PEF3200

Aula 5

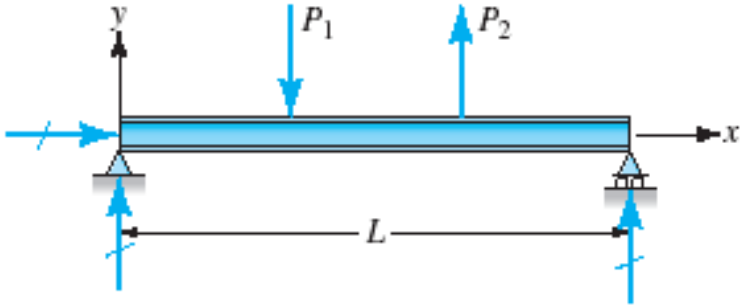
26 abr

PROF. NAKAO

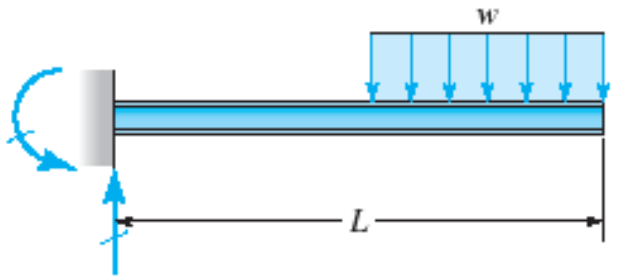
# AGENDA

- **Diagramas de esforços solicitantes de estruturas planas.**

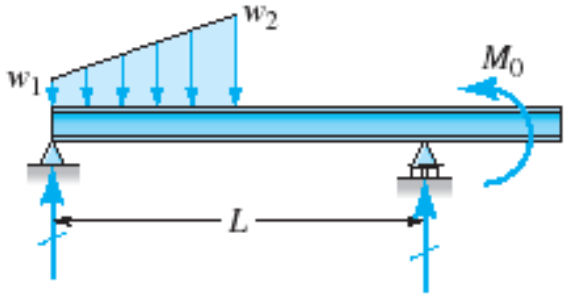
# Types of beams




Simple beam
(Viga biapoiada)




Cantilever beam
(Viga em balanço/engastada)




Overhang beam
(viga biapoiada com balanço)

# Example (cantilever beam)

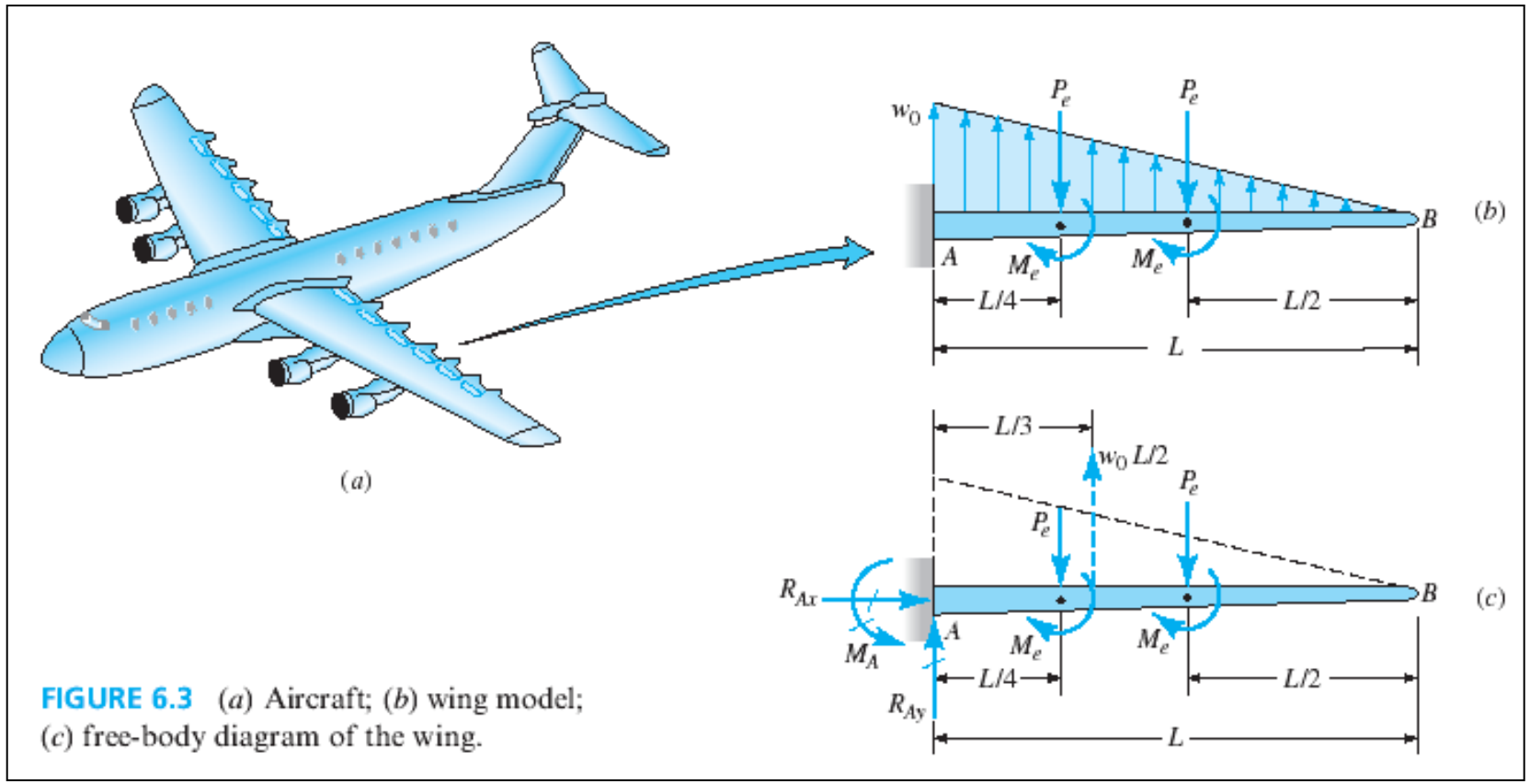



**FIGURE 6.3** (*a*) Aircraft; (*b*) wing model; (*c*) free-body diagram of the wing.

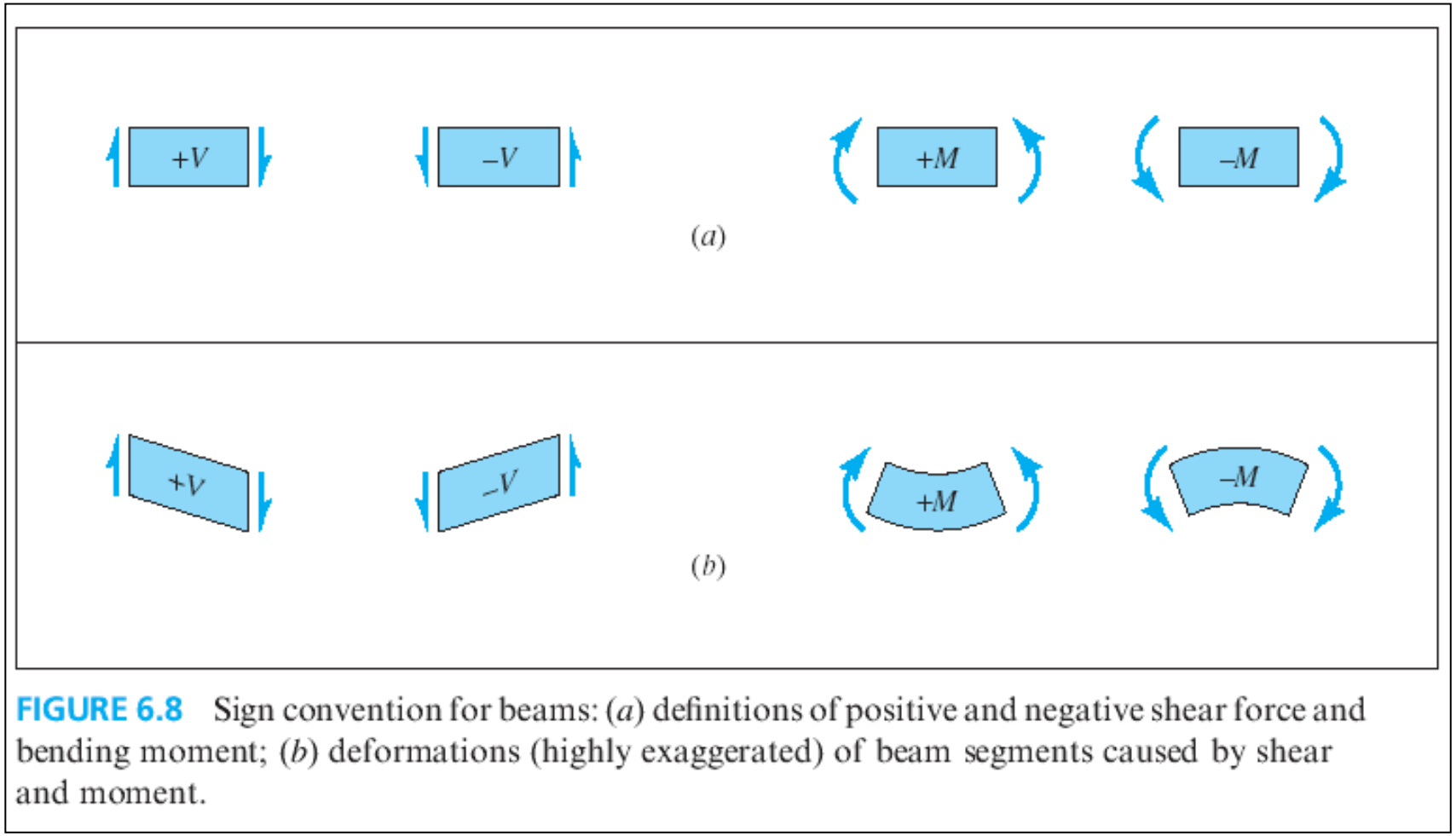
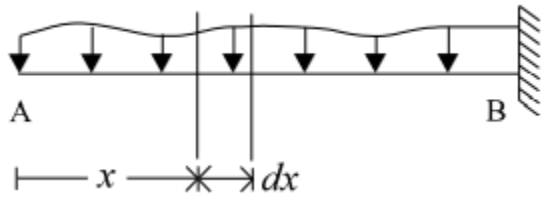
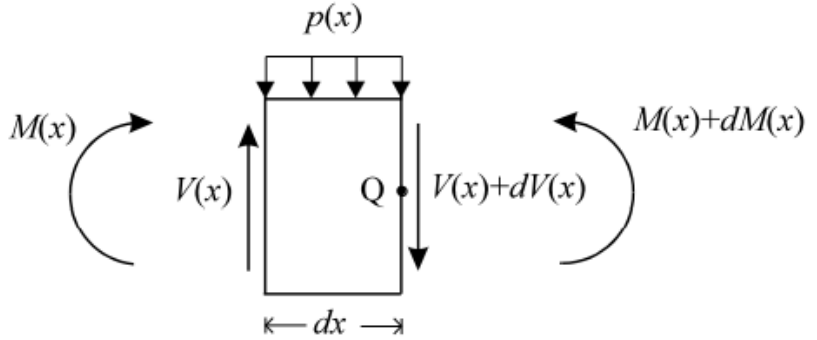



**FIGURE 6.8** Sign convention for beams: (*a*) definitions of positive and negative shear force and bending moment; (*b*) deformations (highly exaggerated) of beam segments caused by shear and moment.

Shear force = força cortante
Bending moment = momento fletor

# EQUAÇÕES DIFERENCIAIS DE EQUILÍBRIO



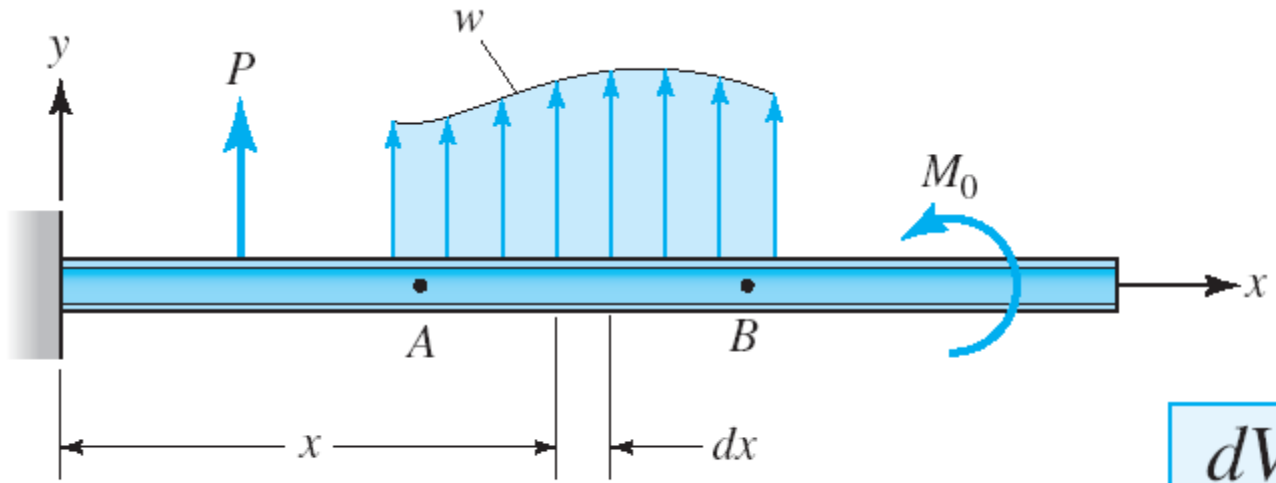

1. $\sum Y = 0 = V(x) - p(x) * dx - (V(x) + dV(x)) \Longrightarrow \frac{\boldsymbol{dV(x)}}{\boldsymbol{dx}} = -\boldsymbol{p(x)}$
2. $M(S_Q) = 0 = -M(x) - V(x) * dx + p(x) * dx * \frac{dx}{2} + (M(x) + dM(x))$

$$\Longrightarrow \frac{\boldsymbol{dM(x)}}{\boldsymbol{dx}} = \boldsymbol{V(x)}$$

$$\frac{\boldsymbol{dV(x)}}{\boldsymbol{dx}} = -\boldsymbol{p(x)}$$

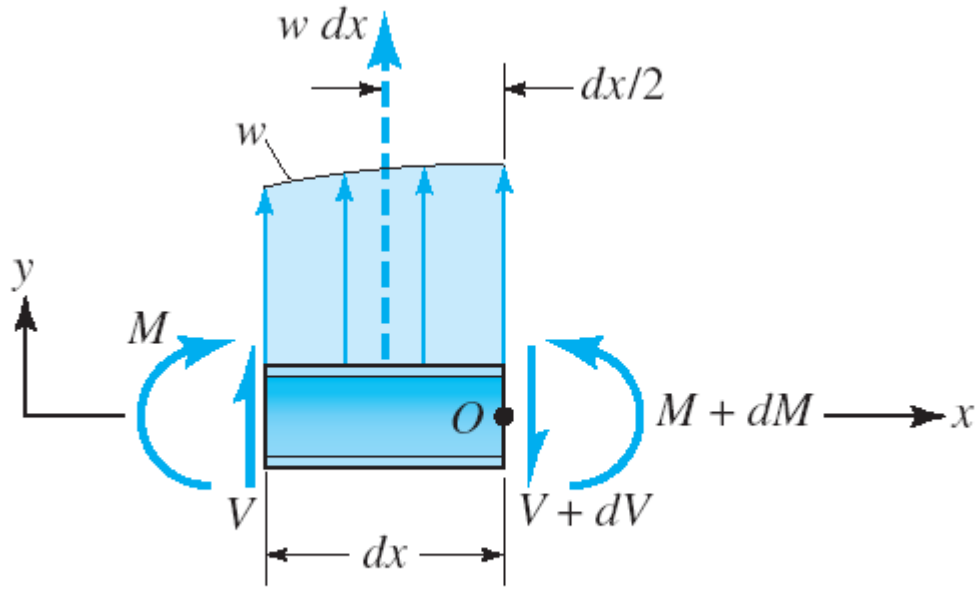
$$\frac{\boldsymbol{dM(x)}}{\boldsymbol{dx}} = \boldsymbol{V(x)}$$

$\boldsymbol{V}$ $\boldsymbol{-}$ $\boldsymbol{força\ cortante}$
$\boldsymbol{M}$ $\boldsymbol{-momento\ fletor}$
$\boldsymbol{p}$ $\boldsymbol{-}$ $\boldsymbol{força\ distribuída}$
$\boldsymbol{x}$ $\boldsymbol{-}$ $\boldsymbol{origem\ em\ A}$




(a)

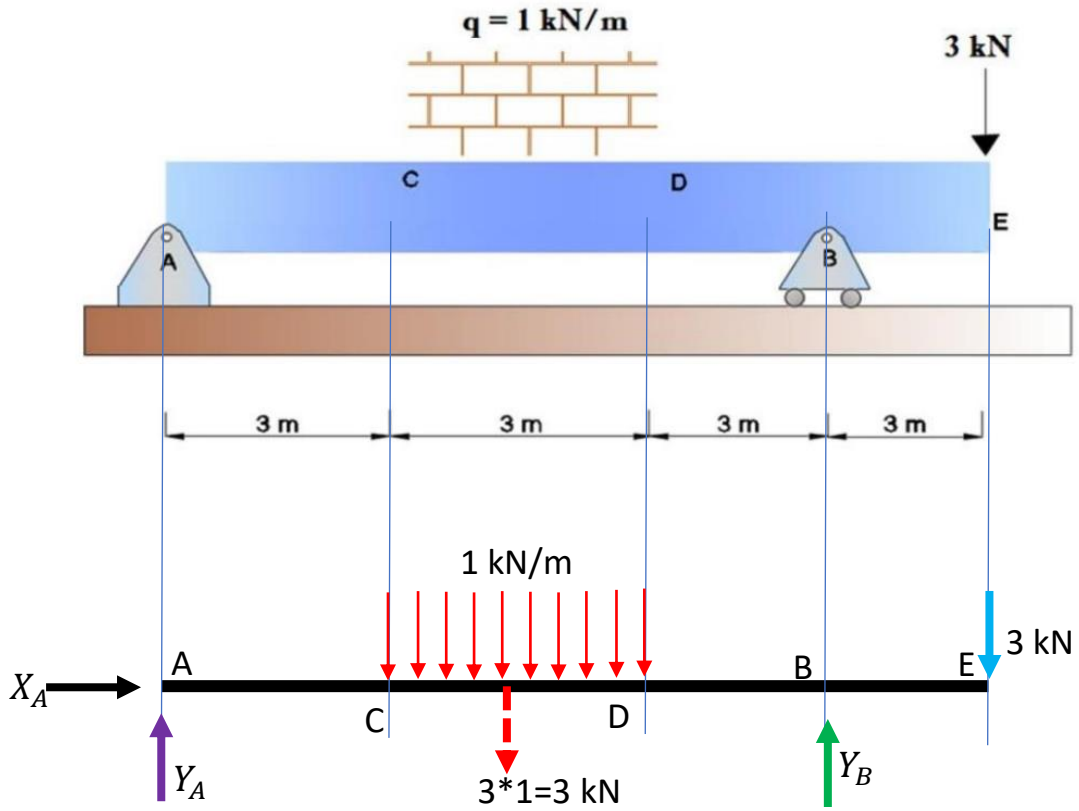
$$\frac{dV}{dx} = w$$




(b)

$$\frac{dM}{dx} = V$$

Determinar os diagramas de esforços solicitantes



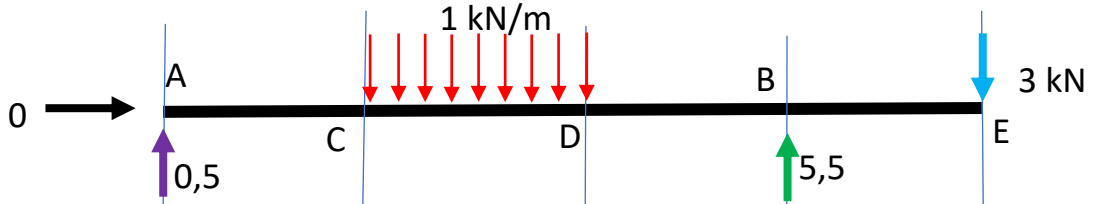

## 1. REAÇÕES NOS APOIOS

$$M_{(A)} = 0 = -3 * 4,5 + Y_B * 9 - 3 * 12 \Rightarrow Y_B = 5,5\ kN$$

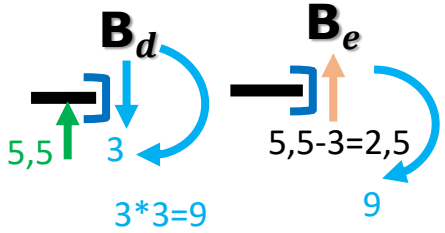
$$M_{(B)} = 0 = -Y_A * 9 + 3 * 4,5 - 3 * 3 \Rightarrow Y_A = 0,5\ kN$$

$$\sum X = 0 = +X_A \Rightarrow X_A = 0\ kN$$

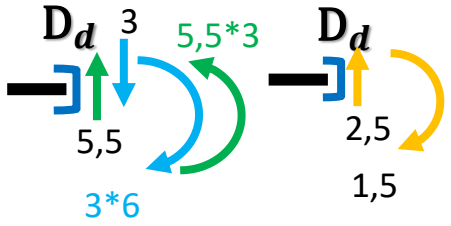
## 2. DIAGRAMA DO CORPO LIVRE



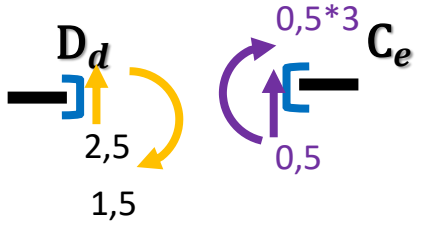

## 2. SECÇÃO B



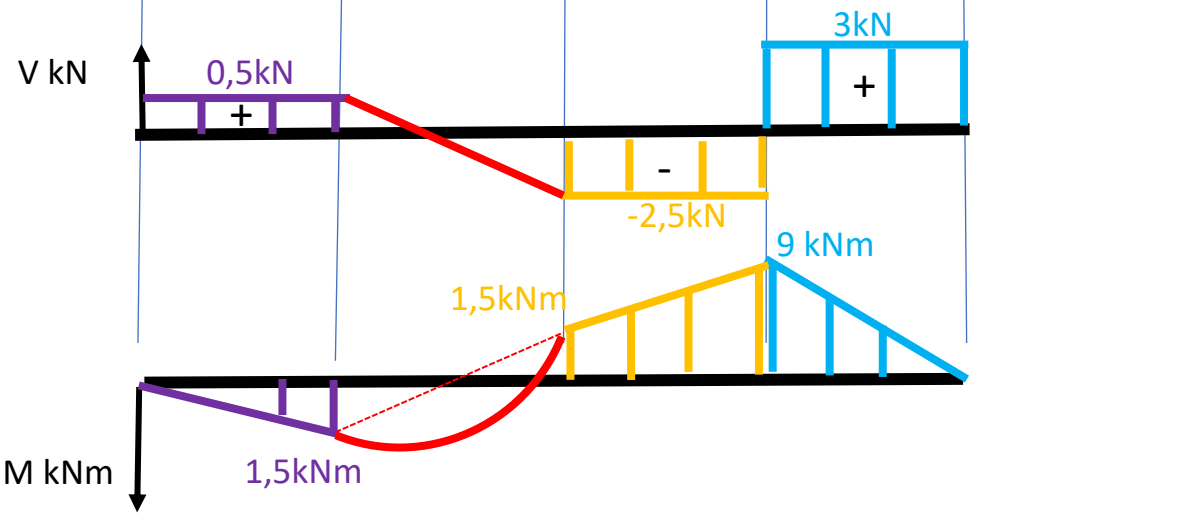

## 3. SECÇÃO D




## 4. SECÇÃO C

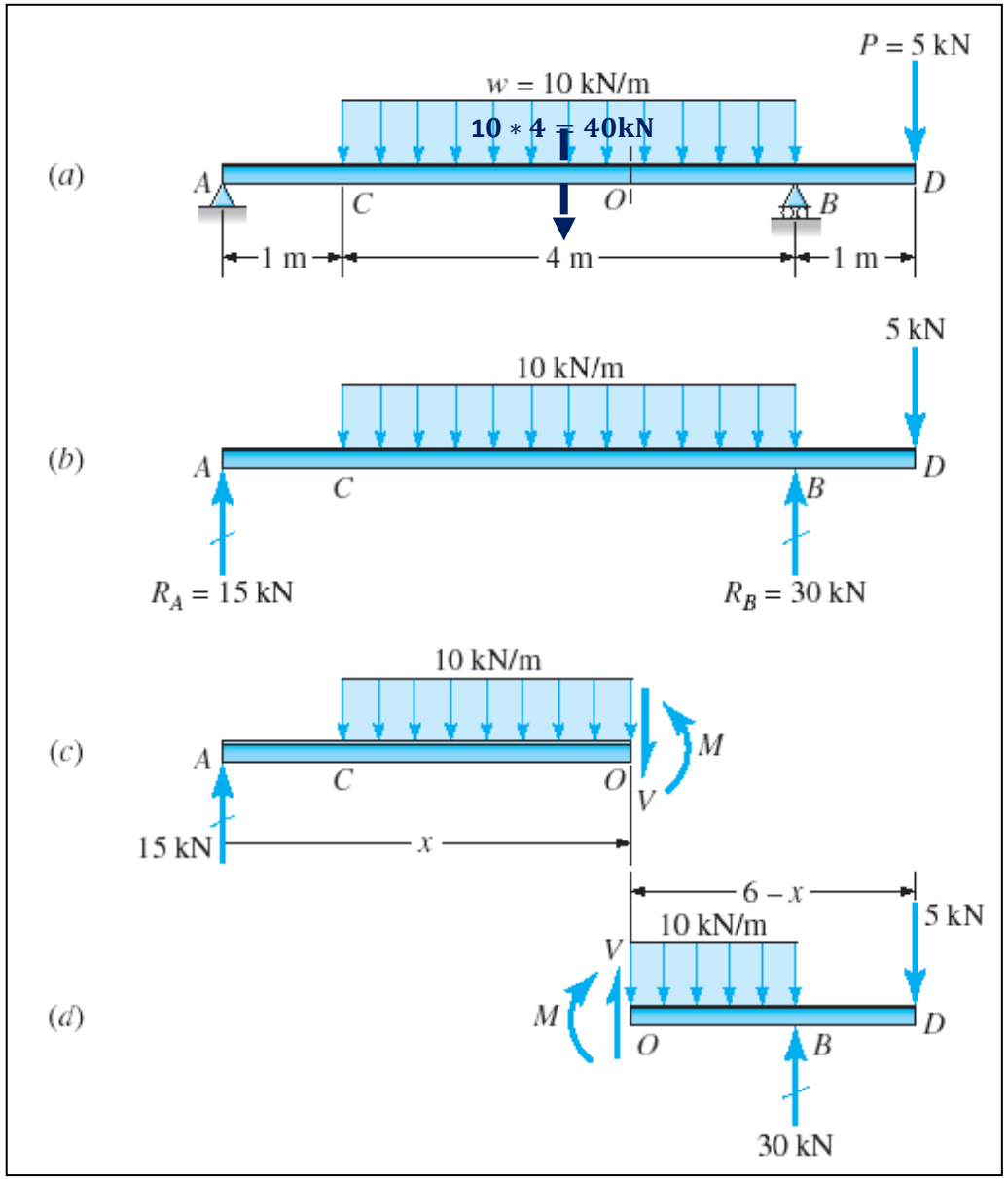



## 5. DIAGRAMAS DA CORTANTE E DO MOMENTO FLETOR



# EXERCÍCIO 1.

## OBTER OS ESFORÇOS SOLICITANTES NA SEÇÃO O

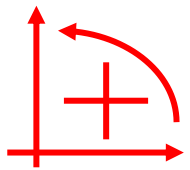



**Reações nos apoios**:

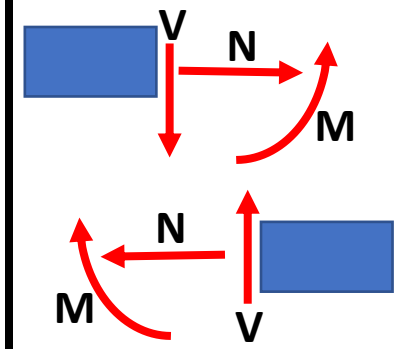
$$\sum M(A) = 0 = -40 * 3 + R_B * 5 - 5 * 6 \Rightarrow R_B = 30\ kN$$

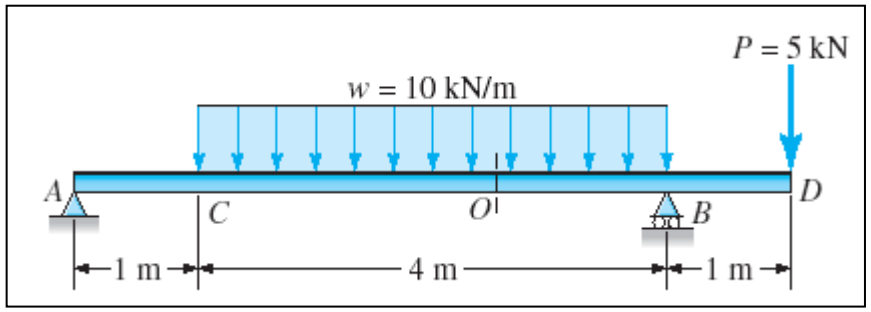
$$\sum M(B) = 0 = -R_A * 5 + 40 * 2 - 5 * 1 \Rightarrow R_A = 15\ kN$$

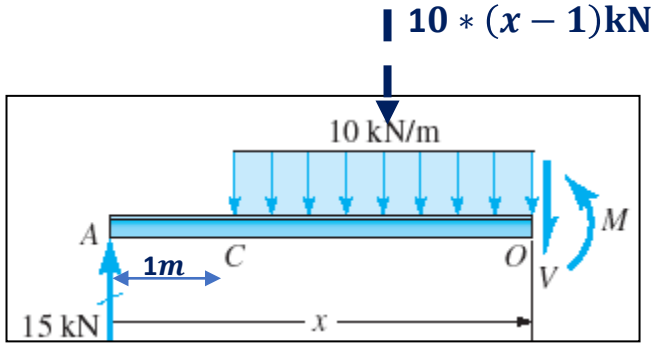
Convenção para o equilíbrio: GRINTER

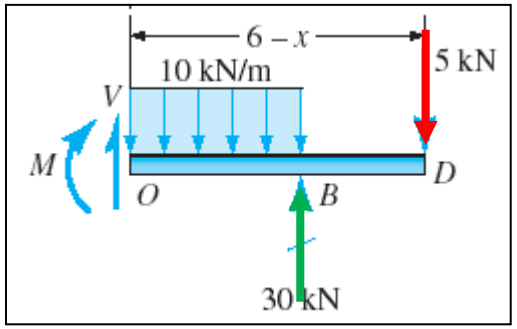


Convenção para esforços solicitantes: +





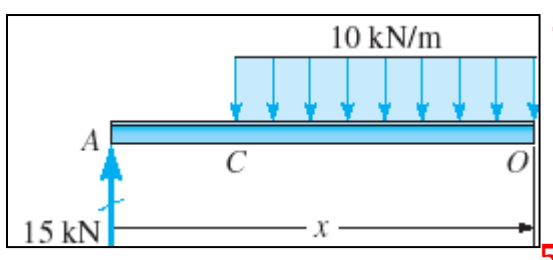



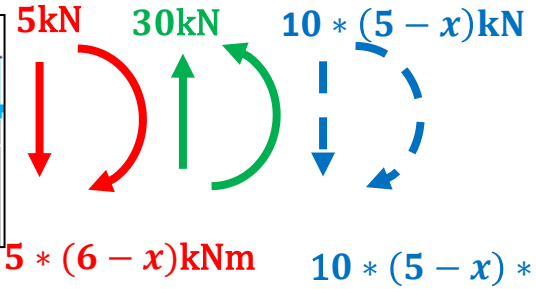




**1. *Equilíbrio*:**

$$\sum Y = 0 = 15 - 10 * (x-1) - V \Longrightarrow V(x) = -10x + 25$$

$$\sum M(O) = -15 * x + 10 * (x-1) * \frac{(x-1)}{2} + M \Longrightarrow M(x) = -5x^2 + 25x - 5$$




**2. *Teorema do corte*:**

$$V = 5 - 30 + 10 * (5-x) = -10x + 25$$

$$M = -5 * (6-x) + 30 * (5-x) - 10 * (5-x) * \frac{(5-x)}{2} = -5x^2 + 25x - 5$$

$$ou\ M = \int V dx = \int (-10x + 25) dx = -10\frac{x^2}{2} + 25x + c$$

$$condição\ de\ contorno:\ M_B(x=5) = -5\ kNm = -10\frac{5^2}{2} + 25.(5) + c \Longrightarrow c = -5$$

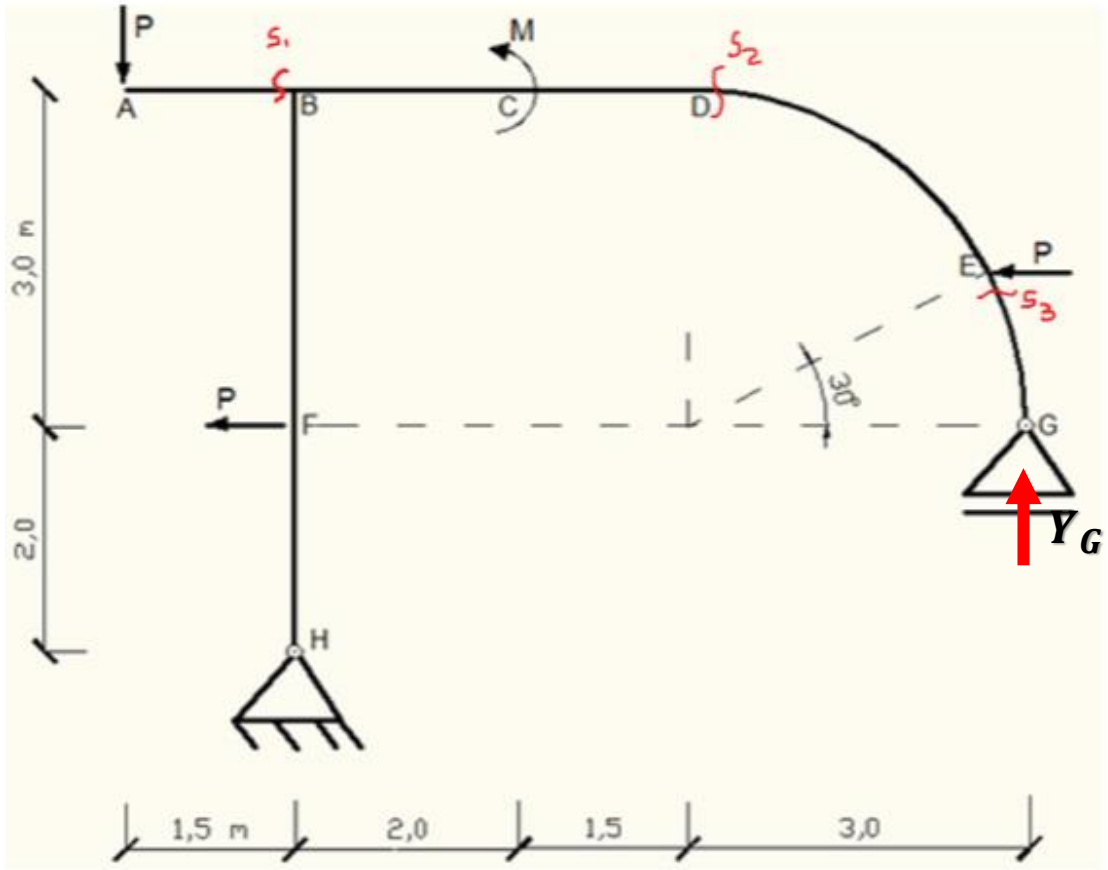
# EXERCÍCIO 2.

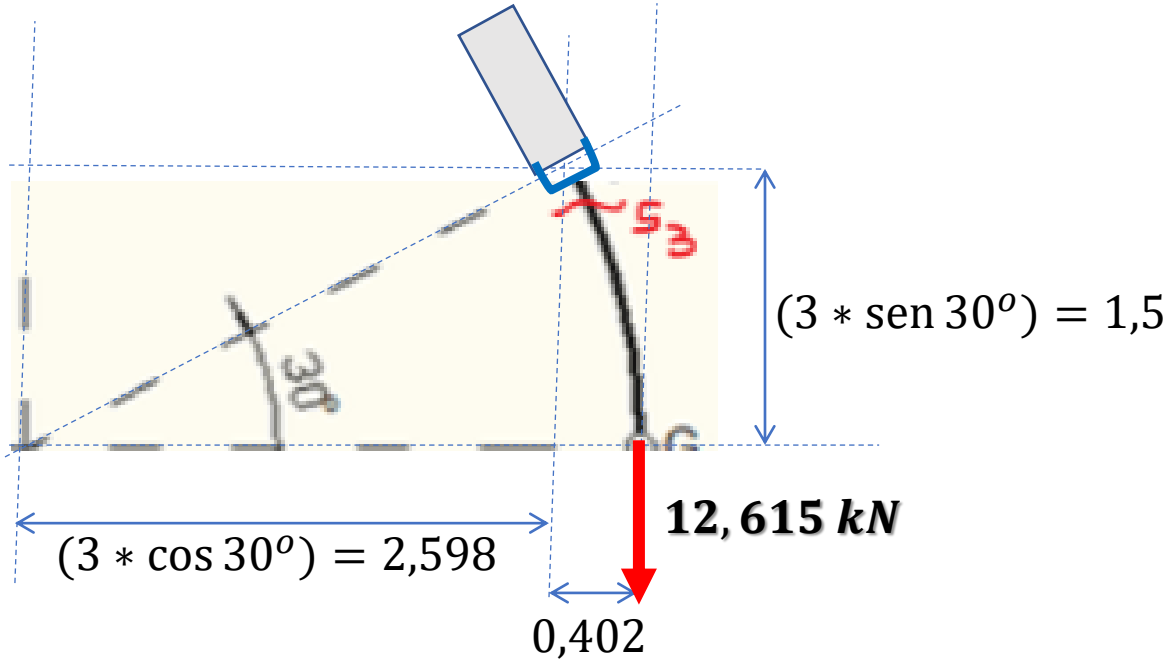
## ADOTE P=10 kN e M=12 kNm. OBTER OS ESFORÇOS SOLICITANTES EM $S_1$, $S_2$ E $S_3$

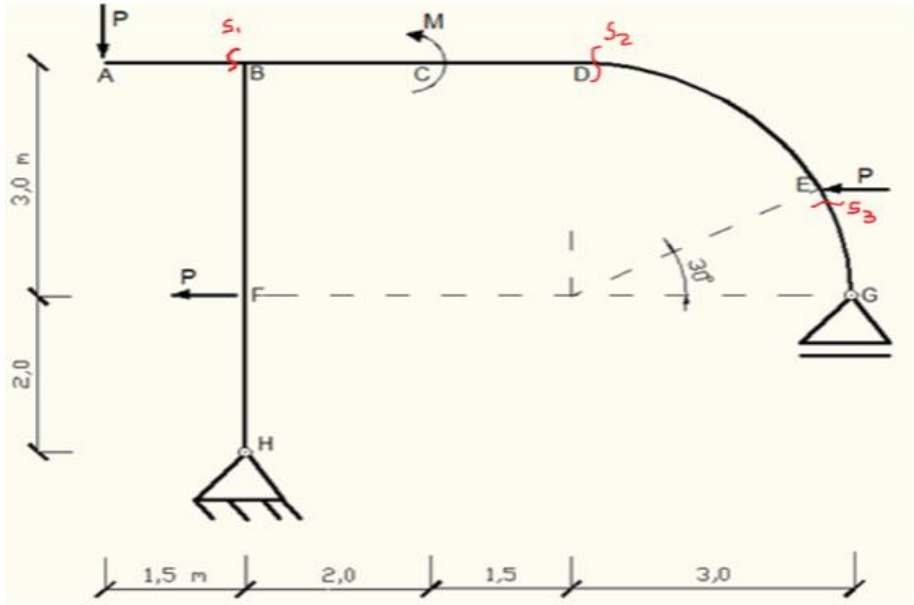
### 1. *REAÇÃO NO APOIO G*

$$M_{(H)} = 0 = +P * 2,0 + P * 1,5 + M + P * (3 * sen30^o + 2,0) + Y_G * (6,5)$$

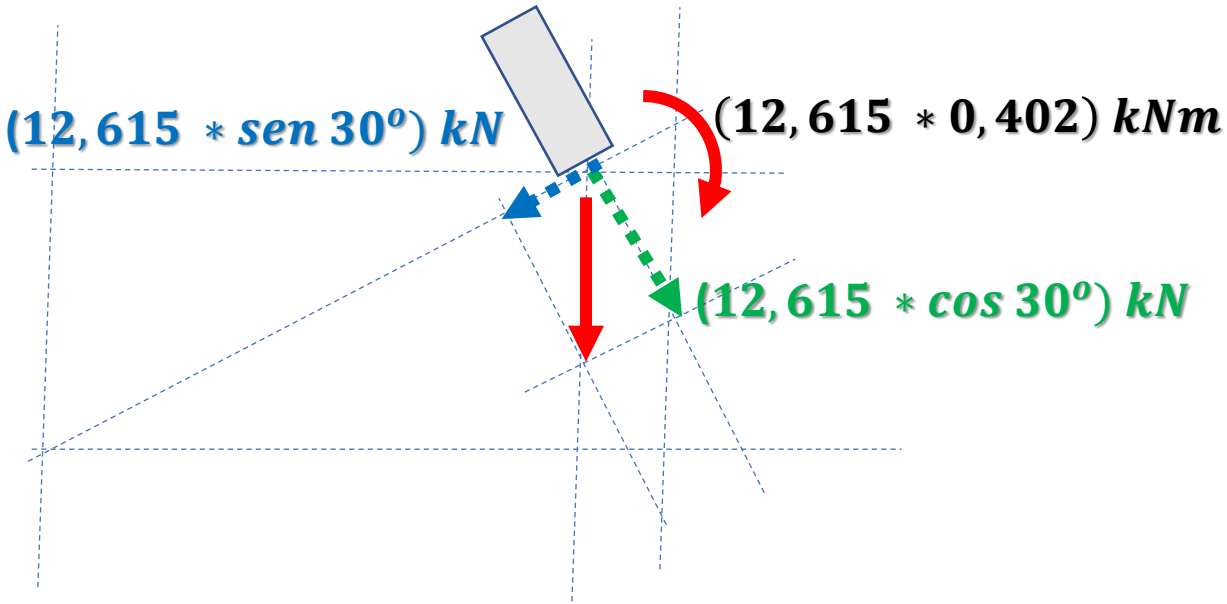
$$Y_G = \frac{-7 * P - M}{6,5} = -12,615\ \text{kN}$$





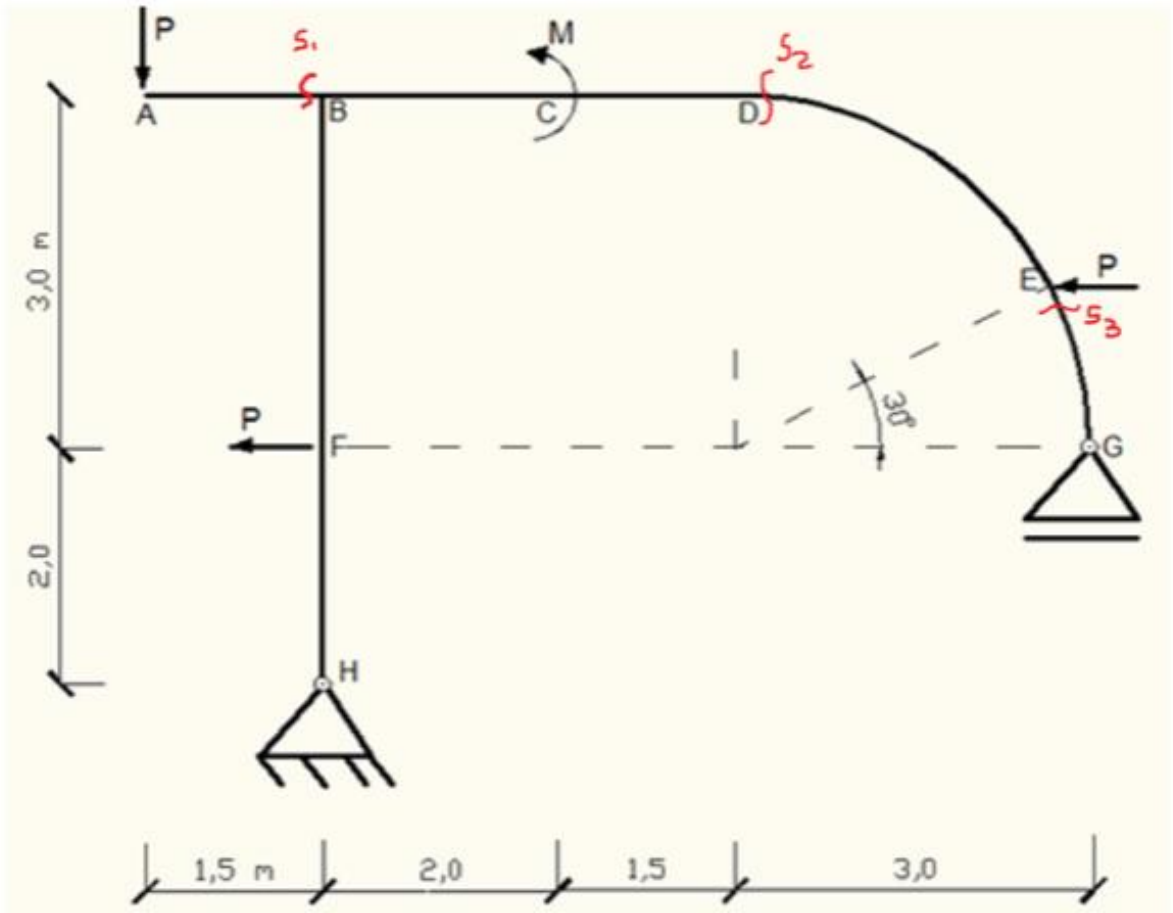

## 2. SEÇÃO $S_3$



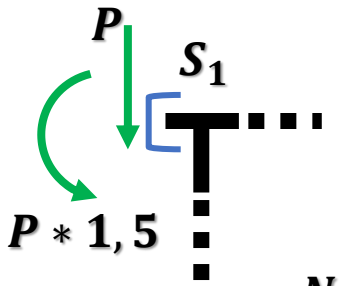

$$N_{S_3} = +(12{,}615 * cos\, 30^o = 10{,}93)\ kN$$

$$V_{S_3} = +(12{,}615 * sen\, 30^o = 6{,}30)\ kN$$

$$M_{S_3} = -(12{,}615 * 0{,}402 = 5{,}07)\ kNm$$




### 3. $SEÇÃO\ S_1$

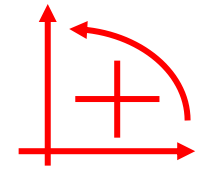



$$N_{S_1} = 0$$

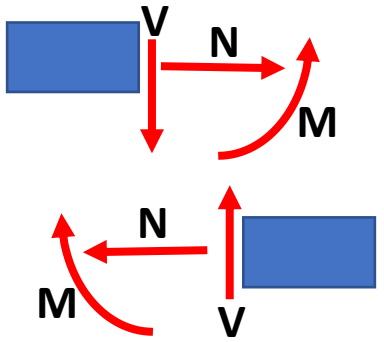
$$V_{S_1} = -P = -10\ kN$$

$$M_{S_1} = -P * 1,5 = -15\ kNm$$

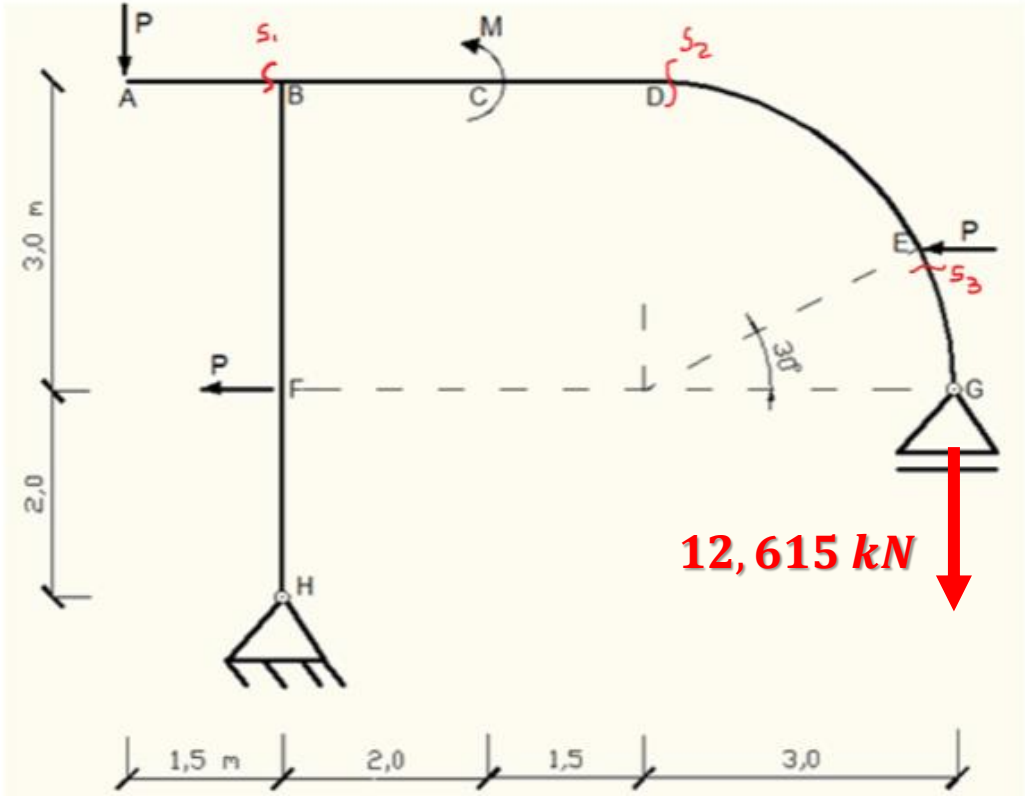
Convenção para o equilíbrio: GRINTER

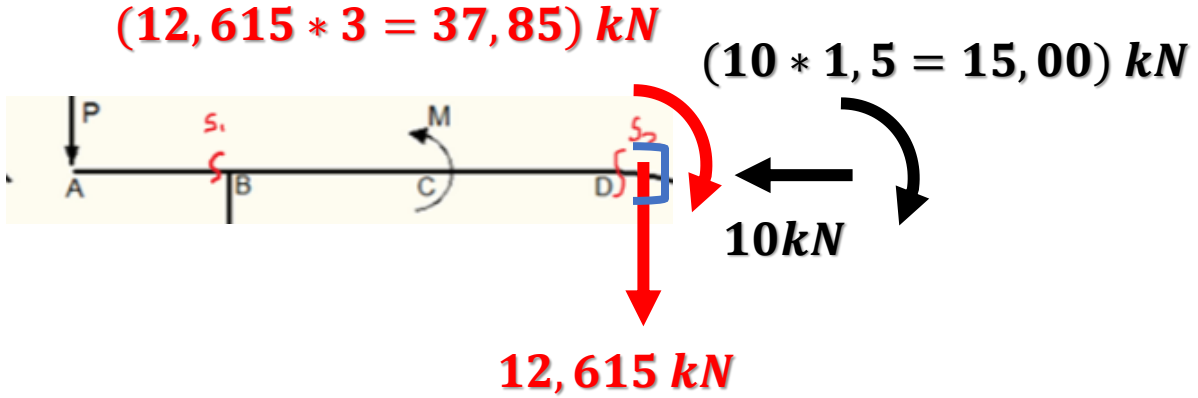


Convenção para esforços solicitantes: +



## 4. SEÇÃO $S_2$



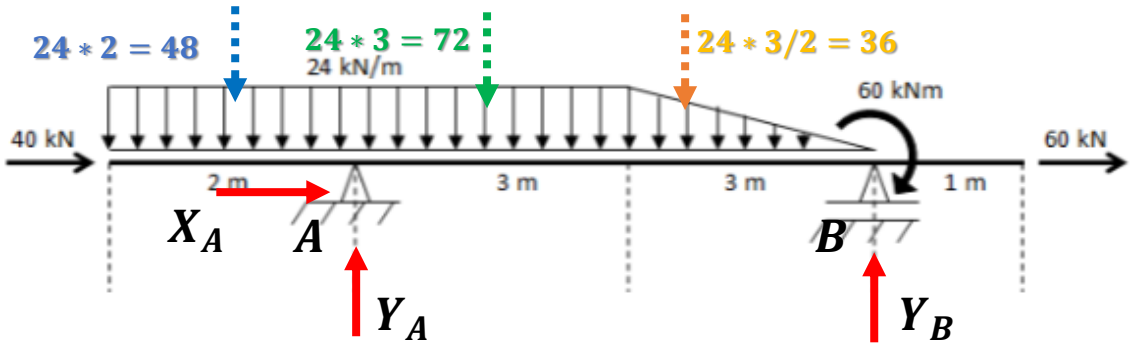

$$N_{S_2} = -10\ kN$$

$$V_{S_2} = +12{,}615\ kN$$

$$M_{S_2} = -(37{,}85 + 15{,}00 = 52{,}85)\ kNm$$

# EXERCÍCIO 3.

## Calcular as reações na viga a seguir



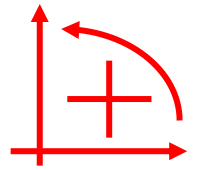

***REAÇÕES NOS APOIOS***

$$M_{(A)} = 0 = +48 * 1 - 72 * 1,5 - 36 * 4 - 60 + Y_B * 6 \Longrightarrow Y_B = 44\ kN$$

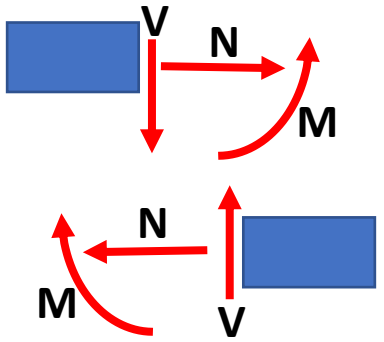
$$M_{(B)} = 0 = +48 * 7 - Y_A * 6 + 72 * 4,5 + 36 * 2 - 60 \Longrightarrow Y_A = 112\ kN$$

$$\sum X = 0 = 40 + X_A + 60 \Longrightarrow X_A = -100\ kN$$

Convenção para o equilíbrio: GRINTER



Convenção para esforços solicitantes: +



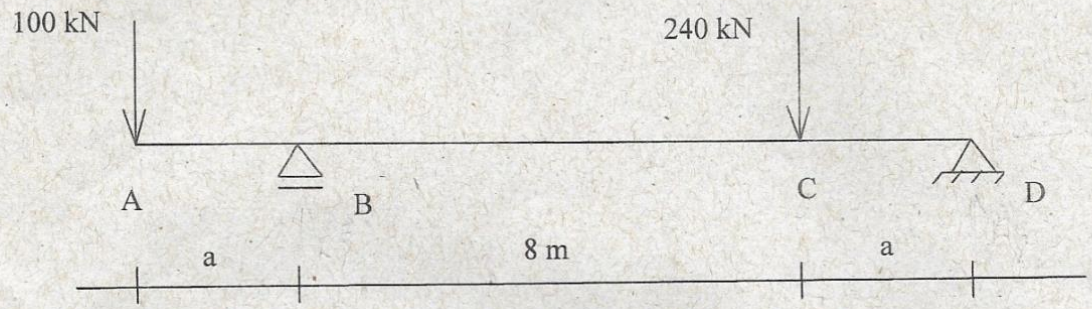
# EXERCÍCIO 4.
## P1 2017

PEF-3200 – 1ª. Prova – 19/04/2017

Nº USP:_____ Nome:_____

**1ª Questão (3,0 pontos)**

Dada a viga simplesmente apoiada com um balanço à esquerda da figura, determinar o valor de "a" que leva o maior momento fletor positivo e o maior momento fletor negativo a serem iguais em módulo.

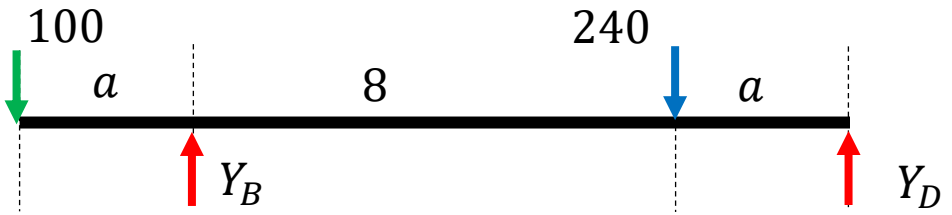


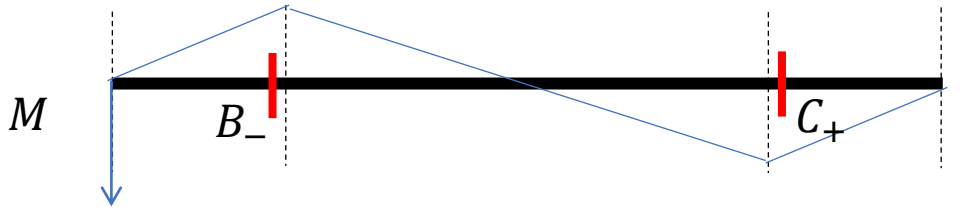

1. $\sum M_{(B)} = 0 = 100 * a - 240 * 8 + Y_D * (8 + a)$

$$\Rightarrow Y_D = \frac{1920 - 100a}{8 + a}$$

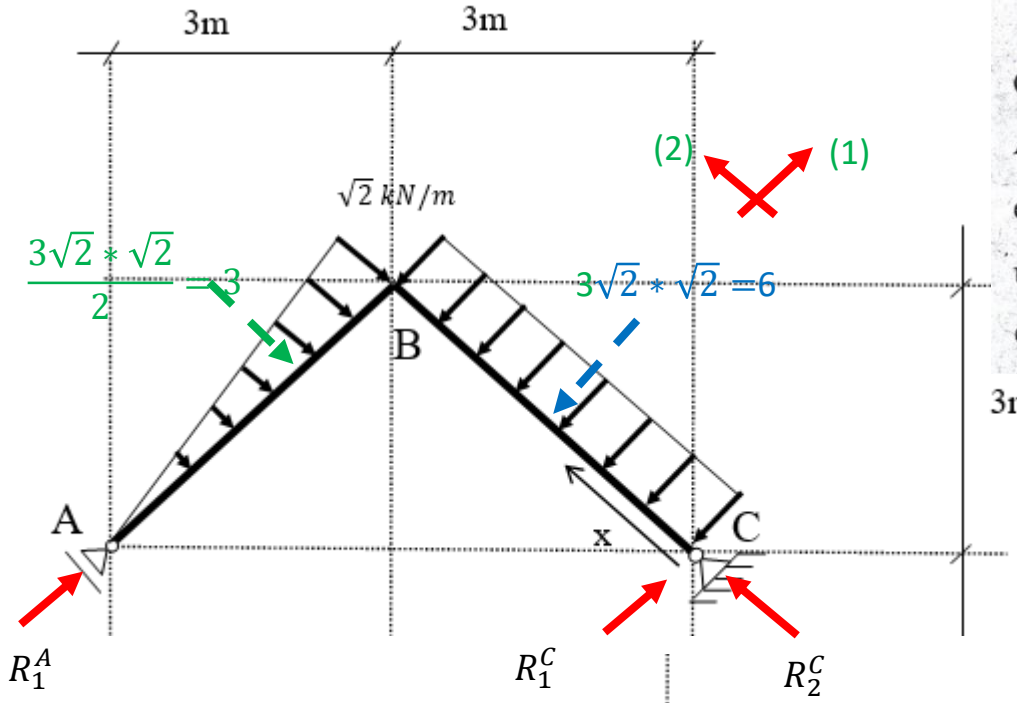
2. $M_{B_-} = -100 * a \qquad M_{C_+} = Y_D * a$

3. $|M_{B_-}| = |M_{C_+}| \Rightarrow 100 * a = \frac{1920 - 100a}{8 + a} * a \Rightarrow a = 5,6\ m$



# EXERCÍCIO 5.
## P1 2018




PEF-3200 – 1ª. Prova – 11/04/2018

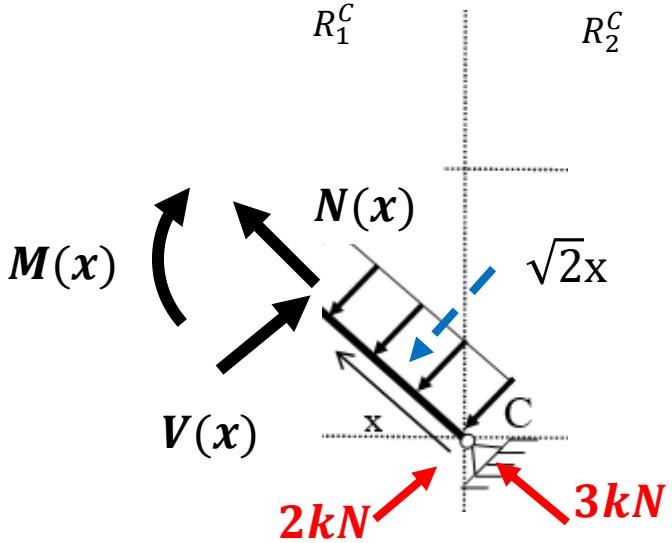
Nº USP: _____ Nome: _____

**Questão 1 (3,0 pontos)** – A estrutura plana ABC da figura é uma viga poligonal com articulação móvel em A e articulação fixa em C. A barra AB está submetida à carga distribuída uniformemente variada de 0 kN/m em A a $\sqrt{2}\ kN/m$ em B aplicada perpendicularmente a AB. A barra BC está submetida à carga distribuída uniforme de $\sqrt{2}\ kN/m$ aplicada perpendicularmente a BC. **Determine as reações no apoio C e as funções dos esforços solicitantes na barra BC com a variável x** de origem em C.

*1. Reações no apoio C*

$$\sum F_2 = 0 = -3 + R_2^C \Longrightarrow \boldsymbol{R_2^C = 3kN}$$

$$\sum M_{(B)} = 0 = 3 * \sqrt{2} - 6 * \frac{3\sqrt{2}}{2} + R_1^C * 3\sqrt{2} \Longrightarrow \boldsymbol{R_1^C = 2kN}$$




*2. Seção S na barra CB*

$$\sum F_2 = 0 = 3 + N(x) \Longrightarrow = \boldsymbol{N(x) = -3kN}$$

$$\sum F_1 = 0 = V(x) - \sqrt{2}x + 2 \Longrightarrow = \boldsymbol{V(x) = (\sqrt{2}x - 2)kN}$$

$$\sum M_{(S)} = 0 = -M(x) - \sqrt{2}x\frac{x}{2} + 2x \Longrightarrow = \boldsymbol{M(x) = (-\frac{\sqrt{2}x^2}{2} + 2x)kNm}$$

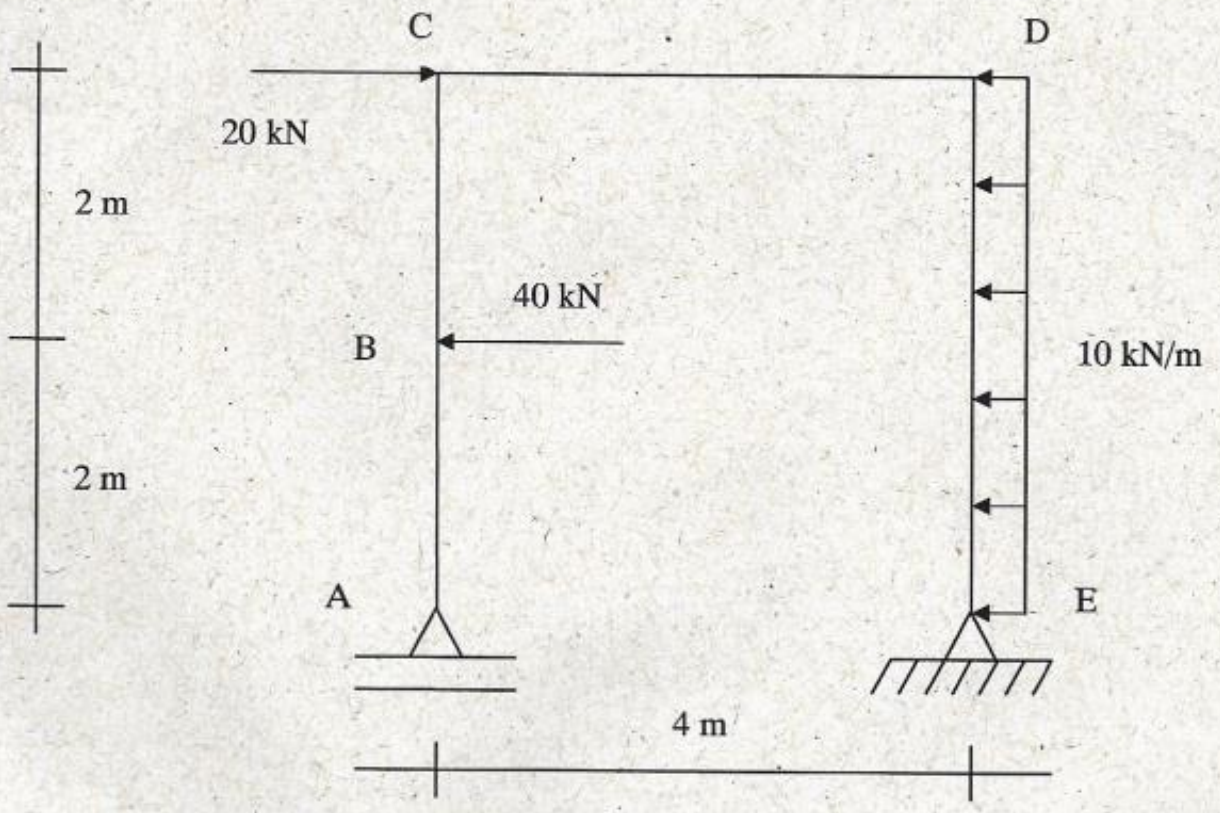
# EXERCÍCIO 6. P1-2019

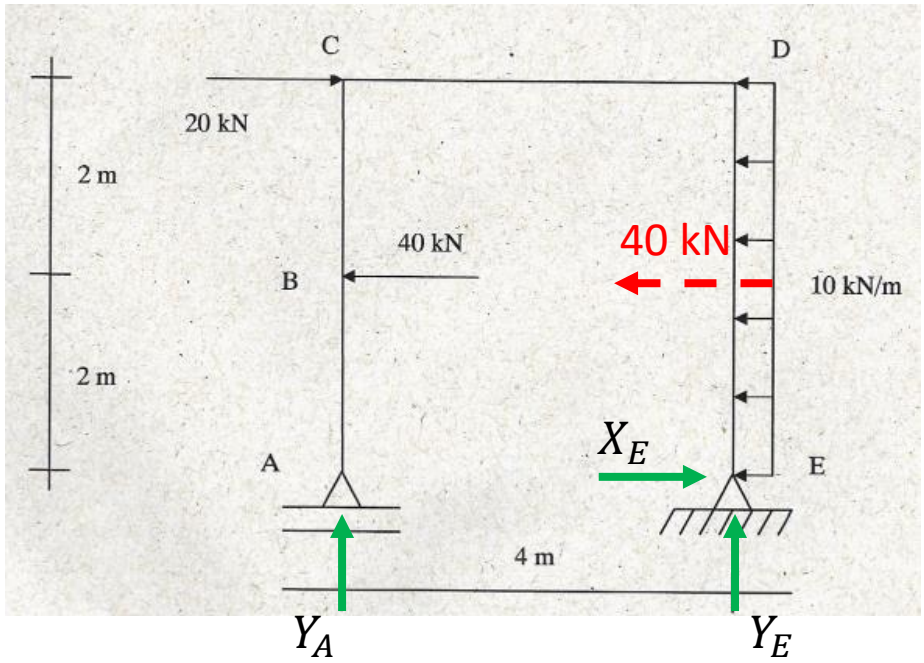
PEF-3200 – 1ª Prova – 28.3.2019

Nº USP:_____ Nome:_____

**Questão 3 (3,5 pontos)**

Traçar os diagramas de esforços solicitantes da estrutura da figura





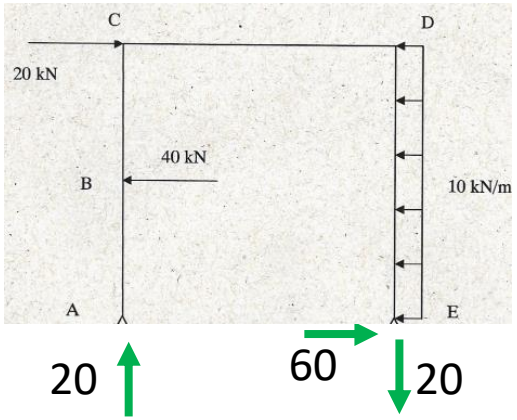

## 1. REAÇÕES NOS APOIOS

$$\sum X = 0 = X_E - 40 + 20 - 40 \Rightarrow X_E = 60\ kN$$

$$\sum M_{(E)} = 0 = -Y_A * 4 + 40 * 2 - 20 * 4 + 40 * 2 \Rightarrow Y_A = 20\ kN$$

$$\sum M_{(A)} = 0 = +40 * 2 - 20 * 4 + 40 * 2 + Y_E * 4 \Rightarrow Y_E = -20\ kN$$

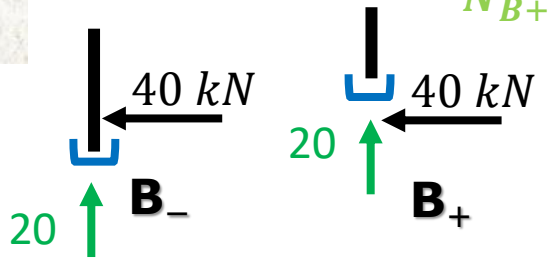
## 2. DIAGRAMA DO CORPO LIVRE




## 3. SEÇÃO B

$N_{B-} = -20\ kN\ (compressão); V_{B-} = 0; M_{B-} = 0$

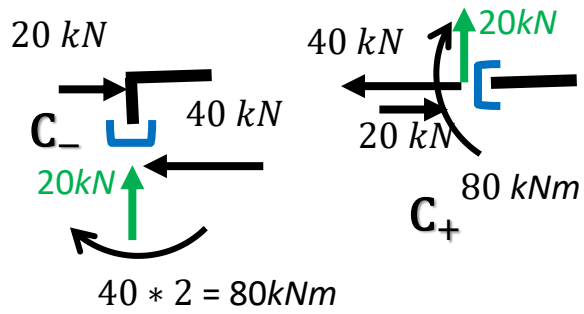
$N_{B+} = -20\ kN (compressão); V_{B+} = +40; M_{B-} = 0$



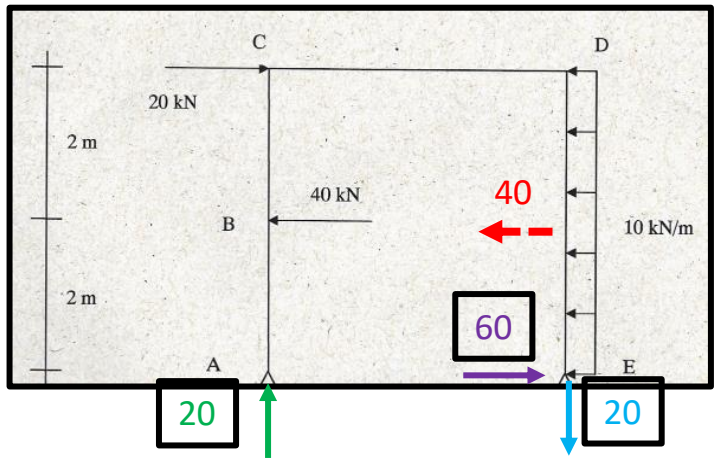

## 4. SEÇÃO C

$N_{C-} = -20\ kN\ (compressão); V_{C-} = +40\ kN;$
$M_{C-} = 80\ kNm\ (traciona\ fibra\ da\ direita)$

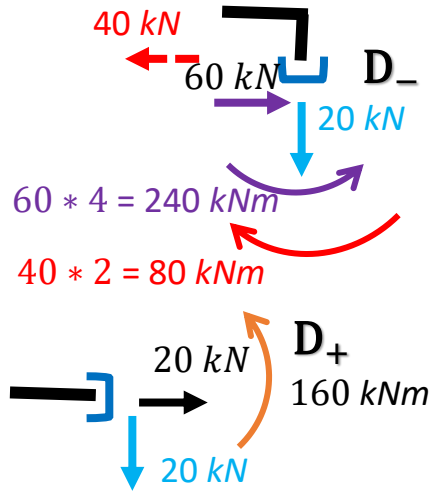
$N_{C+} = +20\ kN\ (tração); V_{C+} = +20\ kN;$
$M_{C+} = 80\ kNm\ (traciona\ fibra\ de\ baixo)$



## 2. DIAGRAMA DO CORPO LIVRE




## 5. SEÇÃO D



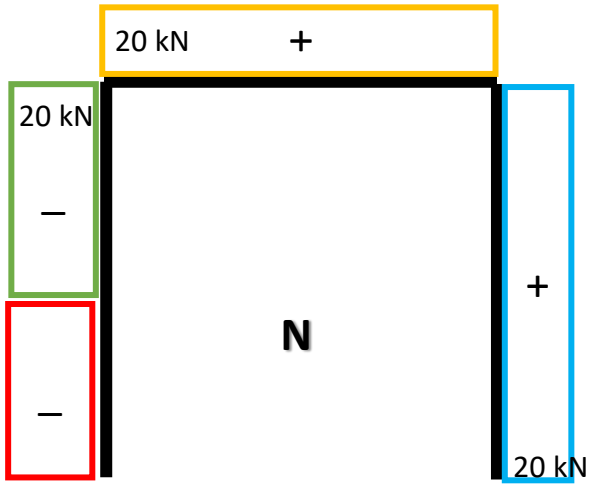

$$N_{D-} = +20\ kN\ (tração); V_{D-} = -20\ kN;$$
$$M_{D-} = 160\ kNm\ (traciona\ fibra\ da\ esquerda)$$

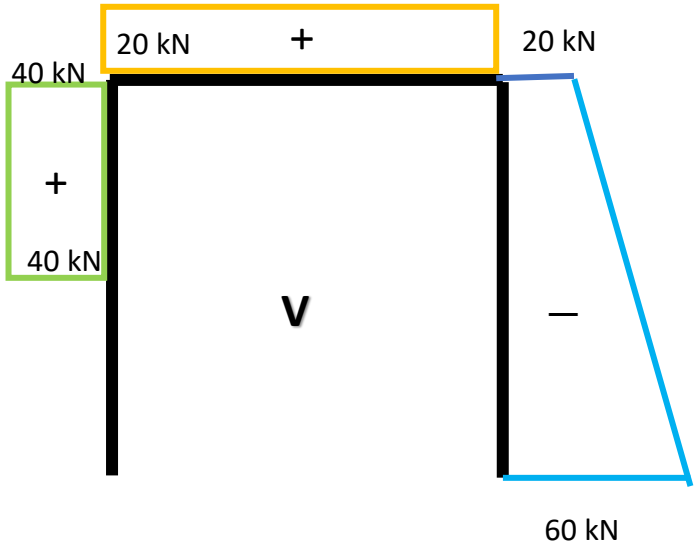
$$N_{D+} = +20\ kN\ (tração); V_{D+} = +20\ kN;$$
$$M_{D+} = 160\ kNm\ (traciona\ fibra\ de\ baixo)$$

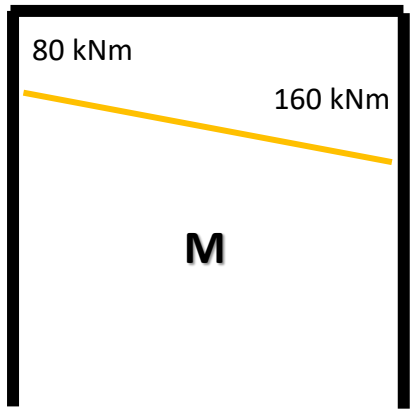
$$N_A = -20\ kN (compressão); V_A = 0; M_A = 0$$

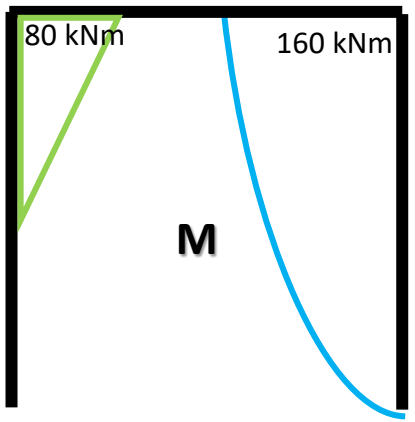
$$N_E = +20\ kN (tração); V_E = -60\ kN; M_A = 0$$

## 6. DIAGRAMAS DOS ESFORÇOS SOLICITANTES



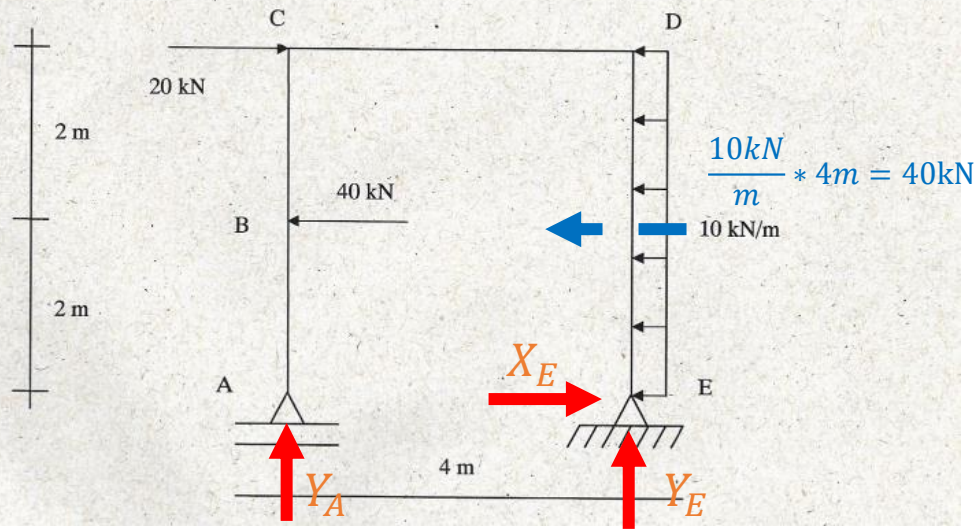



PEF-3200 – 1ª Prova – 28.3.2019

Nº USP:_____ Nome:_____

**Questão 3 (3,5 pontos)**

Traçar os diagramas de esforços solicitantes da estrutura da figura



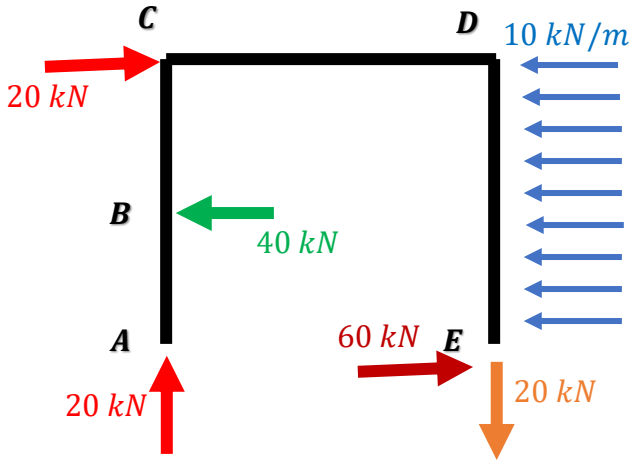

## 1. Reações nos apoios

$$\sum X = 0 = 20 - 40 - 40 + X_E \Longrightarrow X_E = 60\ kN$$

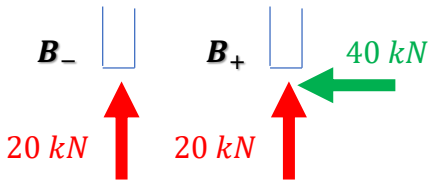
$$\sum M_{(A)} = 0 = 40 * 2 - 20 * 4 + 40 * 2 + Y_E * 4 \Longrightarrow Y_E = -20kN$$

$$\sum M_{(E)} = 0 = 40 * 2 - 20 * 4 + 40 * 2 - Y_A * 4 \Longrightarrow Y_A = +20kN$$

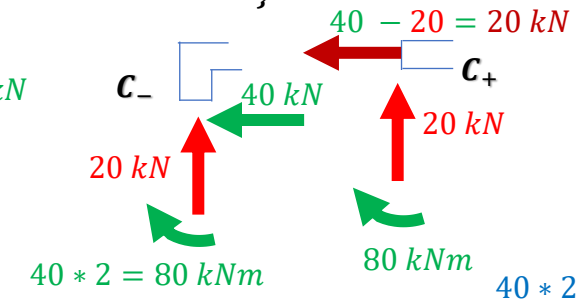
## 2. Diagrama do corpo livre



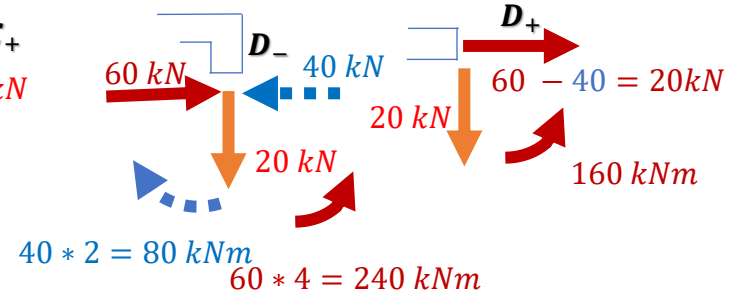

## 3. Seção B



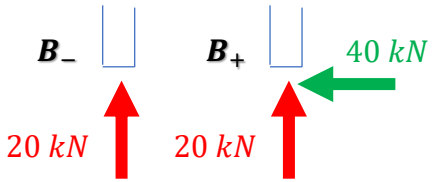

## 4. Seção C



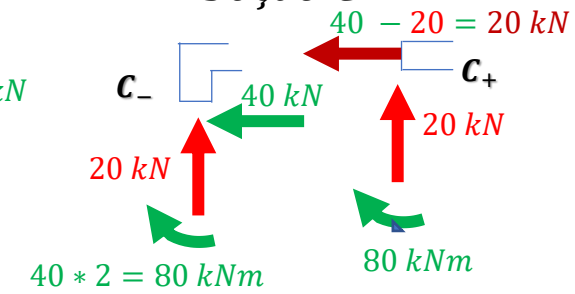

## 5. Seção D



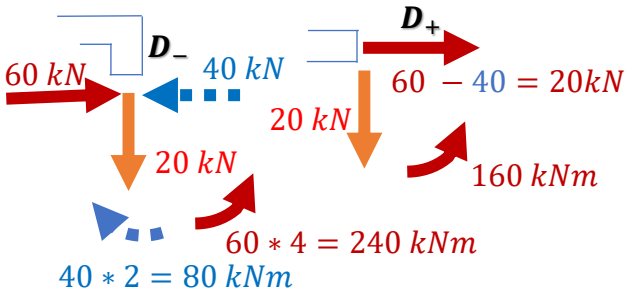
## 3. Seção B



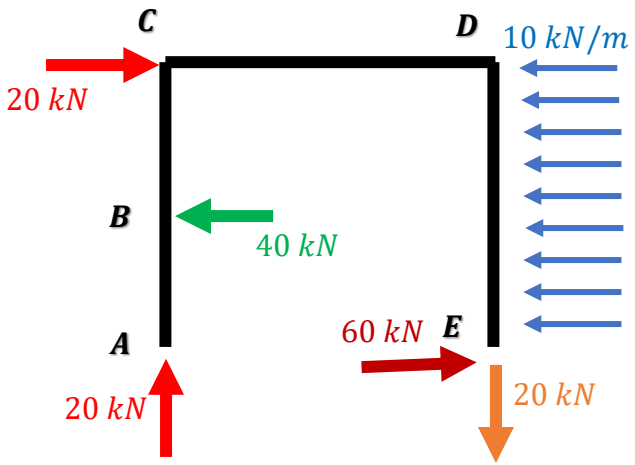

## 4. Seção C

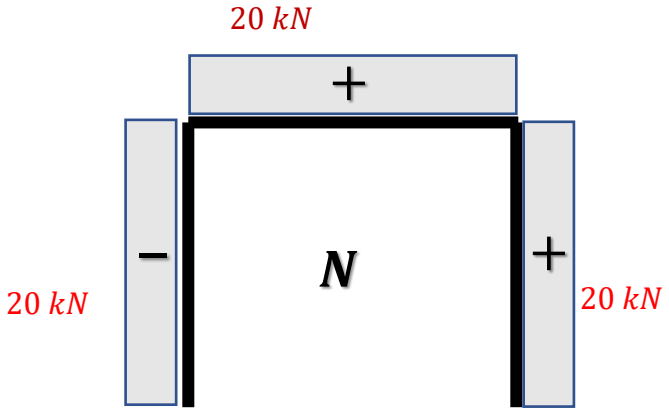


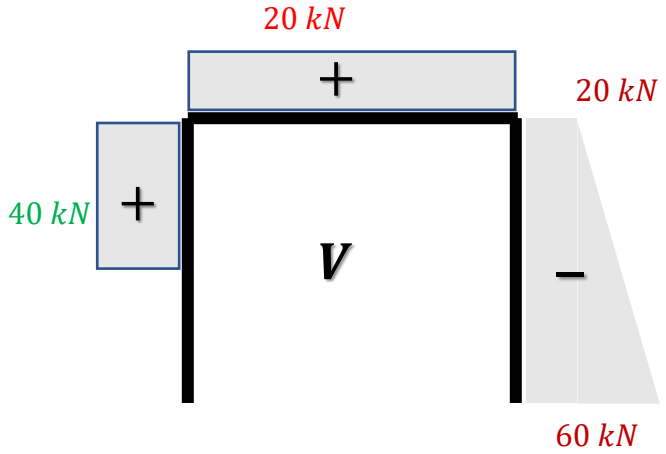

## 5. Seção D

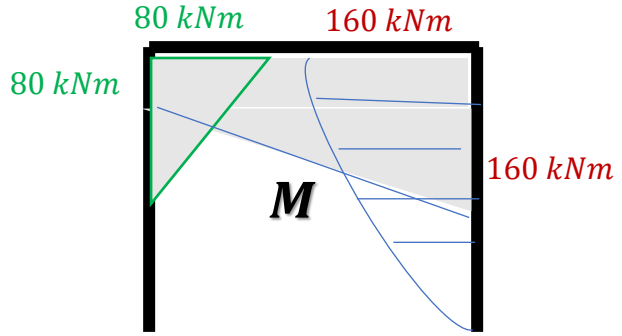



## 2. Diagrama do corpo livre



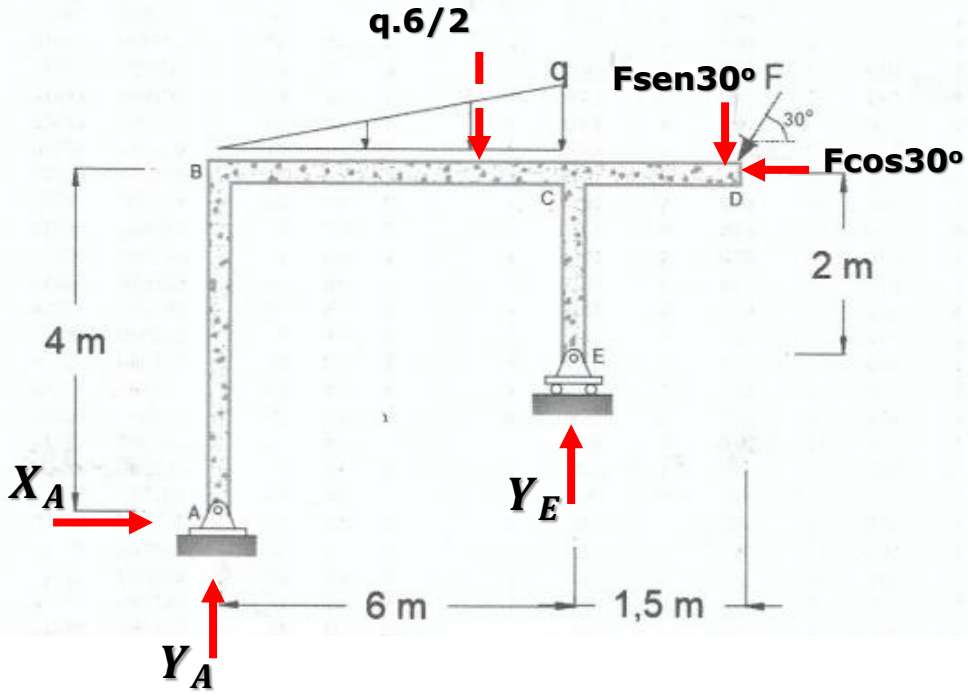



# EXERCÍCIO 7.
# P1-2020

PEF-3200 – 1ª Prova – 27/5/2020

Nº USP:_____ Nome:_____

**2ª Questão( pontos)** Para a estrutura a seguir, sabendo que F = 10 kN e q = 10 kN/m, obtenha os diagramas de esforços solicitantes em todos os trechos, indicando os valores e posições dos extremos dos esforços em cada trecho.



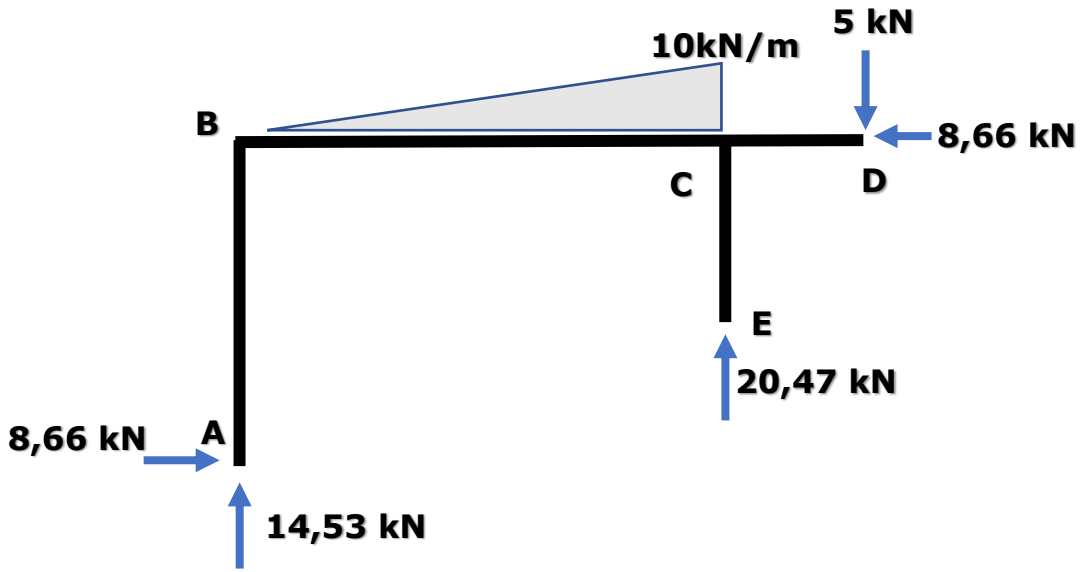

## 1. REAÇÕES NOS APOIOS

$$\sum X = 0 = X_A - 5\sqrt{3} \Longrightarrow X_A = 5\sqrt{3} = 8{,}66$$

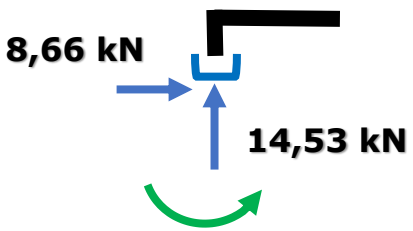
$$\sum M_{(A)} = 0 = -30 * 4 - 5 * 7{,}5 + 5\sqrt{3} * 4 + Y_E * 6 \Longrightarrow Y_E = 20{,}47$$

$$\sum Y = 0 = Y_A - 30 - 5 + Y_E \Longrightarrow Y_A = 14{,}53$$

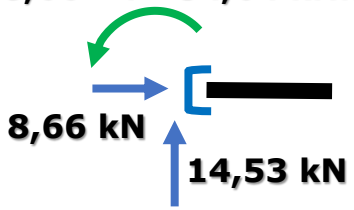
## 2. DIAGRAMA DO CORPO LIVRE



## 3. SEÇÃO B

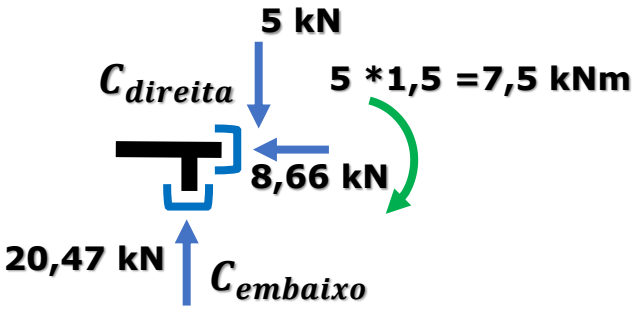



8,66 *4 =34,64 kNm

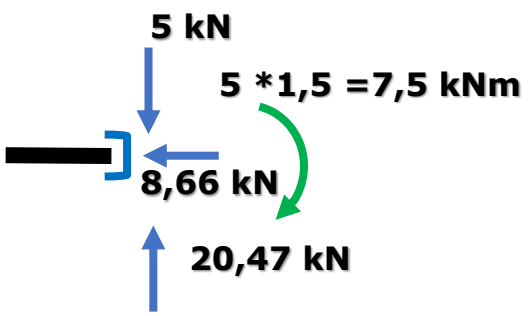



8,66 *4 =34,64 kNm

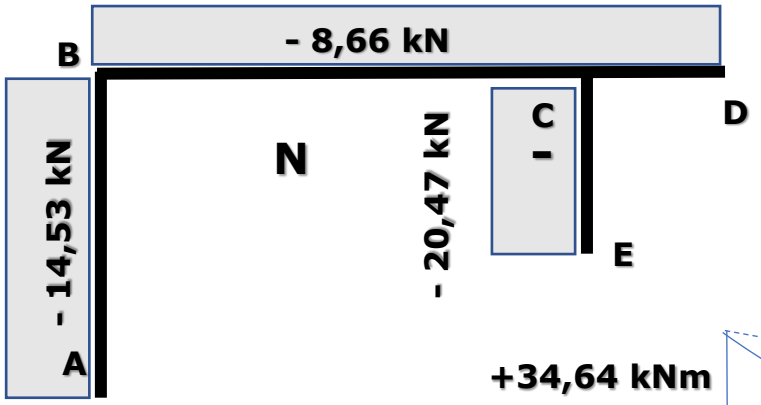
## 4. SEÇÃO C

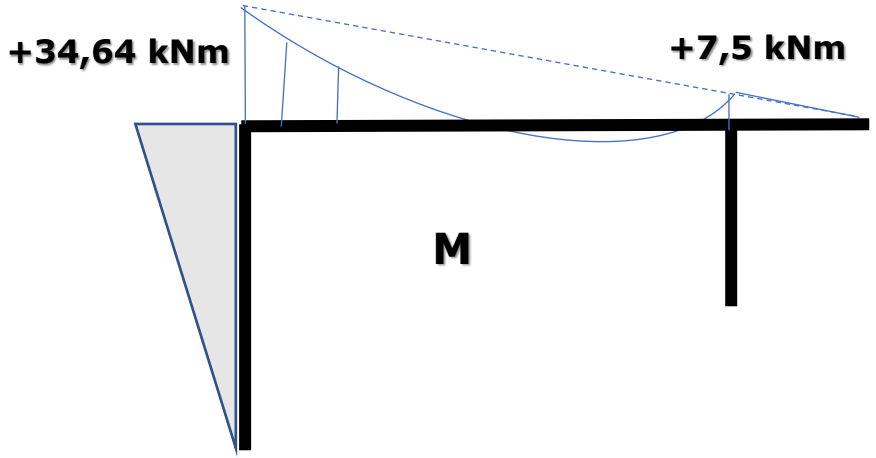


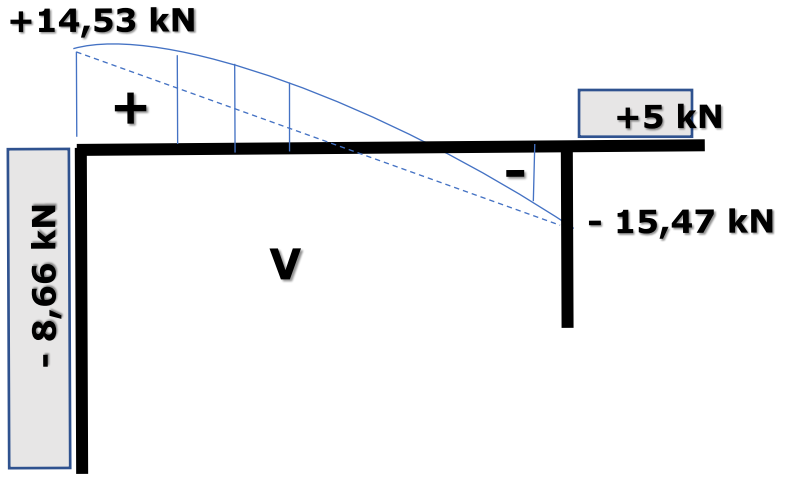

$C_{esquerda}$

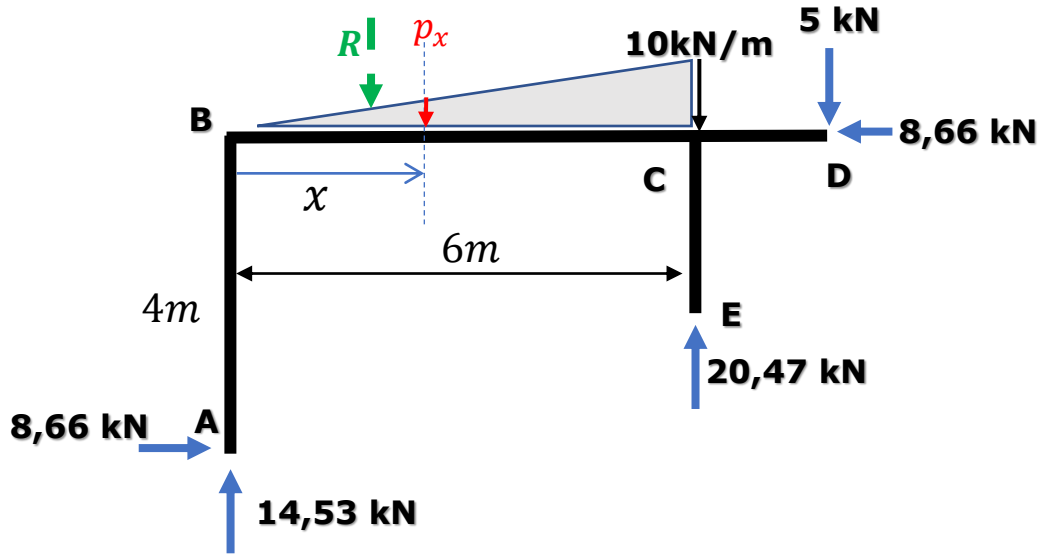


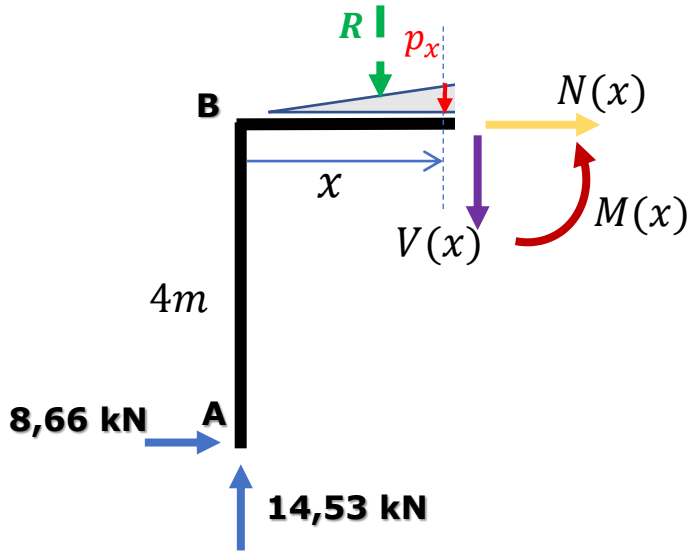

## 5. DIAGRAMAS DOS ESFORÇOS SOLICITANTES






## 6. SEÇÃO S

$$A)\ \frac{p_x}{x} = \frac{10}{6} \Longrightarrow p_x = \frac{5}{3}x$$

$$B)\ R = \frac{p_x * x}{2} = \frac{5.x^2}{6}$$

$$C)\ Equilíbrio\ em\ S$$

$$\sum X = 0 = 8,66 + N(x) \Longrightarrow N(x) = -8,66$$

$$\sum Y = 0 = 14,53 - \frac{5.x^2}{6} - V(x) \Longrightarrow V(x) = 14,53 - \frac{5.x^2}{6}$$

$$\sum M_{(S)} = 0 = 8,66 * 4 - 14,53 * x + \frac{5.x^2}{6} * \frac{x}{3} + M(x) \Longrightarrow$$

$$M(x) = -8,66 * 4 + 14,53 * x - \frac{5.x^2}{6} * \frac{x}{3}$$

$$D)\ valores\ notáveis: V = 0\ para\ x = 4,17; M(4,17) = 5,81$$

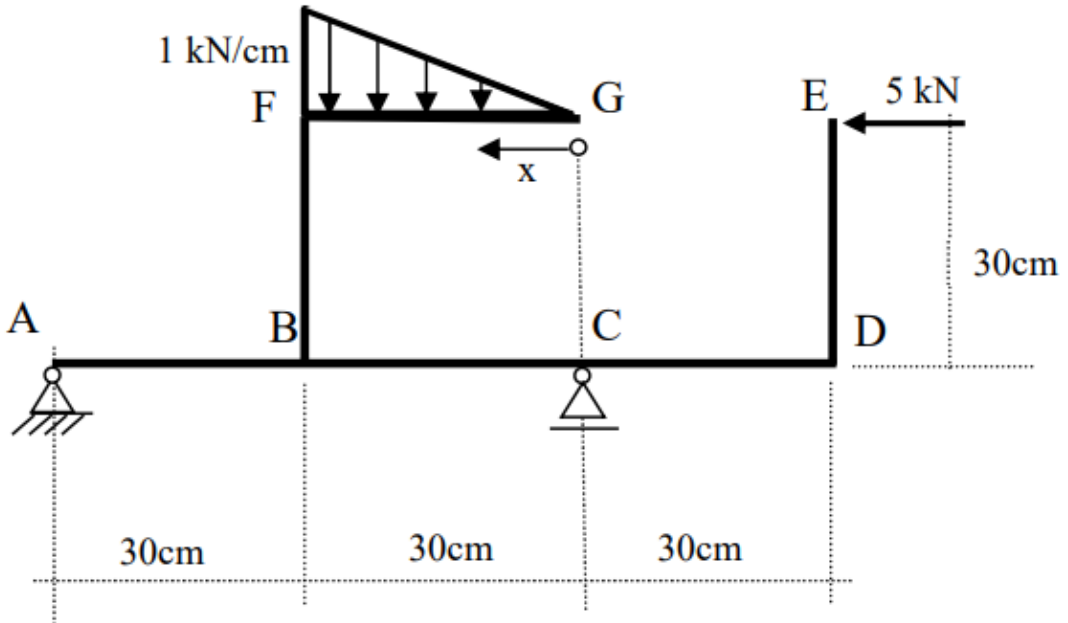
# EXERCÍCIO 8.
# P1 2021

PEF-3200 – 1ª Prova – 09/06/2021

Nº USP:_____Nome:_____

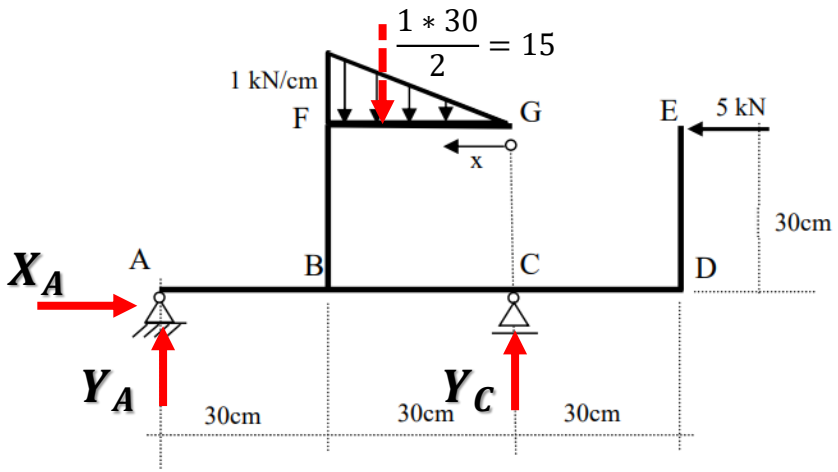
**1ª Questão (3,5 pontos)** A estrutura ABCDEFG, plana, está submetida a uma força uniformemente variada de zero em G a 1 kN/cm em F no trecho GF e a uma força horizontal de 5 kN aplicada em E. Determine

a) as reações nos apoios A (articulação fixa) e C (articulação móvel).
b) as funções de x dos esforços solicitantes no trecho GF, com G sendo a origem de x.
c) o diagrama do momento fletor no trecho ABCDE.



**1ª Questão (3,5 pontos)** A estrutura ABCDEFG, plana, está submetida a uma força uniformemente variada de zero em G a 1 kN/cm em F no trecho GF e a uma força horizontal de 5 kN aplicada em E. Determine

a) as reações nos apoios A (articulação fixa) e C (articulação móvel).
b) as funções de x dos esforços solicitantes no trecho GF, com G sendo a origem de x.
c) o diagrama do momento fletor no trecho ABCDE.



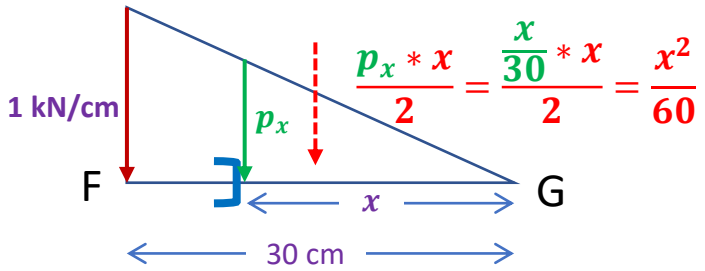

## a) REAÇÕES NOS APOIOS

$$\sum X = 0 = X_A - 5 \Longrightarrow X_A = 5\ kN$$

$$\sum M_{(A)} = 0 = -15 * 40 + Y_C * 60 + 5 * 30 \Longrightarrow Y_C = 7,5\ kN$$

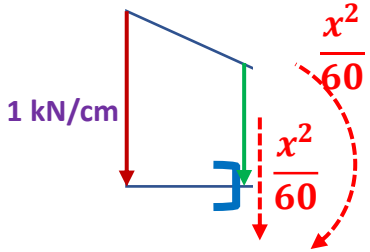
$$\sum M_{(C)} = 0 = -Y_A * 60 + 15 * 20 + 5 * 30 \Longrightarrow Y_A = 7,5\ kN$$

## b) FUNÇÕES DE $x$ NO TRECHO GF




$$\frac{p_x * x}{2} = \frac{\frac{x}{30} * x}{2} = \frac{x^2}{60}$$

$$\frac{p_x}{x} = \frac{1}{30} \Longrightarrow p_x = \frac{x}{30}$$

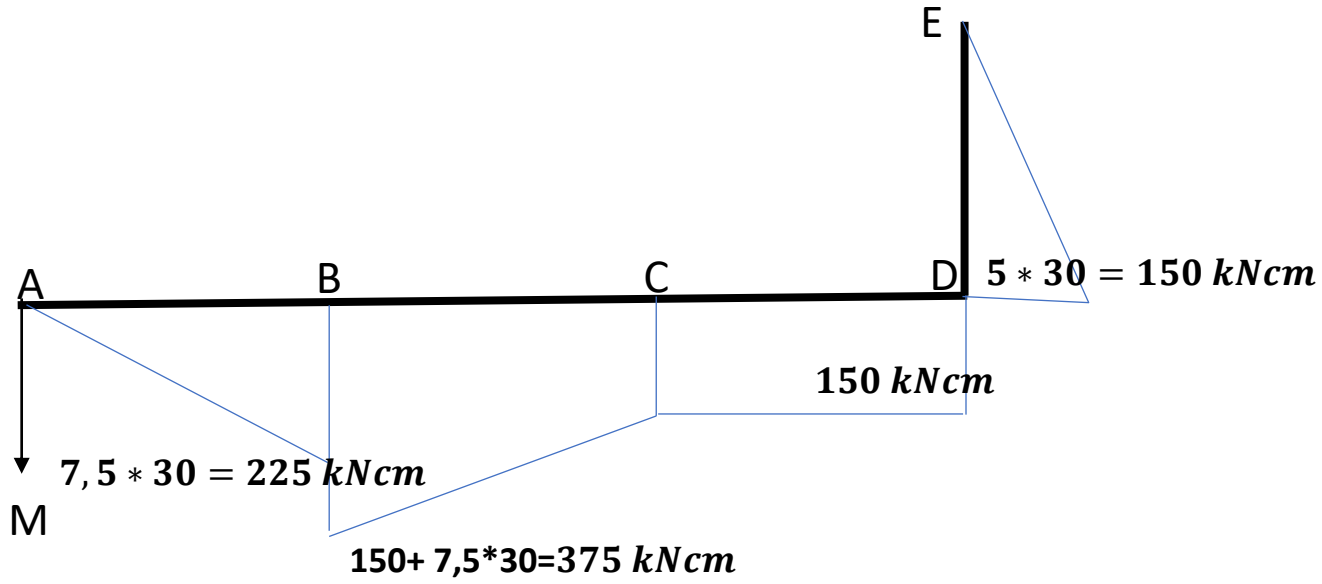



$$\frac{x^2}{60} * \frac{x}{3} = \frac{x^3}{180}$$

$$V(x) = +\frac{x^2}{60}\ kN$$

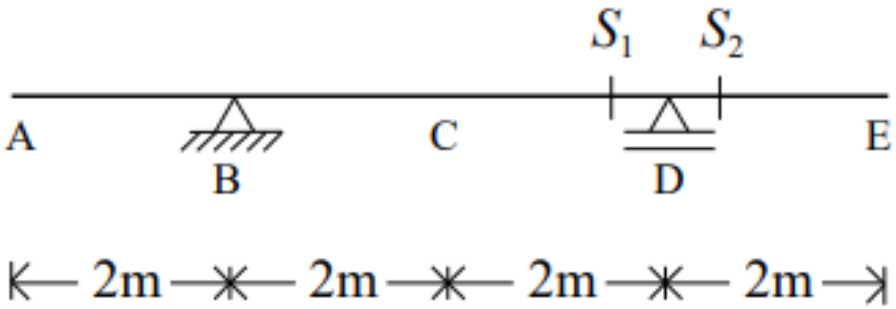
$$M(x) = -\frac{x^3}{180}\ kNcm$$

## c) DIAGRAMA DO MOMENTO FLETOR

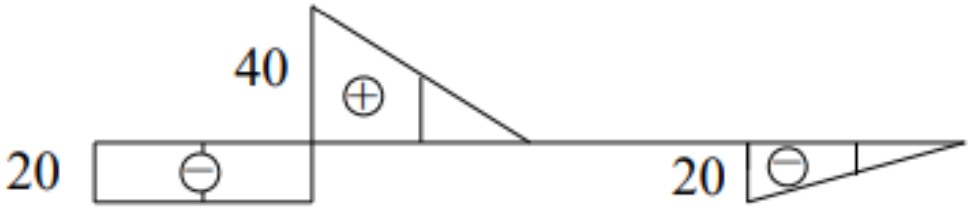


# EXERCÍCIO 9.

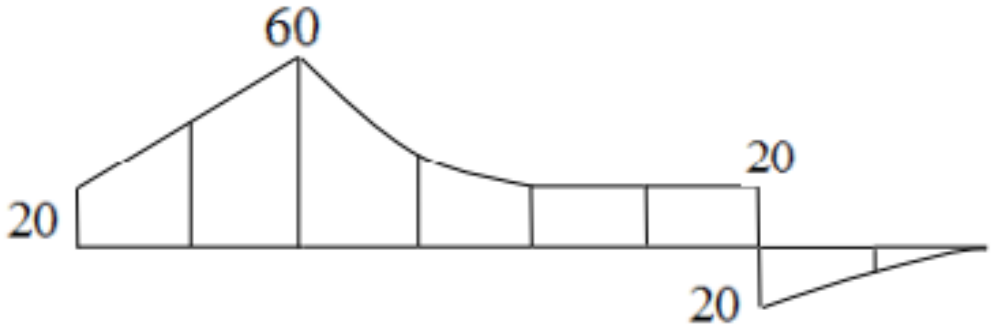
## A PARTIR DOS DIAGRAMAS DE ESFORÇOS SOLICITANTES, DETERMINAR OS ESFORÇOS EXTERNOS QUE ATUAM NA VIGA

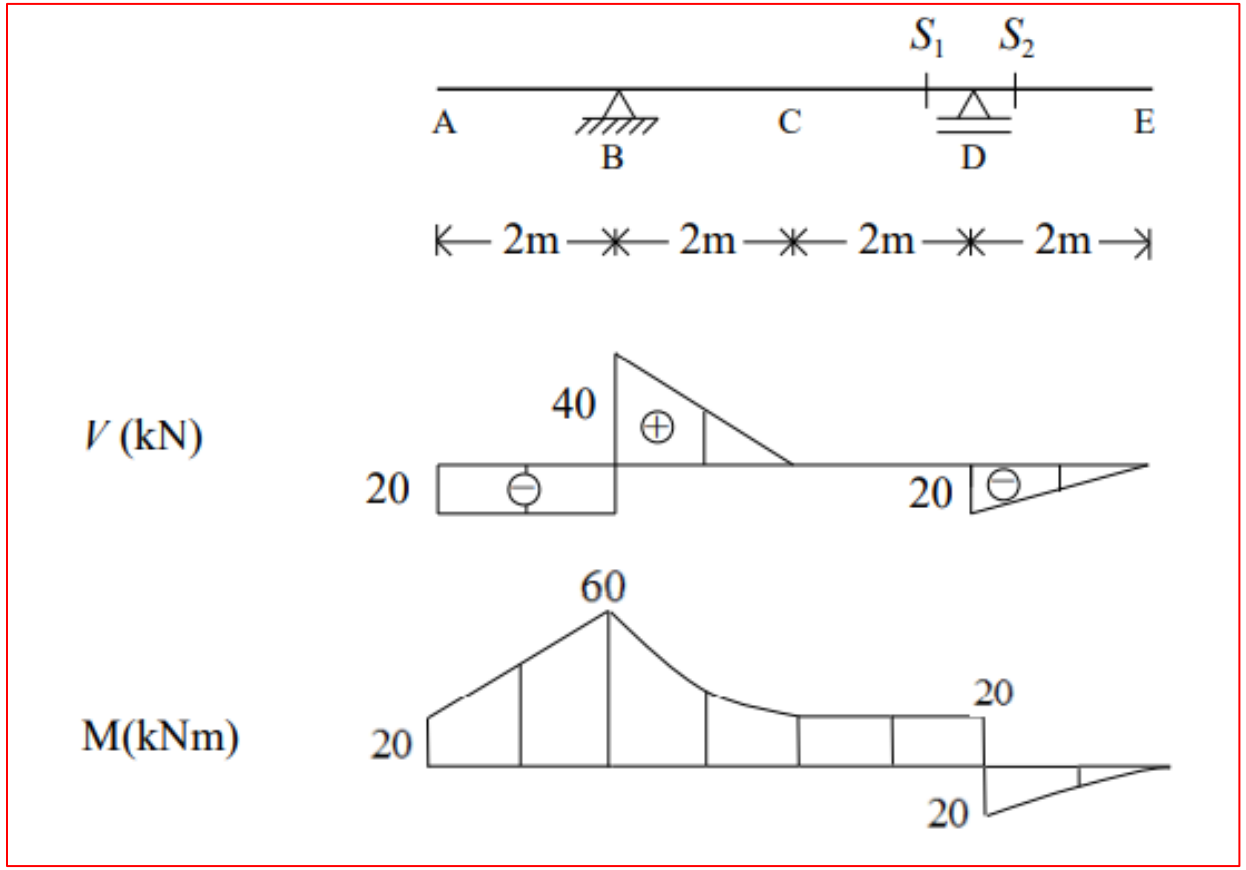


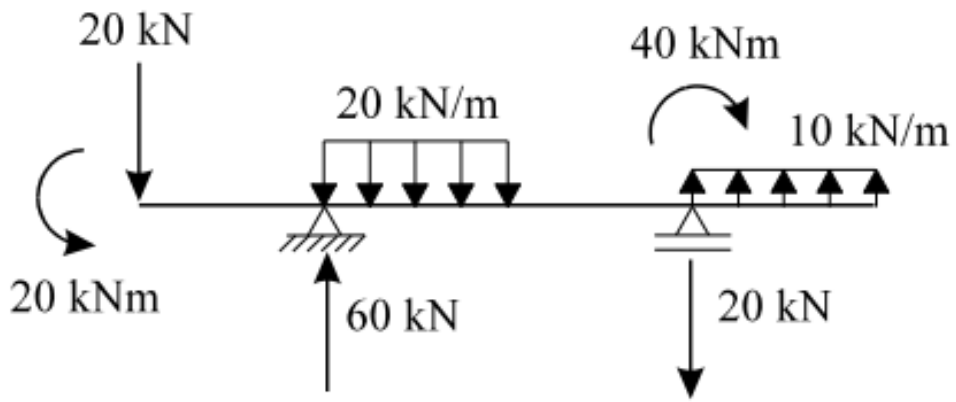

$V$ (kN)



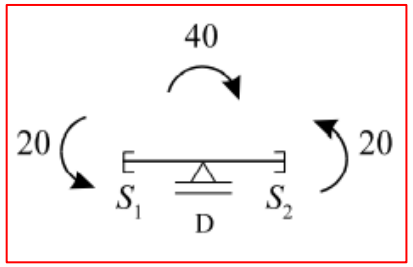

M(kNm)






1. **Descontinuidade no diagrama de forças cortantes indica que neste ponto é aplicada uma força concentrada**: existe uma força concentrada para cima de 60 kN aplicada em B e uma de 20 kN para baixo aplicada em D.
2. **Trechos lineares (variando constantemente) no diagrama de forças cortantes, indica que neles se tem forças uniformemente distribuídas**: no trecho BC a cortante varia 40 kN a 0 kN em 2m (a força distribuída aplicada é de 20 kN/m). Em DE a força uniformemente distribuída é de 10 kN/m.
3. **"regra do barbante"** (sentidos das forças distribuídas se ligam às concavidades dos **momentos fletores**): em BC o carregamento distribuído é para baixo e em DE, é para cima.
4. **Descontinuidade no diagrama de momentos fletores indica que neste ponto existe aplicado um momento concentrado**: em A, 20 kNm no sentido anti-horário e em D, 40 kNm no sentido horário.



# EXERCÍCIO 10.

P1 2022

2ª. Questão (3,5 pontos): Um muro de arrimo vertical de 4 metros de altura está submetido a uma força vertical de 20 kN e a um momento de 30 kNm na extremidade livre C e a uma carga horizontal uniformemente variada de 18 kN/m em A a zero kN/m em B (a 3 metros de A e a 1 metro de C). Considerando A como a origem da variável x, determine a função do momento fletor no trecho AB.

# EXERCÍCIO 10.

P1 2022

2ª. Questão (3,5 pontos): Um muro de arrimo vertical de 4 metros de altura está submetido a uma força vertical de 20 kN e a um momento de 30 kNm na extremidade livre C e a uma carga horizontal uniformemente variada de 18 kN/m em A a zero kN/m em B (a 3 metros de A e a 1 metro de C). Considerando A como a origem da variável x, determine a função do momento fletor no trecho AB.

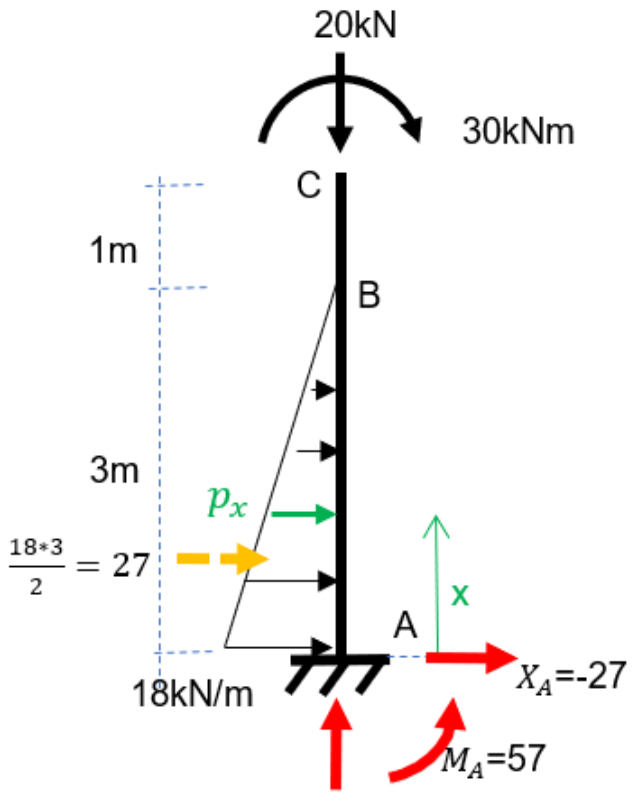



Figura 1

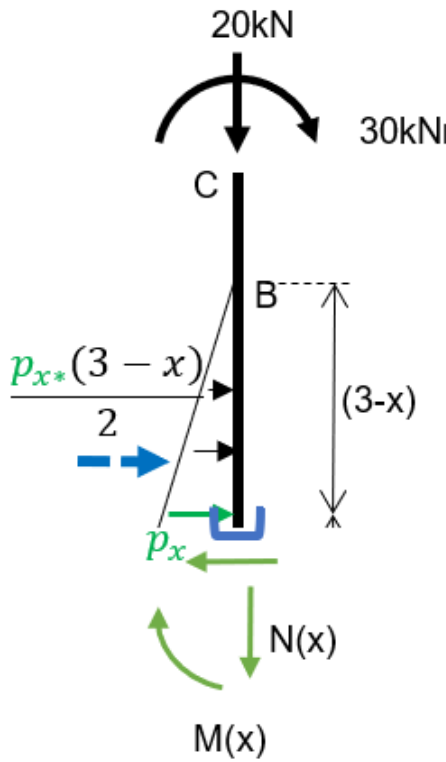



Figura 2

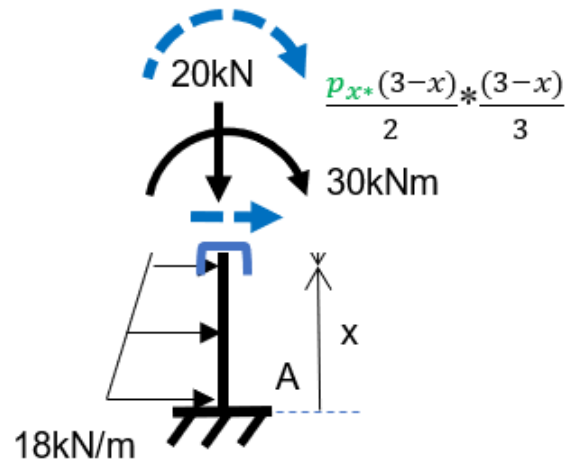



Figura 3

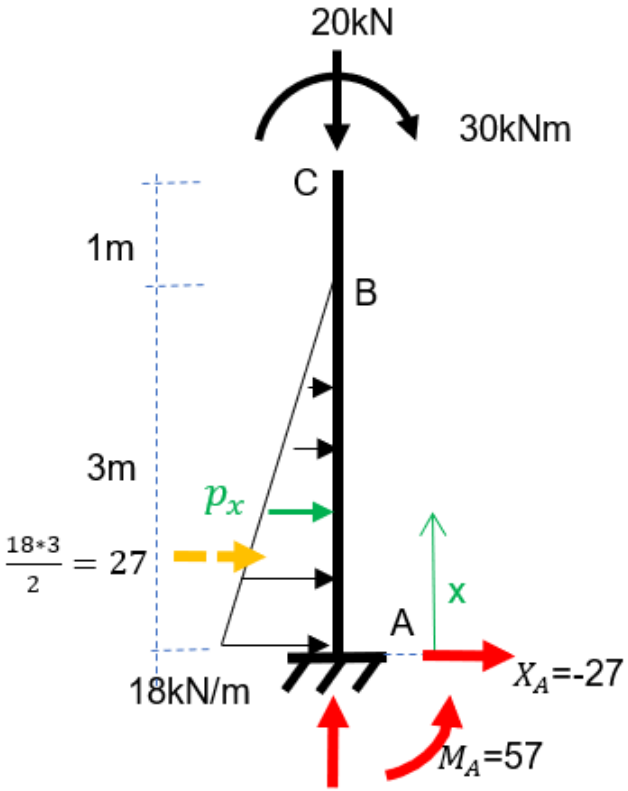



Figura 1

1. *Reações no apoio A* (Figura 1)

$$\sum F_H = 0 = 27 + X_A \Longrightarrow X_A = -27\ kN$$

$$\sum F_V = 0 = -20 + Y_A \Longrightarrow Y_A = 20\ kN$$

$$\sum M_{(A)} = 0 = -27 * 1 - 30 + M_A \Longrightarrow M_A = 57\ kNm$$

2. Obtenção de $p_x$ (Figura 1)

$$\frac{p_x}{(3-x)} = \frac{18}{3} \Longrightarrow p_x = 6 * (3-x)$$

3. Aplicando o teorema do corte (Figura 3)

$$M(x) = -30 - \frac{p_{x*}(3-x)}{2} * \frac{(3-x)}{3} = -30 - \frac{6*(3-x)*(3-x)*(3-x)}{2*3} = \text{-30 - } (3-x)^3 \Longrightarrow$$

$$M(x) = x^3 - 9x^2 + 27x - 57 \Longrightarrow$$

$$\boldsymbol{M(x) = +x^3 - 9x^2 + 27x - 57}$$

4. Aplicando o equilíbrio (Figura 2)

$$\sum M(S) = 0 = -M(x) - 30 - \frac{p_{x*}(3-x)}{2} * \frac{(3-x)}{3} = -M(x) - 30 - \frac{6*(3-x)*(3-x)*(3-x)}{2*3} \Longrightarrow$$

$$\Longrightarrow \text{M(x)} = \text{-30 - } (3-x)^3 = x^3 - 9x^2 + 27x - 57$$

$$\boldsymbol{M(x) = +x^3 - 9x^2 + 27x - 57}$$

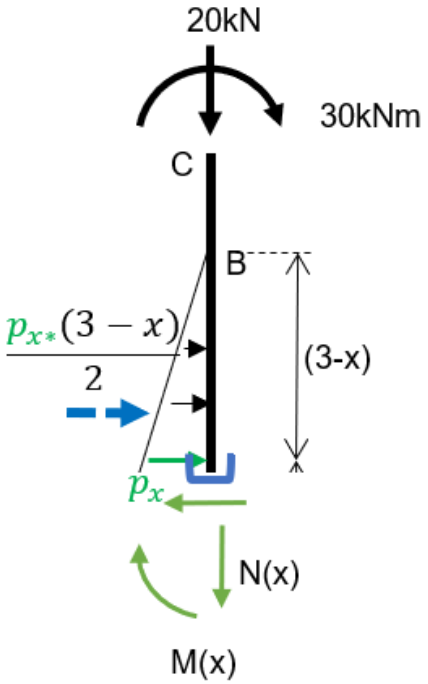



Figura 2

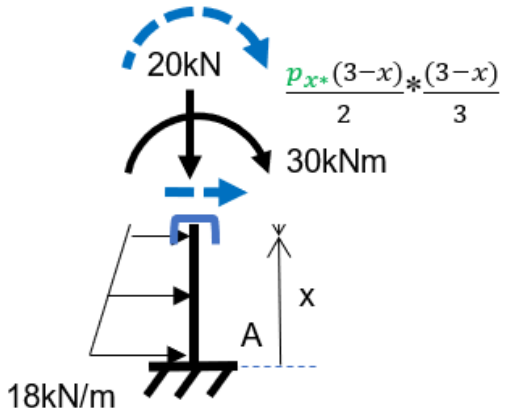



Figura 3

1. *Reações no apoio A* (Figura 1)

$$\sum F_H = 0 = 27 + X_A \Longrightarrow X_A = -27\ kN$$

$$\sum F_V = 0 = -20 + Y_A \Longrightarrow Y_A = 20\ kN$$

$$\sum M_{(A)} = 0 = -27 * 1 - 30 + M_A \Longrightarrow M_A = 57\ kNm$$

2. Obtenção de $p_x$ (Figura 1)

$$\frac{p_x}{(3-x)} = \frac{18}{3} \Longrightarrow p_x = 6 * (3-x)$$

3. Aplicando o teorema do corte (Figura 3)

$$M(x) = -30 - \frac{p_{x*}(3-x)}{2} * \frac{(3-x)}{3} = -30 - \frac{6*(3-x)*(3-x)*(3-x)}{2*3} = \text{-30 - } (3-x)^3 \Longrightarrow$$

$$M(x) = x^3 - 9x^2 + 27x - 57 \Longrightarrow$$

$$\boldsymbol{M(x) = +x^3 - 9x^2 + 27x - 57}$$

4. Aplicando o equilíbrio (Figura 2)

$$\sum M(S) = 0 = -M(x) - 30 - \frac{p_{x*}(3-x)}{2} * \frac{(3-x)}{3} = -M(x) - 30 - \frac{6*(3-x)*(3-x)*(3-x)}{2*3} \Longrightarrow$$

$$\Longrightarrow \text{M(x)} = \text{-30 - } (3-x)^3 = x^3 - 9x^2 + 27x - 57$$

$$\boldsymbol{M(x) = +x^3 - 9x^2 + 27x - 57}$$

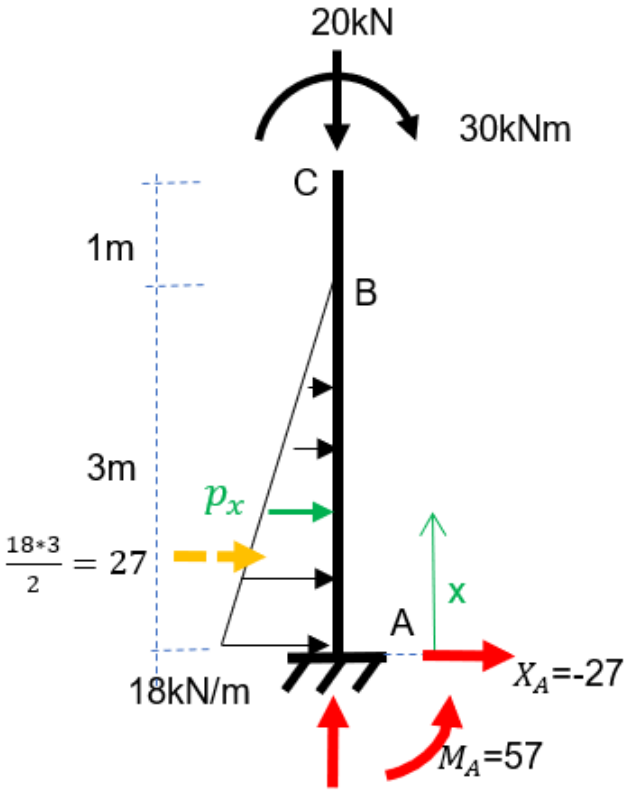



Figura 1

2. Obtenção de $p_x$ (Figura 1)

$$\frac{p_x}{(3-x)} = \frac{18}{3} \Longrightarrow p_x = 6 * (3-x)$$

5. Por integração

$p_x = 6 * (3-x) \Longrightarrow V(x) = \int -p_x(x)\,dx = \int -6 * (3-x)\,dx = -6 * (3x - \frac{x^2}{2}) + c_1$

$\Longrightarrow \boldsymbol{V(x) = -6 * (3x - \frac{x^2}{2}) + c_1}$

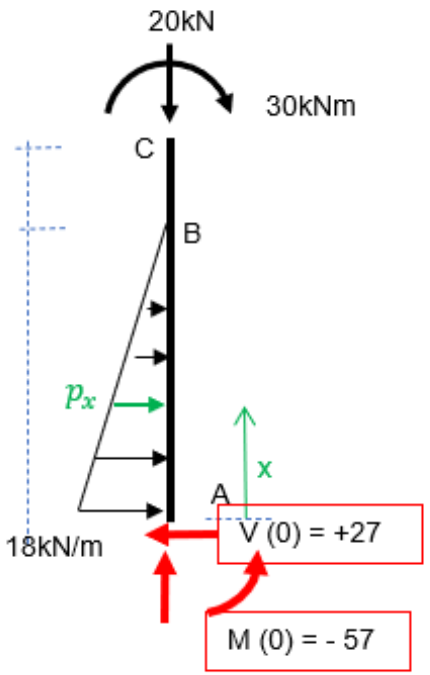
$M(x) = \int V(x)\,dx = \int (-6 * (3x - \frac{x^2}{2}) + c_1)\,dx = \int (-18x + 3x^2 + c_1)\,dx \Longrightarrow$

$M(x) = \int (-18x + 3x^2 + c_1)\,dx = -18\frac{x^2}{2} + 3\frac{x^3}{3} + c_1 x + c_2 = -9x^2 + x^3 + c_1 x + c_2$

$\Longrightarrow \boldsymbol{M(x) = +x^3 - 9x^2 + c_1 x + c_2}$

Condições de contorno

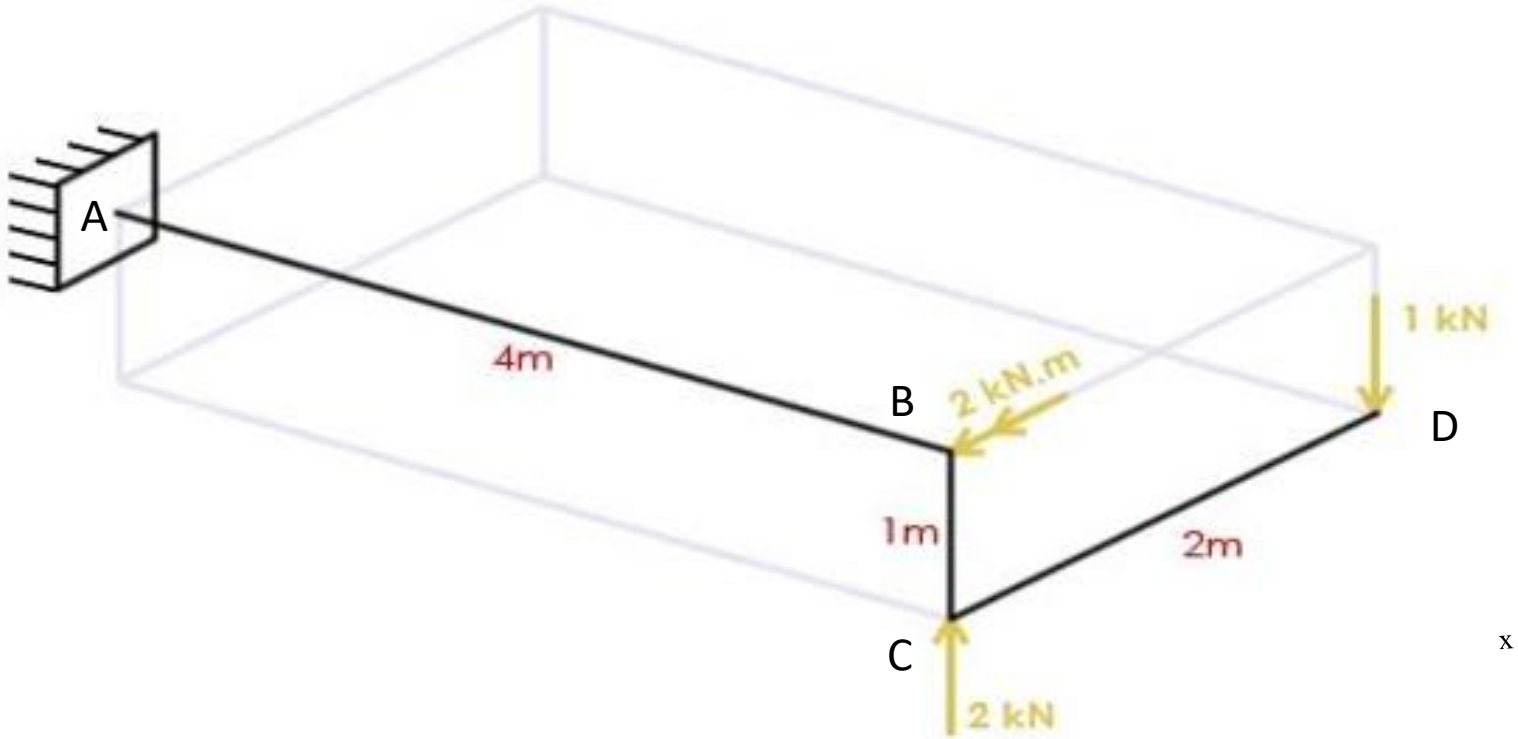
$V(0) = 27\ e\ M(0) = -57 \Longrightarrow \boldsymbol{c_1 = 27\ \ e\ \ c_2 = -57}$

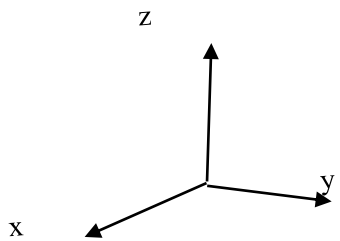


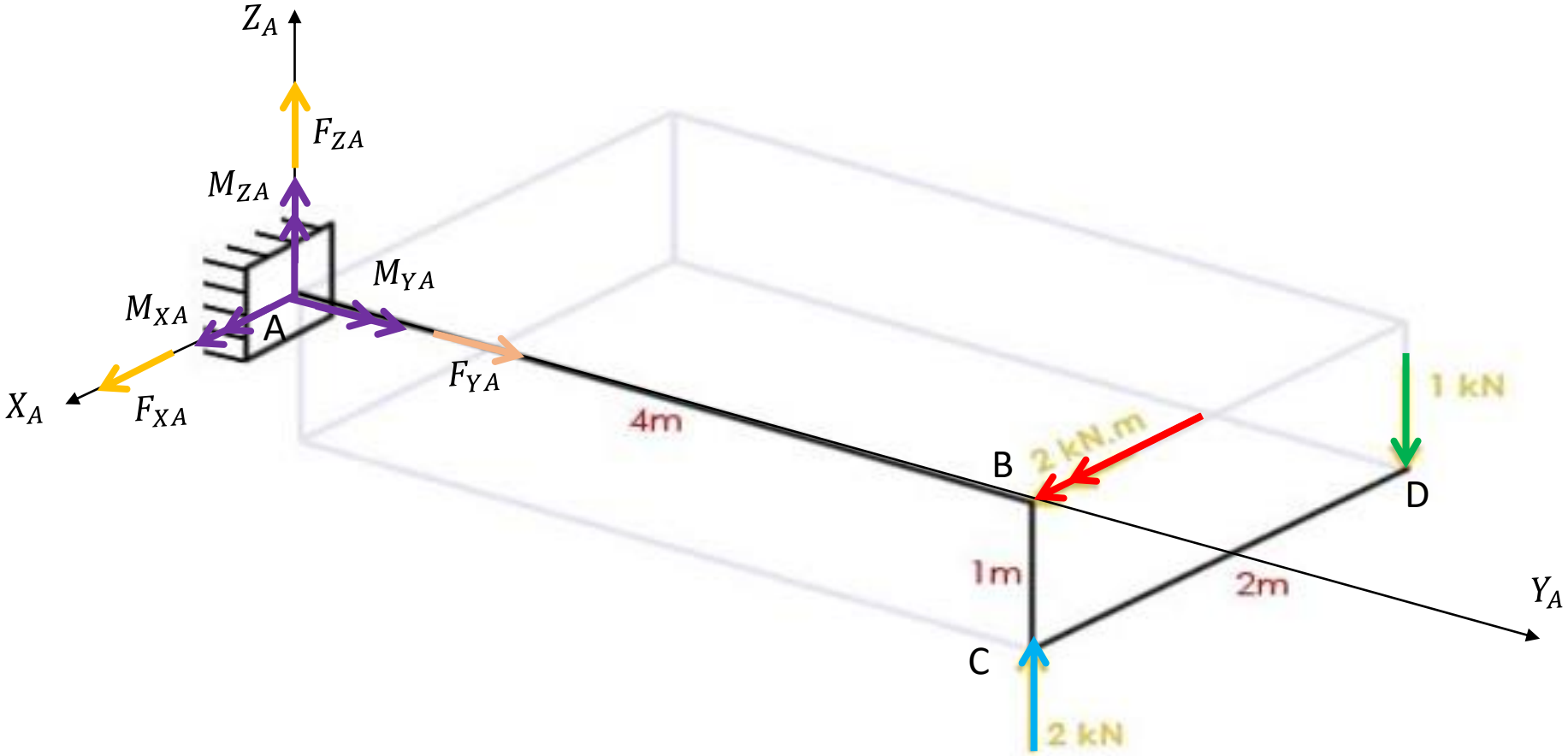

$\boldsymbol{M(x) = +x^3 - 9x^2 + 27x - 57}$

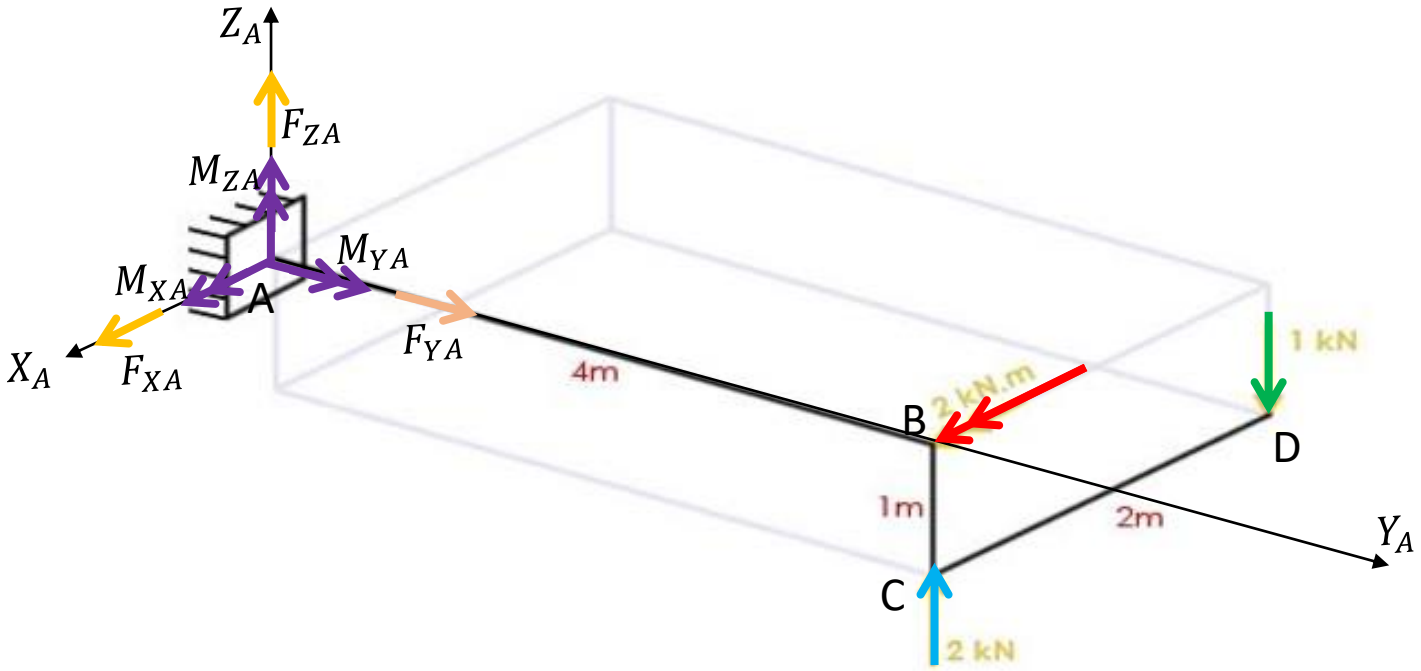
# EXERCÍCIO 11.

**Determinar as reações no engastamento da estrutura espacial da figura**



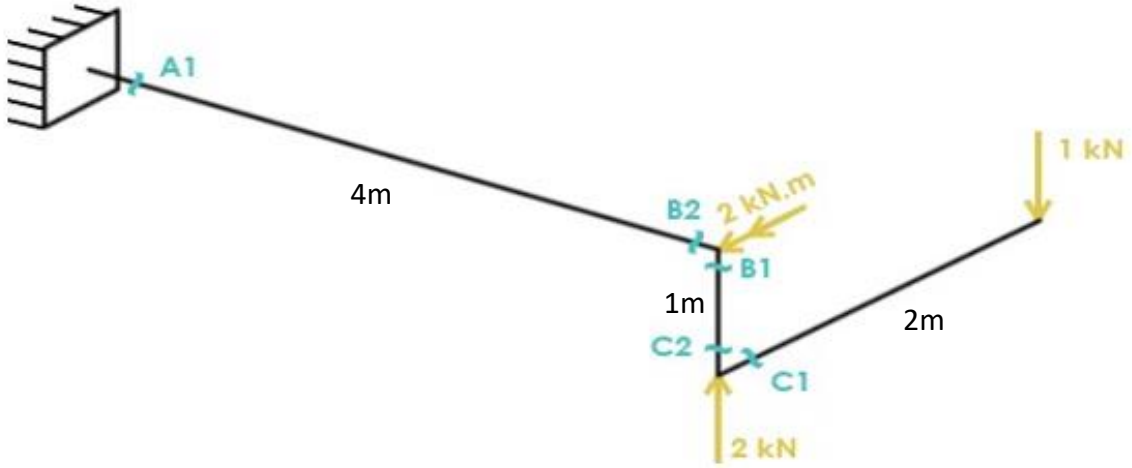



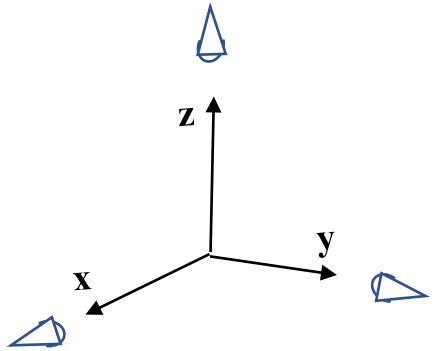

$Z_A$
$F_{ZA}$
$M_{ZA}$
$M_{YA}$
$M_{XA}$
A
$F_{YA}$
$X_A$
$F_{XA}$
4m
B
2 kN.m
1 kN
D
1m
2m
$Y_A$
C
2 kN

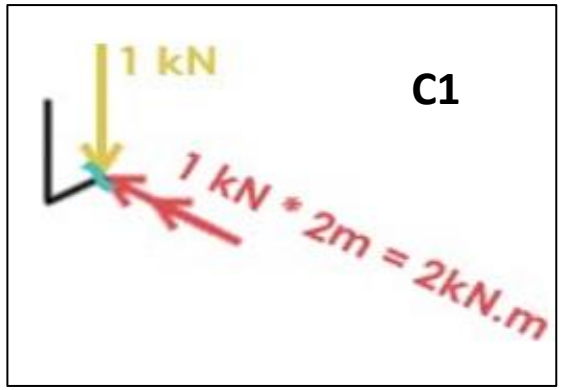


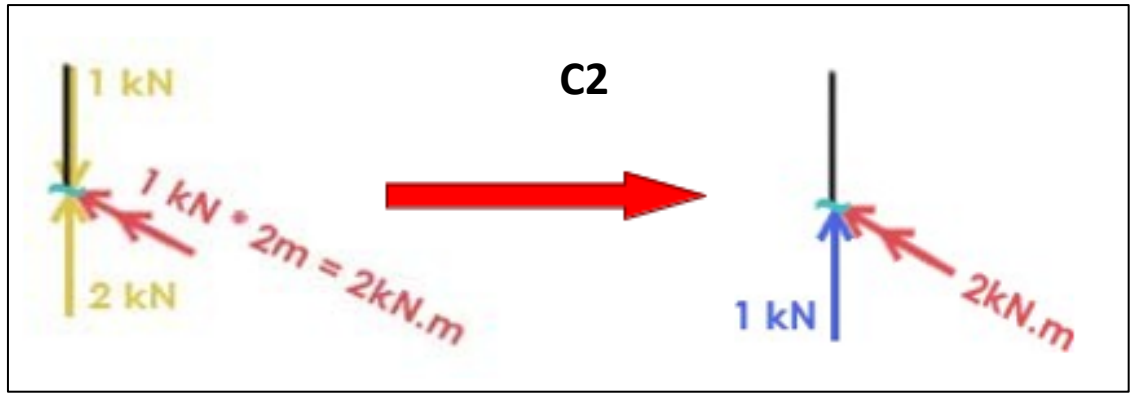

1. $\sum F_{(X)} = 0 = F_{XA} \rightarrow F_{XA} = 0\ kN$
2. $\sum F_{(Y)} = 0 = F_{YA} \rightarrow F_{YA} = 0\ kN$
3. $\sum F_{(Z)} = 0 = F_{ZA} + 2 - 1 \rightarrow F_{ZA} = -1\ kN$
4. $\sum M_{(XA)} = 0 = M_{XA} + 2 - 1 * 4 + 2 * 4 \rightarrow M_{XA} = -6\ kNm$
5. $\sum M_{(YA)} = 0 = M_{YA} - 1 * 2 \rightarrow M_{YA} = 2\ kNm$
6. $\sum M_{(ZA)} = 0 = M_{ZA} \rightarrow M_{ZA} = 0\ kNm$

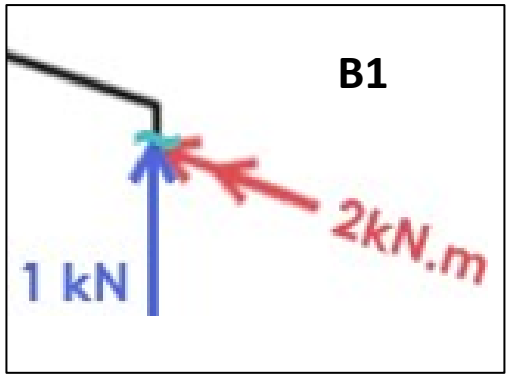
# TEOREMA DO CORTE: SEÇÕES C1, C2, B1, B2, A1

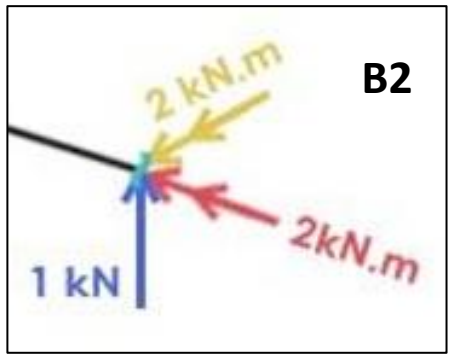


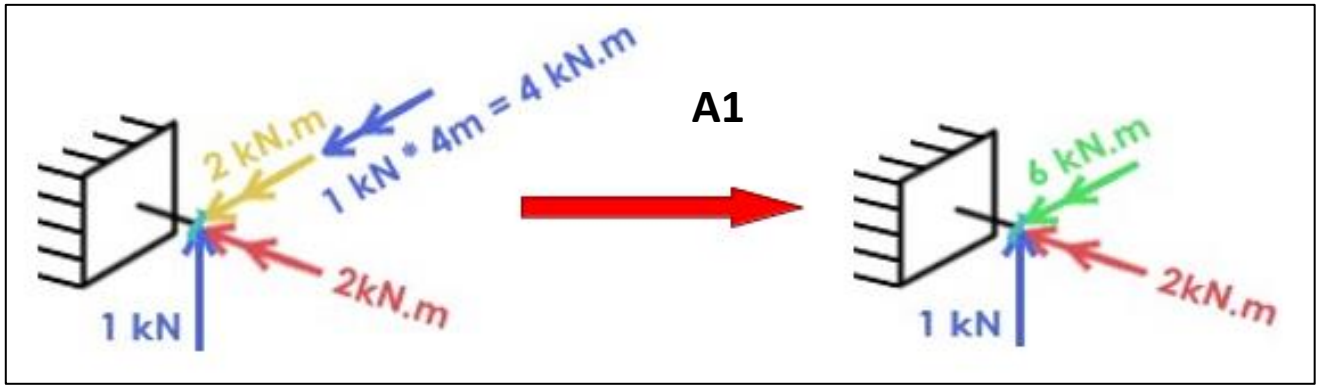



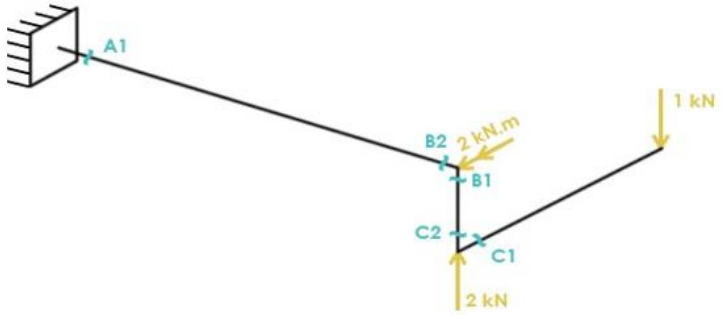



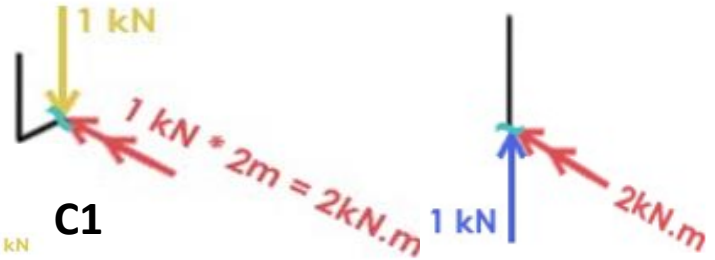



# DIAGRAMAS DOS ESFORÇOS SOLICITANTES: FORÇA NORMAL E FORÇA CORTANTE

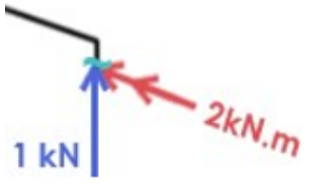


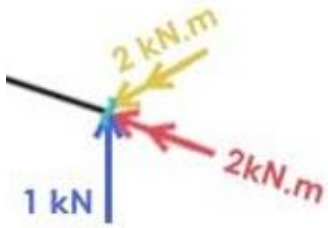

C1



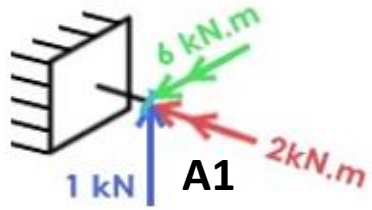

C2




B1

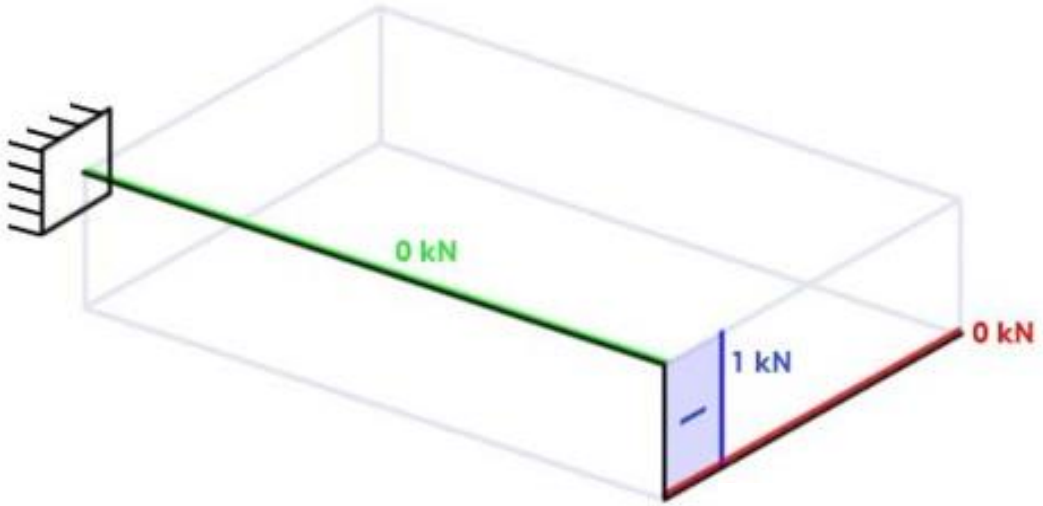



B2

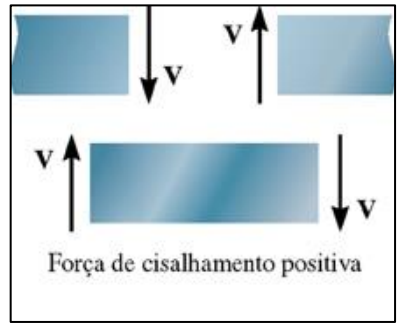


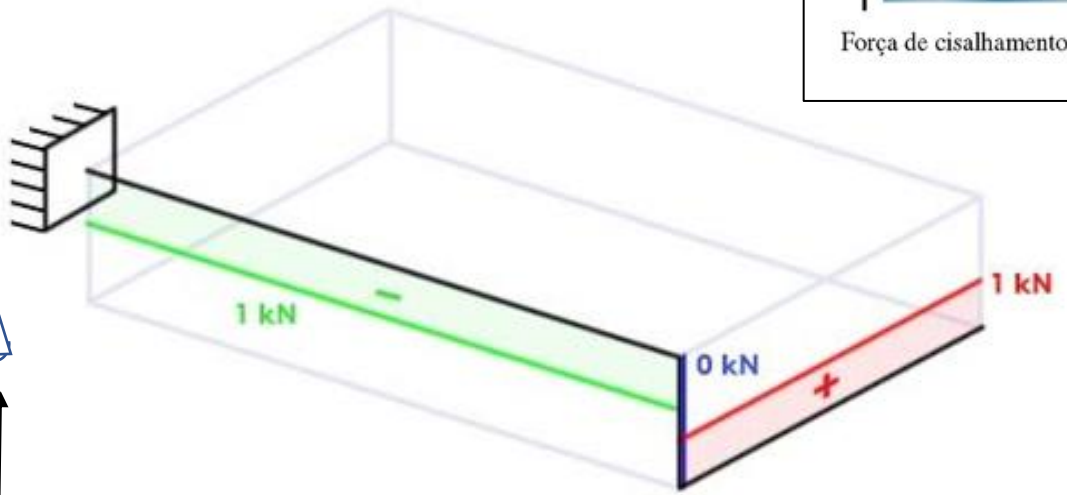

A1

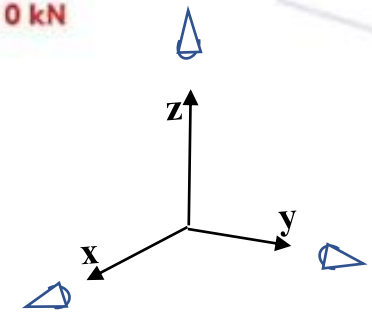
## FORÇA NORMAL



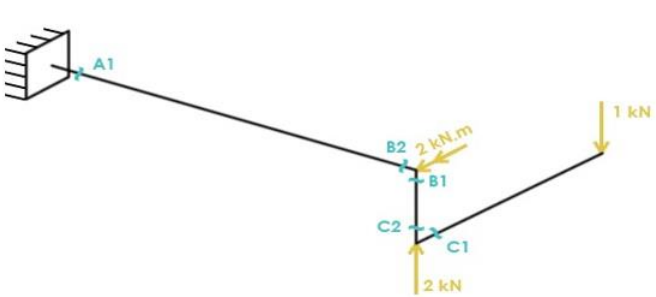

## FORÇA CORTANTE

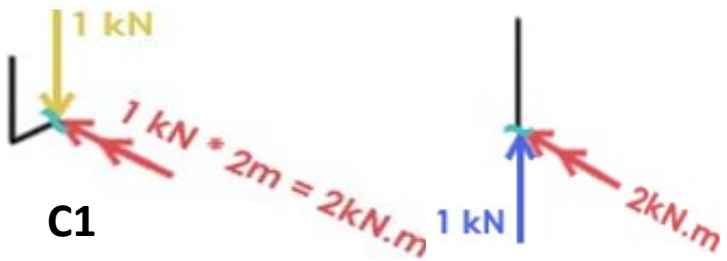


# DIAGRAMAS DOS ESFORÇOS SOLICITANTES: MOMENTO FLETOR E MOMENTO DE TORÇÃO

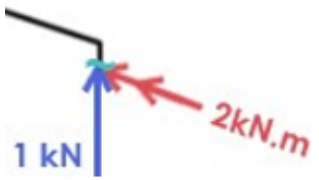


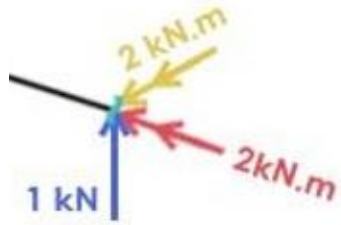

C1



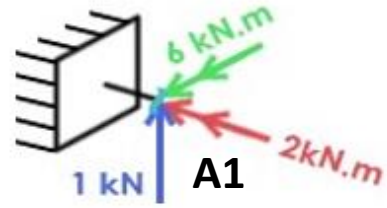

C2




B1

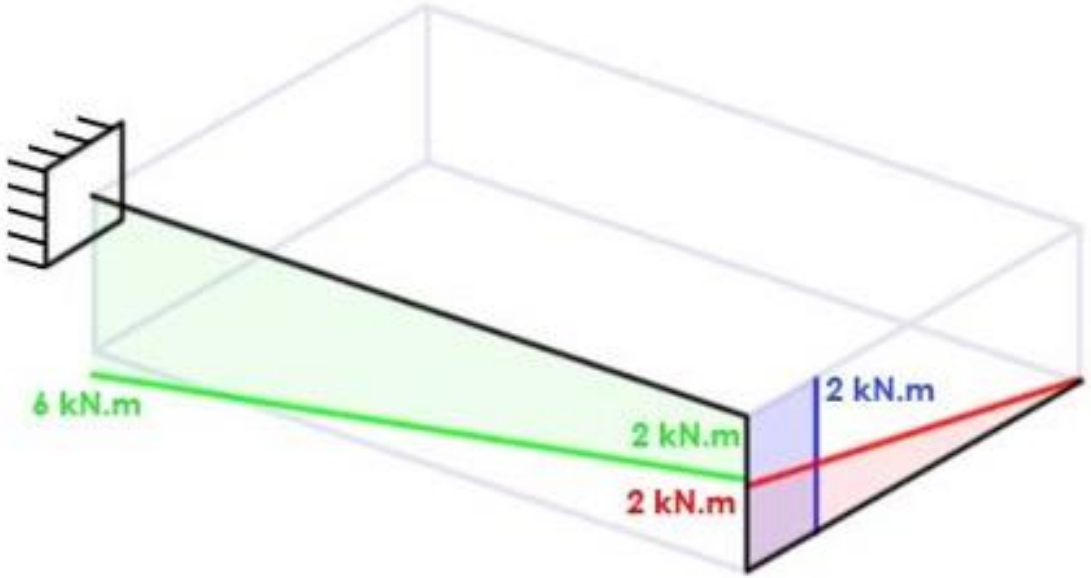



B2

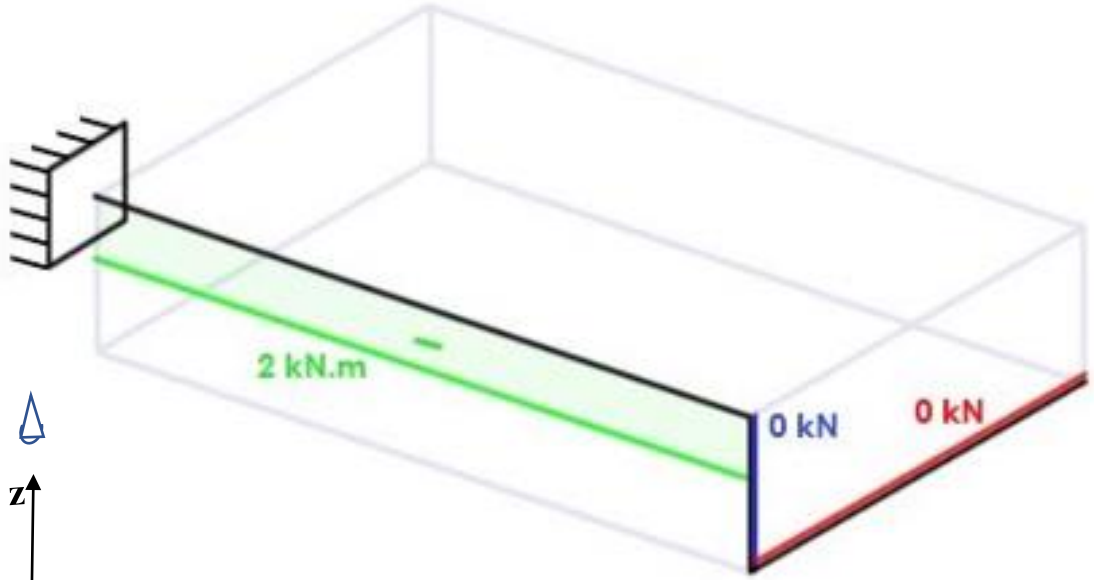


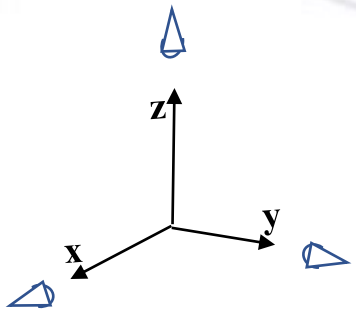

A1

## MOMENTO FLETOR



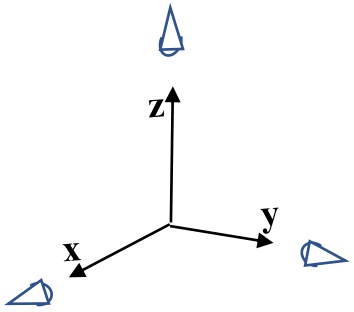

## MOMENTO DE TORÇÃO

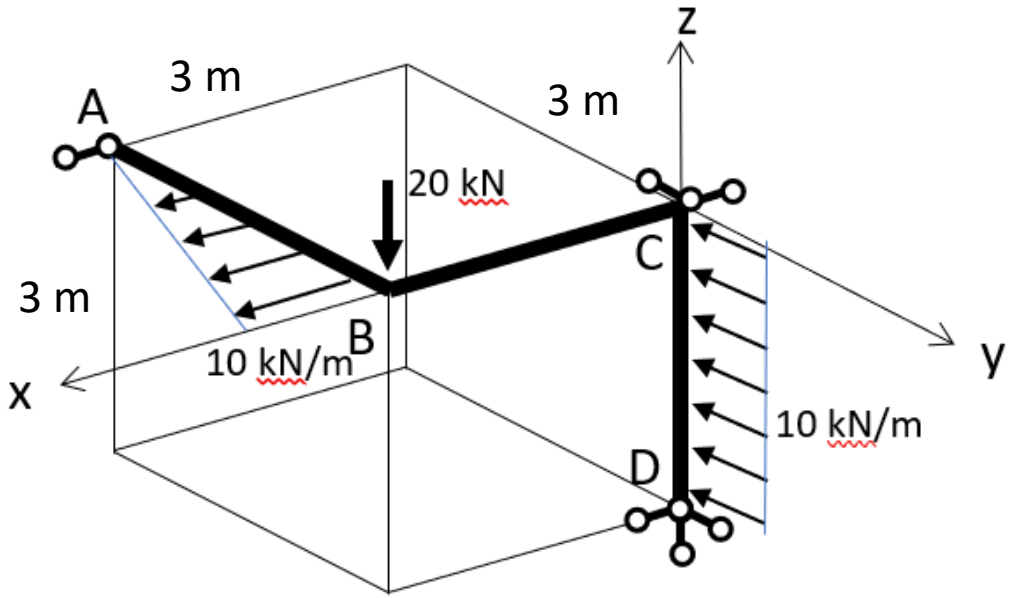


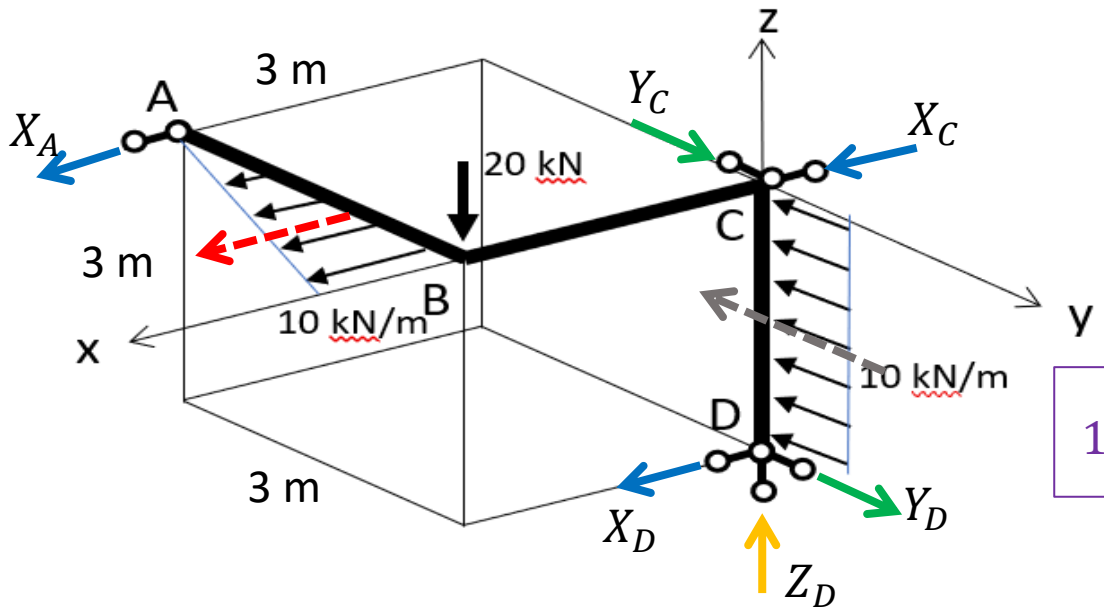
# EXERCÍCIO 12.
# P1 2020

*A viga principal ABCD da figura está apoiada em A, C e D por barras curtas nas direções dos eixos ortogonais x, y e z. A barra AB na direção do eixo y está submetida a um carregamento uniformemente variado de zero a* $10\frac{kN}{m}$ *na direção do eixo x; a barra BC está na direção do eixo x; a barra CD na direção do eixo z está submetida a um carregamento uniforme de* $10\frac{kN}{m}$ *na direção do eixo y ; em B há uma força concentrada de* $20\ kN$ *na direção z. Determine: a) as reações dos apoios A, C e D; b) os diagramas dos esforços solicitantes na barra CD, considerando o observador de frente aos eixos.*



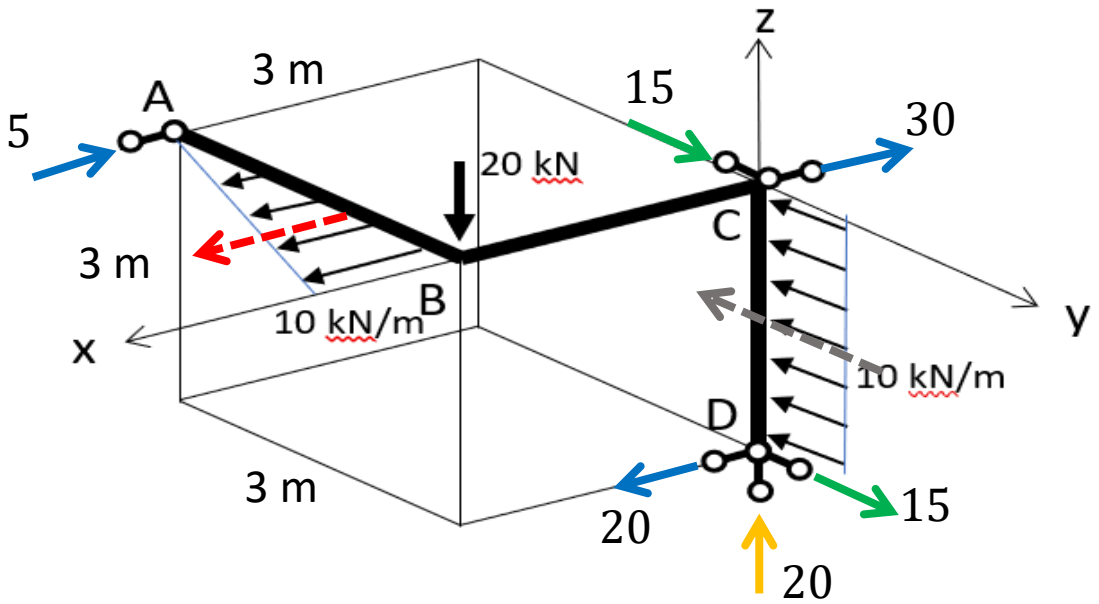



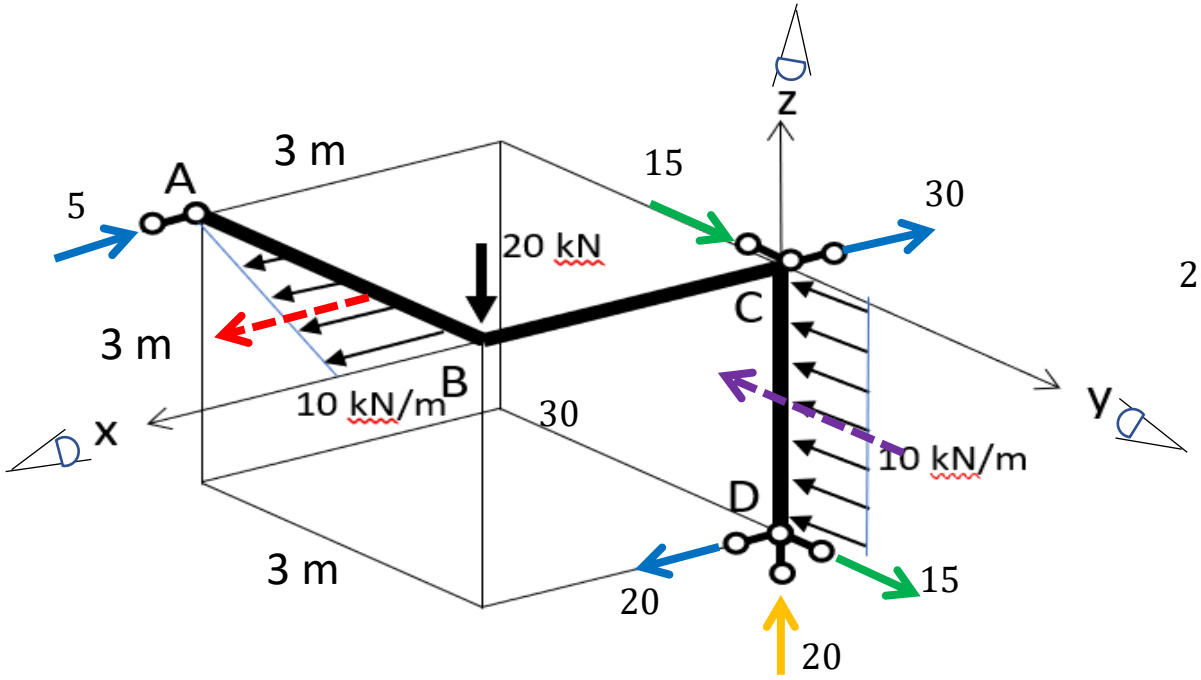
$$10\frac{kN}{m} * 3m \div 2 = 15\ kN$$

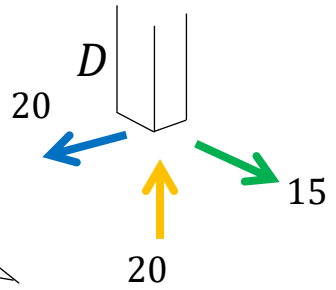


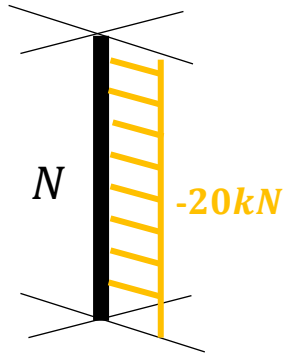

$$10\frac{kN}{m} * 3m = 30\ kN$$

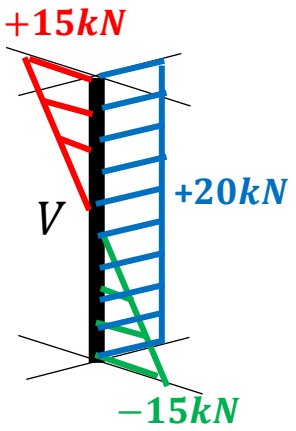
$$\begin{cases} \sum X = 0 = X_A + 15 + X_C + X_D \Longrightarrow X_C = -30kN \\ \sum Y = 0 = Y_C - 30 + Y_D \Longrightarrow Y_C = 15kN \\ \sum Z = 0 = -20 + Z_D \Longrightarrow Z_D = 20kN \\ \sum M_x = 0 = -30 * 1{,}5 + Y_D * 3 \Longrightarrow Y_D = 15kN \\ \sum M_y = 0 = 20 * 3 - X_D * 3 \Longrightarrow X_D = 20kN \\ \sum M_z = 0 = X_A * 3 + 15 * 1 \Longrightarrow X_A = -5kN \end{cases}$$

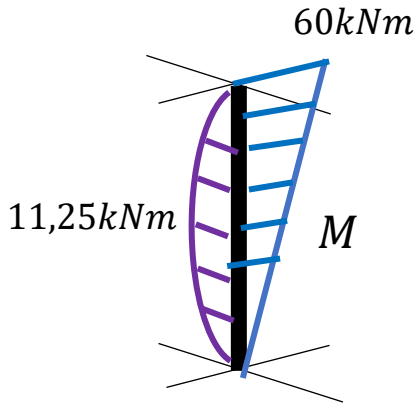


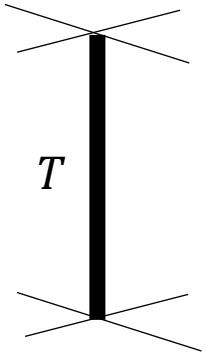


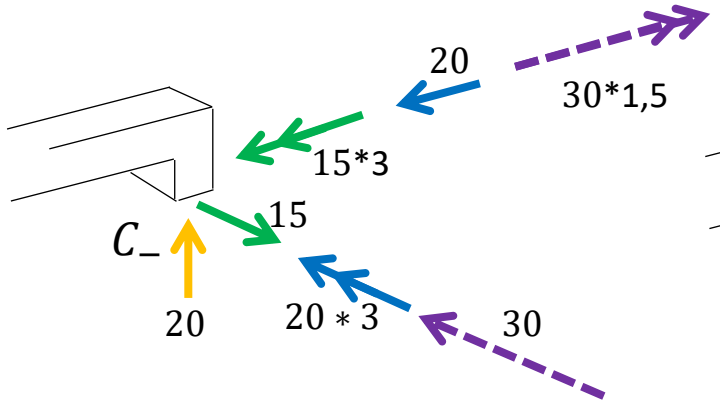



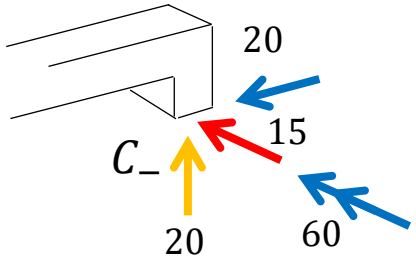




$$\frac{pl^2}{8} = \frac{10 * 3^2}{8} = 11{,}25kNm$$

Site oficial do KIT ESTRUTURAL MOLA

## Provas Antigas e Gabaritos

1. Reações de Apoio
2. Teoria
3. Esforços Solicitantes em Estruturas Planas
4. Diagramas de Esforços Solicitantes em Estruturas Espaciais
5. Linhas de Influência
6. Treliças

# EXTRA

1ª Questão: Uma escada reta está apoiada em um ponto A de uma laje (horizontal) com muito atrito e em um ponto B (4 m acima da laje) de uma parede lisa (vertical). Se o comprimento da escada é 5 m e seu peso é 100 N então determine

1) um modelo matemático para a situação classificando os apoios A e B (em engastamento, articulação fixa ou articulação móvel) e determine as reações nesses apoios;
2) as equações do momento fletor e da força cortante na barra AB, considerando o ponto A como a origem da variável x que segue no eixo da escada, a partir dos valores das reações obtidos em 1).