Monitoria - 24/05

- Limite fundamental
- Sequências
- Limites.

Limite Fundamental

$$\lim_{x \to 0} \frac{\operatorname{sen}(x)}{x} = 1$$

Exercícios (3.8)

1. Calcule

$3x = u$

c) $\lim_{x \to 0} \frac{\text{sen}(3x)}{x} = \lim_{x \to 0} \left[\frac{\text{sen}(3x)}{3x} \cdot 3 \right] = 3 \lim_{u \to 0} \frac{\text{sen}(u)}{u} = 3$

$u = 3x$

$x \to 0, \ u \to 0$

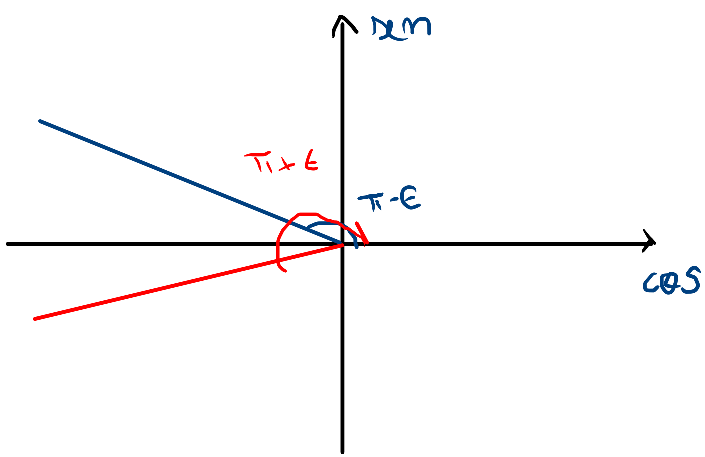
$d)$ $\lim_{x \to \pi} \frac{\text{sen } x}{x - \pi} = \lim_{u \to 0} \frac{\text{sen}(u+\pi)}{u} = \lim_{u \to 0} -\frac{\text{sen}(u)}{u} = -1$.

$u = x - \pi \Rightarrow x = u + \pi$

$x \to \pi,\ u \to 0$

$$\boxed{\begin{aligned} \cos(a \pm b) &= \cos(a)\cos(b) \mp \text{sen}(a)\,\text{sen}(b) \\ \text{sen}(a \pm b) &= \text{sen}(a)\cos(b) \pm \text{sen}(b)\cos(a) \end{aligned}}$$

$$\text{sen}(u+\pi) = \text{sen}(u)\cdot(-1) + \cos(u)\cdot 0 = -\text{sen}(u)$$



$x < \pi$: $\text{sen}(x) > 0$

$x - \pi < 0$

$\frac{\text{sen}(x)}{x - \pi} < 0$

$x > \pi$: $\text{sen}(x) < 0$

$x - \pi > 0$

g) $\lim\limits_{x \to 0} \dfrac{\operatorname{tg} 3x}{\operatorname{sen} 4x}$

$$\frac{\operatorname{tg}(3x)}{\operatorname{sen}(4x)} = \frac{\operatorname{sen}(3x)}{3x} \cdot \frac{1}{\cos(3x)} \cdot \frac{4x}{\operatorname{sen}(4x)} \cdot \frac{3}{4}$$

$$\lim_{x \to 0} \frac{4x}{\operatorname{sen}(4x)} = \lim_{u \to 0} \frac{u}{\operatorname{sen}(u)} = \lim_{u \to 0} \frac{1}{\left(\frac{\operatorname{sen}(u)}{u}\right)} = 1$$

$u = 4x$

$x \to 0,\ u \to 0$

$$\lim_{x \to 0} \frac{\operatorname{tg} 3x}{\operatorname{sen}(4x)} = \frac{3}{4}.$$

j) $\lim_{x \to 0} x \operatorname{sen} \frac{1}{x} = 0$ $\left(\operatorname{sen}\left(\frac{1}{x}\right) \text{ é limitada, } x \to 0\right)$

Propriedade: Se f, g funções, tq $\forall x \in \mathbb{R}$, $|g(x)| \leq M$, para $M > 0$ (ou seja, g é limitada) e $\lim_{x \to p} f(x) = 0$, então

$$\lim_{x \to p} f(x) \cdot g(x) = 0$$

$$o)\ \lim_{x\to 0} \frac{x + \operatorname{sen} x}{x^2 - \operatorname{sen} x} = \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}} \left[\frac{1 + \frac{\operatorname{sen}(x)}{x}}{x - \frac{\operatorname{sen}(x)}{x}} \right] =$$

$$= \lim_{x\to 0} \frac{\left(1 + \operatorname{sen}(x)/x\right)}{\left(x - \operatorname{sen}(x)/x\right)} = -2.$$

1

-1

Exercícios (4.2)

1. Calcule

b) $\lim\limits_{x \to +\infty} \dfrac{x + \sqrt{x+3}}{2x-1} = \lim\limits_{x \to +\infty} \dfrac{\cancel{x}}{\cancel{x}} \left[\dfrac{1 + \sqrt{\frac{1}{x} + \frac{3}{x^2}}}{2 - \frac{1}{x}} \right] = \dfrac{1}{2}.$

d) $\lim\limits_{x \to +\infty} (x - \sqrt{3x^3 + 2}) = \lim\limits_{x \to +\infty} \overset{+\infty}{\cancel{x}} \cdot \left(1 - \sqrt{3x + \frac{2}{x^2}}\right) = -\infty$

$-\infty$

g) $\lim\limits_{x \to +\infty} \left(\sqrt{x+\sqrt{x}} - \sqrt{x-1}\right)$

$$\sqrt{x+\sqrt{x}} - \sqrt{x-1} = \left(\sqrt{x+\sqrt{x}} - \sqrt{x-1}\right) \cdot \frac{\left(\sqrt{x+\sqrt{x}} + \sqrt{x-1}\right)}{\left(\sqrt{x+\sqrt{x}} + \sqrt{x-1}\right)}$$

$$= \frac{\overbrace{(x+\sqrt{x}) - (x-1)}^{\sqrt{x}+1}}{\sqrt{x+\sqrt{x}} + \sqrt{x-1}} = \frac{\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}} \frac{\left(1 + \frac{1}{\sqrt{x}}\right)}{\left(\sqrt{1+\frac{1}{\sqrt{x}}} + \sqrt{1-\frac{1}{x}}\right)}$$

$$\lim_{x \to +\infty} \frac{\left(1 + \cancelto{0}{\frac{1}{\sqrt{x}}}\right)}{\sqrt{1+\cancelto{0}{\frac{1}{\sqrt{x}}}} + \sqrt{1-\cancelto{0}{\frac{1}{x}}}} = \frac{1}{2}.$$

6. Dê exemplo de funções f e g tais que $\lim_{x \to +\infty} f(x) = +\infty$, $\lim_{x \to +\infty} g(x) = +\infty$ e $\lim_{x \to +\infty} [f(x) - g(x)] \neq 0$.

(ou seja, $(+\infty)+(-\infty)$ é indeterminado)

- $f(x) = 2x$, $g(x) = x$

$$\lim_{x \to +\infty} [2x - x] = +\infty$$

- $f(x) = x$, $g(x) = 2x$

$$\lim_{x \to +\infty} [x - 2x] = -\infty$$

c real

- $f(x) = x + c$, $g(x) = x$

$$\lim_{x \to +\infty} \left((x+c) - x \right) = c.$$

7. Dê exemplo de funções f e g tais que $\lim_{x \to +\infty} f(x) = +\infty$, $\lim_{x \to +\infty} g(x) = +\infty$ e $\lim_{x \to +\infty} \frac{f(x)}{g(x)} \neq 1$.

($\frac{+\infty}{+\infty}$ é indeterminado)

- $f(x) = c \cdot x$
  $g(x) = x$

$$\frac{f(x)}{g(x)} = c$$

- $f(x) = x^2$
  $g(x) = x$

$$\frac{f(x)}{g(x)} = x$$

- $f(x) = x$
  $g(x) = x^2$

$$\frac{f(x)}{g(x)} = \frac{1}{x}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$2x^2 + x$

$$f(x) = 5 \quad \forall x \in \mathbb{R}$$

$$f(x+h) = 5 \quad \forall x, h \in \mathbb{R}$$

$$\lim_{h \to 0} \frac{\overbrace{5 - 5}^{0}}{h} = 0$$

$$\lim_{h\to 0} \frac{(2(x+h)^2+(x+h))-2x^2-x}{h} =$$

$$\lim_{h\to 0} \frac{\cancel{2x^2}+4xh+2h^2+\cancel{x}+h-\cancel{2x^2}-\cancel{x}}{h} =$$

$$\lim_{h\to 0} \frac{(4x\cancel{h}+2h^{\cancel{2}}+\cancel{h})}{\cancel{h}} = \lim_{h\to 0}(4x+2h+1) =$$

$$(4x+1)$$

(3.4)

1. Calcule, caso exista.

$$i)\ \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} \text{ em que } f(x) = \begin{cases} x^2 & \text{se } x \leqslant 1 \\ 2x - 1 & \text{se } x > 1 \end{cases}$$

4- Calcule

g) $\lim_{x \to 0^+} \dfrac{3}{x^2 - x}$

m) $\lim_{x \to 3^+} \dfrac{x^2 - 3x}{x^2 - 6x + 9}$