Modos TEM, TE, TM
Guias de Onda

SEL 369 Micro-ondas

Tania Regina Tronco
Departamento de Engenharia Elétrica da EESC-USP

Guias de Onda

- São tubos metálicos (ocos ou preenchidos com material dielétrico) utilizados para a transmissão de energia em altas frequências (acima de 1 GHz);
- Menor atenuação e maior potência do que as linhas de transmissão.

Usados como “tubulação de água”

Comparação:

A transmissão é possível?

- **Linha de transmissão:** Sim, desde DC até altas frequências
- **Guia de onda:** DC $\Rightarrow$ Não

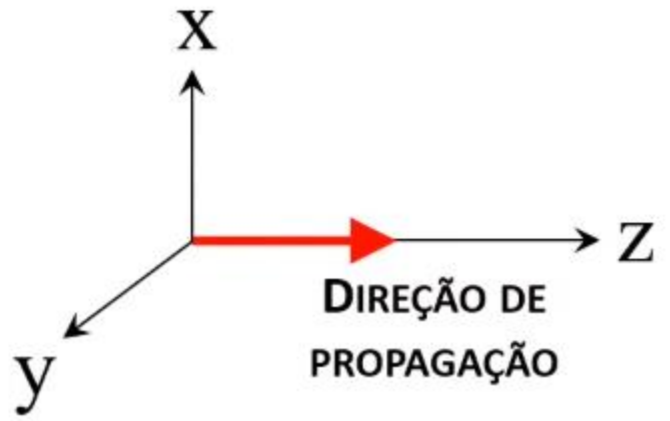
  Luz (10^{14} Hz < f < 10^{15} Hz) $\Rightarrow$ Sim

- A transmissão num guia de onda só é possível acima de uma certa frequência (**frequência de corte do guia**).
- Os guias de onda se comportam como filtros **passa-altas**.

Guiamento

- ✓ Finalidade : Conduzir energia eletromagnética de um ponto para outro
- ✓ Modos de Propagação
    - Arranjo único de campos elétrico e magnético que:
        - Satisfaz todas as equações de Maxwell
        - Satisfaz as condições de contorno impostas pela geometria da estrutura
    - Os vários modos correspondem às diferentes soluções das equações de onda
- ✓ Como obter as soluções
    - Resolvendo a equação de onda

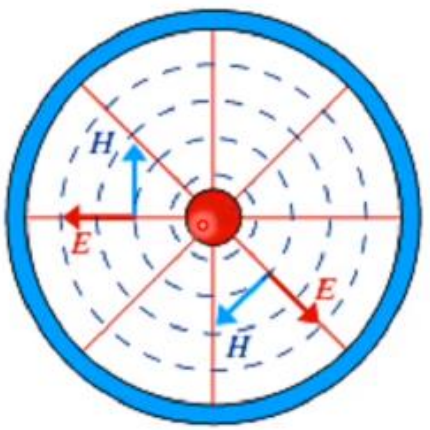
Modos de Propagação:



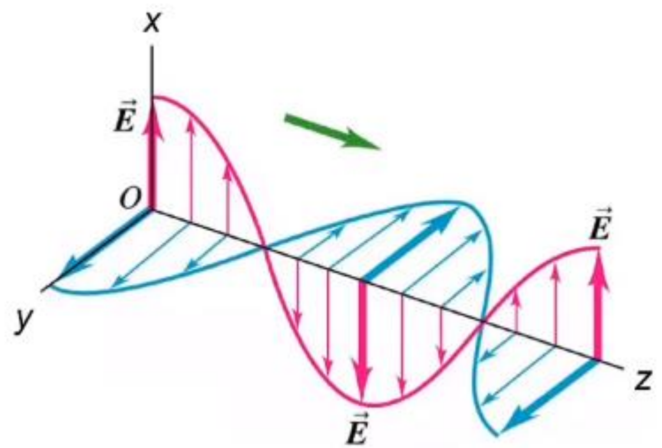
Modo **TEM** (**e**letro **m**agnético **t**ransversal)

$$E_z = H_z = 0$$

Componentes de **E** e **H** diferentes de zero estão no plano transversal ao de propagação



CABO COAXIAL



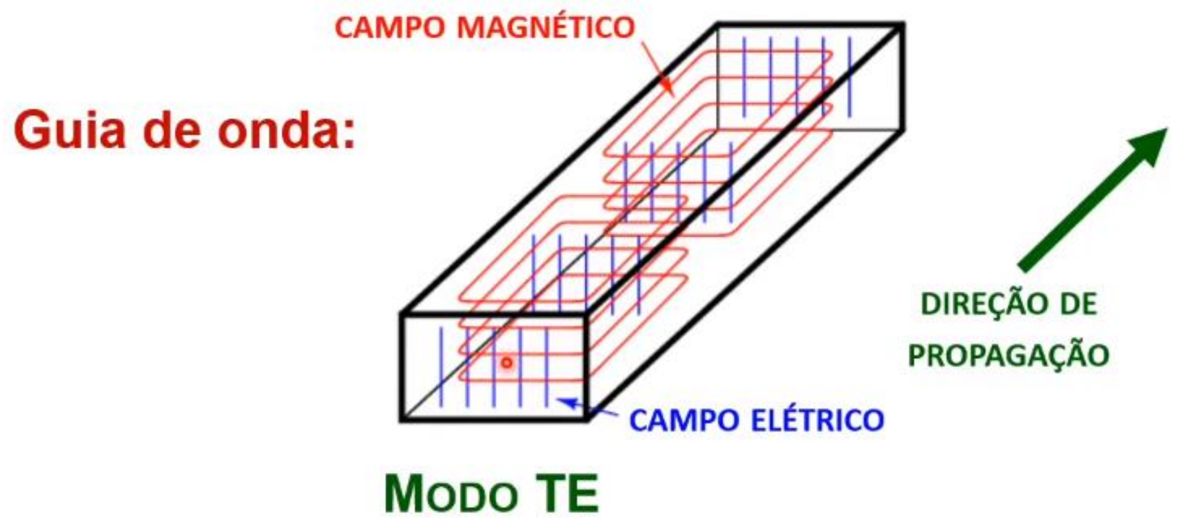
ONDA PLANA UNIFORME

Modos de Propagação

Modo **TE** (**e**létrico **t**ransversal)

$$E_z = 0 \,;\, H_z \neq 0$$

Componentes de **E** diferentes de zero estão no plano transversal ao de propagação

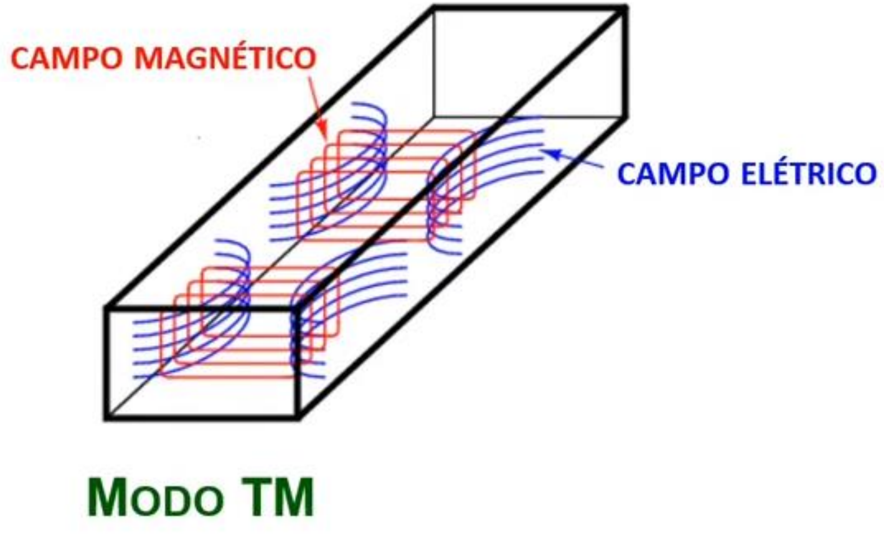


Modos de Propagação

Modo **TM** (**m**agnétrico **t**ransversal)

$$H_z = 0\,;\, E_z \neq 0$$

Componentes de **H** diferentes de zero estão no plano transversal ao de propagação



Modos de Propagação

Modos Híbridos **HE** ou **EH**

$$H_z \neq 0\,;\,E_z \neq 0$$

- ✓ Guias de onda suportam apenas modos TE e TM ou híbridos, nuncaTEM

Forma e Notação dos Campos-1

Suposições

- Guias sem perdas:
- Dielétricos ideais (sem perdas)
- Metais condutores perfeitos

Fator de variação temporal: $\exp(j\omega\, t)$

Fator de variação espacial: $\exp(-jk_z\, z)$

Os campos são da forma

$$\overline{E}\;\; x,y,z\;\; = \left[E_x\;\; x,y\;\; \hat{x} + E_y(x,y)\,\hat{y} + E_z\;\; x,y\;\; \hat{z}\right] e^{-jk_z z}$$

Campos com variação senoidal no tempo $\Rightarrow$ análise usando fasores

Vetores de campo: $\vec{E} = E_x \vec{i} + E_y \vec{j} + E_z \vec{k}$ $\quad$ $\vec{H} = H_x \vec{i} + H_y \vec{j} + H_z \vec{k}$

Para uma onda que se propaga na direção +z:

$$\vec{E} = \vec{E}(x, y, z) = \vec{E}(x, y)e^{-\gamma z}$$

$$\vec{H} = \vec{H}(x, y, z) = \vec{H}(x, y)e^{-\gamma z}$$

Assim: $\dfrac{\partial \vec{E}}{\partial z} = -\gamma \vec{E}(x, y)e^{-\gamma z} = -\gamma \vec{E}$ e $\dfrac{\partial \vec{H}}{\partial z} = -\gamma \vec{H}(x, y)e^{-\gamma z} = -\gamma \vec{H}$

Lei de Ampère:

$$\vec{\nabla} \times \vec{H} = \cancel{\sigma_d \vec{E}} + j\omega\varepsilon\vec{E} \Rightarrow \vec{\nabla} \times \vec{H} = j\omega\varepsilon\vec{E}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left(E_x\vec{i} + E_y\vec{j} + E_z\vec{k}\right)$$

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x$$

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

Lei de Faraday: $\vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & -\gamma \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\left(H_x\vec{i} + H_y\vec{j} + H_z\vec{k}\right)$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

Escrevendo as componentes transversais (E_x, E_y, H_x e H_y) em termos das componentes longitudinais (E_z e H_z):

$$E_x = \frac{-1}{\left(\gamma^2 + \omega^2 \mu \varepsilon\right)} \left[\gamma \frac{\partial E_z}{\partial x} + j\omega\mu \frac{\partial H_z}{\partial y} \right]$$

$$E_y = \frac{1}{\left(\gamma^2 + \omega^2 \mu \varepsilon\right)} \left[-\gamma \frac{\partial E_z}{\partial y} + j\omega\mu \frac{\partial H_z}{\partial x} \right]$$

$$H_x = \frac{1}{\left(\gamma^2 + \omega^2 \mu \varepsilon\right)} \left[j\omega\varepsilon \frac{\partial E_z}{\partial y} - \gamma \frac{\partial H_z}{\partial x} \right]$$

$$H_y = \frac{-1}{\left(\gamma^2 + \omega^2 \mu \varepsilon\right)} \left[j\omega\varepsilon \frac{\partial E_z}{\partial x} + \gamma \frac{\partial H_z}{\partial y} \right]$$

Equação de onda:

$$\vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H} \qquad \vec{\nabla} \times \vec{H} = j\omega\varepsilon\vec{E}$$

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = -j\omega\mu\,\vec{\nabla} \times \vec{H} \quad = \quad \omega^2\mu\varepsilon\vec{E}$$

Usando a identidade vetorial $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{A}\right) = \nabla\left(\vec{\nabla} \cdot \vec{A}\right) - \nabla^2\vec{A}$:

$$\nabla\left(\vec{\nabla} \cdot \vec{E}\right) - \nabla^2\vec{E} = \omega^2\mu\varepsilon\vec{E}$$

Como o meio não tem cargas livres: $\vec{\nabla} \cdot \vec{D} = 0 \quad \Rightarrow \quad \vec{\nabla} \cdot \vec{E} = 0$

Portanto: $\nabla^2\vec{E} = -\omega^2\mu\varepsilon\vec{E}$

Considerando apenas a componente z: $\dfrac{\partial^2 E_z}{\partial x^2} + \dfrac{\partial^2 E_z}{\partial y^2} + \dfrac{\partial^2 E_z}{\partial z^2} + \omega^2\mu\varepsilon E_z = 0$

Portanto: $\nabla^2 \vec{E} = -\omega^2 \mu \varepsilon \vec{E}$

Considerando apenas a componente z: $$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + \omega^2 \mu \varepsilon E_z = 0$$

Analogamente, obtém-se: $$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} + \omega^2 \mu \varepsilon H_z = 0$$

Modo TE

Transverse electric (TE) waves, (also referred to as H-waves) are characterized by $E_z = 0$ and $H_z \neq 0$. Equations (3.5) then reduce to

$$H_x = \frac{-j\beta}{k_c^2} \frac{\partial H_z}{\partial x},$$

$$H_y = \frac{-j\beta}{k_c^2} \frac{\partial H_z}{\partial y},$$

$$E_x = \frac{-j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y},$$

$$E_y = \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x}.$$

In this case $k_c \neq 0$, and the propagation constant $\beta = \sqrt{k^2 - k_c^2}$ is generally a function of frequency and the geometry of the line or guide.

Equação de Helmholtz

$$\nabla \times \bar{E} = -j\omega\mu\bar{H},$$
$$\nabla \times \bar{H} = j\omega\epsilon\bar{E},$$

$$\nabla \times \nabla \times \bar{E} = -j\omega\mu\nabla \times \bar{H} = \omega^2\mu\epsilon\bar{E}$$

$$\nabla \times \hat{\nabla} \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2\bar{A}$$

$$\nabla^2\bar{E} + \omega^2\mu\epsilon\bar{E} = 0.$$

$\nabla \cdot \bar{E} = 0$ in a source-free region.

$$\nabla^2\bar{H} + \omega^2\mu\epsilon\bar{H} = 0.$$

$k = \omega\sqrt{\mu\epsilon}$ is defined and called the *propagation constant*

Modo TE

Transverse electric (TE) waves, (also referred to as H-waves) are characterized by $E_z = 0$ and $H_z \neq 0$. Equations (3.5) then reduce to

$$H_x = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial x},$$

$$H_y = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial y},$$

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y},$$

$$E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}.$$

In this case $k_c \neq 0$, and the propagation constant $\beta = \sqrt{k^2 - k_c^2}$ is generally a function of frequency and the geometry of the line or guide. To apply (3.19), one must first find H_z from the Helmholtz wave equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right) H_z = 0,$$

which, since $H_z(x, y, z) = h_z(x, y)e^{-j\beta z}$, can be reduced to a two-dimensional wave equation for h_z:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) h_z = 0,$$

since $k_c^2 = k^2 - \beta^2$. This equation must be solved subject to the boundary conditions of the specific guide geometry.

The TE wave impedance can be found as

$$Z_{\mathrm{TE}} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta},$$

which is seen to be frequency dependent. TE waves can be supported inside closed conductors, as well as between two or more conductors.

Modo TM

Transverse magnetic (TM) waves (also referred to as E-waves) are characterized by $E_z \neq 0$ and $H_z = 0$. Equations (3.5) then reduce to

$$H_x = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial y},$$

$$H_y = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial x},$$

$$E_x = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial x},$$

$$E_y = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial y}.$$

As in the TE case, $k_c \neq 0$, and the propagation constant $\beta = \sqrt{k^2 - k_c^2}$ is a function of frequency and the geometry of the line or guide. E_z is found from the Helmholtz wave equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) E_z = 0, \qquad (3.24)$$

which, since $E_z(x, y, z) = e_z(x, y)e^{-j\beta z}$, can be reduced to a two-dimensional wave equation for e_z:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) e_z = 0, \qquad (3.25)$$

since $k_c^2 = k^2 - \beta^2$. This equation must be solved subject to the boundary conditions of the specific guide geometry.

The TM wave impedance can be found as

$$Z_{\text{TM}} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\beta}{\omega\epsilon} = \frac{\beta\eta}{k}, \qquad (3.26)$$

which is frequency dependent. As for TE waves, TM waves can be supported inside closed conductors, as well as between two or more conductors.

Equações de Onda

$$\left.\begin{array}{c}\left(\nabla_t^2 + k_1^2\right) H_z = 0 \\ E_z = 0\end{array}\right\}$$ Conjunto de soluções **TE**

$$\left.\begin{array}{c}\left(\nabla_t^2 + k_1^2\right) E_z = 0 \\ H_z = 0\end{array}\right\}$$ Conjunto de soluções **TM**

nas quais $\nabla_t^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$

The procedure for analyzing TE and TM waveguides can be summarized as follows:

1. Solve the reduced Helmholtz equation, (3.21) or (3.25), for h_z or e_z. The solution will contain several unknown constants and the unknown cutoff wave number, k_c.
2. Use (3.19) or (3.23) to find the transverse fields from h_z or e_z.
3. Apply the boundary conditions to the appropriate field components to find the unknown constants and k_c.
4. The propagation constant is given by (3.6) and the wave impedance by (3.22) or (3.26).

$$\bar{E}(x, y, z) = [\bar{e}(x, y) + \hat{z}e_z(x, y)]e^{-j\beta z},$$

$$\bar{H}(x, y, z) = [\bar{h}(x, y) + \hat{z}h_z(x, y)]e^{-j\beta z},$$

Equações de Maxwell

$$\nabla \times \bar{E} = -j\omega\mu\bar{H},$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E}.$$

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x,$$

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y,$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z,$$

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\epsilon E_x,$$

$$-j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y,$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z.$$

$$H_x = \frac{j}{k_c^2}\left(\omega\epsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right),$$

$$H_y = \frac{-j}{k_c^2}\left(\omega\epsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right),$$

$$E_x = \frac{-j}{k_c^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right),$$

$$E_y = \frac{j}{k_c^2}\left(-\beta\frac{\partial E_z}{\partial y} + \omega\mu\frac{\partial H_z}{\partial x}\right),$$

$$k_c^2 = k^2 - \beta^2$$

$$k = \omega\sqrt{\mu\epsilon} = 2\pi/\lambda$$