Monitoria - 09/06

4.1

3. *a*) Calcule $\lim_{x \to +\infty} \dfrac{x^3 + 3x - 1}{2x^3 - 6x + 1}$

b) Mostre que existe $r > 0$ tal que

$$x > r \Rightarrow \frac{1}{4} < \frac{x^3 + 3x - 1}{2x^3 - 6x + 1} < \frac{3}{4}$$

$f(x)$ =

a) $\lim_{x \to +\infty} \dfrac{(x^3 + 3x - 1)}{2x^3 - 6x + 1} = \lim_{x \to +\infty} \dfrac{1 + \frac{3}{x^2} - \frac{1}{x^3}}{2 - \frac{6}{x^2} + \frac{1}{x^3}} = \dfrac{1}{2}$

b) $\forall \epsilon > 0, \exists r > 0$ tq

$x > r \Rightarrow |f(x) - L| < \epsilon \Rightarrow L - \epsilon < f(x) < L + \epsilon$

$1/4 = L - \epsilon$, $L + \epsilon = 1/4$

Para $\epsilon = \frac{1}{4}$, existe $r > 0$ tal que

$x > r$, então $\frac{1}{4} < f(x) < \frac{3}{4}$.

6.3

$$\lim_{u\to 0}(1+u)^{1/u}=e$$

2. Seja $a > 0$, $a \neq 1$. Mostre que

$$\boxed{\lim_{h\to 0}\frac{a^h-1}{h}=\ln\ a.}$$

$\ln(a)$

$$\lim_{h\to 0}\frac{a^h-1}{h}\overset{(*)}{=}\lim_{u\to 0}\frac{u\ln(a)}{\ln(1+u)}=\lim_{u\to 0}\frac{\ln(a)}{\ln\left[1+u\right]^{1/u}}=\frac{\ln(a)}{\ln(e)}\ //$$

$$\underbrace{u=a^h-1}\Rightarrow a^h=1+u\Rightarrow h\ln(a)\overset{(*)}{=}\ln(1+u)$$

$h\to 0,\ u\to 0$

6.3

ex. anterior $= \ln(e) = 1$

3. Calcule.

a) $\lim_{x \to 0} \frac{e^{2x} - 1}{x} = \lim_{u \to 0} \left(\frac{e^{u} - 1}{u} \right) \cdot 2 = 2.$

$u = 2x$

$x \to 0, \ u \to 0$

d) $\lim_{x \to 0^+} \frac{3^x - 1}{x^2}$

7.2

• g é contínua em $p = 1$?

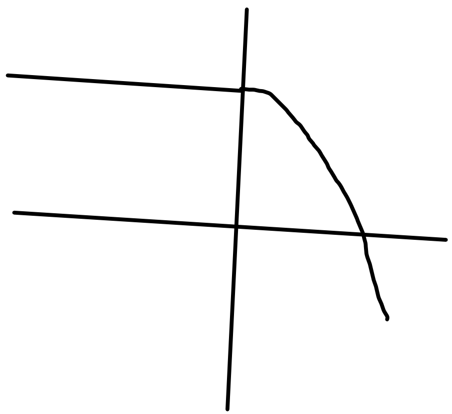
15. Seja $g(x) = \begin{cases} x^2 + 2 & \text{se } x < 1 \\ 2x + 1 & \text{se } x \geqslant 1 \end{cases}$

a) Mostre que g é derivável em $p = 1$ e calcule $g'(1)$.

b) Esboce o gráfico de g.

• 1) $1 \in dom(g)$ ✓

2) $\lim_{x \to 1} g(x) = g(1)$



2) $\lim_{x \to 1} g(x)$

$$\lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} (2x+1) = 3$$

$$\lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} (x^2+2) = 3$$

$=$

$\therefore$ O limite $\lim_{x \to 1} g(x)$ existe e dá 3.

$$g(1) = 3 = \lim_{x \to 1} g(x)$$

g é contínua.

a) $\lim\limits_{x \to 1} \dfrac{g(x) - g(1)}{x-1}$

$$\lim_{x \to 1^+} \frac{g(x)-g(1)}{x-1} = \lim_{x \to 1^+} \frac{(2x+1)-3}{x-1} = \lim_{x \to 1^+} \frac{\overbrace{2x-2}^{2}}{x-1} = 2.$$

$$\lim_{x \to 1^-} \frac{g(x)-g(1)}{x-1} = \lim_{x \to 1^-} \frac{(x^2+2)-3}{(x-1)} = \lim_{x \to 1^-} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 2$$

$\therefore$ O limite existe e dá 2 : $g'(1) = 2$

7.11

28. A função diferenciável $y = f(x)$ é tal que, para todo $x \in D_f$, o ponto $(x, f(x))$ é solução da equação $xy^3 + 2xy^2 + x = 4$. Sabe-se que $f(1) = 1$. Calcule $f'(1)$.

7.6

2. Seja $f(x) = \begin{cases} x^2 & \text{se } x \leqslant 0 \\ -x^2 & \text{se } x > 0 \end{cases}$.

a) f é derivável em 0? Justifique.
b) f é contínua em 0? Justifique.

a) $$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$

$$\lim_{x \to 0^+} \frac{-x^2 - 0}{x - 0} = \lim_{x \to 0^+} (-x) = 0$$

$$\lim_{x \to 0^-} \frac{(x^2 - 0)}{x - 0} = \lim_{x \to 0^-} (x) = 0$$

$= \therefore$

O limite existe, $f'(0)=0$

b) Como f é derivável em 0,

f é contínua em 0.

$$\boxed{\operatorname{arc\,sen}' x = \frac{1}{\sqrt{1-x^2}},\ -1 < x < 1.}$$

$$\boxed{\operatorname{arctg}' x = \frac{1}{1+x^2},\quad x \in \mathbb{R}}$$

$$\boxed{\arccos' x = \frac{-1}{\sqrt{1-x^2}},\quad -1 < x < 1}$$

8.2

1. Determine a derivada.

$$h)\ y = \frac{\operatorname{sen} 3x}{\operatorname{arc\,tg} 4x}$$

4.2

4. Calcule.

i) $\lim\limits_{x \to \frac{1}{2}^+} \dfrac{3x + 1}{4x^2 - 1}$

$m)\ \lim_{x \to 3^+} \frac{x^2 - 3x}{x^2 - 6x + 9} = \lim_{x \to 3^+} \frac{x}{(x-3)} = \lim_{x \to 3^+} \overset{\to 3}{x} \cdot \overset{\to +\infty}{\frac{1}{(x-3)}} = +\infty$.

$\forall x \neq 3:$

$(x^2 - 3x) = x(x-3)$

$(x^2 - 6x + 9) = (x-3)^2$

s) $\lim\limits_{x \to 0^+} \frac{\operatorname{sen} x}{x^3 - x^2}$

$$\frac{\operatorname{sen}(x)}{x^3 - x^2} = \left(\frac{\operatorname{sen}(x)}{x}\right) \cdot \frac{1}{(x^2 - x)} = \frac{1}{x} \cdot \frac{1}{(x-1)} \cdot \frac{\operatorname{sen}(x)}{x}$$

$$\lim_{x \to 0^+} \frac{1}{(x-1)} \cdot \frac{\operatorname{sen}(x)}{x} = -1$$

$$\lim_{x \to 0^+} \frac{\operatorname{sen}(x)}{x^3 - x^2} = \lim_{x \to 0^+} \left[\left(\overbrace{\frac{1}{(x-1)} \cdot \frac{\operatorname{sen}(x)}{x}}^{-1} \right) \cdot \overbrace{\frac{1}{x}}^{+\infty} \right] = -\infty.$$

$$\lim_{x \to 1} \frac{\operatorname{sen}(x^2-1)}{(x-1)}$$

$$\forall x \neq 1, \quad \frac{\operatorname{sen}(x^2-1)}{(x-1)} = \frac{\operatorname{sen}(x^2-1)}{(x^2-1)} \cdot (x+1)$$

$$\lim_{x \to 1} \frac{\operatorname{sen}(x^2-1)}{(x^2-1)} = \lim_{u \to 0} \frac{\operatorname{sen}(u)}{u} = 1$$

$u = (x^2-1)$

$x \to 1, \quad u \to 0$

$$\therefore \lim_{x \to 1} \left[\frac{\sin(x^2-1)}{(x-1)} \right] = 2.$$

$$f(x) = e^{\arctan(x^4)} \cdot \sin(x^2) + \arcsin(\ln(x))$$

$$f'(x) = \left[e^{\arctan(x^4)} \right]' \cdot \sin(x^2) + e^{\arctan(x^4)} \cdot (\sin(x^2))'$$

$$+ \frac{1}{\sqrt{1-\ln(x)^2}} \cdot \frac{1}{x}$$

$$\arctan(x)' = \frac{1}{1+x^2}$$

$$\left(e^{\arctan(x^4)} \right)' = e^{\arctan(x^4)} \cdot (\arctan(x^4))' =$$

$$= e^{\arctan(x^4)} \cdot \frac{1}{1+(x^4)^2} \cdot (4x^3)$$

$$(\sin(x^2))' = \cos(x^2) \cdot 2x$$

$$f'(x) = e^{\arctan(x^4)} \cdot \frac{1}{1+x^8} \cdot 4x^3 \cdot \sin(x^2) + e^{\arctan(x^4)} \cdot \cos(x^2) \cdot 2x$$

$$+ \frac{1}{\sqrt{1-\ln(x)^2}} \cdot \frac{1}{x}$$