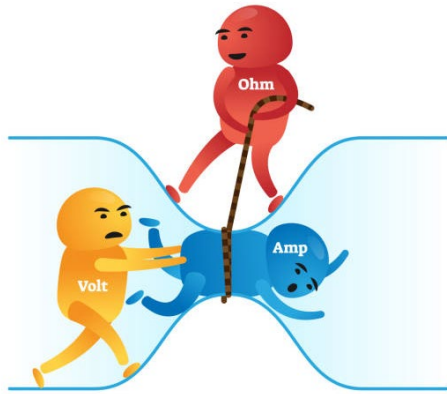
$$Q = \sigma \cdot A \cdot \Sigma(T^4 - T_{\infty}^4) \rightarrow Q = \bar{h}_{rad} \cdot A \cdot \Delta T$$

$$R_{term} = \frac{\Delta T}{Q}$$



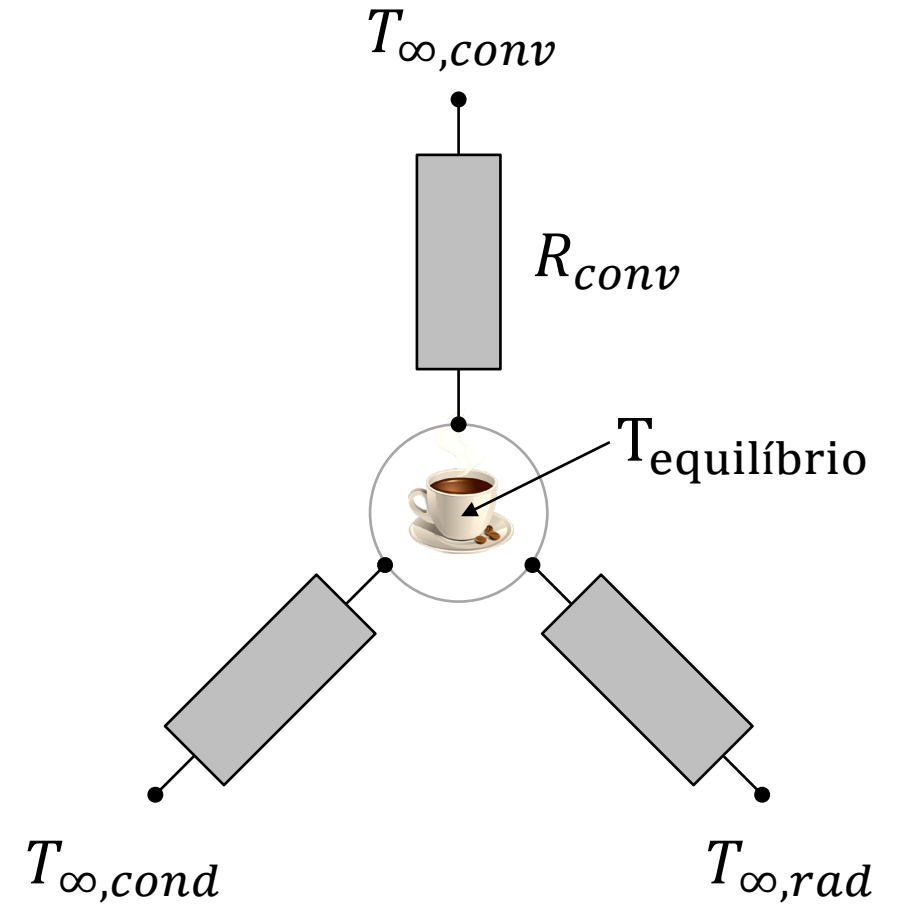
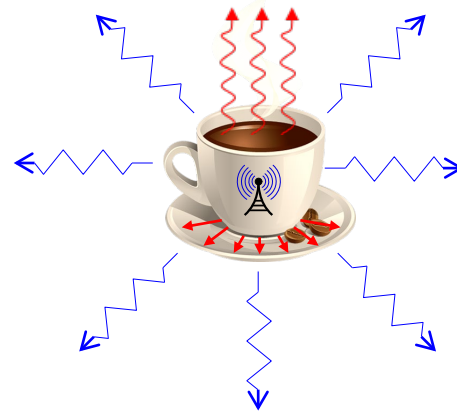


# DEFINIÇÃO DE UMA RESISTÊNCIA TÉRMICA → CIRCUITOS TÉRMICOS EQUIVALENTES



$$R_{elec} = \frac{\Delta V}{i}$$

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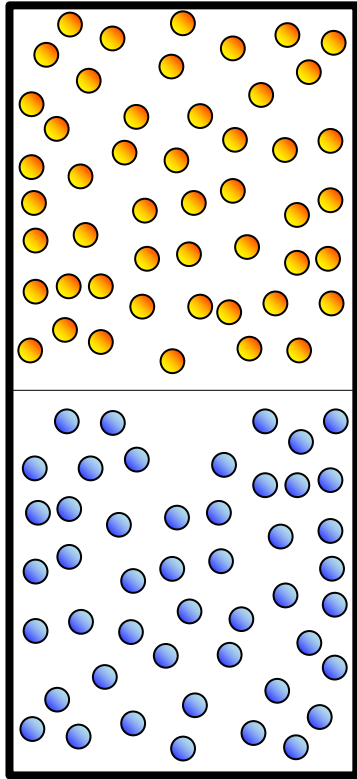
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# CONVECÇÃO NATURAL × CONVECÇÃO FORÇADA...

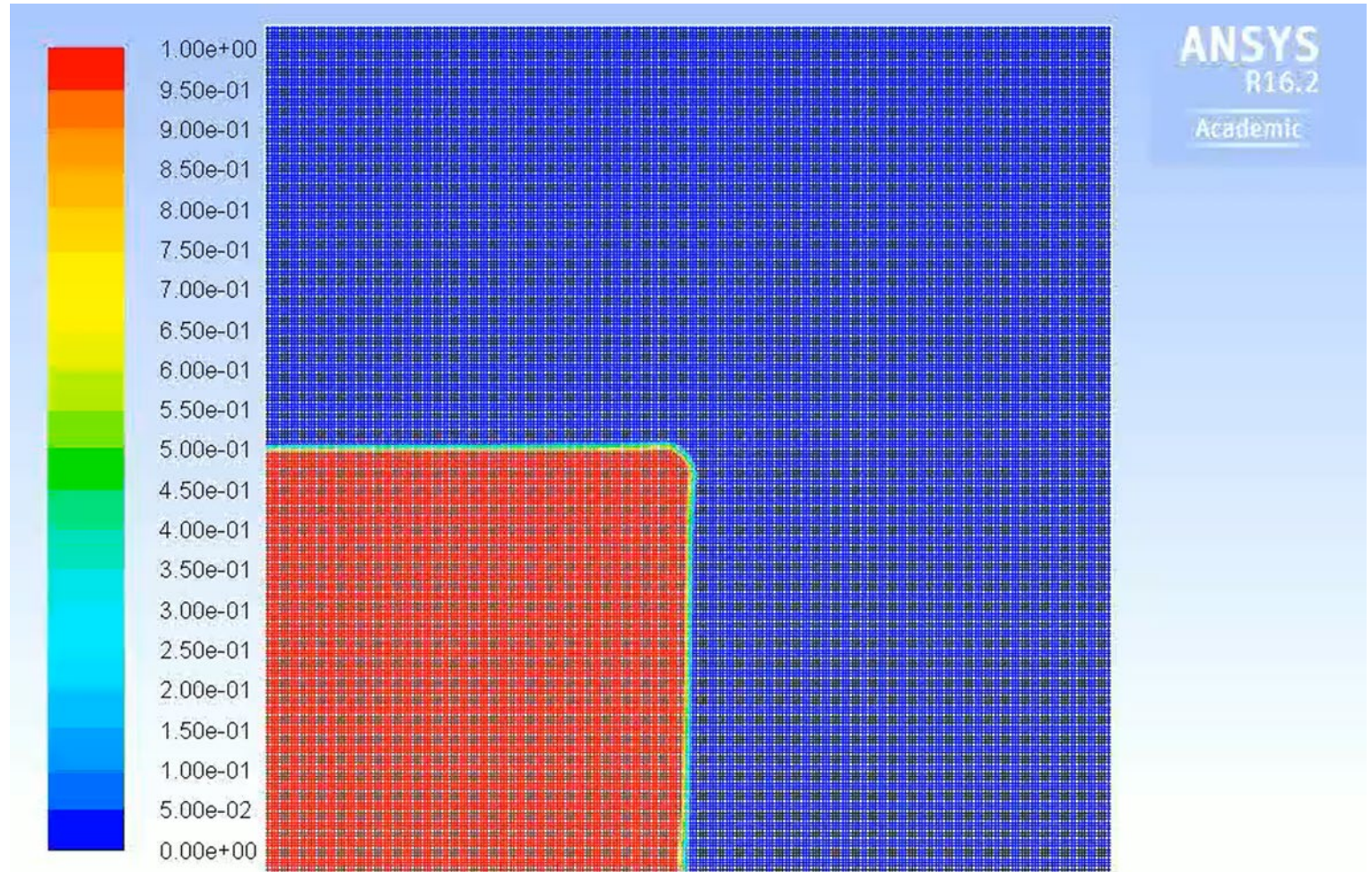
movimentação do fluido ↔ transporte de energia térmica



# difusão



# convecção



Contours of Volume fraction (phase-2) (Time=5.0000e-02)

Mar 07, 2016

ANSYS Fluent Release 16.2 (2d, dp, pbns, vof, lam, transient)



Jingwei Zhu  
26 subscribers

Oil-red; water-blue  
Software: ANSYS Fluent  
Oil density: 900kg/m<sup>3</sup>; viscosity: 0.1kg/m-s  
Water density: 1000kg/m<sup>3</sup>; viscosity: 0.005kg/m-s

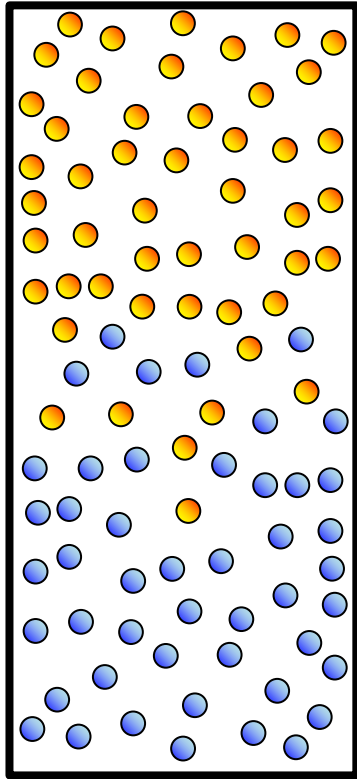


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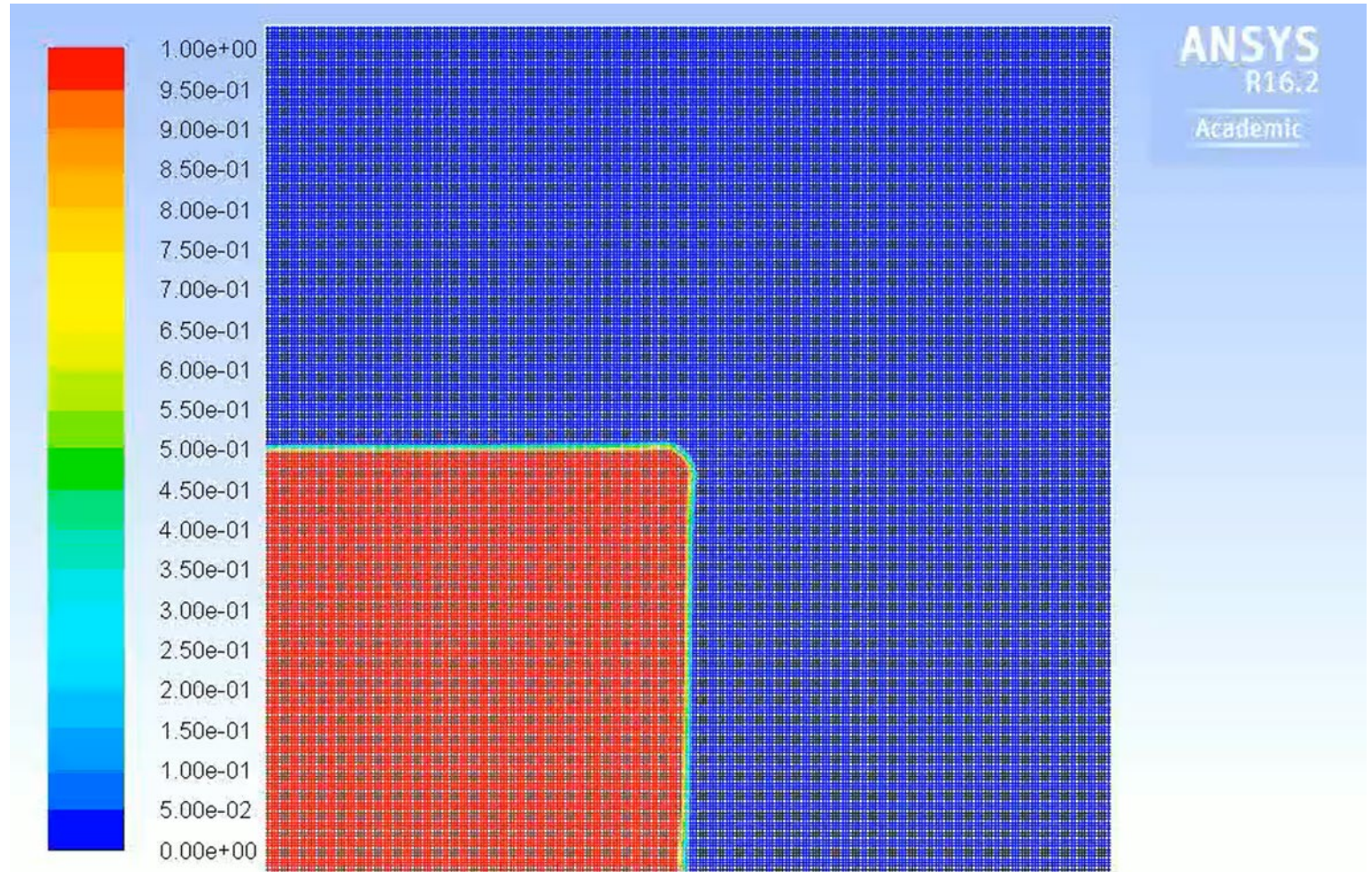
<https://youtu.be/kNiUPuRLCXw>



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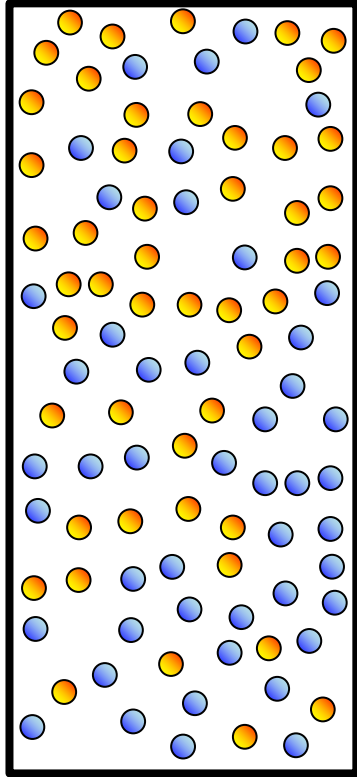


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difusão



convecção



$$J_M = -D \vec{\nabla} C$$

lei de Fick:



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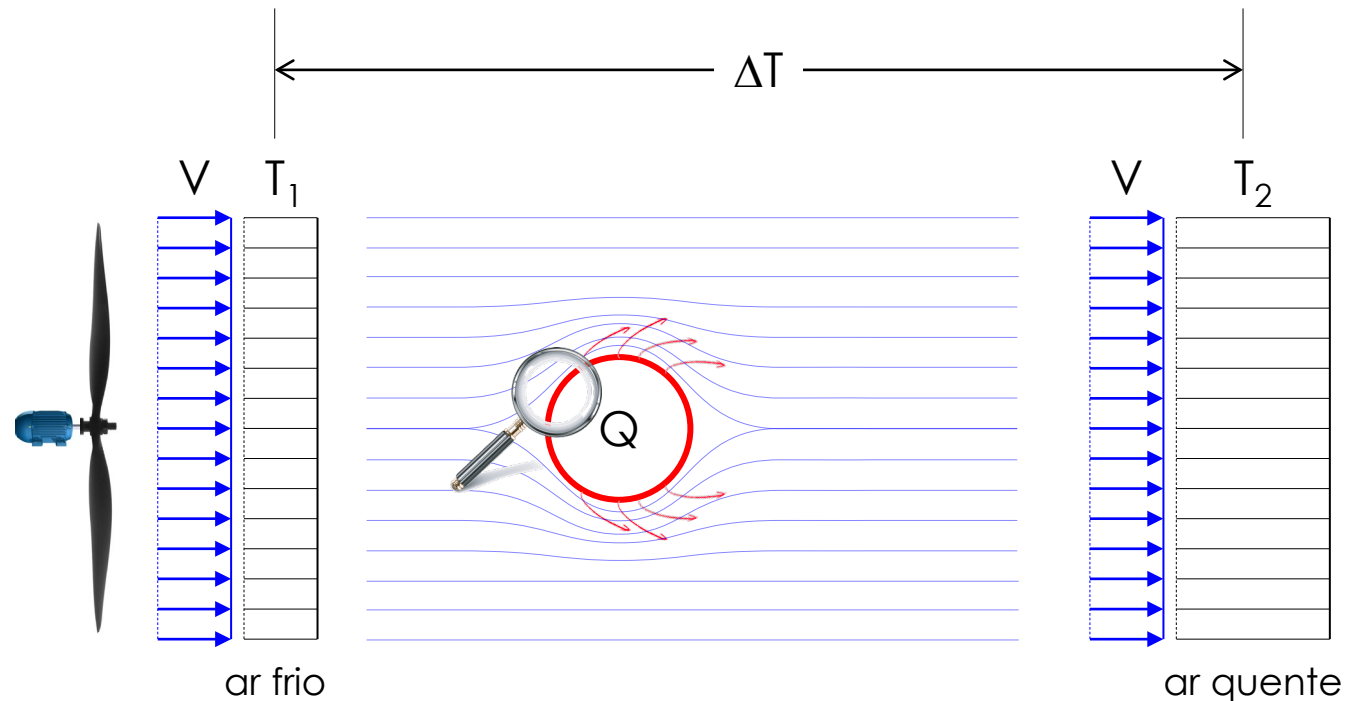
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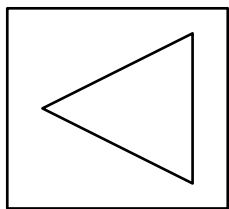
<https://youtu.be/kNiUPuRLCXw>

# Convecção natural × convecção forçada...

**Forçada:** a movimentação do fluido é induzida por forças **externas**...



Estes são os efeitos globais.  
Qual é o mecanismo de transferência de calor da resistência para o ar ?  
(Efeitos locais)



$$\Delta E_{\text{Cin}} = 0$$

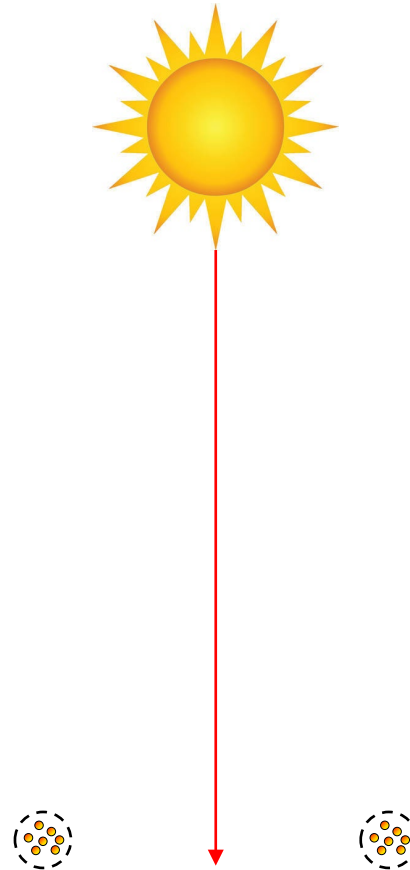
$$\Delta E_{\text{Ter}} = Q = \dot{m}_{\text{ar}} C_{P,\text{ar}} \Delta T$$



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# Convecção natural × convecção forçada...

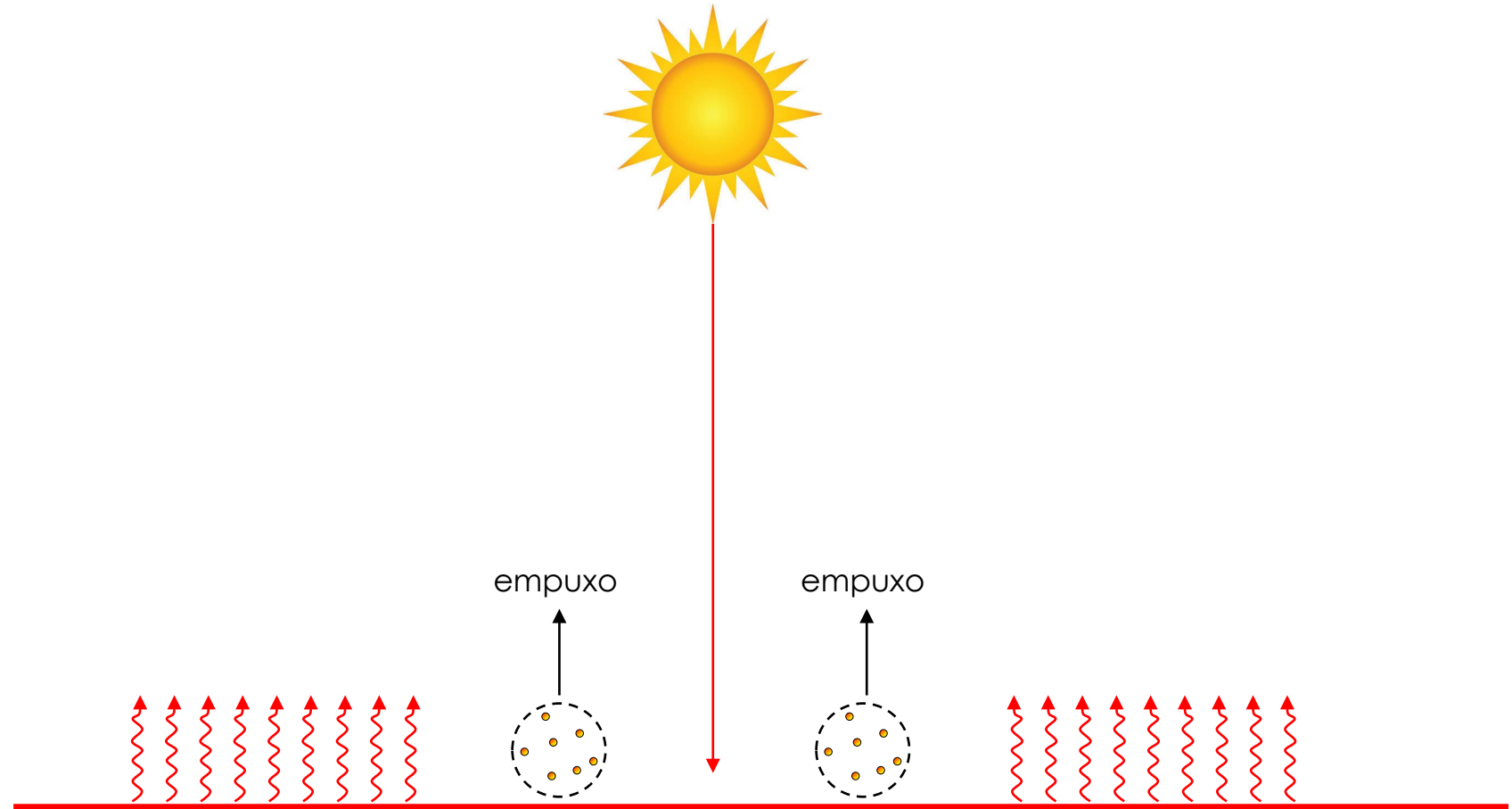
**Natural:** a movimentação do fluido é induzida por forças de **empuxo**...



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# Convecção natural × convecção forçada...

**Natural:** a movimentação do fluido é induzida por forças de **empuxo**...

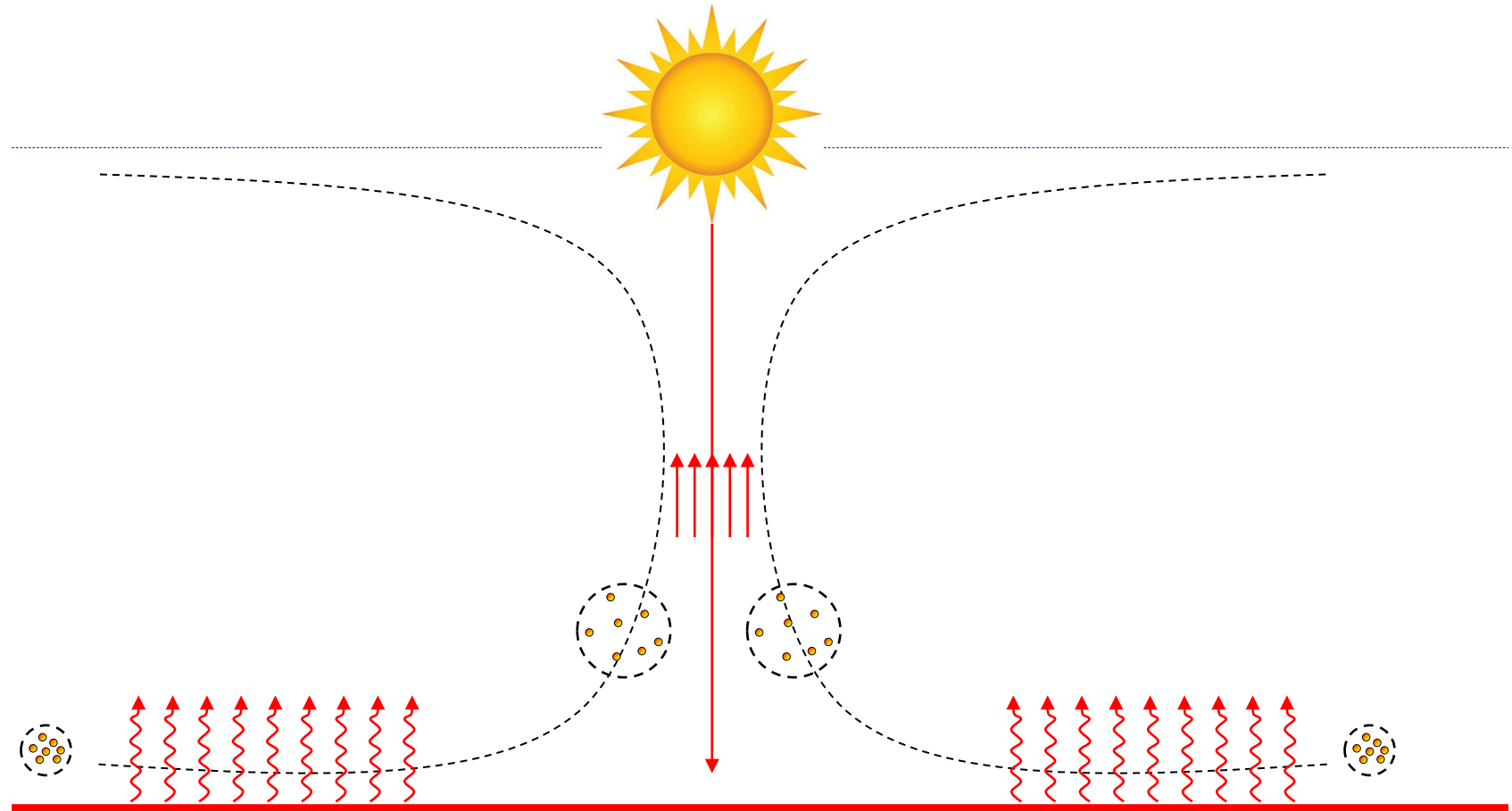


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# Convecção natural × convecção forçada...

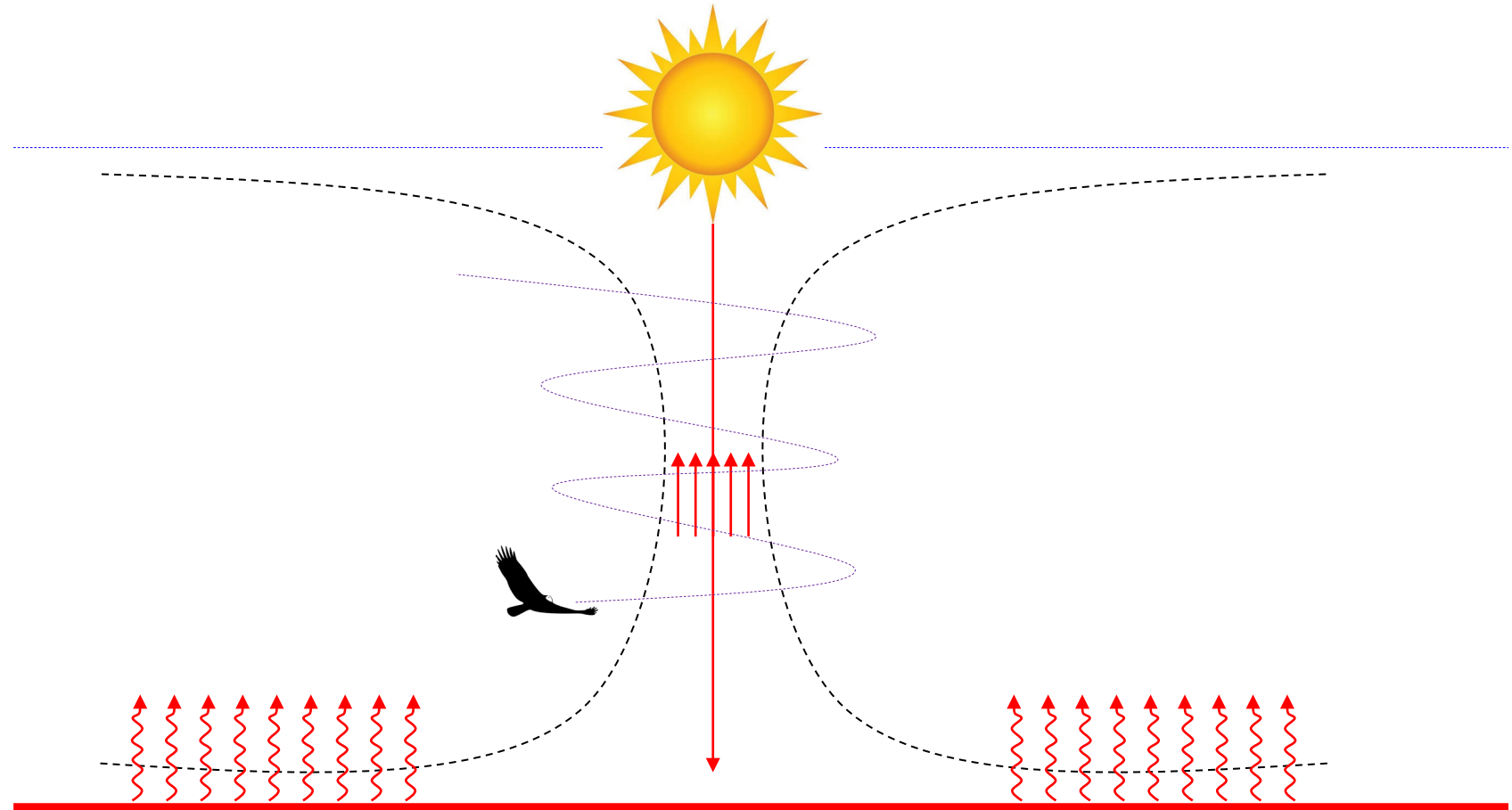
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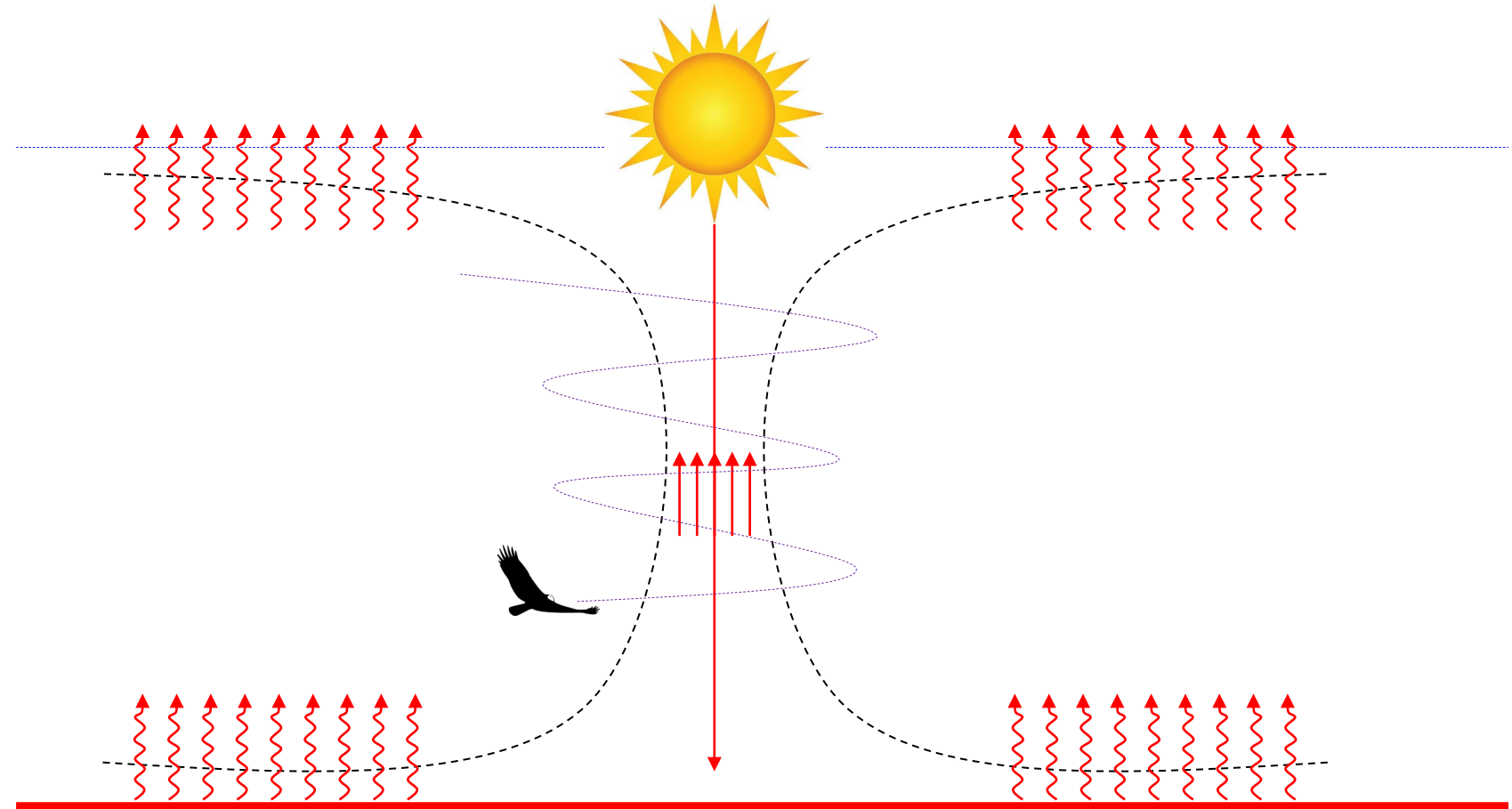
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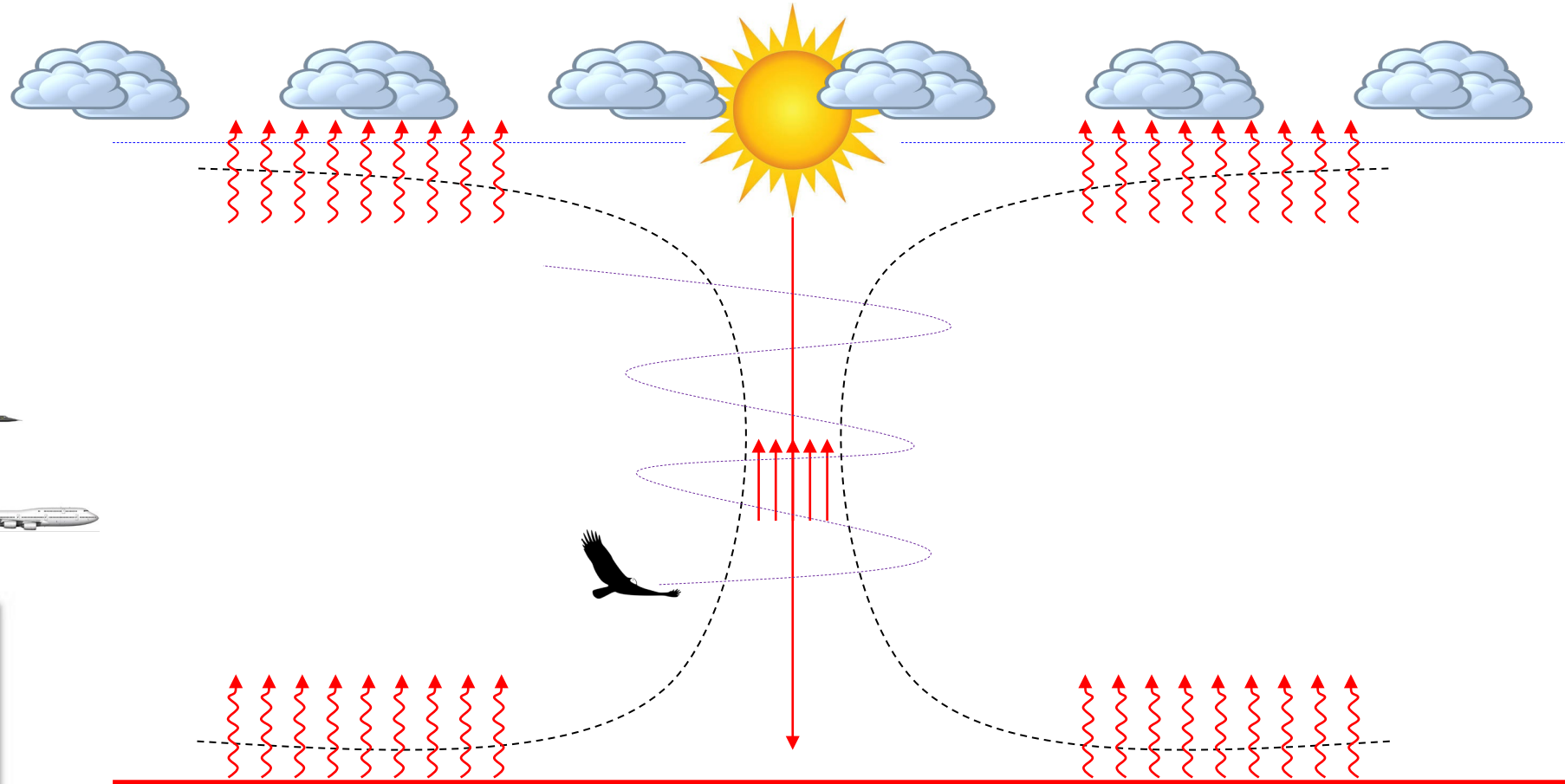
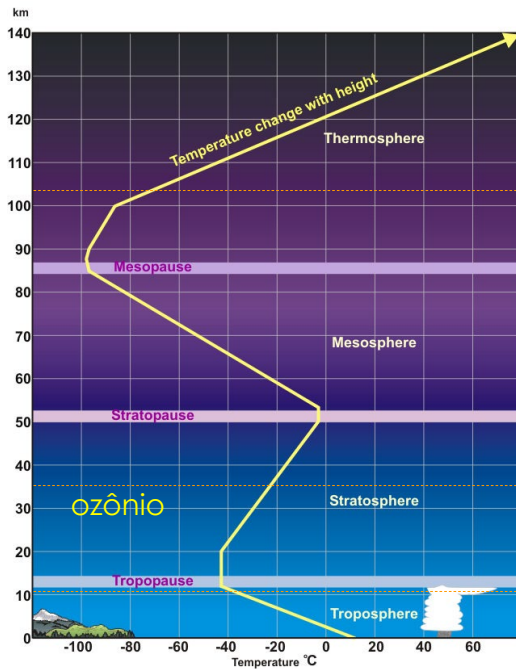
**Natural:** a movimentação do fluido é induzida por forças de **empuxo**...



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# Convecção natural × convecção forçada...

**Natural:** a movimentação do fluido é induzida por forças de **empuxo**...



**Cumulonimbus !**



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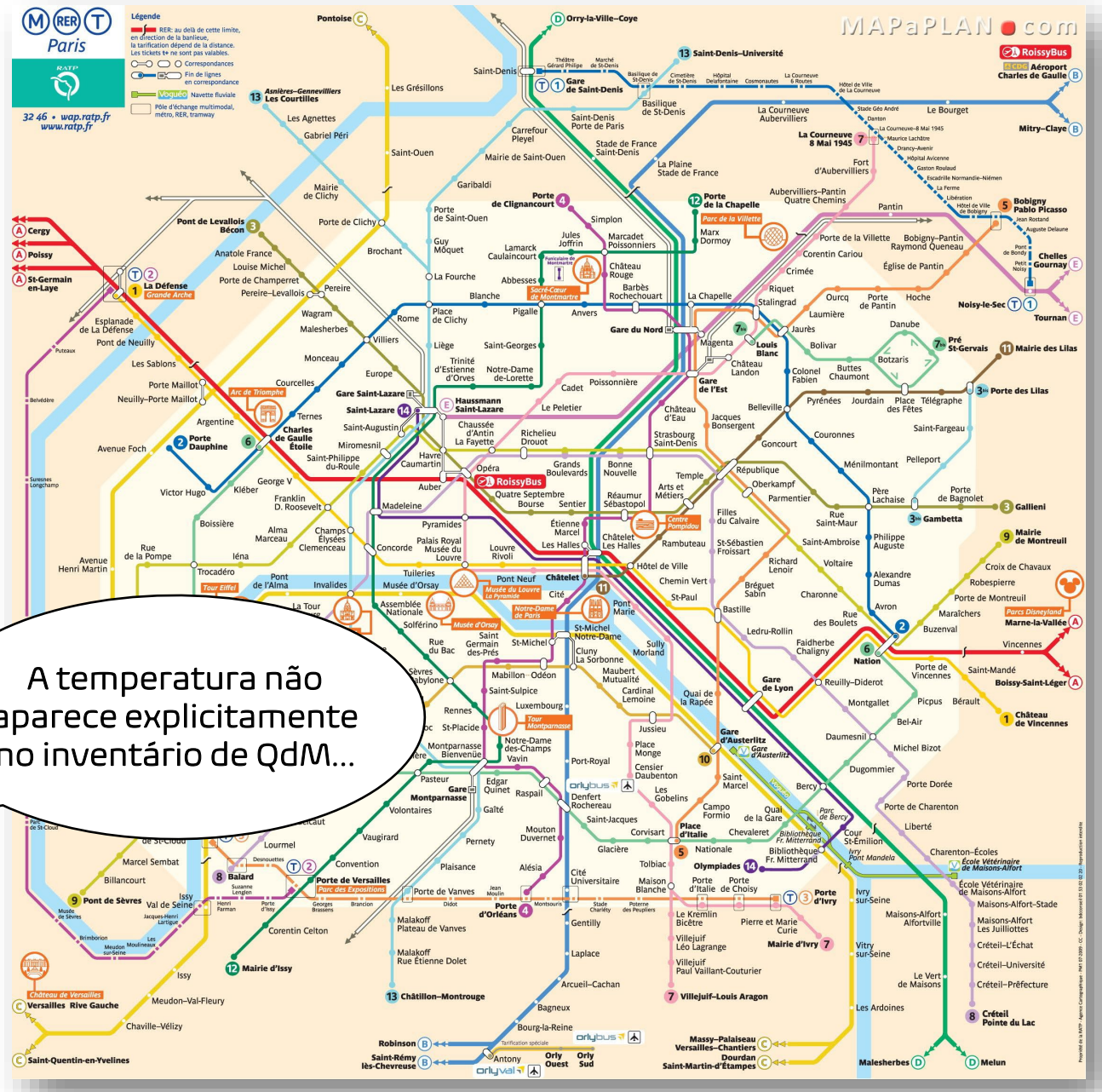






A CONVECÇÃO DE CALOR É UM FENÔMENO ACOPLADO...

movimentação do fluido ↔ transporte de energia térmica



$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$\left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} \right) = -\vec{\nabla} P + \frac{1}{Re} \nabla^2 \vec{u} + \sum \frac{1}{R_k} F_k$$

$$\left( \frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T \right) = \frac{1}{Re Pr} \vec{\nabla} \cdot (k \vec{\nabla} T) + \frac{Ec}{Re} \Phi(\vec{u})$$

A temperatura não aparece explicitamente no inventário de QdM...



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# Escoamento incidindo sobre uma placa plana... CAMADA LIMITE



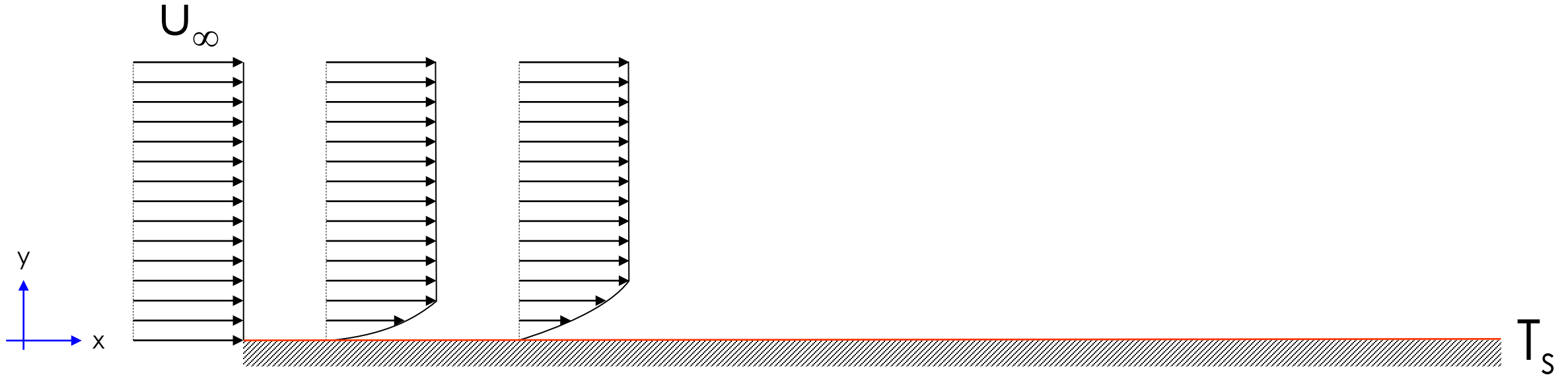
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# Escoamento incidindo sobre uma placa plana... CAMADA LIMITE



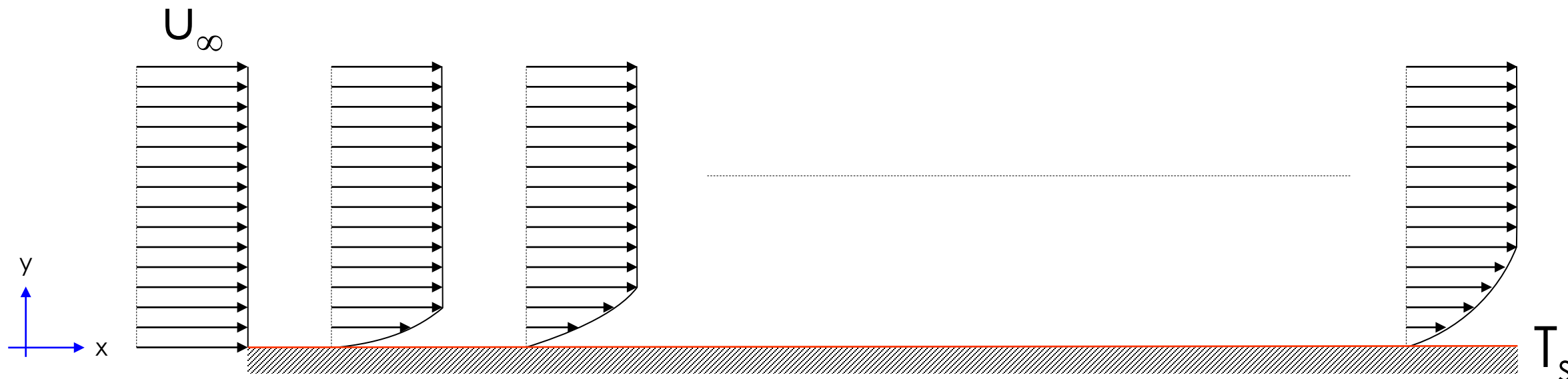
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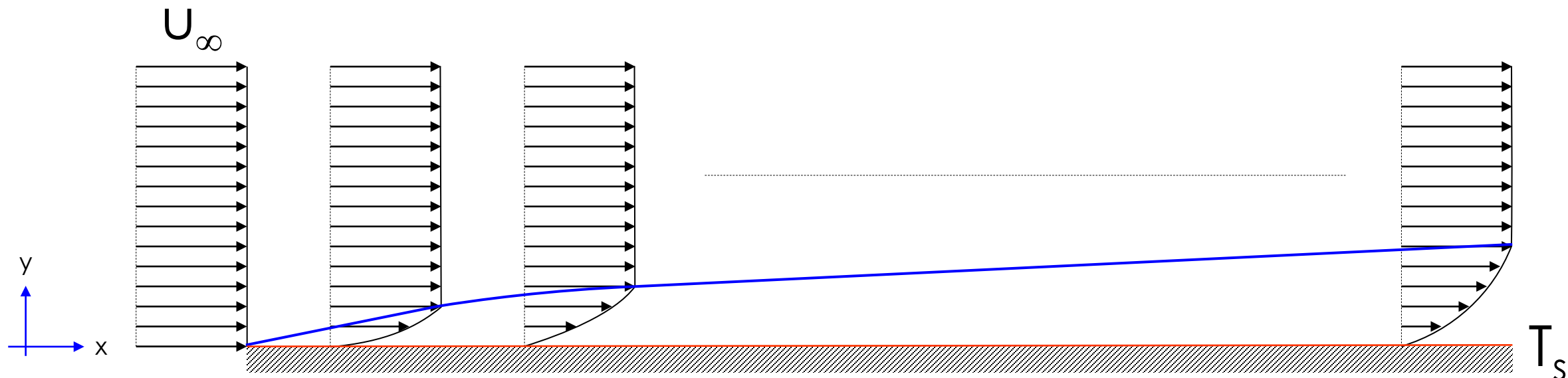
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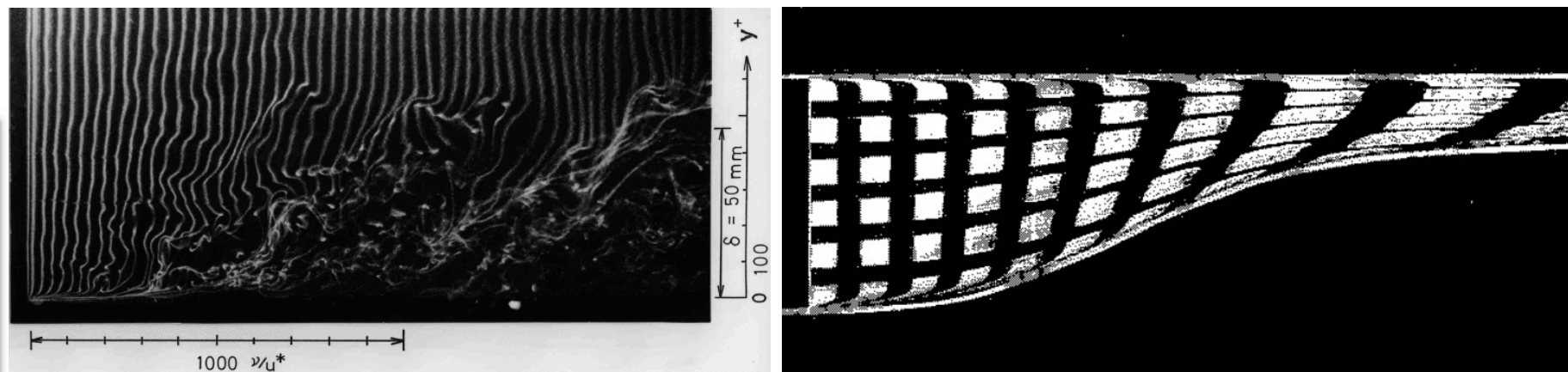
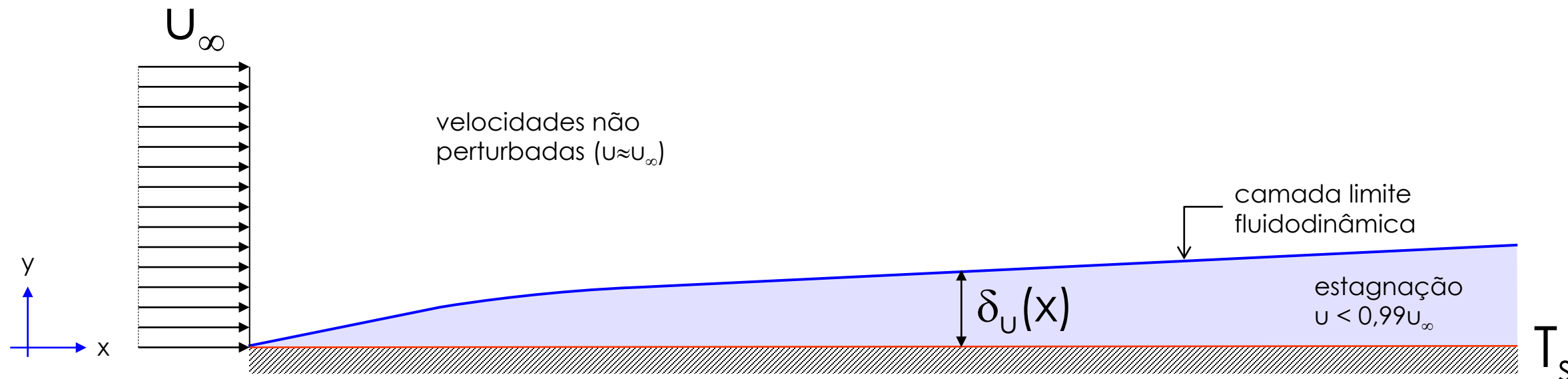
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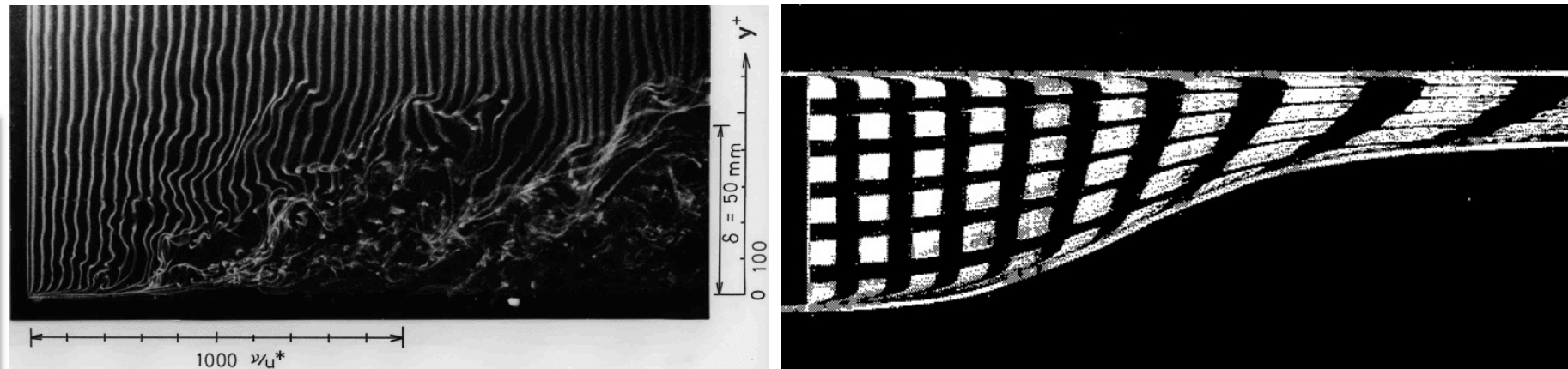
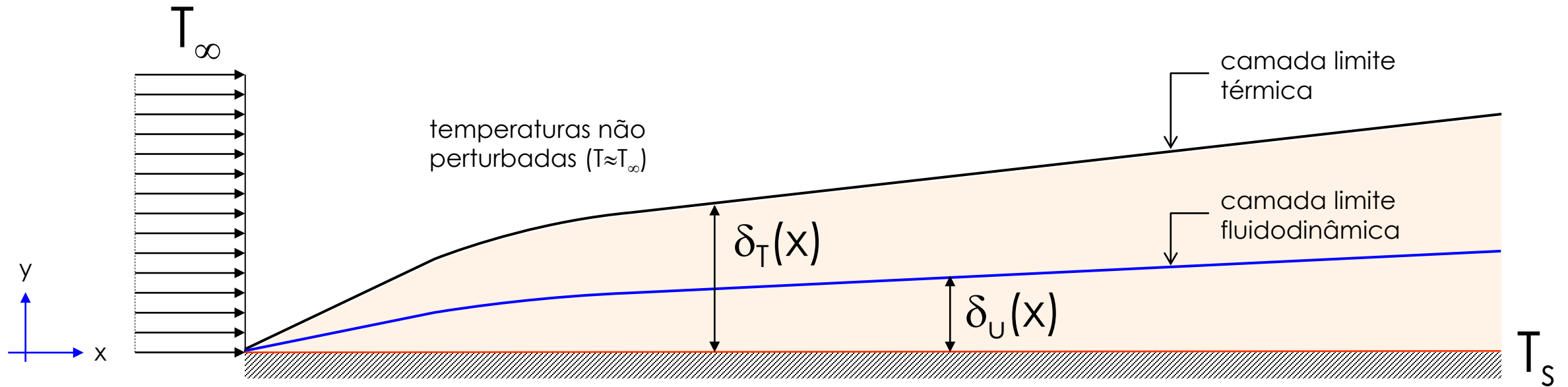


# Escoamento incidindo sobre uma placa plana... CAMADA LIMITE



Y. IRITANI, N. KASAGI and M. HIRATA

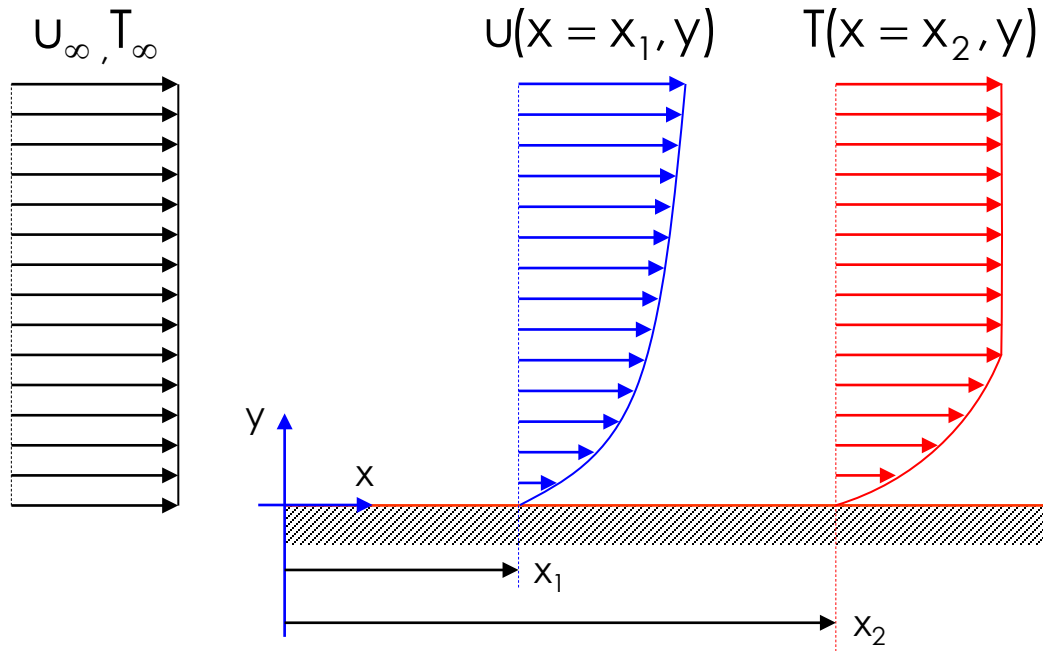
# Escoamento incidindo sobre uma placa plana... CAMADA LIMITE



Y. IRITANI, N. KASAGI and M. HIRATA



# Cálculo das camadas limites hidrodinâmica e térmica...



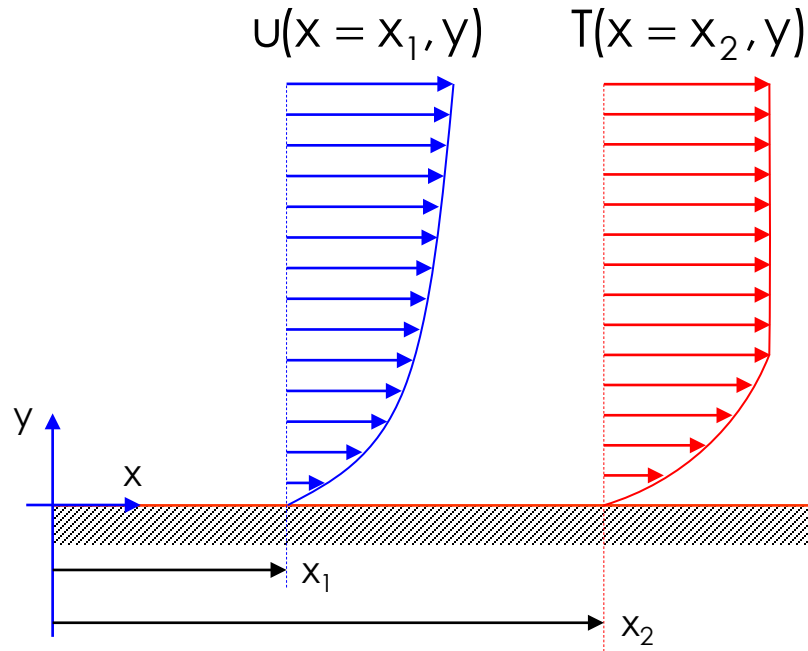
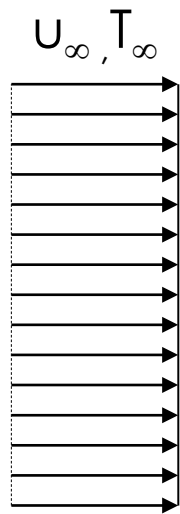
the following slides are  
**ANTI-EDUCATIONAL**

Mind the logical sequence not  
necessarily the mathematical  
manipulation...



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# Cálculo das camadas limites hidrodinâmica e térmica...



$$U \frac{\partial U}{\partial x} + v \frac{\partial U}{\partial y} = 0$$

← massa

$$U \frac{\partial U}{\partial x} + v \frac{\partial U}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 U}{\partial y^2}$$

← q. de movimento

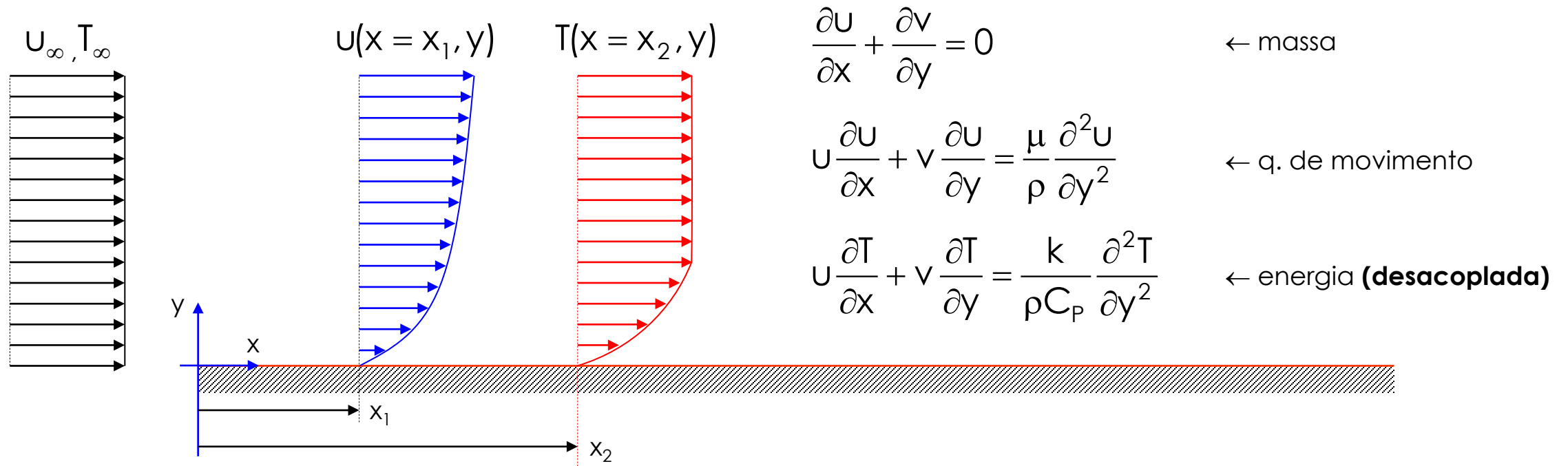
$$U \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$$

← energia



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# Cálculo das camadas limites hidrodinâmica e térmica...

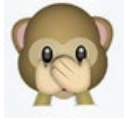


$$p / x = 0 \rightarrow u(0, y) = u_\infty \text{ e } T(0, y) = T_\infty$$

$$p / y = 0 \rightarrow u(x, 0) = 0, v(x, 0) = 0 \text{ e } T(x, 0) = T_s$$

$$p / y \rightarrow \infty \rightarrow u(x, \infty) = u_\infty \text{ e } T(x, \infty) = T_\infty$$

# Solução de Blasius (1908)... velocidades

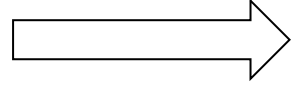


$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

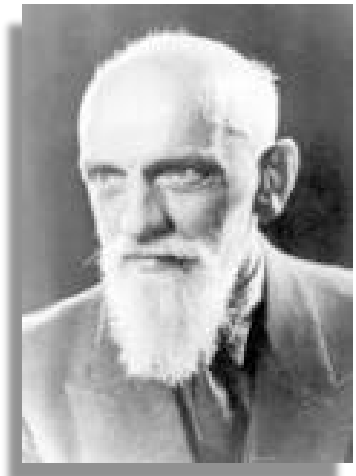


$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$

transformação



$$a_0 \cdot f(\eta) + a_1 \frac{f(\eta)}{d\eta} + a_2 \frac{f^2(\eta)}{d\eta^2} + a_3 \frac{f^3(\eta)}{d\eta^3} + \dots$$

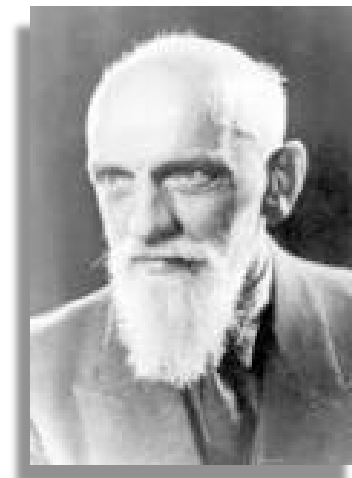


Paul R.H. Blasius  
(1883 – 1970)

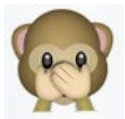


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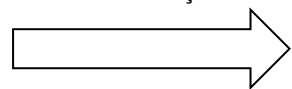


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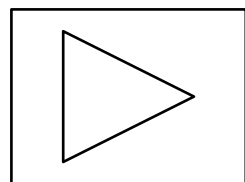
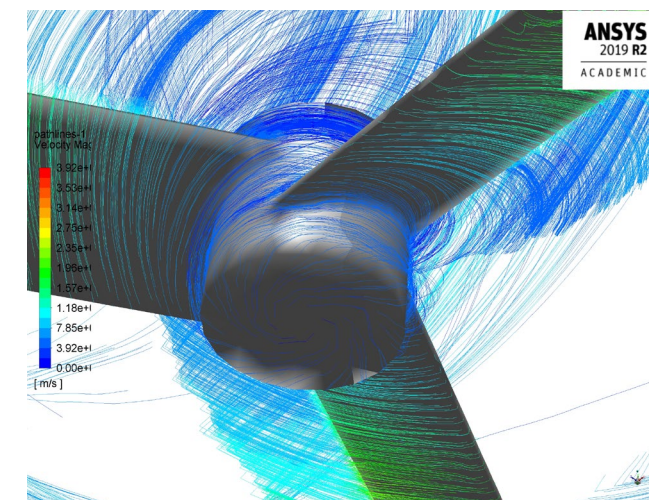
variável de similaridade



$$\eta \stackrel{\text{def}}{=} y \cdot \sqrt{\frac{U_\infty}{x \cdot \mu / \rho}}$$

$$f(\eta) \stackrel{\text{def}}{=} \frac{\psi}{U_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{U_\infty}}}$$

$\Psi$  = função de corrente



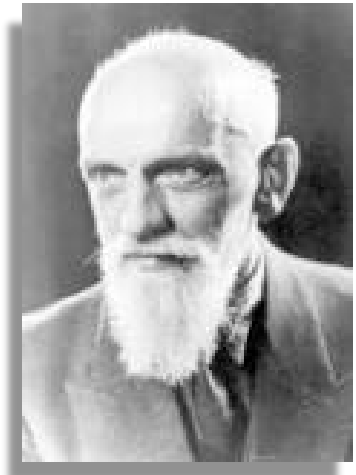
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# Solução de Blasius (1908)... **velocidades**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\psi(x, y) \stackrel{\text{def}}{=} \begin{cases} u = +\frac{\partial \psi}{\partial y} \\ v = -\frac{\partial \psi}{\partial x} \end{cases} \Rightarrow \text{🐒} \equiv 0$$

função de corrente

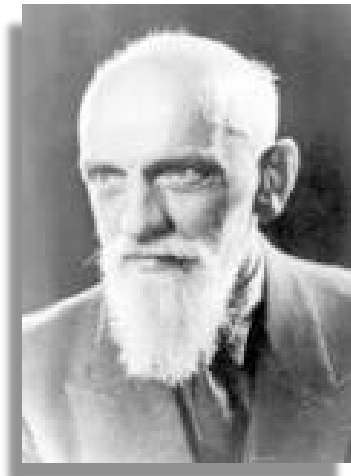


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# Solução de Blasius (1908)... **velocidades**



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função de corrente

As componentes da velocidade se escrevem então como:

$$\eta = y \cdot \sqrt{\frac{U_\infty}{x \cdot \rho / \mu}}$$

$$\psi = U_\infty \sqrt{x \cdot \frac{\mu / \rho}{U_\infty}} \cdot f(\eta)$$

$$U = + \frac{\partial \psi}{\partial y} = + \left( \frac{\partial \psi}{\partial \eta} \right) \cdot \left( \frac{\partial \eta}{\partial y} \right) = \left( U_\infty \sqrt{\frac{xv}{U_\infty}} \cdot \frac{df}{d\eta} \right) \cdot \left( \sqrt{\frac{U_\infty}{xv}} \right) = U_\infty \frac{df}{d\eta}$$

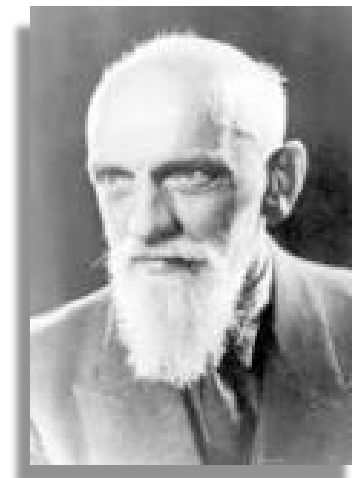
$$v = \mu / \rho$$



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# Solução de Blasius (1908)... **velocidades**



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função de corrente

As componentes da velocidade se escrevem então como:

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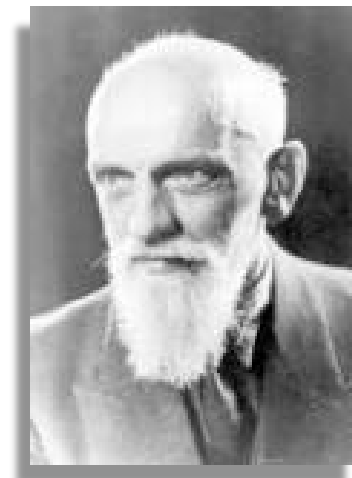
$$U = + \frac{\partial \psi}{\partial y} = + \left( \frac{\partial \psi}{\partial \eta} \right) \cdot \left( \frac{\partial \eta}{\partial y} \right) = \left( U_\infty \sqrt{\frac{xv}{U_\infty}} \cdot \frac{df}{d\eta} \right) \cdot \left( \sqrt{\frac{U_\infty}{xv}} \right) = U_\infty \frac{df}{d\eta}$$

$$v = - \frac{\partial \psi}{\partial x} = - U_\infty \sqrt{\frac{v}{U_\infty}} \cdot f(\eta) \cdot \left( \frac{x^{-1/2}}{2} \right) - \left( U_\infty \sqrt{x \cdot \frac{v}{U_\infty}} \cdot \frac{\partial f}{\partial x} \right) = \frac{1}{2} \sqrt{\frac{U_\infty v}{x}} \cdot \left( \eta \frac{df}{d\eta} - f \right)$$

$$v = \mu / \rho$$



# Solução de Blasius (1908)... **velocidades**



Paul R.H. Blasius  
(1883 – 1970)

$$\psi(x, y) \stackrel{\text{def}}{=} \begin{cases} U = + \frac{\partial \psi}{\partial y} \\ v = - \frac{\partial \psi}{\partial x} \end{cases} \Rightarrow \text{🙈} \equiv 0$$

função de corrente

As componentes da velocidade se escrevem então como:

$$\eta = y \cdot \sqrt{\frac{U_\infty}{x \cdot \rho / \mu}}$$

$$\psi = U_\infty \sqrt{x \cdot \frac{\mu / \rho}{U_\infty}} \cdot f(\eta)$$

$$U = + \frac{\partial \psi}{\partial y} = + \left( \frac{\partial \psi}{\partial \eta} \right) \cdot \left( \frac{\partial \eta}{\partial y} \right) = \left( U_\infty \sqrt{\frac{xv}{U_\infty}} \cdot \frac{df}{d\eta} \right) \cdot \left( \sqrt{\frac{U_\infty}{xv}} \right) = U_\infty \frac{df}{d\eta}$$

$$v = - \frac{\partial \psi}{\partial x} = - U_\infty \sqrt{\frac{v}{U_\infty}} \cdot f(\eta) \cdot \left( \frac{x^{-1/2}}{2} \right) - \left( U_\infty \sqrt{x \cdot \frac{v}{U_\infty}} \cdot \frac{\partial f}{\partial x} \right) = \frac{1}{2} \sqrt{\frac{U_\infty v}{x}} \cdot \left( \eta \frac{df}{d\eta} - f \right)$$

$$v = \mu / \rho$$

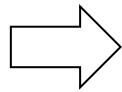
$$\begin{matrix} U = \dots \\ v = \dots \end{matrix} \Rightarrow U \frac{\partial U}{\partial x} + v \frac{\partial U}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 U}{\partial y^2} \text{ 🙈}$$

# Solução de Blasius (1908)... **velocidades**

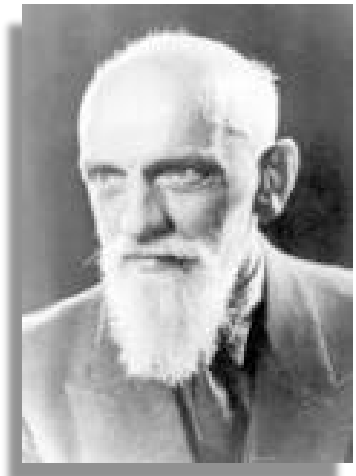
$$\frac{\partial u}{\partial x} = -\frac{u_{\infty}}{2x} \eta \frac{d^2 f}{d\eta^2}$$

$$\frac{\partial u}{\partial y} = u_{\infty} \sqrt{\frac{u_{\infty}}{\nu x}} \frac{d^2 f}{d\eta^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{\infty}^2}{\nu x} \frac{d^3 f}{d\eta^3}$$



$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$



Paul R.H. Blasius  
(1883 – 1970)



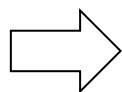
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# Solução de Blasius (1908)... **velocidades**

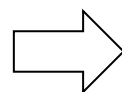
$$\frac{\partial u}{\partial x} = -\frac{u_\infty}{2x} \eta \frac{d^2 f}{d\eta^2}$$

$$\frac{\partial u}{\partial y} = u_\infty \sqrt{\frac{u_\infty}{\nu x}} \frac{d^2 f}{d\eta^2}$$

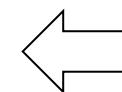
$$\frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^2}{\nu x} \frac{d^3 f}{d\eta^3}$$



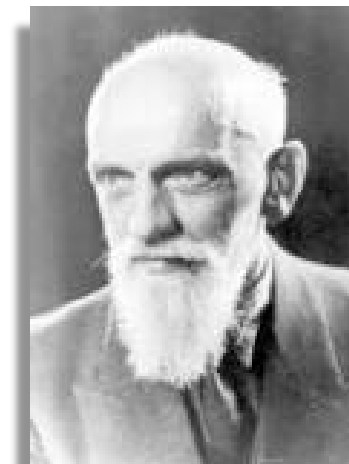
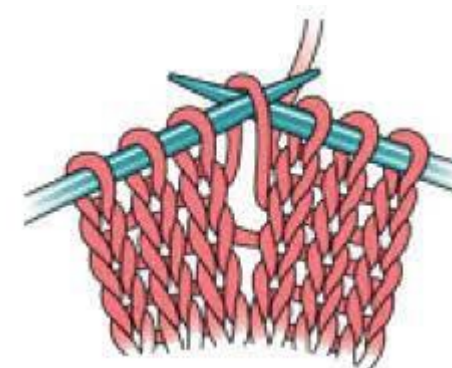
$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$



$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$



DIFERENÇAS FINITAS



Paul R.H. Blasius  
(1883 – 1970)



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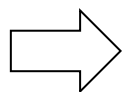


# Solução de Blasius (1908)... **velocidades**

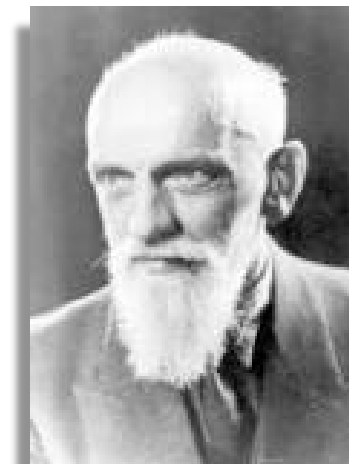
$$\frac{\partial u}{\partial x} = -\frac{u_\infty}{2x} \eta \frac{d^2 f}{d\eta^2}$$

$$\frac{\partial u}{\partial y} = u_\infty \sqrt{\frac{u_\infty}{\nu x}} \frac{d^2 f}{d\eta^2}$$

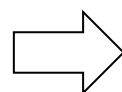
$$\frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^2}{\nu x} \frac{d^3 f}{d\eta^3}$$



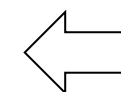
$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$



Paul R.H. Blasius  
(1883 – 1970)



$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$



DIFERENÇAS FINITAS

$$p/x = 0 \rightarrow u(0, y) = u_\infty$$

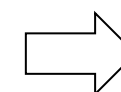
$$p/y = 0 \rightarrow u(x, 0) = 0, v(x, 0) = 0$$

$$p/y \rightarrow \infty \rightarrow u(x, \infty) = u_\infty$$

$$p/\eta = 0 \rightarrow f(\eta) = 0$$

$$p/\eta = 0 \rightarrow df/d\eta = 0$$

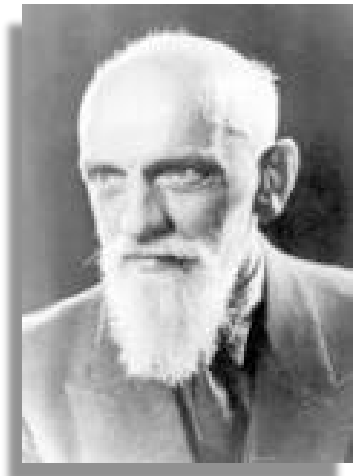
$$p/\eta = \infty \rightarrow df/d\eta = 1$$



# Solução de Blasius (1908)... **c. limite fluidodinâmica**

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$

| $\eta$   | $f$      | $\frac{df}{d\eta} = \frac{u}{u_\infty}$ | $\frac{d^2 f}{d\eta^2}$ |
|----------|----------|---|-------------------------|
| 0        | 0        | 0                                       | 0.332                   |
| 0.5      | 0.042    | 0.166                                   | 0.331                   |
| 1.0      | 0.166    | 0.330                                   | 0.323                   |
| 1.5      | 0.370    | 0.487                                   | 0.303                   |
| 2.0      | 0.650    | 0.630                                   | 0.267                   |
| 2.5      | 0.996    | 0.751                                   | 0.217                   |
| 3.0      | 1.397    | 0.846                                   | 0.161                   |
| 3.5      | 1.838    | 0.913                                   | 0.108                   |
| 4.0      | 2.306    | 0.956                                   | 0.064                   |
| 4.5      | 2.790    | 0.980                                   | 0.034                   |
| 5.0      | 3.283    | 0.992                                   | 0.016                   |
| 5.5      | 3.781    | 0.997                                   | 0.007                   |
| 6.0      | 4.280    | 0.999                                   | 0.002                   |
| $\infty$ | $\infty$ | 1                                       | 0                       |



Paul R.H. Blasius  
(1883 – 1970)



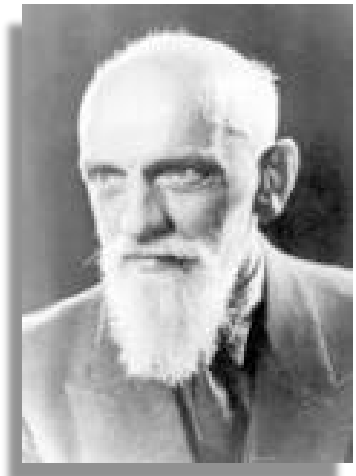
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# Solução de Blasius (1908)... **c. limite fluidodinâmica**

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$

$$\delta \Rightarrow u/u_\infty = 0.99 \Rightarrow \eta = 5.0$$

| $\eta$   | $f$      | $\frac{df}{d\eta} = \frac{u}{u_\infty}$ | $\frac{d^2 f}{d\eta^2}$ |
|----------|----------|---|-------------------------|
| 0        | 0        | 0                                       | 0.332                   |
| 0.5      | 0.042    | 0.166                                   | 0.331                   |
| 1.0      | 0.166    | 0.330                                   | 0.323                   |
| 1.5      | 0.370    | 0.487                                   | 0.303                   |
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| 5.5      | 3.781    | 0.997                                   | 0.007                   |
| 6.0      | 4.280    | 0.999                                   | 0.002                   |
| $\infty$ | $\infty$ | 1                                       | 0                       |



Paul R.H. Blasius  
(1883 – 1970)



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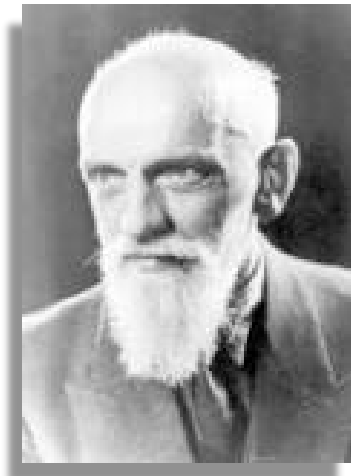
# Solução de Blasius (1908)... **c. limite fluidodinâmica**

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$

$$\delta \Rightarrow u/u_\infty = 0.99 \Rightarrow \eta = 5.0$$

$$\eta = y \cdot \sqrt{\frac{u_\infty}{xv}} \Rightarrow \delta = 5.0 \cdot \sqrt{\frac{xv}{u_\infty}}$$

| $\eta$   | $f$      | $\frac{df}{d\eta} = \frac{u}{u_\infty}$ | $\frac{d^2 f}{d\eta^2}$ |
|----------|----------|---|-------------------------|
| 0        | 0        | 0                                       | 0.332                   |
| 0.5      | 0.042    | 0.166                                   | 0.331                   |
| 1.0      | 0.166    | 0.330                                   | 0.323                   |
| 1.5      | 0.370    | 0.487                                   | 0.303                   |
| 2.0      | 0.650    | 0.630                                   | 0.267                   |
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| 3.5      | 1.838    | 0.913                                   | 0.108                   |
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| 5.0      | 3.283    | 0.992                                   | 0.016                   |
| 5.5      | 3.781    | 0.997                                   | 0.007                   |
| 6.0      | 4.280    | 0.999                                   | 0.002                   |
| $\infty$ | $\infty$ | 1                                       | 0                       |



Paul R.H. Blasius  
(1883 – 1970)



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# Solução de Blasius (1908)... **c. limite fluidodinâmica**

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$

$$\delta \Rightarrow u/u_\infty = 0.99 \Rightarrow \eta = 5.0$$

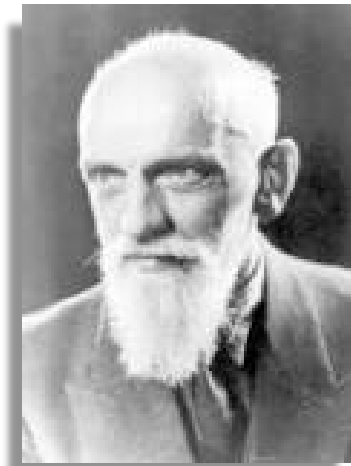
$$\eta = y \cdot \sqrt{\frac{u_\infty}{x\nu}} \Rightarrow \delta = 5.0 \cdot \sqrt{\frac{x\nu}{u_\infty}}$$



$$\delta = \frac{5.0 \cdot x}{\sqrt{Re_x}}$$

$$Re = \frac{\rho u_\infty x}{\mu}$$

| $\eta$   | $f$      | $\frac{df}{d\eta} = \frac{u}{u_\infty}$ | $\frac{d^2 f}{d\eta^2}$ |
|----------|----------|---|-------------------------|
| 0        | 0        | 0                                       | 0.332                   |
| 0.5      | 0.042    | 0.166                                   | 0.331                   |
| 1.0      | 0.166    | 0.330                                   | 0.323                   |
| 1.5      | 0.370    | 0.487                                   | 0.303                   |
| 2.0      | 0.650    | 0.630                                   | 0.267                   |
| 2.5      | 0.996    | 0.751                                   | 0.217                   |
| 3.0      | 1.397    | 0.846                                   | 0.161                   |
| 3.5      | 1.838    | 0.913                                   | 0.108                   |
| 4.0      | 2.306    | 0.956                                   | 0.064                   |
| 4.5      | 2.790    | 0.980                                   | 0.034                   |
| 5.0      | 3.283    | 0.992                                   | 0.016                   |
| 5.5      | 3.781    | 0.997                                   | 0.007                   |
| 6.0      | 4.280    | 0.999                                   | 0.002                   |
| $\infty$ | $\infty$ | 1                                       | 0                       |



Paul R.H. Blasius  
(1883 – 1970)



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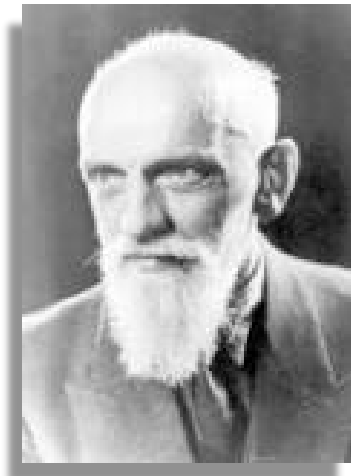
# Solução de Blasius (1908)... **tensões de cisalhamento**

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 0$$

$$\tau_x = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \mu U_\infty \sqrt{\frac{U_\infty}{\nu x}} \left. \frac{d^2 f}{d\eta^2} \right|_{\eta=0}$$

$$\tau_x = 0.332 \frac{\rho U_\infty^2}{\sqrt{Re_x}}$$

| $\eta$   | $f$      | $\frac{df}{d\eta} = \frac{u}{U_\infty}$ | $\frac{d^2 f}{d\eta^2}$ |
|----------|----------|---|-------------------------|
| 0        | 0        | 0                                       | 0.332                   |
| 0.5      | 0.042    | 0.166                                   | 0.331                   |
| 1.0      | 0.166    | 0.330                                   | 0.323                   |
| 1.5      | 0.370    | 0.487                                   | 0.303                   |
| 2.0      | 0.650    | 0.630                                   | 0.267                   |
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| 3.0      | 1.397    | 0.846                                   | 0.161                   |
| 3.5      | 1.838    | 0.913                                   | 0.108                   |
| 4.0      | 2.306    | 0.956                                   | 0.064                   |
| 4.5      | 2.790    | 0.980                                   | 0.034                   |
| 5.0      | 3.283    | 0.992                                   | 0.016                   |
| 5.5      | 3.781    | 0.997                                   | 0.007                   |
| 6.0      | 4.280    | 0.999                                   | 0.002                   |
| $\infty$ | $\infty$ | 1                                       | 0                       |



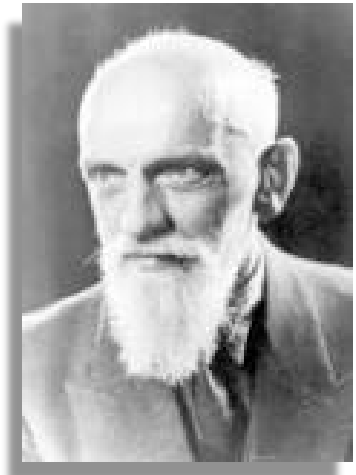
Paul R.H. Blasius  
(1883 – 1970)



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# Solução de Blasius (1908)... **c. limite térmica**

$$U \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$$



Paul R.H. Blasius  
(1883 – 1970)

$$\psi = U_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{U_\infty}} \cdot f(\eta)$$

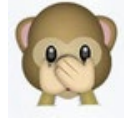


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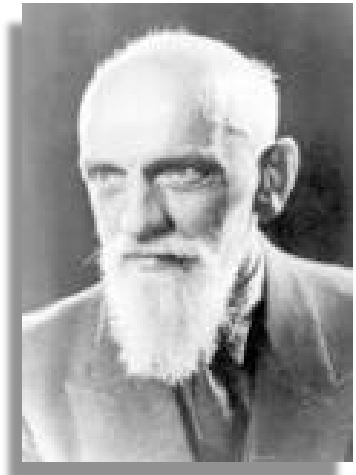


# Solução de Blasius (1908)... **c. limite térmica**

$$\psi(x, y) \stackrel{\text{def}}{=} \begin{cases} u = +\frac{\partial\psi}{\partial y} \\ v = -\frac{\partial\psi}{\partial x} \end{cases}$$



$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$$



Paul R.H. Blasius  
(1883 – 1970)

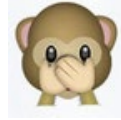
$$\psi = U_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{U_\infty}} \cdot f(\eta)$$



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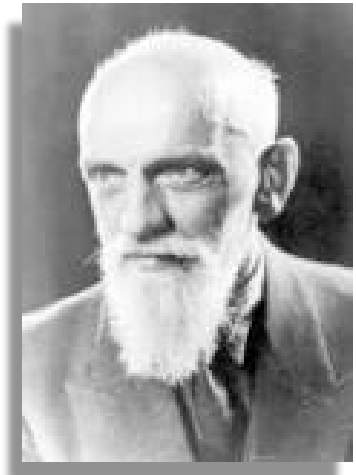
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$$\psi(x, y) \stackrel{\text{def}}{=} \begin{cases} u = +\frac{\partial\psi}{\partial y} \\ v = -\frac{\partial\psi}{\partial x} \end{cases}$$



$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$$

$$\theta(x, y) \stackrel{\text{def}}{=} \frac{T(x, y) - T_s}{T_\infty - T_s} \leftarrow$$



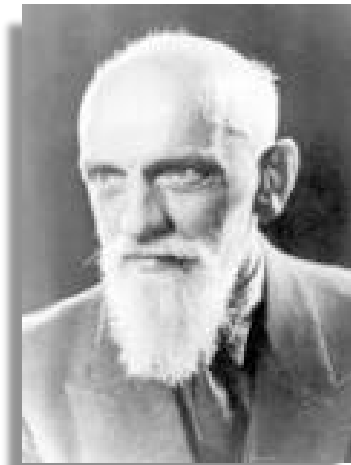
Paul R.H. Blasius  
(1883 – 1970)

$$\psi = U_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{U_\infty}} \cdot f(\eta)$$



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# Solução de Blasius (1908)... c. limite térmica



Paul R.H. Blasius  
(1883 – 1970)

$$\psi(x, y) \stackrel{\text{def}}{=} \begin{cases} u = + \frac{\partial \psi}{\partial y} \\ v = - \frac{\partial \psi}{\partial x} \end{cases}$$



$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$$

$$\theta(x, y) \stackrel{\text{def}}{=} \frac{T(x, y) - T_s}{T_\infty - T_s}$$



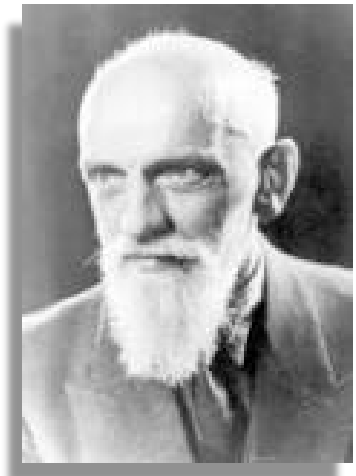
$$\psi = U_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{U_\infty}} \cdot f(\eta)$$

$$U_\infty \frac{df}{d\eta} \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{1}{2} \sqrt{\frac{U_\infty \nu}{x}} \left( \eta \frac{df}{d\eta} - f \right) \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial y} = \frac{k}{\rho C_p} \frac{d^2 \theta}{d\eta^2} \left( \frac{\partial \eta}{\partial y} \right)^2$$



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# Solução de Blasius (1908)... c. limite térmica



Paul R.H. Blasius  
(1883 – 1970)

$$u_{\infty} \frac{df}{d\eta} \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{1}{2} \sqrt{\frac{u_{\infty} \nu}{x}} \left( \eta \frac{df}{d\eta} - f \right) \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial y} = \frac{k}{\rho C_p} \frac{d^2 \theta}{d\eta^2} \left( \frac{\partial \eta}{\partial y} \right)^2$$

⋮

$$Pr = \frac{\mu}{k/C_p} \Rightarrow$$

$$2 \frac{d^2 \theta}{d\eta^2} + Pr \cdot f \frac{d\theta}{d\eta} = 0$$

$$\theta(0) = 0 \text{ e } \theta(\infty) = 1$$

| $\eta$   | $f$      | $\frac{df}{d\eta} = \frac{u}{u_{\infty}}$ | $\frac{d^2 f}{d\eta^2}$ |
|----------|----------|---|-------------------------|
| 0        | 0        | 0   | 0.332                   |
| 0.5      | 0.042    | 0.166                                     | 0.331                   |
| 1.0      | 0.166    | 0.330                                     | 0.323                   |
| 1.5      | 0.370    | 0.487                                     | 0.303                   |
| 2.0      | 0.650    | 0.630                                     | 0.267                   |
| 2.5      | 0.996    | 0.751                                     | 0.217                   |
| 3.0      | 1.397    | 0.846                                     | 0.161                   |
| 3.5      | 1.838    | 0.913                                     | 0.108                   |
| 4.0      | 2.306    | 0.956                                     | 0.064                   |
| 4.5      | 2.790    | 0.980                                     | 0.034                   |
| 5.0      | 3.283    | 0.992                                     | 0.016                   |
| 5.5      | 3.781    | 0.997                                     | 0.007                   |
| 6.0      | 4.280    | 0.999                                     | 0.002                   |
| $\infty$ | $\infty$ | 1   | 0                       |



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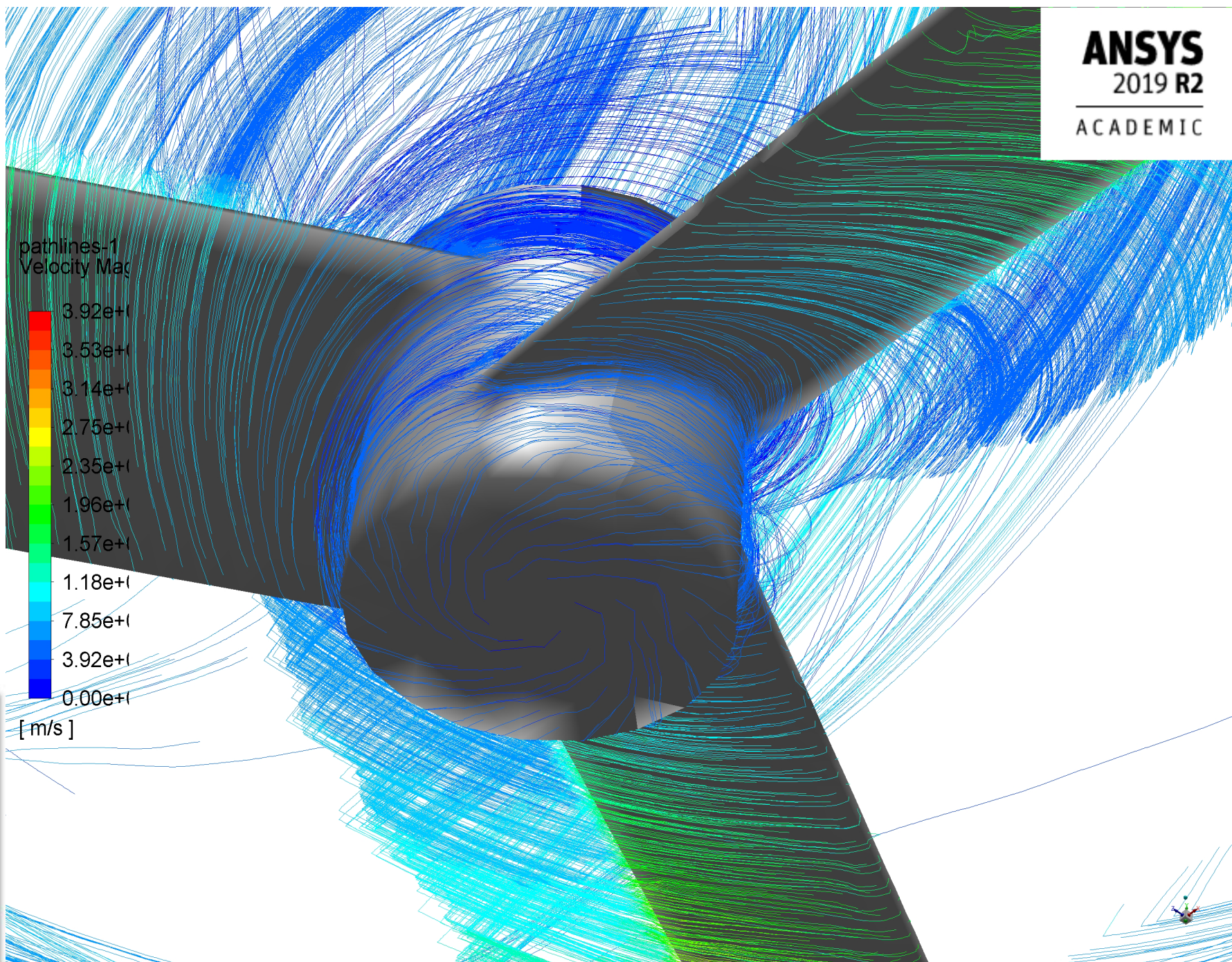




# MÉTODOS DE SOLUÇÃO

↓  
empíricos

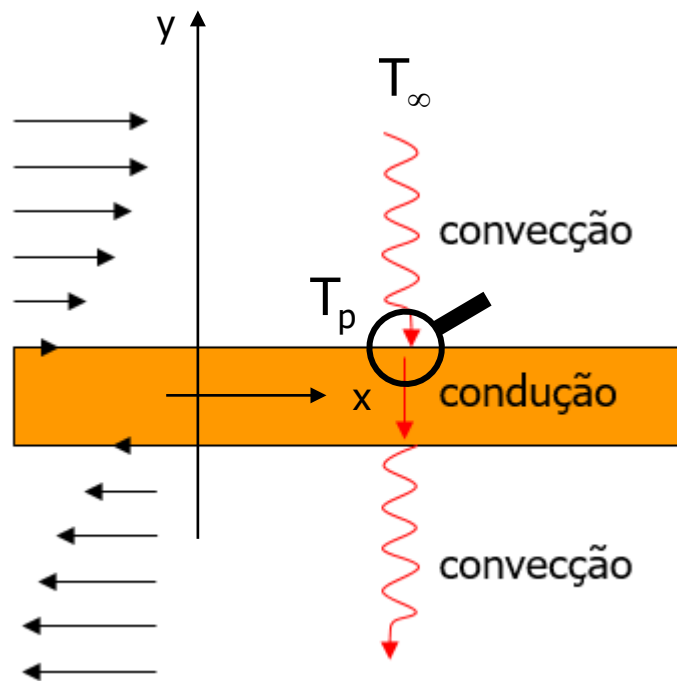
↓  
computacionais



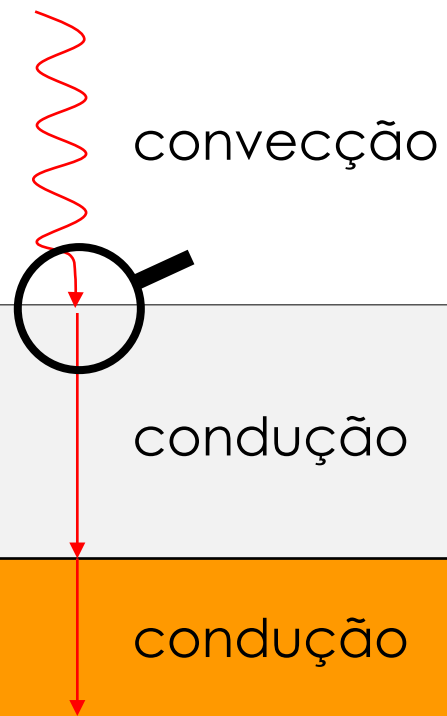


# A CONVECÇÃO DE CALOR – ABORDAGEM EMPÍRICA...

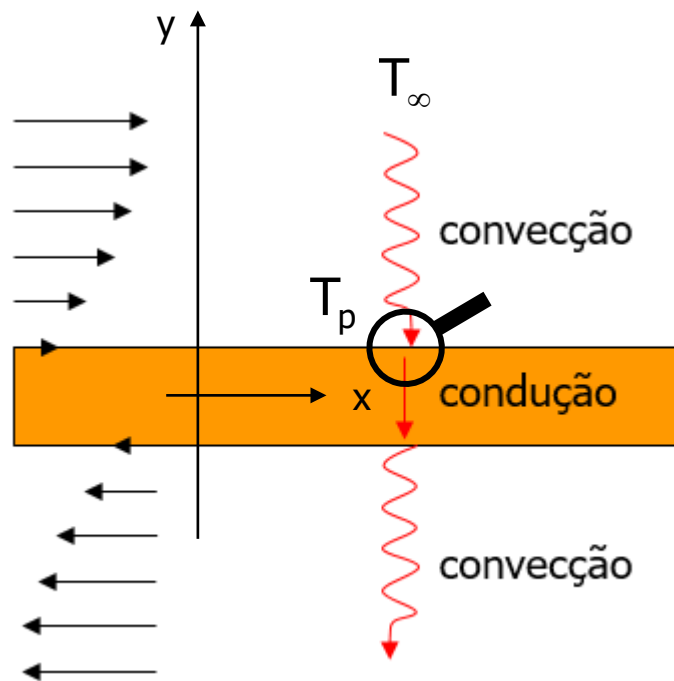
movimentação do fluido  $\leftrightarrow$  transporte de energia térmica



camada em movimento



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$$q = h_q \cdot (T_\infty - T_p) = -k \cdot \frac{dT}{dy} \Big|_{\text{camada limite}}$$

$$y^* \leftarrow y/D \quad T^* \leftarrow \frac{T - T_p}{T_\infty - T_p}$$

$$h_q \cdot (T_\infty - T_p) = -k \cdot \frac{d}{dy} (T_p + (T_\infty - T_p) \cdot T^*)$$

$$h_q \cdot (T_\infty - T_p) = -k(T_\infty - T_p) \cdot \frac{dT^*}{dy}$$

$$h_q = -k \cdot \frac{dT^*}{dy^*} \frac{dy^*}{dy}$$

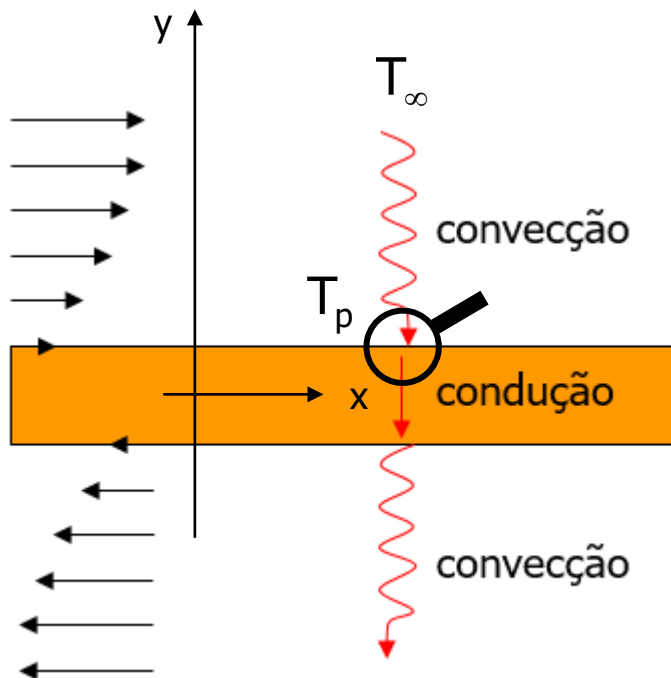
$$\text{Nr. de Nusselt} \rightarrow \frac{h_q}{k/D} = -\frac{dT^*}{dy^*} \leftarrow \frac{\text{convecção}}{\text{condução}}$$



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$$Nu \stackrel{\text{def}}{=} \frac{h_q}{k/D}$$



As equações diferenciais de balanço indicam que:

Solução de Blasius (1908)... c. limite térmica

Paul R.H. Blasius (1883–1970)

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| 0.5      | 0.042    | 0.166                                   | 0.331                  |
| 1.0      | 0.166    | 0.330                                   | 0.323                  |
| 1.5      | 0.370    | 0.487                                   | 0.303                  |
| 2.0      | 0.650    | 0.630                                   | 0.267                  |
| 2.5      | 0.996    | 0.751                                   | 0.217                  |
| 3.0      | 1.397    | 0.846                                   | 0.161                  |
| 3.5      | 1.838    | 0.913                                   | 0.108                  |
| 4.0      | 2.306    | 0.956                                   | 0.064                  |
| 4.5      | 2.790    | 0.980                                   | 0.034                  |
| 5.0      | 3.283    | 0.992                                   | 0.016                  |
| 5.5      | 3.781    | 0.997                                   | 0.007                  |
| 6.0      | 4.280    | 0.999                                   | 0.002                  |
| $\infty$ | $\infty$ | 1                                       | 0                      |

$$\tau_x = 0.332 \frac{\rho u_\infty^2}{\sqrt{Re_x}} \quad -q_x = 0.332 \cdot Pr^{1/3} (T_\infty - T_p) \sqrt{\frac{u_\infty}{\nu x}}$$

$$\delta_{FD} = \frac{5.0 \cdot x}{\sqrt{Re_x}} \quad \delta_{TM} = \frac{5.0 \cdot x}{Pr^{1/3} \sqrt{Re_x}}$$

aula MF2

Adimensionalização das equações governantes...

$$\frac{\partial \bar{u}}{\partial x} = \frac{u_\infty}{L} \frac{\partial \bar{u}^*}{\partial x^*} \quad \frac{\partial \bar{v}}{\partial y} = \frac{u_\infty}{L} \frac{\partial \bar{v}^*}{\partial y^*} \quad \frac{\partial \bar{P}}{\partial x} = \frac{\rho u_\infty^2}{L} \frac{\partial \bar{P}^*}{\partial x^*} \quad \frac{\partial \bar{P}}{\partial y} = \frac{\rho u_\infty^2}{L} \frac{\partial \bar{P}^*}{\partial y^*}$$

$$\frac{\partial \bar{\rho}^*}{\partial t^*} + \bar{\nabla} \cdot (\bar{\rho}^* \bar{U}^*) = 0$$

$$\rho^* \left( \frac{\partial \bar{U}^*}{\partial t^*} + \bar{U}^* \cdot \bar{\nabla} \bar{U}^* \right) = -\bar{\nabla} \bar{P}^* + \frac{1}{Re} \nabla^2 \bar{U}^* + \sum \frac{1}{R_k} F_k^*$$

$$\left( \frac{\partial \bar{T}^*}{\partial t^*} + \bar{U}^* \cdot \bar{\nabla} \bar{T}^* \right) = \frac{1}{Re \cdot Pr} \bar{\nabla} \cdot \bar{\nabla} \bar{T}^* \quad \text{Forças externas}$$



$$Re = \frac{\rho u_0 D}{\mu} \rightarrow \frac{\text{inércia}}{\text{d.viscosa}}$$

$$Pr = \frac{\mu}{k/C_p} \rightarrow \frac{\text{d.viscosa}}{\text{d.térmica}}$$

As equações governam o fenômeno via leis de conservação. Os números adimensionais definem o comportamento na escala do problema modelado...



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$$Nu \stackrel{\text{def}}{=} \frac{h_q}{k/D}$$

As equações diferenciais de balanço indicam que:

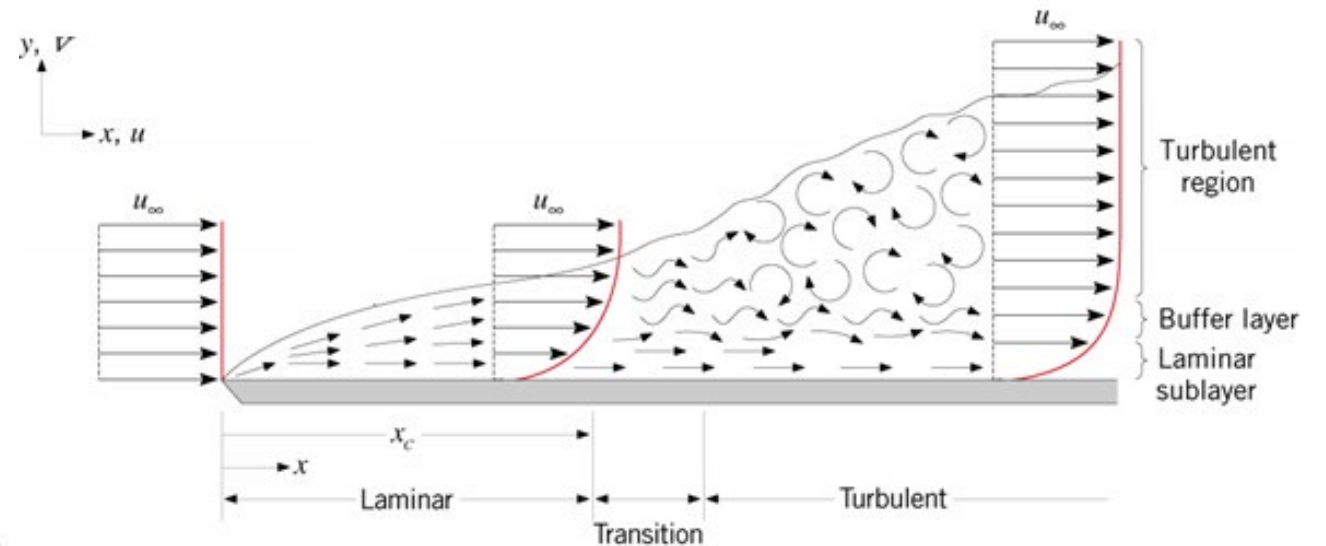
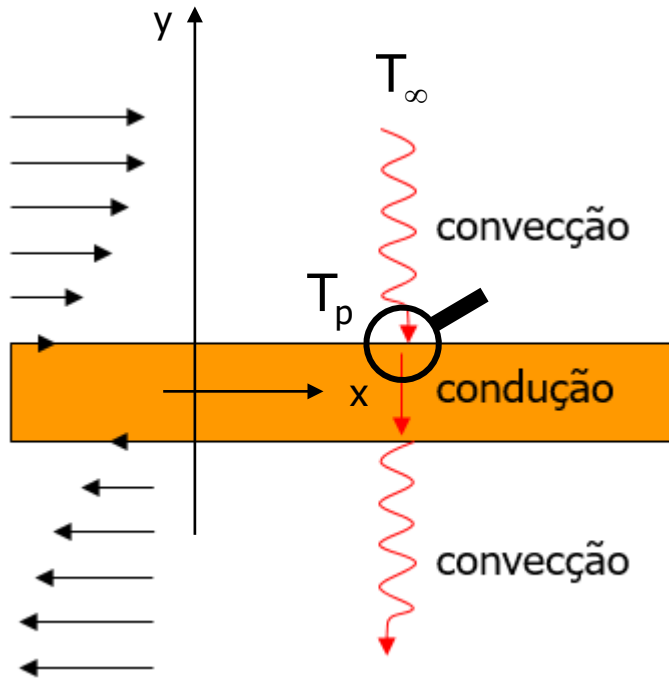
$$Nu = f(Re, Pr)$$

$$Re = \frac{\rho u_0 D}{\mu}$$

caracteriza o escoamento

caracteriza o fluido

$$Pr = \frac{C_p \mu}{k}$$



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$$Nu \stackrel{\text{def}}{=} \frac{h_q}{k/D}$$

As equações diferenciais de balanço indicam que:

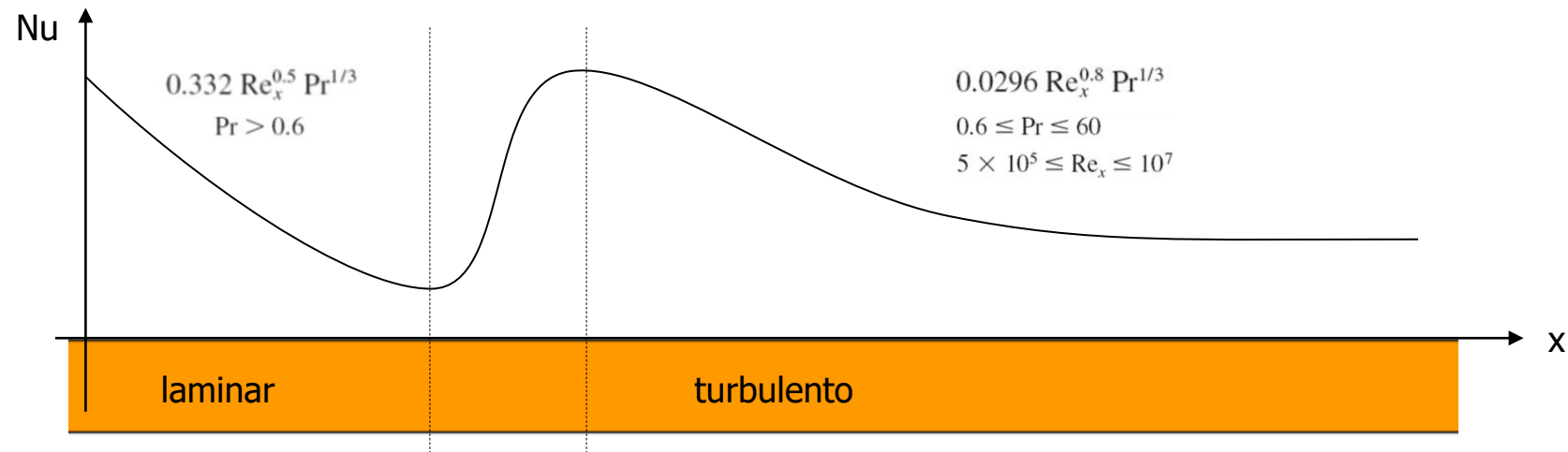
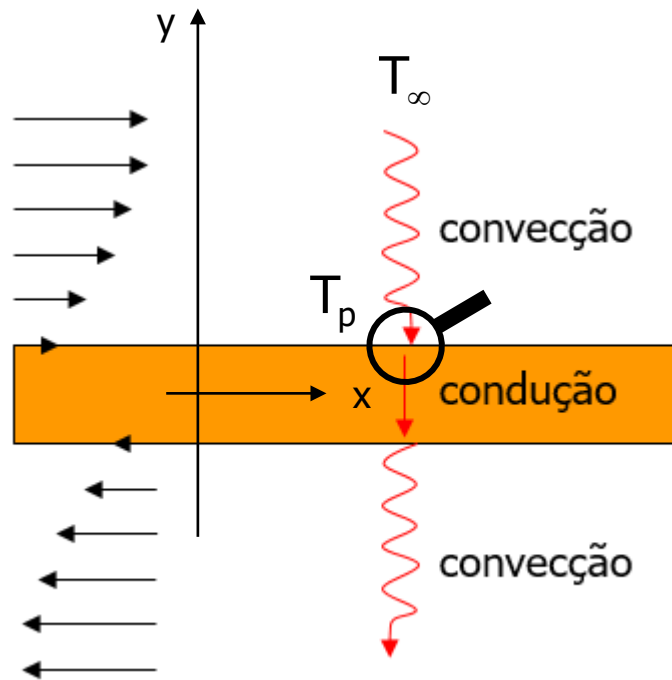
$$Nu = f(Re, Pr)$$

$$Re = \frac{\rho u_0 D}{\mu}$$

caracteriza o escoamento

caracteriza o fluido

$$Pr = \frac{C_p \mu}{k}$$



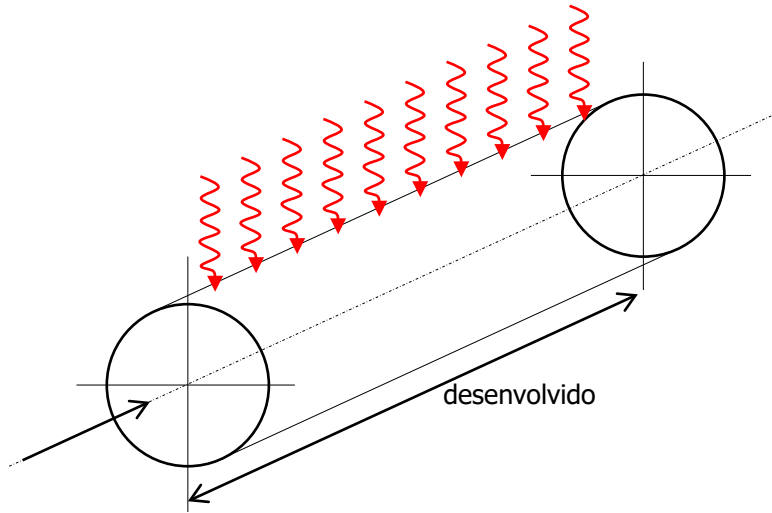
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# Correlações para escoamentos internos...

## Escoamento laminar:

Fluxo de calor constante:  $Nu = 4.36$

Temperatura constante:  $Nu = 3.66$



## Escoamento turbulento desenvolvido ( $Re > 10^4$ ):

Superfície lisa:

$$Nu = 0.023 \cdot Re^{4/5} \cdot Pr^n$$

$$\begin{aligned} n = 0.3 \quad p / T_s < T_m & \quad \left[ \begin{array}{l} 0.7 \leq Pr \leq 160 \\ Re_D \geq 10,000 \end{array} \right] \\ n = 0.4 \quad p / T_s > T_m & \quad \left[ \begin{array}{l} L/D \geq 10 \end{array} \right] \end{aligned}$$

$$Re = \frac{\rho u_0 D}{\mu} \quad Pr = \frac{C_p \mu}{k}$$



HOMEM DE FERRO

(Dittus-Boelter)

AVALIAÇÃO P2



Superfície rugosa:

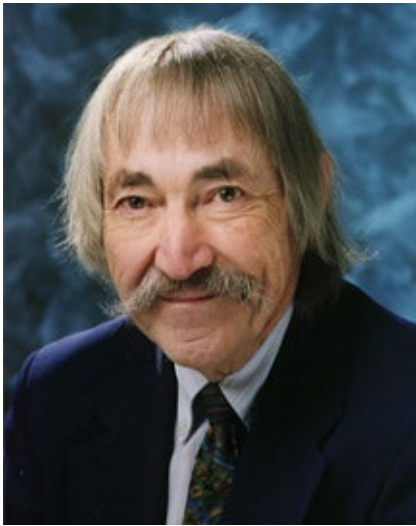
$$Nu = \frac{f}{8} \cdot \frac{(Re - 1000) \cdot Pr}{1 + 12.7(f/8)^{1/2} (Pr^{2/3} - 1)}$$

$f$  = friction factor

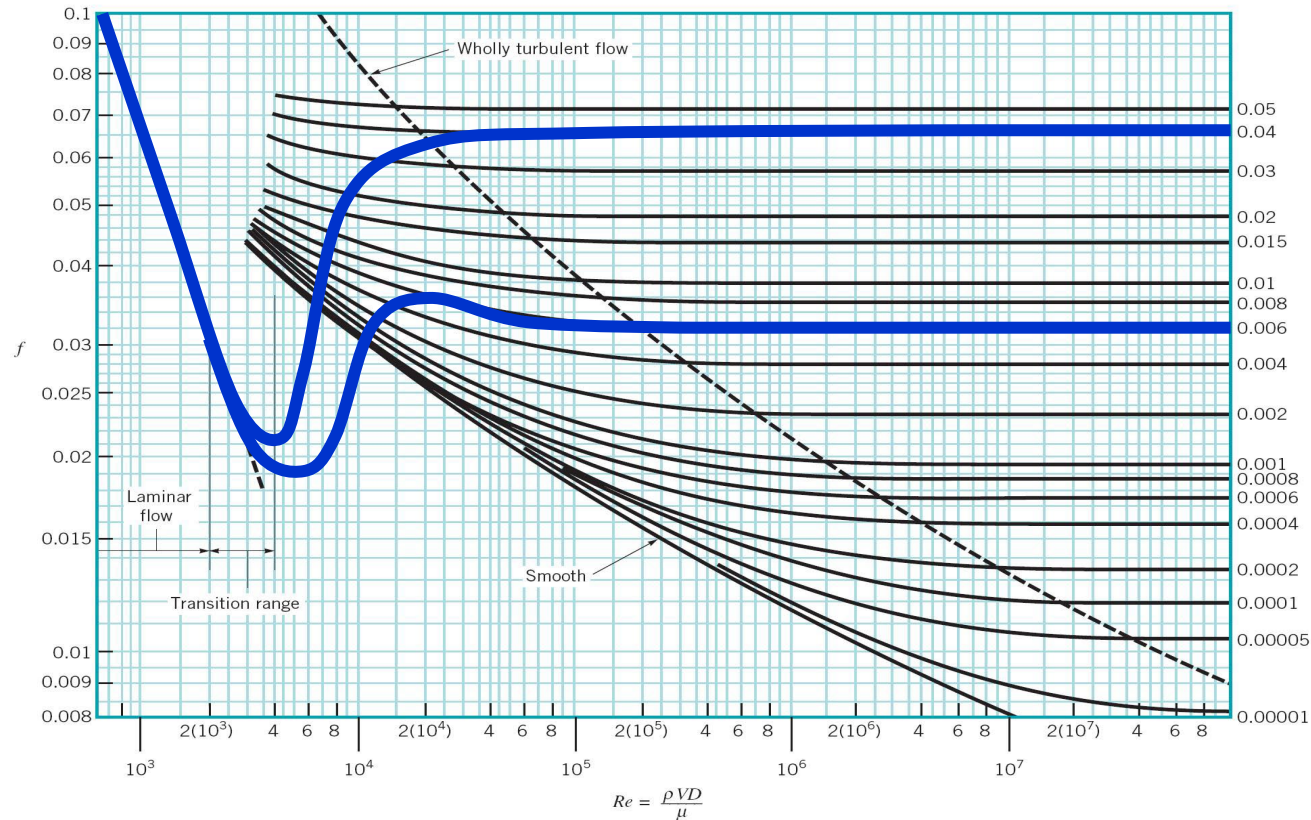


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# A equação de Darcy e cálculo do fator de atrito...



Stuart W. Churchill



**Laminar (Re < 2500)**

$$f = \frac{64}{Re}$$



HOMEM ARANHA

$\frac{\epsilon}{D}$  Turbulento (Re > 4000)

**Colebrook-White**

$$\frac{1}{\sqrt{f}} = -2 \log \left( 3.7 \frac{\epsilon}{D} + \frac{2.51}{Re \sqrt{f}} \right)$$

$$f = 8 \cdot \left[ \left( \frac{8}{Re} \right)^{12} + (A + B)^{-1,5} \right]^{1/12}$$

$$A = \left\{ 2,457 \ln \left( \left( \left( \frac{7}{Re} \right)^{0,9} + 0,27 \cdot \frac{\epsilon}{D} \right)^{-1} \right) \right\}^{16}$$

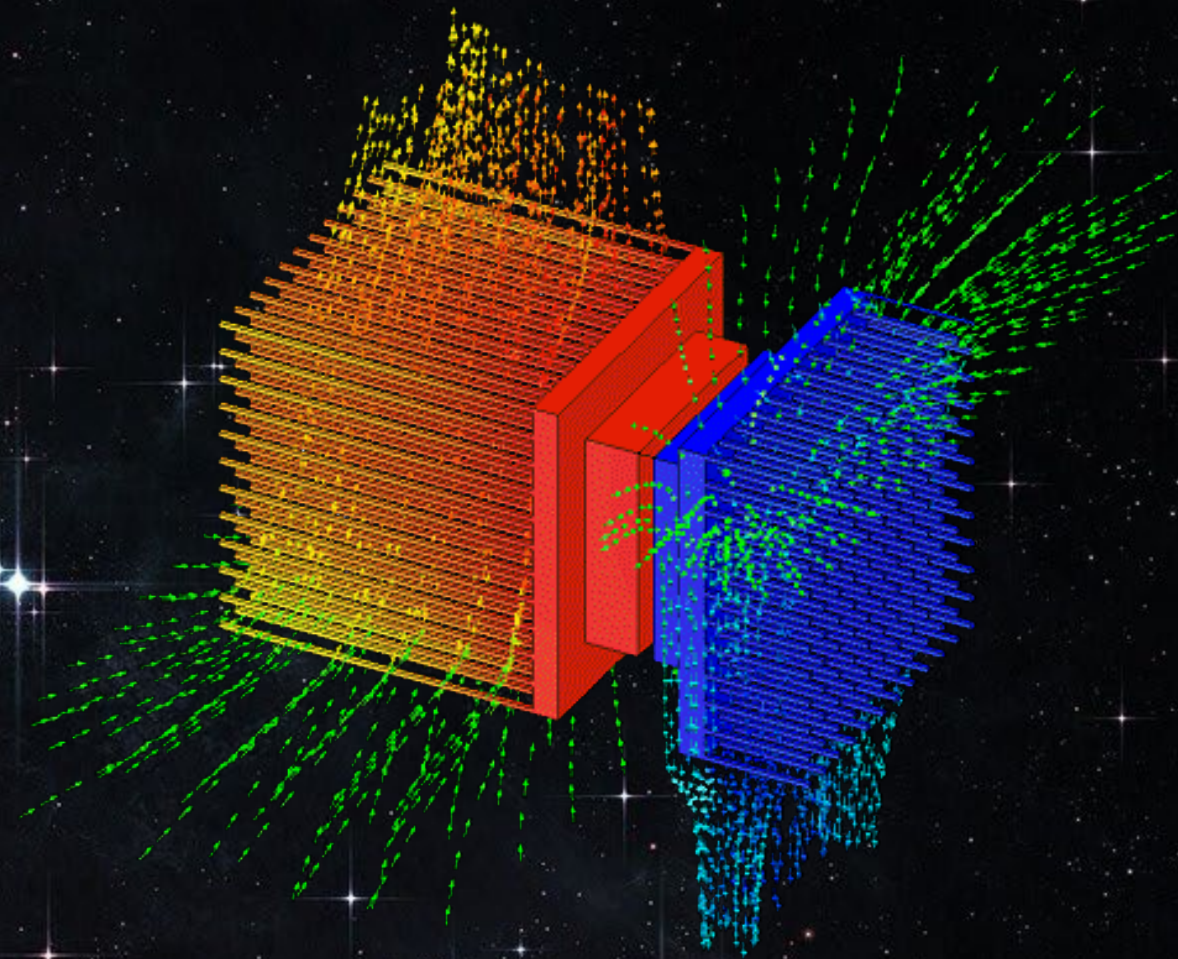
$$B = \left( \frac{37530}{Re} \right)^{16} \quad Re = \frac{4 \cdot m}{\mu \pi D}$$



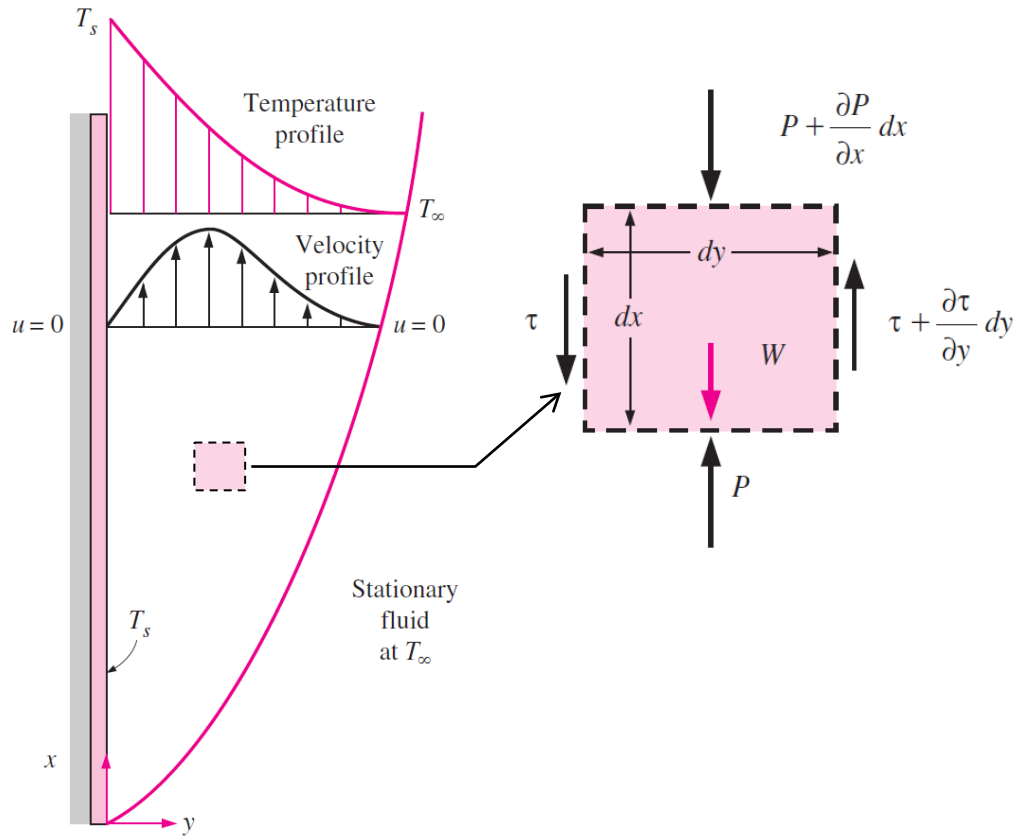
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# Convecção natural / escoamentos externos



# Convecção natural: equações governantes e o número de Grashof...



$$F_x = a_x \cdot dm$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$\frac{\partial P}{\partial y} = 0 \rightarrow \downarrow \leftarrow \frac{\partial P}{\partial x} = -\rho_\infty g$$

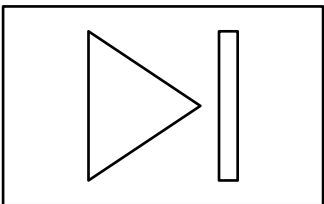
$$\rightarrow \rho \cdot \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g$$

$$\downarrow \leftarrow \Delta \rho = \rho \cdot \beta \cdot \Delta T \quad \text{Coef. expansão volumétrica}$$

$$\rightarrow \rho \cdot \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + g\beta \cdot (T - T_\infty)$$

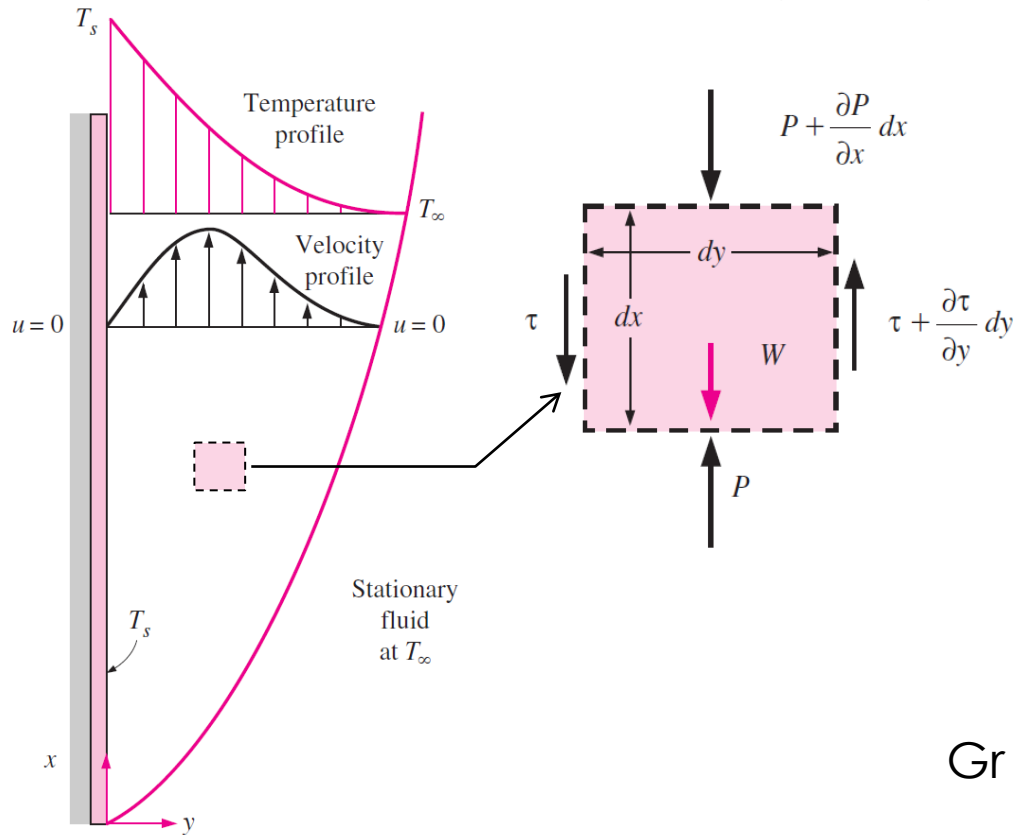
Força de empuxo dependente da temperatura

p/ eq. convecção forçada



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# Convecção natural: equações governantes e o número de Grashof...



$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{U}, \quad v^* = \frac{v}{U}, \quad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

adimensionalização

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}} + \left( \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \right) \cdot \frac{1}{Re} T^*$$

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{\text{força de empuxo}}{\text{dissipação viscosa}}$$

Análogo de Reynolds para convecção natural...

$$Ra = Gr \cdot Pr$$

Forma genérica para o Nr. Nusselt:

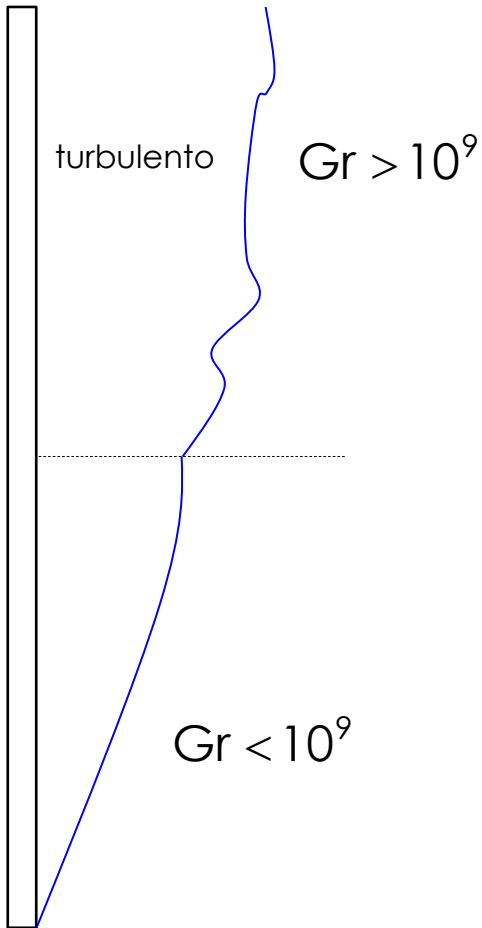
$$Nu = C \cdot (Gr \cdot Pr)^n = C \cdot Ra^n$$



HE-MAN



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## Avaliação P2



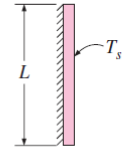
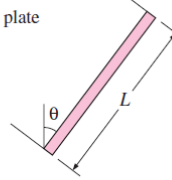
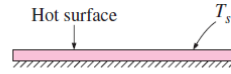

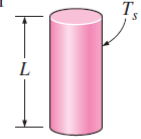
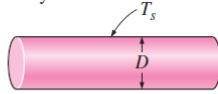
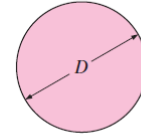
MULHER-MARAVILHA

$$Q = hA_{\text{lateral}} \cdot (T - T_{\infty})$$

$$Nu = C \cdot Ra^n$$

$$\text{properties @ } T_{\text{fluid}} = \left( \frac{T_s + T_{\infty}}{2} \right)$$

### Empirical correlations for the average Nusselt number for natural convection over surfaces

| Geometry  | Characteristic length $L_c$ | Range of Ra                              | Nu  |
|---|-----------------------------|--|---|
| Vertical plate<br>   | $L$                         | $10^4 - 10^9$<br>$10^9 - 10^{13}$        | $Nu = 0.59Ra_l^{1/4}$ (9-19)<br>$Nu = 0.1Ra_l^{1/3}$ (9-20)   |
| Inclined plate<br>   | $L$                         |  | Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate<br>Replace $g$ by $g \cos \theta$ for $Ra < 10^9$ |
| Horizontal plate<br>(Surface area $A$ and perimeter $p$ )<br>(a) Upper surface of a hot plate (or lower surface of a cold plate)<br> | $A_s/p$                     | $10^4 - 10^7$<br>$10^7 - 10^{11}$        | $Nu = 0.54Ra_l^{1/4}$ (9-22)<br>$Nu = 0.15Ra_l^{1/3}$ (9-23)  |
| (b) Lower surface of a hot plate (or upper surface of a cold plate)<br>  |                             | $10^5 - 10^{11}$                         | $Nu = 0.27Ra_l^{1/4}$ (9-24)  |
| Vertical cylinder<br>   | $L$                         |  | A vertical cylinder can be treated as a vertical plate when<br>$D \geq \frac{35L}{Gr_l^{1/4}}$  |
| Horizontal cylinder<br>  | $D$                         | $Ra_D \leq 10^{12}$                      | $Nu = \left\{ 0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$ (9-25)   |
| Sphere<br>   | $D$                         | $Ra_D \leq 10^{11}$<br>( $Pr \geq 0.7$ ) | $Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (9-26)   |

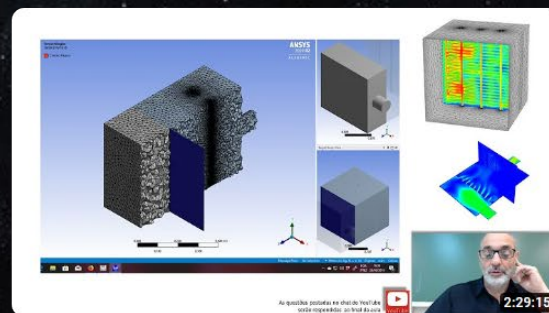


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# A CONVECÇÃO DE CALOR – ABORDAGEM NUMÉRICA...

movimentação do fluido ↔ transporte de energia térmica



TUTORIAL CFD ANSYS/FLUENT: PROJETO DE UM AQUECEDOR DE AR

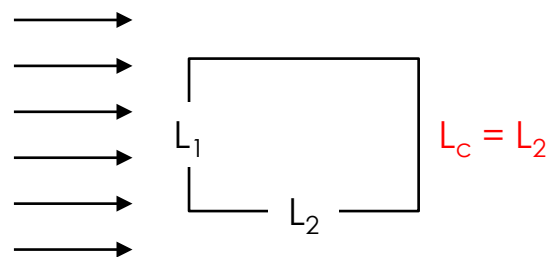
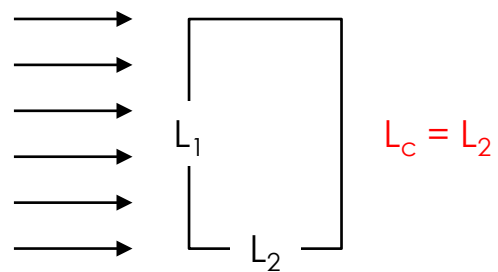
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Prof. P. Seleghim

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<https://youtube.com/live/wgQCpJSJz0Y>





**Table 2.** Empirical correlations for the average Nusselt number for forced convection over isothermal surfaces. The characteristic length,  $L_c$ , is defined in Section 3.1.3.

| Geometry                   | $L_c$ | Range of Validity   | $Nu$                                      |
|----------------------------|-------|---|---|
| <b>Parallel to a plate</b> |       |   |   |
| Laminar flow               | L     | $Re \leq 5 \times 10^5$<br>$0.6 \leq Pr \leq 60$                  | $Nu = 0.664Re^{1/2}Pr^{1/3}$ [33]         |
| Turbulent flow             | L     | $5 \times 10^5 \leq Re \leq \times 10^7$<br>$0.6 \leq Pr \leq 60$ | $Nu = 0.037Re^{4/5}Pr^{1/3}$ [33]         |
| Combined flow              | L     | $5 \times 10^5 \leq Re \leq \times 10^7$<br>$0.6 \leq Pr \leq 60$ | $Nu = (0.037Re^{4/5} - 871)Pr^{1/3}$ [33] |
| <b>Around a sphere</b>     |       |   |   |
|                            | D     | $0 \leq Re < 200$<br>$0 \leq Pr \leq 250$                         | $Nu = 2 + 0.6Re^{1/2}Pr^{1/3}$ [35]       |

[33] Welty, J.; Rorrer, G.L.; Foster, D.G. Fundamentals of Momentum, Heat and Mass Transfer; John Wiley & Sons: New York, NY, USA, 2014.

[35] Ranz, W.E.; Marshall, W.R. Evaporation from Drops. Chem. Eng. Prog. 1952, 48, 141–146.

Maragkos, Georgios, and Tarek Beji. "Review of convective heat transfer modelling in CFD simulations of fire-Driven Flows." *Applied Sciences* 11.11 (2021): 5240.

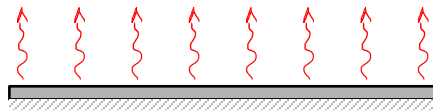
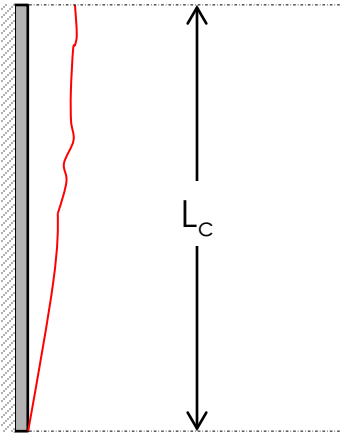
**TUTORIAL CFD ANSYS/FLUENT: PROJETO DE UM AQUECEDOR DE AR**

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$L_c = \text{Area}/\text{perímetro}$

**Table 3.** Empirical correlations for the average Nusselt number for natural convection over isothermal surfaces. The characteristic length,  $L_c$ , is defined in Section 3.1.3.

| Geometry                | $L_c$   | Range of Validity                  | $Nu$  |
|-------------------------|---------|------------------------------------|---|
| <b>Vertical plate</b>   |         |                                    |   |
| Laminar flow            | L       | $10^4 \leq Ra \leq 10^9$           | $Nu = 0.59Ra^{1/4}$ [32]  |
| Turbulent flow          | L       | $10^9 \leq Ra \leq 10^{13}$        | $Nu = 0.1Ra^{1/3}$ [32]   |
| Any type of flow        | L       | $Ra \leq 10^{12}$                  | $Nu = \left(0.825 + \frac{0.387Ra^{1/6}}{(1+(0.492/Pr)^{9/16})^{8/27}}\right)^2$ [36] |
| <b>Horizontal plate</b> |         |                                    |   |
| Laminar flow            | $A_s/p$ | $10^4 \leq Ra \leq 10^7$           | $Nu = 0.54Ra^{1/4}$ [32]  |
| Turbulent flow          | $A_s/p$ | $10^7 \leq Ra \leq 10^{11}$        | $Nu = 0.15Ra^{1/3}$ [32]  |
| <b>Sphere</b>           |         |                                    |   |
|                         | D       | $Ra \leq 10^{11}$<br>$Pr \geq 0.7$ | $Nu = 2 + \frac{0.589Ra^{1/4}}{(1+(0.469/Pr)^{9/16})^{4/9}}$ [37]                     |

[32] McAdams, W.H. Heat Transmission; McGraw-Hill Book Company: New York, NY, USA, 1957.

[36] Churchill, S.W.; Chu, H.H.S. Correlating Equations for Laminar and Turbulent Free Convection from a Vertical Plate. Int. J. Heat Mass Transf. 1975, 18, 1323–1329. [CrossRef]

[37] Churchill, S.W. Free Convection around Immersed Bodies. In Heat Exchanger Design Handbook; Hemisphere Publishing: New York, NY, USA, 1983.

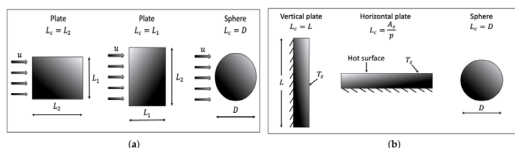
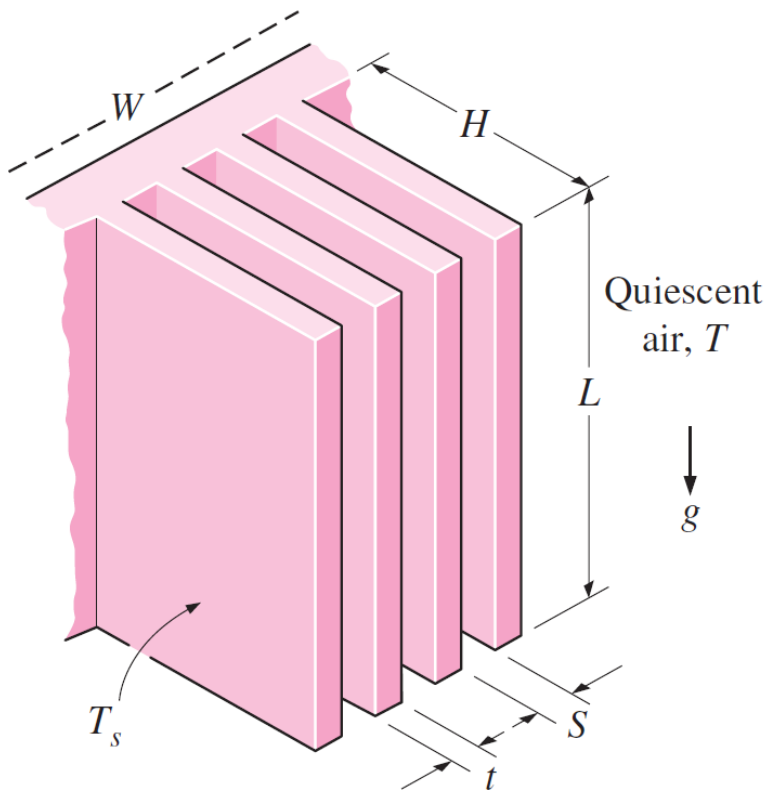


Figure 1. Characteristic length,  $L_c$ , for (a) forced and (b) natural convection scenarios.  $A_s$  and  $p$  are the surface area and perimeter of the plate, respectively.



# Convecção natural em superfícies aletadas @ $T_s = \text{cte} \dots$



$$Ra_S = \frac{g\beta(T_s - T_\infty) \cdot S^3}{\nu^2} Pr \quad \text{ou} \quad Ra_L = \frac{g\beta(T_s - T_\infty) \cdot L^3}{\nu^2} Pr$$

$$Nu = \frac{hS_{opt}}{k} = \left[ \frac{576}{(Ra_S \cdot S/L)^2} + \frac{2.873}{(Ra_S \cdot S/L)^{0.5}} \right]^{-0.5}$$

**AVALIAÇÃO P2**  
Bar-Cohen and Rohsenow

Espaçamento ótimo (trade-off área x vazão):

$$@ T_s = \text{cte} \rightarrow S_{opt} = 2.714 \frac{L}{Ra_L^{0.25}} \rightarrow Nu = 1.307$$

$$Q = h \cdot (2nLH) \cdot (T_s - T_\infty) \quad \leftarrow t \ll S$$

$$\text{properties @ } T_{avg} = (T_s + T_\infty) / 2$$

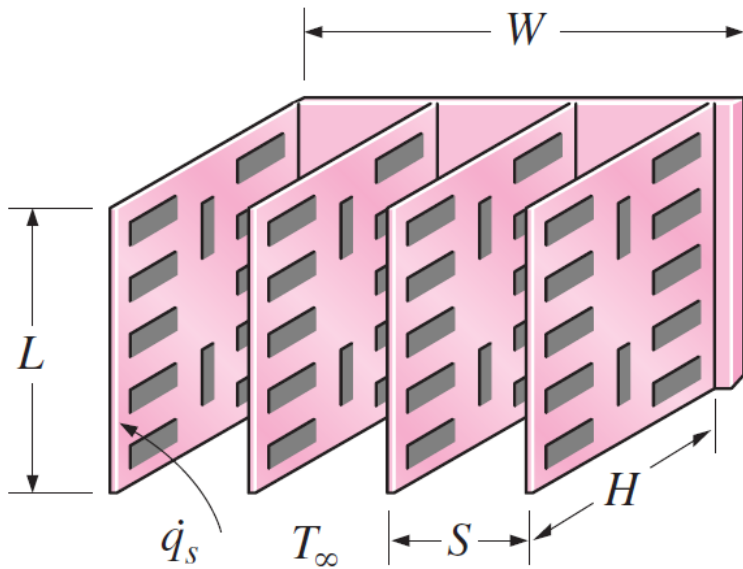


AQUAMAN



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# Convecção natural em superfícies aletadas @ $q_s = \text{cte} \dots$



$$Ra_s = \frac{g\beta q_s \cdot S^4}{k\nu^2} Pr$$

$$Nu = \frac{hL}{k} = \left[ \frac{48}{Ra_s \cdot S/L} + \frac{2.51}{(Ra_s \cdot S/L)^{0.4}} \right]^{-0.5}$$

Espaçamento ótimo (trade-off área x vazão):

$$@ q_s = \text{cte} \rightarrow S_{\text{opt}} = 2.12 \cdot \left( \frac{S^4 L}{Ra_s} \right)^{0.2}$$

$$Q = q_s \cdot (2nLH) \quad \leftarrow t \ll S$$

$$\text{properties @ } T_{\text{avg}} = (T_L + T_\infty)/2 \quad \leftarrow T_L = T_\infty + q_s / h$$

temperatura crítica  
ocorrendo na borda  
superior



**AVALIAÇÃO P2**  
Bar-Cohen and Rohsenow



TARTARUGA  
NINJA



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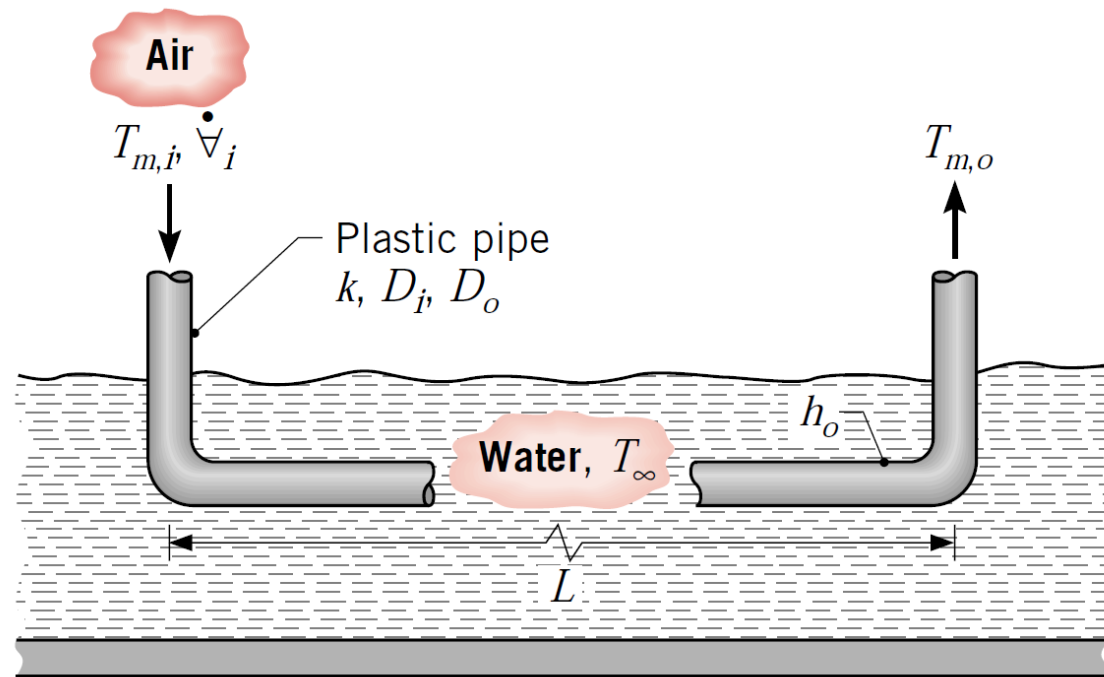
# A CONVECÇÃO DE CALOR – ABORDAGEM EMPÍRICA...

exercício resolvido – tutorial excel





**8.31:** Para resfriar uma casa de verão sem uso de um ciclo frigorífico, ar é encaminhado através de uma tubulação de plástico ( $k=0.15\text{W/m/K}$ ,  $D_i=0.15\text{m}$ ,  $D_o=0.17\text{m}$ ) submersa em um corpo d'água adjacente. A temperatura da água é normalmente de  $T_\infty=17^\circ\text{C}$ , e o coeficiente de convecção é mantido em  $h_o=1500\text{ W/m}^2/\text{K}$  na superfície externa da tubulação. Se ar proveniente da casa entra no tubo a uma temperatura de  $T_{m,i}=29^\circ\text{C}$  e uma vazão volumétrica de  $V_i=0.025\text{m}^3/\text{s}$ , qual extensão  $L$  é necessária para que a temperatura na saída seja de  $T_{m,o}=21^\circ\text{C}$  ?



Próxima aula ?



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

**8.31:** Para resfriar uma casa de verão sem uso de um ciclo frigorífico, ar é encaminhado através de uma tubulação de plástico ( $k=0.15\text{W/m/K}$ ,  $D_i=0.15\text{m}$ ,  $D_o=0.17\text{m}$ ) submersa em um corpo d'água adjacente. A temperatura da água é normalmente de  $T_\infty=17^\circ\text{C}$ , e o coeficiente de convecção é mantido em  $h_o=1500\text{ W/m}^2/\text{K}$  na superfície externa da tubulação. Se ar proveniente da casa entra no tubo a uma temperatura de  $T_{m,i}=29^\circ\text{C}$  e uma vazão volumétrica de  $V_i=0.025\text{m}^3/\text{s}$ , qual extensão  $L$  é necessária para que a temperatura na saída seja de  $T_{m,o}=21^\circ\text{C}$  ?



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REFPROP (air (dry)) - NIST Reference Fluid Properties (DLL version 9,1)

File Edit Options Substance Calculate Plot Window Help Cautions

2: air (dry): Specified state points

|   | Temperature (°C) | Pressure (bar) | Density (kg/m³) | Enthalpy (kJ/kg) | Cp (kJ/kg-K) | Therm. Cond. (mW/m-K) | Viscosity (μPa-s) | Prandtl |
|---|------------------|----------------|-----------------|------------------|--------------|-----------------------|-------------------|---------|
| 1 | 29,000           | 1,0000         | 1,1533          | 428,46           | 1,0064       | 26,544                | 18,641            | 0,70678 |
| 2 | 25,000           | 1,0000         | 1,1688          | 424,44           | 1,0063       | 26,247                | 18,448            | 0,70729 |
| 3 | 21,000           | 1,0000         | 1,1848          | 420,41           | 1,0062       | 25,948                | 18,254            | 0,70781 |
| 4 |                  |                |                 |                  |              |                       |                   |         |

$$Q = \dot{m} \cdot (h_i - h_o) = \rho_i V_i \cdot (h_i - h_o)$$



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|   | Temperature (°C) | Pressure (bar) | Density (kg/m³) | Enthalpy (kJ/kg) | Cp (kJ/kg-K) | Therm. Cond. (mW/m-K) | Viscosity (μPa-s) | Prandtl |
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$$Q = 1.1533 \frac{\text{kg}}{\text{m}^3} 0.025 \frac{\text{m}^3}{\text{s}} \cdot (428.46 - 420.41) \frac{\text{kJ}}{\text{kg}}$$



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$$\dot{m} = 0.02883 \text{kg/s}$$



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$$Q = 1.1533 \frac{\text{kg}}{\text{m}^3} 0.025 \frac{\text{m}^3}{\text{s}} \cdot (428.46 - 420.41) \frac{\text{kJ}}{\text{kg}}$$

$$\dot{m} = 0.02883 \text{ kg/s}$$

$$Q = 0.2321 \text{ kW}$$



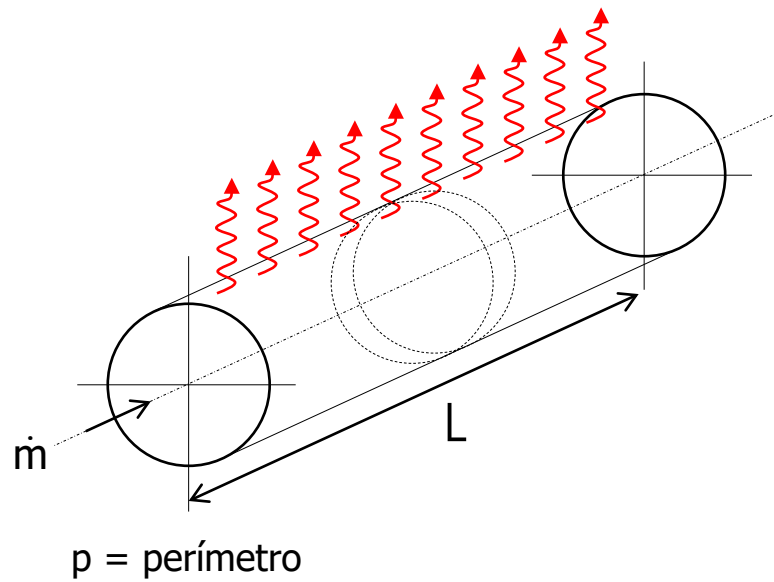
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# Equação do decaimento da temperatura ao longo da tubulação



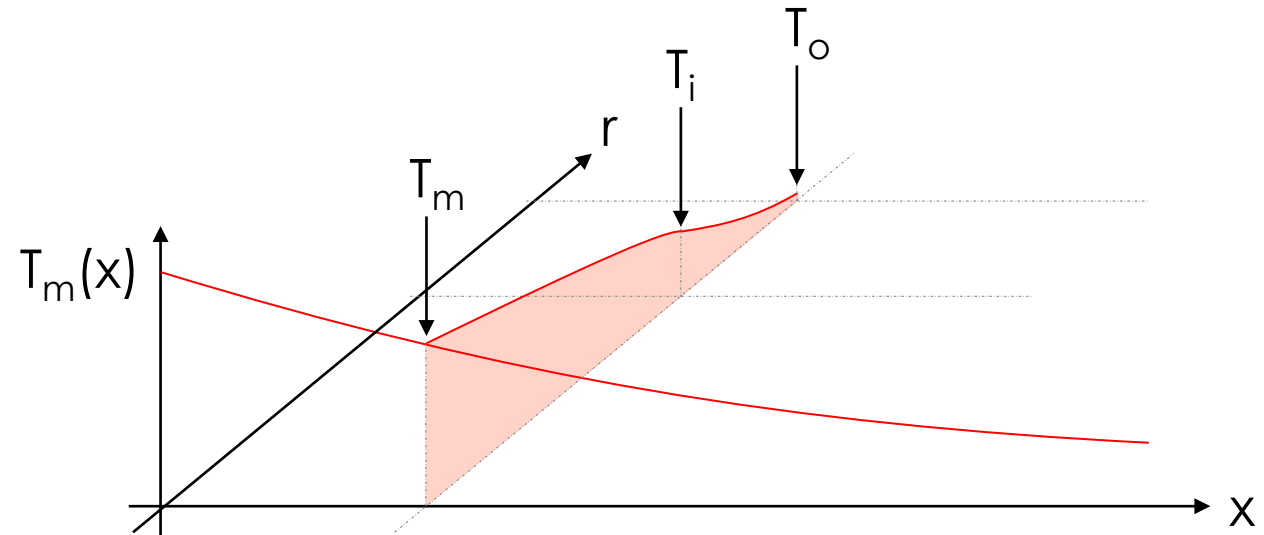
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# Balço global de energia...

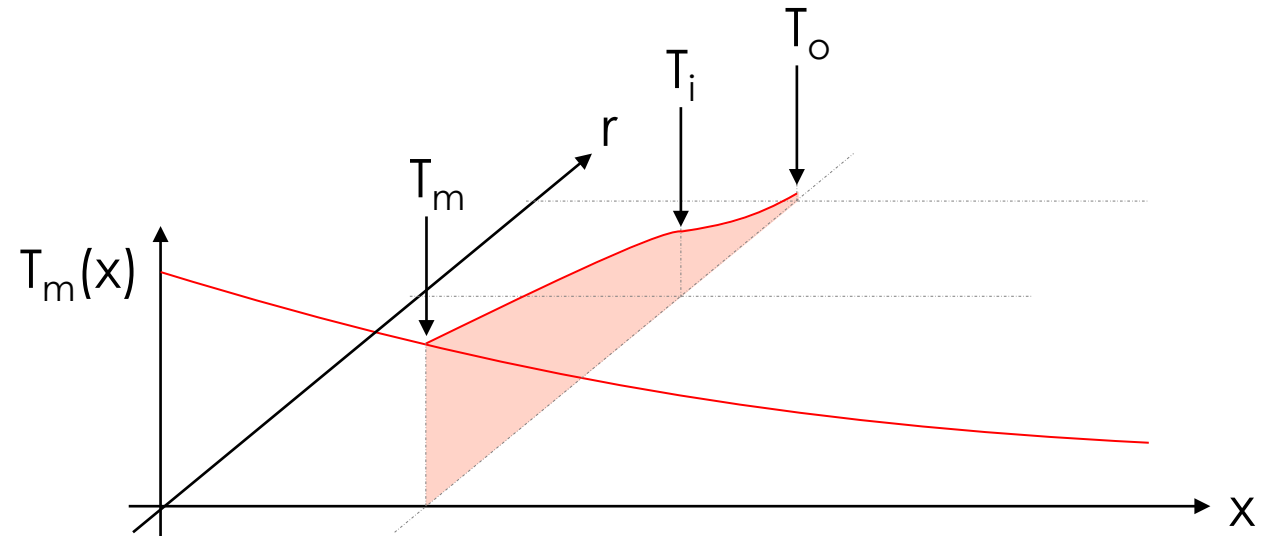
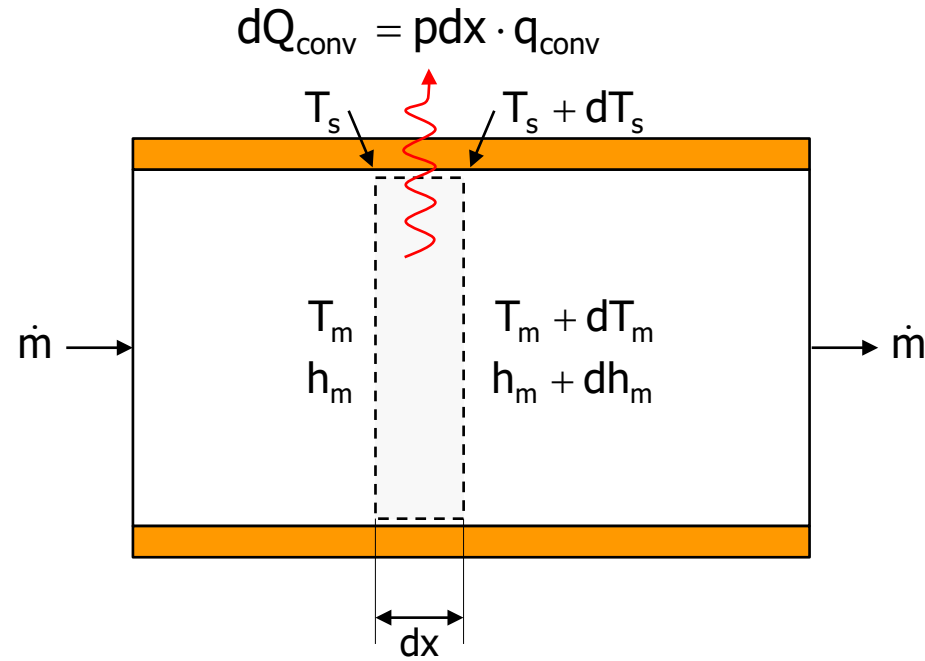
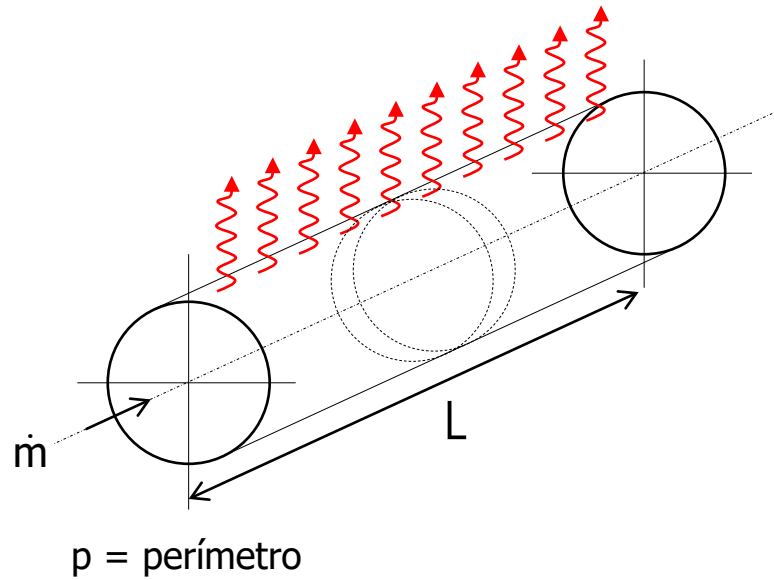


$T_m(x)$  → perfil axial de temperaturas

$T_r(x)$  → perfil radial de temperaturas

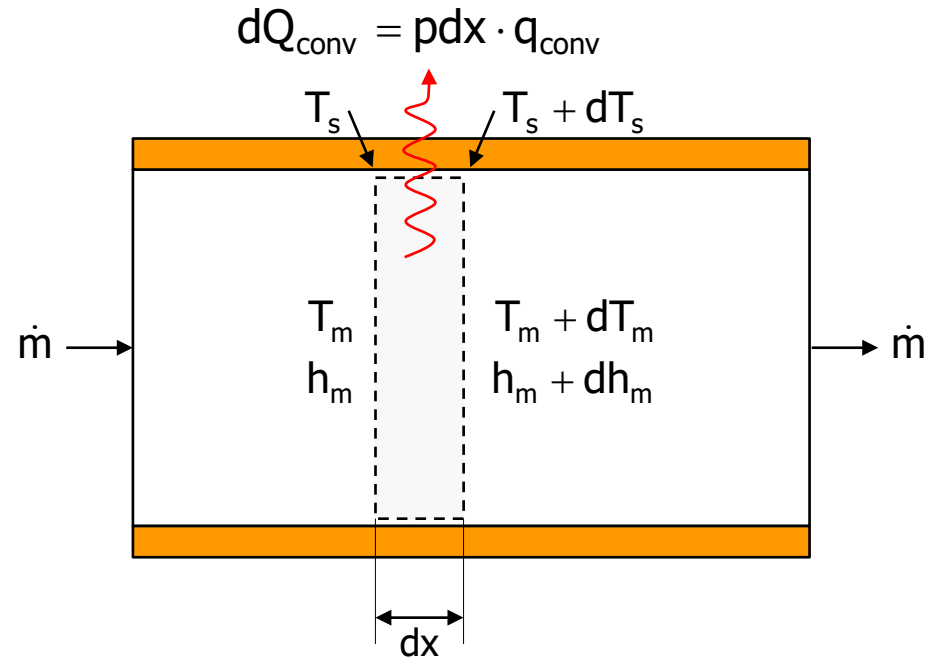
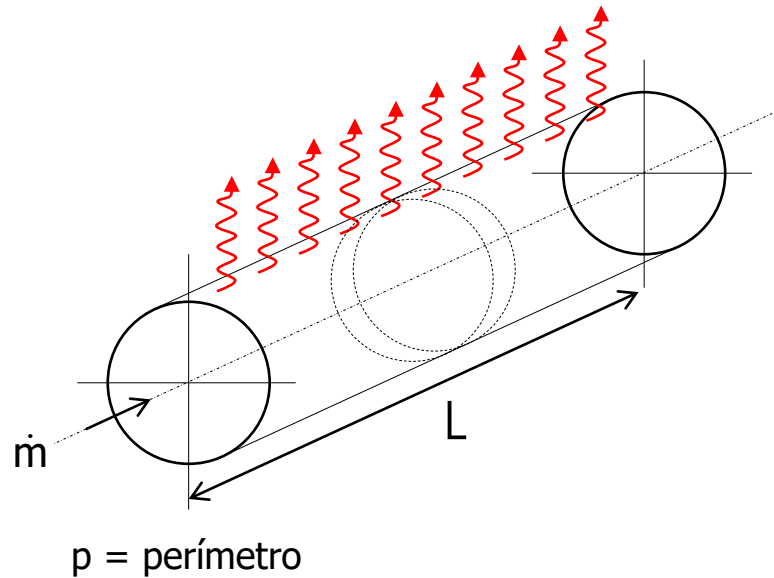


# Balanco global de energia...



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Balanco global de energia...



$$\dot{Q} - \dot{W} = \sum_{\text{sai}} \dot{m}_k \cdot (\cancel{h_k} + \cancel{gz_k} + \cancel{v_k^2 / 2}) - \sum_{\text{entra}} \dot{m}_k \cdot (\cancel{h_k} + \cancel{gz_k} + \cancel{v_k^2 / 2})$$

$$p q_{\text{conv}} dx = \dot{m} \cdot (\cancel{h_m + dh_m}) - \dot{m} \cdot \cancel{h_m}$$

$$q_{\text{conv}} = h_{\text{conv}} \cdot (T_s - T_m) \rightarrow$$

$$\dot{m} \cdot \frac{dh_m}{dx} - p h_{\text{conv}} \cdot (T_s - T_m) = 0$$



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## Balanco global de energia...

$$\dot{m} \cdot \frac{dh_m}{dx} - ph_{\text{conv}} \cdot (T_s - T_m) = 0$$

Solução direta via método numérico de  
solução de equação diferencial

$$h = h(P, T) \downarrow$$

Implementação numérica...



As questões postadas no Chat do YouTube  
serão respondidas ao final da aula.

# Balço global de energia...



$$\dot{m} \cdot \frac{dh_m}{dx} - ph_{conv} \cdot (T_s - T_m) = 0$$

Solução direta via método numérico de solução de equação diferencial

$$h = h(P, T) \downarrow$$

Implementação numérica...

Solução analítica a partir de hipóteses simplificadoras

$$h = C_p \cdot T \downarrow \begin{array}{l} \text{gases perfeitos} \\ \text{"fluido incompressível"} (C_p = C_v) \end{array}$$

$$\dot{m} \cdot \frac{dT_m}{dx} - \frac{ph_{conv}}{C_p} \cdot (T_s - T_m) = 0$$



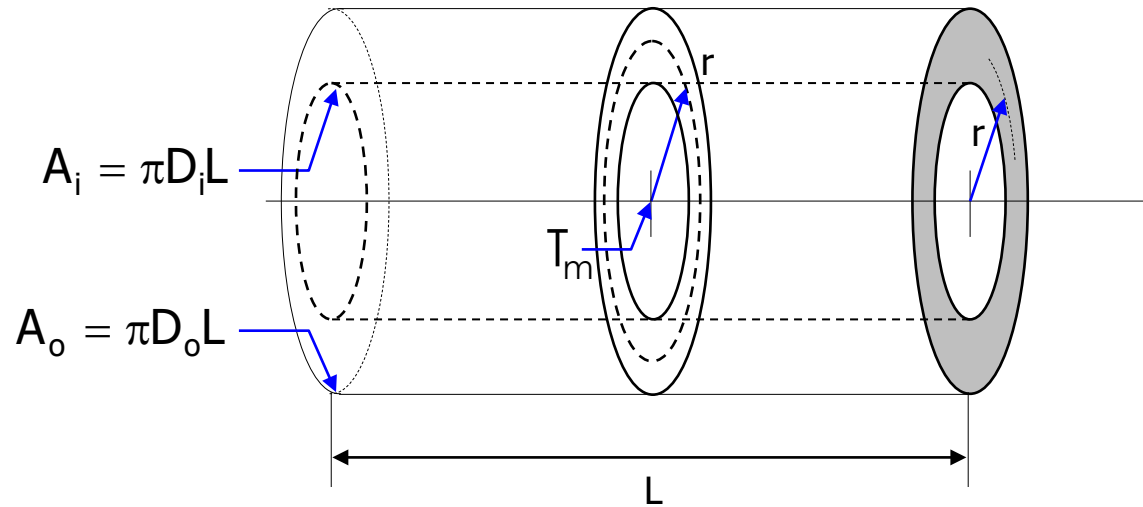
Fluxo de calor constante →

$$T_m(x) = T_{m,i} + \frac{pq_{conv}}{\dot{m}C_p} \cdot x$$

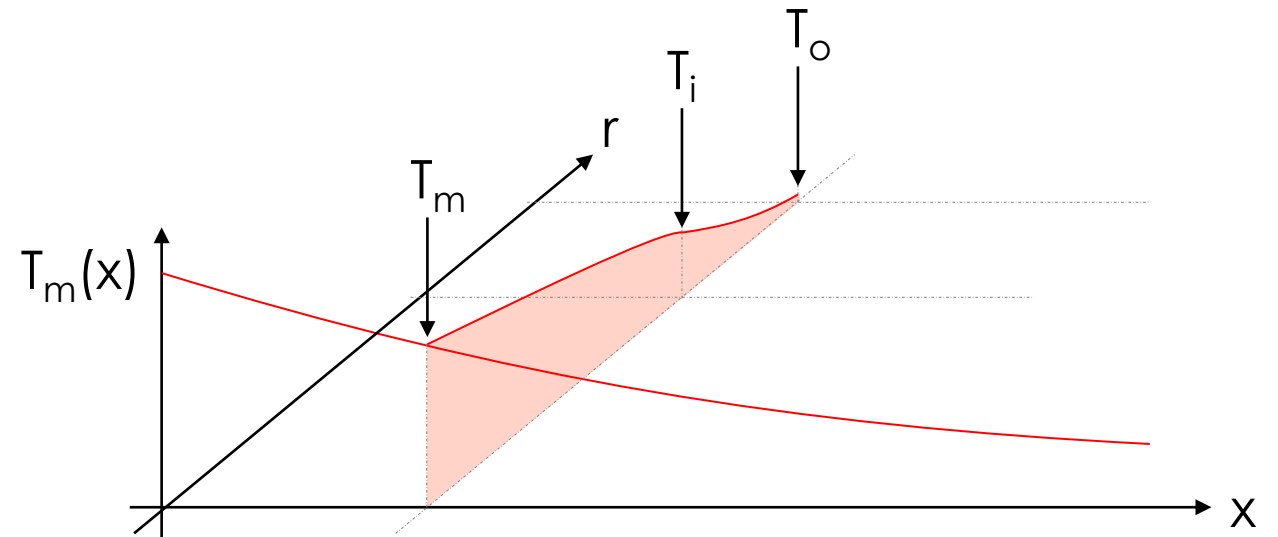
Temperatura superficial constante →

$$T_m(x) = T_s - (T_s - T_{m,i}) \cdot \exp\left(\frac{p\bar{h}_{conv}}{\dot{m}C_p} \cdot x\right)$$

# Geometria cilíndrica... **variação radial da temperatura**

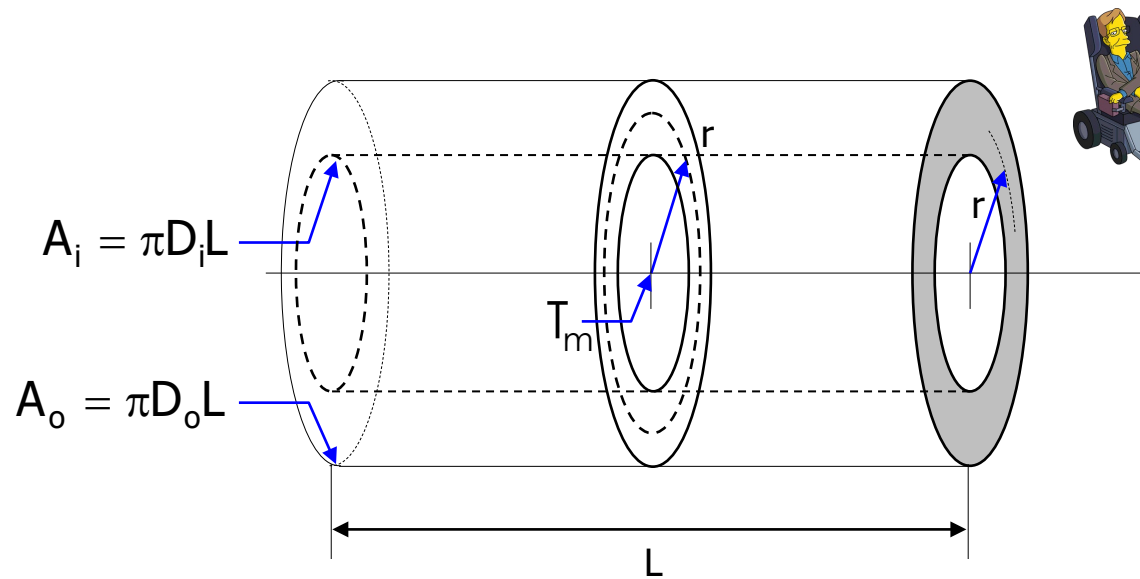


Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k\vec{\nabla}T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

$$\rightarrow \frac{dT}{dr} = C_1 + \frac{C_2}{r}$$

Obs.:  $\lim_{r \rightarrow \infty} \frac{dT}{dr} = 0$

$$\rightarrow dT = \frac{C_2}{r} dr$$

$$\rightarrow \int_{T_i}^{T(r)} dT = \int_{r_i}^r \frac{C_2}{r} dr$$

$$\rightarrow T(r) - T_i = \ln\left(\frac{r}{r_i}\right) \cdot C_2 \quad \leftarrow T(r_o) = T_o$$

Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$

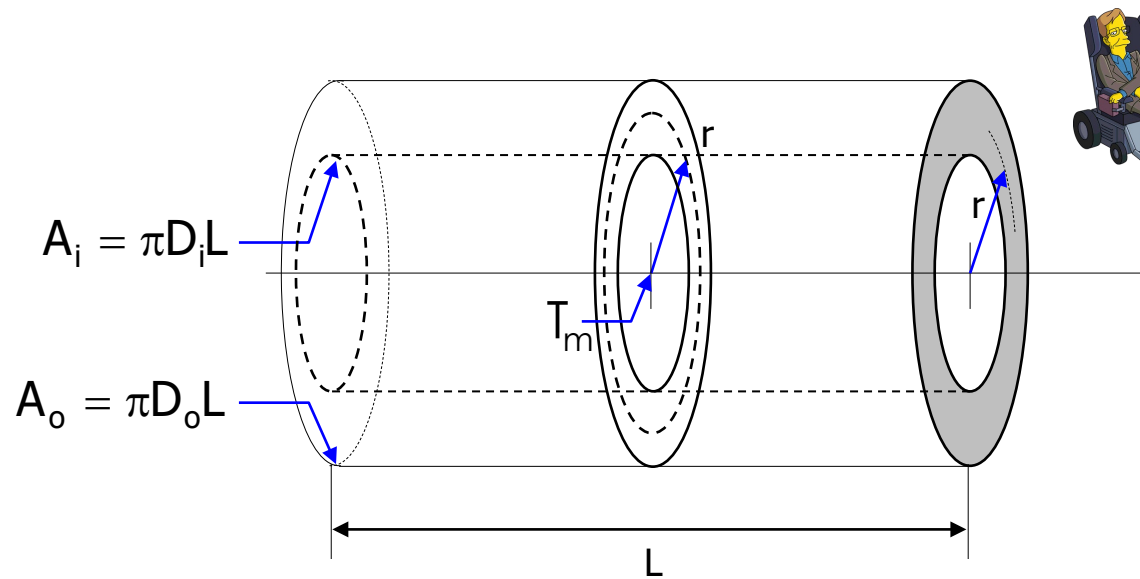
Equação do decaimento radial da temperatura

$$\rightarrow T(r) = T_i + \frac{T_o - T_i}{\ln(r_o / r_i)} \cdot \ln\left(\frac{r}{r_i}\right)$$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k \vec{\nabla} T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

$$T(r) = T_i + \frac{T_o - T_i}{\ln(r_o / r_i)} \cdot \ln\left(\frac{r}{r_i}\right)$$

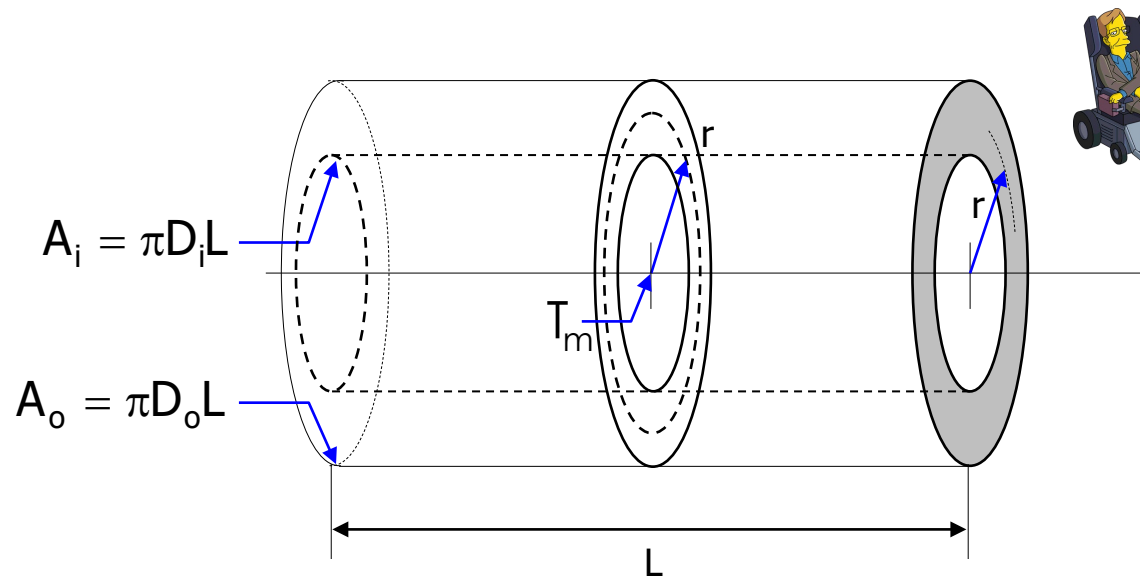
Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$



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# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k\vec{\nabla}T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

$$T(r) = T_i + \frac{T_o - T_i}{\ln(r_o/r_i)} \cdot \ln\left(\frac{r}{r_i}\right)$$

$$Q = -kA(r) \frac{dT}{dr}(r) = \dots = \frac{2\pi Lk}{\ln(r_o/r_i)} \cdot (T_i - T_o)$$

Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$

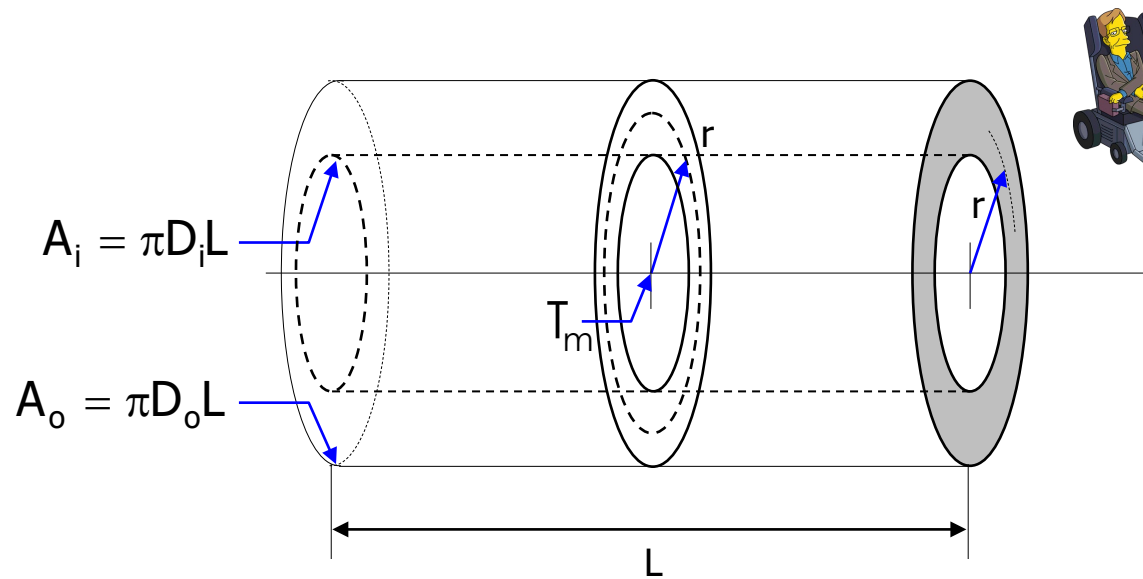
$$Q = h_i A_i (T_i - T_m) \rightarrow T_i = \frac{Q}{h_i A_i} + T_m$$

$$Q = h_o A_o (T_o - T_\infty) \rightarrow T_o = \frac{Q}{h_o A_o} - T_\infty$$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k \vec{\nabla} T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

$$T(r) = T_i + \frac{T_o - T_i}{\ln(r_o / r_i)} \cdot \ln\left(\frac{r}{r_i}\right)$$

$$Q = -kA(r) \frac{dT}{dr}(r) = \dots = \frac{2\pi Lk}{\ln(r_o / r_i)} \cdot (T_i - T_o)$$

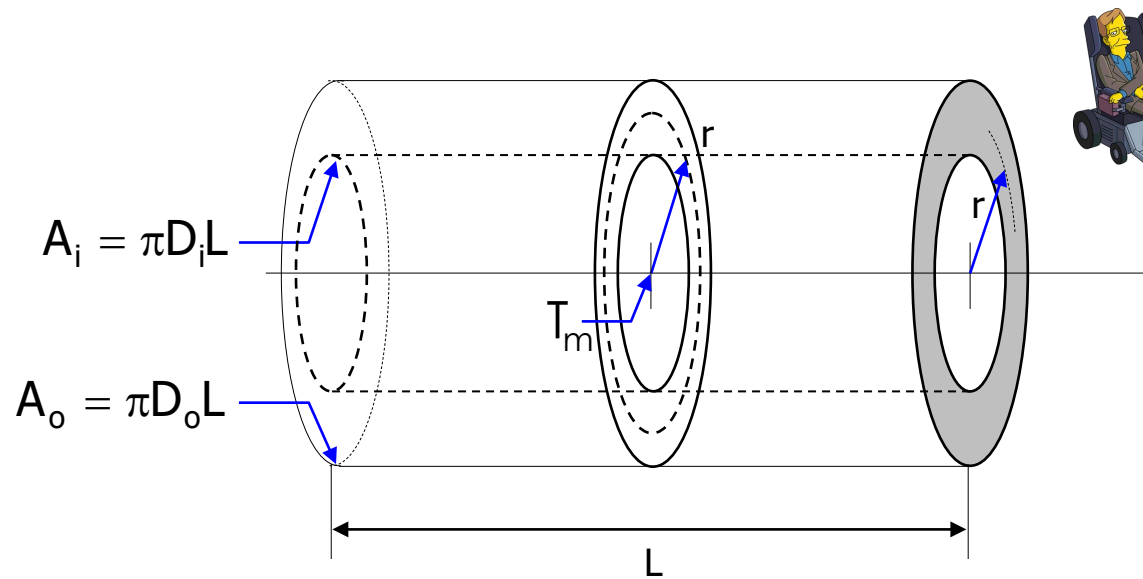
Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$

$$Q = \frac{T_m - T_\infty}{\frac{1}{2\pi r_i L h_i} + \frac{\ln(r_o / r_i)}{2\pi L k} + \frac{1}{2\pi r_o L h_o}}$$



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# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k \vec{\nabla} T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

$$T(r) = T_i + \frac{T_o - T_i}{\ln(r_o / r_i)} \cdot \ln\left(\frac{r}{r_i}\right)$$

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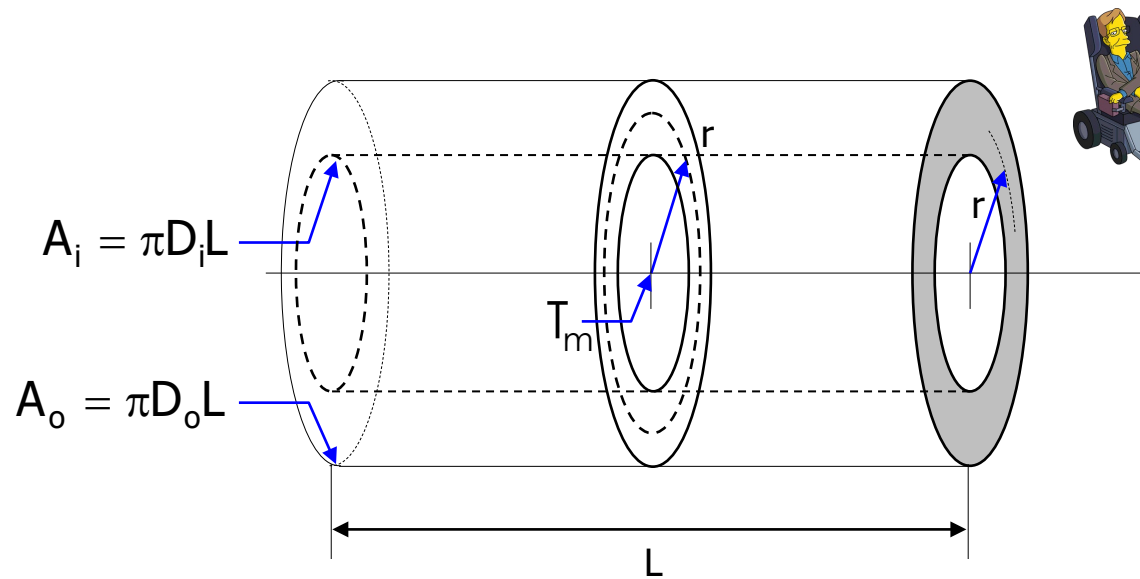
$$Q = \frac{T_m - T_\infty}{\frac{1}{2\pi r_i L h_i} + \frac{\ln(r_o / r_i)}{2\pi L k} + \frac{1}{2\pi r_o L h_o}}$$

$$\dots = \frac{T_m - T_\infty}{R_{\text{total}}} = UA \cdot (T_m - T_\infty)$$



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# Geometria cilíndrica... variação radial da temperatura



$$\vec{\nabla} \cdot (k\vec{\nabla}T) = 0 \rightarrow \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

$$T(r) = T_i + \frac{T_o - T_i}{\ln(r_o/r_i)} \cdot \ln\left(\frac{r}{r_i}\right)$$

$$Q = -kA(r) \frac{dT}{dr}(r) = \dots = \frac{2\pi Lk}{\ln(r_o/r_i)} \cdot (T_i - T_o)$$

Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$

$$Q = \frac{T_m - T_\infty}{\frac{1}{2\pi r_i L h_i} + \frac{\ln(r_o/r_i)}{2\pi L k} + \frac{1}{2\pi r_o L h_o}}$$

$$\dots = \frac{T_m - T_\infty}{R_{\text{total}}} = UA \cdot (T_m - T_\infty)$$

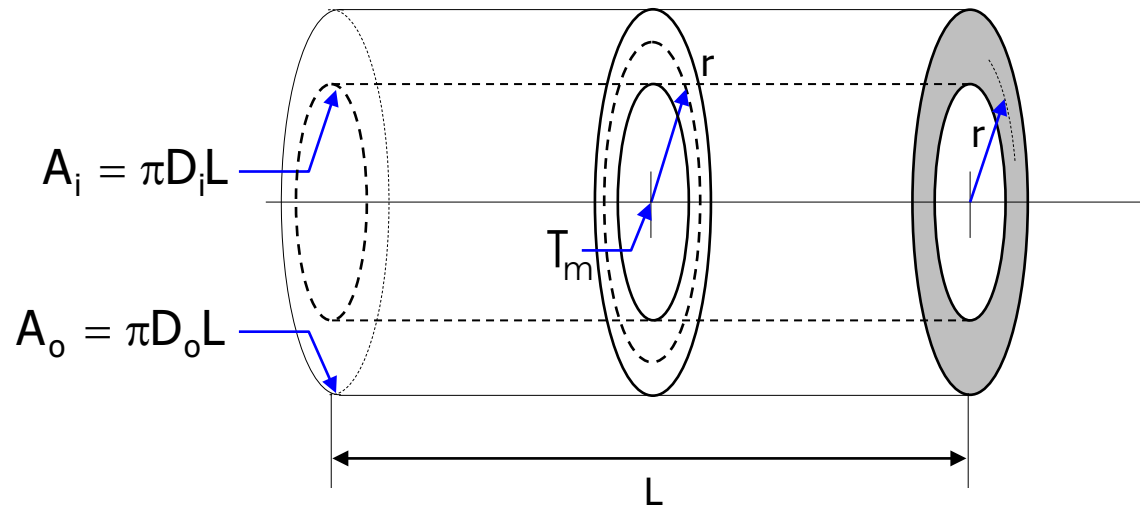
$$A = \pi D_i L \xrightarrow{\text{def}} U = \left[ \frac{1}{h_i} + \frac{r_i}{k} \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o} \cdot \frac{r_i}{r_o} \right]^{-1}$$

coeficiente global de transferência de calor (condutância)



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

# Geometria cilíndrica... variação radial da temperatura



Obs.:  $Q = q_i \cdot A_i = q_o \cdot A_o$

TABLE 13-1

Representative values of the overall heat transfer coefficients in heat exchangers

| Type of heat exchanger                        | $U$ , $W/m^2 \cdot ^\circ C^*$ |
|---|--------------------------------|
| Water-to-water                                | 850–1700                       |
| Water-to-oil                                  | 100–350                        |
| Water-to-gasoline or kerosene                 | 300–1000                       |
| Feedwater heaters                             | 1000–8500                      |
| Steam-to-light fuel oil                       | 200–400                        |
| Steam-to-heavy fuel oil                       | 50–200                         |
| Steam condenser                               | 1000–6000                      |
| Freon condenser (water cooled)                | 300–1000                       |
| Ammonia condenser (water cooled)              | 800–1400                       |
| Alcohol condensers (water cooled)             | 250–700                        |
| Gas-to-gas                                    | 10–40                          |
| Water-to-air in finned tubes (water in tubes) | 30–60 <sup>†</sup>             |
|   | 400–850 <sup>†</sup>           |
| Steam-to-air in finned tubes (steam in tubes) | 30–300 <sup>†</sup>            |
|   | 400–4000 <sup>‡</sup>          |

\*Multiply the listed values by 0.176 to convert them to  $Btu/h \cdot ft^2 \cdot ^\circ F$ .

<sup>†</sup>Based on air-side surface area.

<sup>‡</sup>Based on water- or steam-side surface area.

$$A = \pi D_i L \xrightarrow{\text{def}} U = \left[ \frac{1}{h_i} + \frac{r_i}{k} \ln \left( \frac{r_o}{r_i} \right) + \frac{1}{h_o} \cdot \frac{r_i}{r_o} \right]^{-1}$$

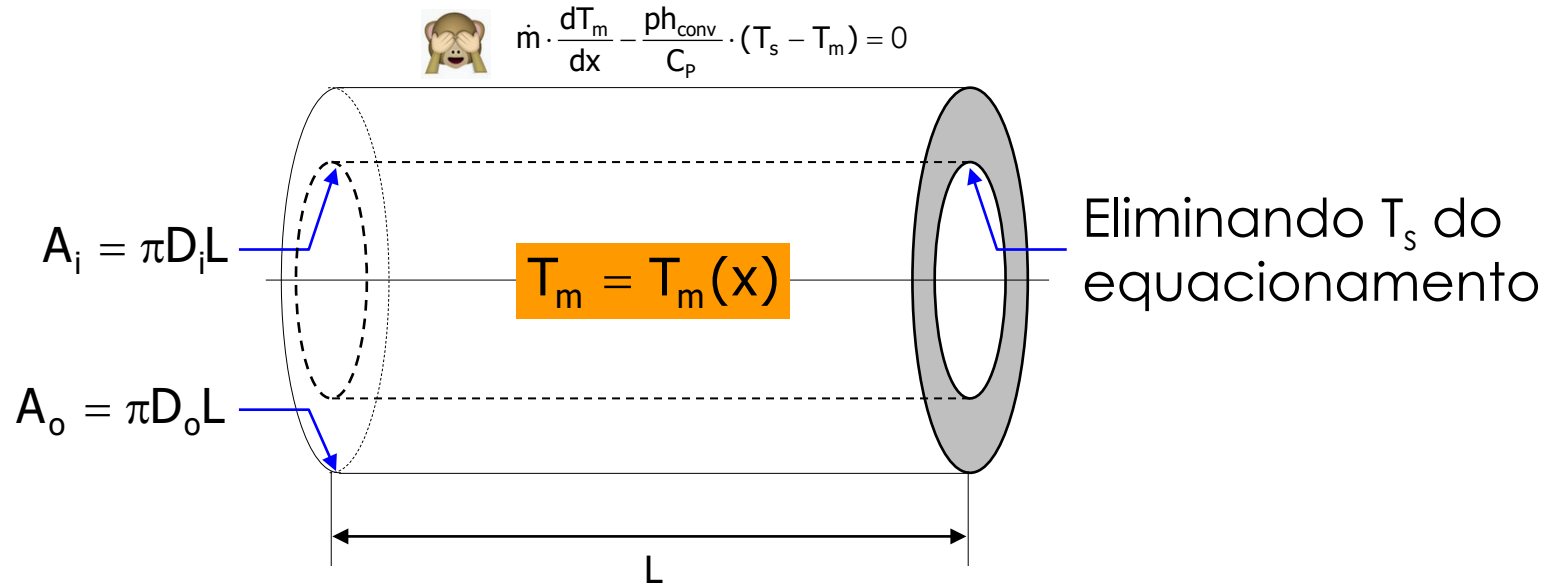
coeficiente global de transferência de calor (condutância)



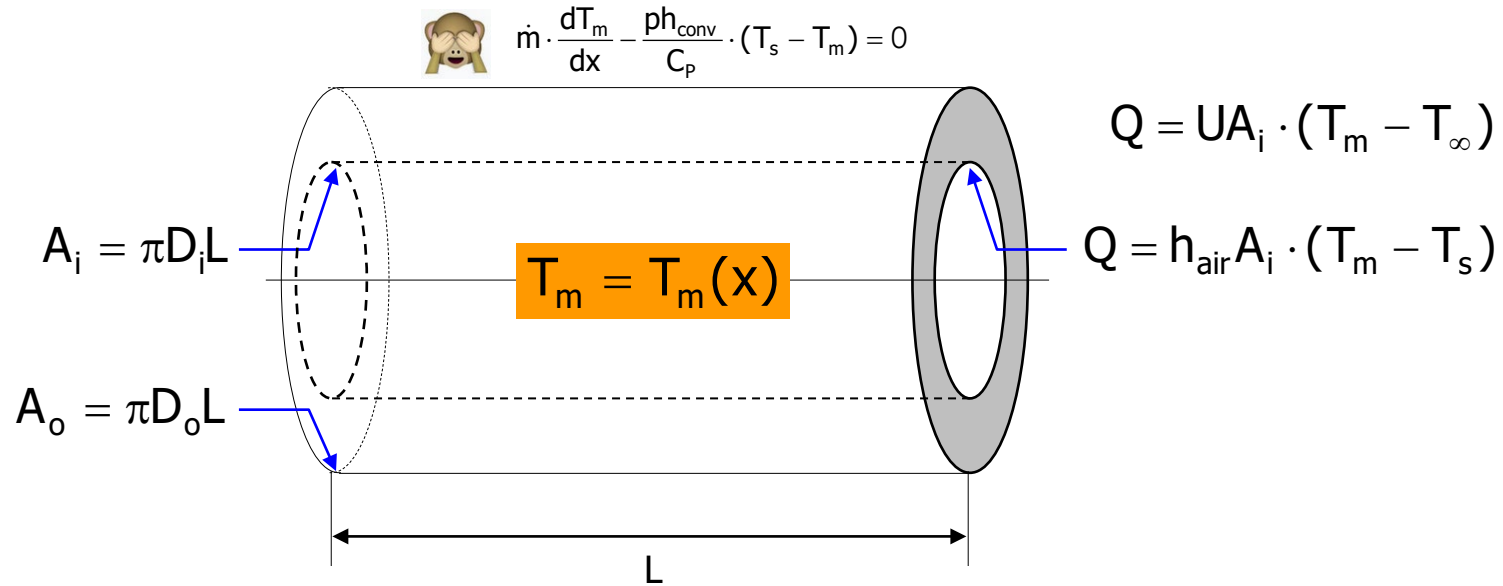
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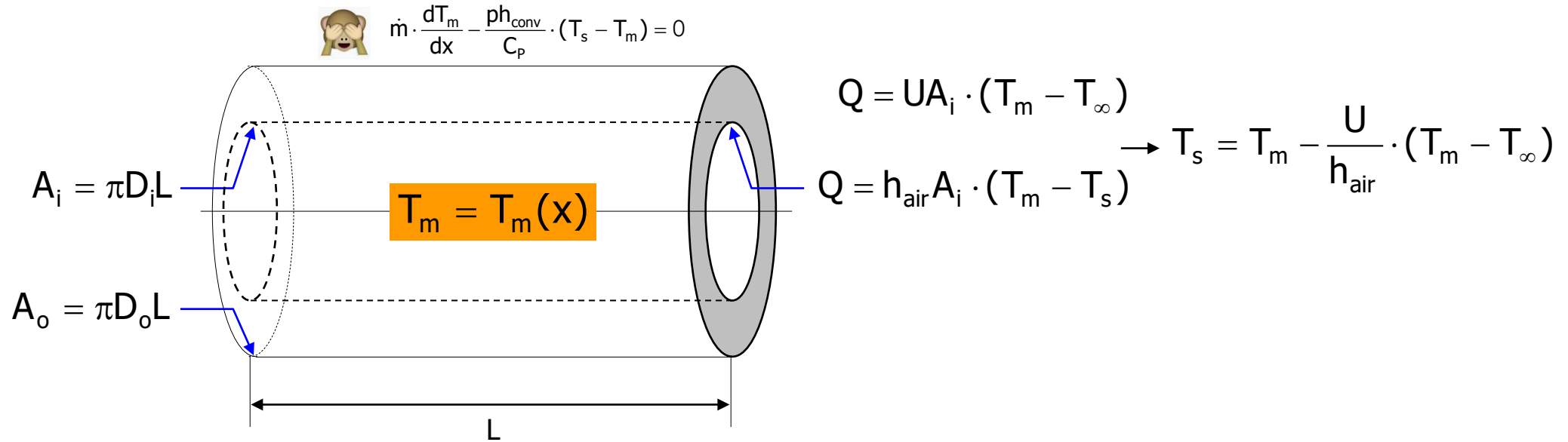
# Geometria cilíndrica... **variação axial da temperatura**



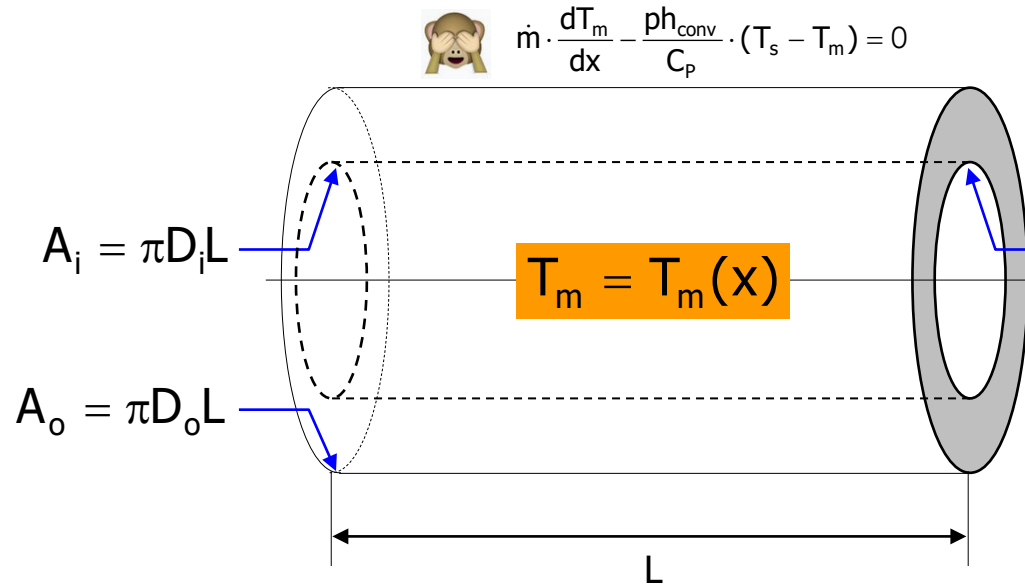
# Geometria cilíndrica... variação axial da temperatura



# Geometria cilíndrica... variação axial da temperatura



# Geometria cilíndrica... variação axial da temperatura



$$Q = UA_i \cdot (T_m - T_\infty)$$

$$Q = h_{air} A_i \cdot (T_m - T_s)$$

$$\rightarrow T_s = T_m - \frac{U}{h_{air}} \cdot (T_m - T_\infty)$$

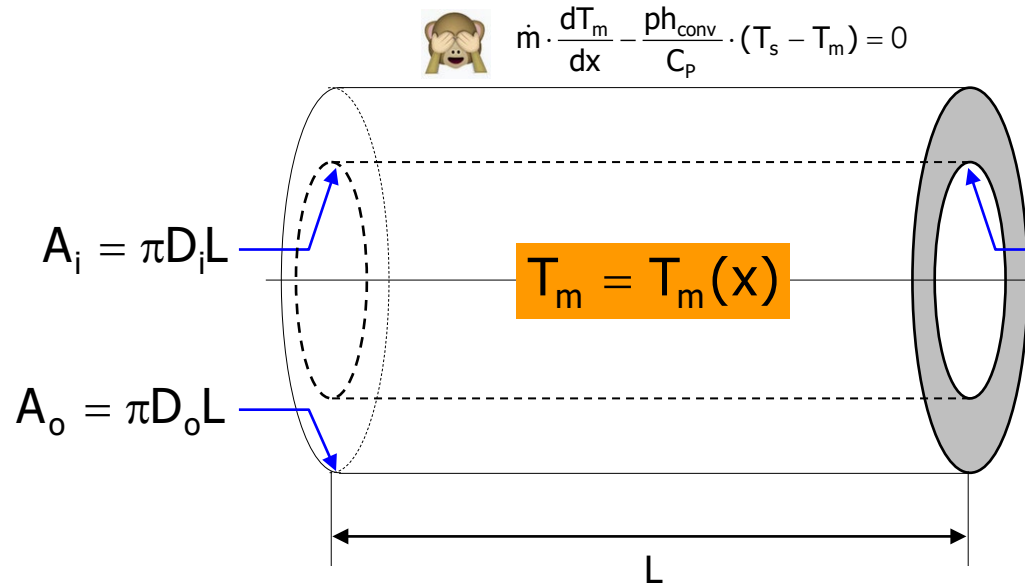
equação do balanço global de energia

$$\dot{m}C_p \cdot \frac{dT_m}{dx} - ph_{air} \cdot (T_s - T_m) = 0$$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

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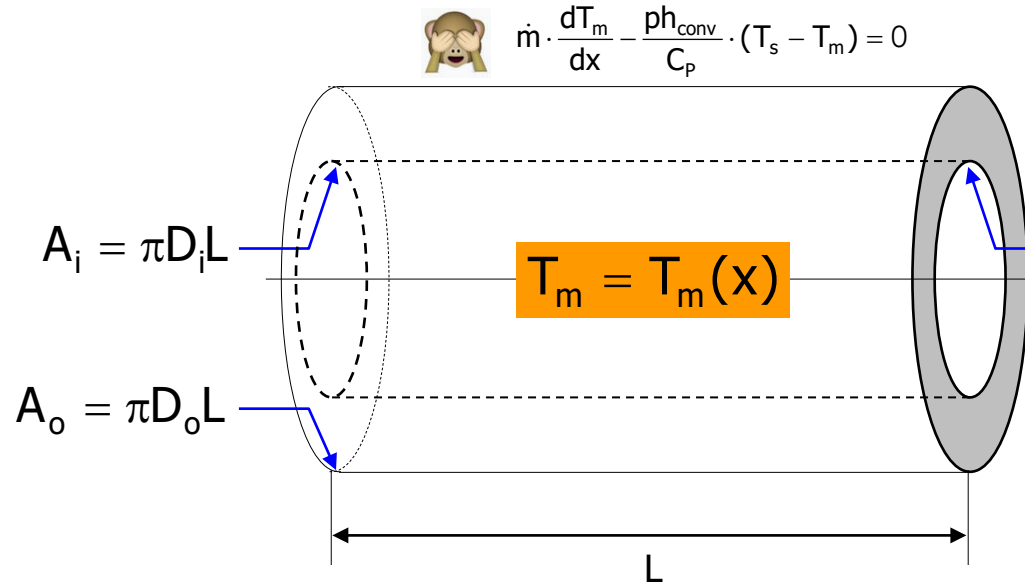
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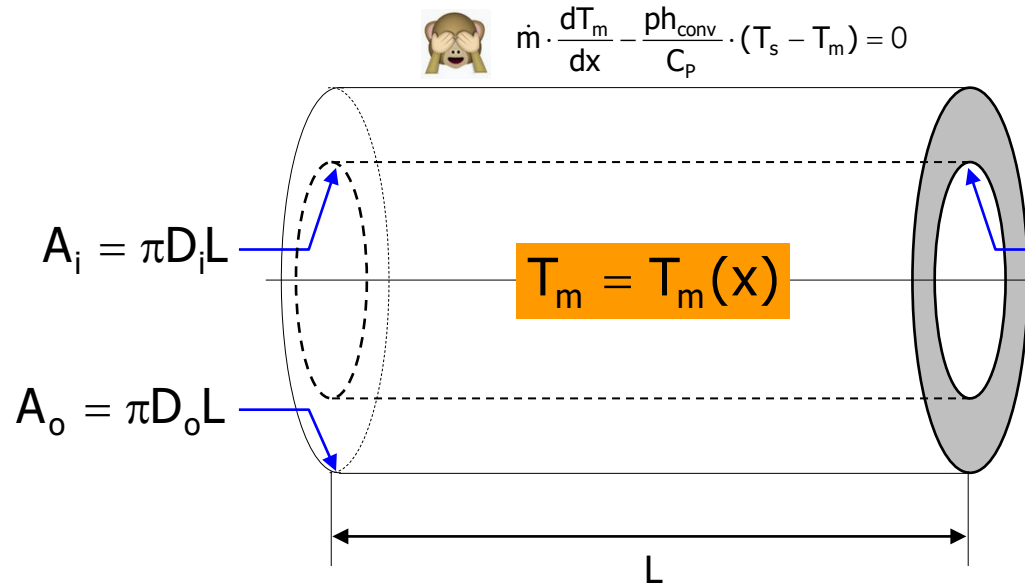
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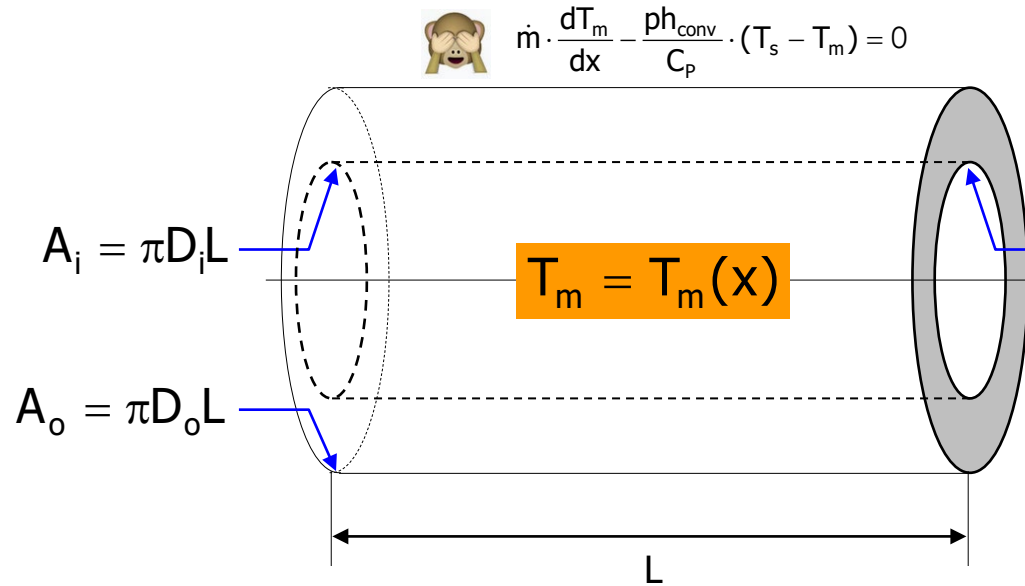


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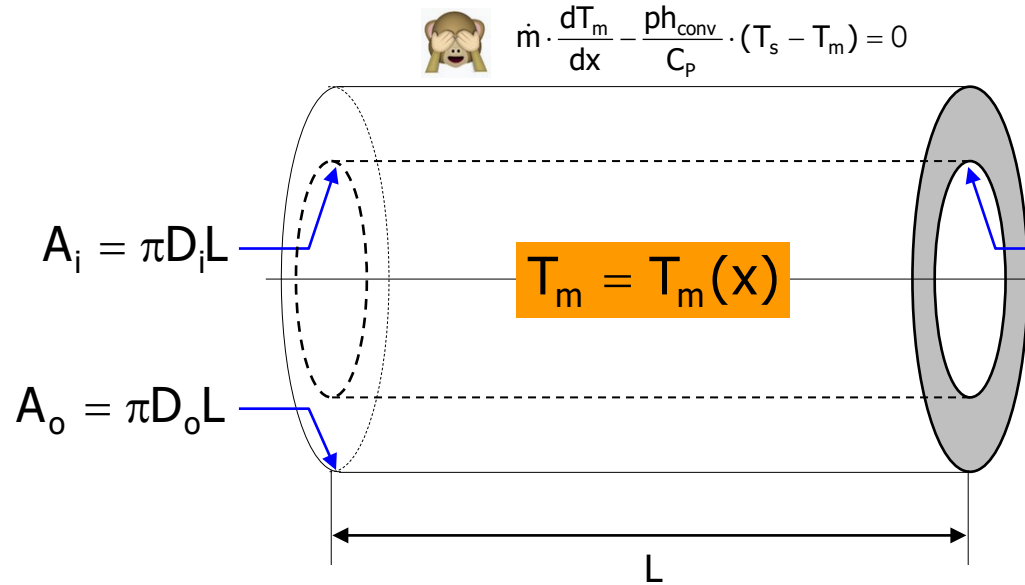


$$\int_{T_{m,e}}^{T_{m,s}} \frac{d(T_m - T_\infty)}{T_m - T_\infty} = -\frac{pL}{\dot{m}C_p} \cdot \frac{1}{L} \int_0^L U dx$$



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$$\frac{T_{m,s} - T_\infty}{T_{m,e} - T_\infty} = \exp\left(-\frac{\bar{U}A}{\dot{m}C_p}\right)$$

... calculado @ temperatura axial média

$$\frac{T_{m,s} - T_{\infty}}{T_{m,e} - T_{\infty}} = \exp\left(-\frac{\bar{U}A}{\dot{m}C_p}\right) \rightarrow$$

$$(\bar{U}A_i)^{-1} = \frac{1}{2\pi r_i L h_i} + \frac{\ln(r_o / r_i)}{2\pi L k} + \frac{1}{2\pi r_o L h_o}$$

Dittus-Boelter ?

=cte

=cte



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Escoamento turbulento desenvolvido ( $Re > 10^4$ ):

Superfície lisa:

$$Nu = 0.023 \cdot Re^{4/5} \cdot Pr^n$$

(Dittus-Boelter)

$$n = 0.3 \quad p / T_s < T_m$$

$$n = 0.4 \quad p / T_s > T_m$$

$$\left[ \begin{array}{l} 0.7 \leq Pr \leq 160 \\ Re_D \geq 10,000 \\ \frac{L}{D} \geq 10 \end{array} \right]$$



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$$Re = \frac{\rho U D}{\mu} = \frac{\rho D}{\mu} \frac{V_i}{\pi D_i^2 / 4} = \frac{1.1688 \cdot 0.15}{18.448 \cdot 10^{-6}} \cdot \frac{0.025}{\pi \cdot 0.15^2 / 4} = 1.344 \cdot 10^4$$



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$$Nu = 0.023 \cdot Re^{4/5} \cdot Pr^{0.3} = 0.023 \cdot (1.344 \cdot 10^4)^{4/5} \cdot (0.70729)^{0.4} = 40.206$$



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$$Nu = \frac{h_i D_i}{k} \rightarrow h_i = \frac{26.247 \cdot 10^{-3} \cdot 40.206}{0.15} = 7.035 \frac{\text{W}}{\text{m}^2 \text{K}}$$

$$\frac{T_{m,s} - T_{\infty}}{T_{m,e} - T_{\infty}} = \exp\left(-\frac{\bar{U}A}{\dot{m}C_p}\right) \rightarrow$$

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$$7.035 \frac{W}{m^2 K}$$

$$0.15 \frac{W}{mK}$$

$$1500 \frac{W}{m^2 K}$$



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$$\frac{21-17}{29-17} = \exp\left(-\frac{2.295 \cdot L}{0.02883 \cdot 1006.3}\right)$$



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$$L = 13.88 \text{ m}$$



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Considerando propriedades termofísicas variáveis...



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$$Re = \frac{\rho u_0 D}{\mu}$$

$$Pr = \frac{C_p \mu}{k}$$

# Balço global de energia...

$$\dot{m} \cdot \frac{dh_m}{dx} - ph_{conv} \cdot (T_s - T_m) = 0$$

Soluço direta via mto numrico de soluço de equaço diferencial

$$h = h(P, T) \downarrow$$

Implementaço numrica...

Soluço analtica a partir de hipoteses simplificadoras

$$h = C_p \cdot T \downarrow \begin{array}{l} \text{gases perfeitos} \\ \text{"fluido incompressvel"} (C_p=C_v) \end{array}$$

$$\dot{m} \cdot \frac{dT_m}{dx} - \frac{ph_{conv}}{C_p} \cdot (T_s - T_m) = 0$$



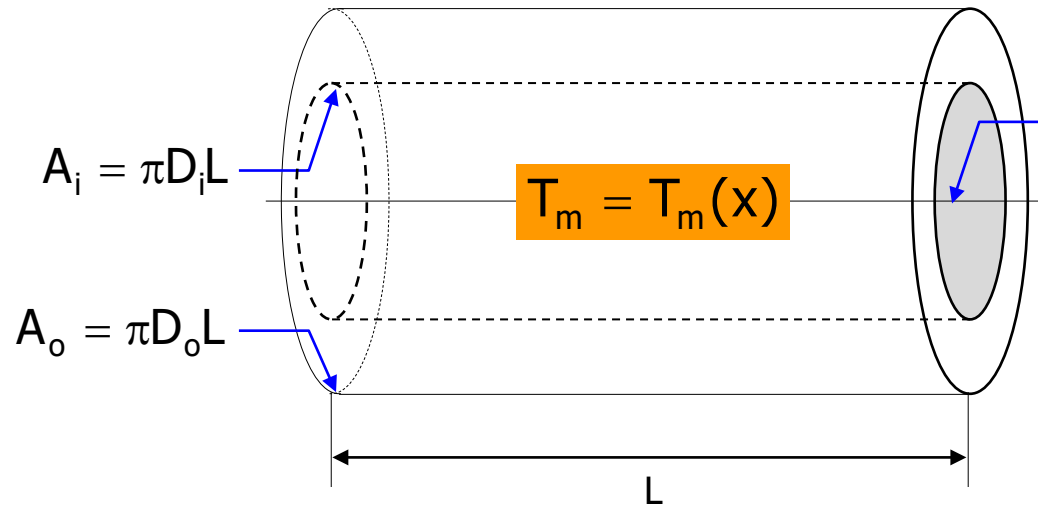
$$\mu = \mu(P, T), \dots \downarrow$$

Implementaço numrica...





# Geometria cilíndrica... variação axial da temperatura



$$\dot{m}C_p \cdot \frac{dT_m}{dx} + pU \cdot (T_m - T_\infty) = 0$$

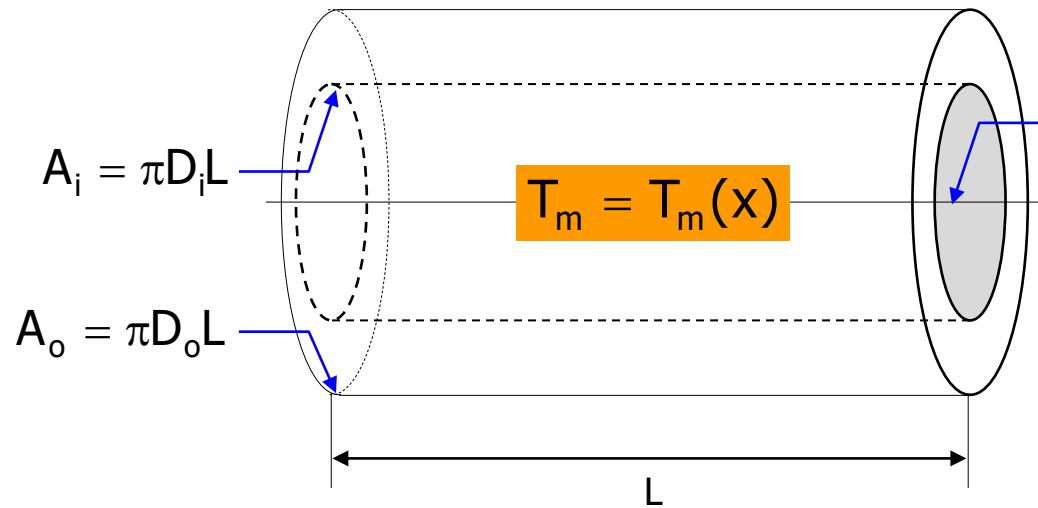


Eliminada  $T_s$  da eq.



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# Geometria cilíndrica... variação axial da temperatura

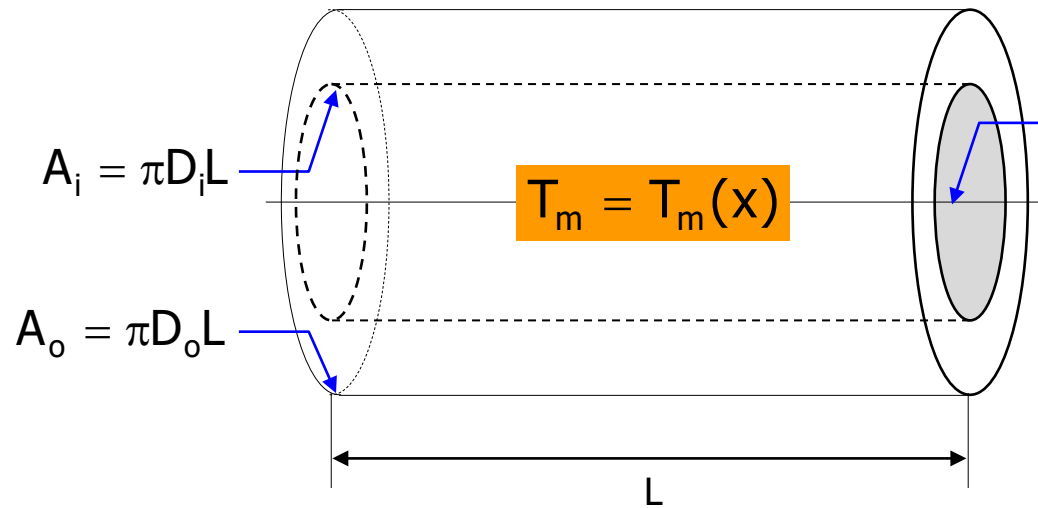


$$\dot{m}C_p \cdot \frac{dT_m}{dx} + pU \cdot (T_m - T_\infty) = 0$$

$$U = \left[ \frac{1}{h_i} + \frac{r_i}{k} \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o} \cdot \frac{r_i}{r_o} \right]^{-1}$$



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Dittus-Boelter

=cte

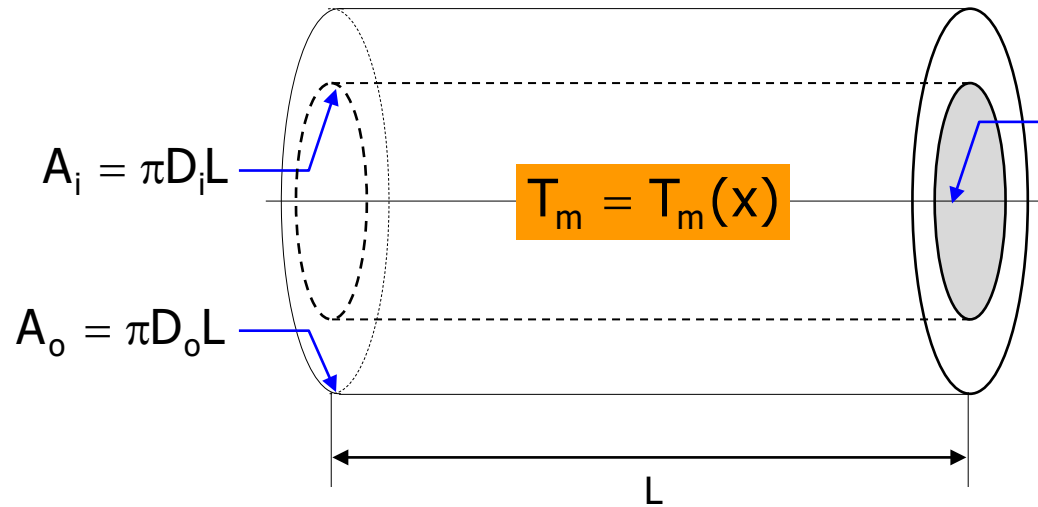
=cte

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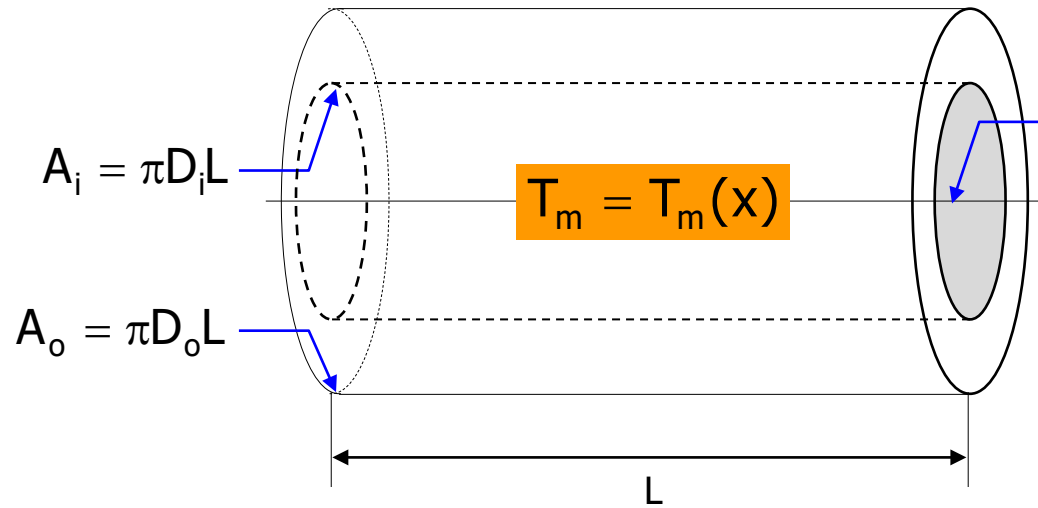
Dittus-Boelter  $\frac{r_i}{k} \ln\left(\frac{r_o}{r_i}\right) = \text{cte}$   $\frac{1}{h_o} \cdot \frac{r_i}{r_o} = \text{cte}$

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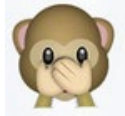
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$$\dot{m} C_p \cdot \frac{dT_m}{dx} + pU \cdot (T_m - T_\infty) = 0$$

$$\frac{dT_m}{dx} + \alpha(T_m) \cdot (T_m - T_\infty) = 0 \quad \alpha(T_m) = \frac{\rho}{\dot{m}} \frac{U(T_m)}{C_p(T_m)}$$


$$\frac{T_{m,j+1} - T_{m,j}}{dx} + \alpha_j \cdot (T_{m,j} - T_\infty) = 0$$

$$T_{m,j+1} = T_{m,j} - \alpha_j \cdot (T_{m,j} - T_\infty) \cdot dx$$


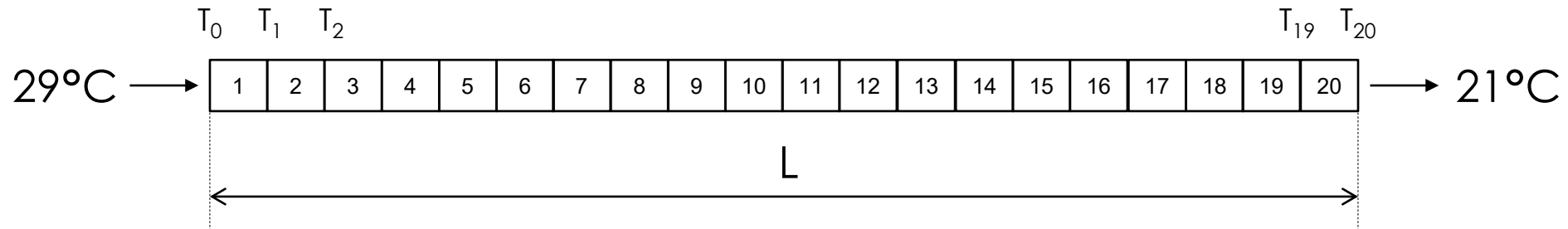
$$\alpha(T_m) = \frac{\rho}{\dot{m}} \frac{U(T_m)}{C_p(T_m)}$$

$$T_{m,j+1} = T_{m,j} - \alpha_j \cdot (T_{m,j} - T_\infty) \cdot dx$$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

**8.31:** Para resfriar uma casa de verão sem uso de um ciclo frigorífico, ar é encaminhado através de uma tubulação de plástico ( $k=0.15\text{W/m/K}$ ,  $D_i=0.15\text{m}$ ,  $D_o=0.17\text{m}$ ) submersa em um corpo d'água adjacente. A temperatura da água é normalmente de  $T_\infty=17^\circ\text{C}$ , e o coeficiente de convecção é mantido em  $h_o=1500\text{ W/m}^2/\text{K}$  na superfície externa da tubulação. Se ar proveniente da casa entra no tubo a uma temperatura de  $T_{m,i}=29^\circ\text{C}$  e uma vazão volumétrica de  $V_i=0.025\text{m}^3/\text{s}$ , qual extensão  $L$  é necessária para que a temperatura na saída seja de  $T_{m,o}=21^\circ\text{C}$ ?



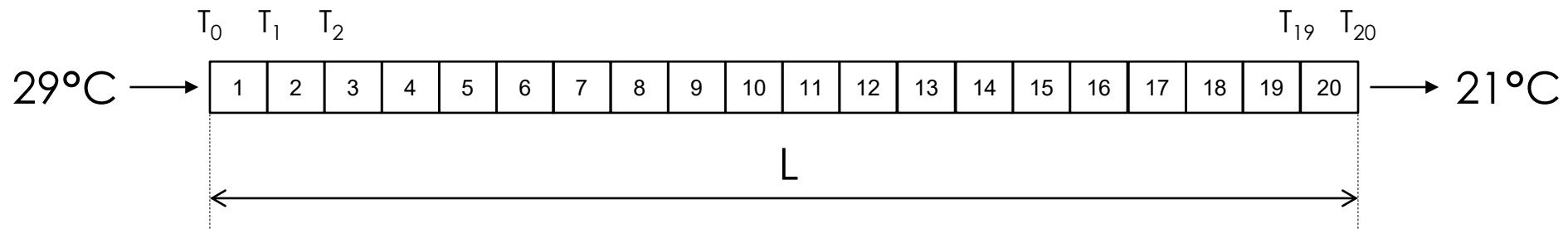
$$T_{m,j+1} = T_{m,j} - \alpha_j \cdot (T_{m,j} - T_\infty) \cdot dx \rightarrow$$



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$$T_{m,j+1} = T_{m,j} - \alpha_j \cdot (T_{m,j} - T_\infty) \cdot dx \rightarrow \begin{cases} T_{m,1} = T_{m,0} - \alpha_0 \cdot (T_{m,0} - T_\infty) \cdot dx \\ \rightarrow T_{m,2} = T_{m,1} - \alpha_1 \cdot (T_{m,1} - T_\infty) \cdot dx \\ \rightarrow T_{m,3} = T_{m,2} - \alpha_2 \cdot (T_{m,2} - T_\infty) \cdot dx \\ \vdots \\ \rightarrow T_{m,20} = T_{m,19} - \alpha_{19} \cdot (T_{m,19} - T_\infty) \cdot dx \end{cases}$$



As questões postadas no Chat do YouTube serão respondidas ao final da aula.

Salvamento Automático | exemplo aula TC3.xlsm - Excel | Entrar | Compartilhar

Arquivo | Página Inicial | Inserir | Layout da Página | Fórmulas | Dados | Revisão | Exibir | Diga-me o que você deseja fazer

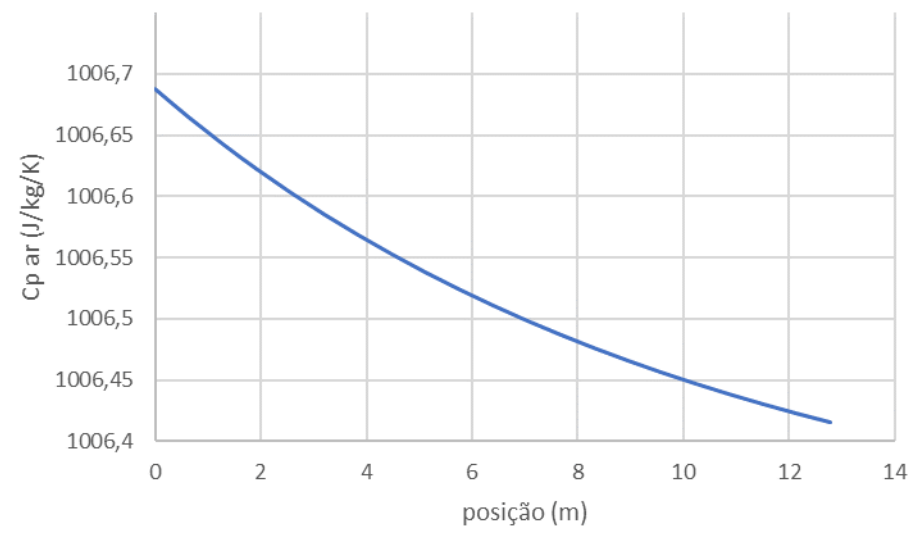
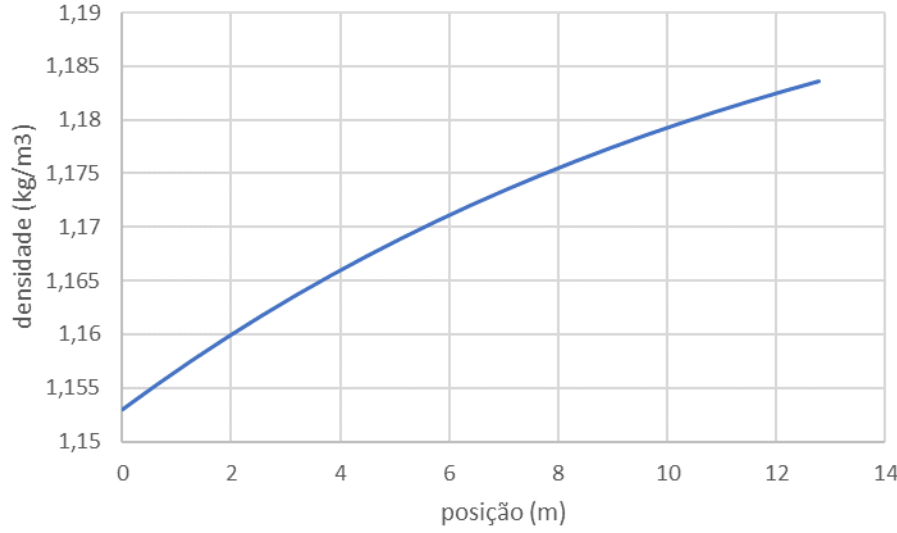
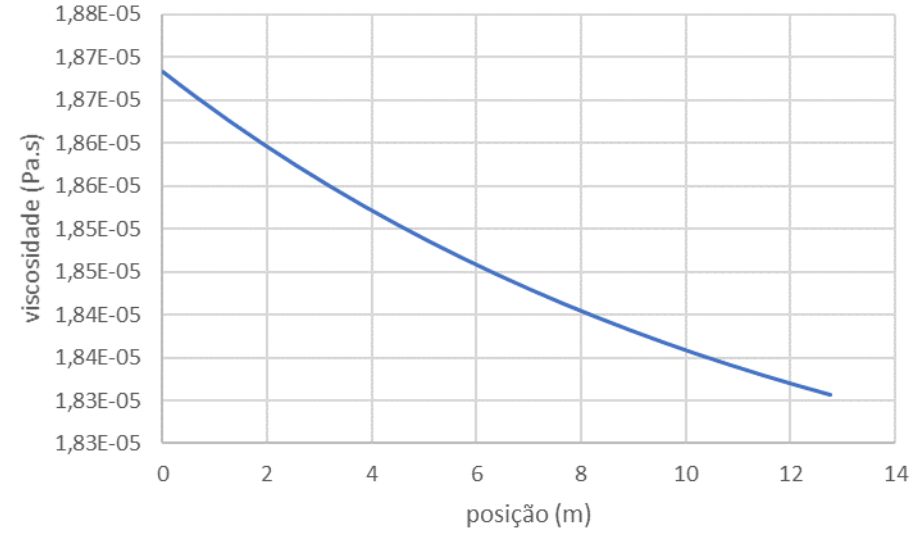
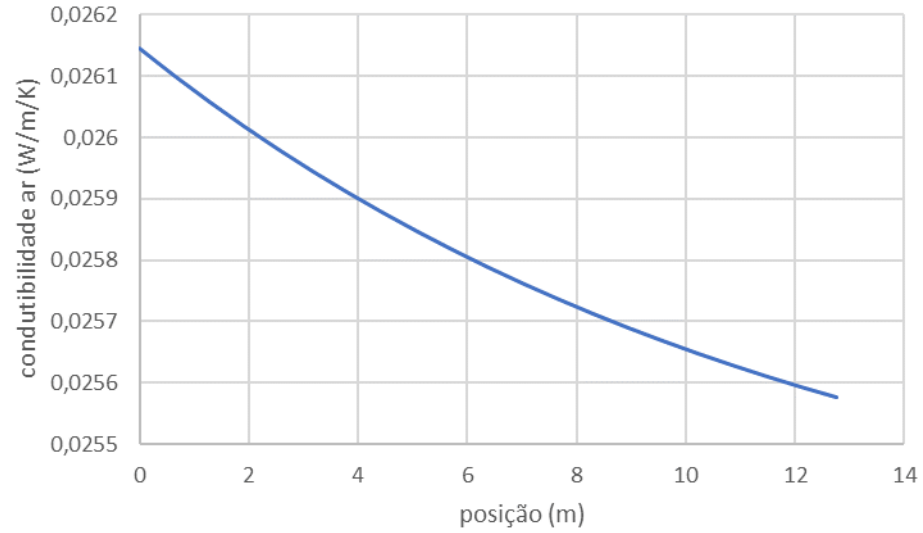
Recortar | Copiar | Pincel de Formatação | Área de Transferência

Calibri 11 | Quebrar Texto Automaticamente | Geral | Formatação Condicional | Formatar como Tabela

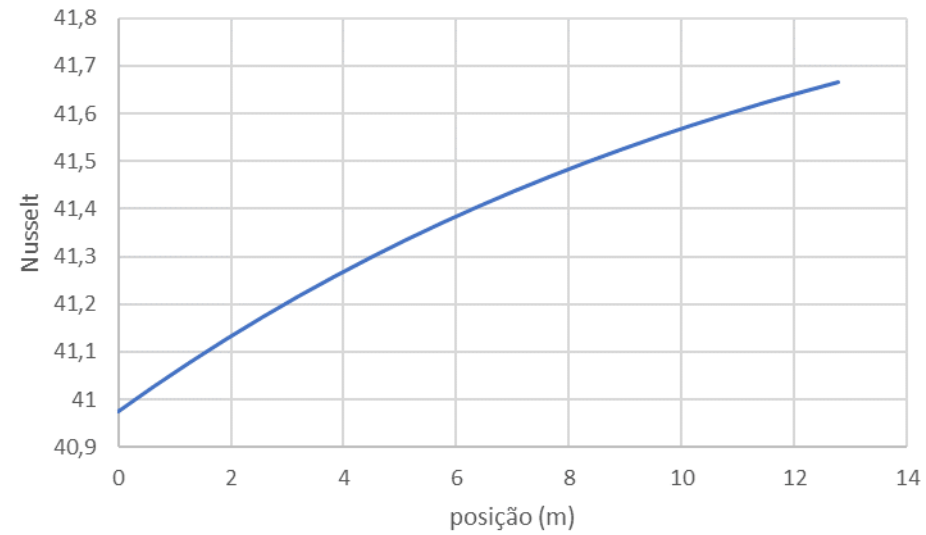
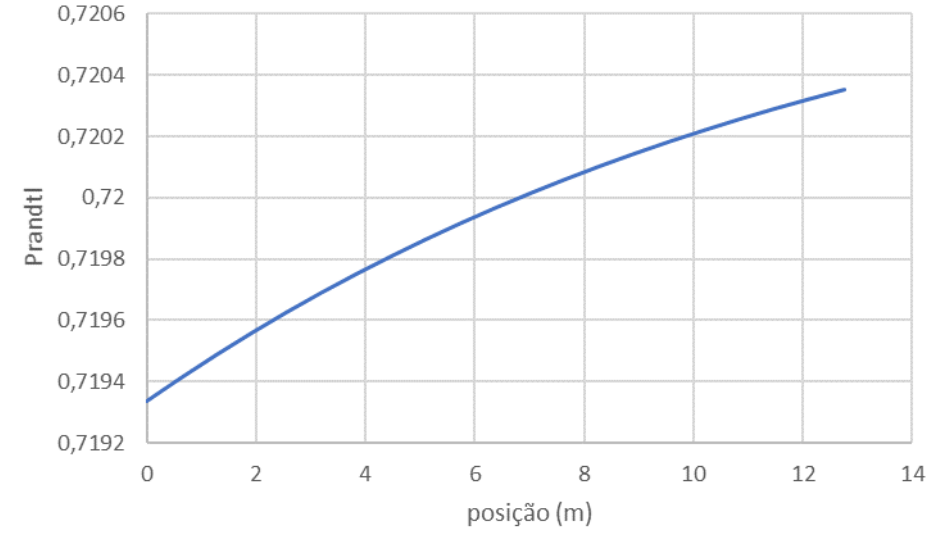
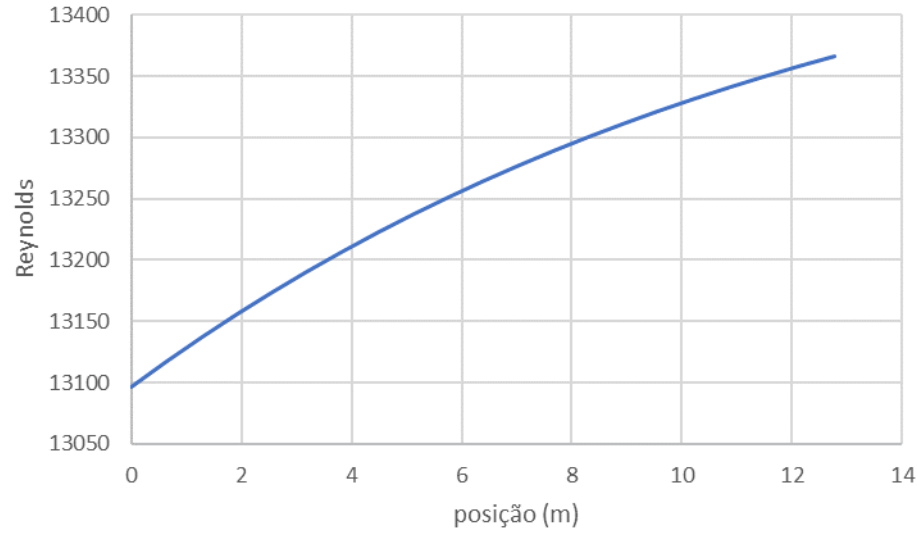
Normal | Bom | Neutro | Ruim | Cálculo | Célula de Ve... | Célula Vincul... | Entrada

Inserir | Excluir | Formatar | AutoSoma | Preencher | Limpar | Classificar e Filtrar | Localizar e Selecionar | Edição

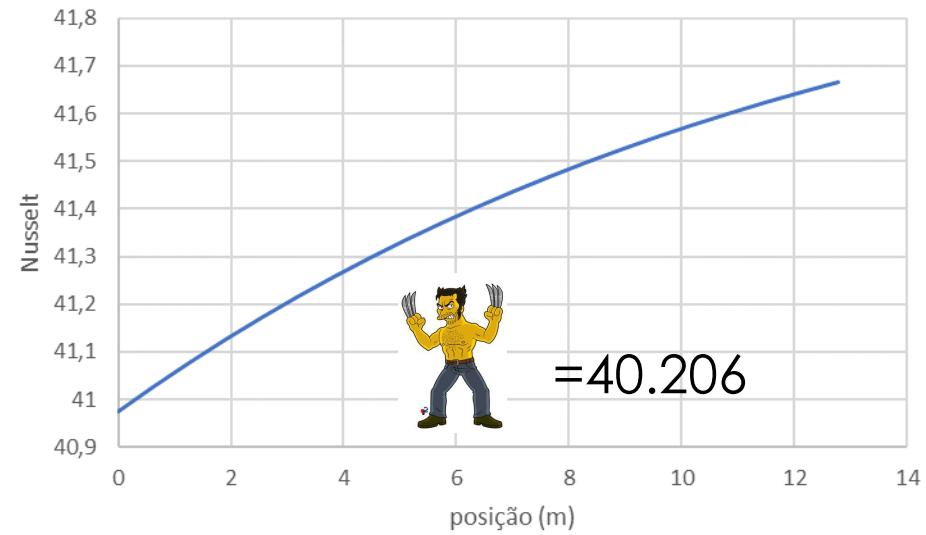
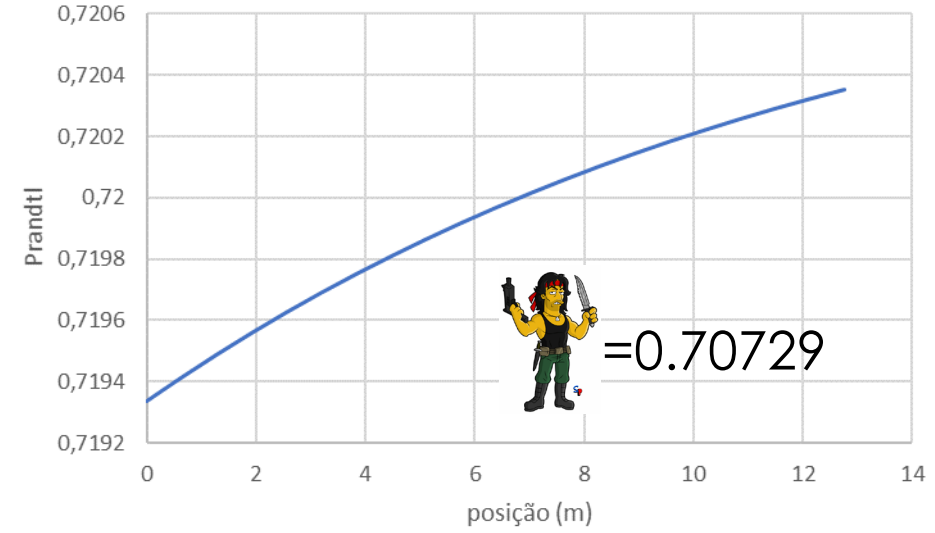
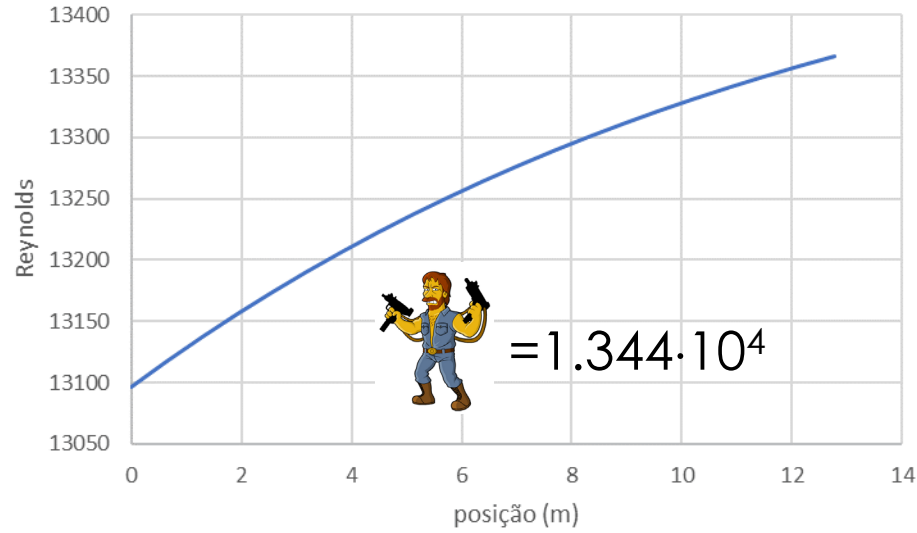
|    | A       | B            | C | D | E       | F     | G       | H       | I       | J            | K       | L       | M       | N       | O       | P       | Q       | R           | S       | T |
|----|---------|--------------|---|---|---------|-------|---------|---------|---------|--------------|---------|---------|---------|---------|---------|---------|---------|-------------|---------|---|
| 1  | P       | 1 bar        |   |   | posição | x     | Tj      | $\rho$  | U (m/s) | $\mu$ (Pa.s) | kar     | Cp      | Re      | Pr      | Nu      | hi      | U       | $\alpha(T)$ | Tj+1    |   |
| 2  | ho      | 1500 W/m2/K  |   |   | 0       | 0     | 29      | 1,15301 | 1,41471 | 1,8683E-05   | 0,02615 | 1006,69 | 13096,2 | 0,71934 | 40,9754 | 7,14239 | 4,92177 | 0,07993     | 28,3574 |   |
| 3  | k       | 0,15 W/m/K   |   |   | 1       | 0,67  | 28,3574 | 1,15548 | 1,41169 | 1,8652E-05   | 0,0261  | 1006,66 | 13117,9 | 0,71942 | 41,0312 | 7,13928 | 4,92029 | 0,0799      | 27,7494 |   |
| 4  | Di      | 0,15 m       |   |   | 2       | 1,34  | 27,7494 | 1,15782 | 1,40884 | 1,8623E-05   | 0,02606 | 1006,64 | 13138,6 | 0,71949 | 41,0842 | 7,13634 | 4,9189  | 0,07988     | 27,174  |   |
| 5  | Do      | 0,17 m       |   |   | 3       | 2,01  | 27,174  | 1,16004 | 1,40614 | 1,8595E-05   | 0,02601 | 1006,62 | 13158,2 | 0,71957 | 41,1345 | 7,13355 | 4,91757 | 0,07986     | 26,6296 |   |
| 6  | Tinf    | 17 oC        |   |   | 4       | 2,68  | 26,6296 | 1,16215 | 1,40358 | 1,8569E-05   | 0,02597 | 1006,6  | 13176,8 | 0,71964 | 41,1823 | 7,13091 | 4,91631 | 0,07985     | 26,1145 |   |
| 7  | Tmi     | 29 oC        |   |   | 5       | 3,35  | 26,1145 | 1,16416 | 1,40117 | 1,8544E-05   | 0,02594 | 1006,58 | 13194,5 | 0,7197  | 41,2276 | 7,1284  | 4,91512 | 0,07983     | 25,627  |   |
| 8  | Vz @ i  | 0,025 m3/s   |   |   | 6       | 4,02  | 25,627  | 1,16606 | 1,39888 | 1,852E-05    | 0,0259  | 1006,56 | 13211,3 | 0,71977 | 41,2707 | 7,12603 | 4,91399 | 0,07981     | 25,1657 |   |
| 9  | VzM @ i | 0,02883 kg/s |   |   | 7       | 4,69  | 25,1657 | 1,16787 | 1,39671 | 1,8498E-05   | 0,02587 | 1006,55 | 13227,2 | 0,71983 | 41,3116 | 7,12378 | 4,91292 | 0,07979     | 24,7291 |   |
| 10 |         |              |   |   | 8       | 5,36  | 24,7291 | 1,16959 | 1,39466 | 1,8477E-05   | 0,02583 | 1006,53 | 13242,4 | 0,71989 | 41,3504 | 7,12164 | 4,91191 | 0,07978     | 24,316  |   |
| 11 |         |              |   |   | 9       | 6,03  | 24,316  | 1,17122 | 1,39272 | 1,8457E-05   | 0,0258  | 1006,52 | 13256,7 | 0,71994 | 41,3872 | 7,11962 | 4,91095 | 0,07976     | 23,925  |   |
| 12 | Tmo     | 21 oC        |   |   | 10      | 6,7   | 23,925  | 1,17276 | 1,39089 | 1,8438E-05   | 0,02578 | 1006,5  | 13270,4 | 0,71999 | 41,4221 | 7,11771 | 4,91004 | 0,07975     | 23,555  |   |
| 13 | L       | 13,4 m       |   |   | 11      | 7,37  | 23,555  | 1,17423 | 1,38915 | 1,842E-05    | 0,02575 | 1006,49 | 13283,3 | 0,72004 | 41,4553 | 7,1159  | 4,90917 | 0,07974     | 23,2048 |   |
| 14 | N       | 20 nd        |   |   | 12      | 8,04  | 23,2048 | 1,17562 | 1,38751 | 1,8403E-05   | 0,02572 | 1006,48 | 13295,6 | 0,72009 | 41,4867 | 7,11418 | 4,90836 | 0,07973     | 22,8733 |   |
| 15 | dx      | 0,67 m       |   |   | 13      | 8,71  | 22,8733 | 1,17694 | 1,38595 | 1,8387E-05   | 0,0257  | 1006,47 | 13307,2 | 0,72013 | 41,5165 | 7,11255 | 4,90758 | 0,07971     | 22,5597 |   |
| 16 |         |              |   |   | 14      | 9,38  | 22,5597 | 1,17819 | 1,38448 | 1,8372E-05   | 0,02567 | 1006,46 | 13318,2 | 0,72017 | 41,5448 | 7,11101 | 4,90685 | 0,0797      | 22,2628 |   |
| 17 |         |              |   |   | 15      | 10,05 | 22,2628 | 1,17938 | 1,38308 | 1,8357E-05   | 0,02565 | 1006,45 | 13328,7 | 0,72021 | 41,5716 | 7,10955 | 4,90615 | 0,07969     | 21,9818 |   |
| 18 |         |              |   |   | 16      | 10,72 | 21,9818 | 1,18051 | 1,38176 | 1,8343E-05   | 0,02563 | 1006,44 | 13338,6 | 0,72025 | 41,597  | 7,10817 | 4,9055  | 0,07968     | 21,7158 |   |
| 19 |         |              |   |   | 17      | 11,39 | 21,7158 | 1,18157 | 1,38051 | 1,8331E-05   | 0,02561 | 1006,43 | 13348,1 | 0,72029 | 41,6211 | 7,10686 | 4,90487 | 0,07967     | 21,4641 |   |
| 20 |         |              |   |   | 18      | 12,06 | 21,4641 | 1,18259 | 1,37933 | 1,8318E-05   | 0,02559 | 1006,42 | 13357   | 0,72032 | 41,644  | 7,10562 | 4,90428 | 0,07966     | 21,2258 |   |
| 21 |         |              |   |   | 19      | 12,73 | 21,2258 | 1,18355 | 1,37821 | 1,8307E-05   | 0,02558 | 1006,42 | 13365,4 | 0,72035 | 41,6656 | 7,10444 | 4,90372 | 0,07966     | 21,0003 |   |
| 22 |         |              |   |   | 20      | 13,4  | 21,0003 | 1,18446 | 1,37716 | 1,8296E-05   | 0,02556 | 1006,41 | 13373,5 | 0,72038 | 41,6861 | 7,10333 | 4,90319 | 0,07965     | 20,7868 |   |
| 23 |         |              |   |   |         |       |         |         |         |              |         |         |         |         |         |         |         |             |         |   |
| 24 |         |              |   |   |         |       |         |         |         |              |         |         |         |         |         |         |         |             |         |   |
| 25 |         |              |   |   |         |       |         |         |         |              |         |         |         |         |         |         |         |             |         |   |
| 26 |         |              |   |   |         |       |         |         |         |              |         |         |         |         |         |         |         |             |         |   |



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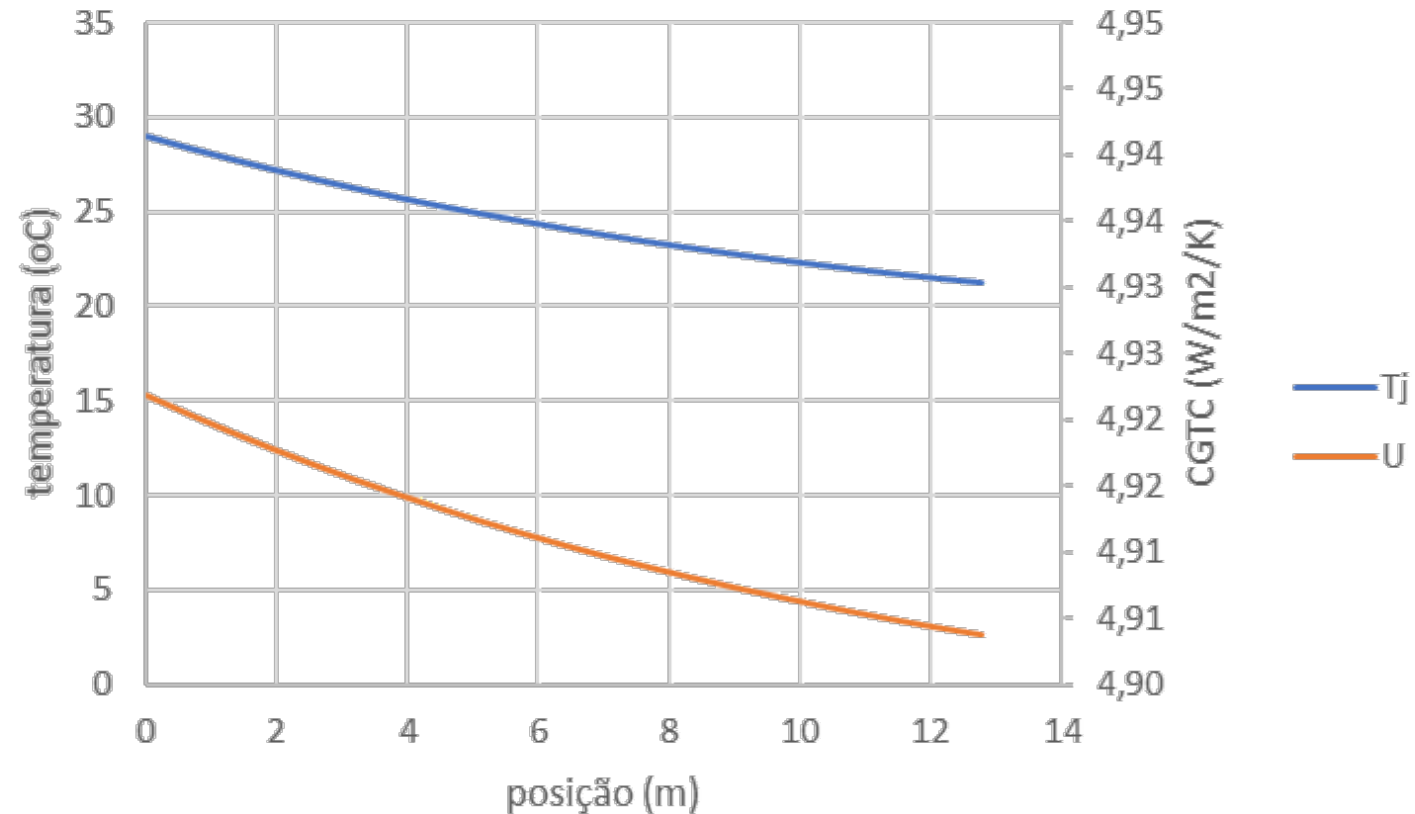


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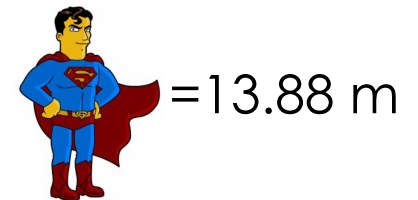
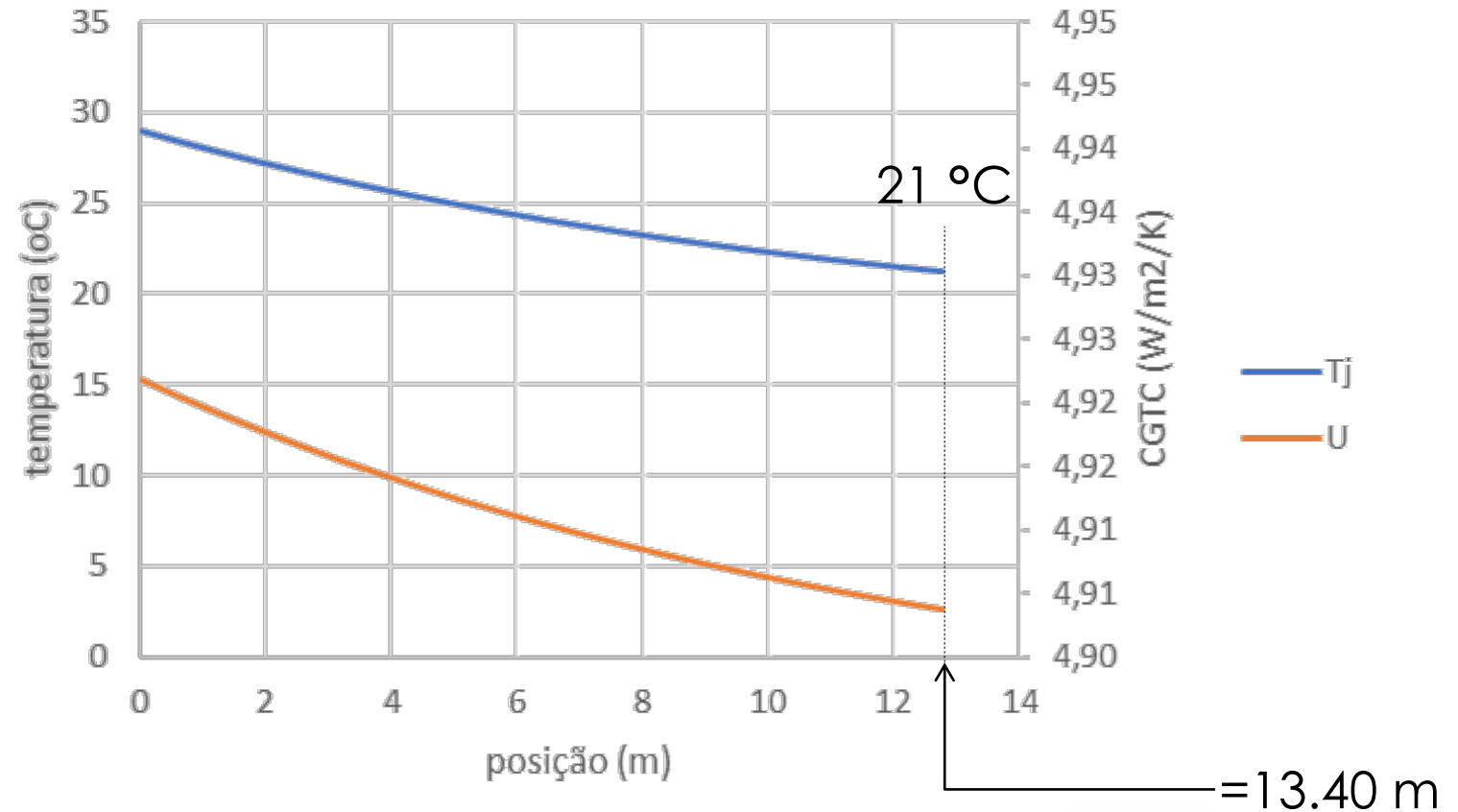
# Tutorial: montagem da planilha de simulação...



As questões postadas no Chat do YouTube serão respondidas ao final da aula.



# Tutorial: montagem da planilha de simulação...



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# Dúvidas ?



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# CONVECÇÃO DE CALOR

