PEF 3200

Diagramas de esforços solicitantes de estruturas planas

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Abril/2023

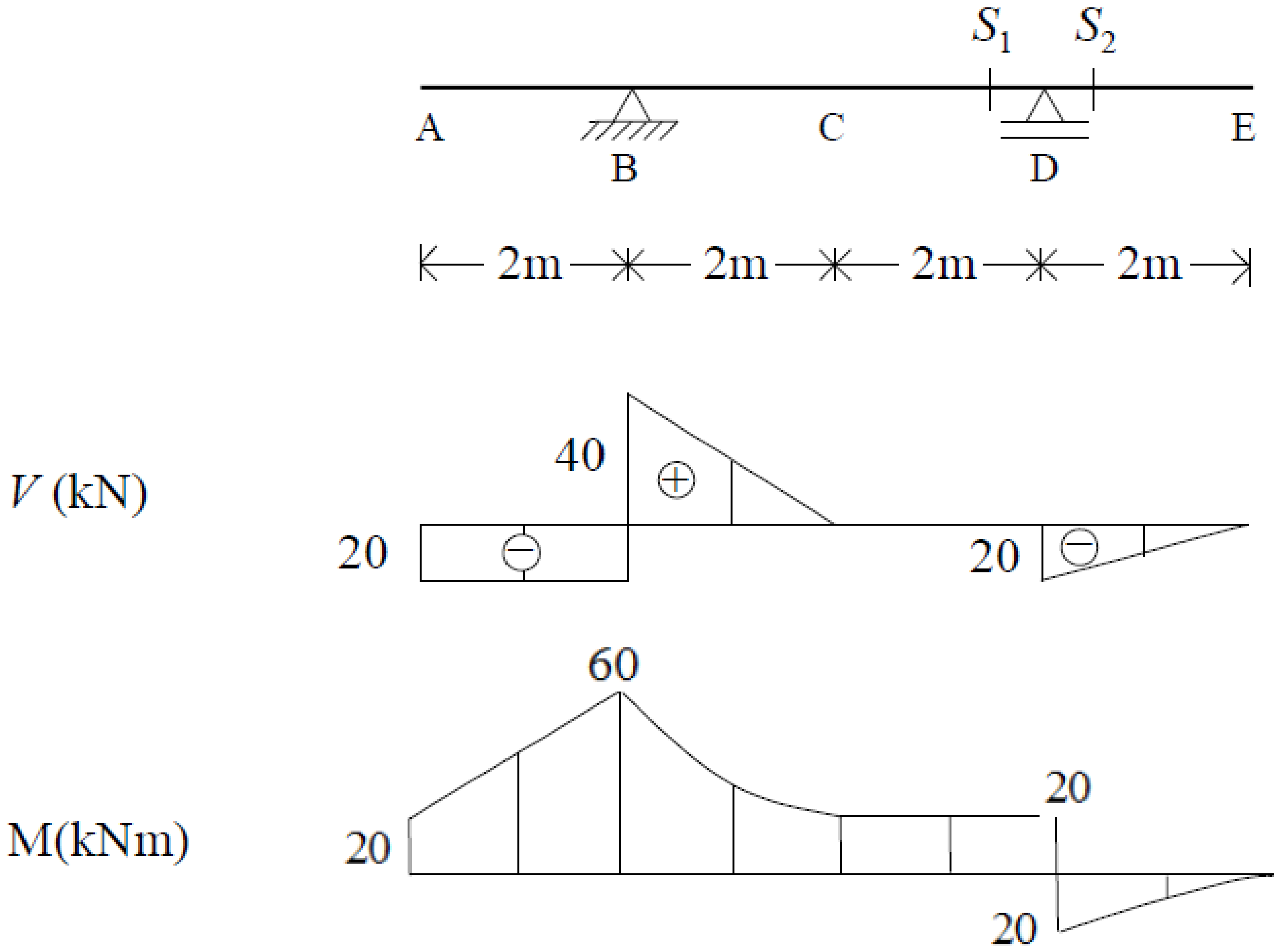
Conteúdo da aula

» Diagramas de esforços solicitantes de estruturas planas

Bibliografia: Apostila de Teoria (pdf no e-disciplinas)

Capítulo 5: Diagramas de esforços solicitantes de estruturas planas

Exemplo 14: A partir dos diagramas de esforços solicitantes, determinar os esforços externos que atuam na viga



Equações Diferenciais de Equilíbrio

$$\frac{dN(x)}{dx} = -p_x(x) \qquad \frac{dV(x)}{dx} = -p(x) \qquad \frac{d^2M(x)}{dx^2} = -p(x) \qquad \frac{dM(x)}{dx} = V(x)$$

a) Caso $p(x) = 0$

Sem carga distribuída no trecho $x_1 < x < x_2$

$V(x) = C_1 = cte \rightarrow$ Função (diagrama) de esforço cortante constante

$M(x) = C_1 \cdot x + C_2 \rightarrow$ Função (diagrama) de momento fletor linear

b) Caso $p(x) = p = cte$

Carga distribuída uniforme no trecho $x_1 < x < x_2$

$V(x) = -\,px + C_1 \rightarrow$ Função (diagrama) de esforço cortante linear

$M(x) = -\,p\frac{x^2}{2} + C_1 \cdot x + C_2 \rightarrow$ Função (diagrama) de momento fletor é parábola

c) Generalização para $\forall$ p(x) é imediata

d) Caso $p_x(x)=0$

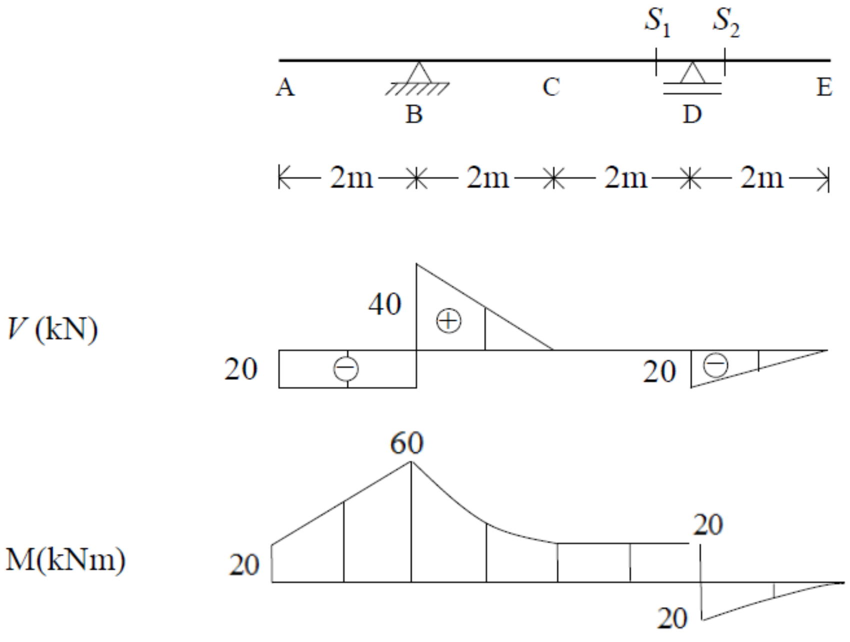
Sem carga distribuída no trecho $x_1 < x < x_2$

N(x) = constante

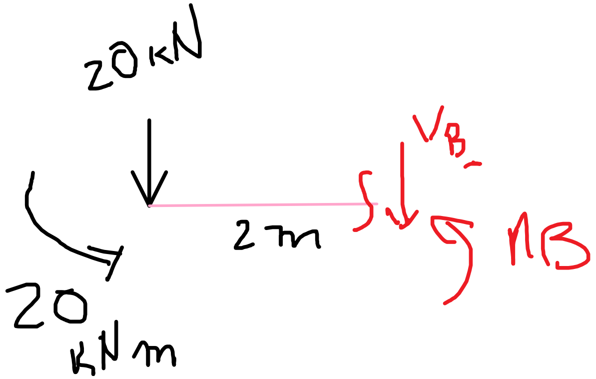
e) Caso $p_x(x)=cte$

Carga distribuída uniforme no trecho $x_1 < x < x_2$

N(x) = linear (reta)



Não há carga horizontal, sem diagramas de N



$V_{B-} = -20$

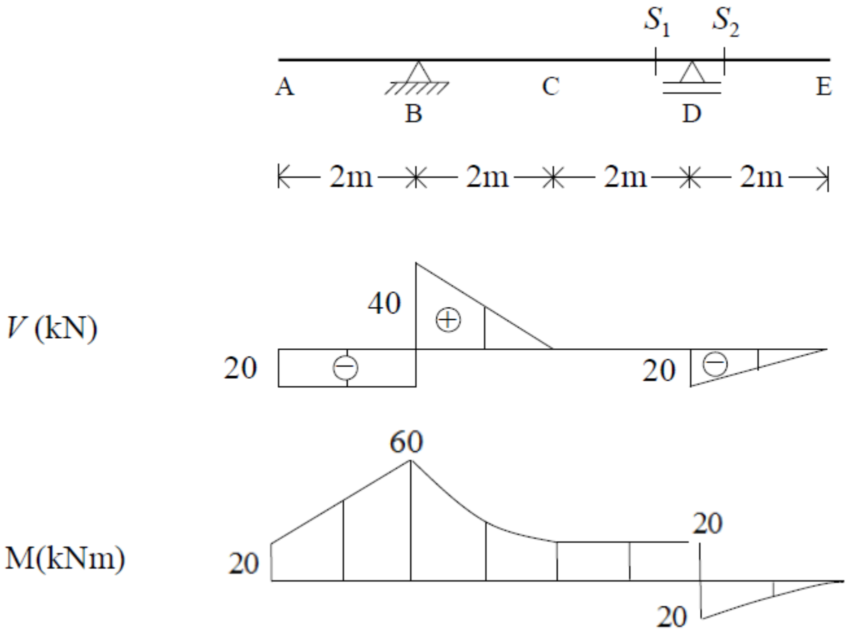
$M_B + 20 \cdot 2 + 20 = 0$

$M_B = -60$

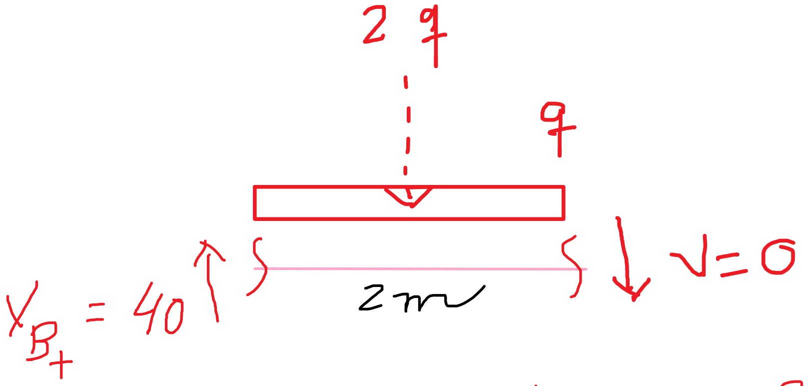
Trecho AB:

V: cte e M : linear: força concentrada. Em A, V = -20 kN, portanto Fvertical em A para baixo de 20 kN

Em A, M = 20 kNm, tração em cima, MA = 20 kNm

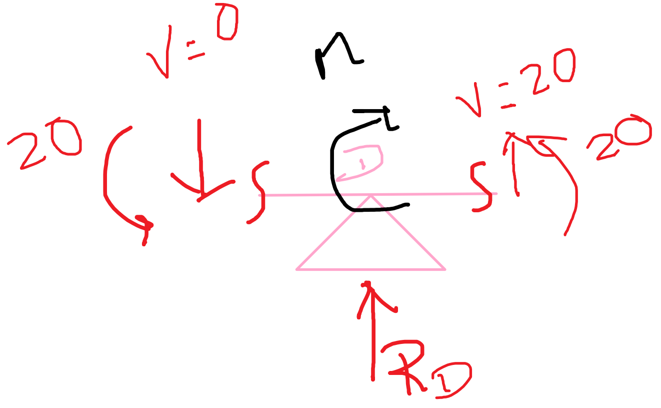


Obter a carga q em BC



$\Sigma F_y = 0. \quad 2q = 40 \Rightarrow q = 20 \text{ kN/m}$

Equilíbrio junto a D

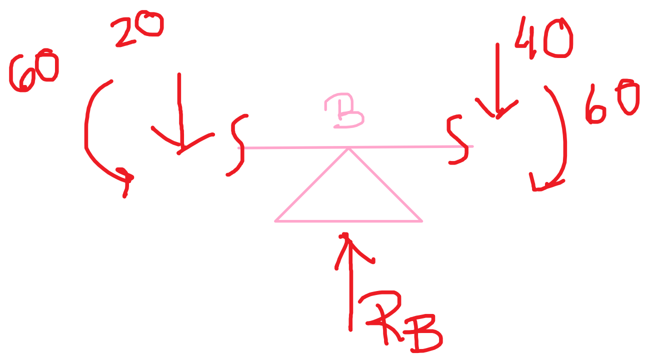


$R_D = -20$

$M - 20 \cdot 20 = 0$

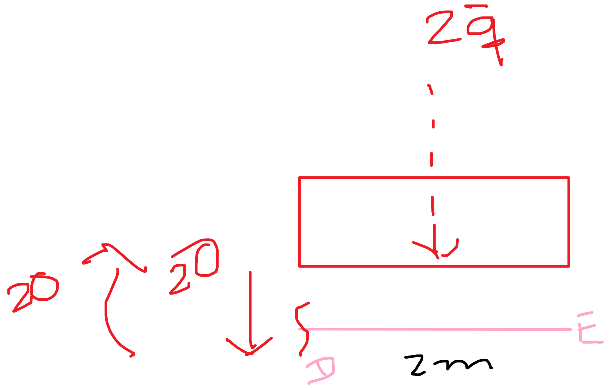
$M = 40 \text{ kNm}$

Equilíbrio junto a B



$R_B = 60 \text{ kN}$

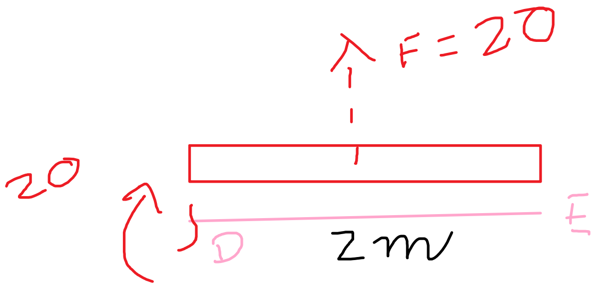
Trecho DE: obter q



$\Sigma F_y = 0$

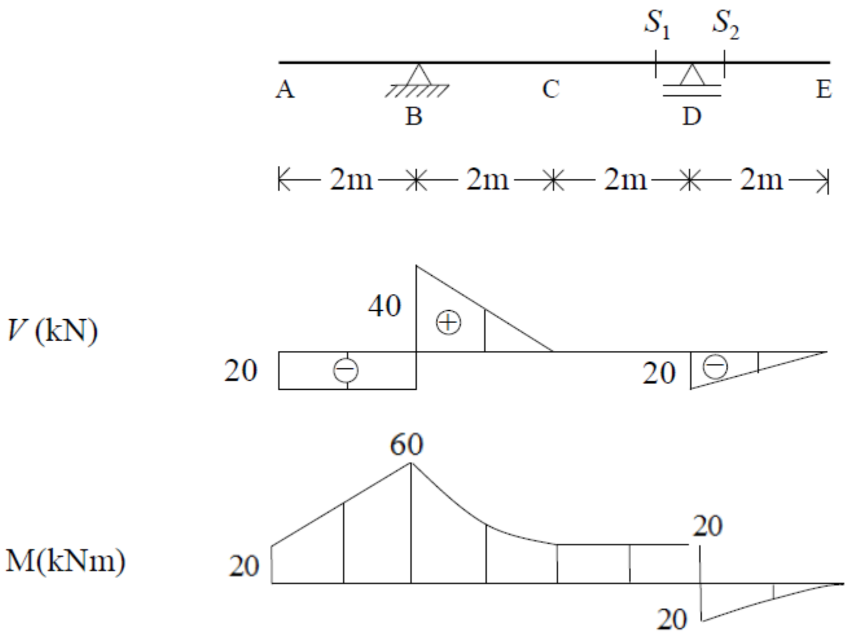
$20 + 2\bar{q} = 0$

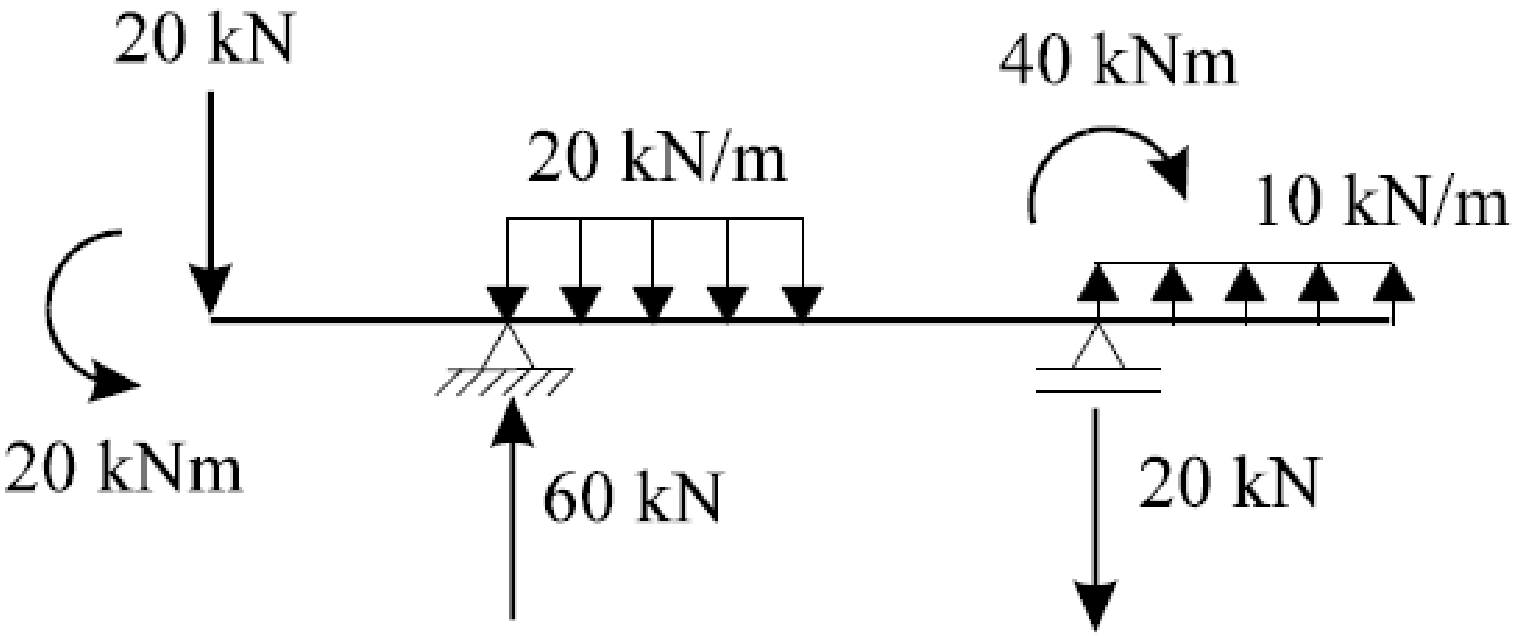
$\bar{q} = -10\ kN/m$



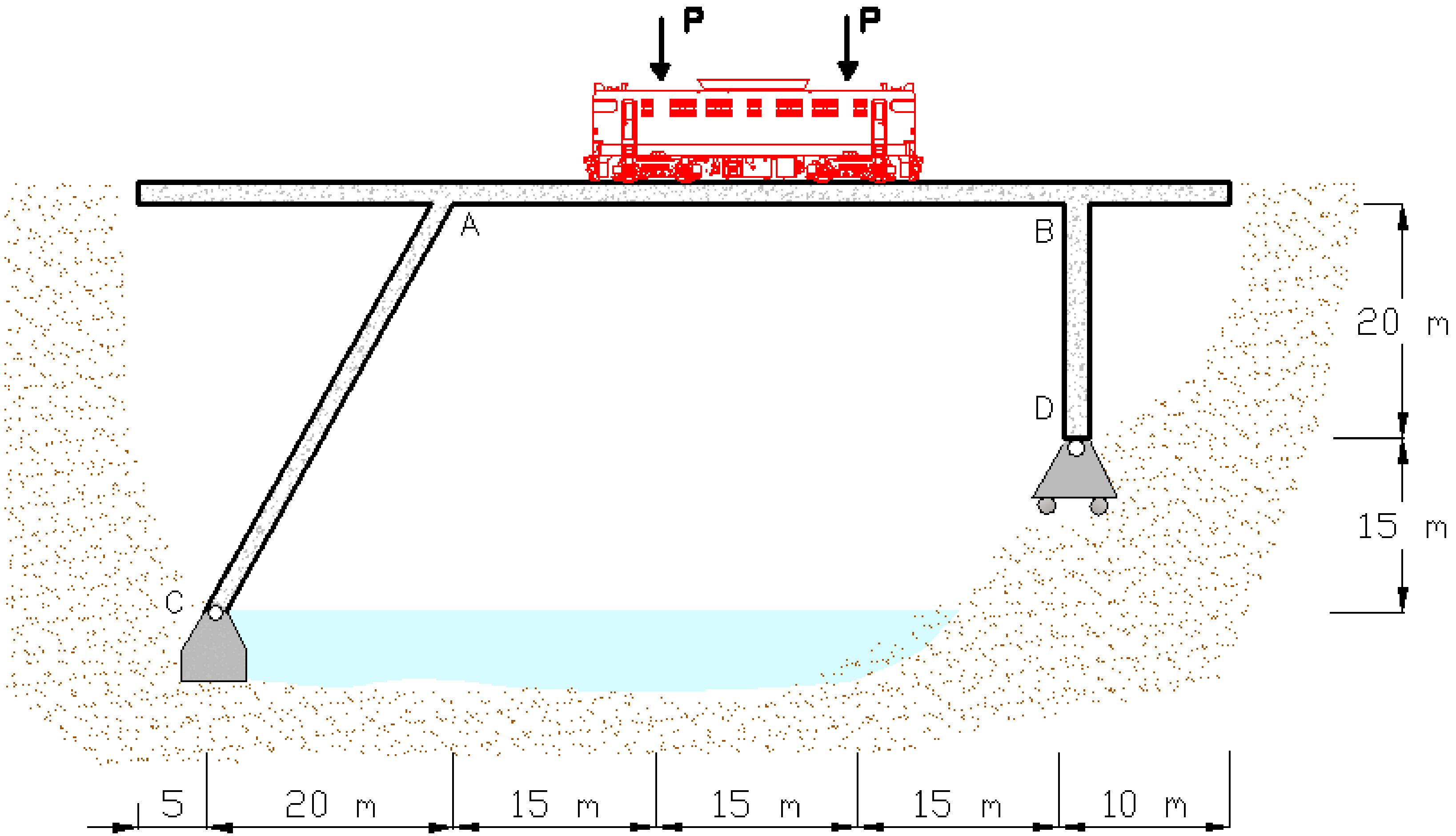
$\Sigma M_D = 0$

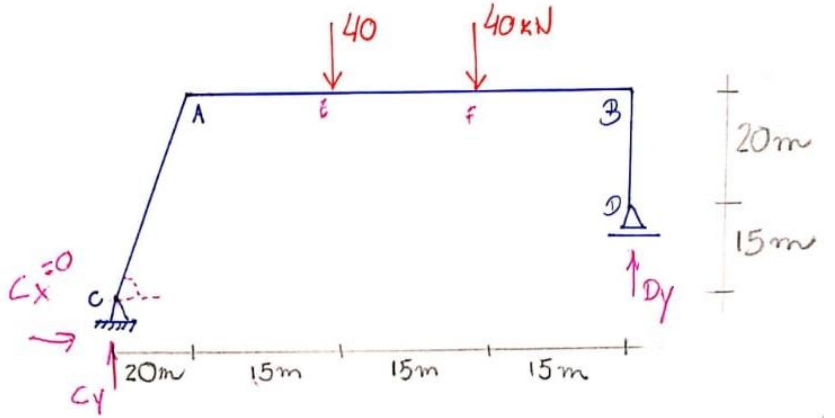
$20 = 20$ ✓ (OK!)


S1
S2
A
B
C
D
E
2m
2m
2m
2m
V (kN)
40
20
20
M(kNm)
60
20
20
20


20 kN
40 kNm
20 kN/m
10 kN/m
20 kNm
60 kN
20 kN

Exemplo 15: A ponte está sujeita ao peso da locomotiva considerado como duas forças concentradas de valor P = 40 kN. Obtenha os diagramas de esforços normal, cortante e momento fletor para o trecho CABD.





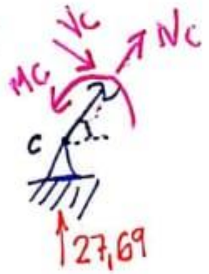
$$+\circlearrowleft \; \Sigma M_C = 0: \quad 40 \cdot 35 + 40 \cdot 50 = D_y \cdot 65 \rightarrow D_y = 52{,}31 \text{ kN}$$

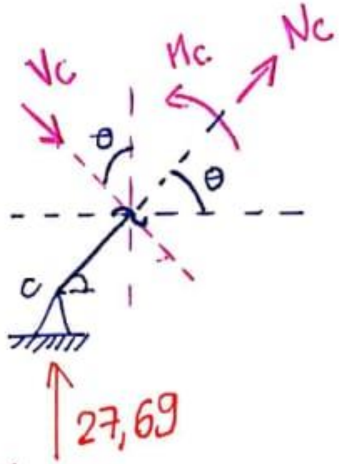
$$\Sigma F_y = 0: \quad C_y + D_y = 80 \rightarrow C_y = 27{,}69 \text{ kN}$$

DIVIDIR EM TRECHOS:
- CA ①
- AE ②
- EF ③
- FB ④
- BD ⑤

$\cos\theta = 0{,}4961$
$\operatorname{sen}\theta = 0{,}8682$

trecho ① (início):





$$\Sigma F_x = 0$$

$$N_C \cdot \cos\theta + V_C \cdot \operatorname{sen}\theta = 0 \quad (1)$$

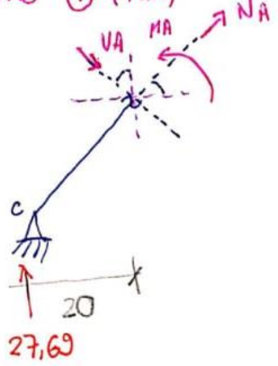
$$\Sigma F_y = 0$$

$$N_C \cdot \operatorname{sen}\theta - V_C \cdot \cos\theta + 27{,}69 = 0 \quad (2)$$

$$\Sigma M_C = 0: \quad M_C = 0$$

$N_C = -24$ kN
$V_C = 13{,}74$ kN

trecho ① (Fim)



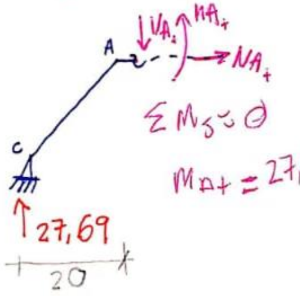
$\Sigma M_S = 0: \; M_A - 27{,}69 \cdot 20 = 0$

$M_A = 553{,}8 \text{ kN.m}$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

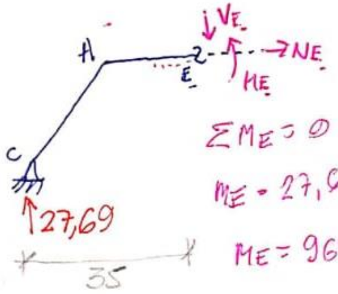
trecho ② (início)



$\Sigma M_S = 0$

$M_{A+} = 27{,}69 \cdot 20$

trecho ② (fim)

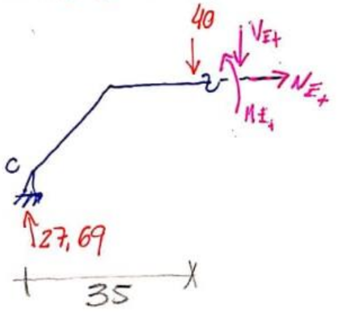


$\Sigma M_E = 0$

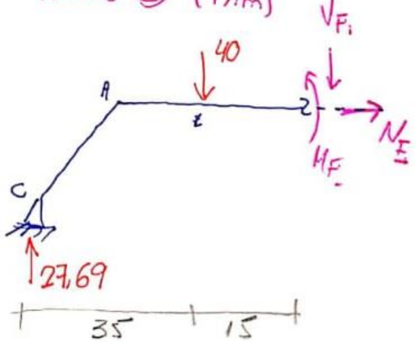
$M_E - 27{,}69 \cdot 35 = 0$

$M_E = 969{,}2 \text{ kN.m}$

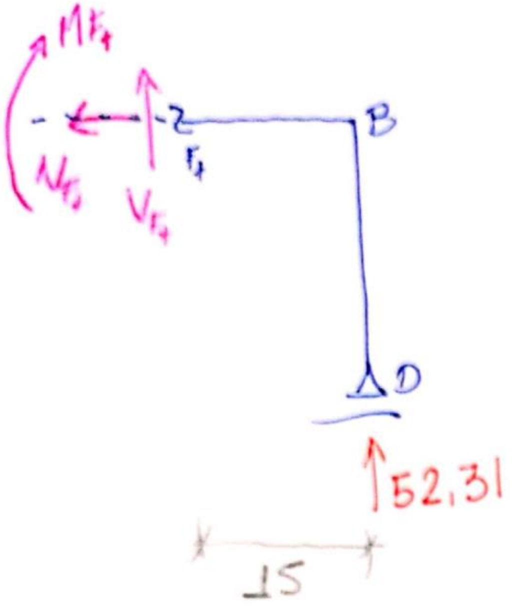
trecho ③ (início)



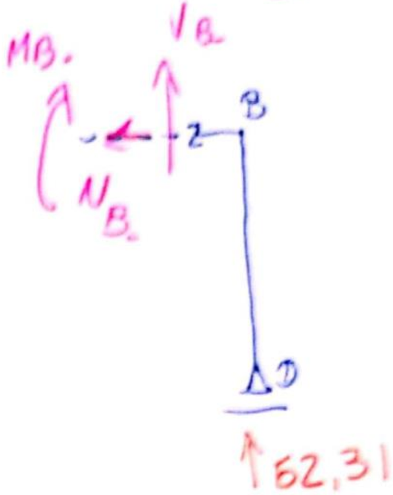
trecho ③ (fim)



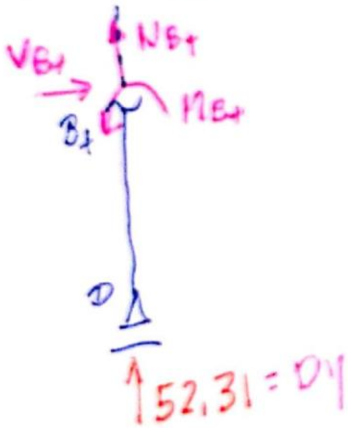
trecho ④ (início)



trecho ④ (Fim)

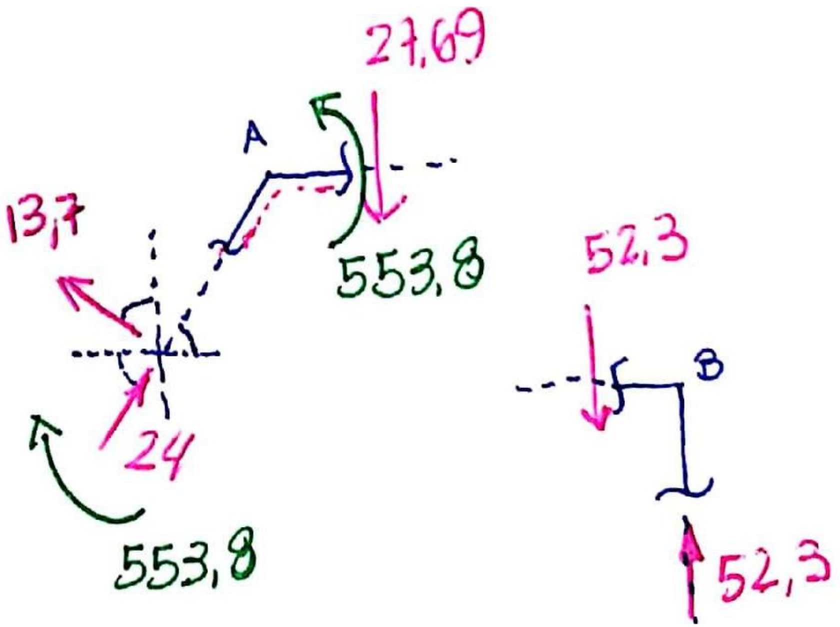


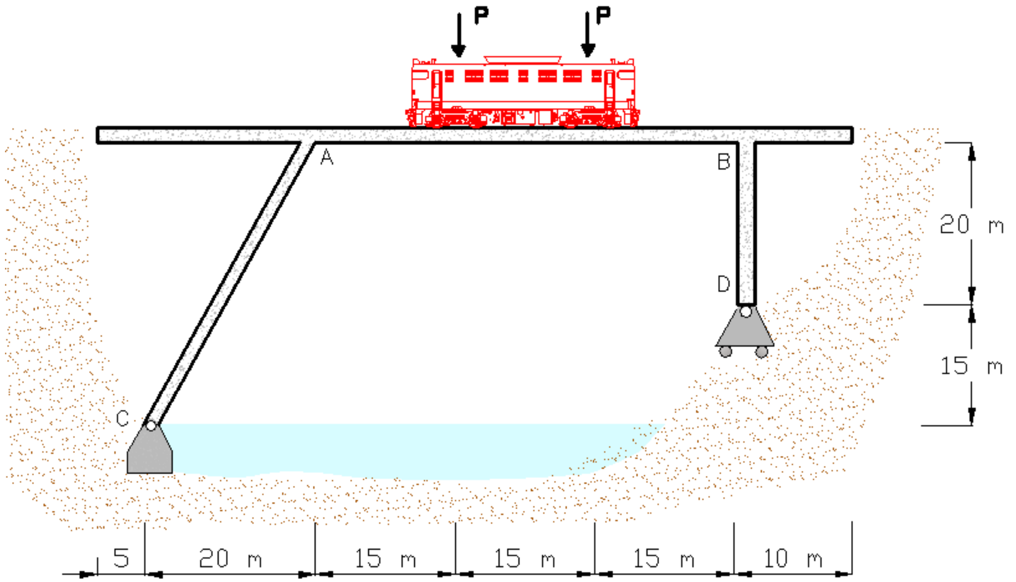
trecho ⑤ (início)

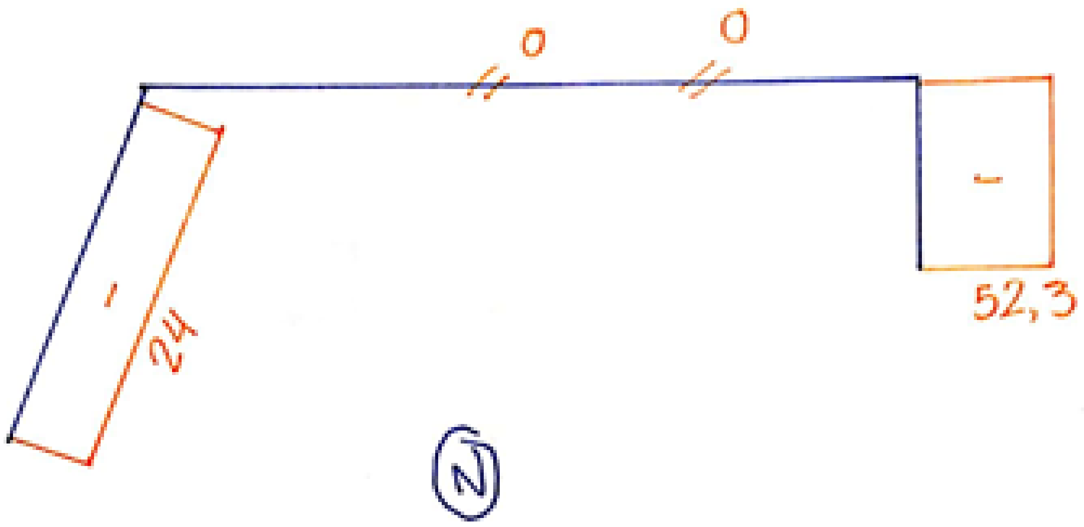


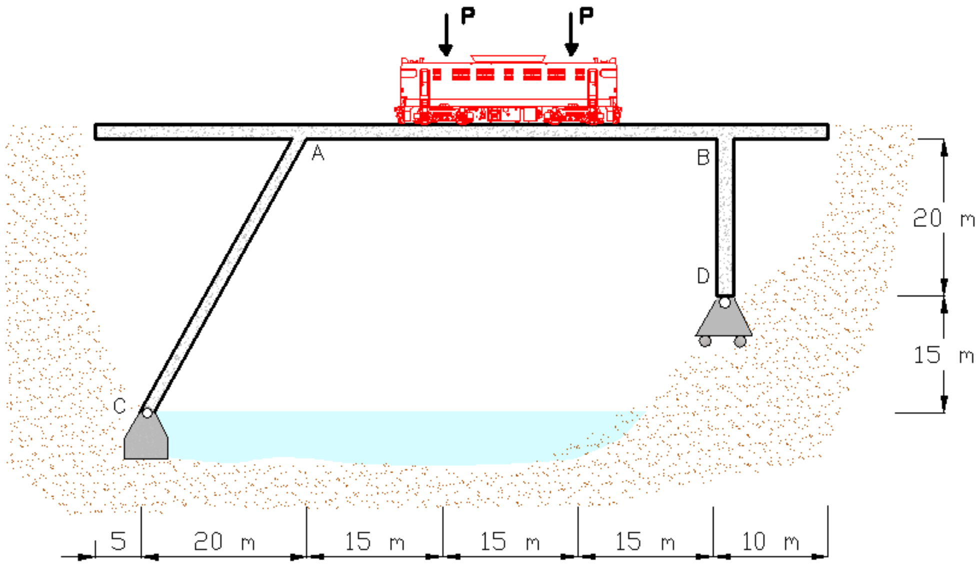
trecho ⑤ (Fim)

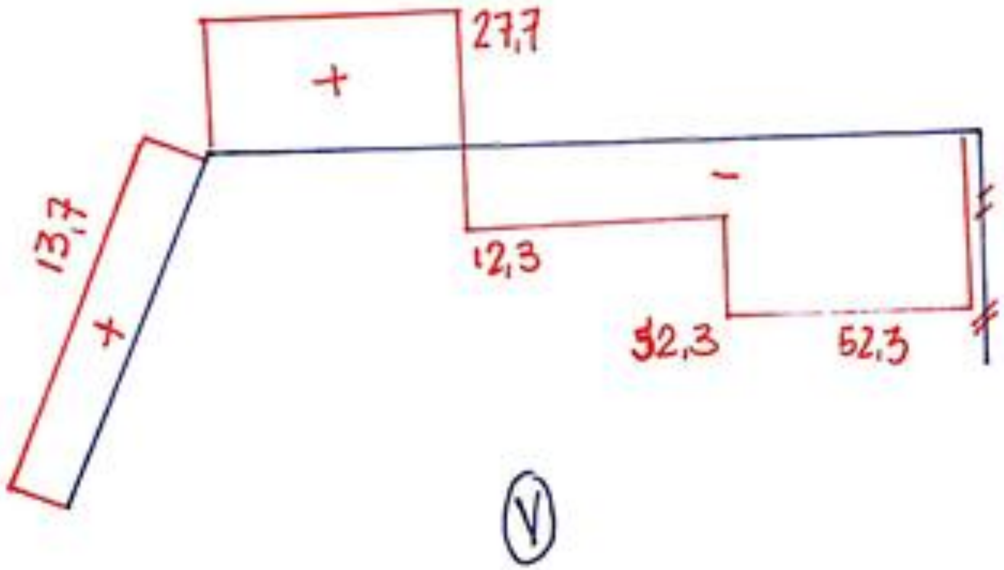
Verifique equilíbrio junto a A e B:

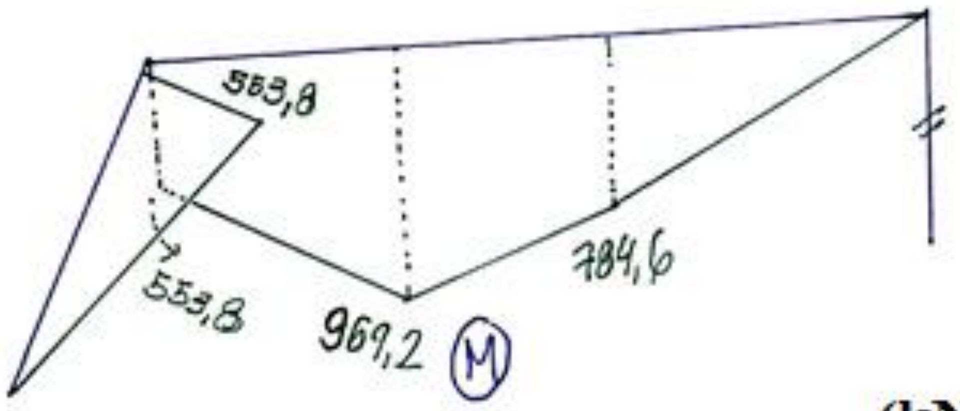






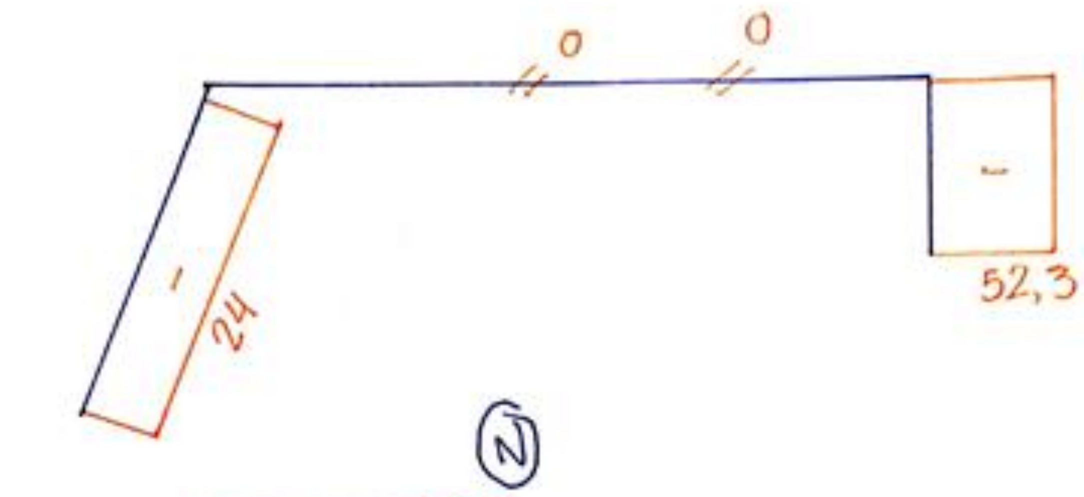

P
P
A
B
D
C
20 m
15 m
5
20 m
15 m
15 m
15 m
10 m

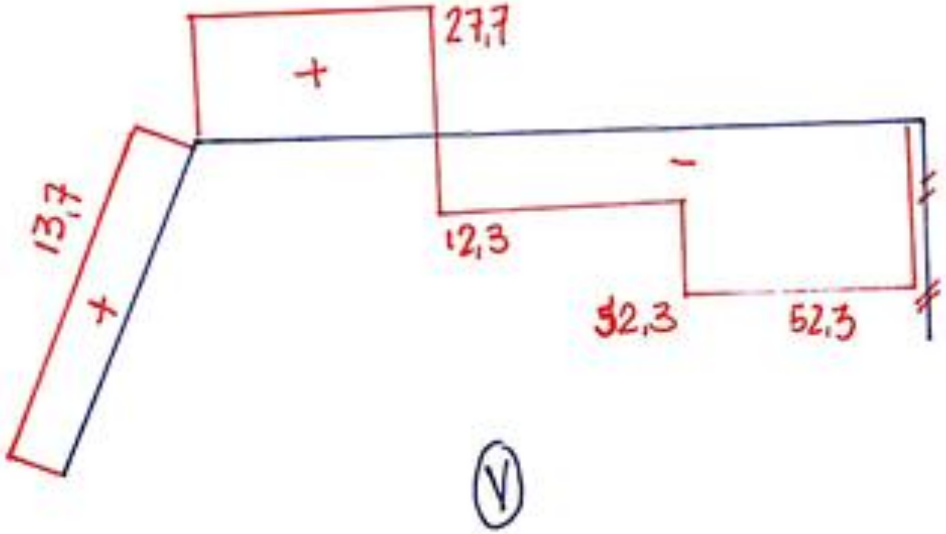

27,7
+
13,7
+
-
12,3
52,3
52,3
V

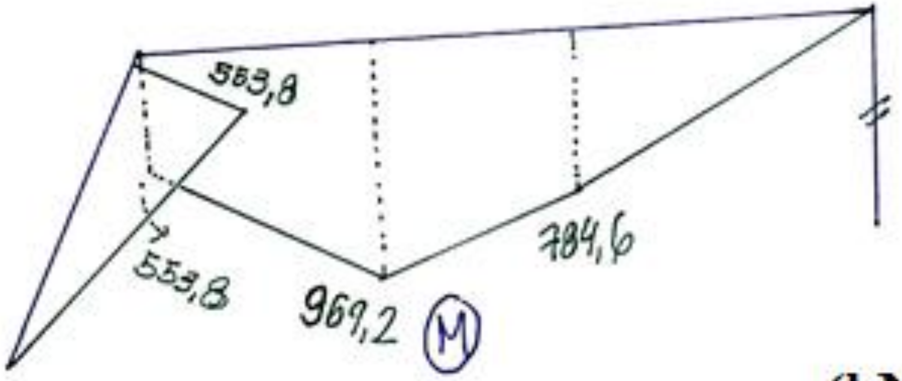

553,8
553,8
969,2
784,6
M

(kN,m)

Diagramas







(kN,m)

Exemplo 16

1.7.21. O pórtico plano ABCD serve como contenção de terra. O solo exerce uma carga no muro CD conforme indicado no desenho e seu valor máximo é dado pela relação $q_e = \gamma_{solo} \cdot H \cdot b \cdot k_0$, com γ_{solo} sendo seu peso específico, k_0 é o coeficiente de empuxo do solo, H a altura do muro e b sua largura. Sobre a viga BC age uma carga constantemente distribuída devido a uma ação permanente de 40 kN/m. Determinar os diagramas de esforço normal, cortante e momento fletor apenas no trecho BC. Considere: $\gamma_{solo} = 22$ kN/m³, H = 6 m, b = 1 m, $k_0 = 0{,}33$.

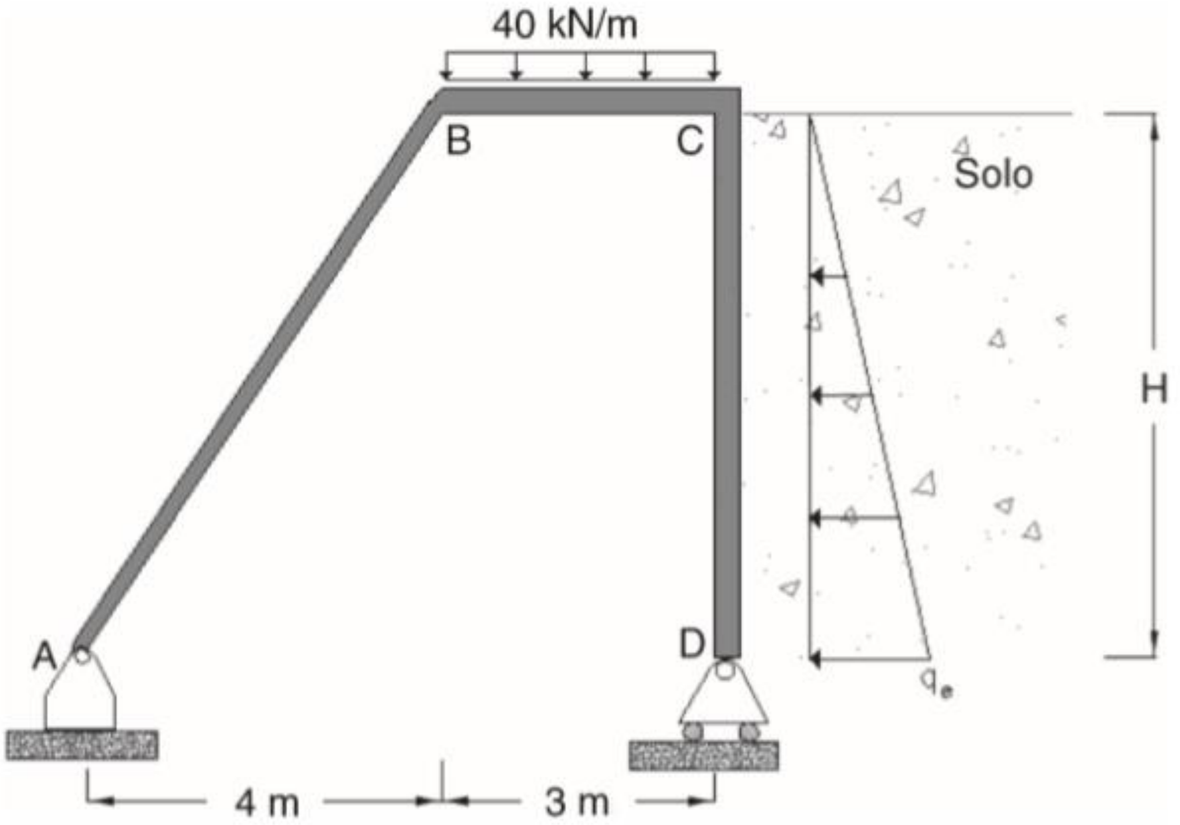


FIGURA 1.60A Pórtico biapoiado submetido às forças distribuídas constante e linear.

Resolução

A carga distribuída é dada por: $q_e = 22 \cdot 6 \cdot 1 \cdot 0 \cdot 33 = 43{,}56$ kN/m, a qual possui resultante de $F_e = 130{,}68$ kN, aplicada a 2 m acima do ponto D. Usando as três equações de equilíbrio, determinam-se as reações:

$$\sum M_A = 0 : \rightarrow 7 \cdot D_y + 2 \cdot 130{,}68 = 40 \cdot 3 \cdot 5{,}5 \rightarrow D_y = 56{,}95 \text{ kN } (\uparrow)$$

$$\sum F_y = 0 : \rightarrow A_y + D_y = 40 \cdot 3 \rightarrow A_y = 63{,}05 \text{ kN } (\uparrow)$$

$$\sum F_X = 0 : \rightarrow A_x = 130{,}68 \text{ kN } (\rightarrow)$$

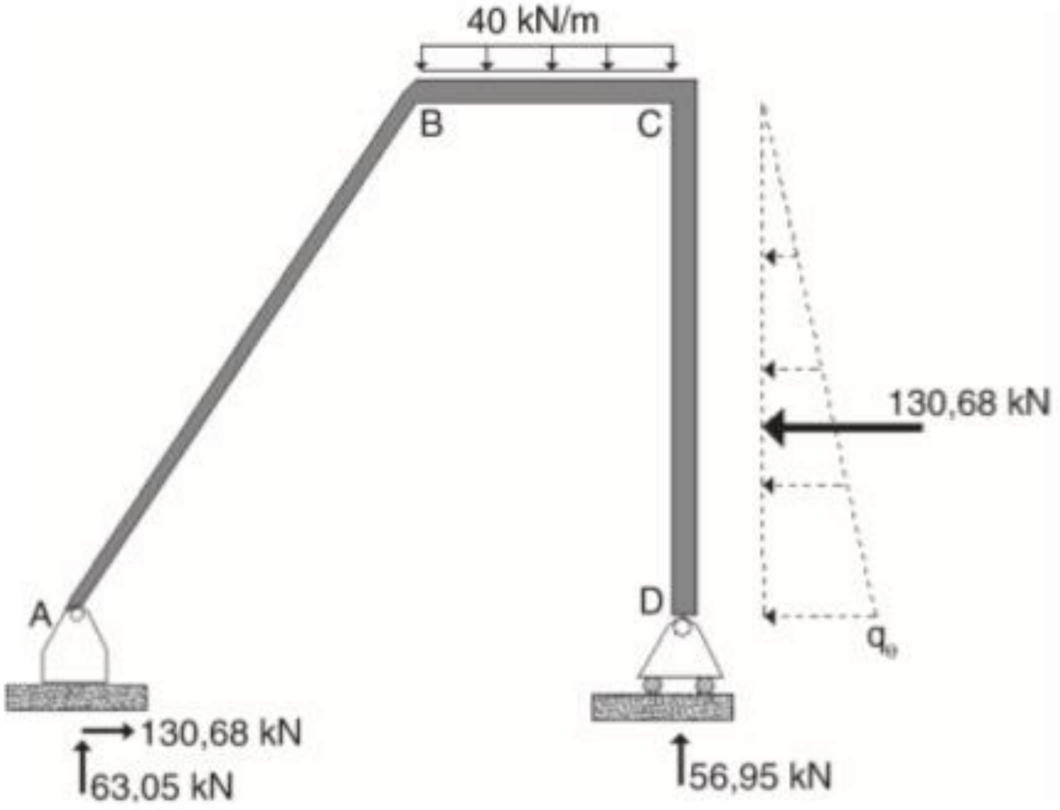
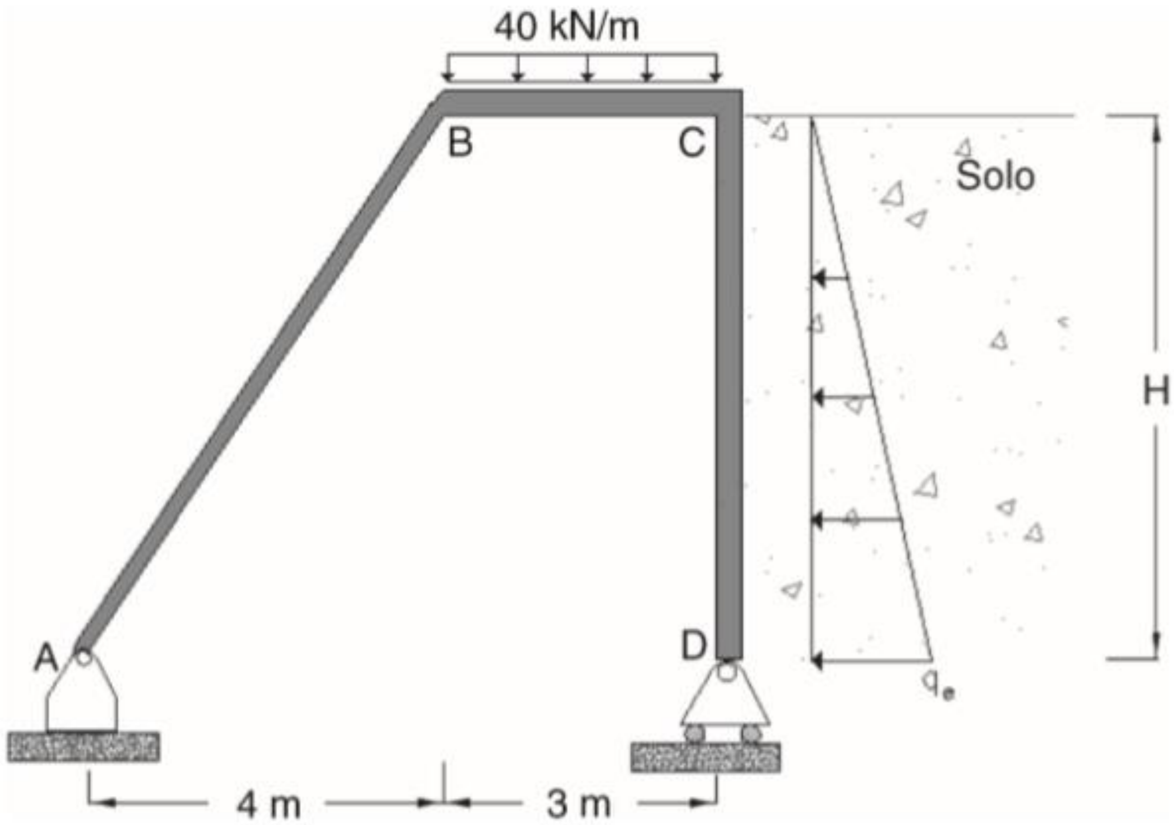
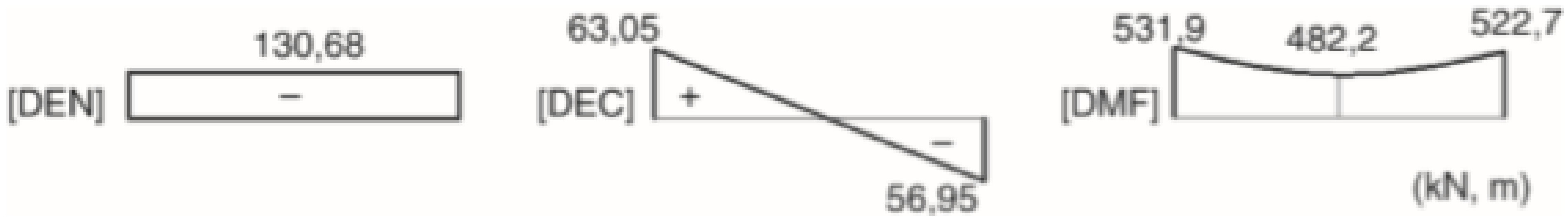


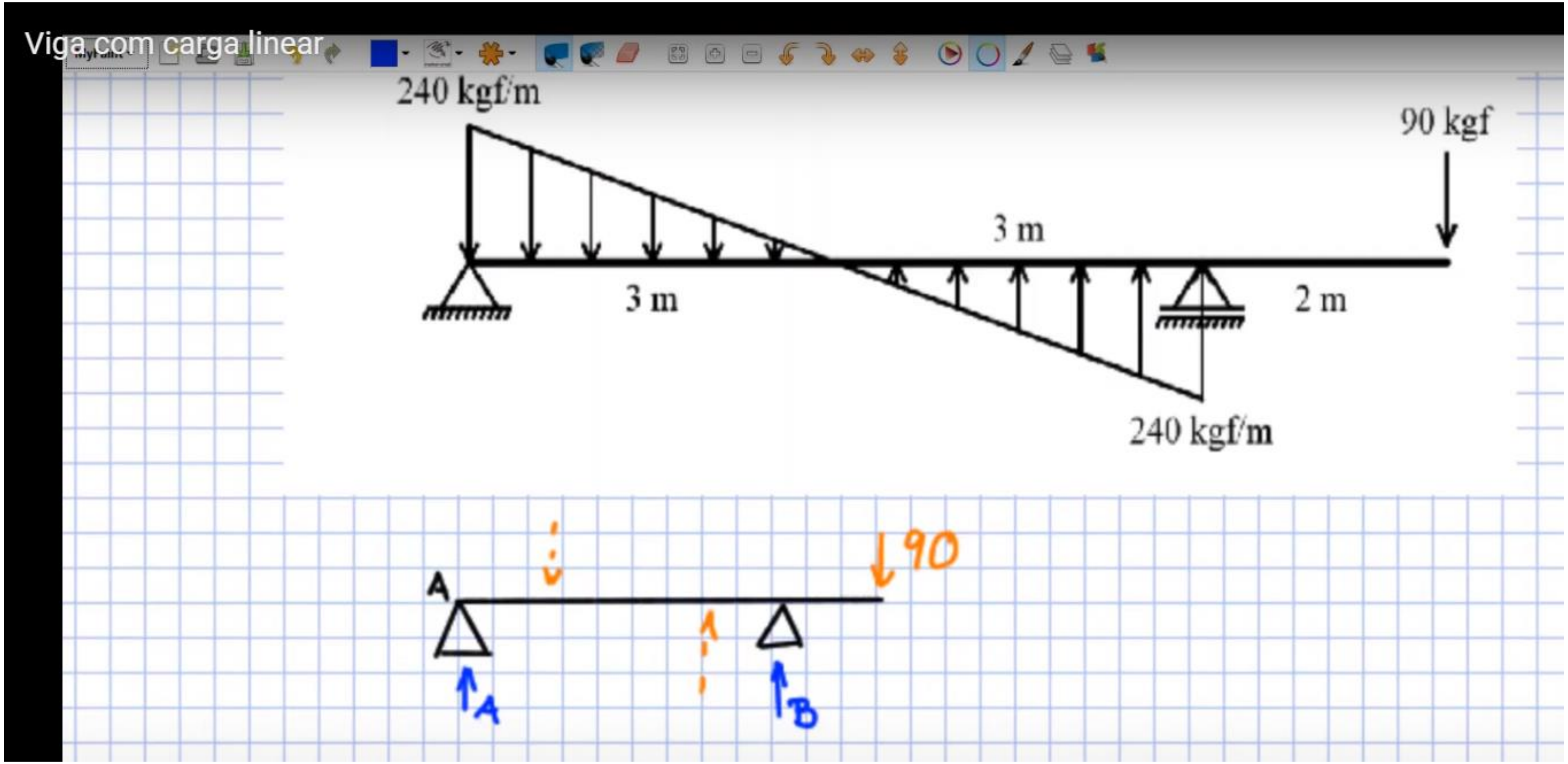
FIGURA 1.60B Indicação das ações resultantes e reações do pórtico biapoiado.


40 kN/m
B
C
Solo
H
A
D
q_e
4 m
3 m

Diagramas

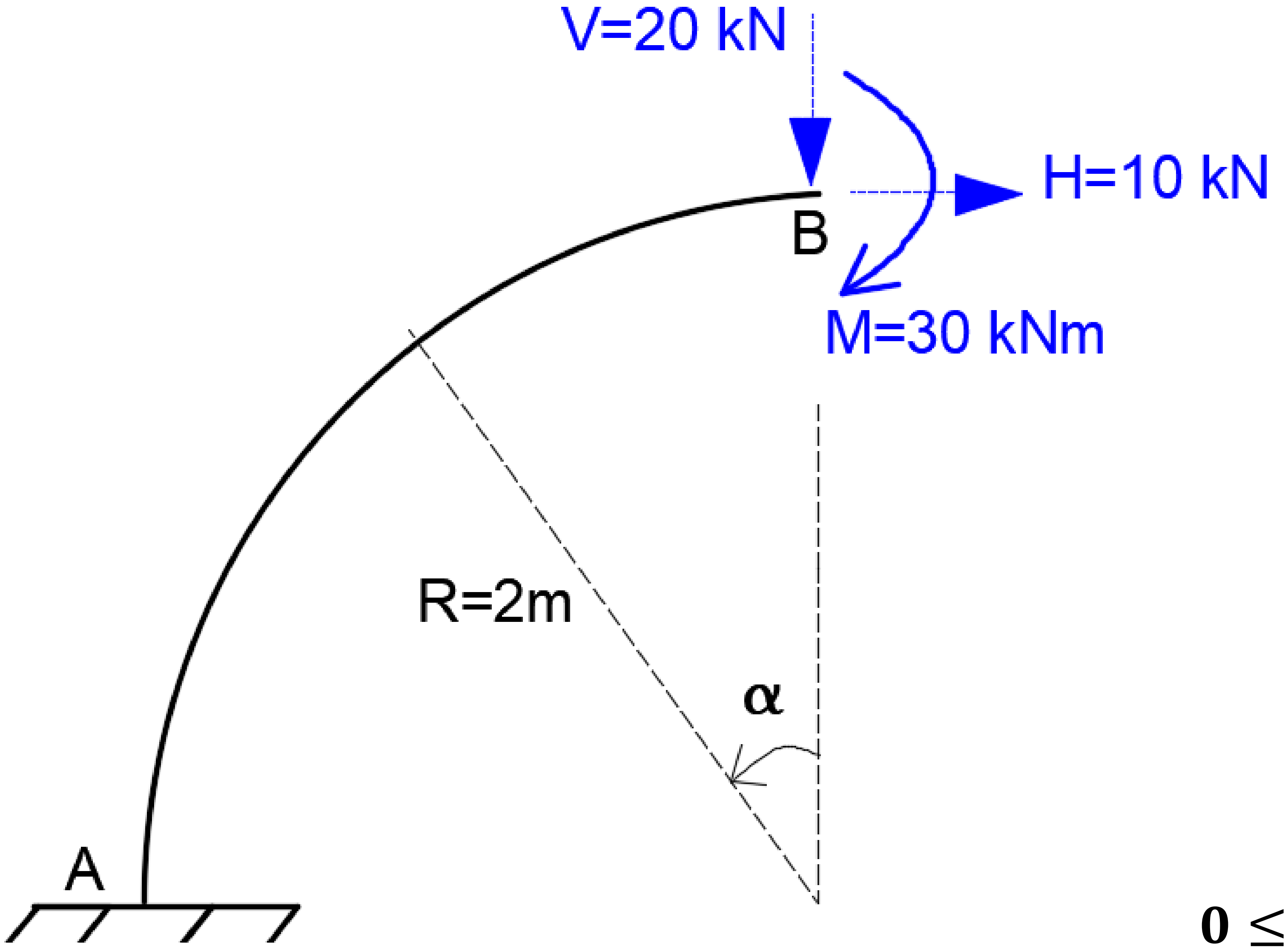

130,68
[DEN]
−
63,05
[DEC]
+
−
56,95
531,9
482,2
522,7
[DMF]
(kN, m)

Exemplo 17: Viga com carga linear – Ver resolução no YouTube!

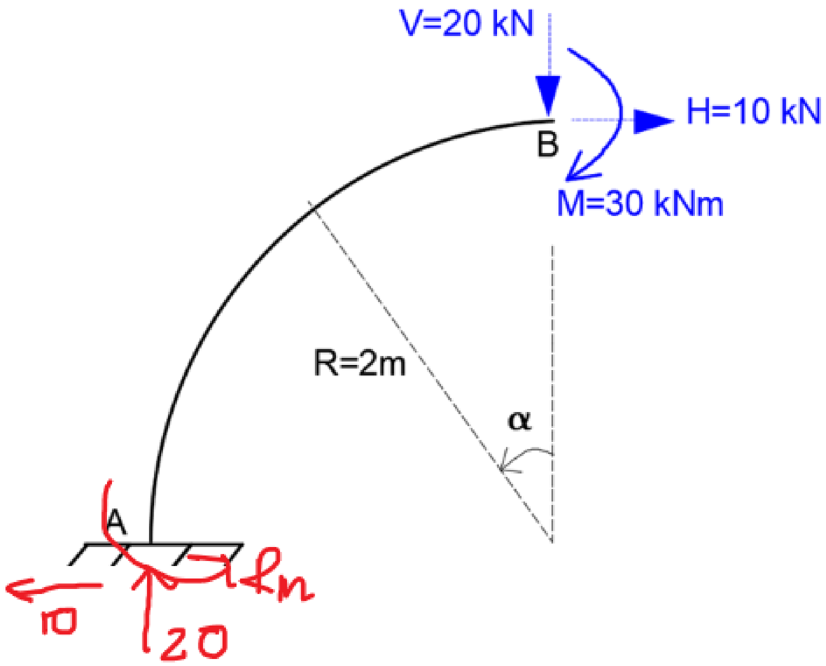


https://www.youtube.com/watch?v=ZKmarvUU5pI

Exemplo 18: Viga com trecho curvo



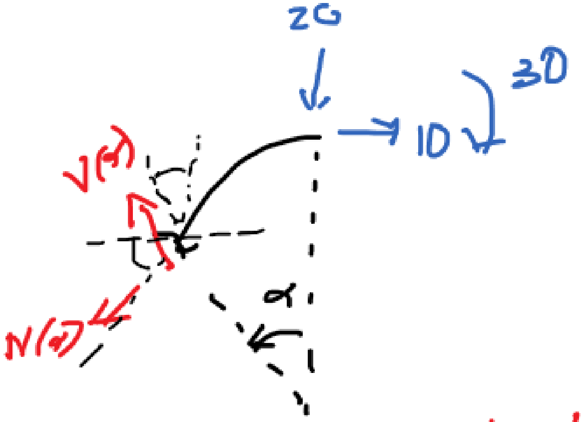
$$0 \leq \alpha \leq 90$$

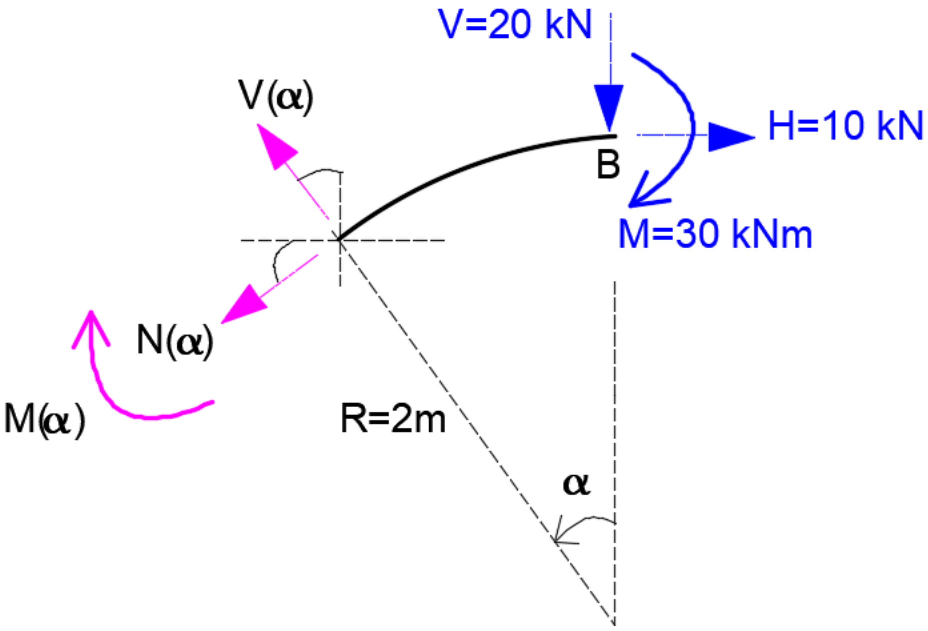


$$\Sigma M_A = 0:$$

$$R_m = 20 \cdot 2 + 10 \cdot 2 + 30$$

$$R_m = 90 \text{ kNm}$$





$$\Sigma F_x = 0: \quad N(\alpha)\cos\alpha + V(\alpha)\,\text{sen}\,\alpha = 10 \quad (1)$$

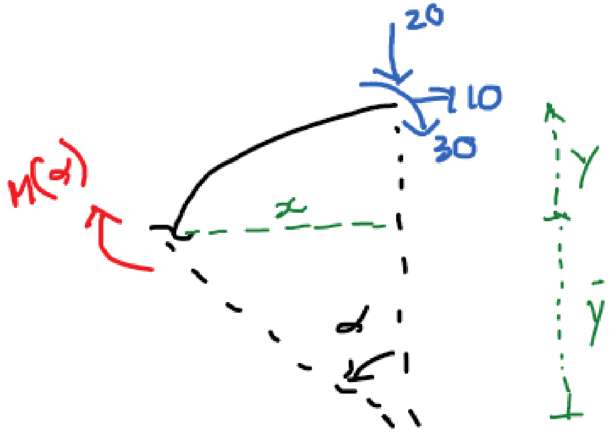
$$\Sigma F_y = 0: \quad -N(\alpha)\,\text{sen}\,\alpha + V(\alpha)\cos\alpha = 20 \quad (2)$$

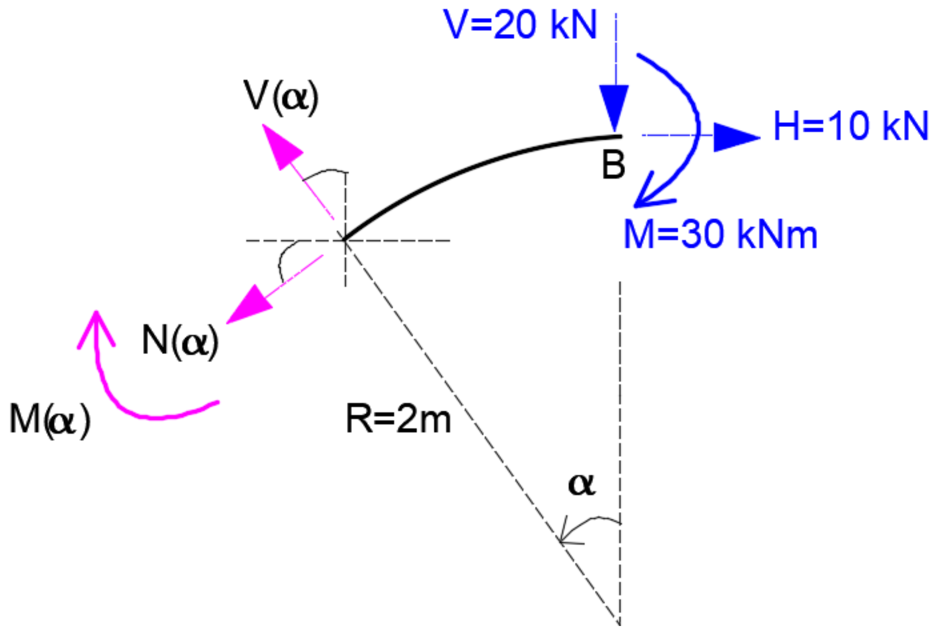
Solving:-

$$V(\alpha) = 10\,\text{sen}\,\alpha + 20\cos\alpha \quad \begin{cases} V(0) = 20 \\ V(90) = 10 \end{cases}$$

$$N(\alpha) = 10\cos\alpha - 20\,\text{sen}\,\alpha \quad \begin{cases} N(0) = 10 \\ N(90) = -20 \end{cases}$$

$$0 \le \alpha \le 90$$





$\operatorname{sen}\alpha = \frac{x}{2} \rightarrow x = 2\operatorname{sen}\alpha$

$\cos\alpha = \bar{y}/2 \rightarrow \bar{y} = 2\cos\alpha$

$\bar{y} + y = 2\,m \rightarrow y = 2 - \bar{y}$

$y = 2 - 2\cos\alpha = 2(1-\cos\alpha)$

$\Sigma M_S = 0:\ M(\alpha) + 30 + 20\cdot x + 10\cdot y = 0$

$M(\alpha) = -30 - 20\cdot 2\operatorname{sen}\alpha - 10\cdot 2(1-\cos\alpha)$

$M(\alpha) = -30 - 40\operatorname{sen}\alpha - 20 + 20\cos\alpha$

$M(\alpha) = -50 + 20\cos\alpha - 40\operatorname{sen}\alpha$ $\begin{cases} M(0) = -50 + 20 = -30 \\ M(90) = -50 - 40 = -90 \end{cases}$

$$0 \leq \alpha \leq 90$$

Extremos locais:

$V'(\alpha) = 10\cos\alpha - 20\,\mathrm{sen}\,\alpha = 0 \rightarrow tg\,\alpha = 1/2 \quad (\alpha = 26{,}56^o)$

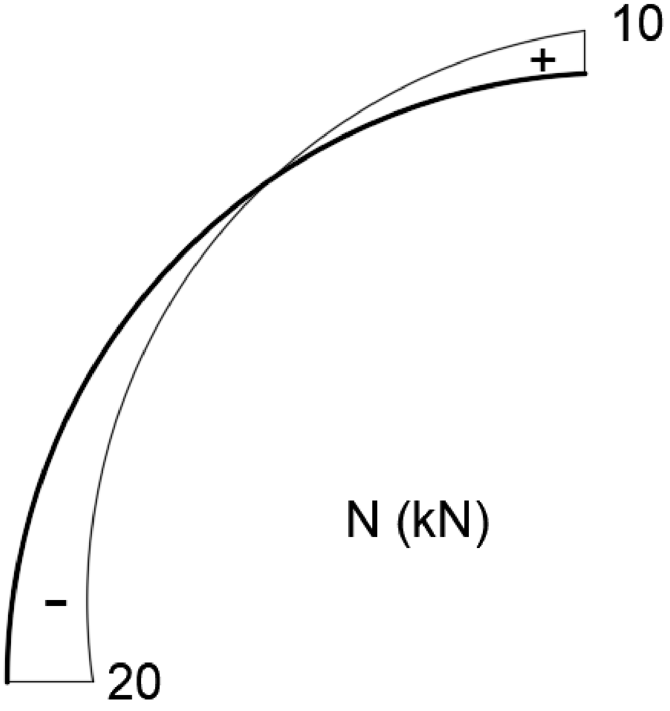
$V''(\alpha) = -10\,\mathrm{sen}\,\alpha - 20\cos\alpha = 0 \rightarrow tg(\alpha) = -2 \quad (\alpha = -63{,}43^o)$

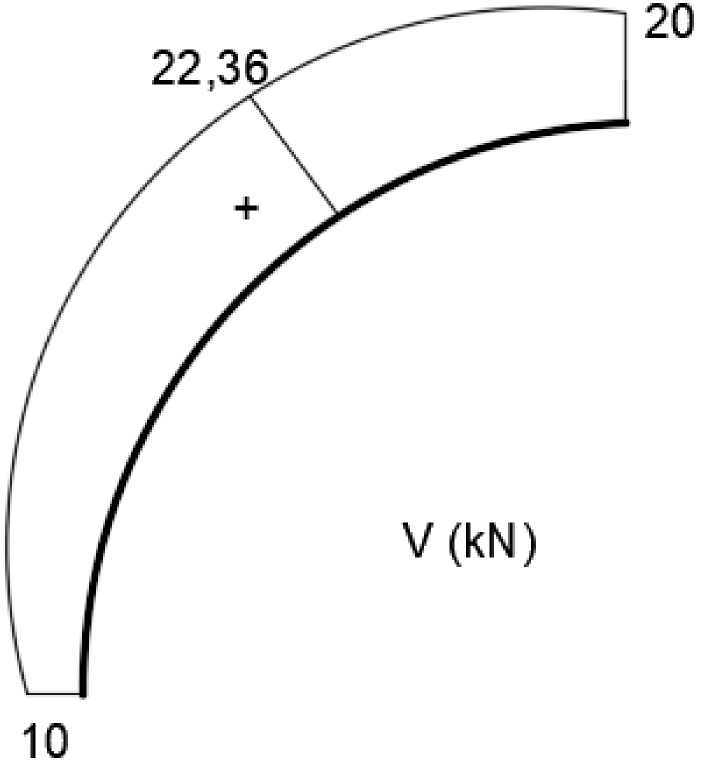
$V(26{,}56^o) = 10 \cdot \mathrm{sen}\, 26{,}56 + 20 \cos 26{,}56 = 22{,}36\ kN$

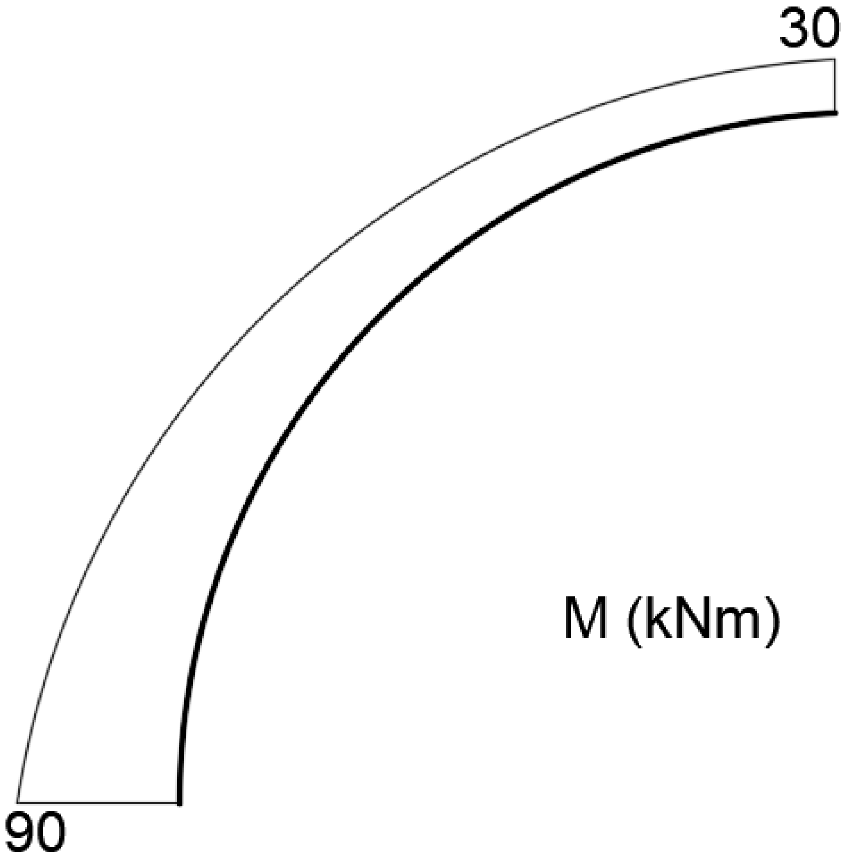
$N'(\alpha) = -10\,\mathrm{sen}\,\alpha - 20\cos\alpha = 0 \rightarrow tg\,\alpha = -2$ (fora intervalo) $(0 \leq \alpha \leq 90)$

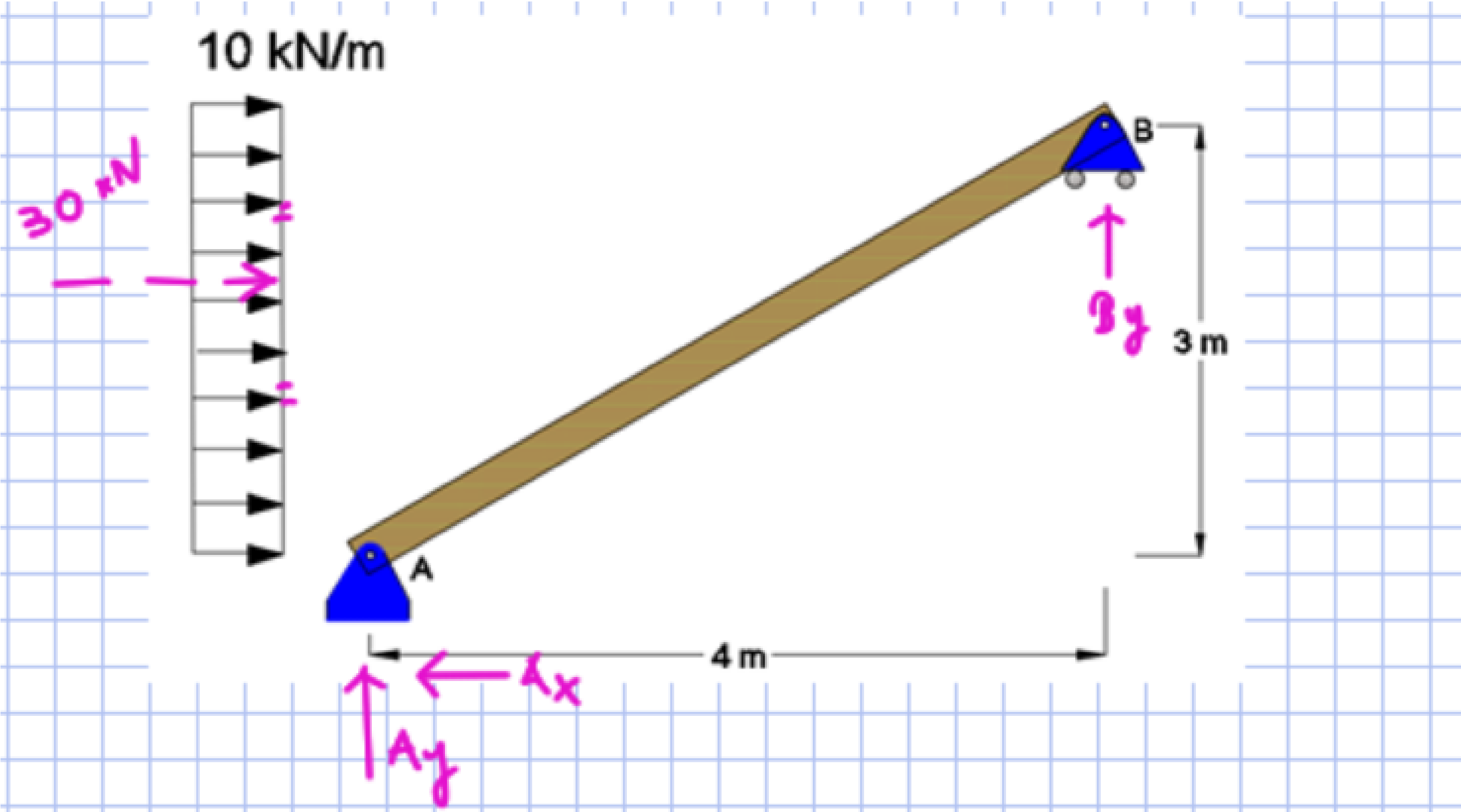
$M'(\alpha) = -20\,\mathrm{sen}\,\alpha - 40\cos\alpha = 0 \rightarrow -\mathrm{sen}\,\alpha = 2\cos\alpha \rightarrow tg\,\alpha = -2$ (fora intervalo)

$$\mathbf{0 \leq \alpha \leq 90}$$


10
+
-
20
N (kN)


22,36
20
+
10
V (kN)


30
90
M (kNm)

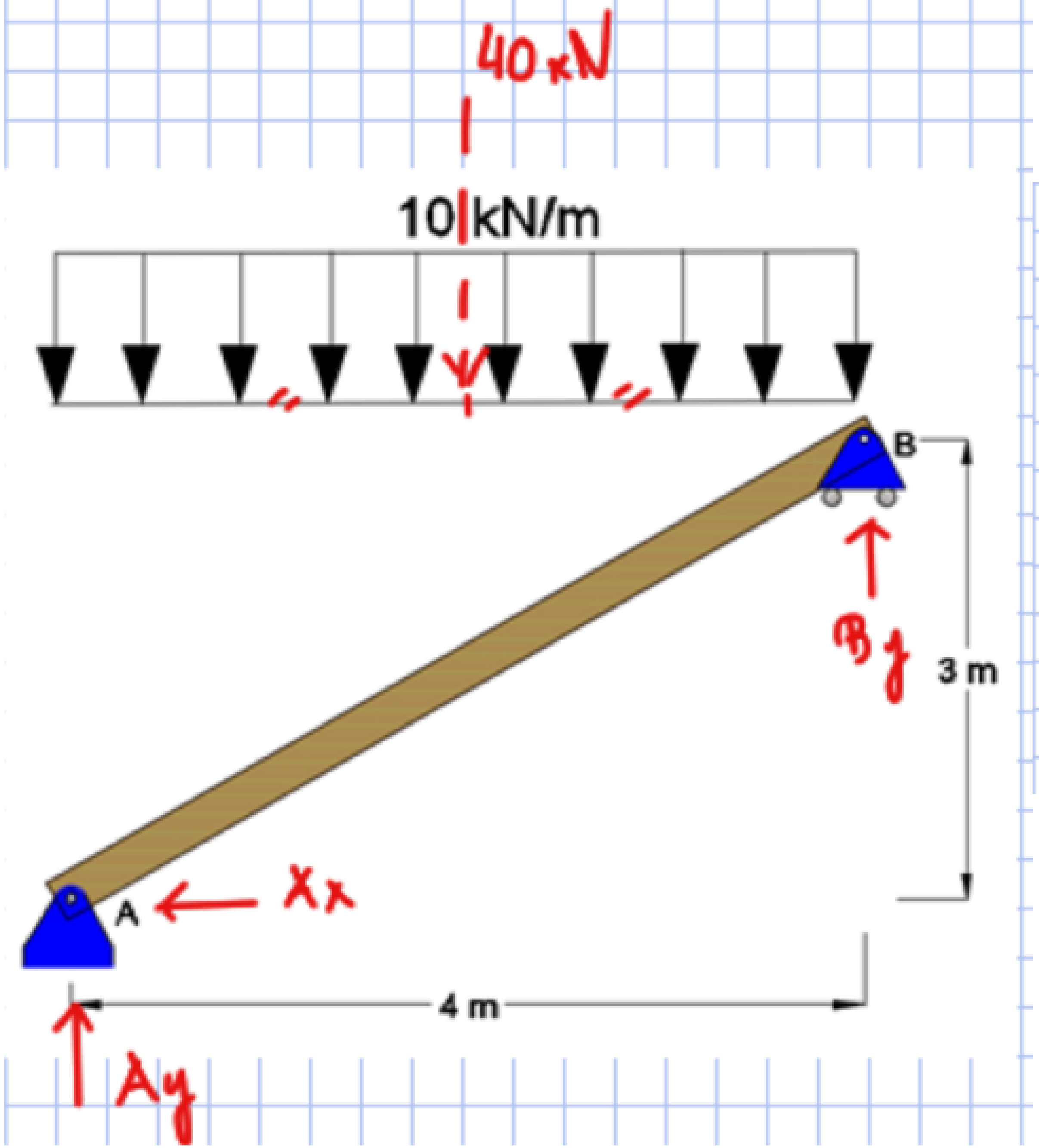


$\Sigma F_x = 0: \quad A_x = 30\ kN$

$\Sigma M_A = 0 \circlearrowleft^{+}: \quad B_y \cdot 4 = 30 \cdot 1{,}5 \ (\circlearrowleft)$

$B_y = 11{,}25\ kN\ (\uparrow)$

$\Sigma F_y = 0: \quad A_y + B_y = 0 \rightarrow A_y = -11{,}25\ kN\ (\downarrow)$



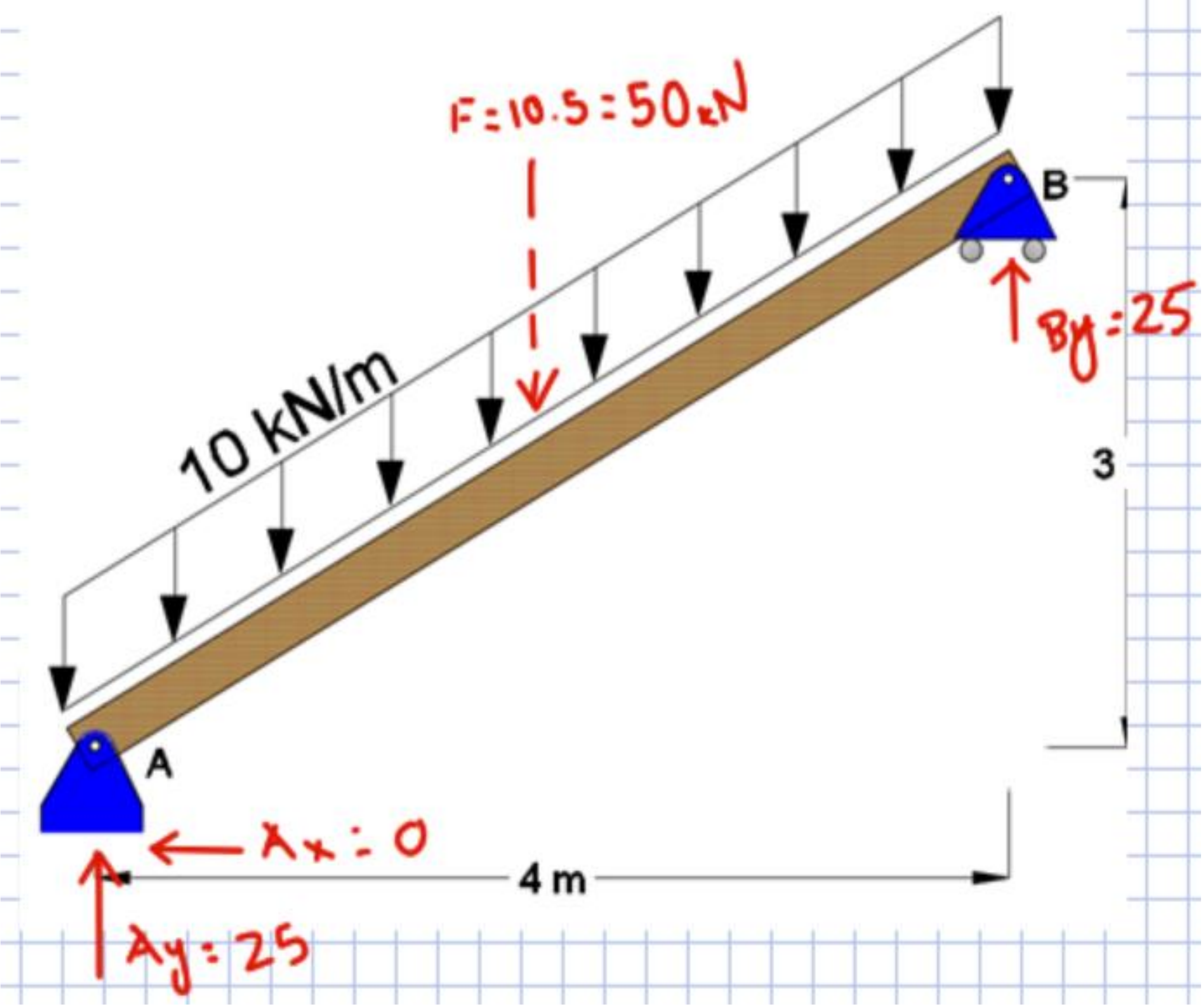
$$\Sigma F_y = 0: \quad A_y + B_y = 40$$

$$\Sigma M_A = 0 \;(\circlearrowleft +): \quad 4 B_y = 40 \cdot 2$$

$$B_y = 20 \text{ kN}$$

$$A_y = 20 \text{ kN}$$

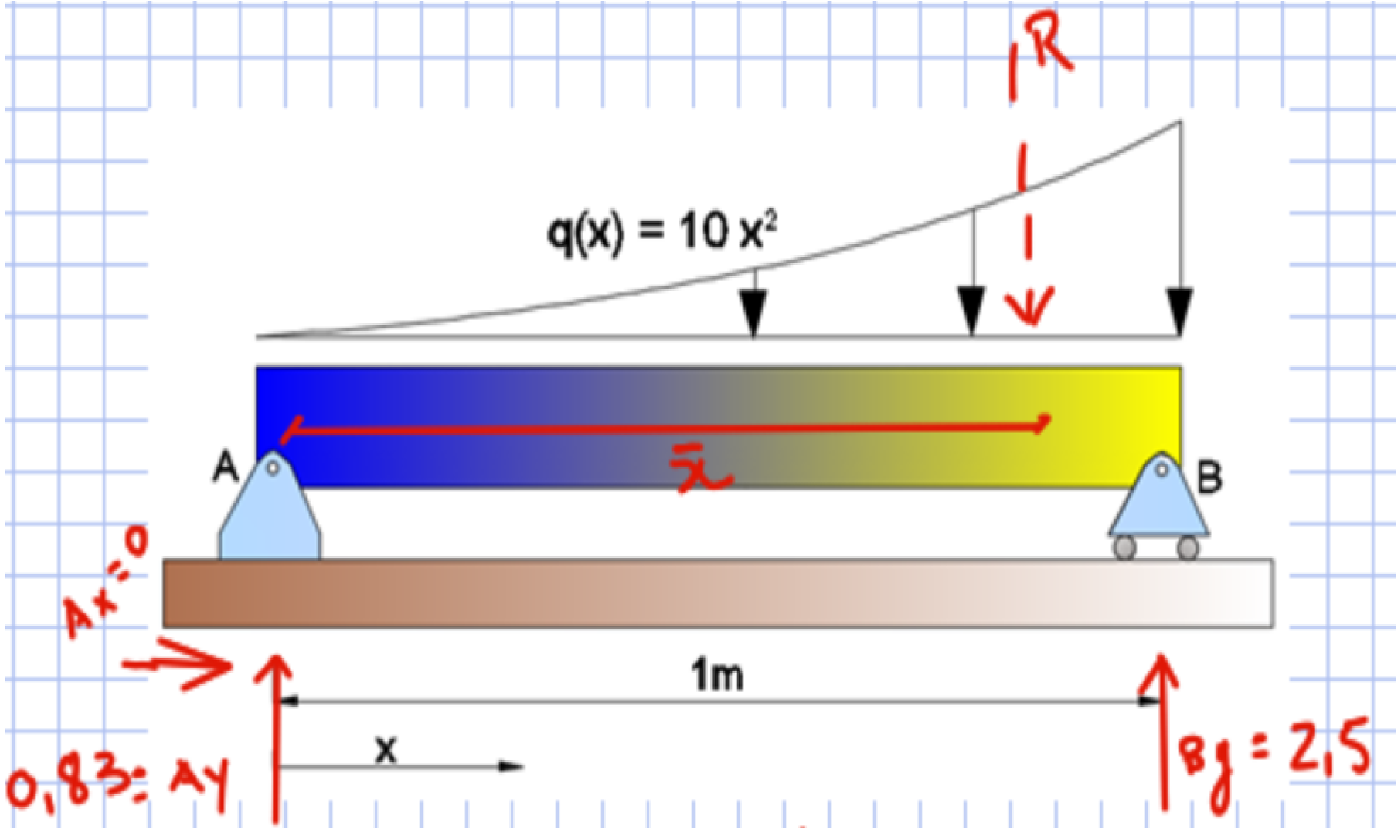
$$\Sigma F_x = 0: \quad A_x = 0$$



$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma M_A = 0 \;(+\circlearrowleft): \quad 4B_y = 50 \cdot 2 \rightarrow B_y = 25\,kN$$

$$\Sigma F_y = 0: \quad A_y + B_y = 50 \rightarrow A_y = 25\,kN$$



Por definição: $R=\int_0^{L=1m} q(x)\,dx$

$$R=\int_0^1 10x^2\,dx = \frac{10\cdot x^3}{3}\Big|_0^1 = \frac{10}{3}\,(kN)$$

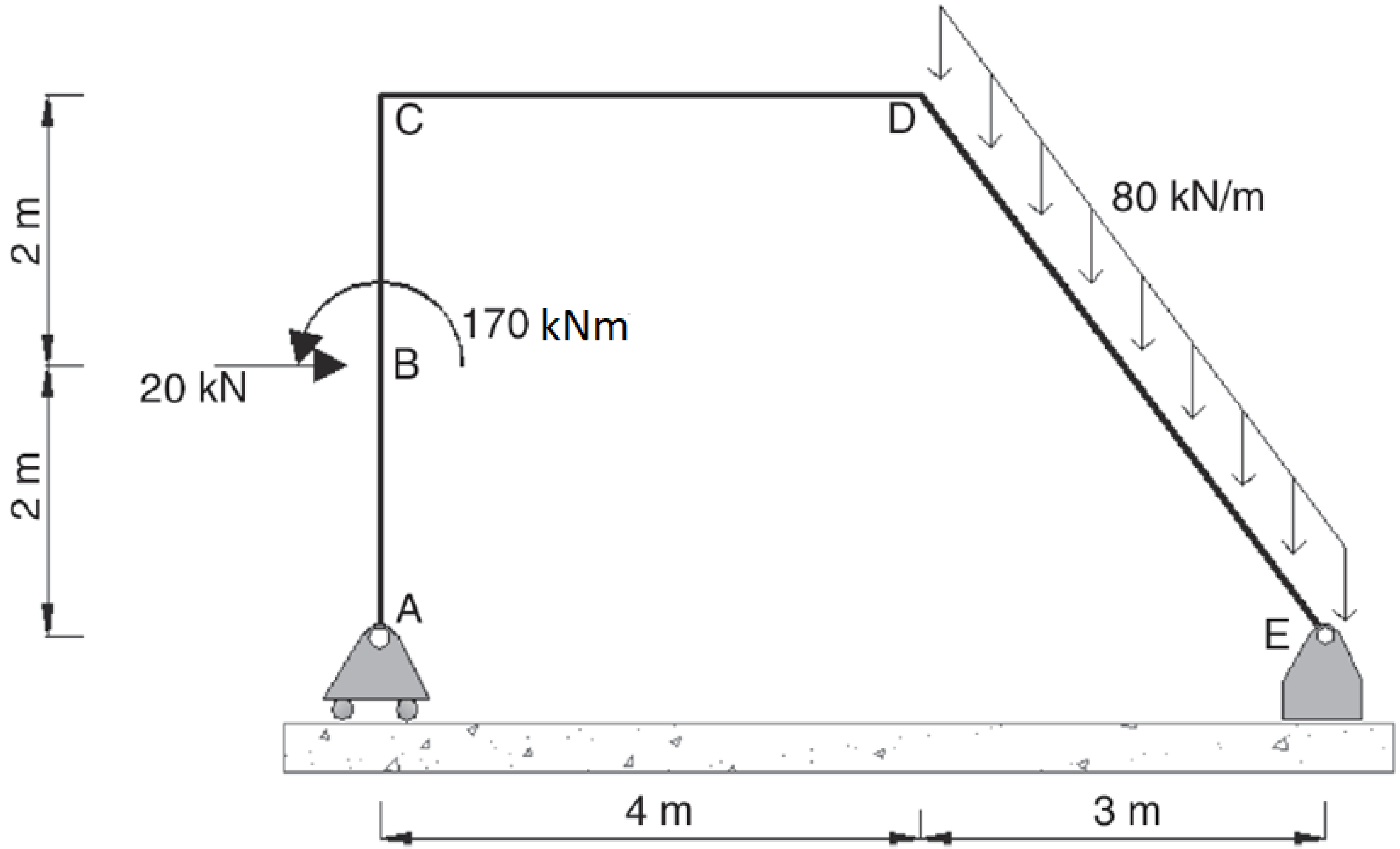
$$\bar{x}=\frac{\int_0^1 q(x)\,x\,dx}{R}=\frac{\int_0^1 (10x^2)\,x\,dx}{10/3}=\frac{\int_0^1 10x^3}{10/3}=$$

$$\bar{x}=\frac{10x^4}{4}\cdot\frac{3}{10}\Big|_0^1=\frac{10}{4}\cdot\frac{3}{10}=0{,}75\,m$$

$$\Sigma M_A=0\ (+\circlearrowleft):\ B_y\cdot 1=\frac{10}{3}\cdot\frac{3}{4}=\frac{5}{2}\rightarrow B_y=2{,}5\,kN$$

$$\Sigma F_y=0:\ A_y+B_y=\frac{10}{3}\rightarrow A_y=\frac{10}{3}-2{,}5=\frac{5}{6}=0{,}83\,kN$$

Exemplo 19: Determinar os diagramas de esforços solicitantes de toda a estrutura plana*



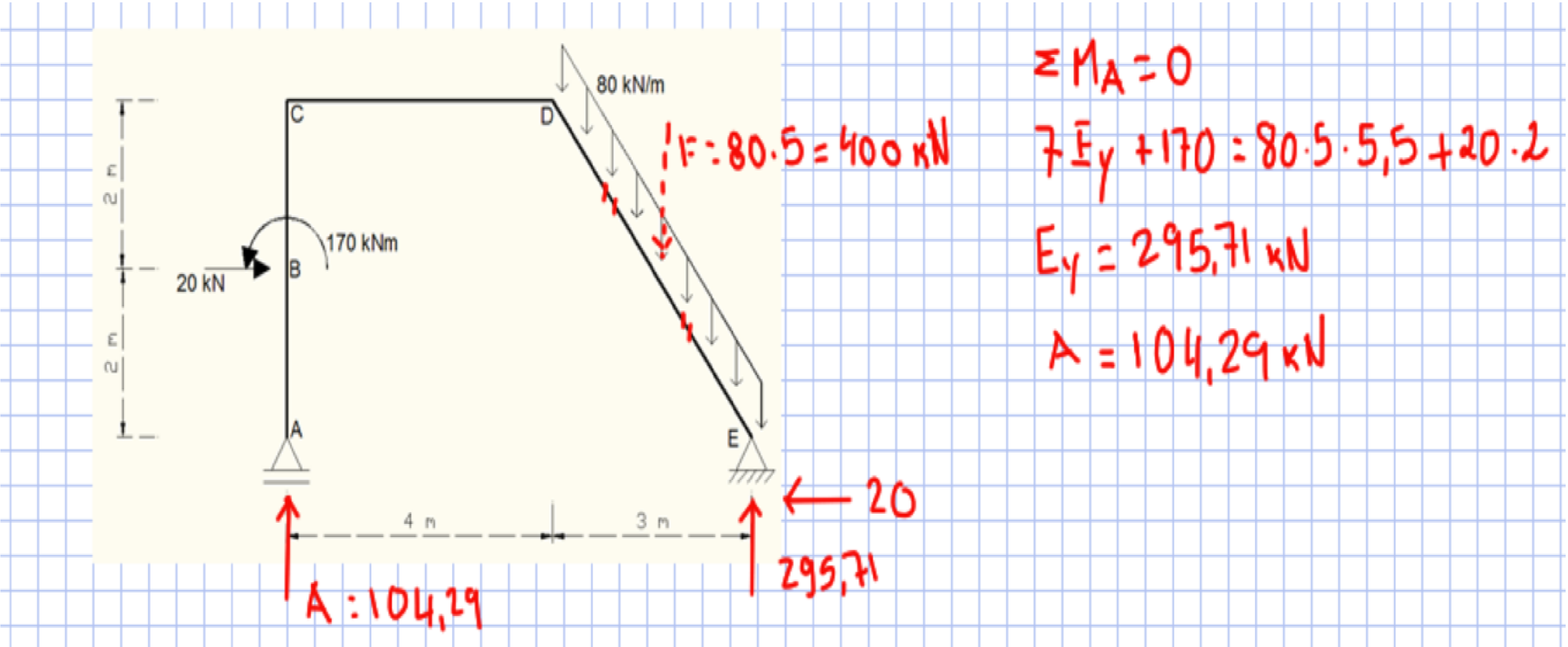
*Livro: Resistência do Materiais: Um guia prático. Valerio Almeida; Marcelo Greco, Daniel Maciel. Elsevier, 2018

Usando as três equações de equilíbrio para determinar as reações:

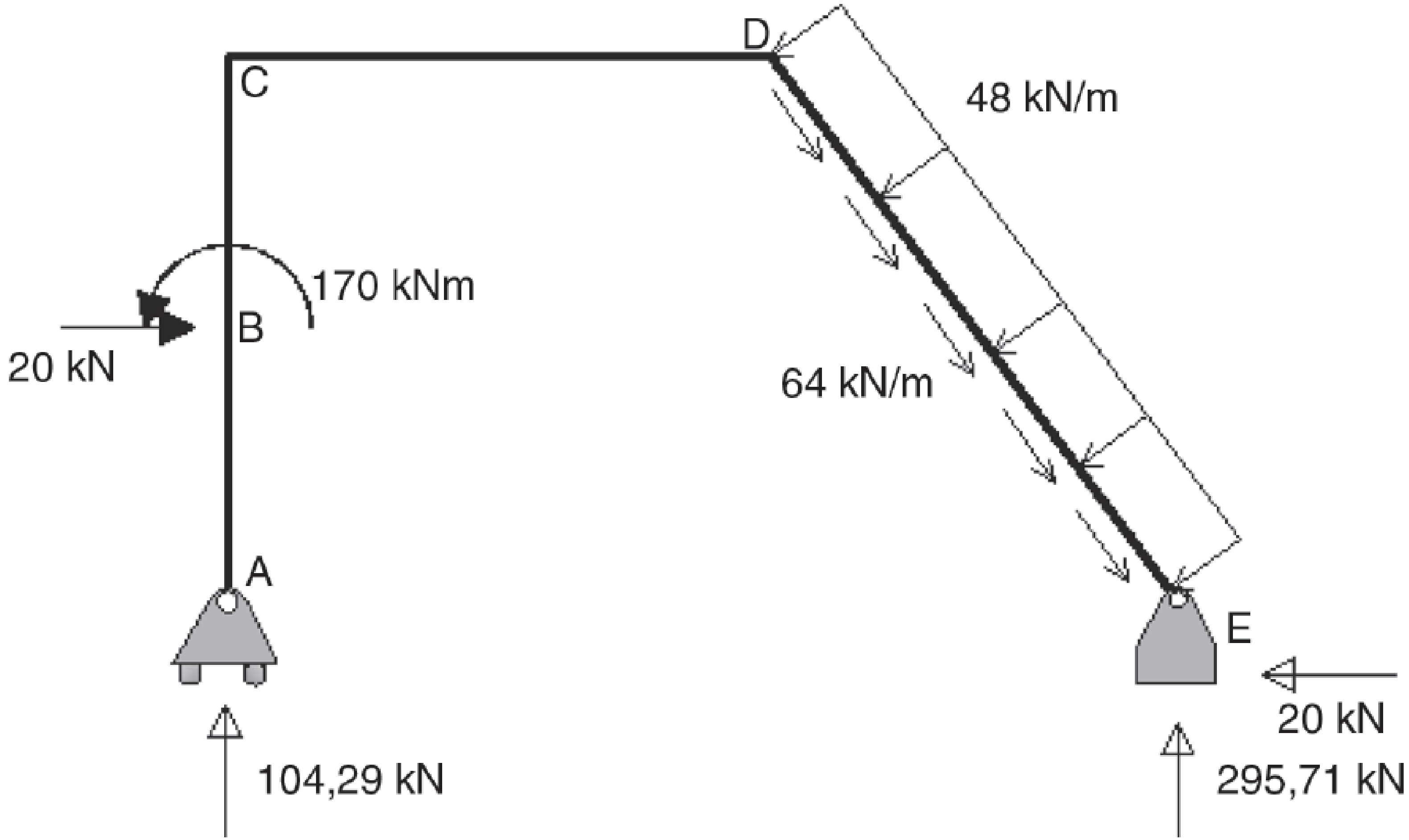
$$\sum F_X = 0 : \rightarrow 20 - E_x = 0 \rightarrow E_x = 20 \ \text{kN} \ (\leftarrow)$$

$$\sum M_A = 0 : \rightarrow 7 \cdot E_y + 170 = 80 \cdot 5 \cdot 5{,}5 + 20 \cdot 2 \rightarrow E_y = 295{,}71 \ \text{kN} \ (\uparrow)$$

$$\sum F_y = 0 : \rightarrow A_y + E_y = 80 \cdot 5 \ \rightarrow A_y = 104{,}29 \ \text{kN} \ (\uparrow)$$



Decompor a carga distribuída paralelo (x') e perpendicular (y') ao seu eixo. A sua resultante é de $80 \cdot 5 = 400$ kN, que a decompondo fica: $F_{x'} = 320$ kN, $F_{y'} = 240$ kN, e pode-se obter a força distribuída como: $q_{x'} = 320/5 = 64$ kN/m, $q_{y'} = 240/5 = 48$ kN/m.



Para os trechos: AB, BC, CD e DE, fazer cortes no início e fim, de modo a aplicar as três equações de equilíbrio, no sentido da convenção positiva, veja trecho AB:

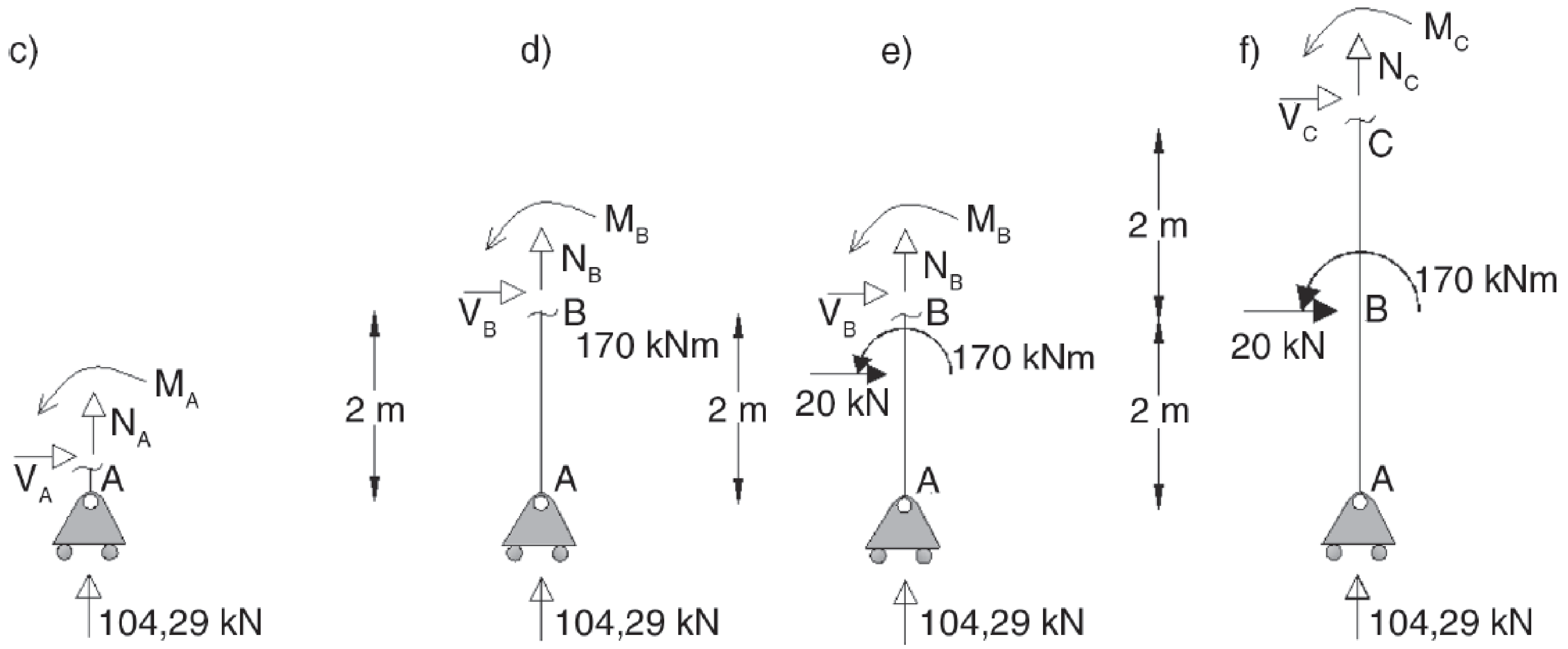


FIGURA 1.58C-F **Esquema do corte: (c) início do trecho AB; (d) fim do trecho AB; (e) início do trecho BC; (f) fim do trecho BC.**

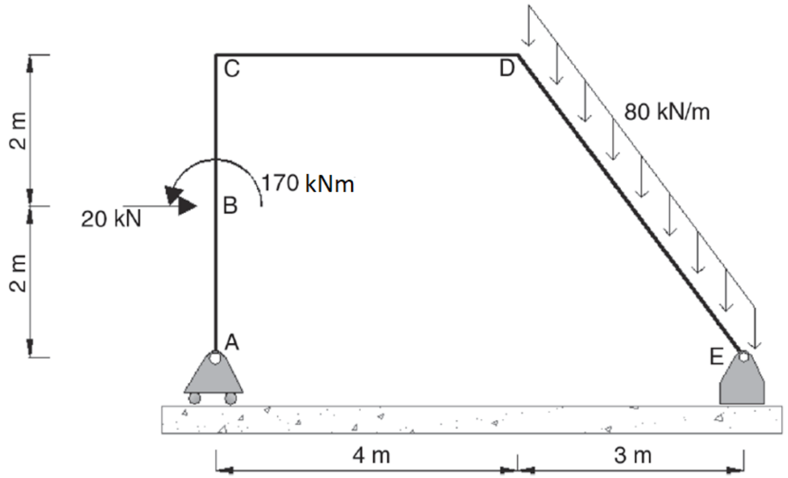
Conforme Figura 1.58C: $N_A = -104{,}29\ \text{kN}; V_A = 0; M_A = 0$

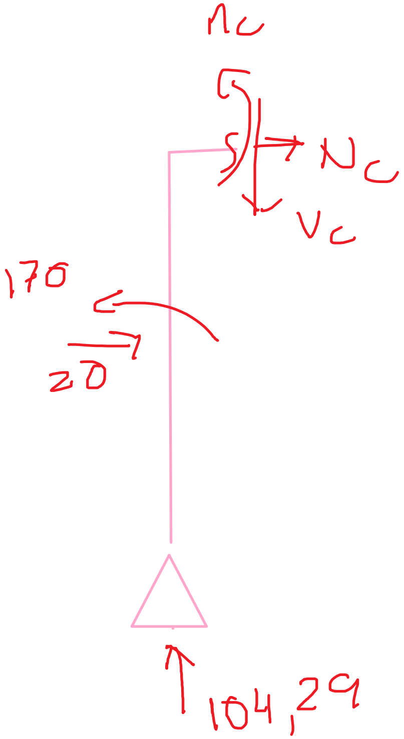
Conforme Figura 1.58D: $N_B = -104{,}29\ \text{kN}; V_B = 0; M_B = 0$

Trecho BC:

Conforme Figura 1.58E: $N_B = -104{,}29\ \text{kN}; V_B = -20\ \text{kN}; M_B = -170\ \text{kNm}$

Conforme Figura 1.58F: $N_C = -104{,}29\ \text{kN}; V_C = -20\ \text{kN}; M_C = -210\ \text{kNm}$





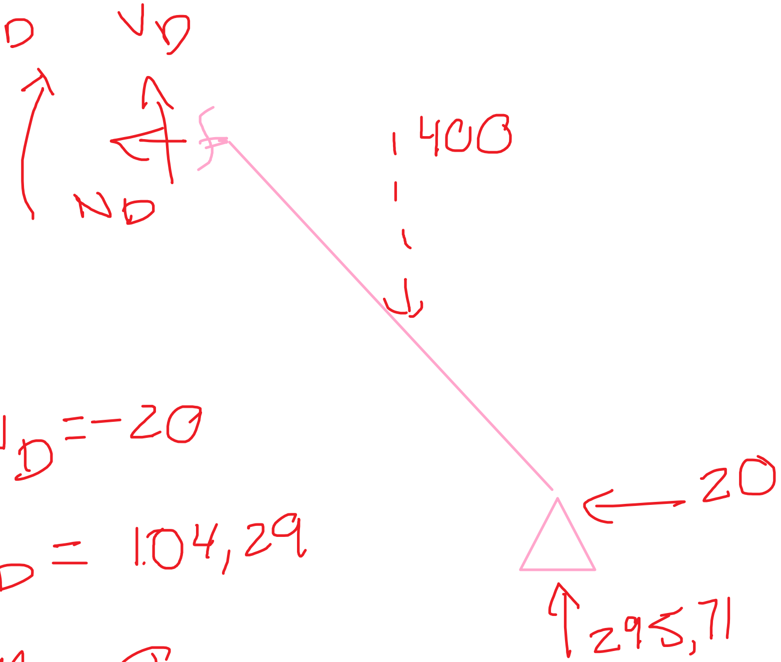
$N_C = -20$

$V_C = 104{,}29$

$\Sigma M_S = 0$:

$M_C + 20 \cdot 2 + 170 = 0$

$M_C = -210$



$N_D = -20$

$V_D = 104{,}29$

$\Sigma M_D = 0$

$M_D + 400 \cdot 1{,}5 + 20 \cdot 4 = 295{,}71 \cdot 3$

$M_D = 207{,}14$

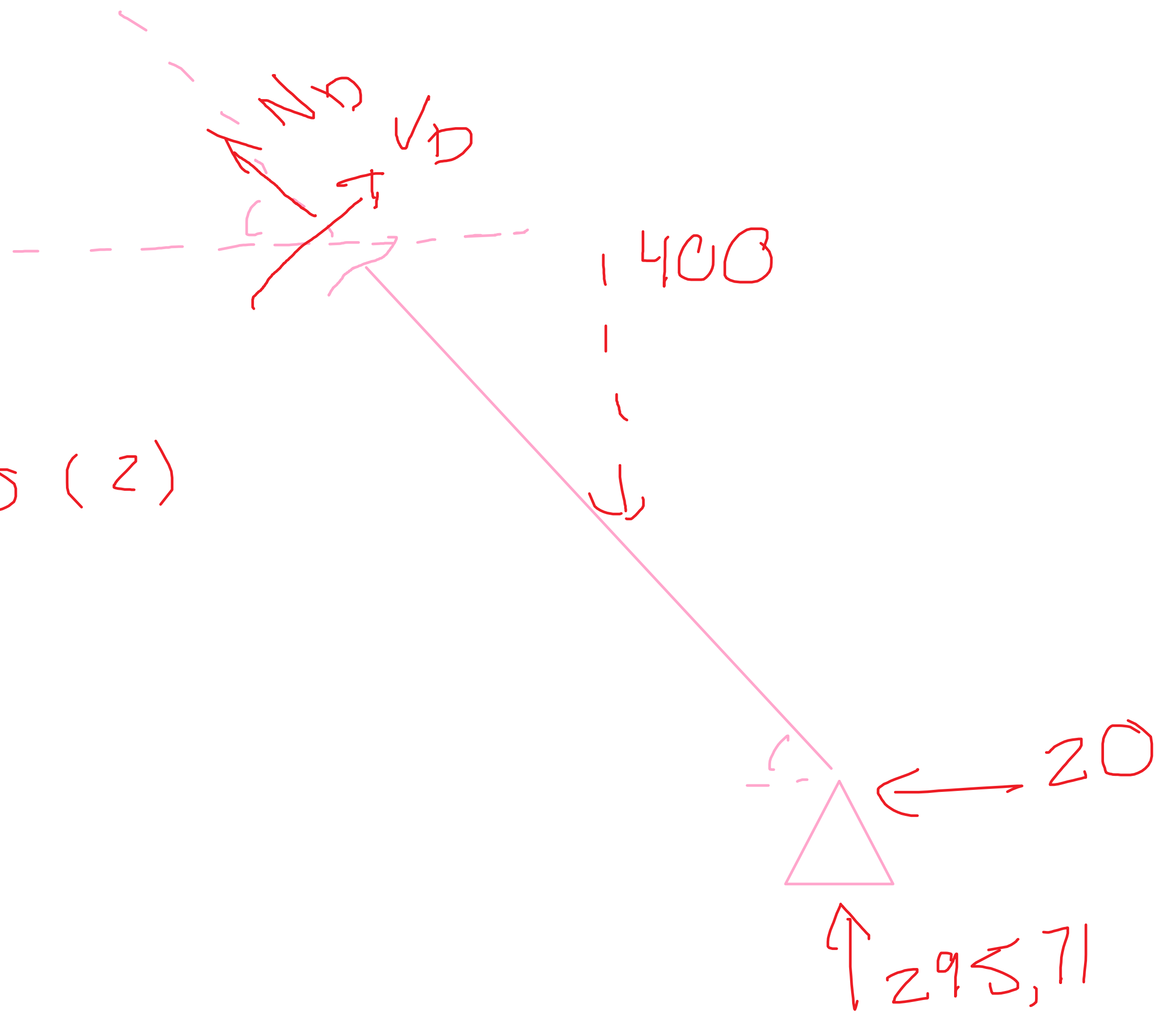
$$\Sigma F_x = 0$$

$$N_D \cos\alpha + 20 - V_D \,\text{sen}\,\alpha = 0 \quad (1)$$

$$\Sigma F_y = 0$$

$$N_D \,\text{sen}\,\alpha + V_D \cos\alpha + 295{,}71 - 400 = 0 \quad (2)$$

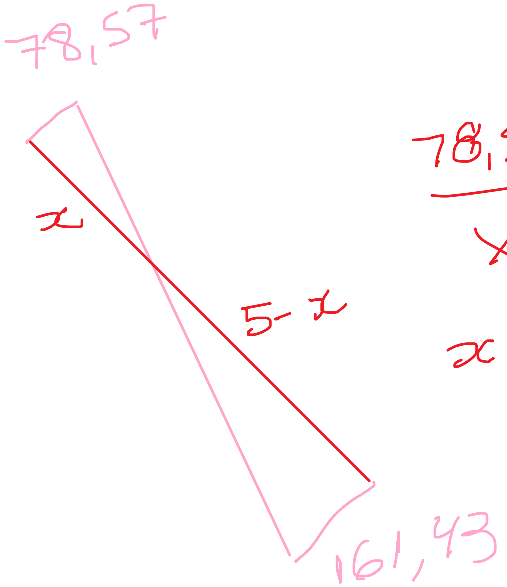
$$N_D = 71{,}43 \;;\; V_D = 78{,}57$$



$N_E = -248,57$

$V_E = -161,43$

$M_E = 0$



$$\frac{78,57}{x} = \frac{161,43}{5-x}$$

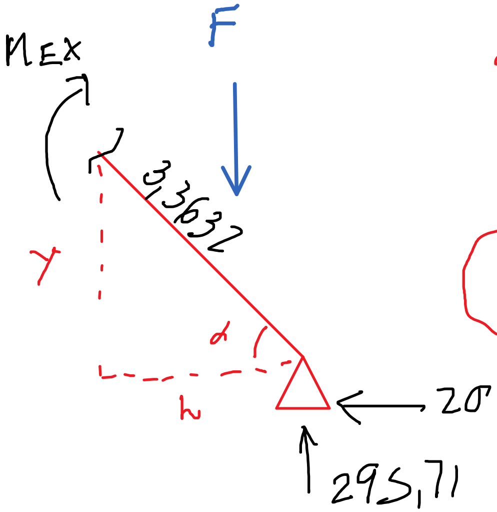
$$x = 1,6368\,m \quad (5-x = 3,3632\,m)$$

$$F = 80 \cdot 3,3632$$

$$F = 269,06\,kN$$

$$\text{sen}\,\alpha = 0,8 \rightarrow y = 2,6906\,m$$

$$\cos\alpha = 0,6 \rightarrow h = 2,0179\,m$$



$$\Sigma M_S = 0$$

$$M_{EX} + F \cdot \frac{h}{2} + 20 \cdot y = 295,71 \cdot h$$

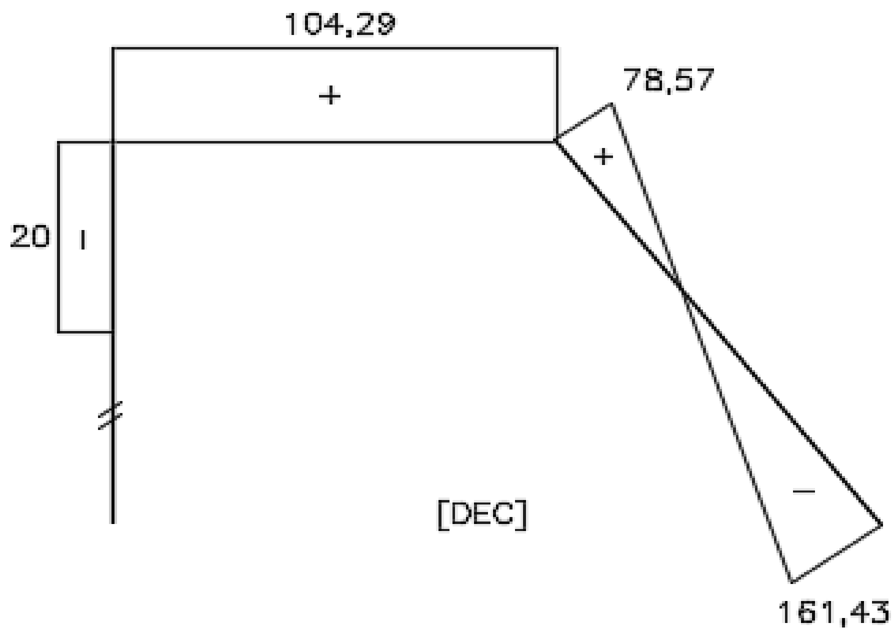
$$M_{EX} = 271,44\,kNm$$

$$\frac{dM}{dx} = V(x) \Rightarrow \int_{x_1}^{x_2} \frac{dM}{dx} = \int_{x_1}^{x_2} V(x)$$

$$M_{x_2} = \int_{x_1}^{x_2} V(x)\,dx + C$$

$$M_{x_2} = \text{Área}(V)\Big|_{x_1}^{x_2} + \sum_{i=1}^{conc.} M_i$$

* Área(V) : ALGÉBRICO (tem sinal!!)

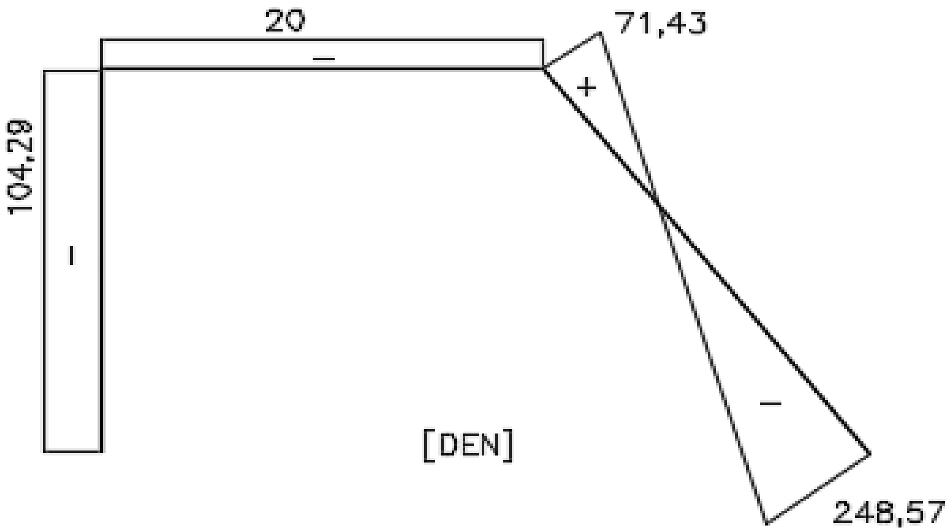


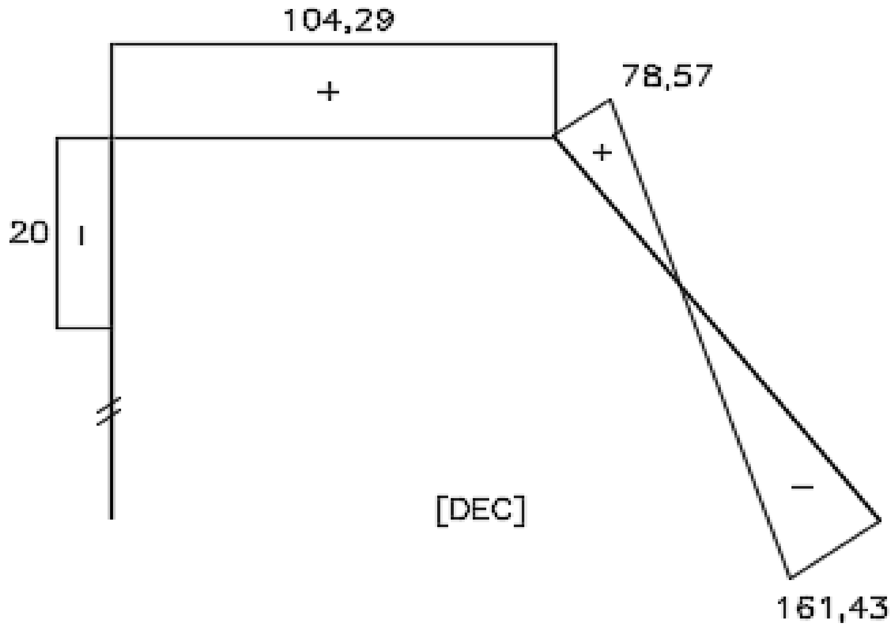
$$M_x = (-20 \cdot 2) + (104{,}29 \cdot 4) + \frac{78{,}57}{2} x - 170$$

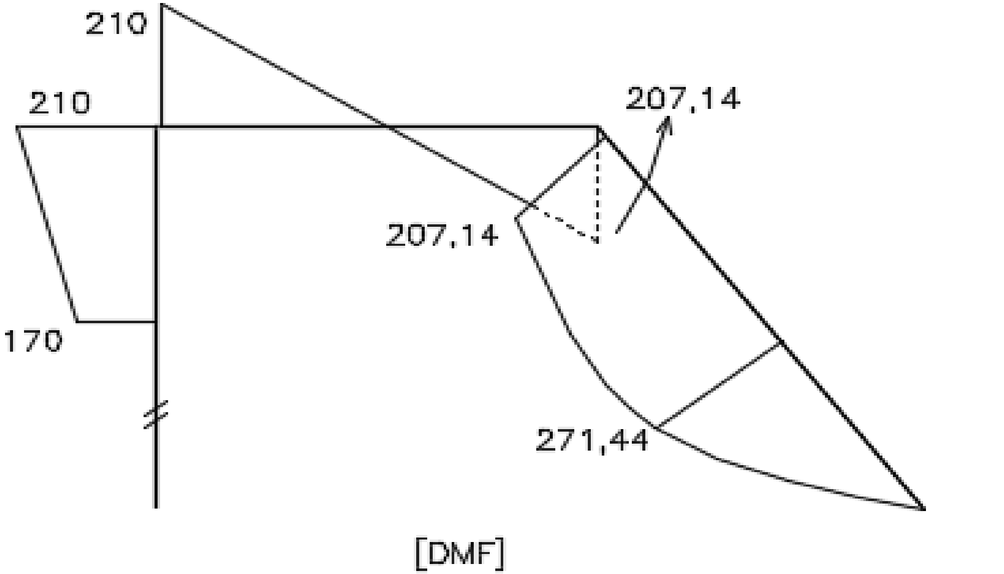
$$M_x = 271{,}44 \text{ kNm}$$

$$M_x = -\left[-\frac{161{,}43}{2} \cdot (5 - x)\right] = 271{,}44$$

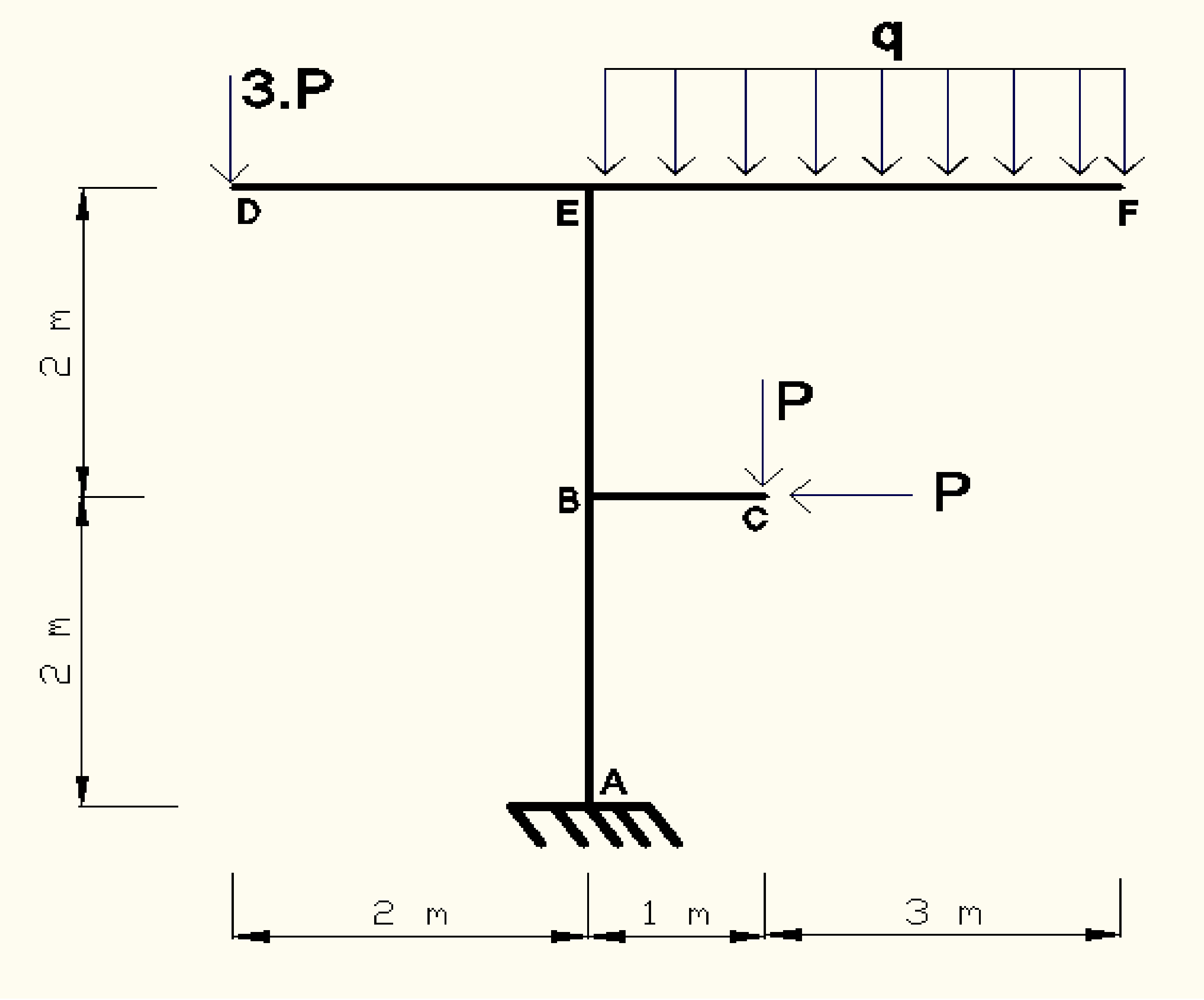
(–) : integrando da direita p/ esquerda)


20
–
71,43
+
104,29
–
–
248,57
[DEN]


104,29
+
78,57
+
20
–
–
161,43
[DEC]


210
210
207,14
207,14
170
271,44
[DMF]

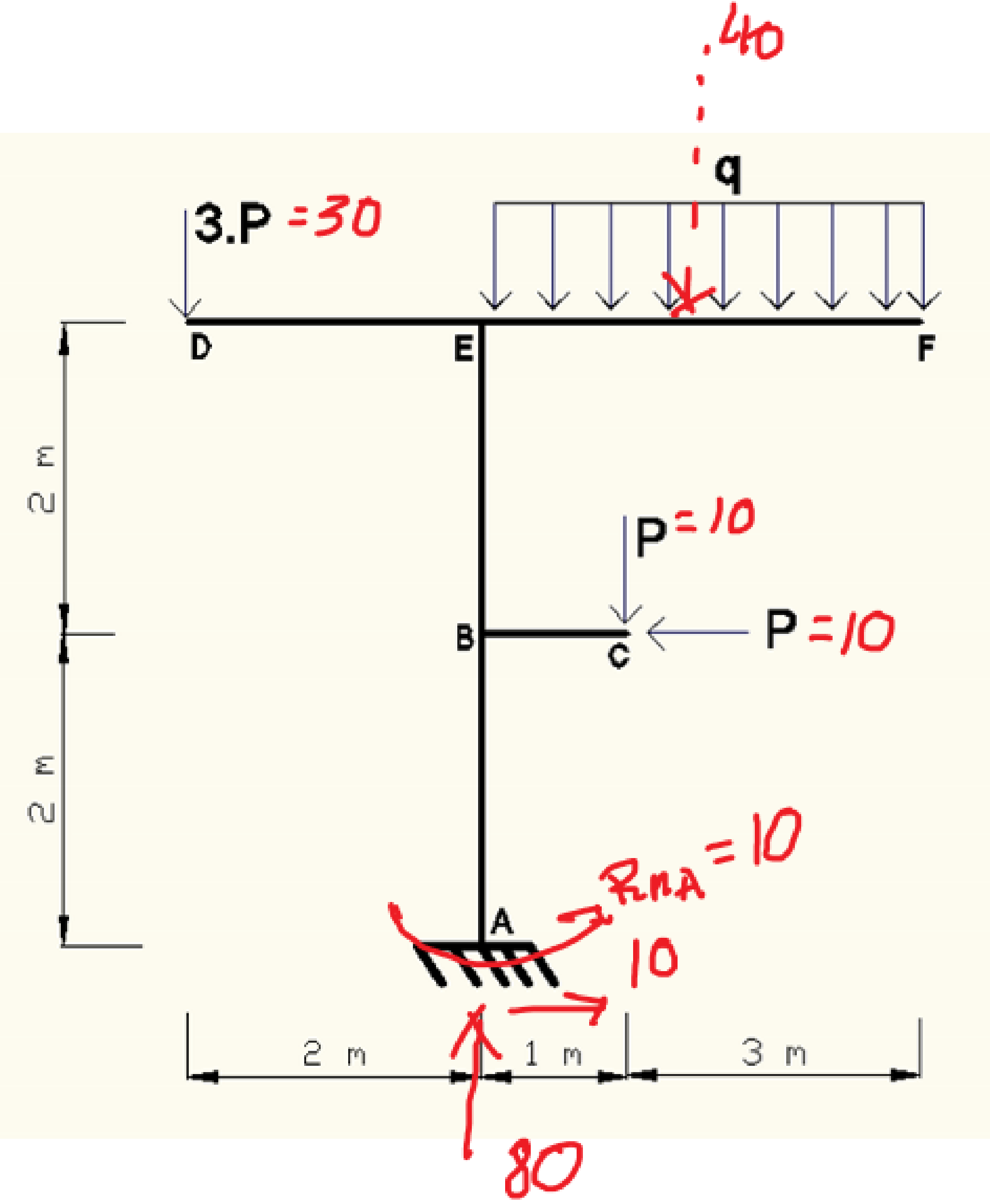
Exemplo 20: Para a estrutura plana a seguir, obtenha os esforços solicitantes para os trechos ABE e DEF. Sabe-se que P = 10 kN e q = 10 kN/m. Indicar os diagramas nos desenhos em destaque.



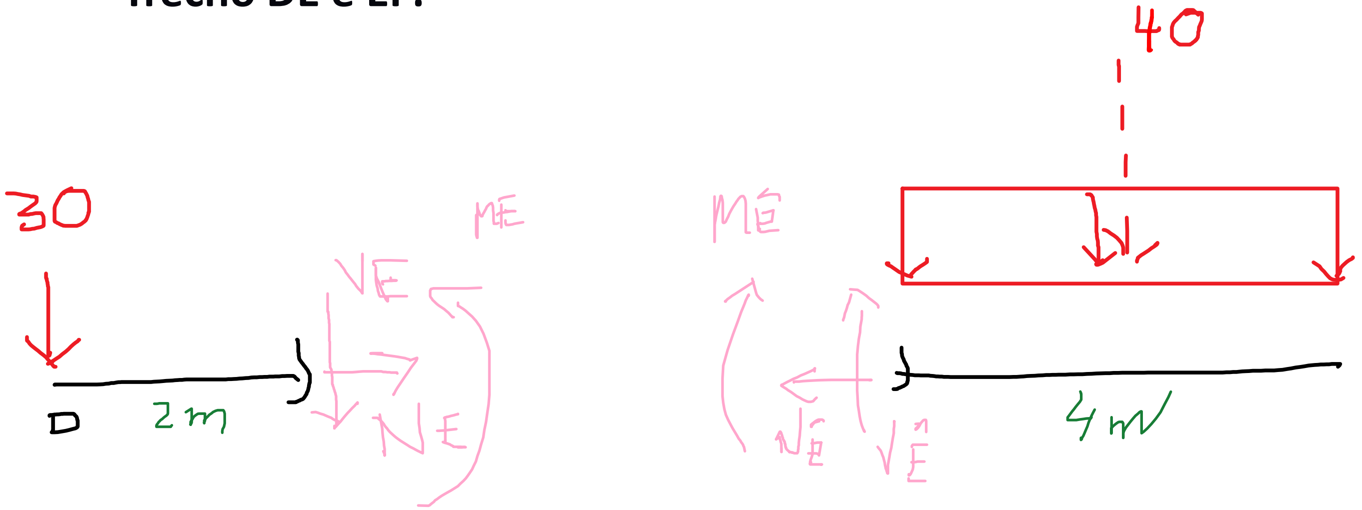
Reações:

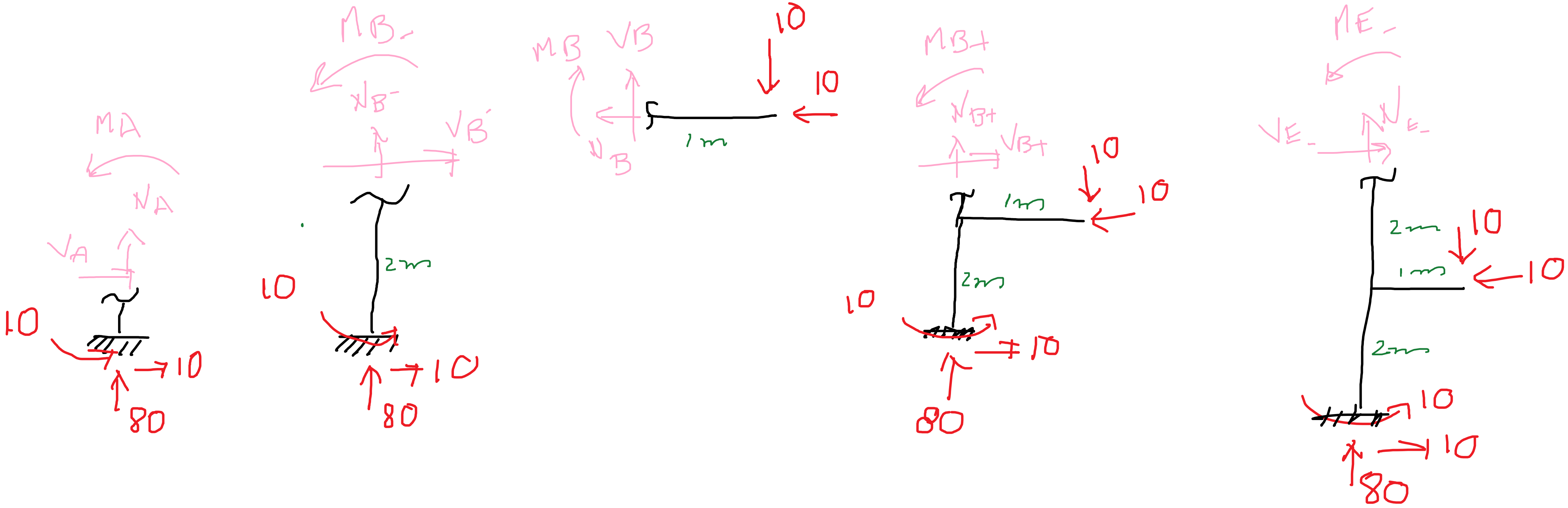
$$\Sigma M_A = 0 : R_{MA} + 10 \cdot 2 + 30 \cdot 2 = 40 \cdot 2 + 10 \cdot 1$$

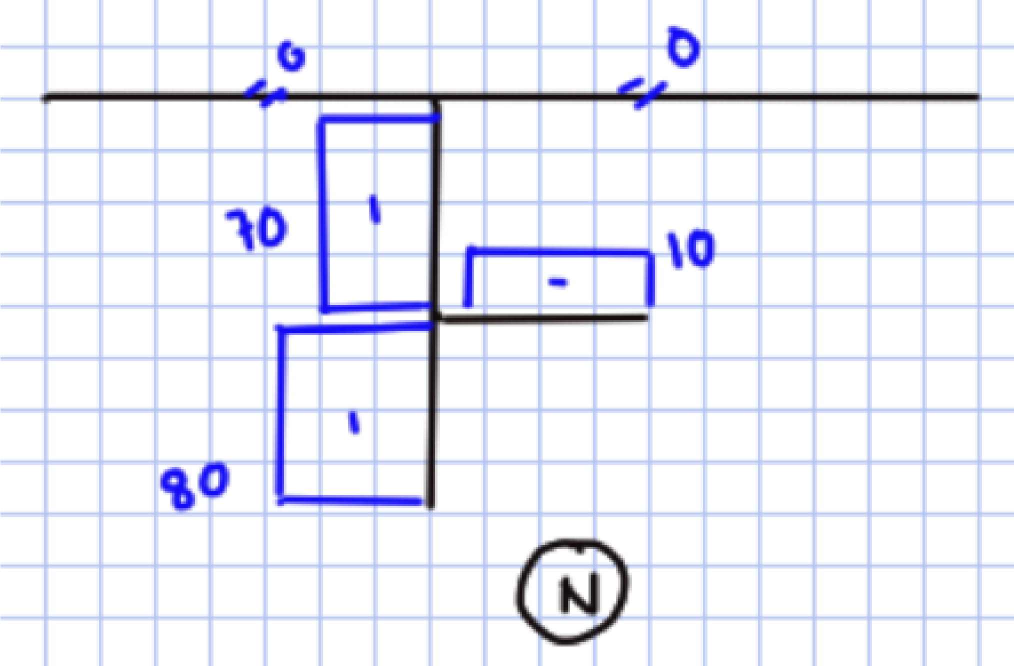
$$R_{MA} = 10 \text{ kNm}$$

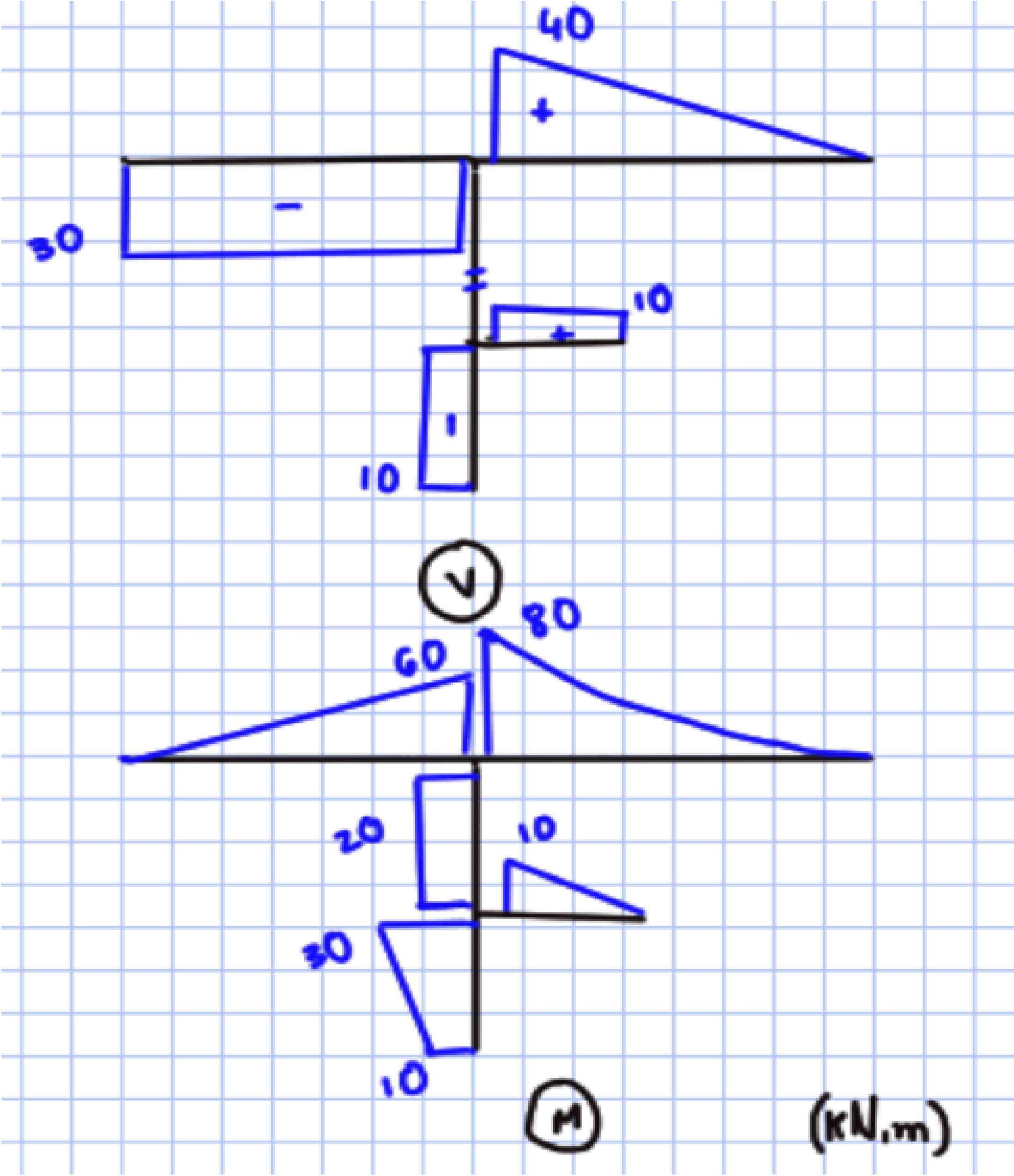


Trecho DE e EF:

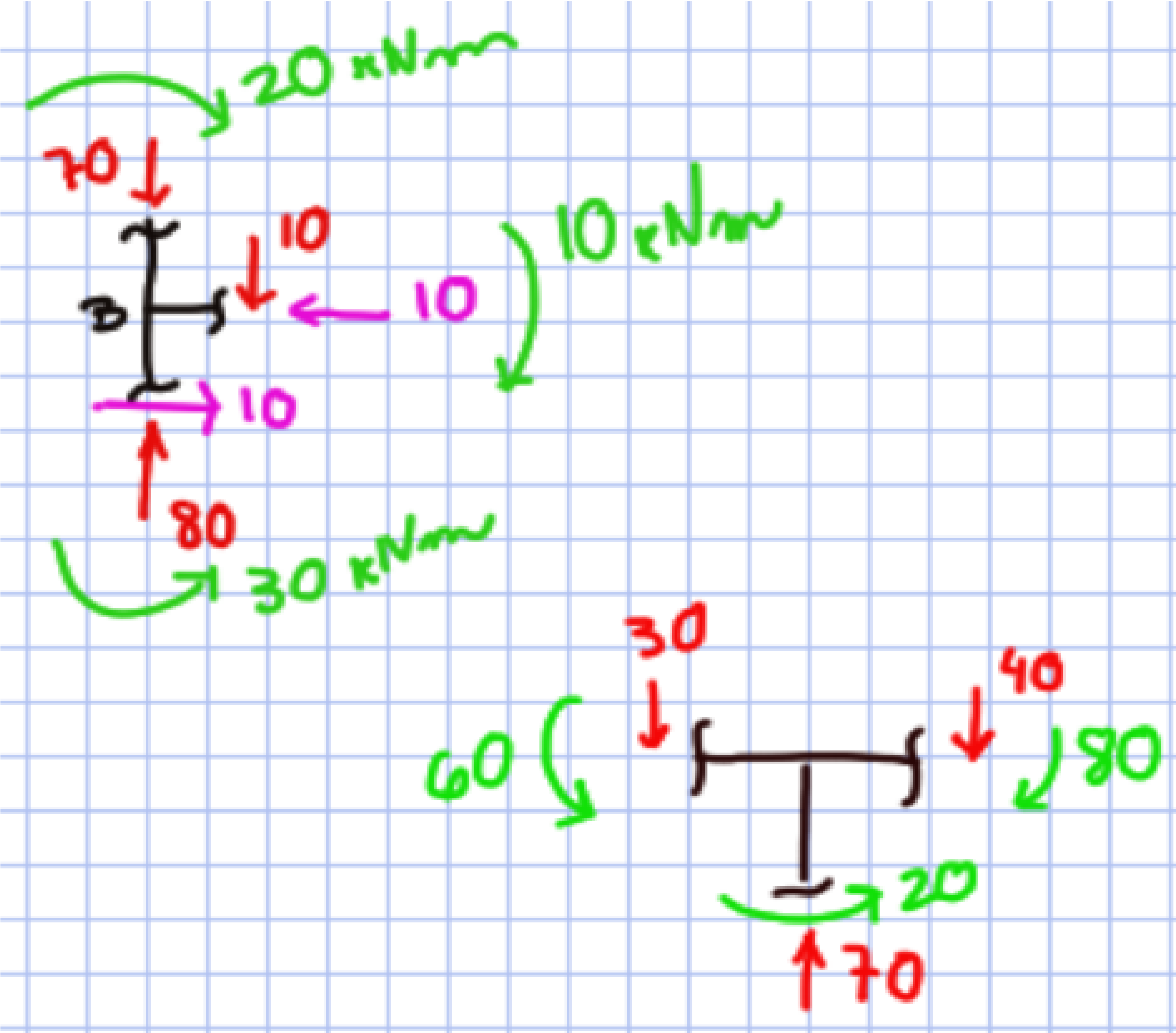



MA
NA
VA
10
10
80
MB-
NB-
VB-
2m
10
10
80
MB
VB
NB
10
10
1m
MB+
NB+
VB+
1m
10
10
2m
10
10
80
ME-
NE-
VE-
2m
10
1m
10
2m
10
10
80

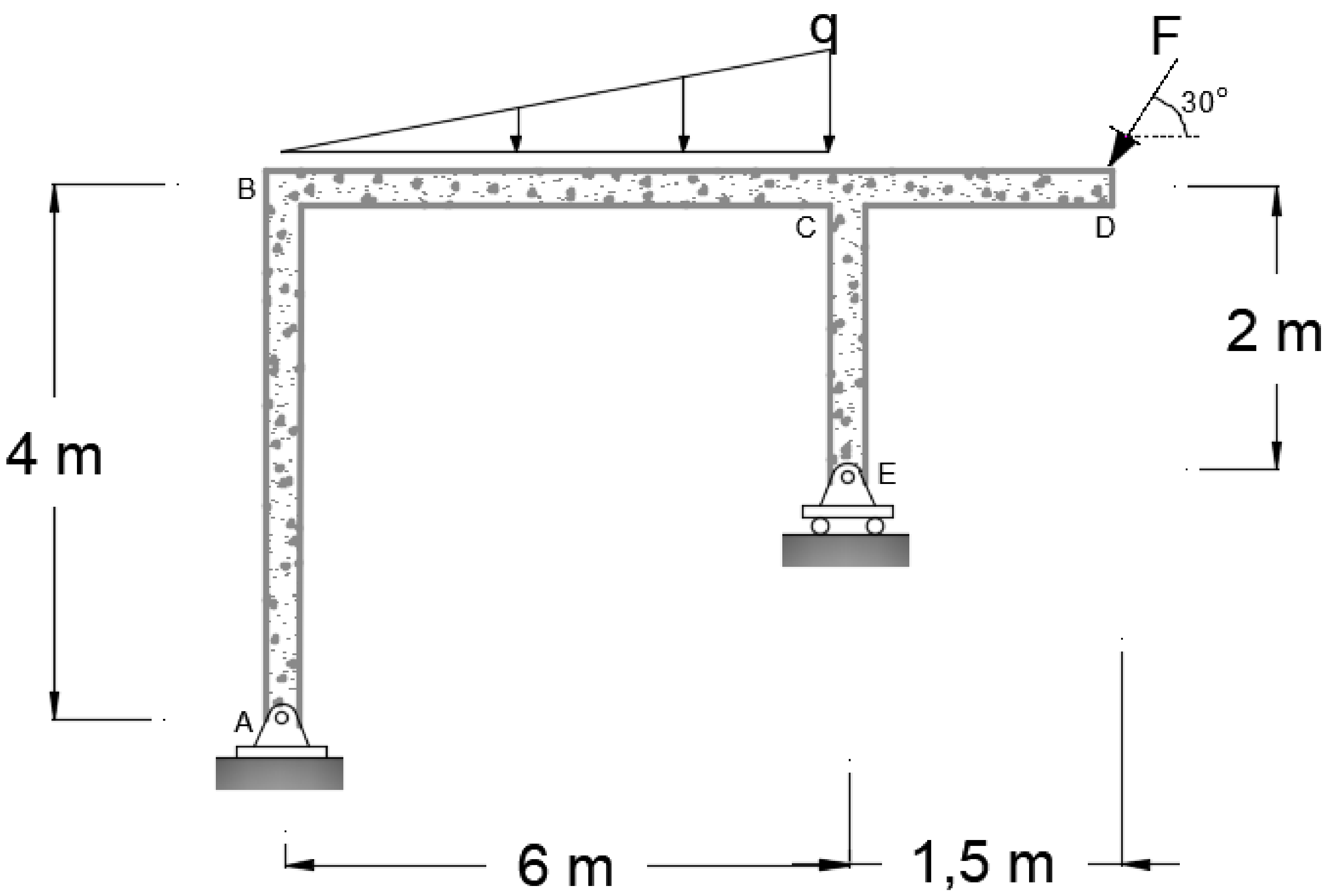

0
0
70
10
80
N


40
+
30
−
10
+
10
V
80
60
20
10
30
10
M
(kN,m)

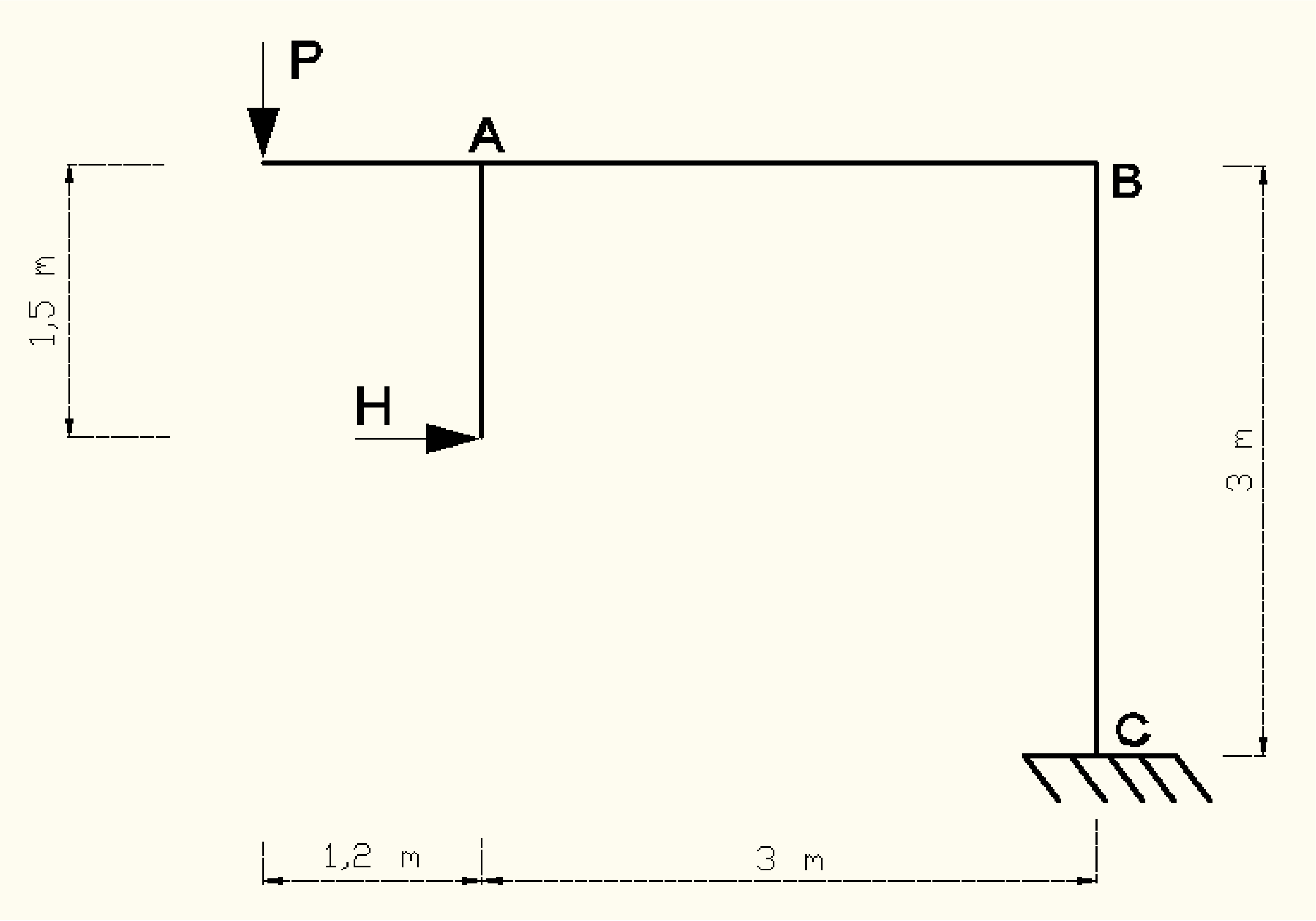
Verificação de equilíbrio nos pontos B e E.

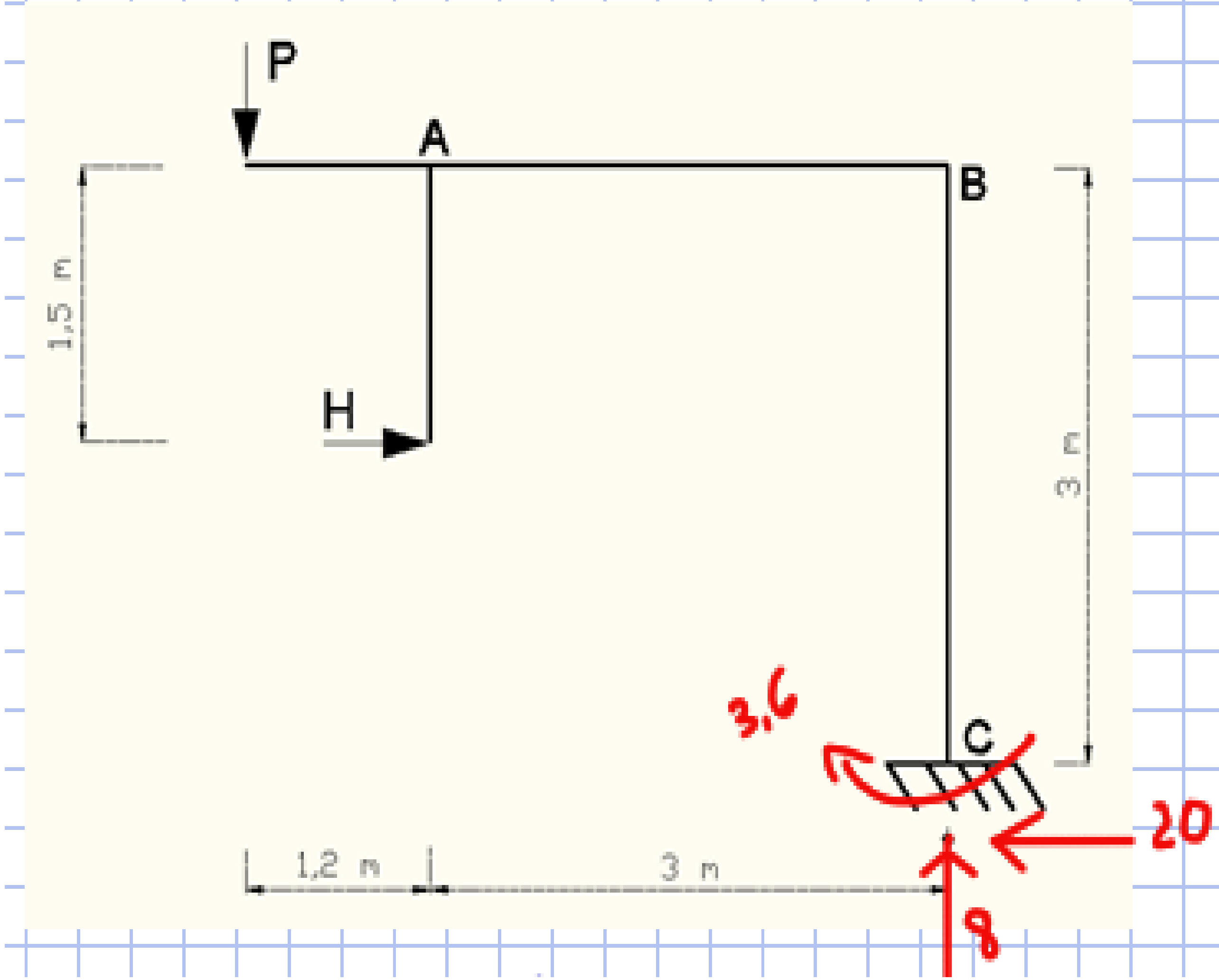


Para a estrutura a seguir, sabendo que F = 10 kN e q = 10 kN/m, obtenha os diagramas de esforços solicitantes em todos os trechos, indicando os valores e posições dos extremos dos esforços em cada trecho.



Exemplo 21: Determinar os esforços solicitantes (N, V e M) na estrutura esquematizada a seguir, sob a ações das cargas indicadas. Indique explicitamente os valores e os pontos de momentos extremos nos desenhos em destaque. Dados: P = 8 kN; H = 20 kN.



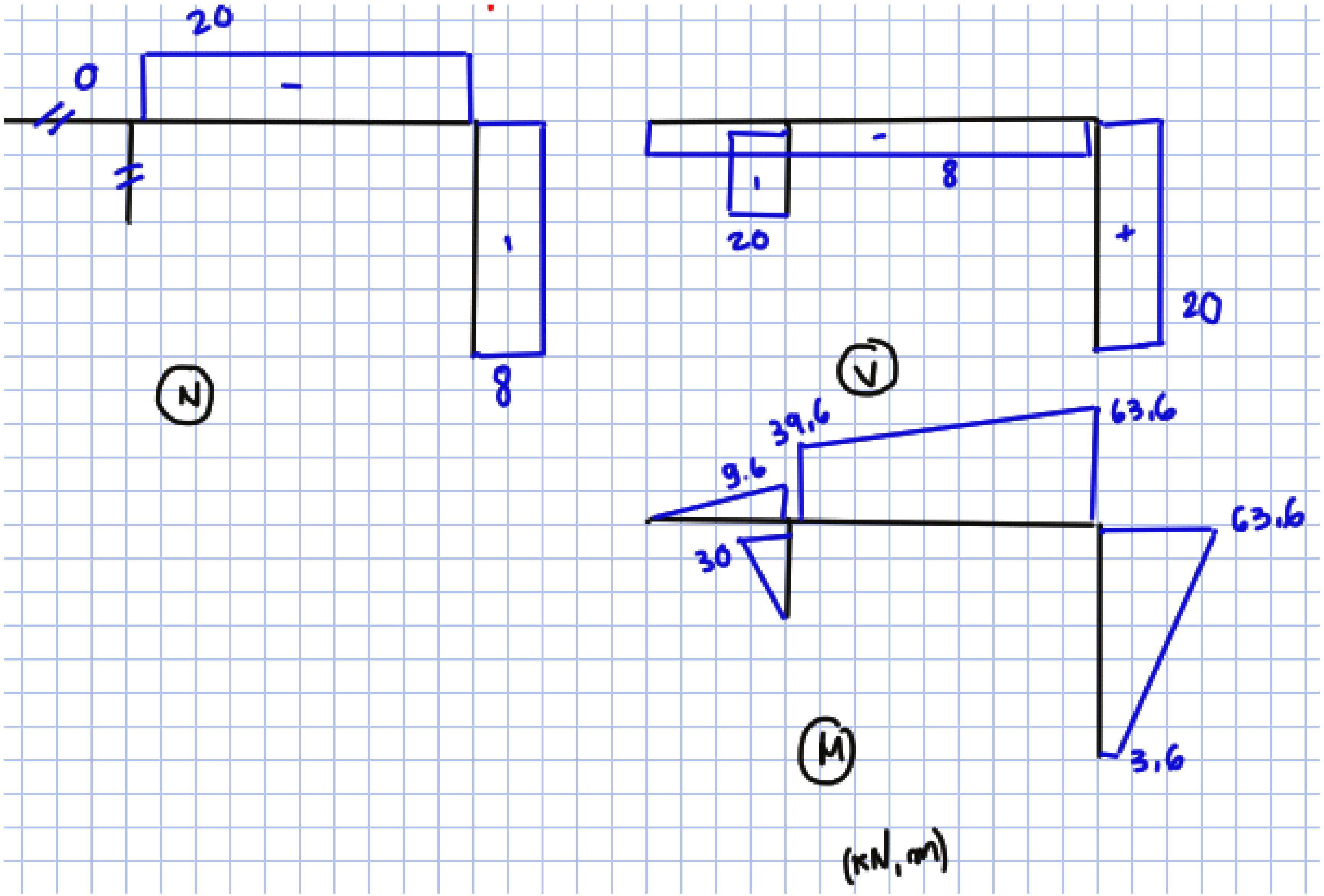


$$\Sigma M_C = 0:$$

$$M_C + H \cdot 1{,}5 = P \cdot 4{,}2$$

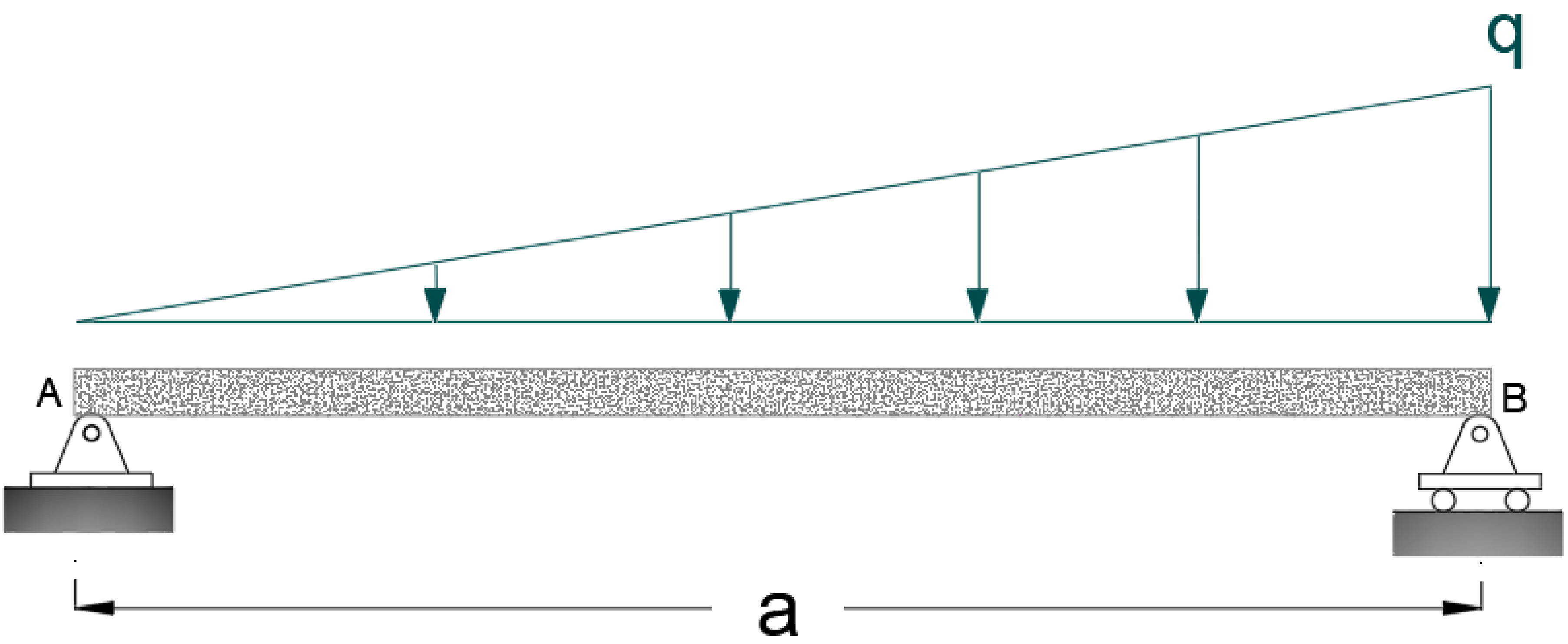
$$M_C = 8 \cdot 4{,}2 - 20 \cdot 1{,}5$$

$$M_C = 3{,}6 \text{ kNm}$$



20
0
-
1
8
N
-
8
1
20
+
20
V
39,6
63,6
9.6
30
63,6
M
3,6
(kN,m)

Para a estrutura a seguir, sabendo que q = 12 kN/m e a = 6 m, obtenha os diagramas de esforços solicitantes



$q(x) = 2x$

$Ay = 12$ kN

$By = 24$ kN

$dV/dx = -q(x)$

$V(x) = -x^2 + 12$

$V(0) = 12$ kN; $V(6) = -24$ kN

$V(x) = -x^2 + 12 = 0$; $x = 2\sqrt{3} = 3{,}46\ m$

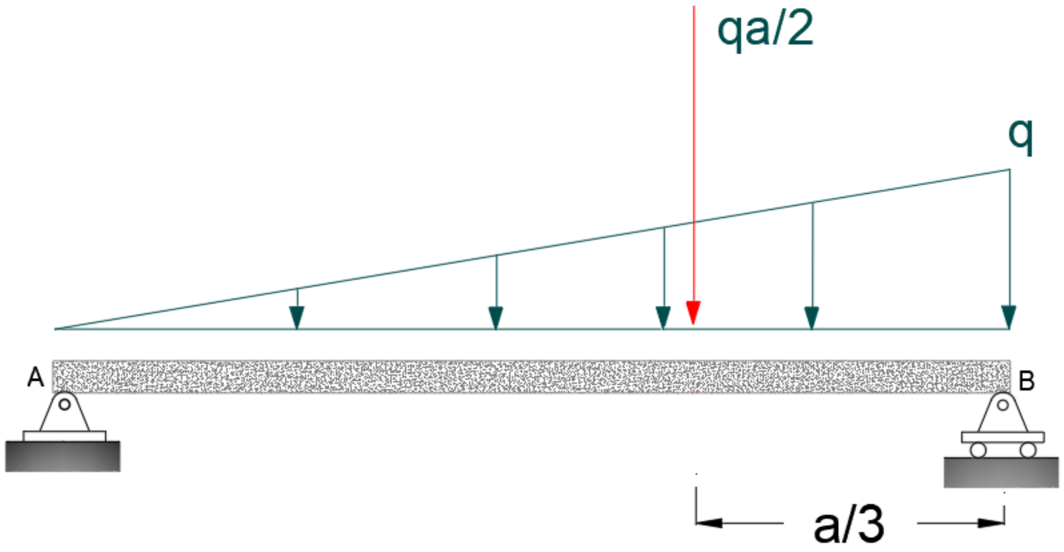
$M(x) = -x^3/3 + 12x$

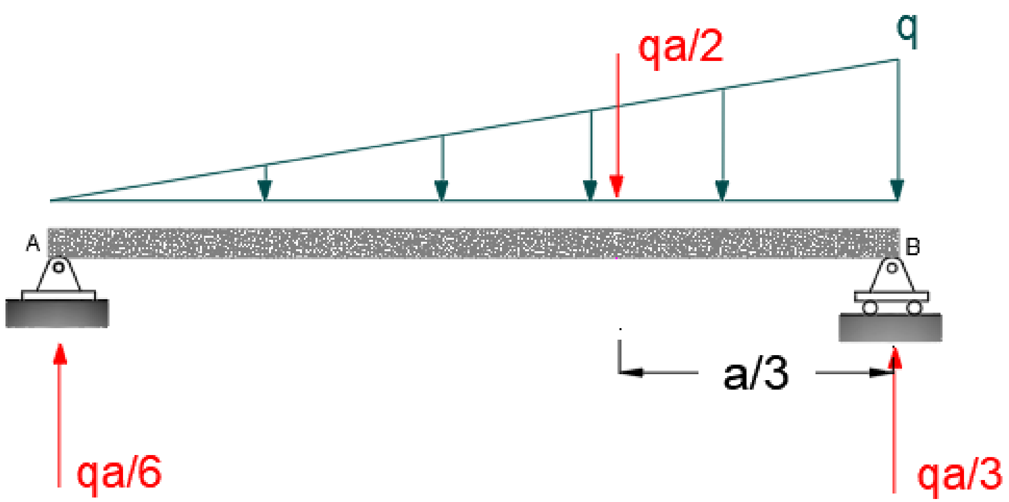
$M(0) = 0$; $M(6) = -x^3/3 + 12x = 0$

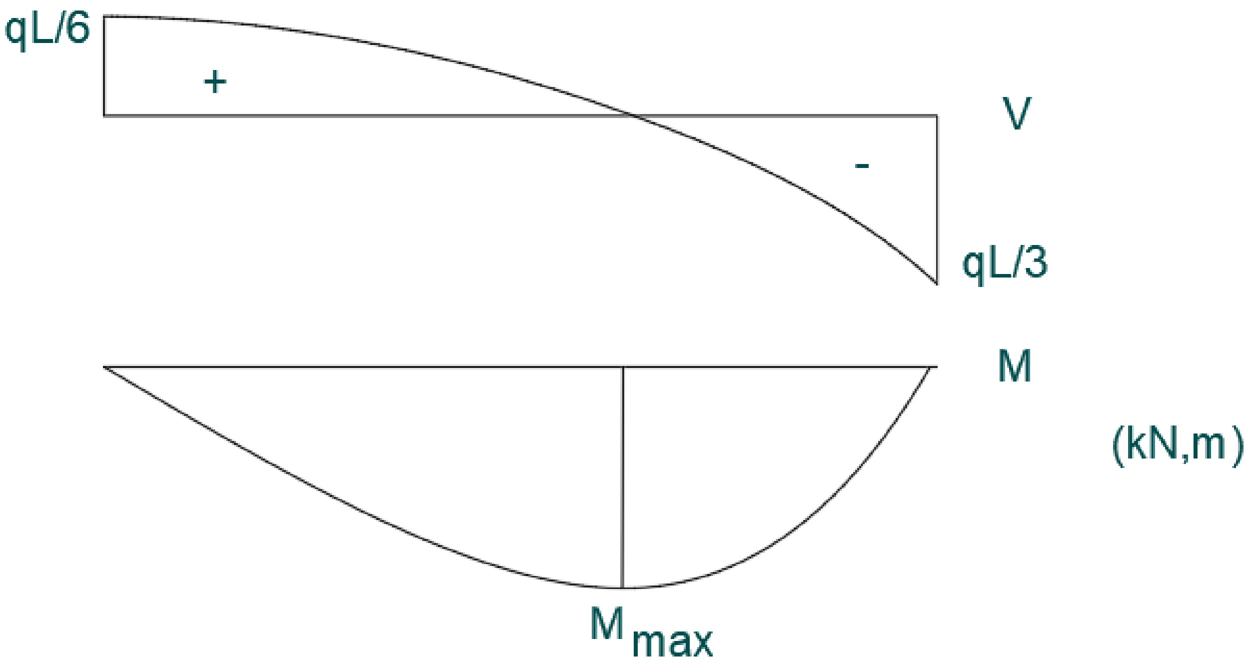
$M'(x) = -3x^2/3 + 12 = 0$; $x = \pm 2\sqrt{3} = \pm 3{,}46\ m$

$M(3{,}46) = -x^3/3 + 12x = 27{,}71$ kNm

$M''(x) = -6x^2/3 = -2x$ (concavidade para cima entre $x=0$ e $x = 6$)




q
qa/2
A
B
a/3
qa/6
qa/3


qL/6
+
V
-
qL/3
M
(kN,m)
M max