PEF 3200
Quadros isostáticos espaciais

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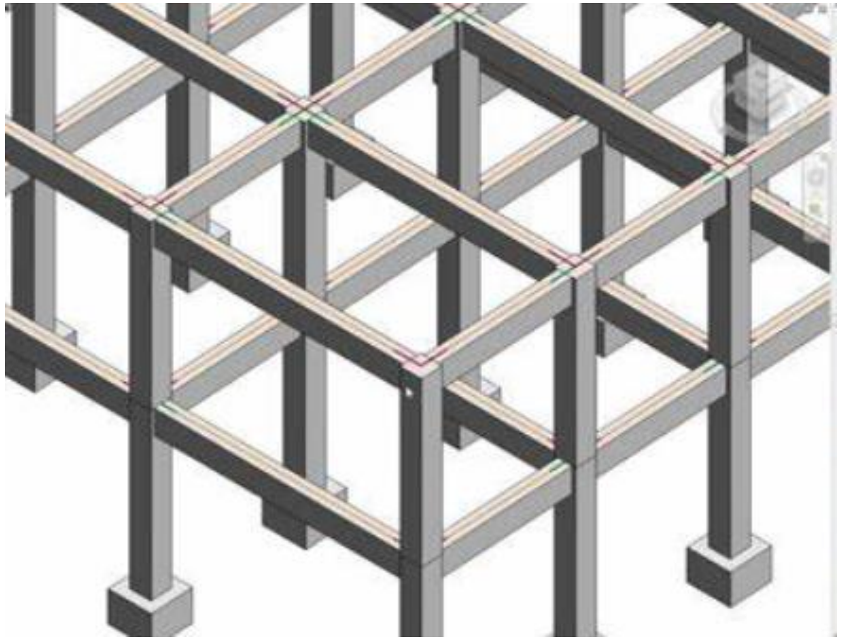
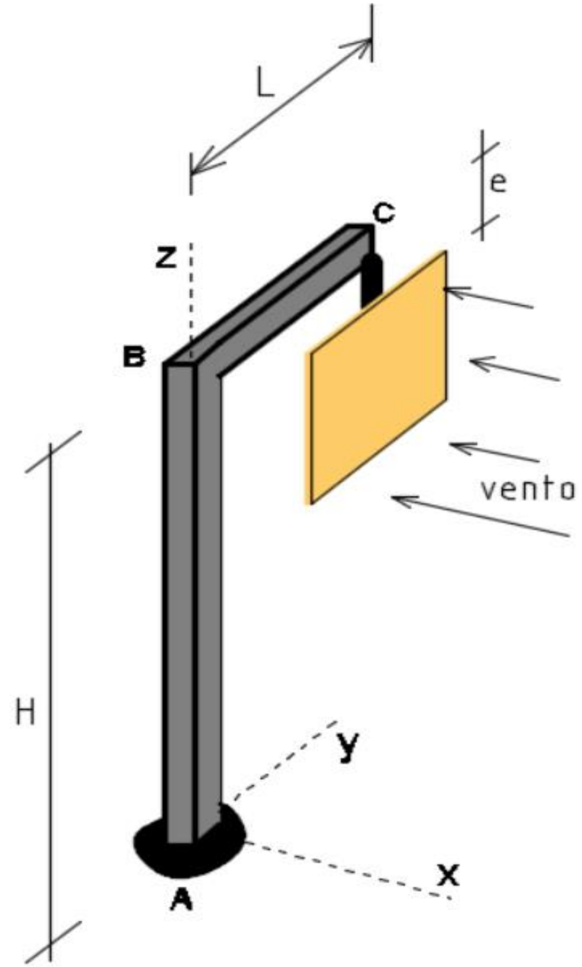
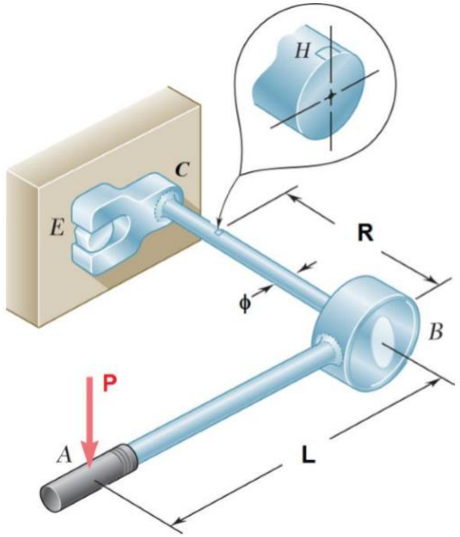
Maio/2023

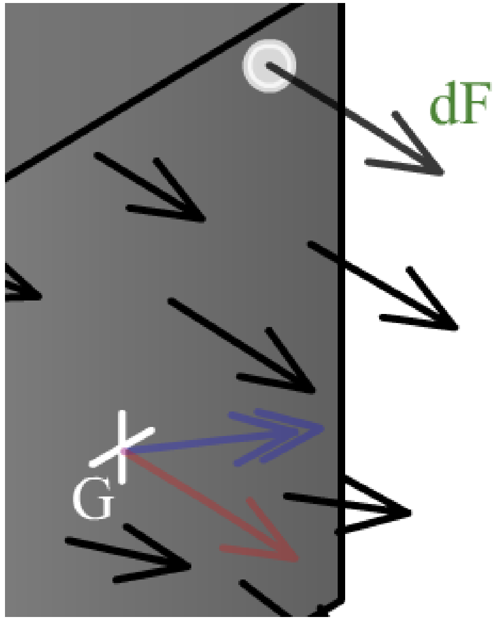
Quadros espaciais





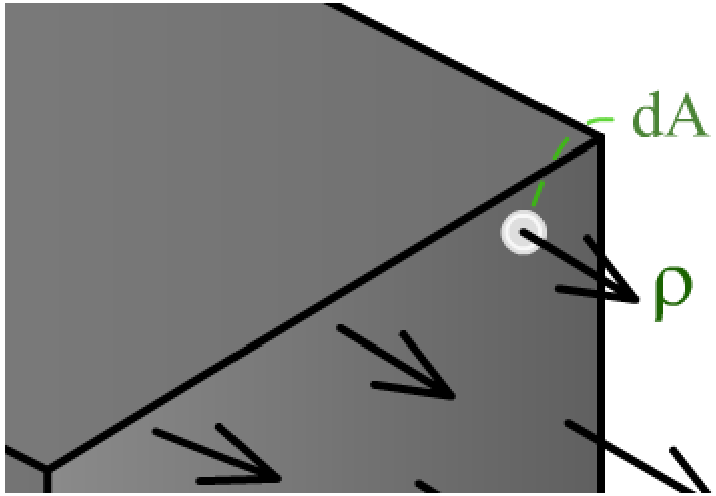


ESFORÇOS SOLICITANTES 3D



Tensão: $\vec{\rho} = \vec{\sigma} + \vec{\tau}$

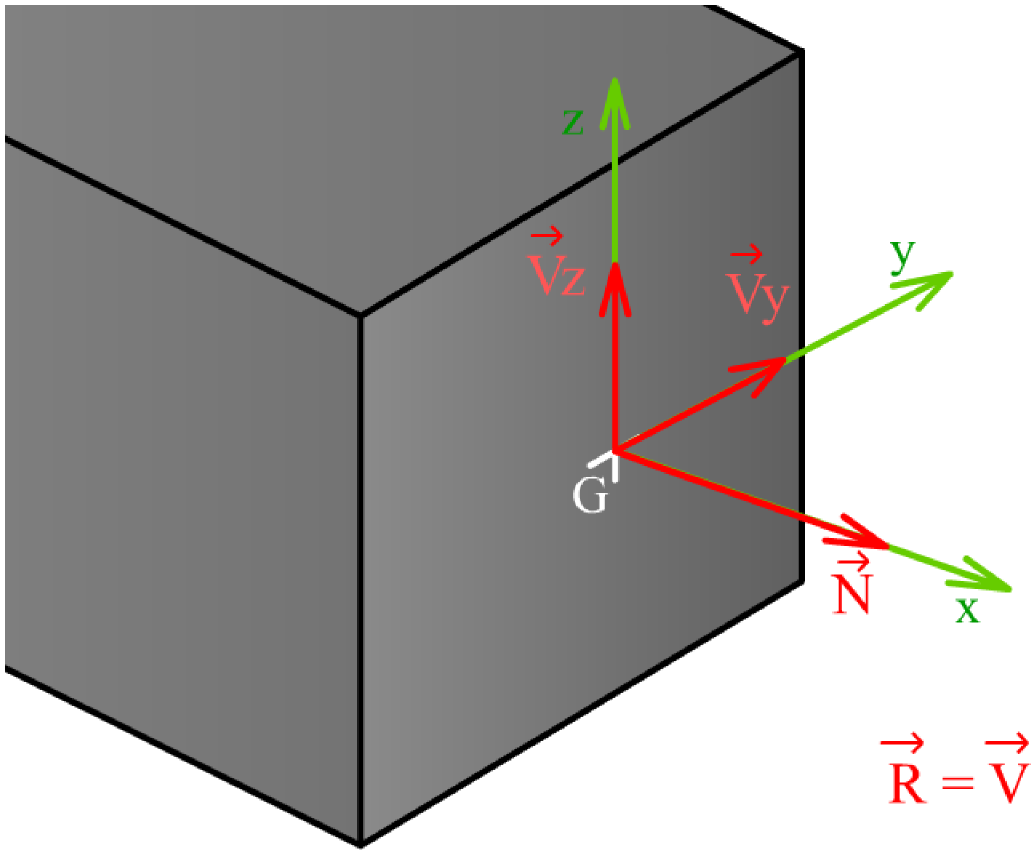
Tensão normal a seção: $\vec{\sigma}$



Tensão paralela a seção: $\vec{\tau}$

$$\vec{\rho}_{média} = \lim_{\Delta A \to 0} \frac{\overrightarrow{\Delta F}}{\Delta A}$$

ESFORÇOS SOLICITANTES 3D



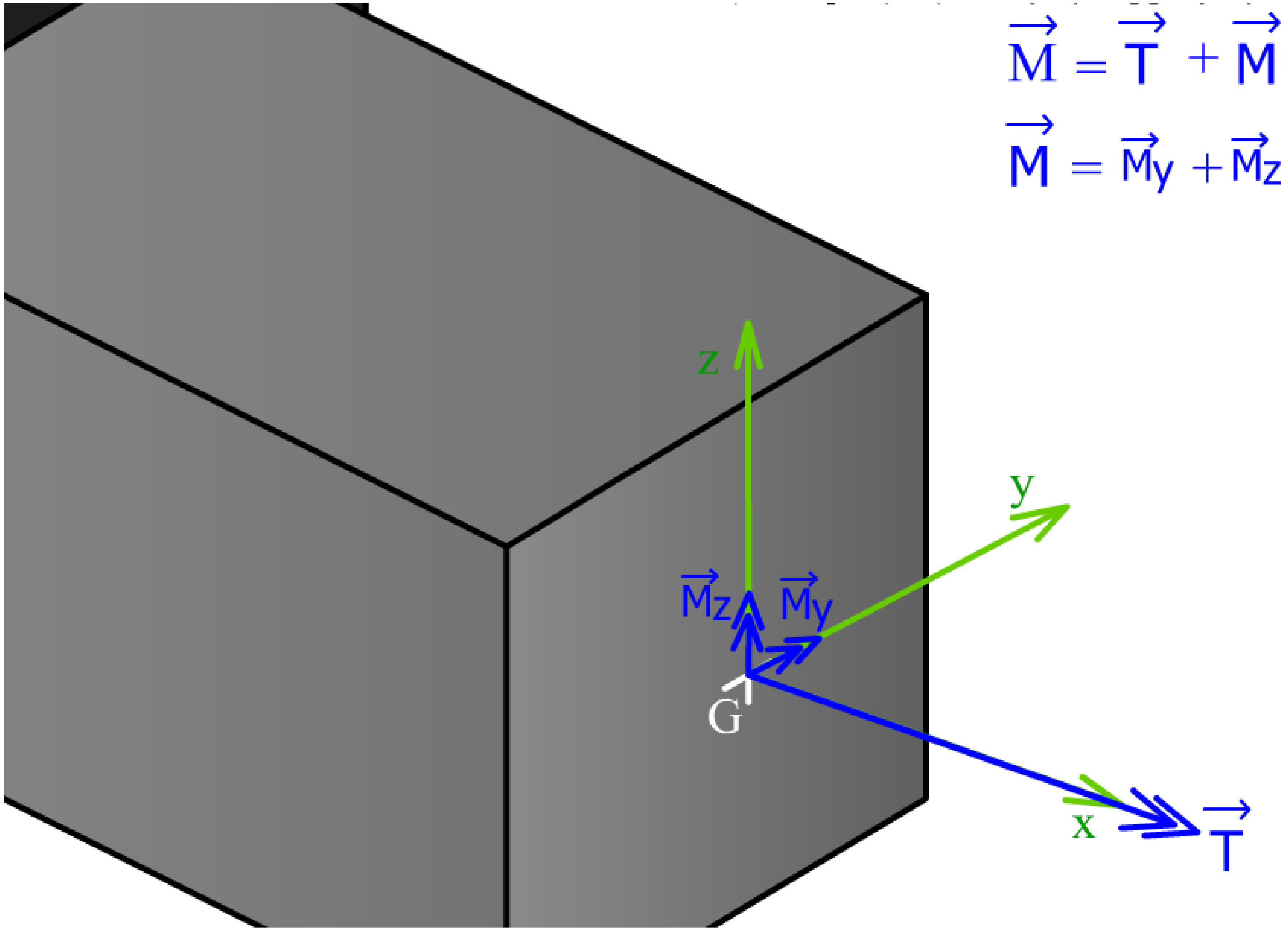
$$\vec{R} = \vec{V} + \vec{N}$$

$$\vec{V} = \vec{Vy} + \vec{Vz}$$

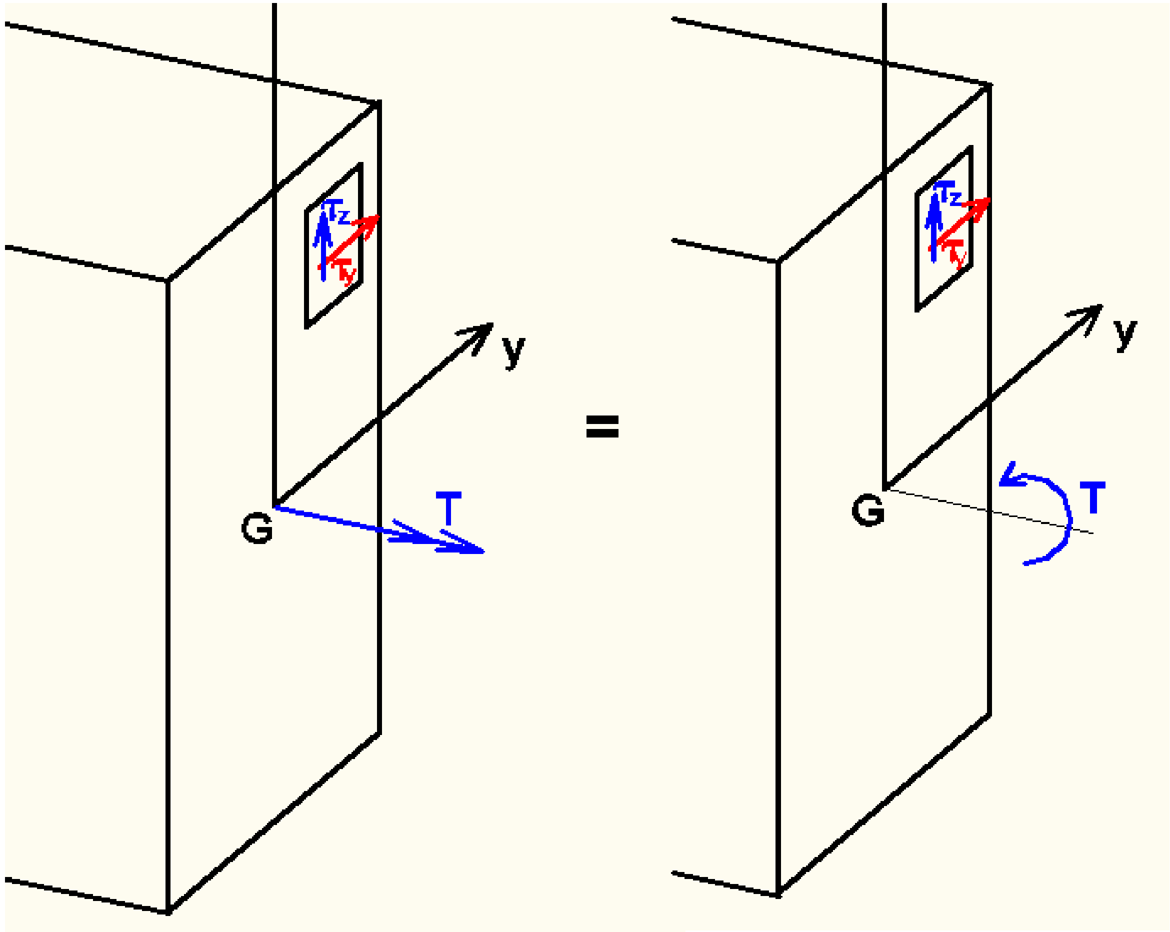
N: Esforço Normal

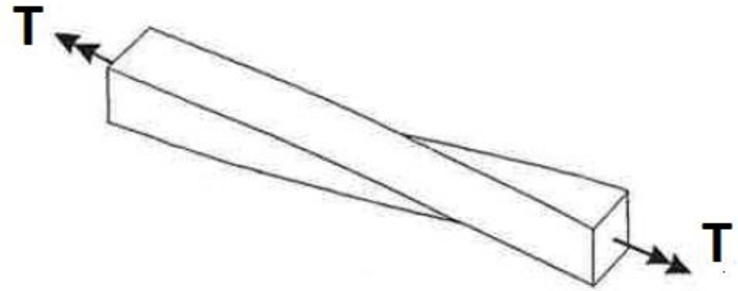
V: Esforço Cisalhante
ou Cortante

ESFORÇOS SOLICITANTES 3D

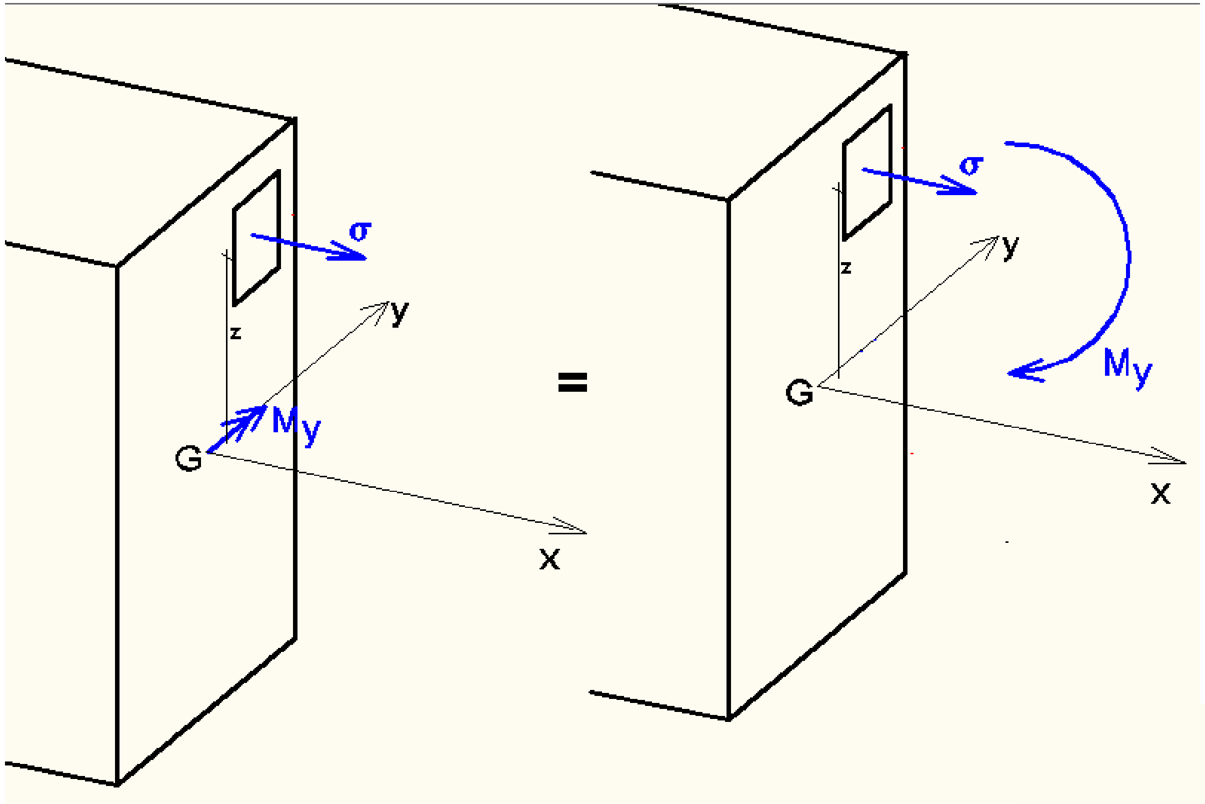


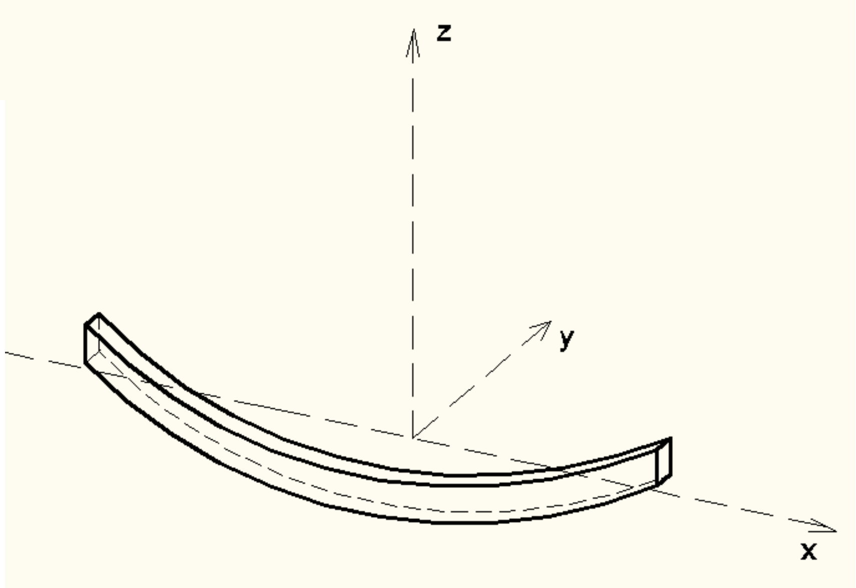
MOMENTO TORÇOR (T)





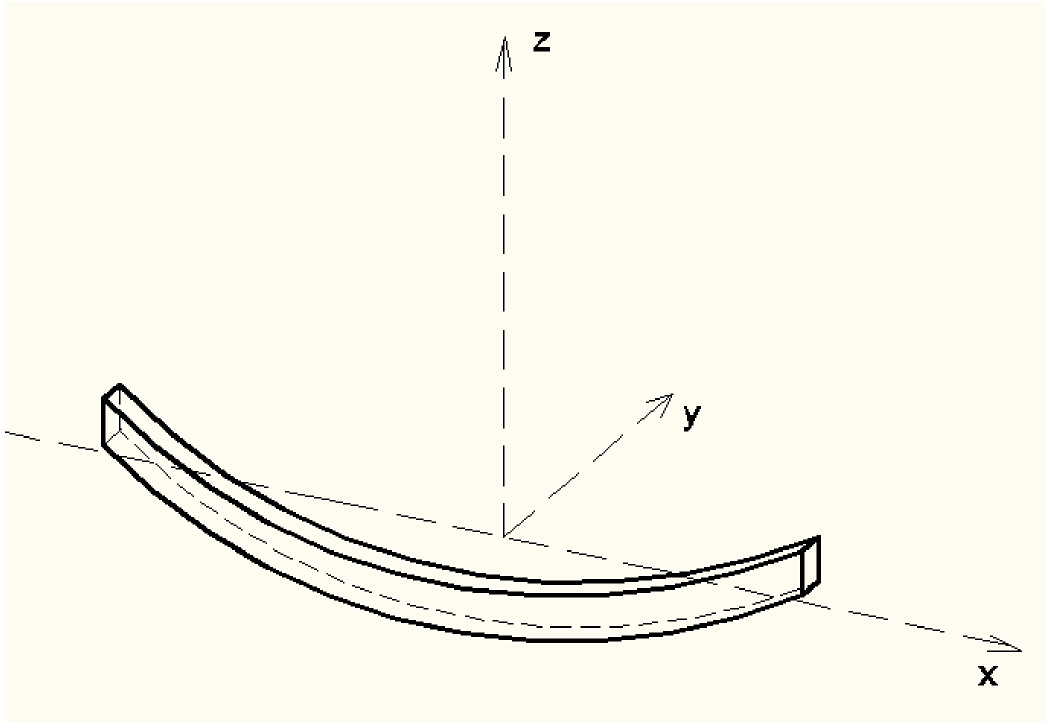
MOMENTO FLETOR (My)

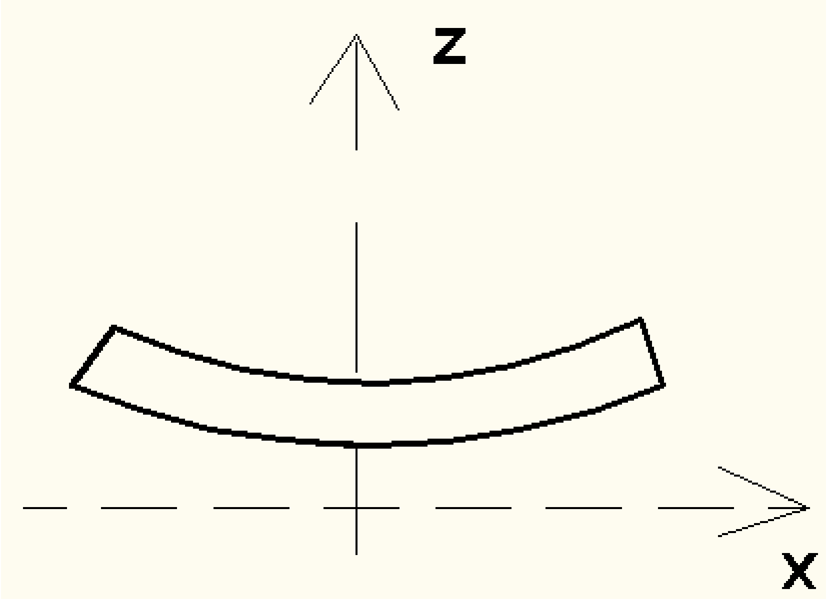




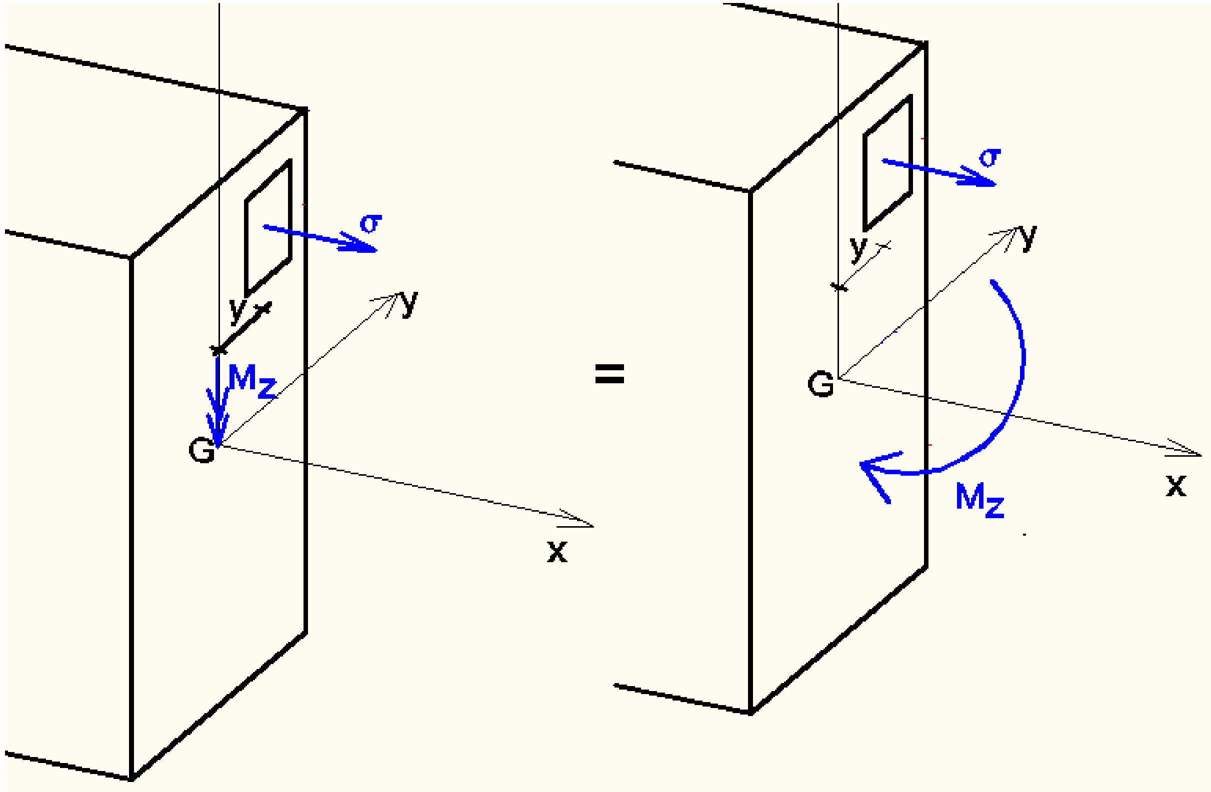
MOMENTO FLETOR (My)

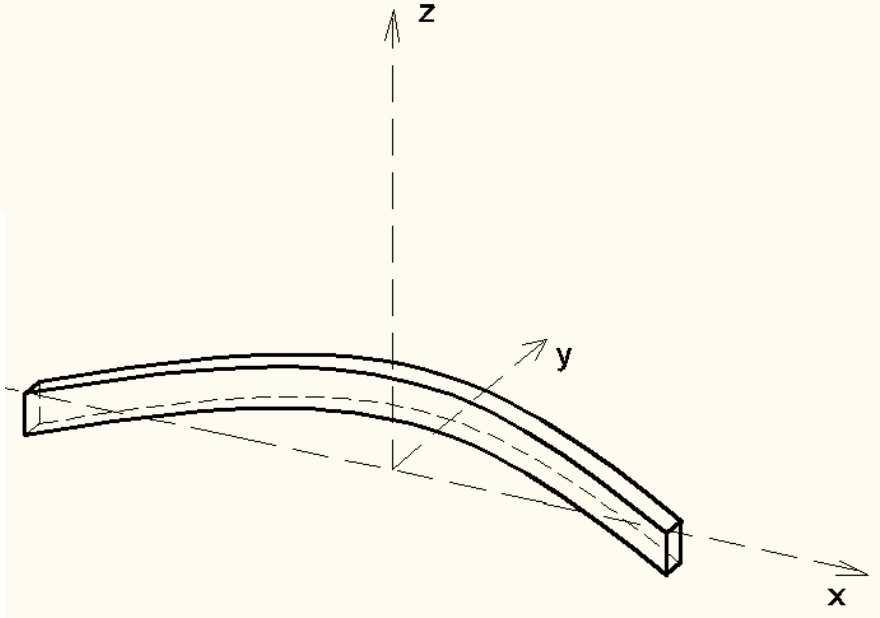
Curvatura em torno de y



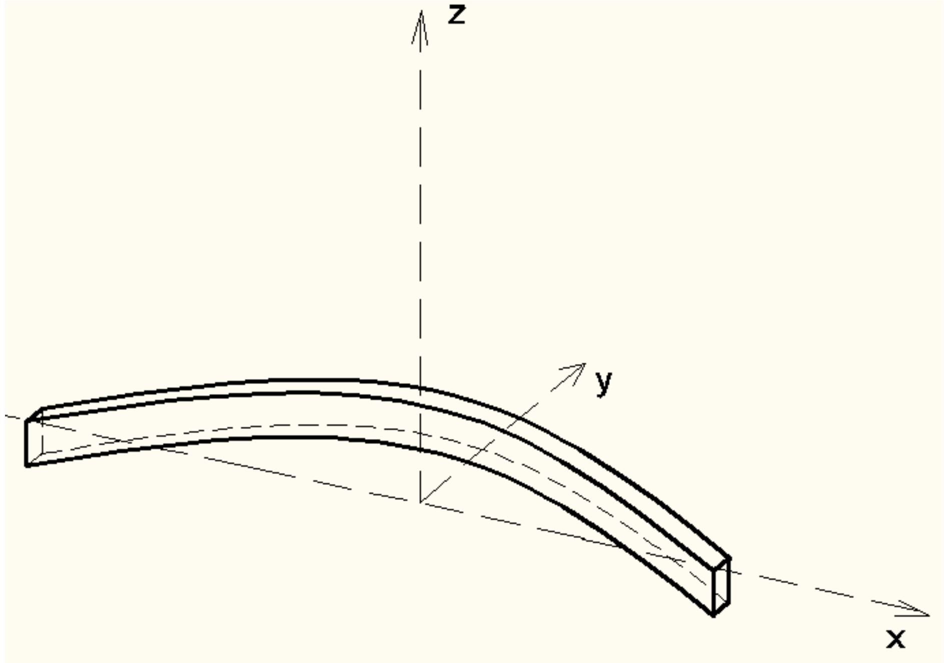


MOMENTO FLETOR (Mz)

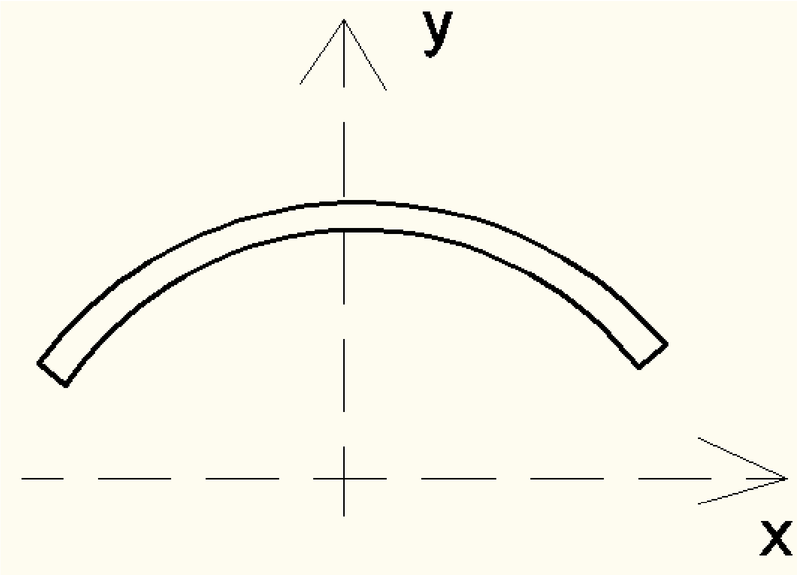




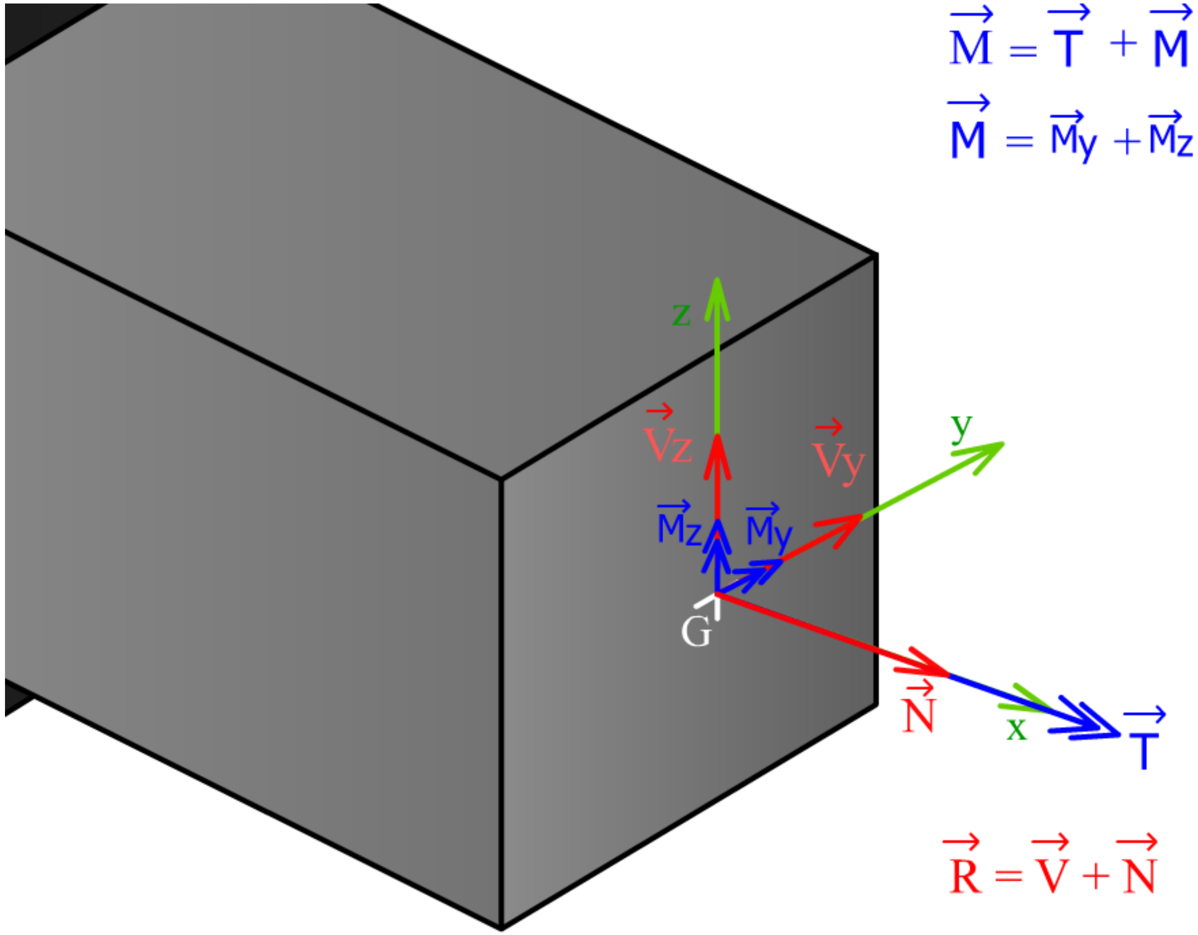
MOMENTO FLETOR (Mz)



Curvatura em torno de z



ESFORÇOS 3D



$$\vec{M} = \vec{T} + \vec{M}$$

$$\vec{M} = \vec{M}y + \vec{M}z$$

$$\vec{R} = \vec{V} + \vec{N}$$

$$\vec{V} = \vec{V}y + \vec{V}z$$

$$N = \int_A \sigma \, dA$$

$$V_y = \int_A \tau_y \, dA$$

$$V_z = \int_A \tau_z \, dA$$

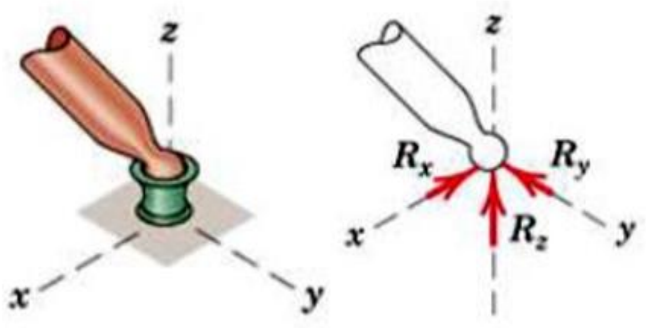
$$M_z = \int_A \sigma \cdot y \, dA$$

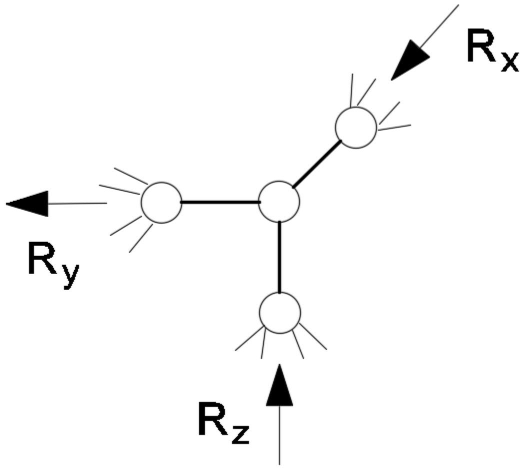
$$M_y = \int_A \sigma \cdot z \, dA$$

$$T = \int_A \left(\tau_z \, y - \tau_y \, z\right) dA$$

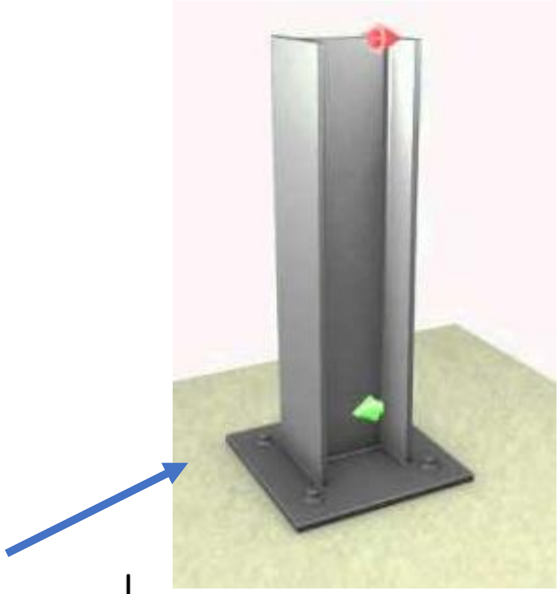
VÍNCULOS

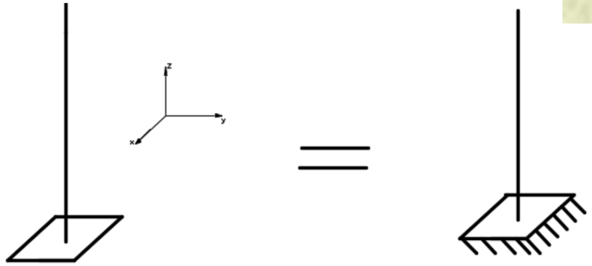
Apoios fixos

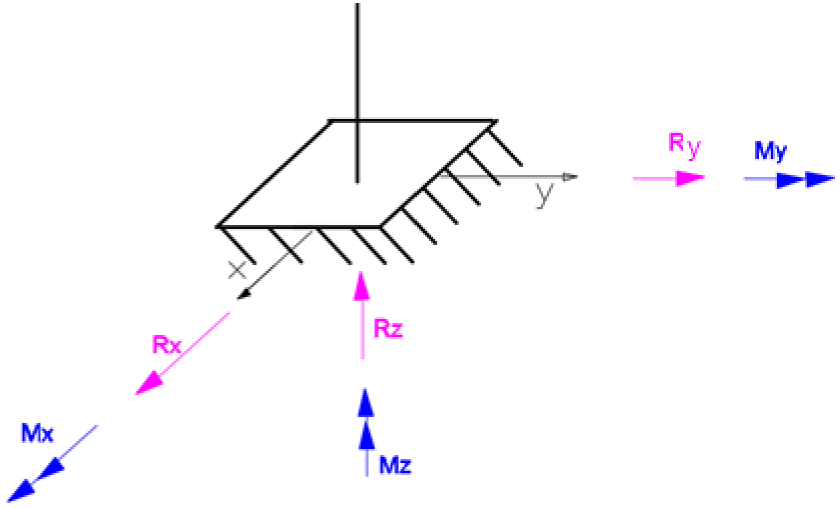




Engaste



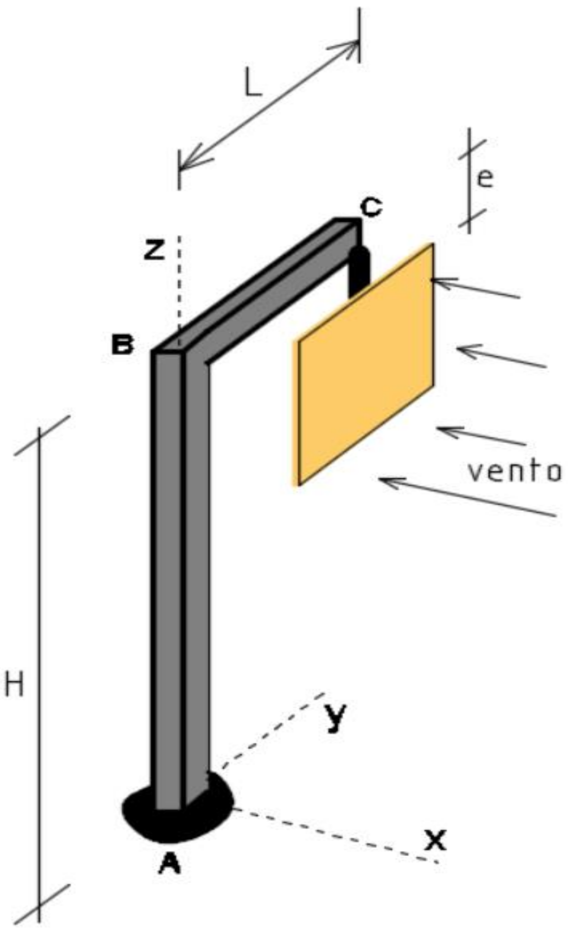


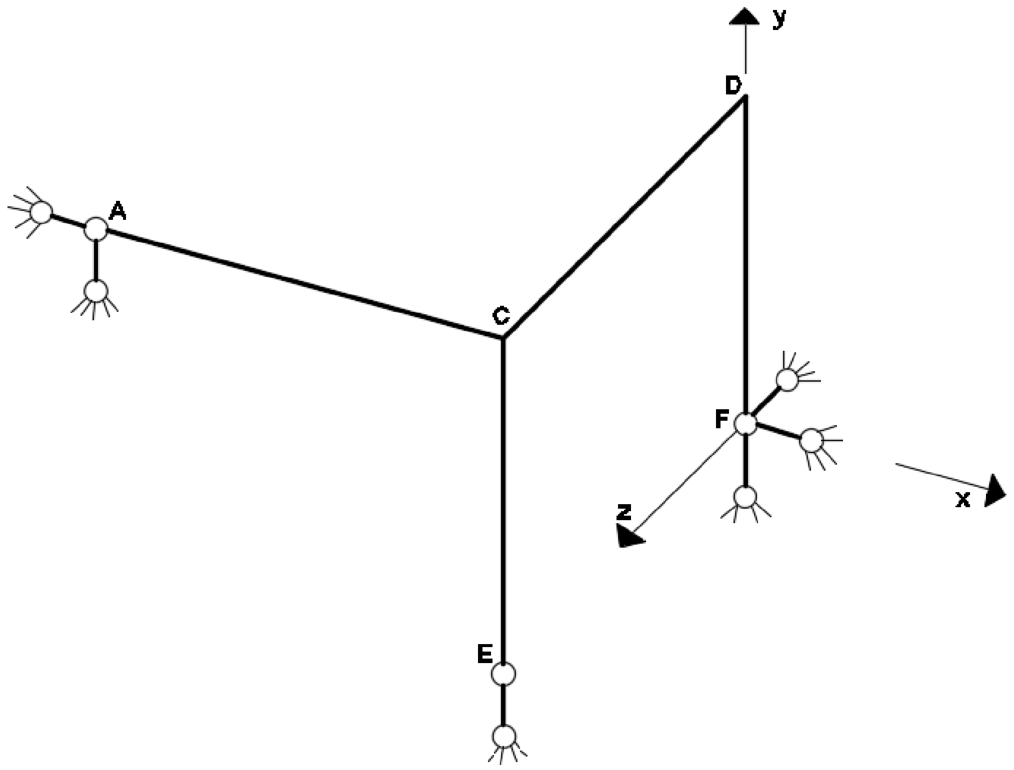


Quadros isostáticos

Vínculos suficiente para manter estrutura em equilíbrio

Obtendo reações e esforços apenas com as equações da estática





EQUAÇÕES DE EQUILÍBRIO DA ESTÁTICA

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$
$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0$$

VETORES – PRODUTO VETORIAL

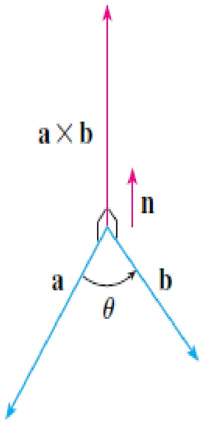
Definição: Se $a = \langle a_1, a_2, a_3 \rangle$ e $b = \langle b_1, b_2, b_3 \rangle$ então o produto vetorial de a e b é o vetor:

$$a \times b = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

Para facilitar o cálculo e tornar mais fácil de lembrar esta definição o produto vetorial $(a \times b)$ é representado com a notação de determinante da seguinte forma:

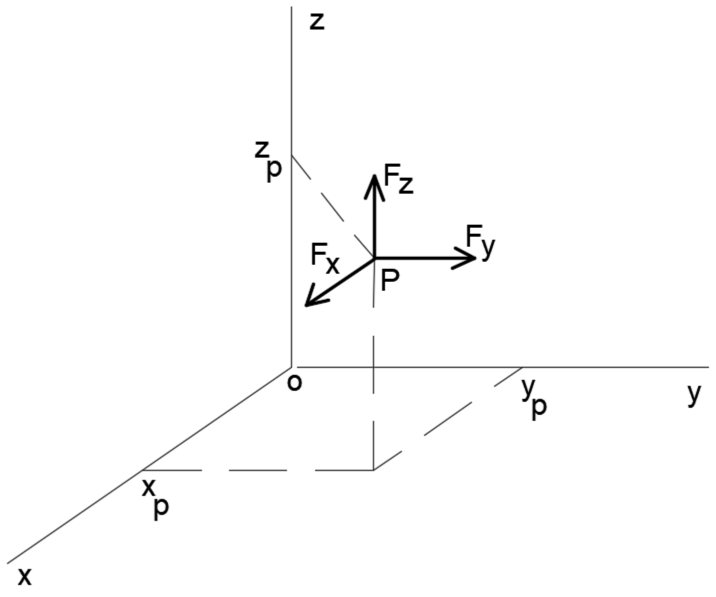
$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\mathbf{a} \times \mathbf{b} = \underbrace{(a_y b_z - a_z b_y)}_{c_x}\mathbf{i} + \underbrace{(a_z b_x - a_x b_z)}_{c_y}\mathbf{j} + \underbrace{(a_x b_y - a_y b_x)}_{c_z}\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



Momento e Força – Produto vetorial

As forças em P geram que momento em “O”?



$$M_o = r \, x \, F$$

$$r = (x_p - x_o)i + (y_p - y_o)j + (z_p - z_o)k$$

$$F = F_x i + F_y j + F_z k$$

$$M_o = [(x_p - x_o)i + (y_p - y_o)j + (z_p - z_o)k] x \, [F_x i + F_y j + F_z k]$$

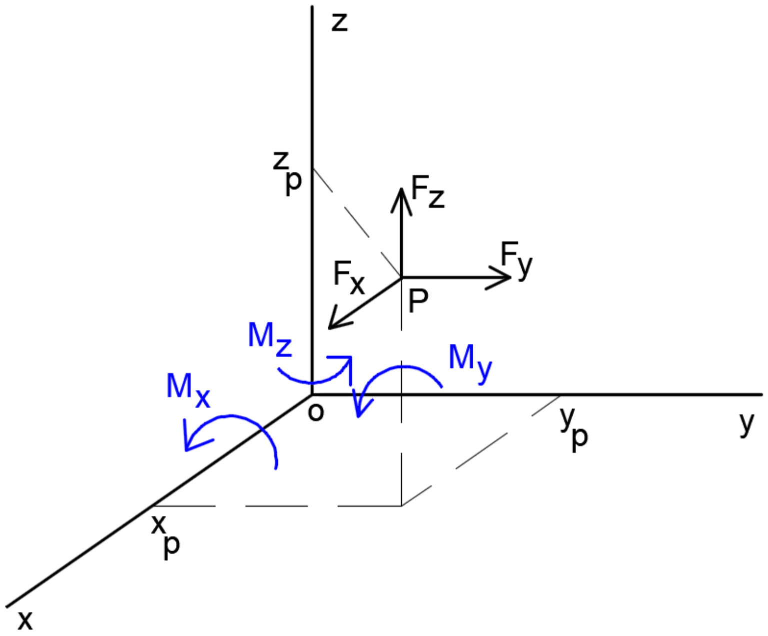
Momento e Força – Produto vetorial

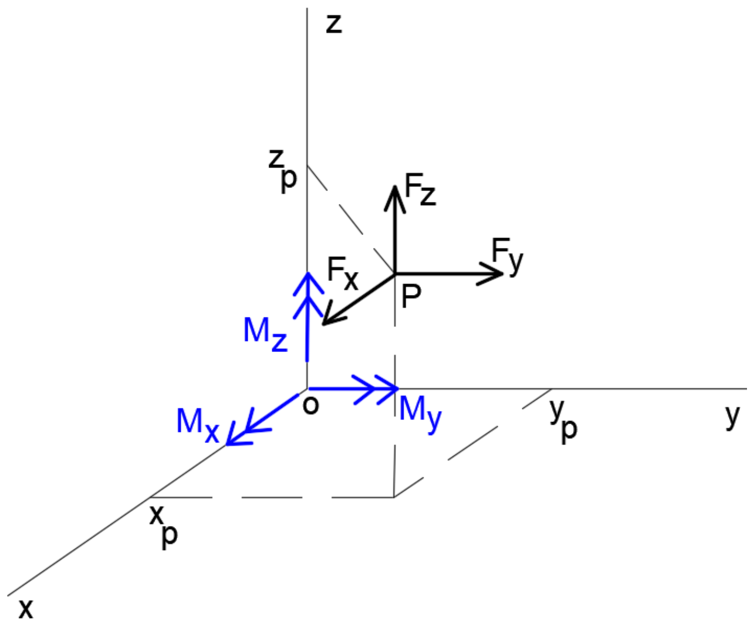
$$M_o = M_x i + M_y j + M_z k$$

$$M_x = (y_p - y_o)F_z - (z_p - z_o)F_y$$

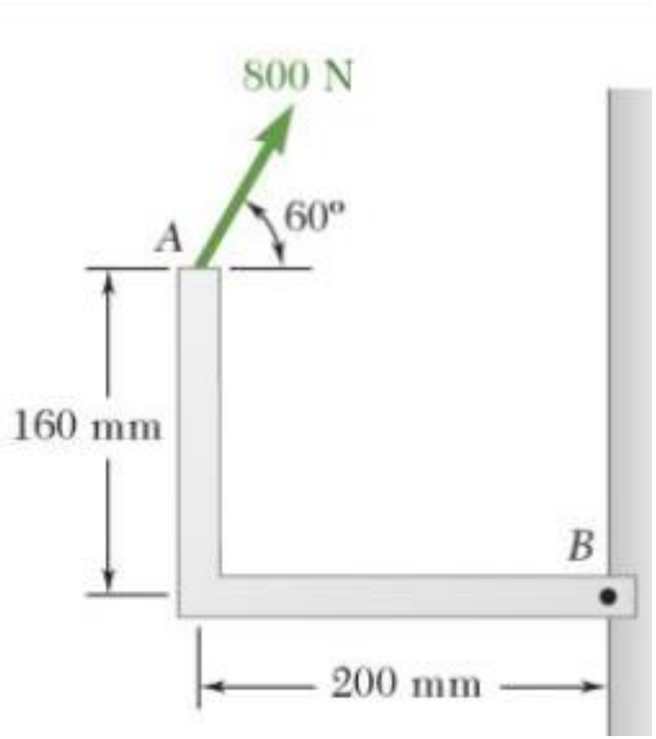
$$M_y = (z_p - z_o)F_x - (x_p - x_o)F_z$$

$$M_z = (x_p - x_o)F_y - (y_p - y_o)F_x$$





EXEMPLO 1: Obter a resultante de momento em relação ao pólo B da estrutura plana por meio do produto vetorial.

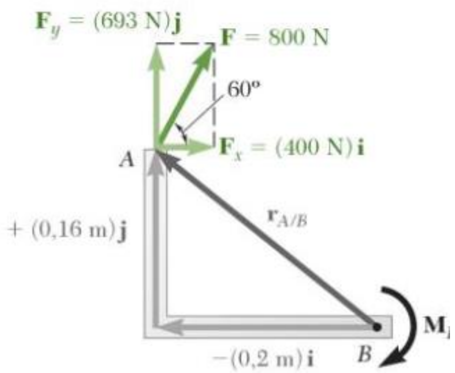


$$M_o = M_x i + M_y j + M_z k$$

$$M_x = (y_p - y_o)F_z - (z_p - z_o)F_y$$

$$M_y = (z_p - z_o)F_x - (x_p - x_o)F_z$$

$$M_z = (x_p - x_o)F_y - (y_p - y_o)F_x$$



$$\mathbf{r}_{A/B} = -(0{,}2\text{ m})\mathbf{i} + (0{,}16\text{ m})\mathbf{j}$$

$$\mathbf{F} = (800\text{ N})\cos 60°\mathbf{i} + (800\text{ N})\text{ sen } 60°\mathbf{j}$$

$$= (400\text{ N})\mathbf{i} + (693\text{ N})\mathbf{j}$$

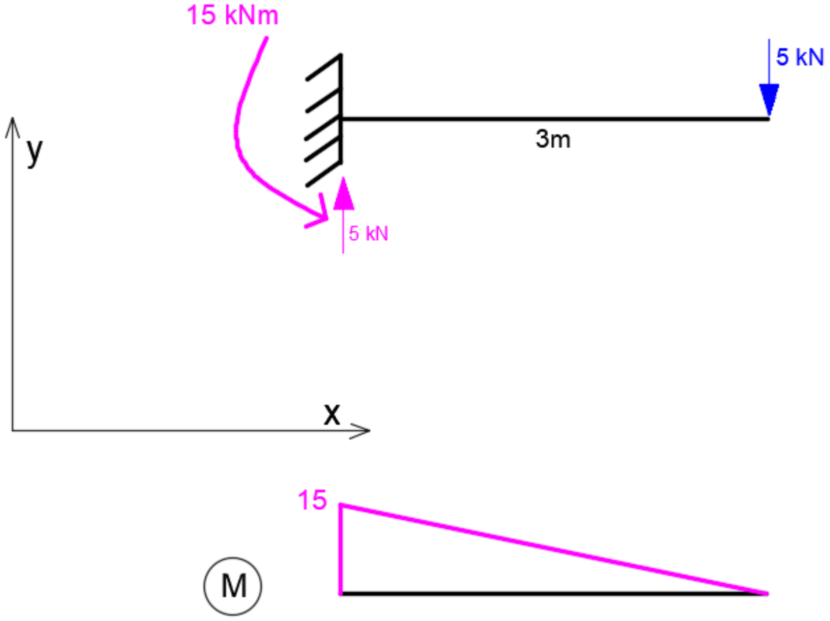
$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F} = [-(0{,}2\text{ m})\mathbf{i} + (0{,}16\text{ m})\mathbf{j}] \times [(400\text{ N})\mathbf{i} + (693\text{ N})\mathbf{j}]$$

$$= -(138{,}6\text{ N}\cdot\text{m})\mathbf{k} - (64{,}0\text{ N}\cdot\text{m})\mathbf{k}$$

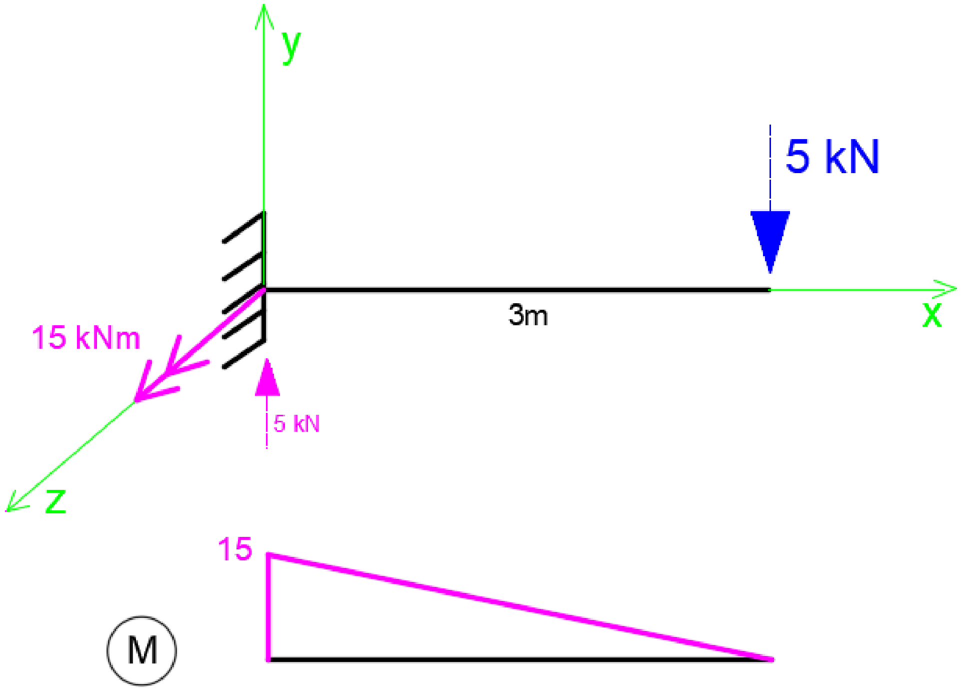
$$= -(202{,}6\text{ N}\cdot\text{m})\mathbf{k} \qquad \mathbf{M}_B = 203\text{ N}\cdot\text{m} \circlearrowright$$

Representação de momento no espaço 3D

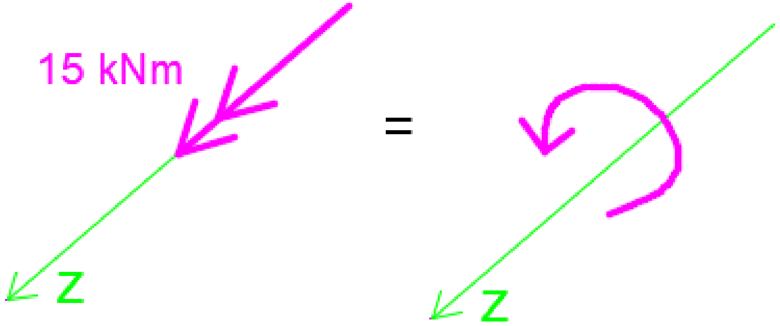
2D:



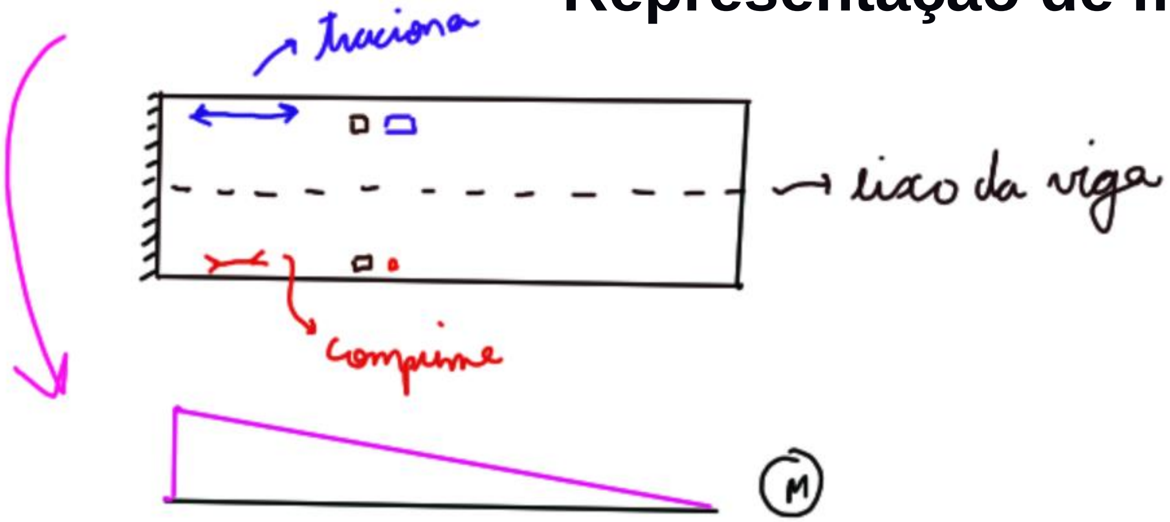
3D:

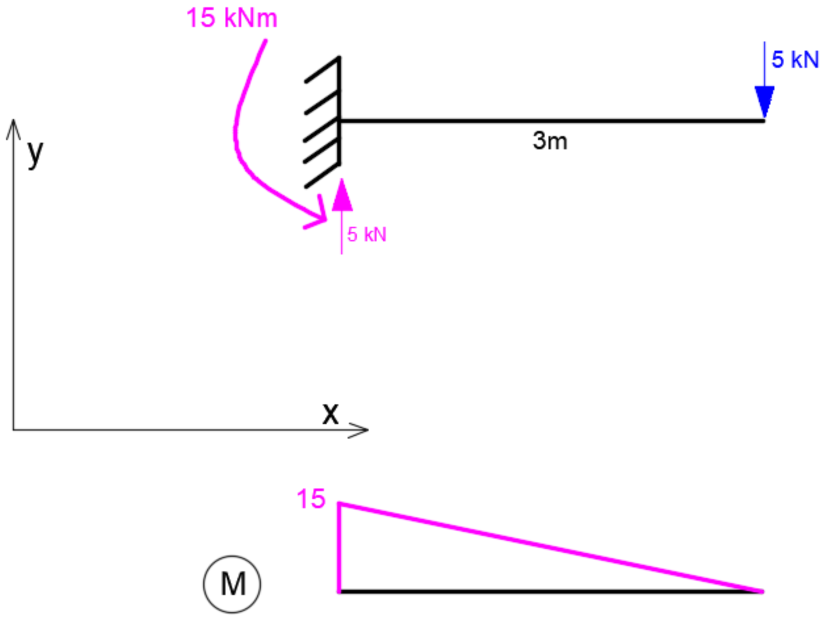


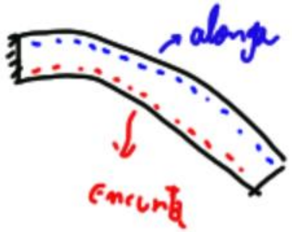
Equivalência de representação

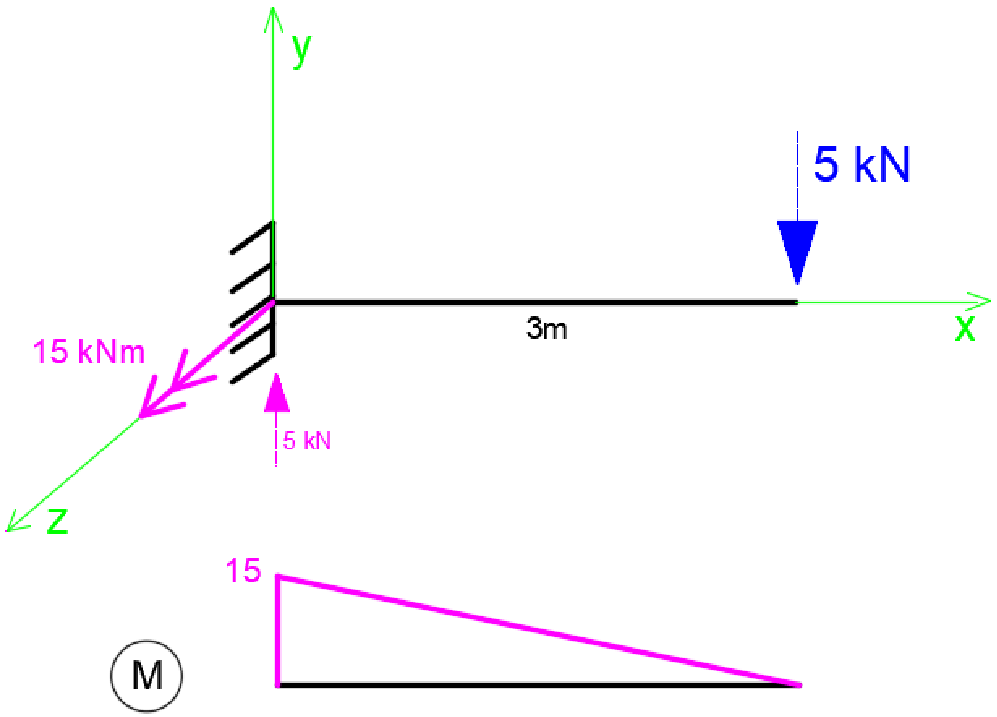


Representação de momento no espaço 3D

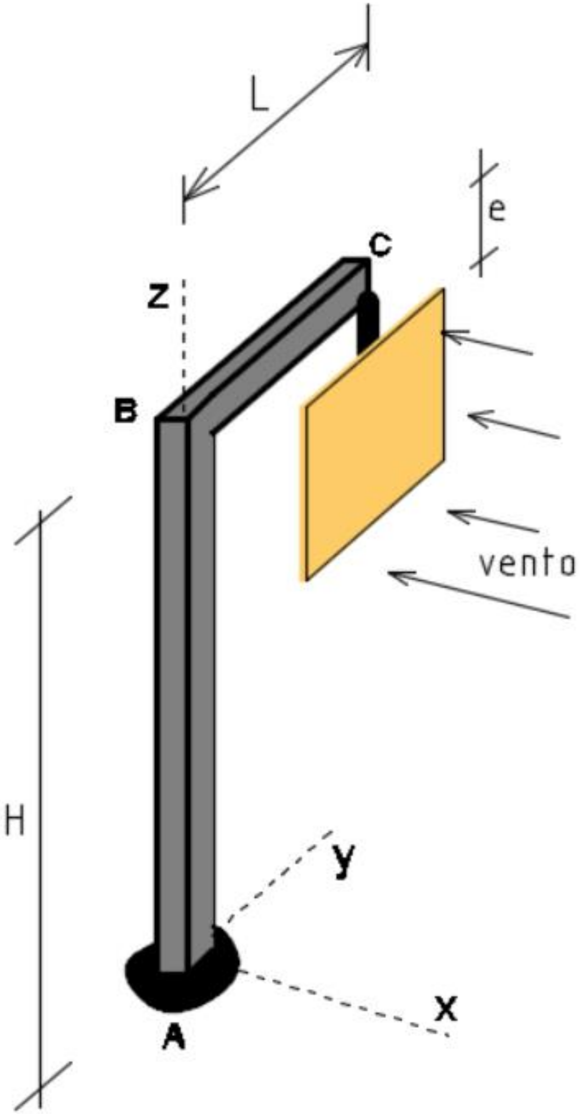


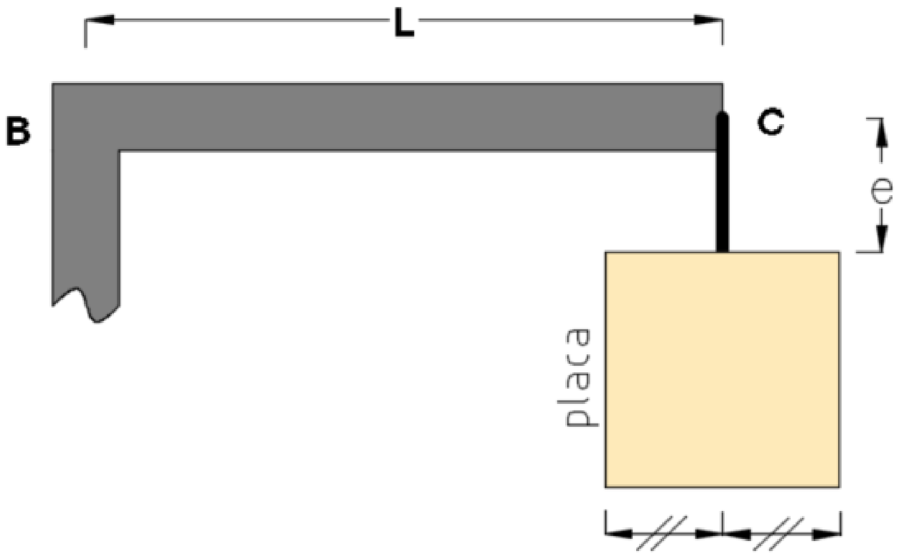






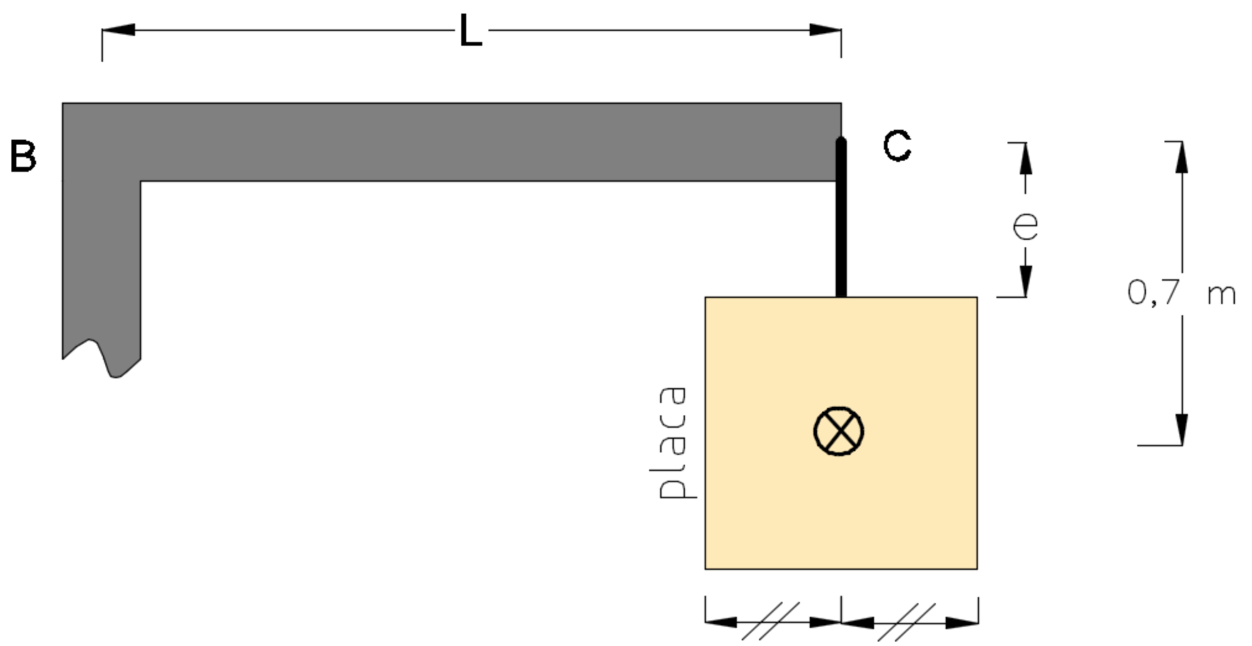
Exemplo 2: O poste está engastando em A. Peso da placa quadrada de lado 800 mm é de 1.000 N, a pressão na placa é 700 N/m^2 .
Dados: L = 2m, H = 5m, e = 30 cm. Obtenha as reações de momento no engaste.

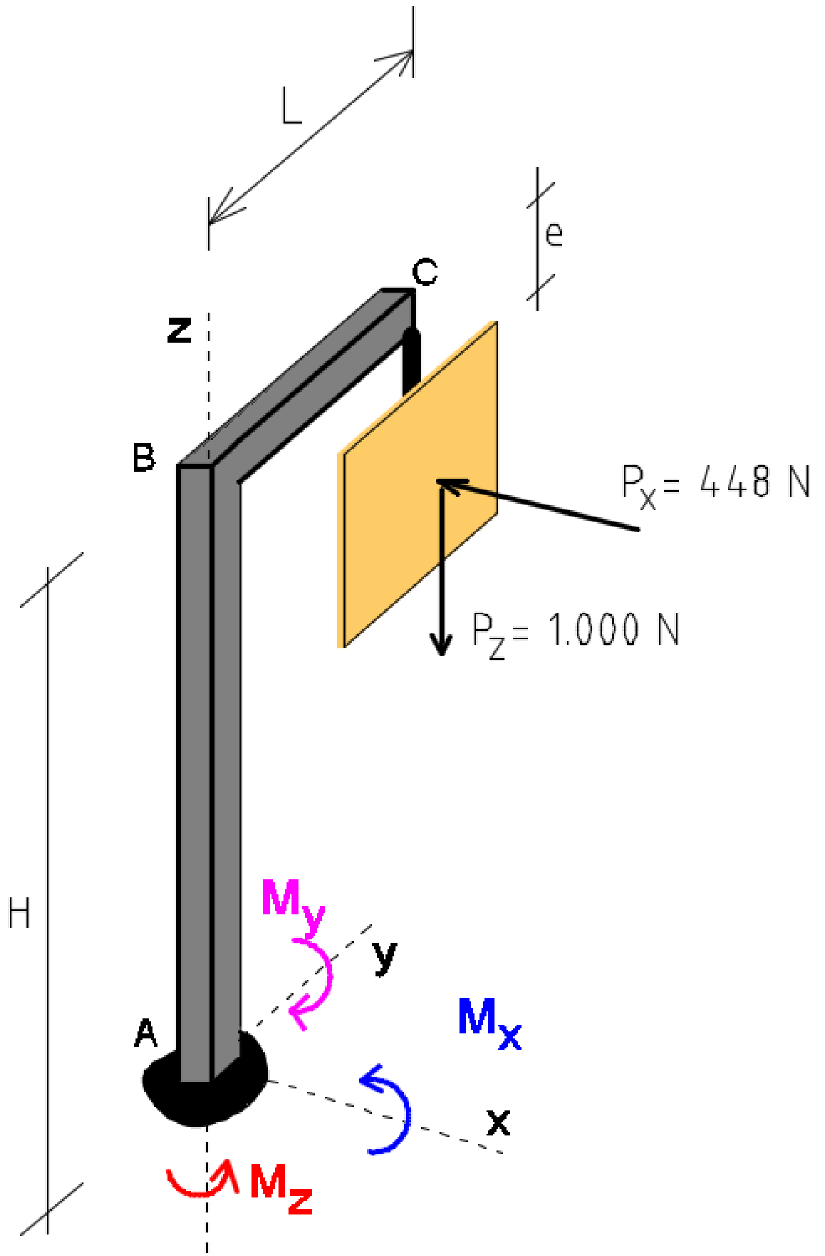




Exemplo 2

Px = 700 N/m². 0,8² (m²)= 448 N





Exemplo 2

$$M_o = M_x i + M_y j + M_z k$$

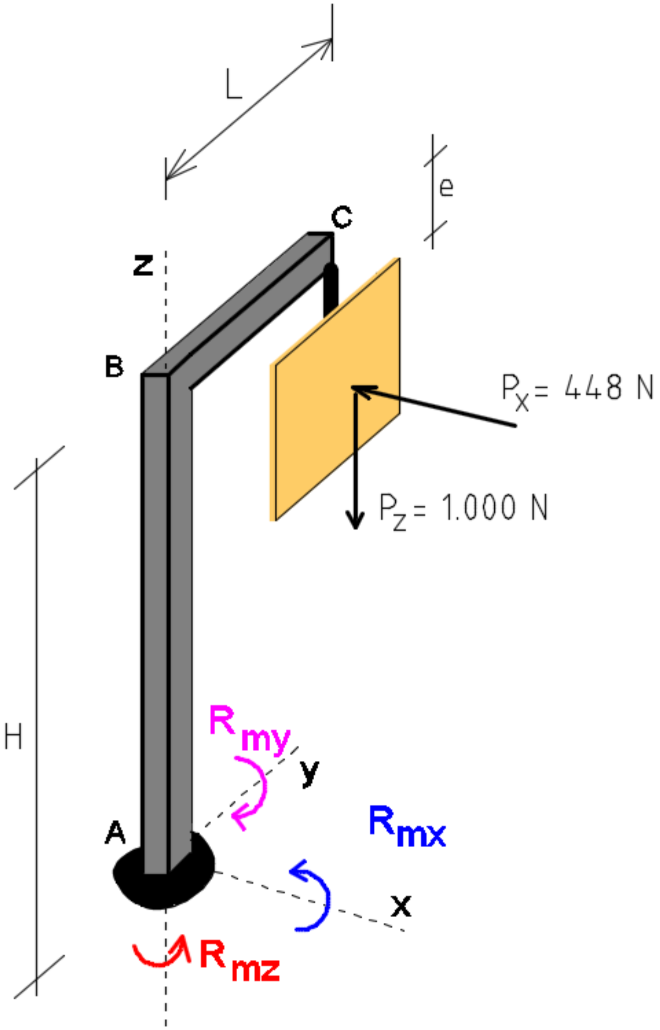
$$M_x = (y_p - y_o)F_z - (z_p - z_o)F_y$$

$$M_y = (z_p - z_o)F_x - (x_p - x_o)F_z$$

$$M_z = (x_p - x_o)F_y - (y_p - y_o)F_x$$

$x_p = 0$
$y_p = L = 2$ m
$z_p = H – 0,7 = 4,3$ m
$(x_0,y_0,z_0) = (0,0,0)$

$P_x = -448$ N
$P_y = 0$
$P_z = -1.000$ N



$M_{xA} = (2-0).(-1.000) – (4,3-0).0 = -2.000$ N.m
$M_{yA} = (4,3-0).(-448) – (0-0).(-1.000) = -1.926,4$ N.m
$M_{zA} = (0-0).(0) – (2-0).(-448) = 896$ N.m

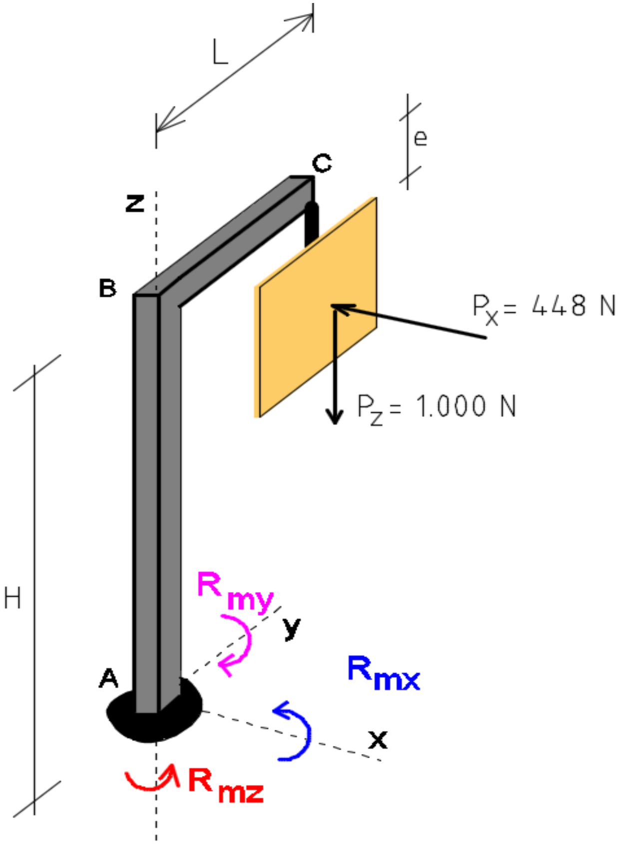
Exemplo 2

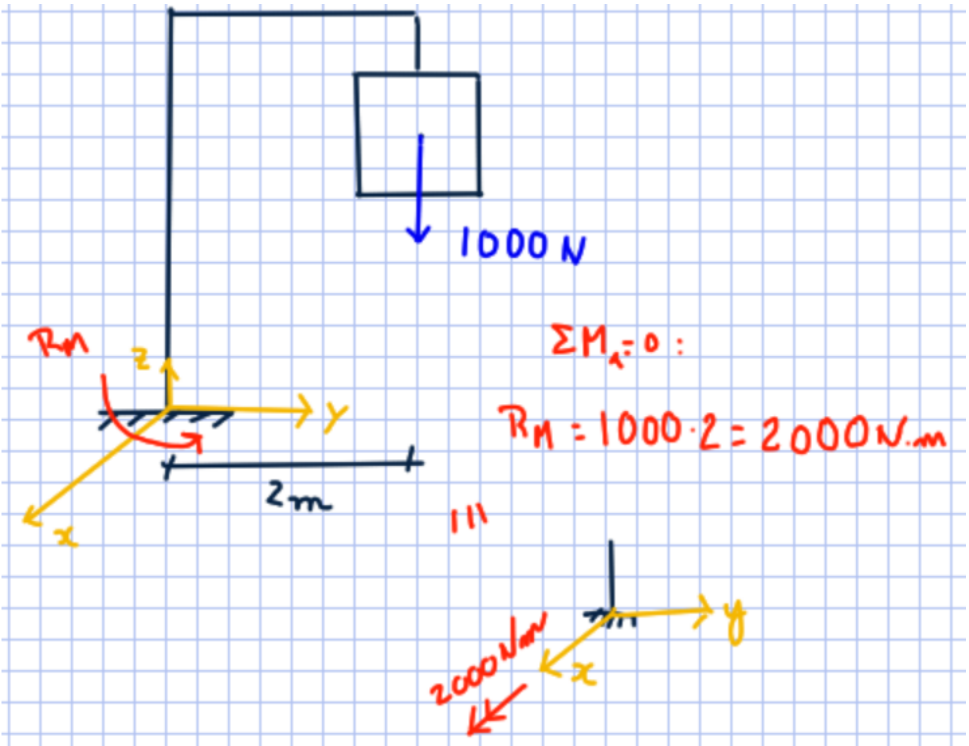
M_{xA} = -2.000 N.m
M_{yA} = -1.926,4 N.m
M_{zA} = 896 N.m

$$\sum M_x = 0: \quad R_{mx} + M_x = 0 \rightarrow R_{mx} - 2000 = 0 \rightarrow \boldsymbol{R_{mx} = 2.000\ N.m}$$

$$\sum M_y = 0: \quad R_{my} + M_y = 0 \rightarrow R_{my} - 1926{,}4 = 0 \rightarrow \boldsymbol{R_{my} = 1.926{,}4\ N.m}$$

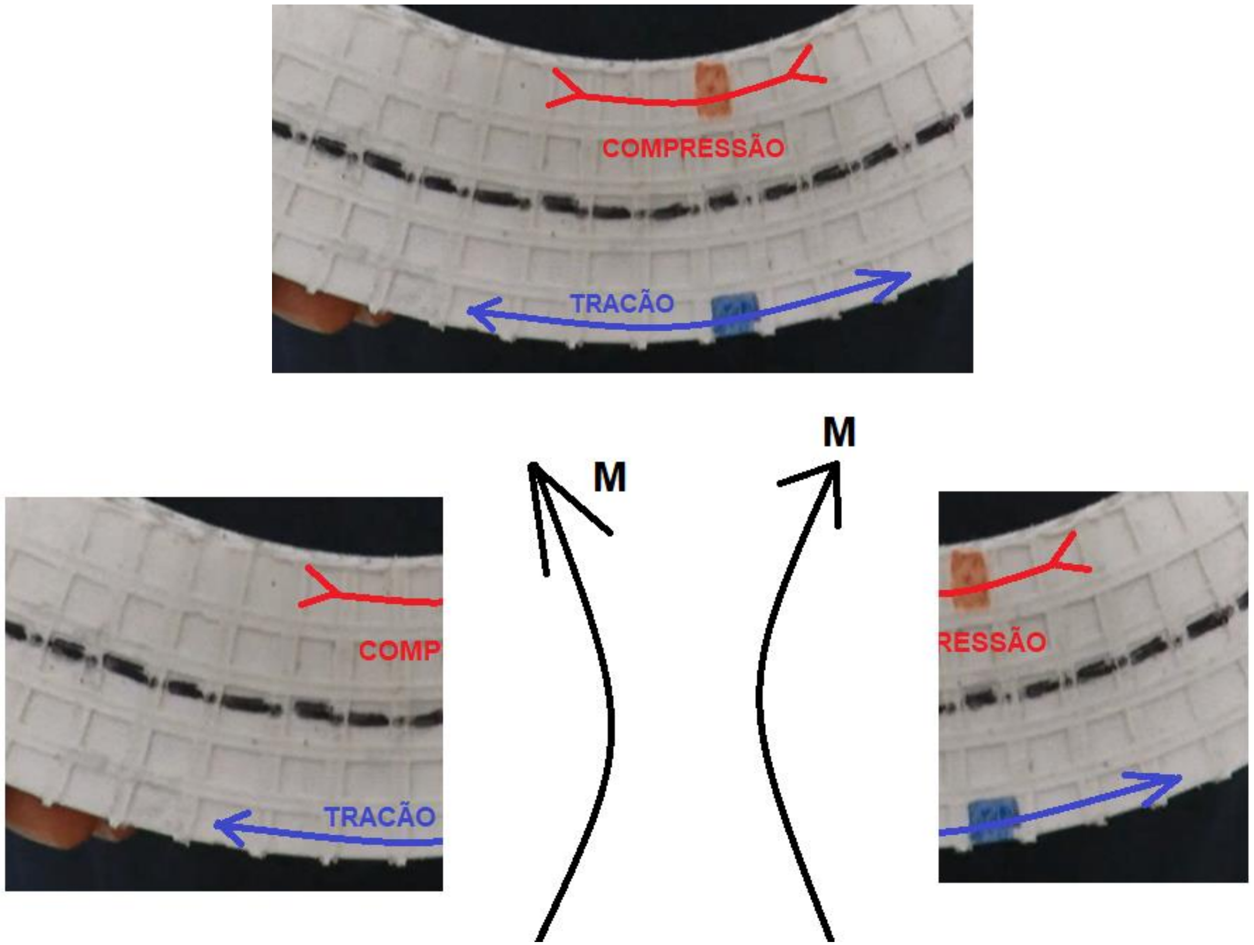
$$\sum M_z = 0: \quad R_{mz} + M_z = 0 \rightarrow R_{mz} + 896 = 0 \rightarrow \boldsymbol{R_{mz} = -896\ N.m}$$





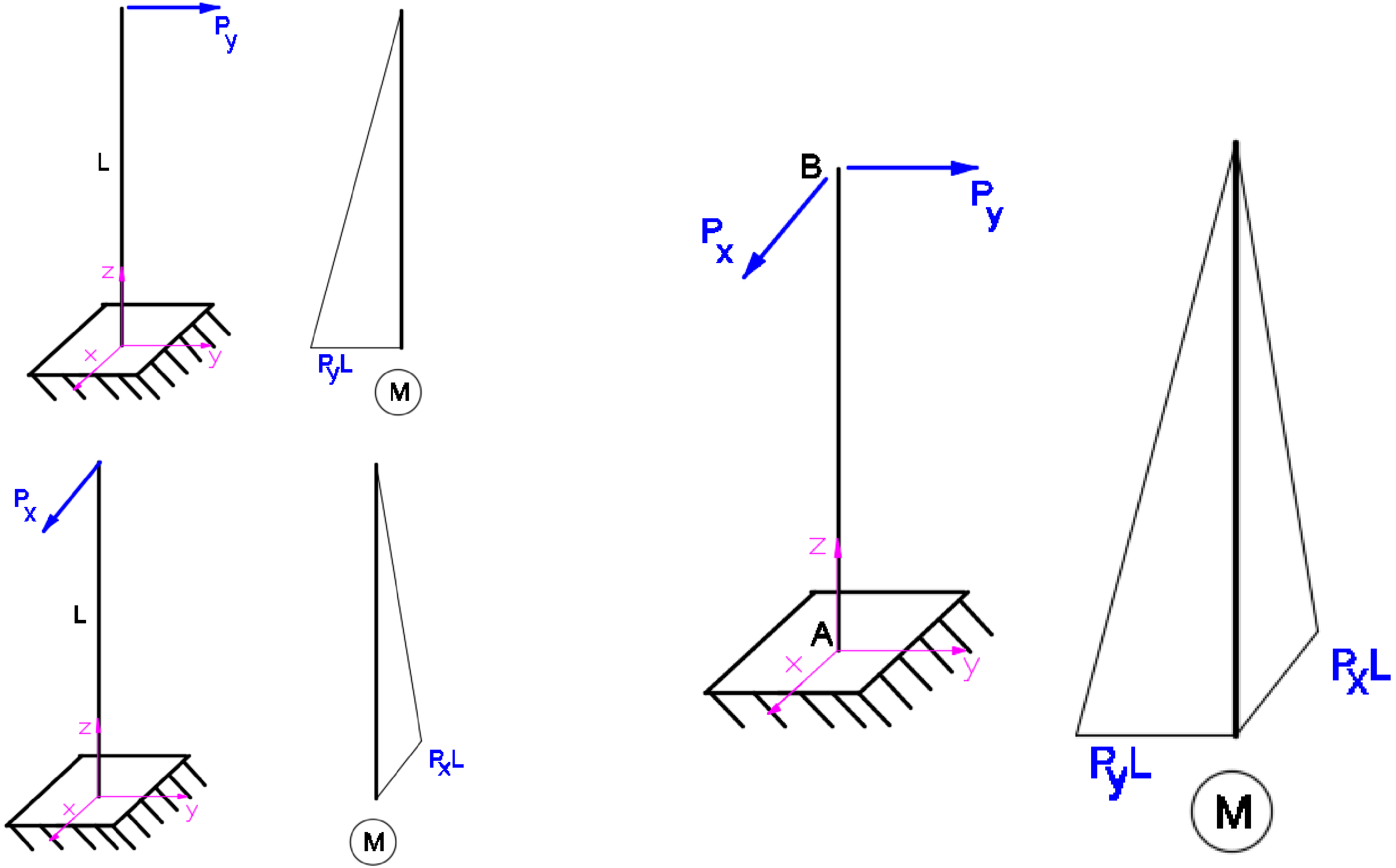
Convenção de sinais para diagramas

Momentos fletores: desenhar lado que traciona



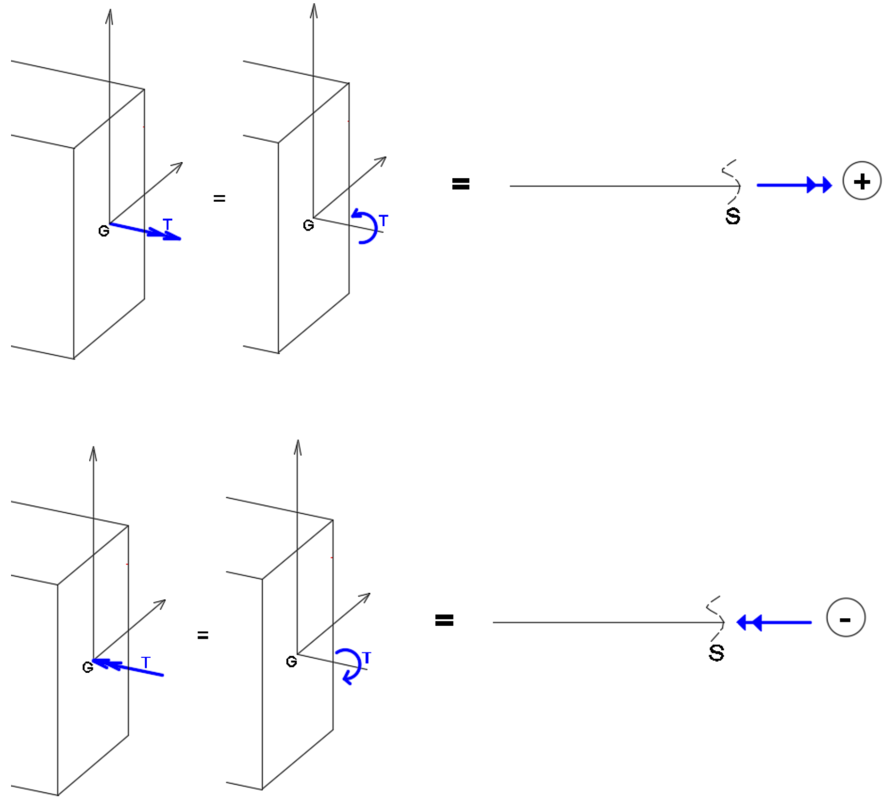
Convenção de sinais para diagramas

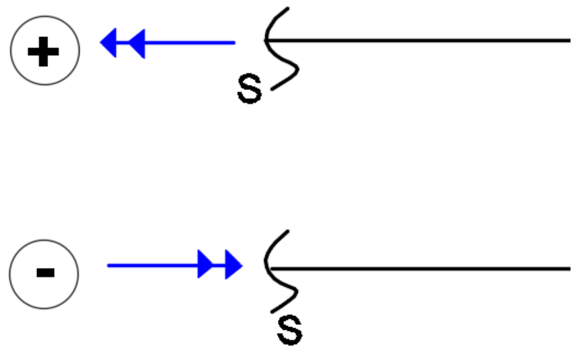
Momentos fletores: desenhar lado que traciona



Convenção de sinais para diagramas

Momento torçor (T) > 0 se vetor sai da seção



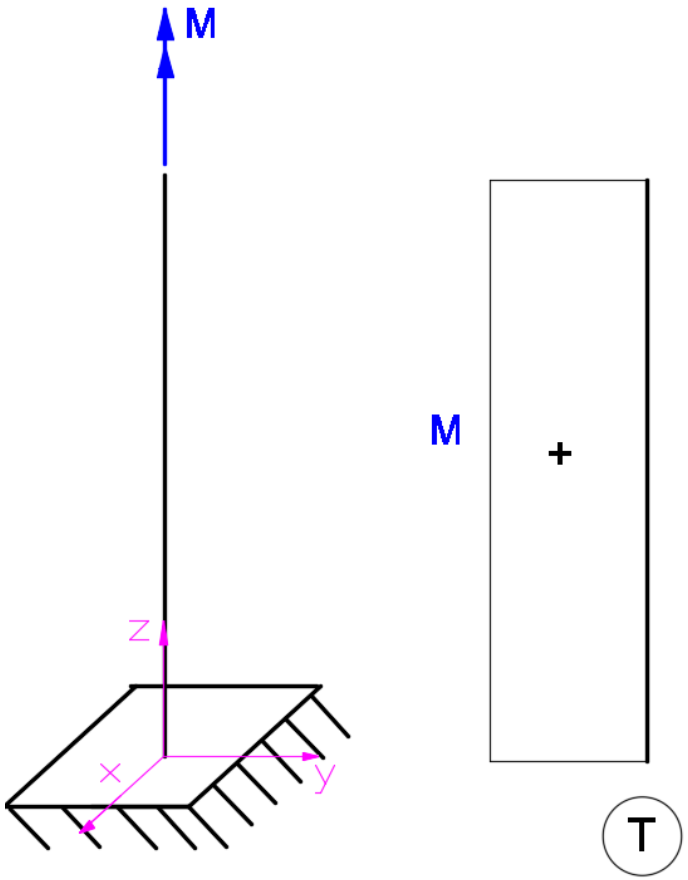


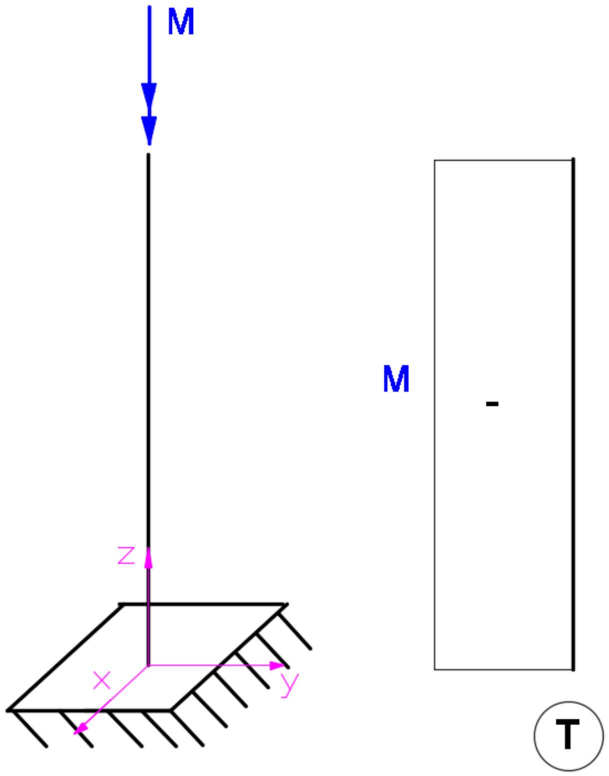
Momento de torção[2]	O vetor momento tem o sentido da normal externa à seção transversal em que atua	O vetor momento tem sentido contrário ao da normal externa à seção tranversal em que atua

Rotacionar a seção no sentido anti-horário (T>0)

Convenção de sinais para diagramas

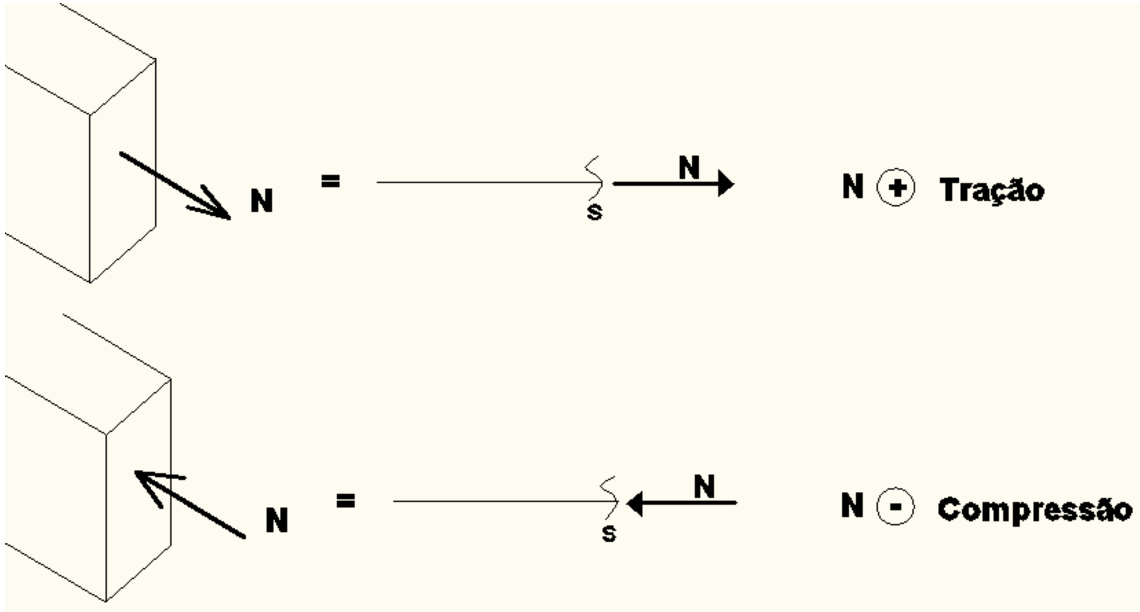
Momento torçor (T) > 0 se vetor sai da seção

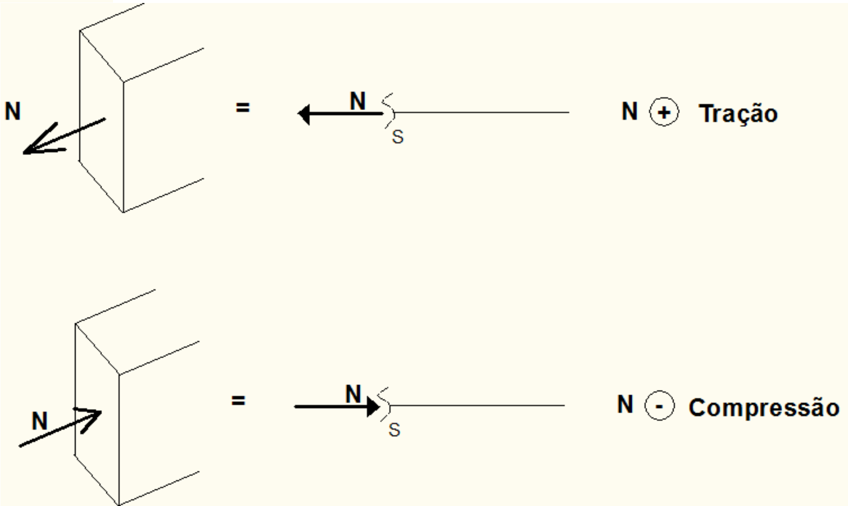




Convenção de sinais para diagramas

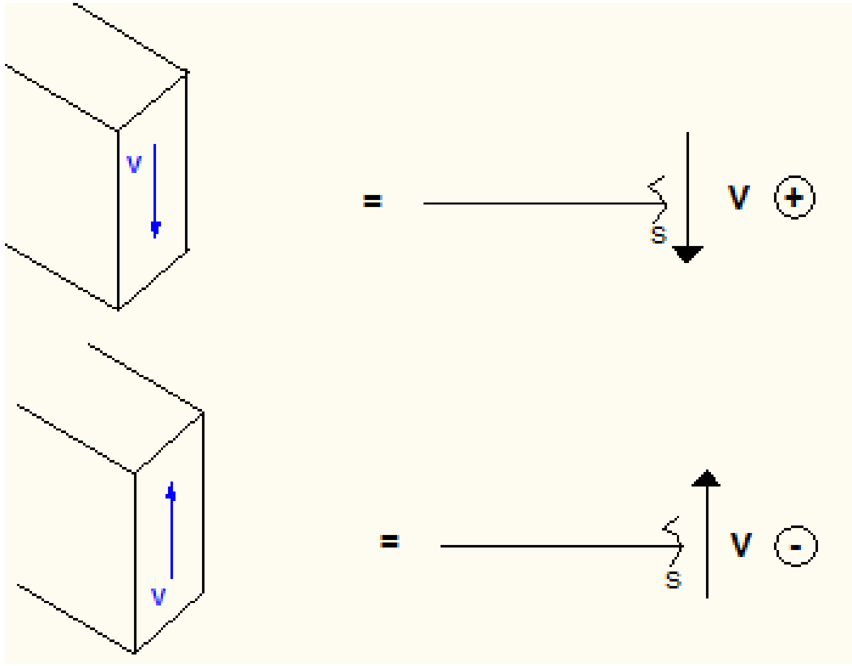
Normal (N) > 0 se vetor sai da seção

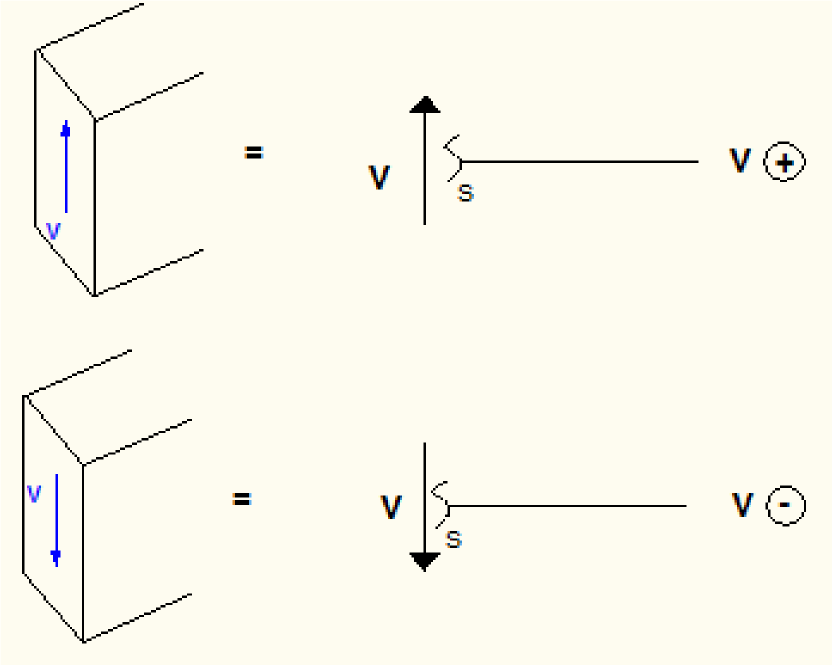




Convenção de sinais para diagramas

Cortantes > 0 se gira a barra no sentido horário



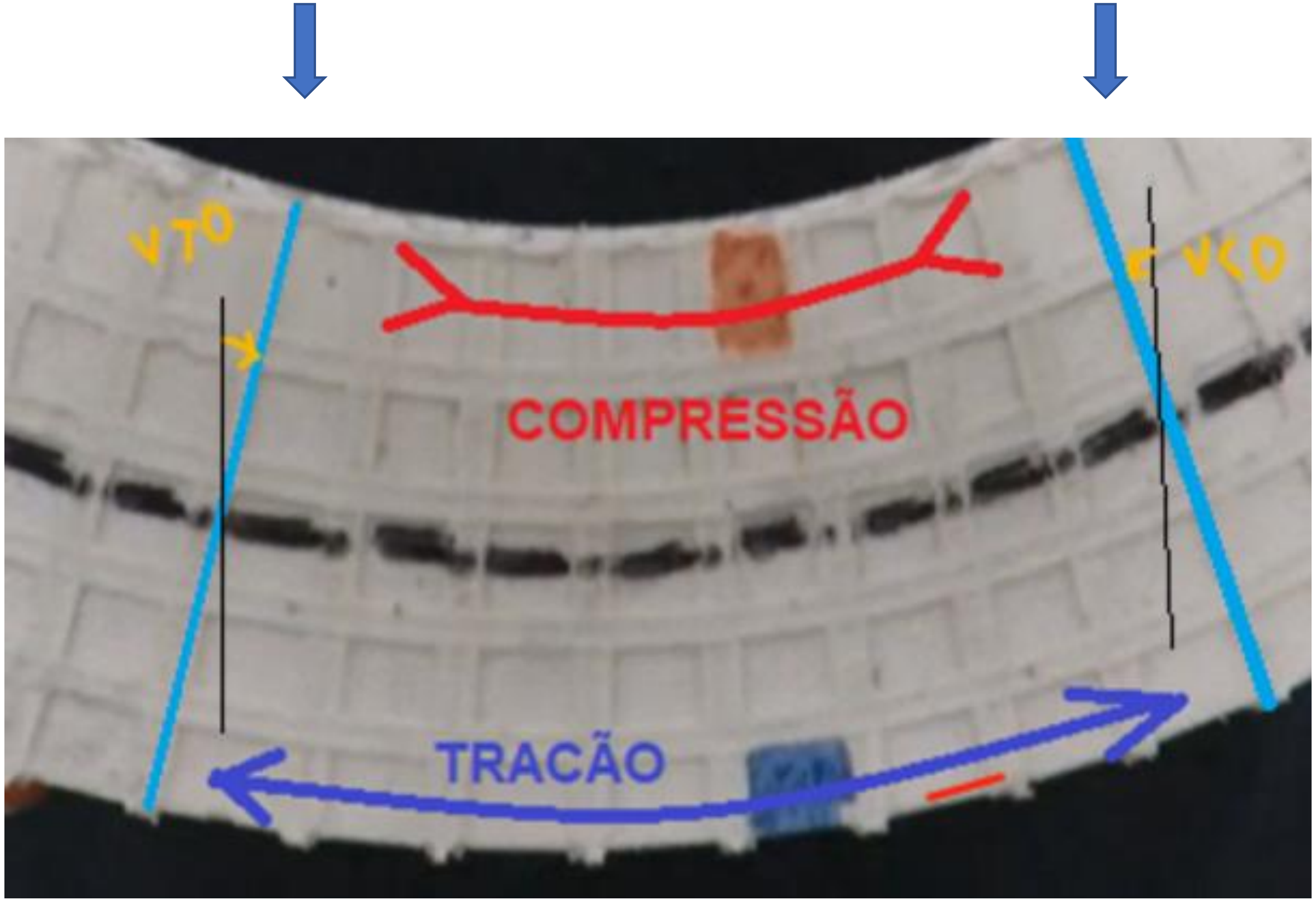


Força cortante	Gira o trecho de barra em que atua no sentido horário	Gira o trecho de barra em que atua no sentido anti-horário

Convenção de sinais para diagramas

Cortantes > 0 se gira a barra no sentido horário

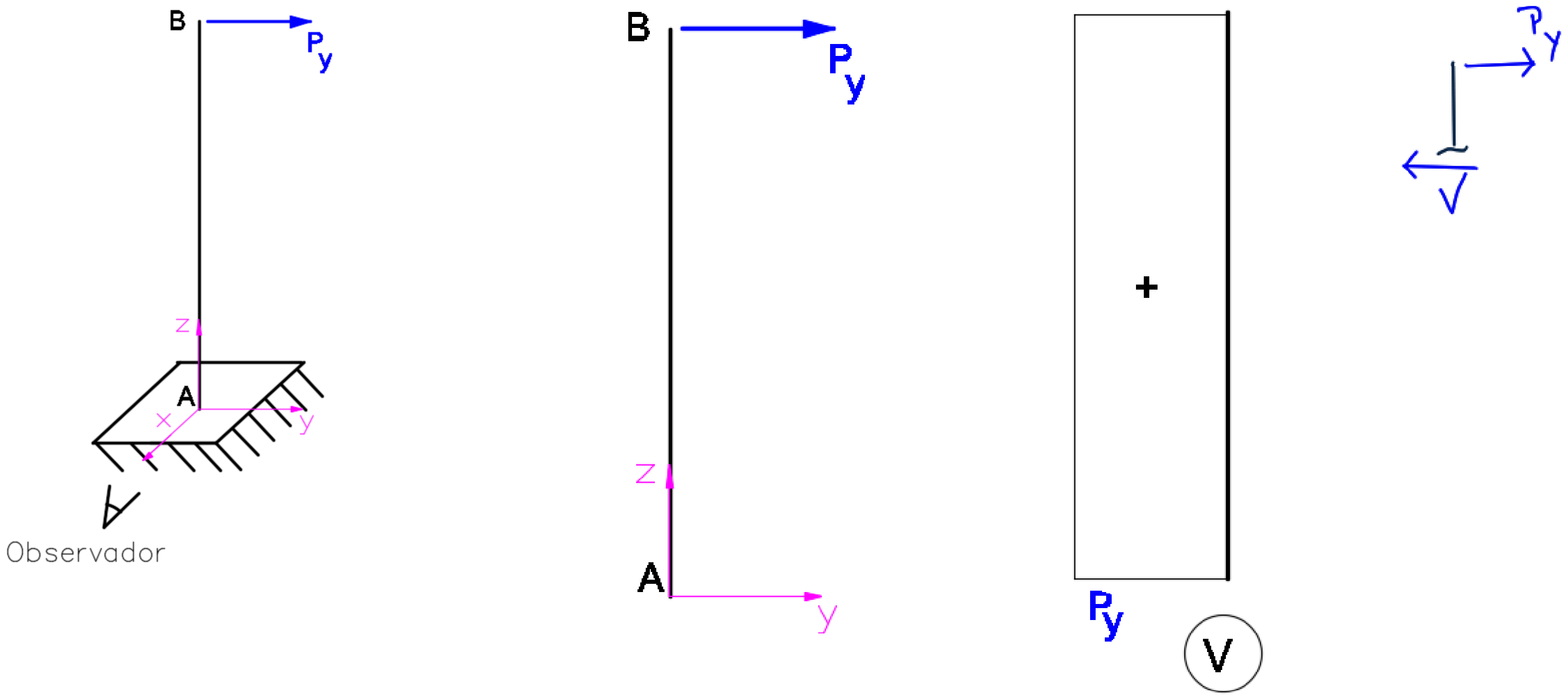
V > 0 **V < 0**



Convenção de sinais para diagramas

Cortantes > 0 se gira a barra no sentido horário

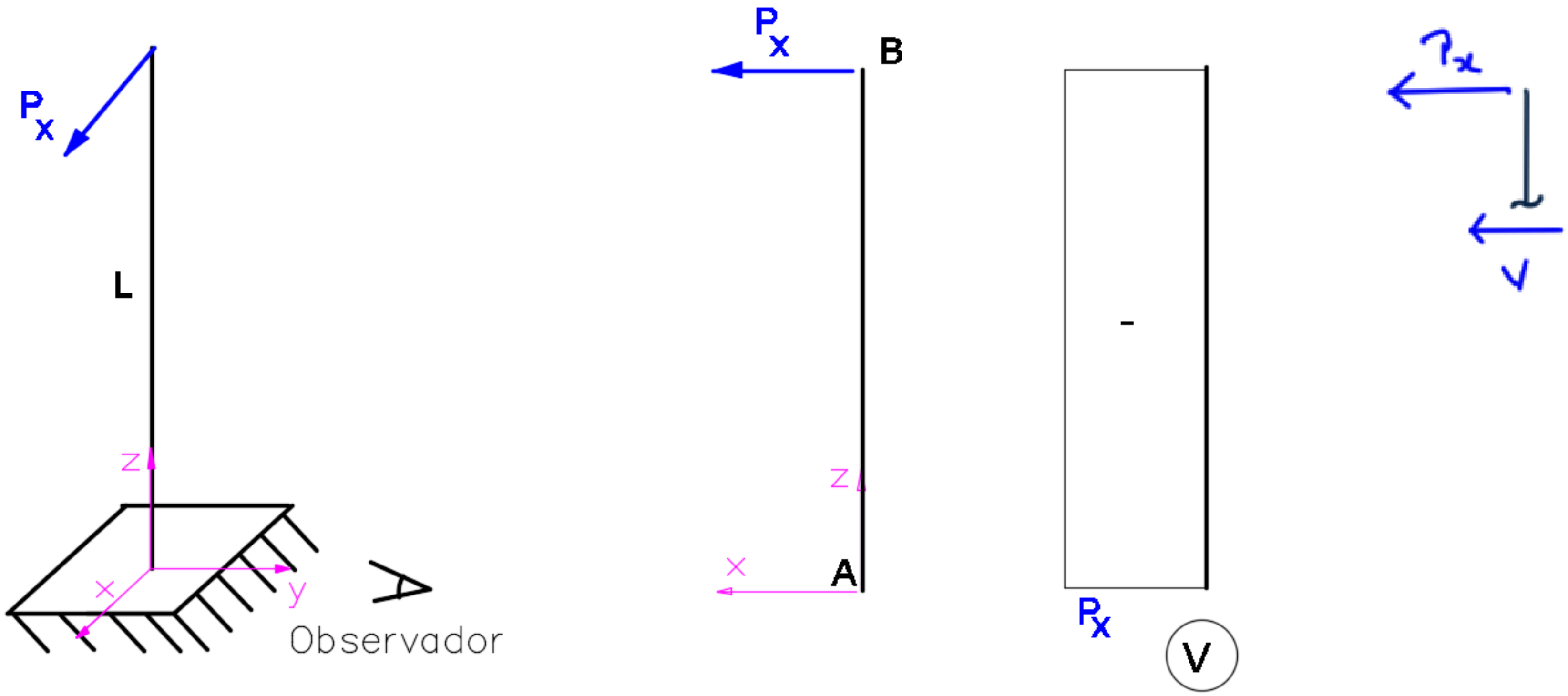
Visualizar a barra no plano pelo lado que está o eixo ortogonal positivo



Convenção de sinais para diagramas

Cortantes > 0 se gira a barra no sentido horário

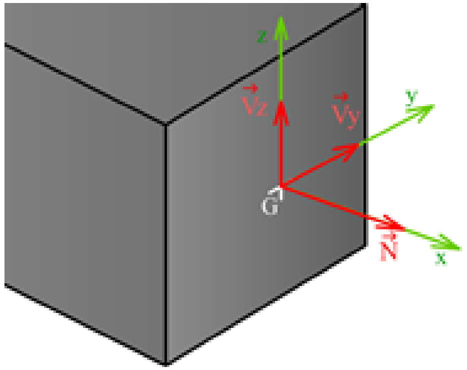
Visualizar a barra no plano pelo lado que está o eixo ortogonal positivo

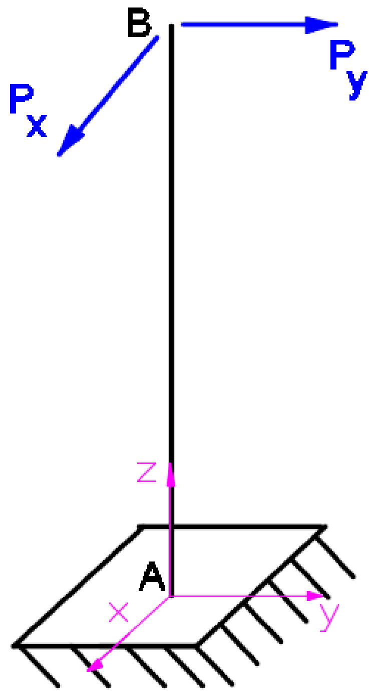


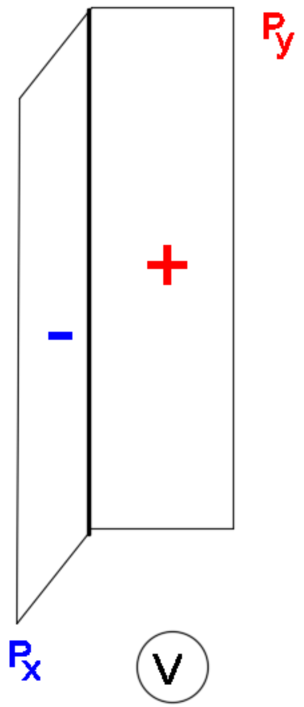
Convenção de sinais para diagramas

Cortantes > 0 se gira a barra no sentido horário

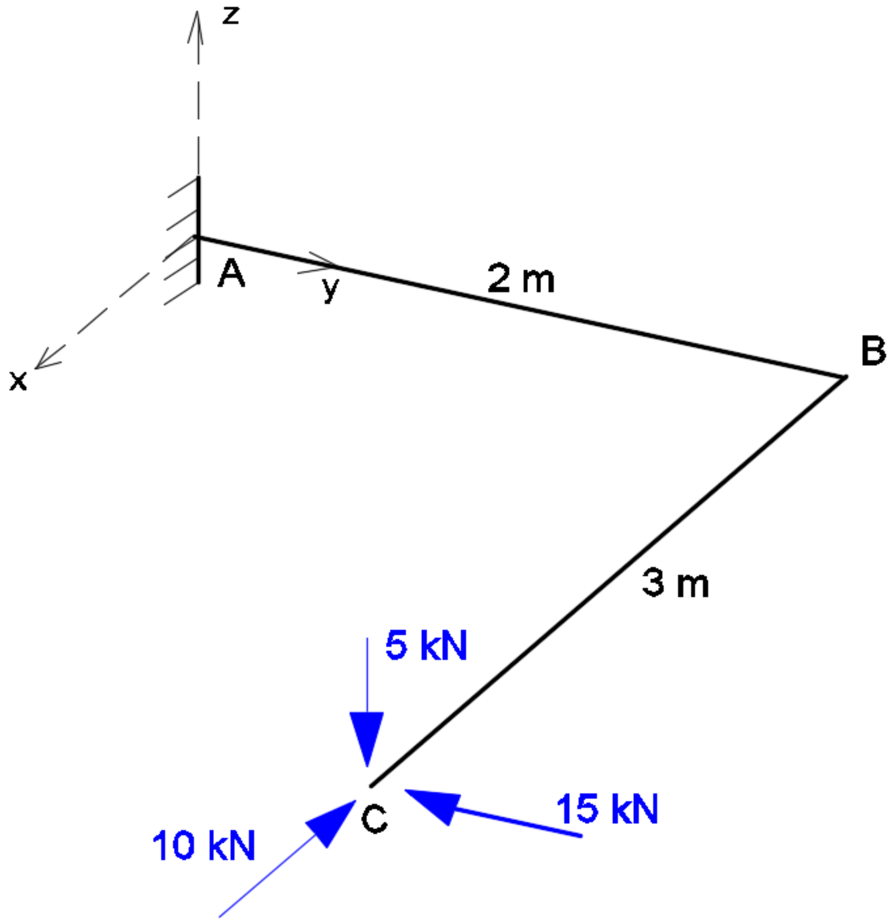
Visualizar a barra no plano pelo lado que está o eixo ortogonal positivo

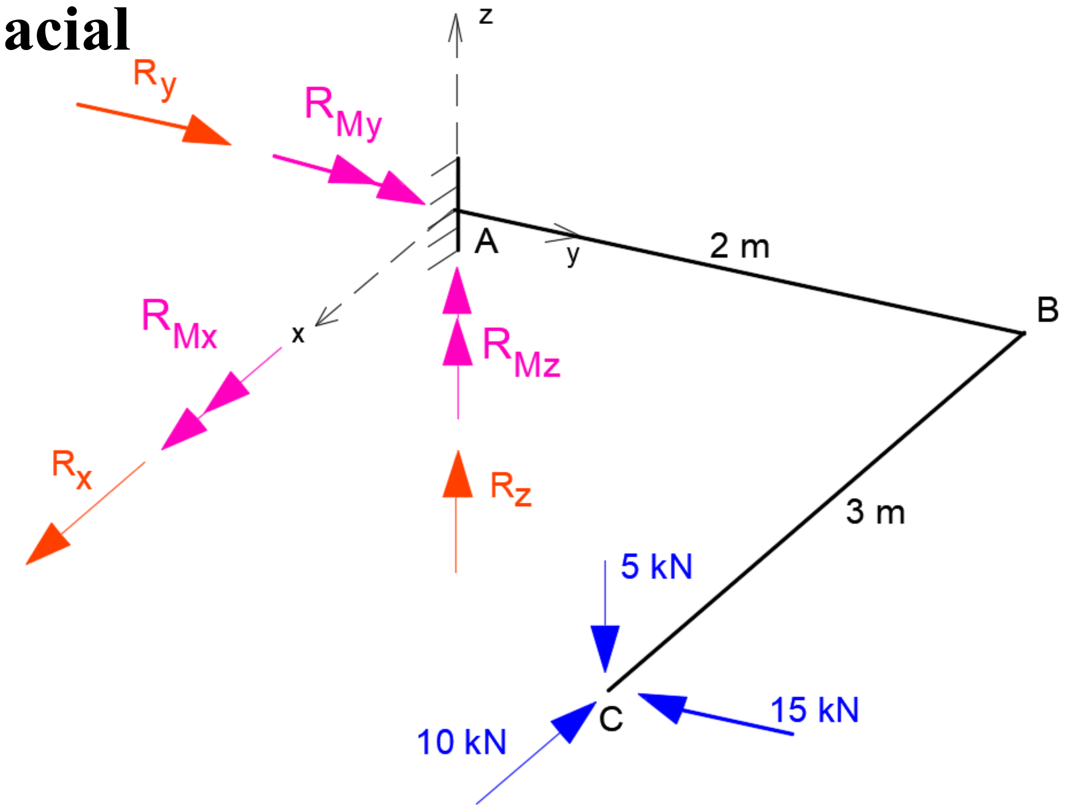






Exemplo 3: Determinar os esforços na estrutura espacial

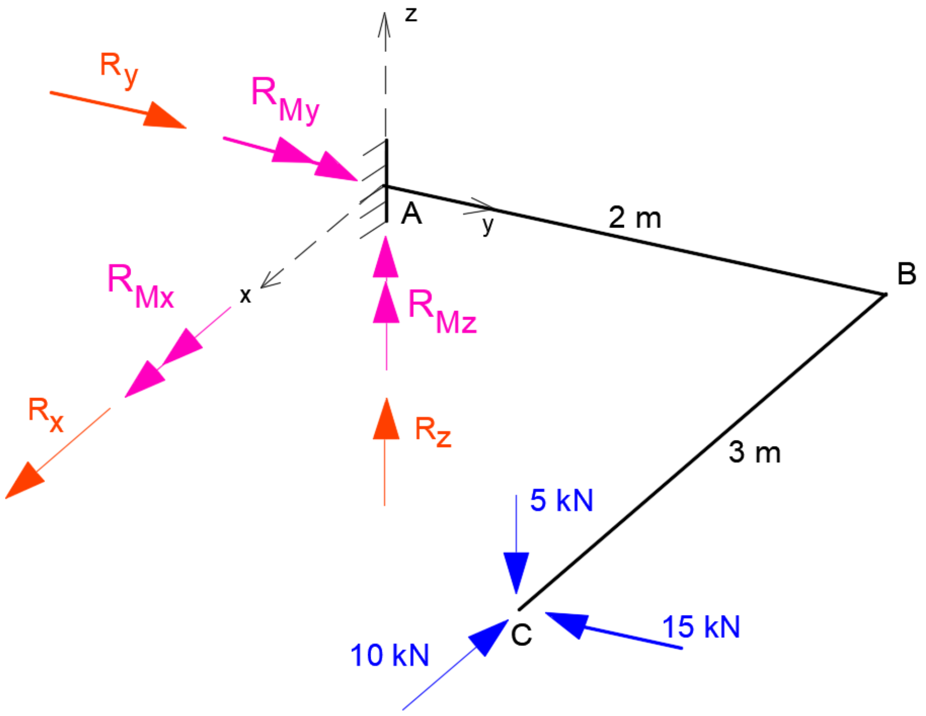




$$\sum F_x = 0: R_x - 10 = 0 \rightarrow R_x = 10\ kN$$

$$\sum F_y = 0: R_y - 15 = 0 \rightarrow R_y = 15\ kN$$

$$\sum F_z = 0: R_z - 5 = 0 \rightarrow R_z = 5\ kN$$



$$M_o = M_x i + M_y j + M_z k$$

$$M_x = (y_p - y_o)F_z - (z_p - z_o)F_y$$

$$M_y = (z_p - z_o)F_x - (x_p - x_o)F_z$$

$$M_z = (x_p - x_o)F_y - (y_p - y_o)F_x$$

$$\mathbf{F_x = -10\ kN; F_y = -15\ kN; F_z = -5kN}$$

$$\mathbf{x_p - x_0 = 3m} \qquad \mathbf{y_p - y_0 = 2m} \qquad \mathbf{z_p - z_0 = 0}$$

$$R_{My} + \sum_{i=1}^{\text{nr. forças}} (M_y)_i = 0$$

$$R_{My} + (0m)(-10kN) - (3m)(-5kN) = 0$$

$$\mathbf{R_{My} = -15\ kNm}$$

$$R_{Mx} + \sum_{i=1}^{\text{nr. forças}} (M_x)_i = 0$$

$$R_{Mx} + (2m)(-5kN) - (0m)(-15kN) = 0$$

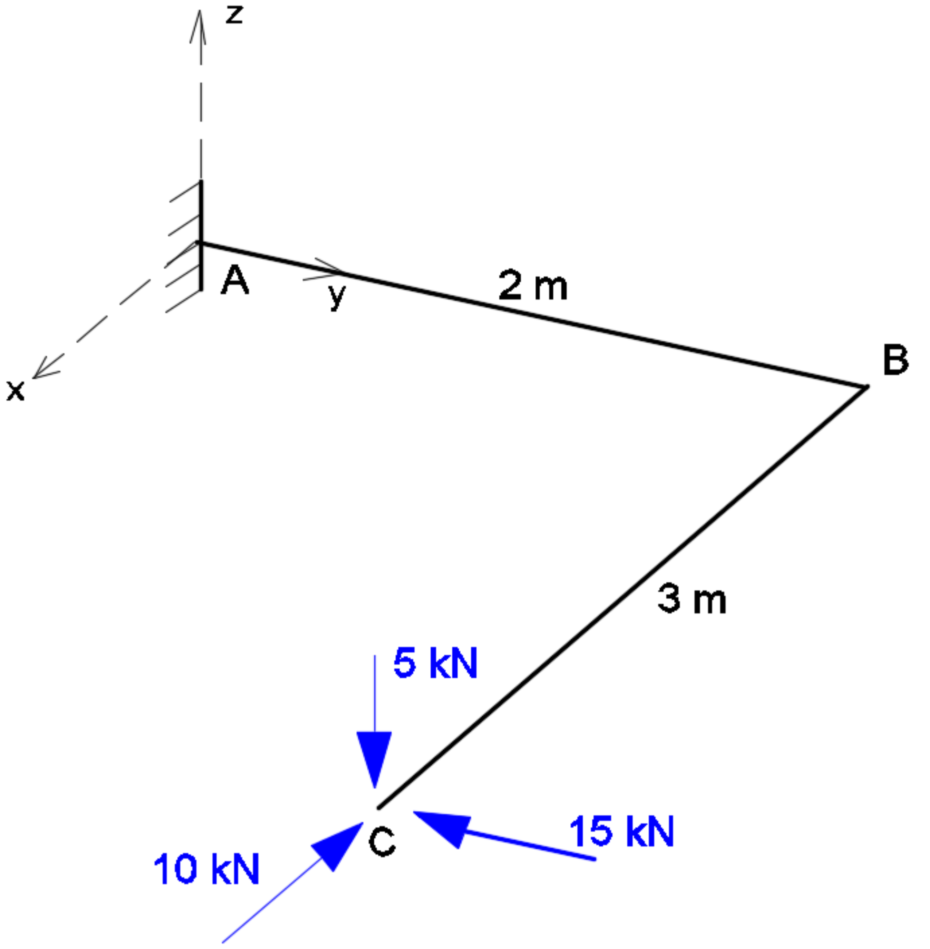
$$\mathbf{R_{Mx} = 10\ kNm}$$

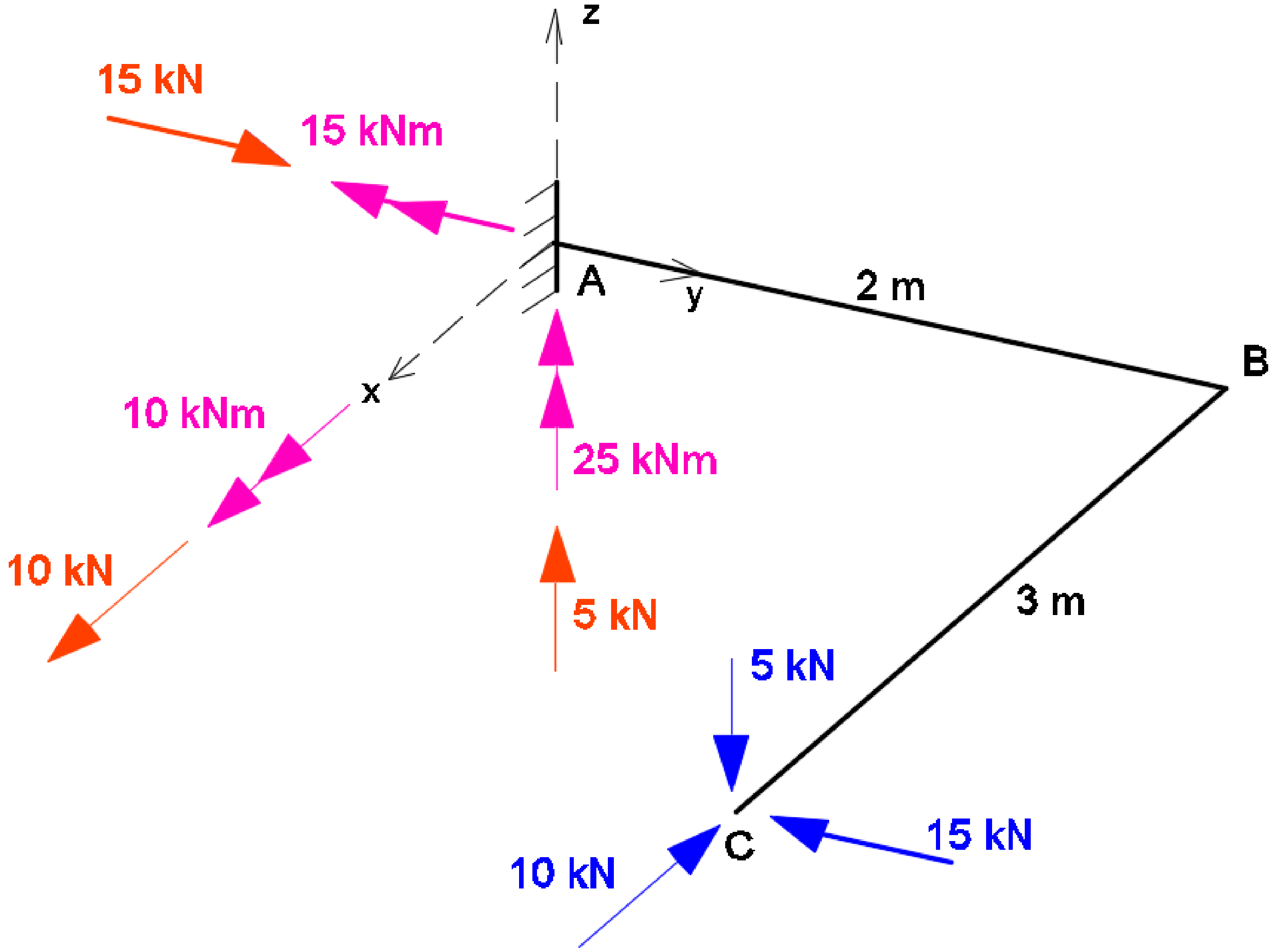
$$R_{Mz} + \sum_{i=1}^{\text{nr. forças}} (M_z)_i = 0$$

$$\mathbf{R_{My} = 25\ kNm}$$

$$R_{Mz} + (3m)(-15kN) - (2m)(-10kN) = 0$$

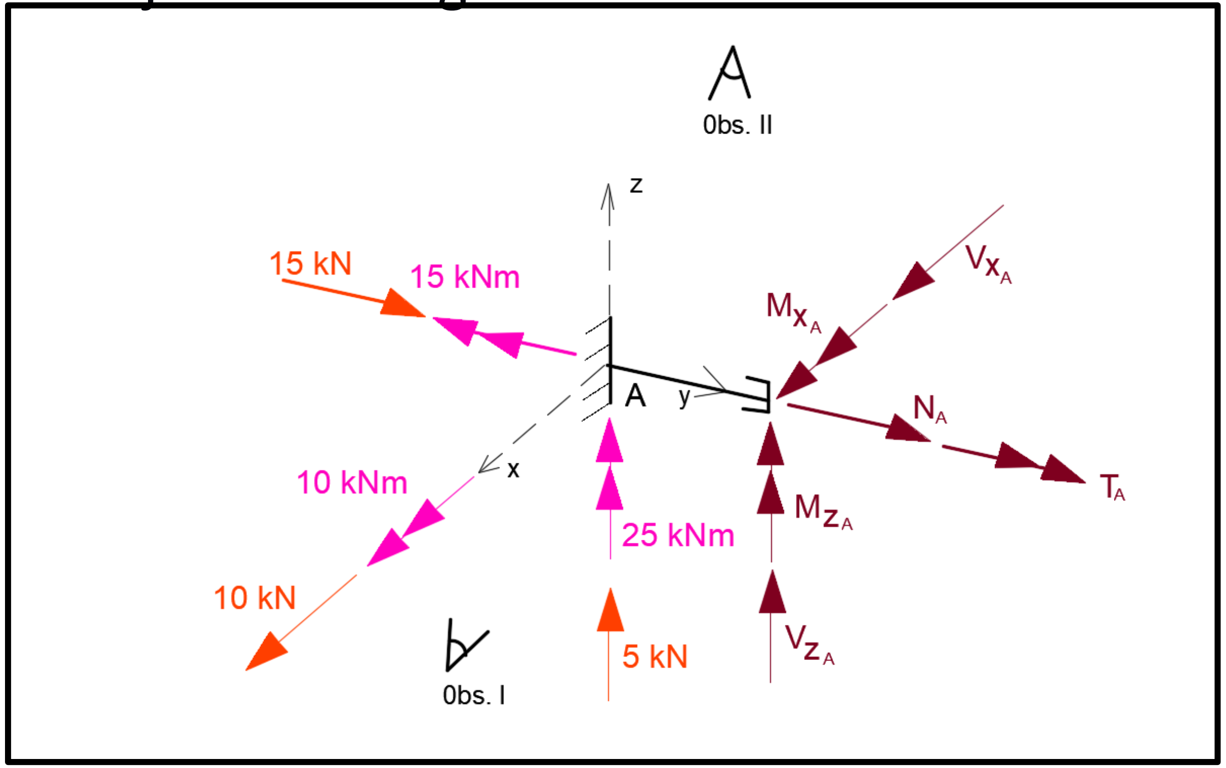
Reações


z
x
A
y
2 m
B
3 m
5 kN
C
10 kN
15 kN


z
15 kN
15 kNm
A
y
2 m
B
x
10 kNm
25 kNm
10 kN
5 kN
3 m
5 kN
C
10 kN
15 kN

Esforços

a) Corte junto ao engaste em A



$\Sigma M_{x_A} = 0$

$M_{x_A} + 10 = 0$

$M_{x_A} = -10\ kNm$

$\Sigma M_{y_A} = 0$

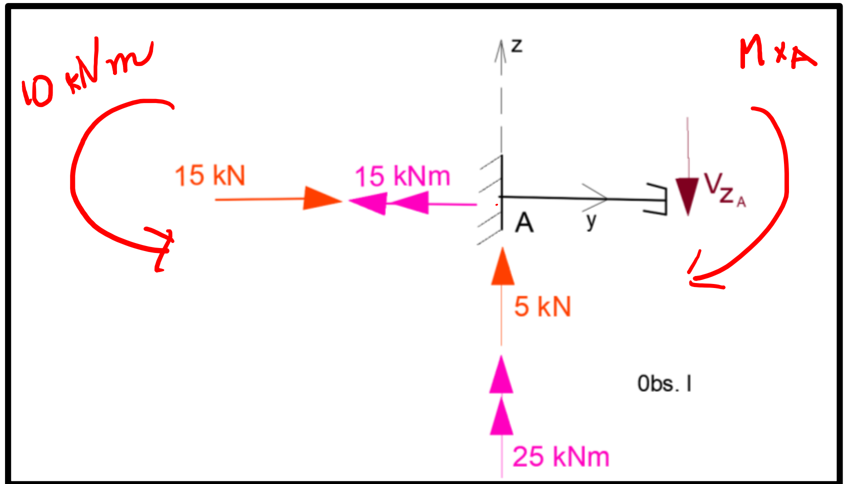
$T_A - 15 = 0$

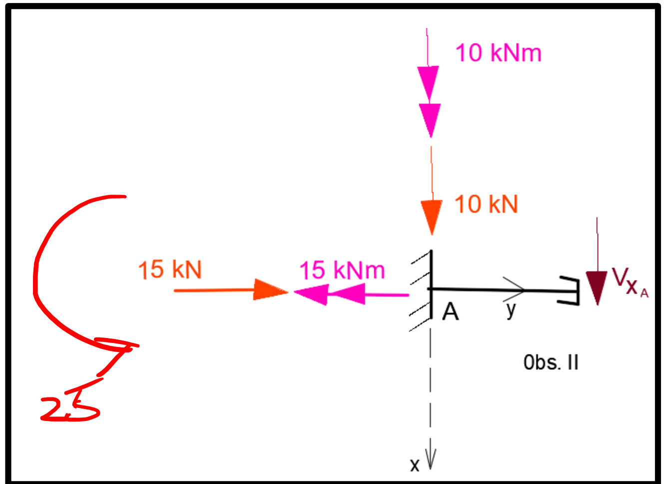
$T_A = 15\ kNm$

$\Sigma M_{z_A} = 0$

$M_{z_A} + 25 = 0$

$M_{z_A} = -25\ kNm$





$\Sigma F_z = 0$

$V_{z_A} - 5 = 0$

$V_{z_A} = 5\ kN$

$\Sigma F_x = 0$

$V_{x_A} + 10 = 0$

$V_{x_A} = -10$

$\Sigma F_{y_A} = 0$

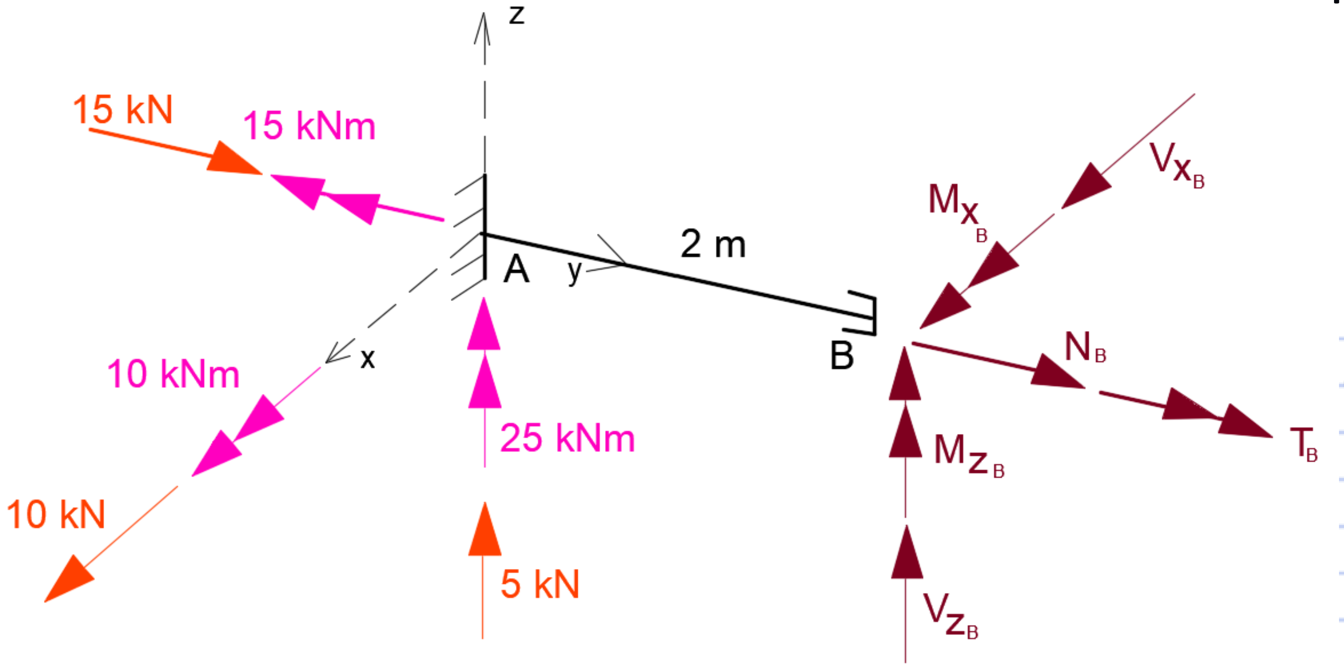
$N_A + 15 = 0$

$N_A = -15\ kN$

Exemplo 3

b) Corte junto a B

N_b, V_{zb}, V_{xb}: idem à seção em A



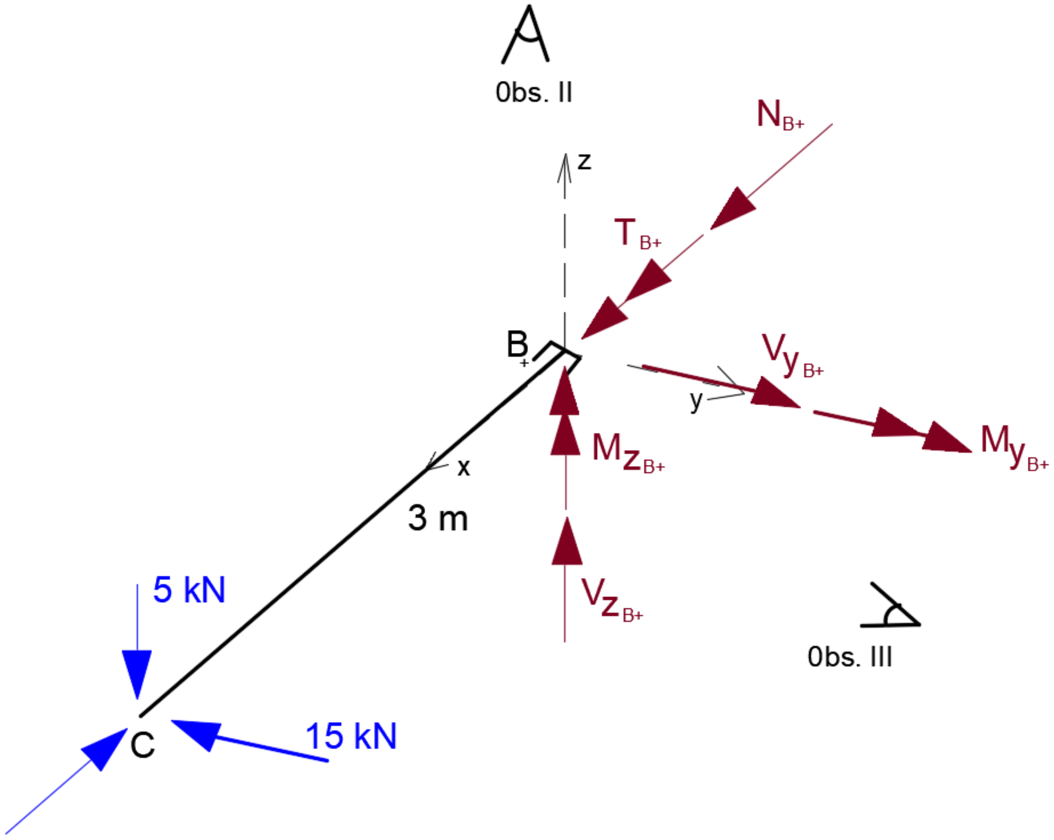
$$\Sigma M_y = 0$$
$$T_B - 15 = 0$$
$$T_B = 15 \text{ kNm}$$

$$\Sigma M_x = 0$$
$$M_{x_B} + 10 - 5 \cdot 2 = 0$$
$$M_{x_B} = 0$$

$$\Sigma M_{z_B} = 0 :$$
$$M_{z_B} + 25 + (10) \cdot 2 = 0$$
$$M_{z_B} = -45 \text{ kNm}$$

Exemplo 3

c) Corte junto a B+



$$\Sigma M_{y_{B+}} = 0$$

$$M_{y_{B+}} + 5 \cdot 3 = 0$$

$$M_{y_{B+}} = -15\,kNm$$

$$\Sigma M_{z_{B+}} = 0$$

$$M_{z_{B+}} - 15 \cdot 3 = 0$$

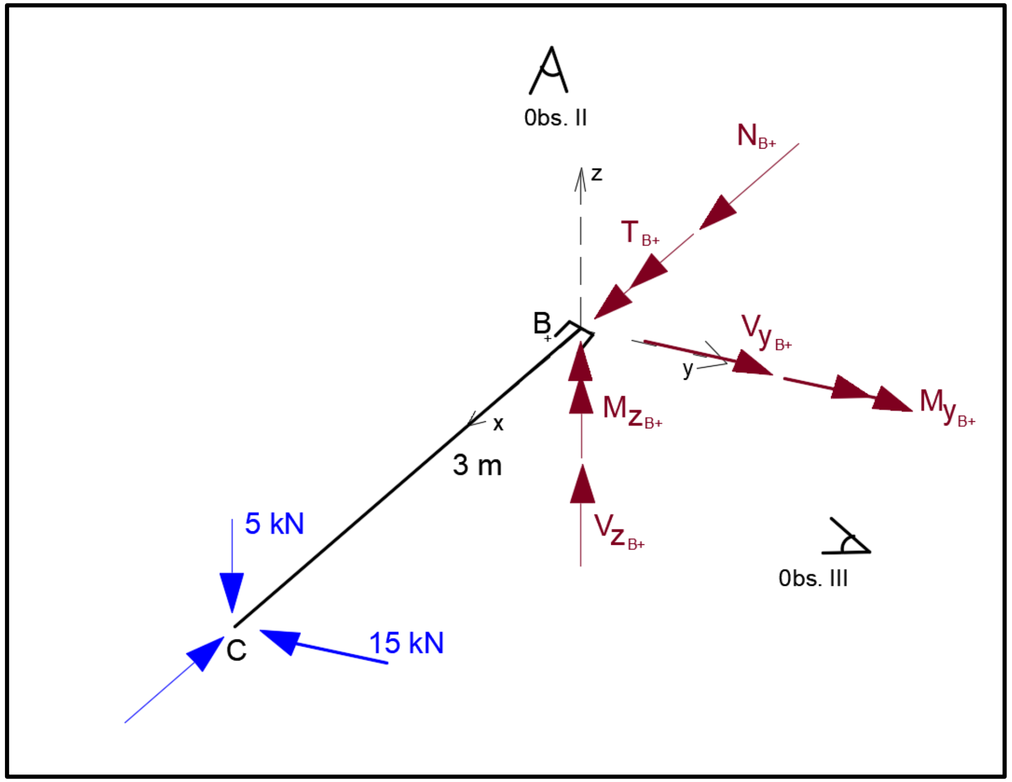
$$M_{z_{B+}} = 45\,kNm$$

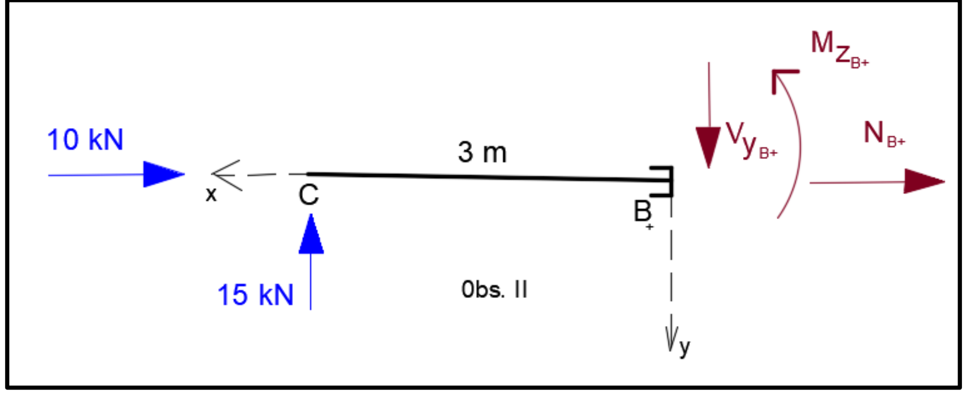
$$\Sigma M_{x_{B+}} = 0$$

$$T_{B+} = 0$$

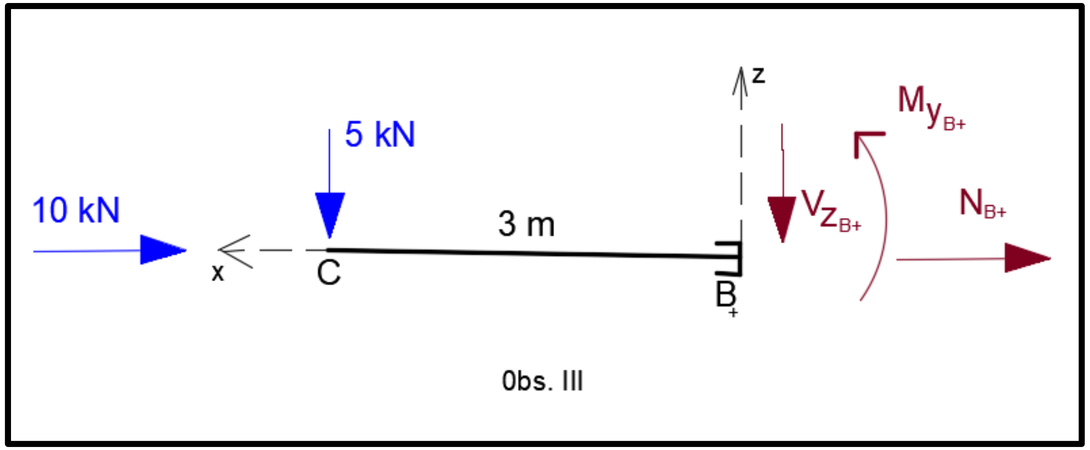
Exemplo 3

c) Corte junto a B+



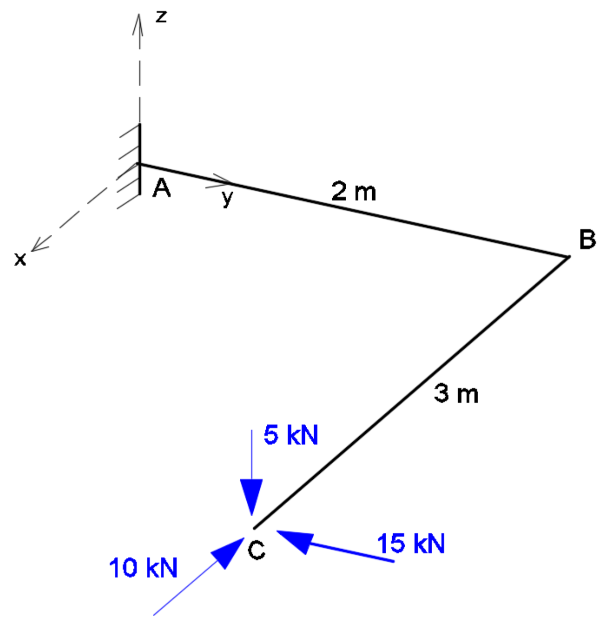


$$\Sigma F_x = 0$$
$$N_{B+} = -10\,kN$$
$$\Sigma F_y = 0$$
$$V_{y_{B+}} = 15\,kN$$
$$\Sigma M_{B+} = 0$$
$$M_{z_{B+}} = 15 \cdot 3 = 45\,kNm$$

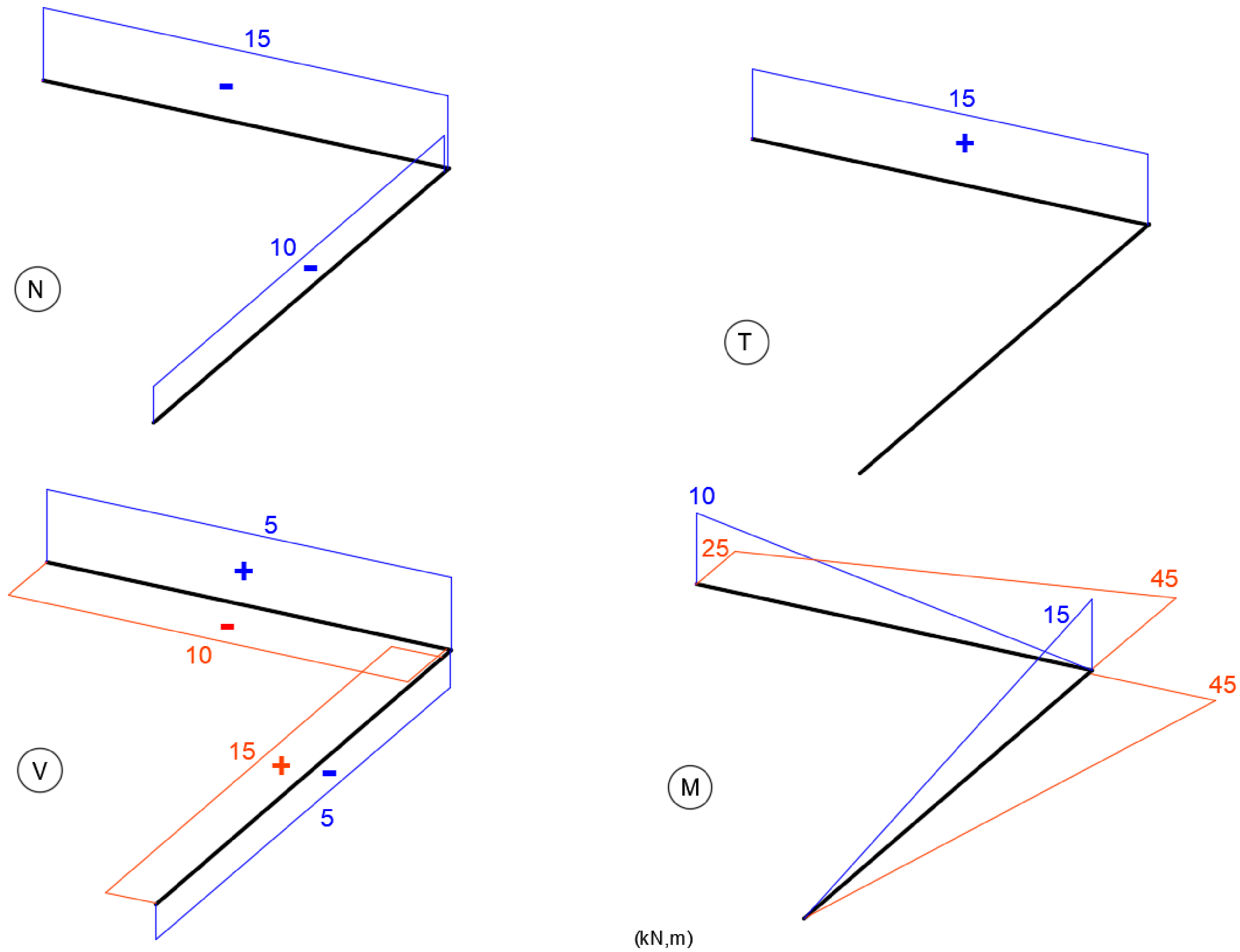


$$\Sigma F_z = 0$$
$$V_{z_{B+}} = -5\,kN$$
$$\Sigma M_{y_{B+}} = 0$$
$$M_{y_{B+}} = -3 \cdot 5 = -15\,kNm$$

Exemplo 3

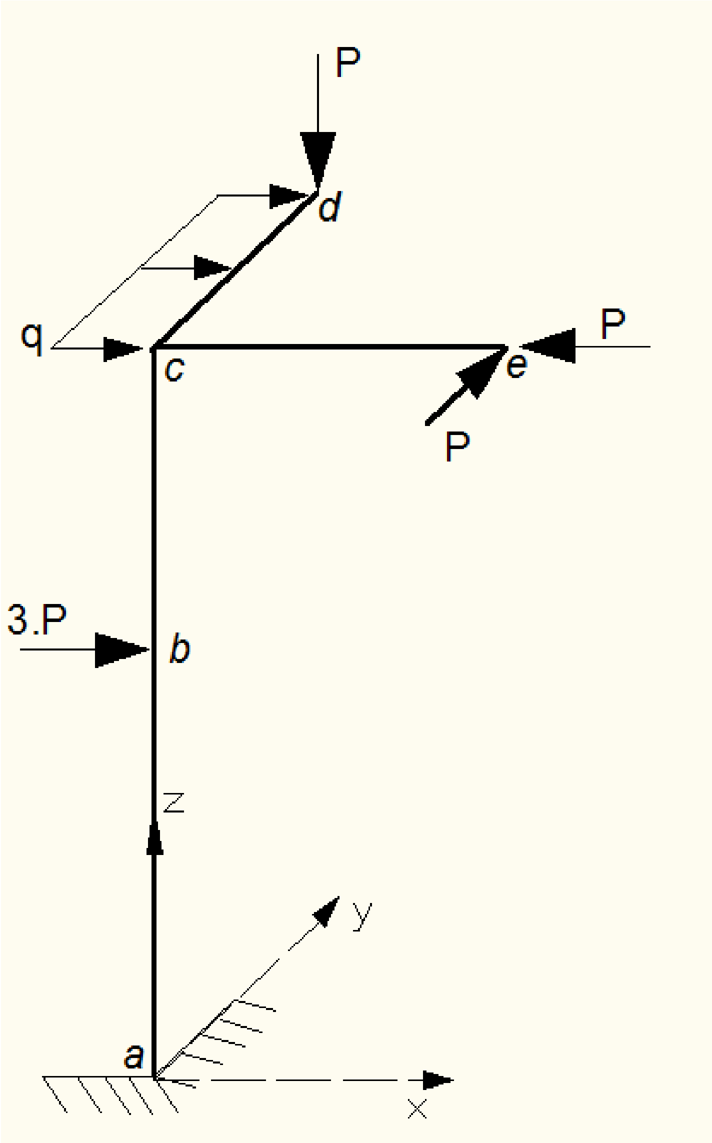


d) Diagramas



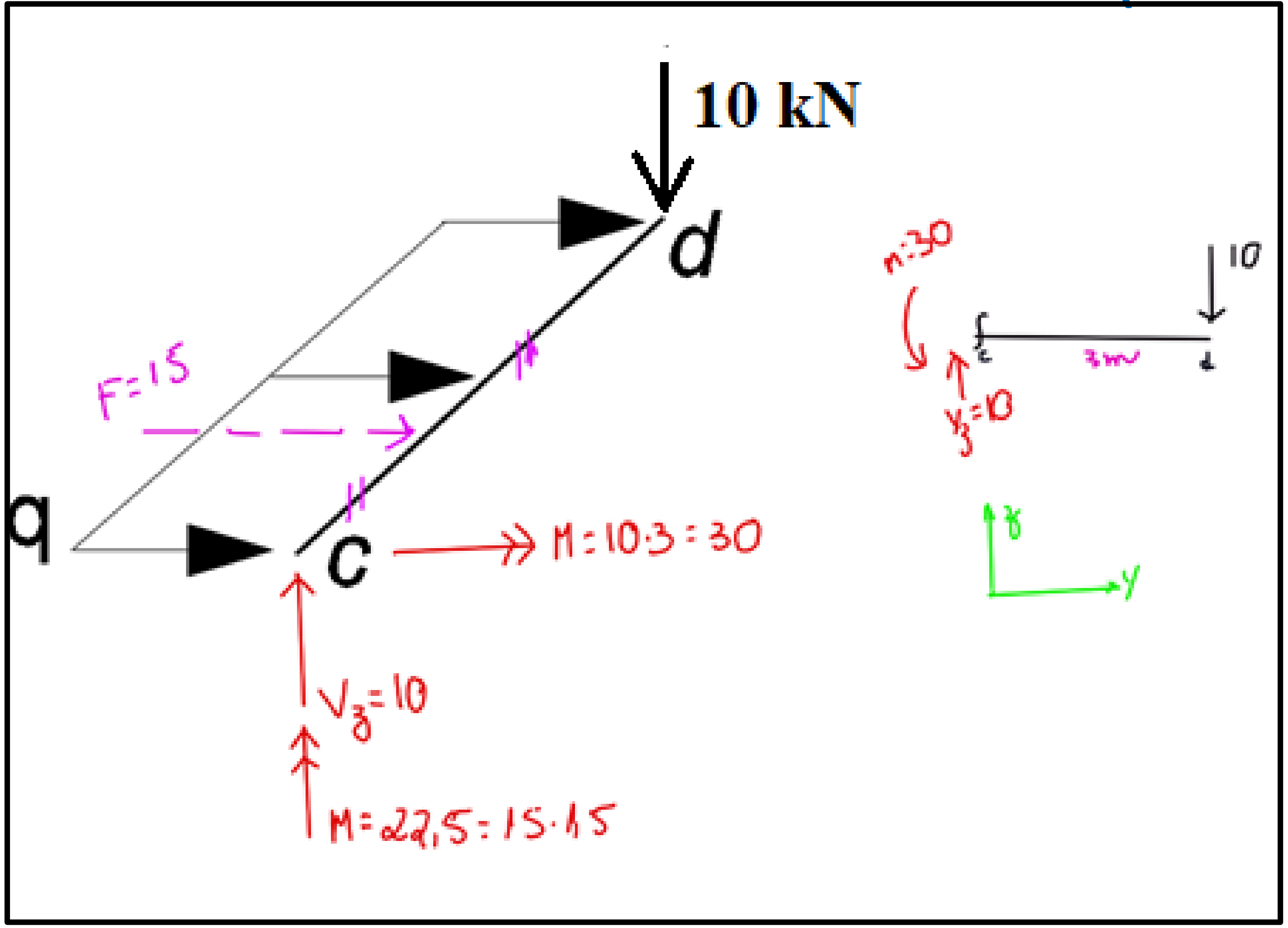
Exemplo 4

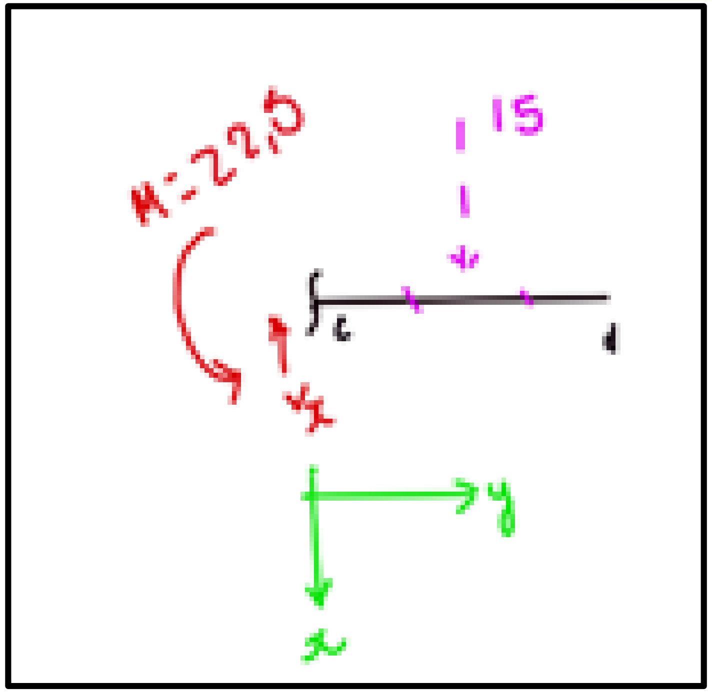
Determinar os esforços solicitantes (M,V, T e N) no pórtico tridimensional. As forças são paralelas aos eixos do sistema xyz, conforme indicado. Dados: As coordenadas dos pontos são, em metros: a(0;0;0), b(0;0;4), c(0;0;8), d(0;3;8), e(2;0;8). P = 10 kN; q = 5 kN/m.

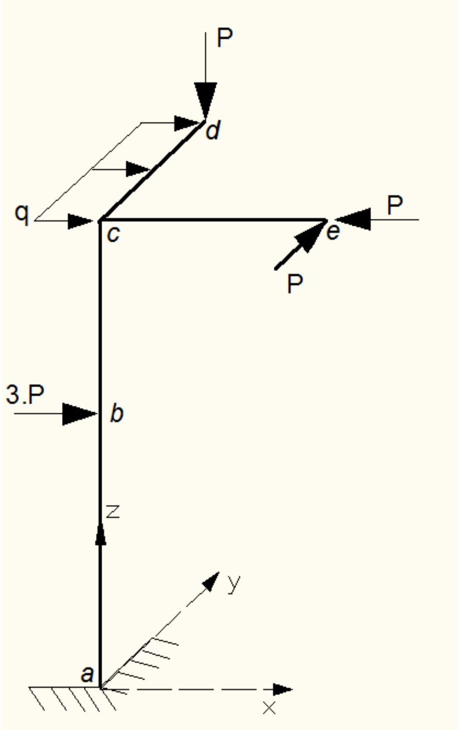


Exemplo 4

Corte em c (trecho cd)

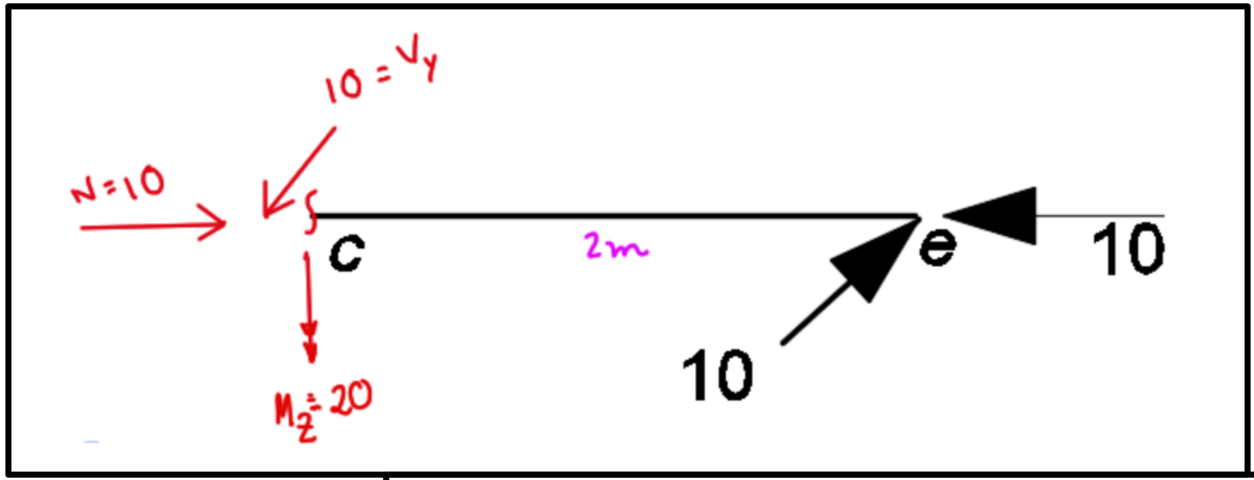




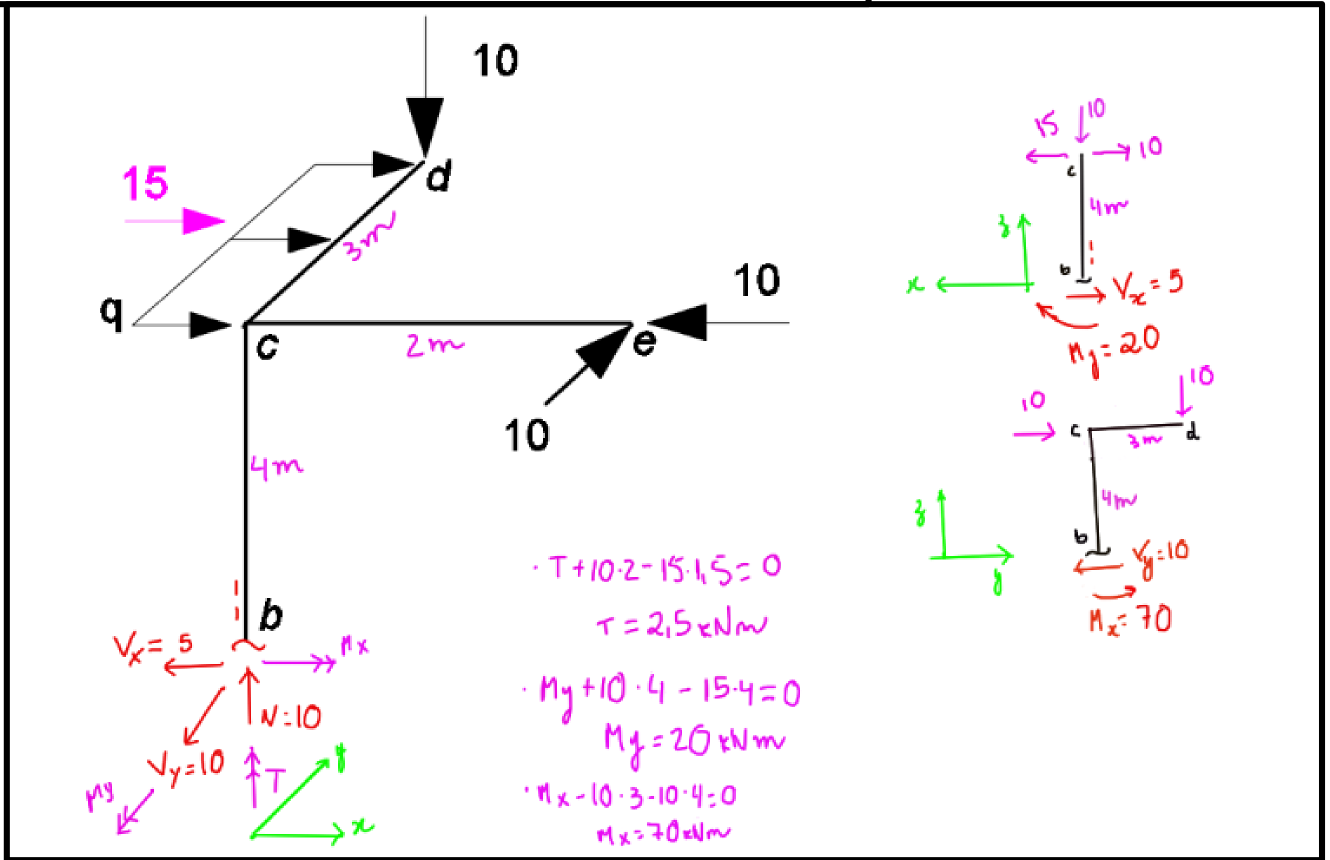


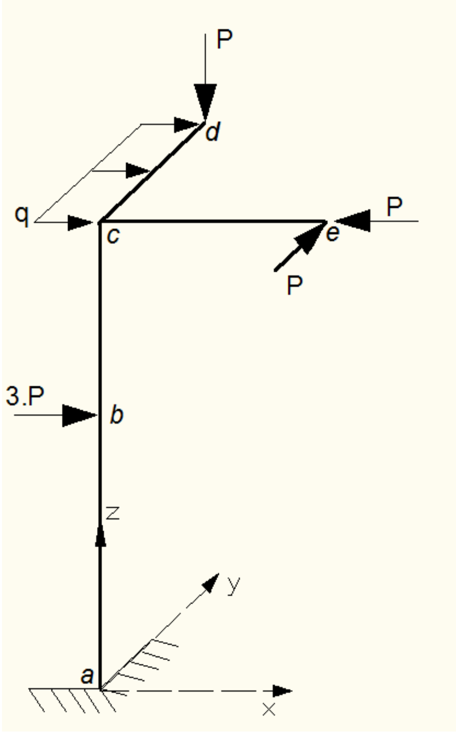
Exemplo 4

Corte em c (trecho ce)



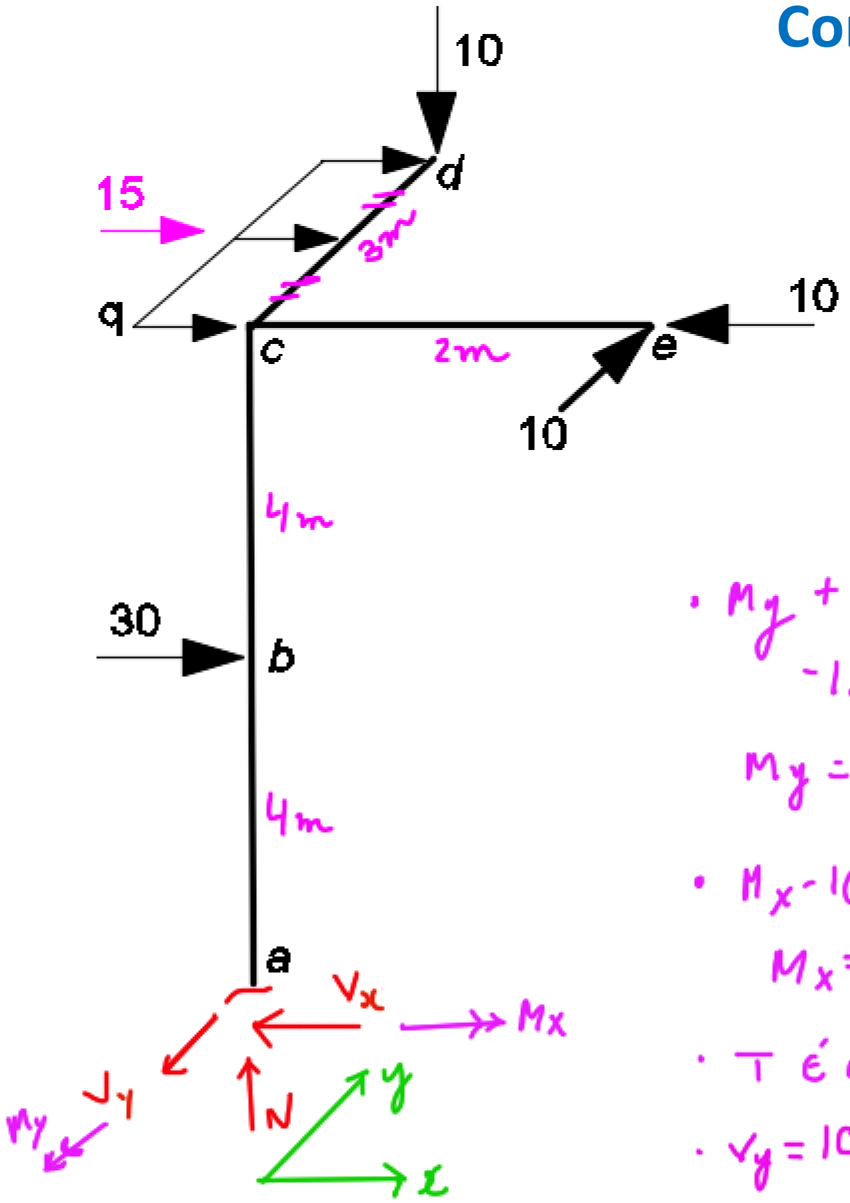
Corte em b

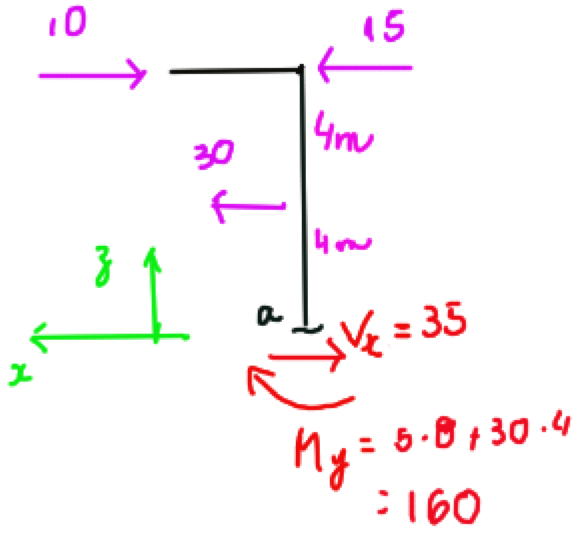


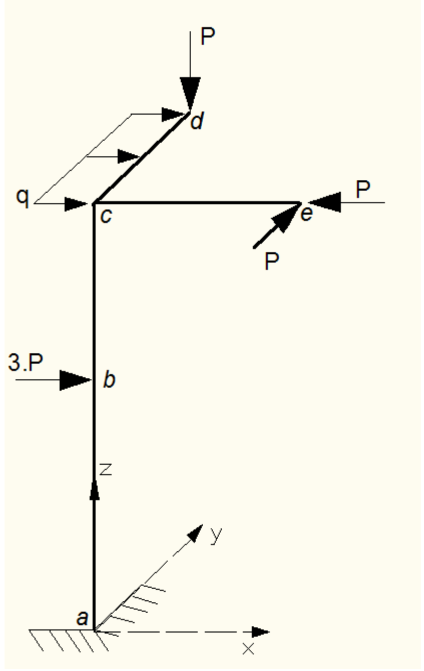


Exemplo 4

Corte em a







- $M_y + 10 \cdot 8 - 30 \cdot 4 - 15 \cdot 8 = 0$

  $M_y = 160$ kNm

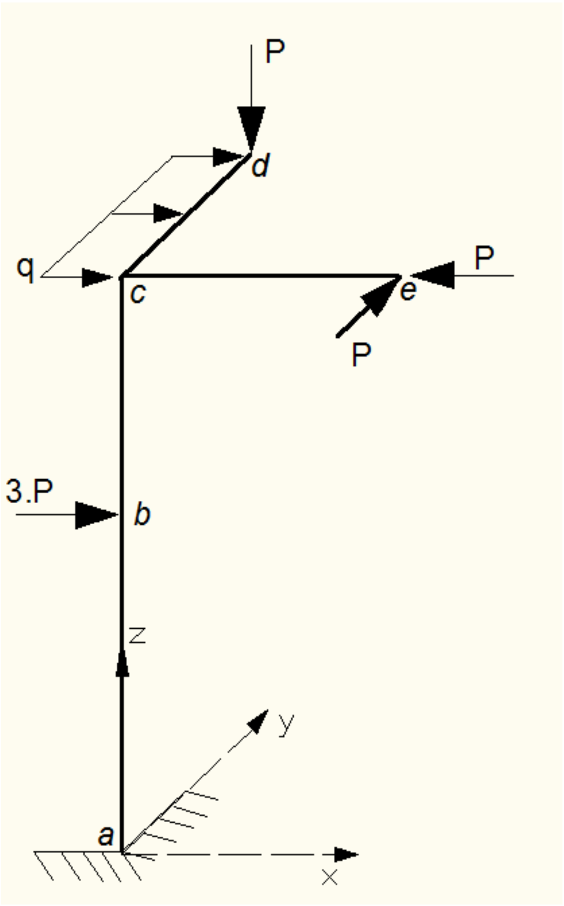
- $M_x - 10 \cdot 3 - 10 \cdot 8 = 0$

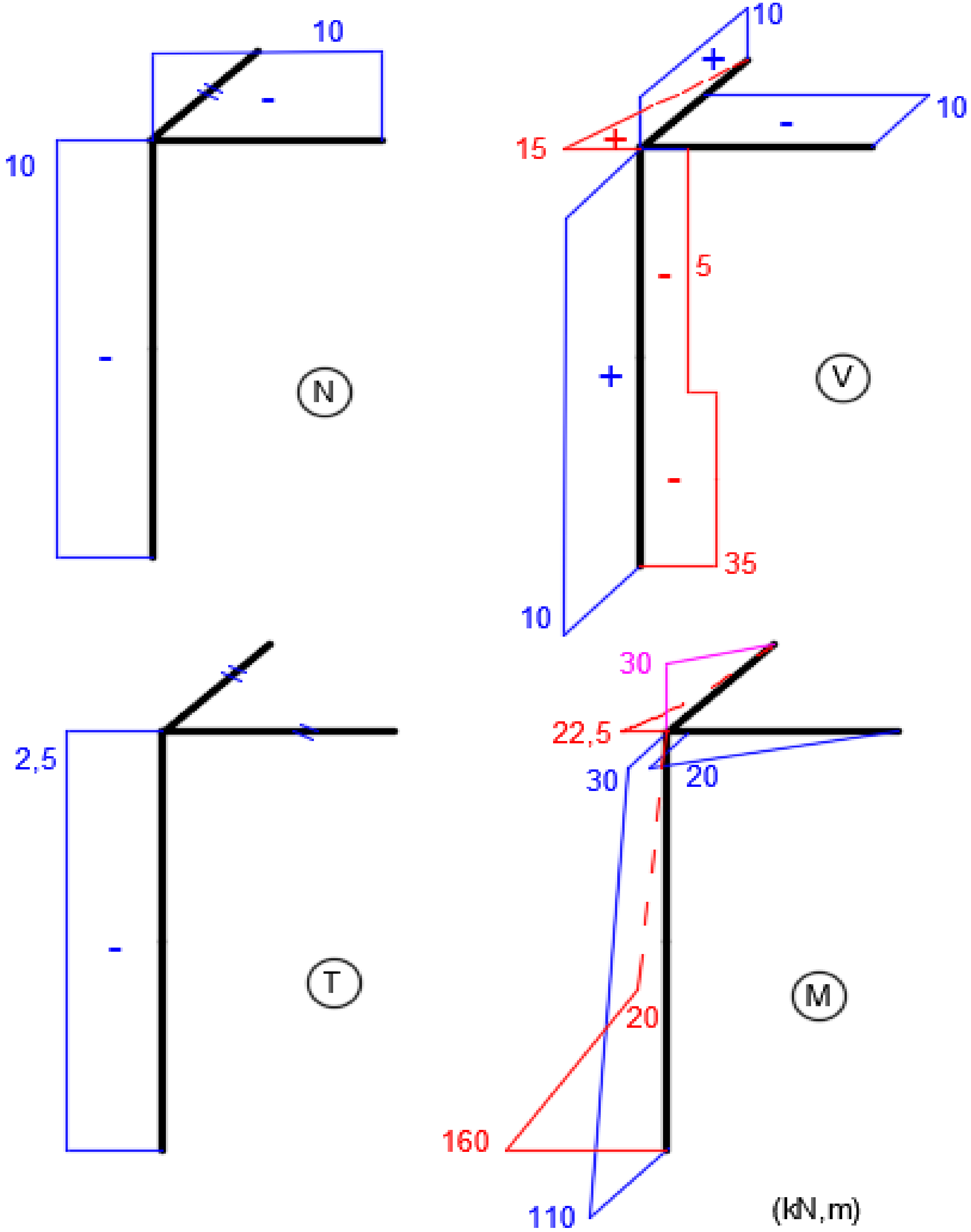
  $M_x = 110$ kNm

- T é CTE

- $V_y = 10$; $N = 10$ (CTE)

Exemplo 4

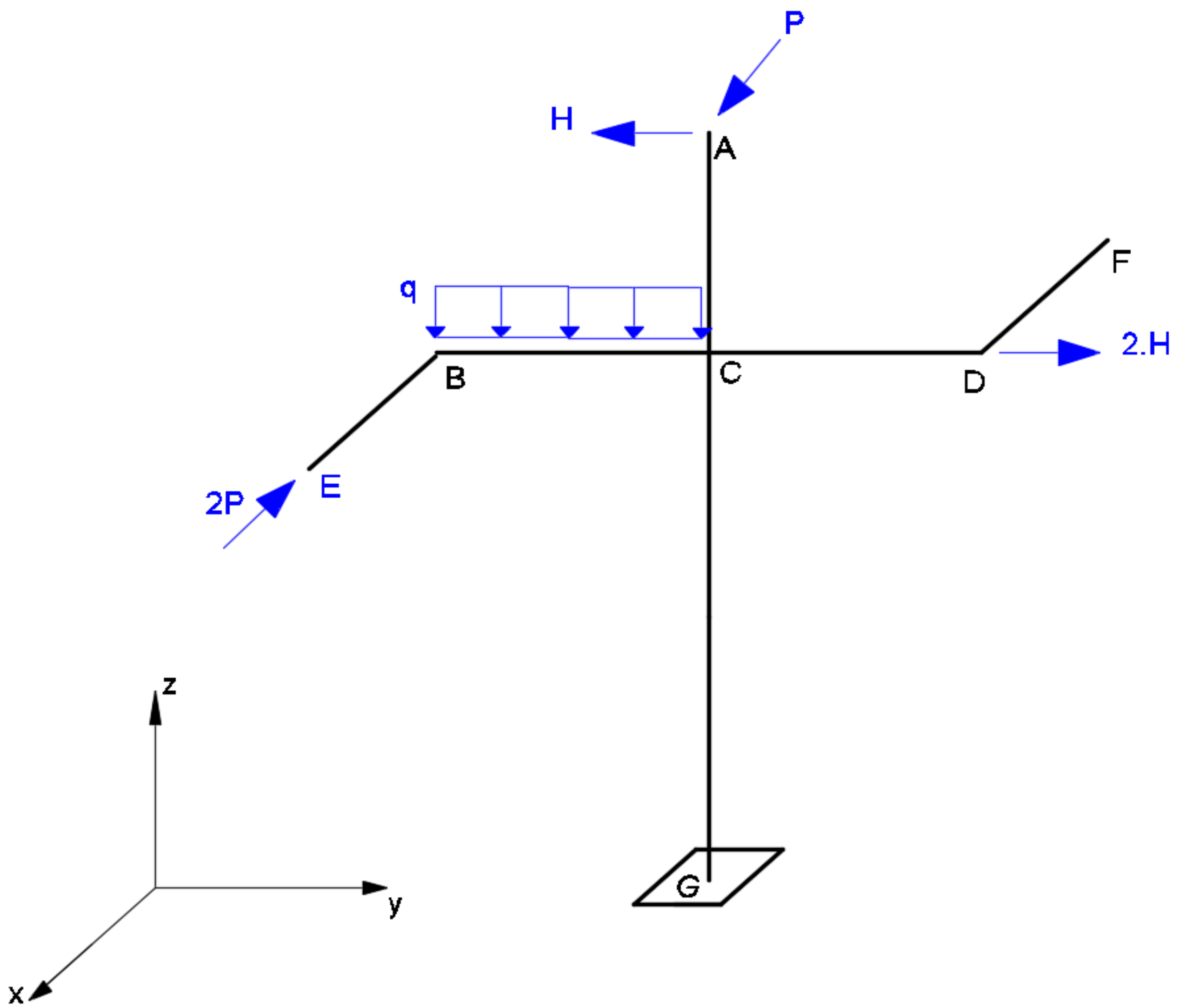




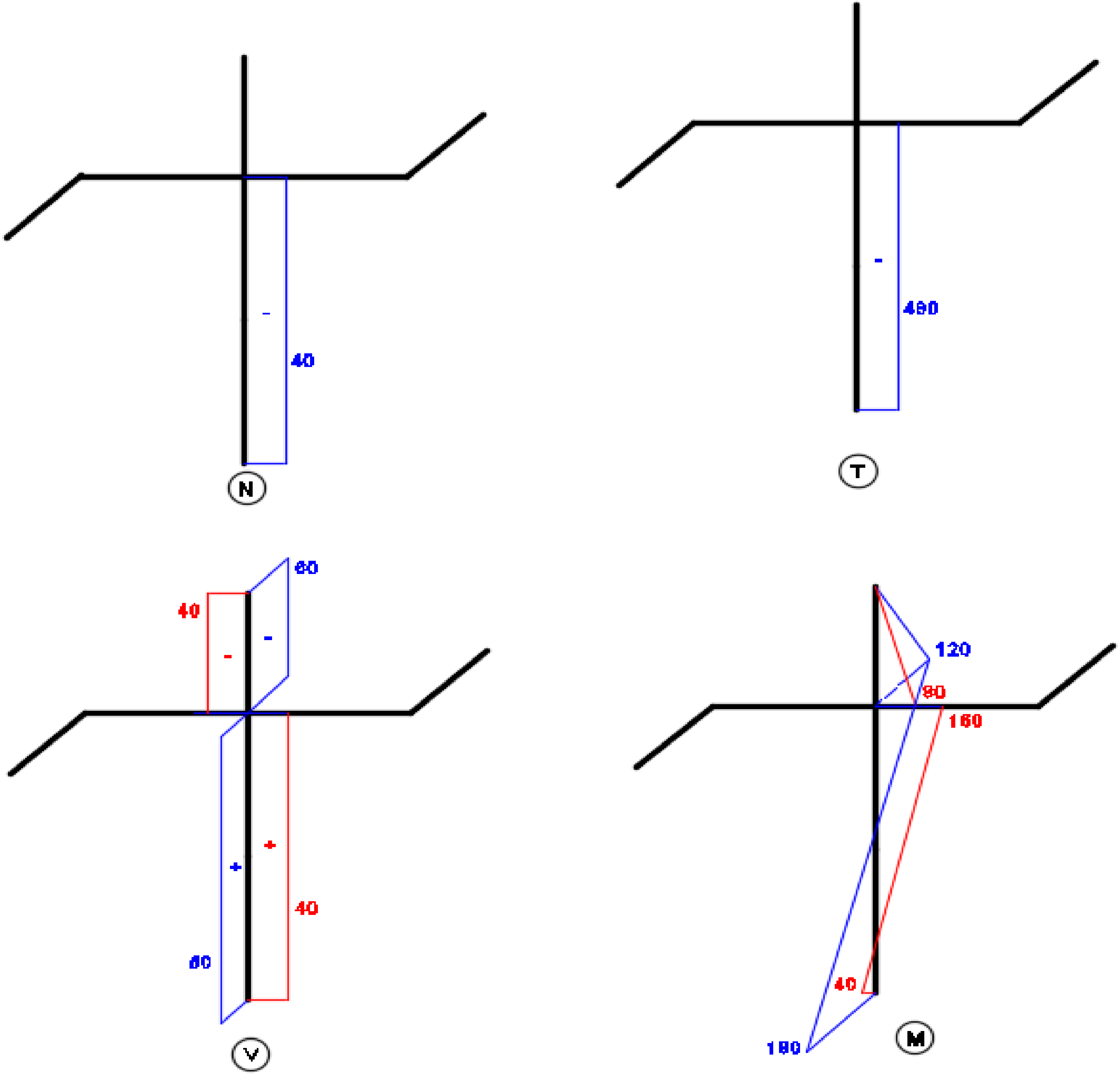
Diagramas

Exemplo 5

Determine para a estrutura a seguir suas reações no engaste G, e os diagramas de esforços nas barras AC e CG. Admita que as barras e forças são paralelas aos eixos do sistema xyz, conforme indicado. Sabe-se que as medidas das barras são: AC = 2m; CG = 5m; EB = DF = 3m; BC = CD = 4m. Adote: H = 40 kN; P = 60 kN e q = 10 kN/m.

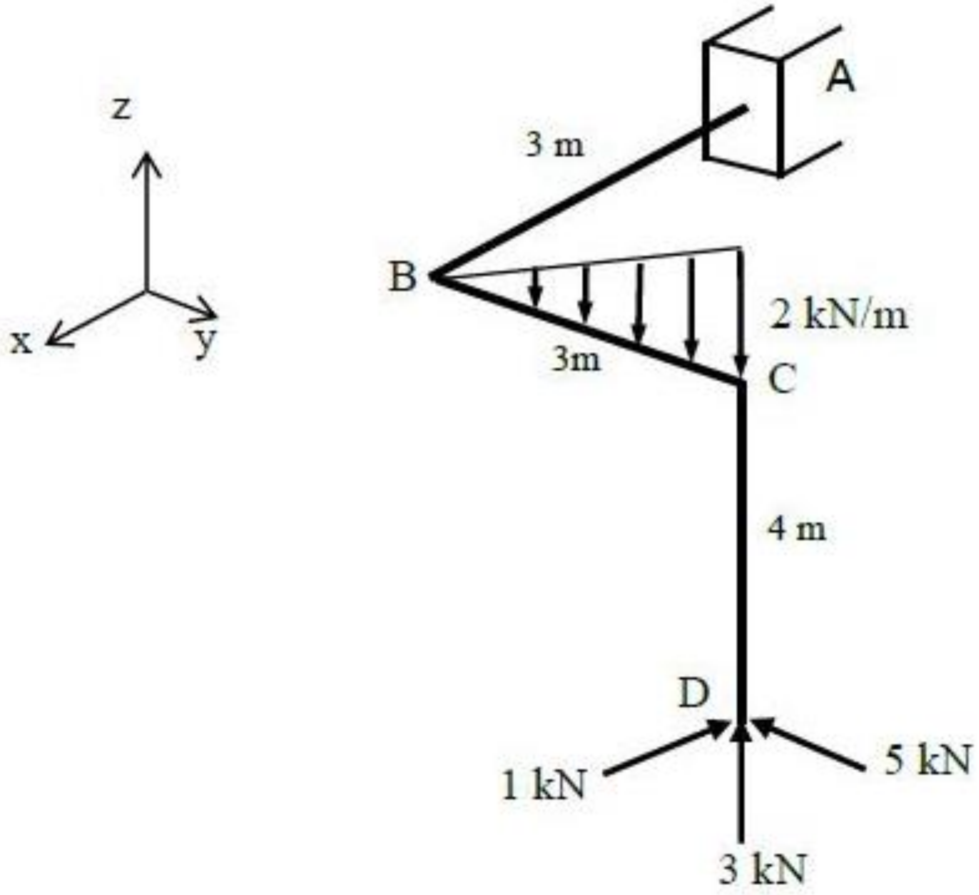


Diagramas

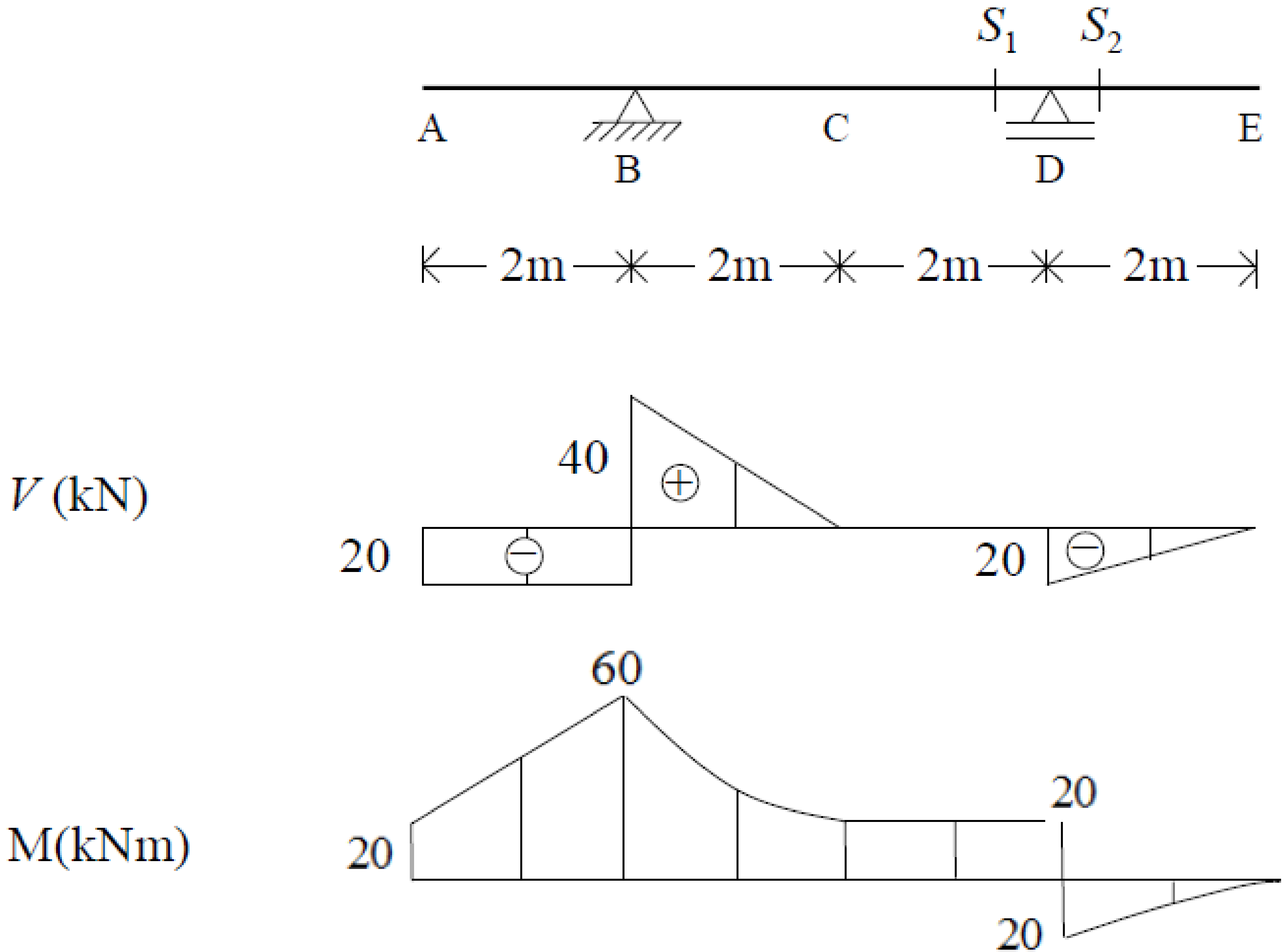


(kN,m)

ª. Questão: Determinar os diagramas dos esforços solicitantes no trecho BC da estrutura espacial ABCD da figura. As barras AB, BC e CD são ortogonais entre si e estão na direção dos eixos. As forças ativas (aplicadas na direção dos eixos) são a força distribuída uniformemente variada de 0 a 2 kN/m em BC e as forças concentradas aplicadas na extremidade livre D. Considere os observadores em frente aos eixos.



Exemplo: A partir dos diagramas de esforços solicitantes, determinar os esforços externos que atuam na viga



Equações Diferenciais de Equilíbrio

$$\frac{dN(x)}{dx} = -p_x(x) \qquad \frac{dV(x)}{dx} = -p(x) \qquad \frac{d^2M(x)}{dx^2} = -p(x) \qquad \frac{dM(x)}{dx} = V(x)$$

a) Caso $p(x) = 0$

Sem carga distribuída no trecho $x_1 < x < x_2$

$V(x) = C_1 = cte \rightarrow$ Função (diagrama) de esforço cortante constante

$M(x) = C_1 \cdot x + C_2 \rightarrow$ Função (diagrama) de momento fletor linear

b) Caso $p(x) = p = cte$

Carga distribuída uniforme no trecho $x_1 < x < x_2$

$V(x) = -\,p x + C_1 \rightarrow$ Função (diagrama) de esforço cortante linear

$M(x) = -\,p\frac{x^2}{2} + C_1 \cdot x + C_2 \rightarrow$ Função (diagrama) de momento fletor é parábola

c) Generalização para ∀ p(x) é imediata

d) Caso $p_x(x)=0$

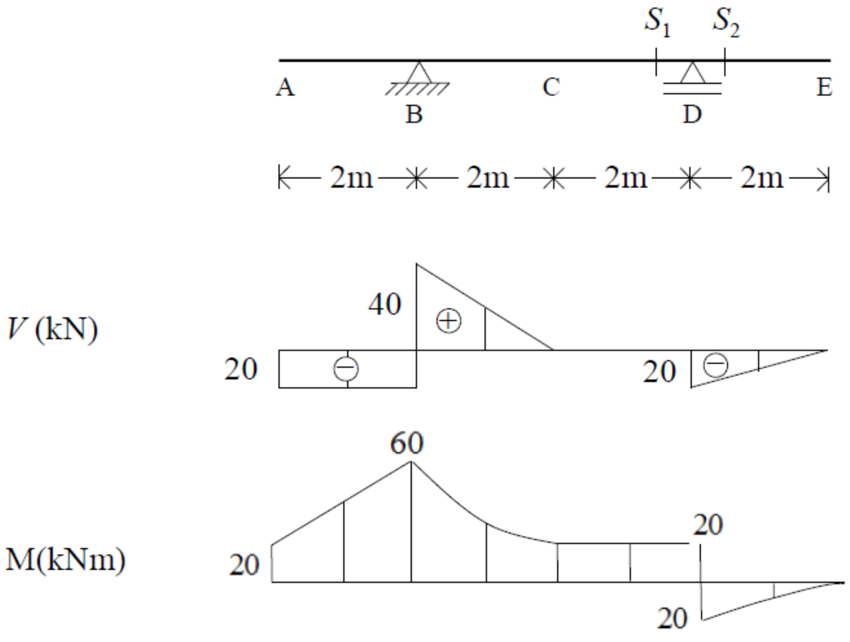
Sem carga distribuída no trecho $x_1 < x < x_2$

N(x) = constante

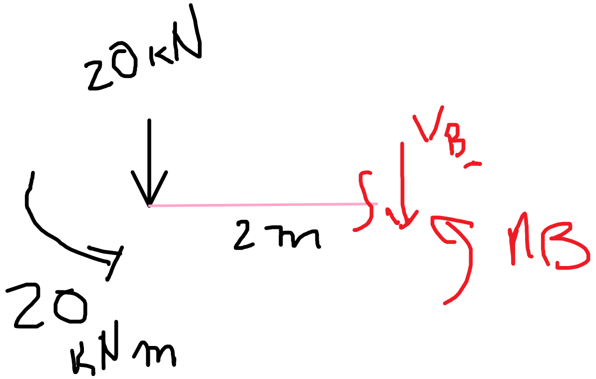
e) Caso $p_x(x)=cte$

Carga distribuída uniforme no trecho $x_1 < x < x_2$

N(x) = linear (reta)



Não há carga horizontal, sem diagramas de N



Trecho AB:

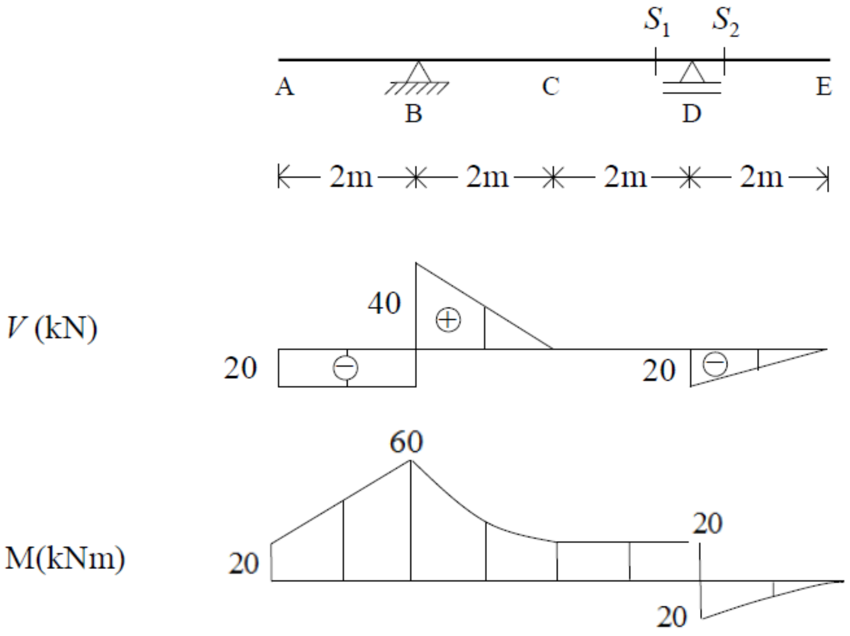
V: cte e M : linear: força concentrada. Em A, V = -20 kN, portanto Fvertical em A para baixo de 20 kN

Em A, M = 20 kNm, tração em cima, MA = 20 kNm

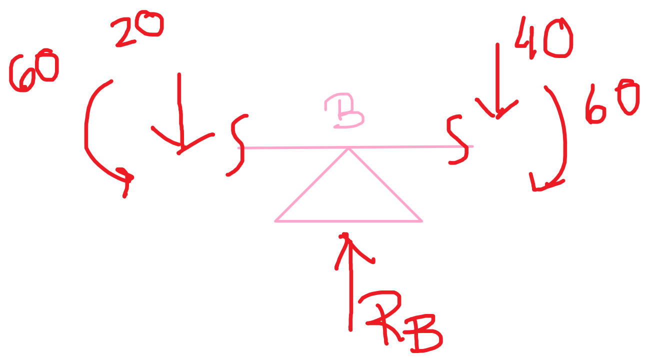
$V_{B-} = -20$

$M_B + 20 \cdot 2 + 20 = 0$

$M_B = -60$

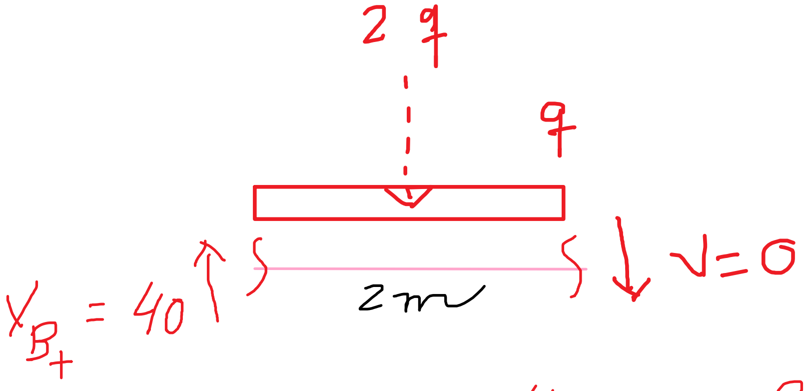


Equilíbrio junto a B



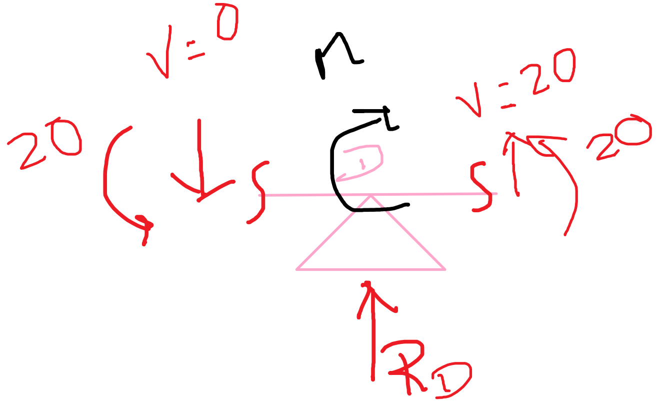
$R_B = 60\ kN$

Obter a carga q em BC



$\Sigma F_y = 0. \quad 2q = 40 \Rightarrow q = 20\ kN/m$

Equilíbrio junto a D

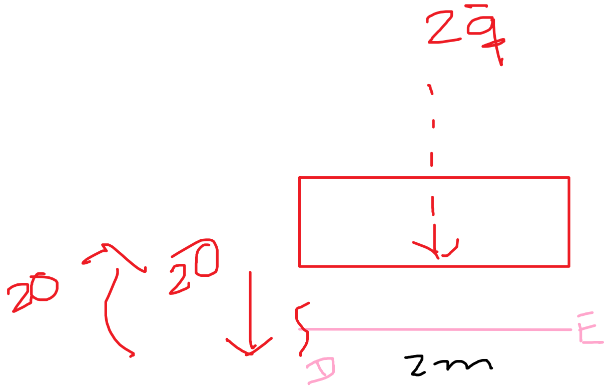


$R_D = -20$

$M - 20 \cdot 20 = 0$

$M = 40\ kNm$

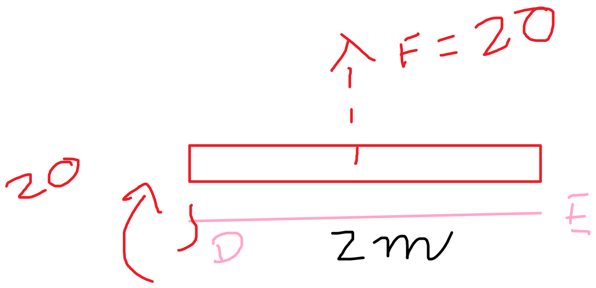
Trecho DE: obter q



$$\Sigma F_y = 0$$

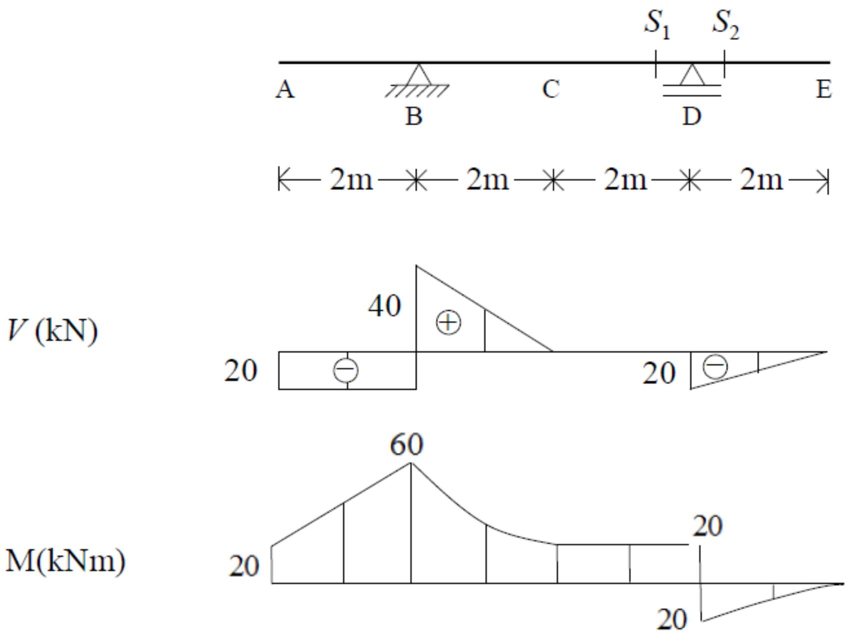
$$20 + 2\bar{q} = 0$$

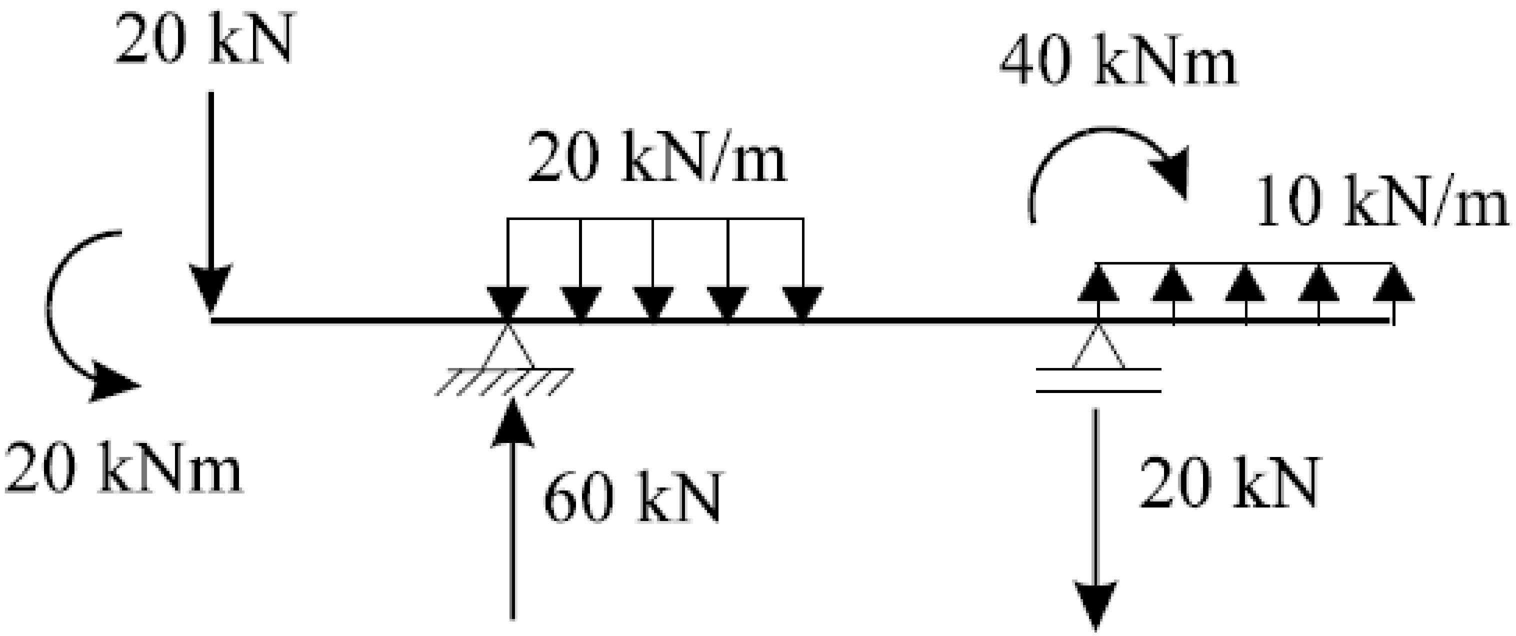
$$\bar{q} = -10\ kN/m$$



$$\Sigma M_D = 0$$

$$20 = 20 \ \text{(OK!)}$$


S1
S2
A
B
C
D
E
2m
2m
2m
2m
V (kN)
40
20
20
60
M(kNm)
20
20
20


20 kN
40 kNm
20 kN/m
10 kN/m
20 kNm
60 kN
20 kN