9.5. 12.

Esboce o gráfico de $f(x)=\dfrac{x^3}{x^2-1}$

$$\left\{ x^2-1=0 \iff x=\pm 1 \right.$$

Dom $f = \mathbb{R} \setminus \{-1,1\}$.

$$f'(x)=\frac{3x^2(x^2-1)-x^3(2x)}{(x^2-1)^2}=\frac{3x^4-3x^2-2x^4}{(x^2-1)^2}$$

$$=\frac{x^4-3x^2}{(x^2-1)^2}$$

$$f''(x)=\frac{(4x^3-6x)(x^2-1)^2-(x^4-3x^2)2(x^2-1).(2x)}{(x^2-1)^4}$$

$$=\frac{[(4x^3-6x)(x^2-1)-(x^4-3x^2)2.2x]}{(x^2-1)^3}\,\frac{\cancel{(x^2-1)}}{\cancel{(x^2-1)}}$$

$$=\frac{\cancel{4x^5}-4x^3-6x^3+6x-\cancel{4x^5}+12x^3}{(x^2-1)^3}=\frac{2x^3+6x}{(x^2-1)^3}$$

$$f'(x) = \frac{x^4 - 3x^2}{(x^2-1)^2}$$

$$f''(x) = \frac{2x^3 + 6x}{(x^2-1)^3}$$

$x^4 - 3x^2 = 0 \iff x^2(x^2-3) = 0$

$\iff x = 0$

$x = \pm\sqrt{3}$

$2x^3 + 6x = 0 \iff 2x(x^2+3) = 0$

$\iff 2x = 0$

$x^2 - 1 = 0 \iff x = \pm 1$

		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$	
x^2	+	+	+	+	+	0	+	+	+	+	+
x^2-3	+	0	−	−	−	−	−	−	−	0	+
$f'(x)$	+	0	−	X	−	0	−	X	−	0	+
	↗	→	↘		↘	→	↘		↘	0	↗
$2x(x^2+3)$	−	−	−	−	−	0	+	+	+	+	+
$(x^2-1)^3$	+	+	+	0	−	−	−	0	+	+	+
$f''(x)$	−	−	−	X	+	0	−	X	+	+	+

assíntota p/ $+\infty$

$$a = \lim_{x\to+\infty} \frac{\left(\frac{x^3}{x^2-1}\right)}{x} = \lim_{x\to+\infty} \frac{x^3}{x\cdot(x^2-1)}$$

$$= \lim_{x\to+\infty} \frac{\cancel{x^3}\cdot 1}{\cancel{x\cdot x^2}\left(1-\frac{1}{x^2}\right)} = 1$$

$$b = \lim_{x\to+\infty} f(x) - ax = \lim_{x\to+\infty} \frac{x^3}{x^2-1} - x$$

$$= \lim_{x\to+\infty} \frac{\cancel{x^3} - \cancel{x^3} + x}{x^2-1} = \lim_{x\to+\infty} \frac{\cancel{x}\cdot 1}{\underbrace{\cancel{x}x}_{+\infty}\left(1-\frac{1}{x^2}\right)} = 0$$

assíntota p/ $+\infty$

$$\boxed{y = x}$$

assíntota p/ $-\infty$

$$a = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{x^3}{x(x^2-1)} = \lim_{x \to -\infty} \frac{\cancel{x^3}}{\cancel{x^3}} \cdot \frac{1}{\left(1 - \frac{1}{x^2}\right)} = 1$$

$$b = \lim_{x \to -\infty} f(x) - x = \lim_{x \to -\infty} \frac{x^3}{x^2-1} - x = \lim_{x \to -\infty} \frac{\cancel{x^3} - \cancel{x^3} + x}{x^2 - 1}$$

$$= \lim_{x \to -\infty} \frac{\cancel{x}}{\cancel{x} \cdot x} \cdot \frac{1}{\left(1 - \frac{1}{x^2}\right)} = 0$$

$x \to -\infty$

assíntota p/ $-\infty$

$$\boxed{y = x}$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{x^3}{x^2-1} = \lim_{x \to -1^-} \frac{\overset{\to -1}{x^3}}{\underset{0^-}{(x+1)}\underset{-2}{(x-1)}} = -\infty$$

$x < -1$

$x + 1 < 0$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \frac{\overset{\to -1}{x^3}}{\underset{0^+}{(x+1)}\underset{-2}{(x-1)}} = +\infty$$

$x > -1$

$x + 1 > 0$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{\overset{\to 1}{x^3}}{\underset{2}{(x+1)}\underset{0^-}{(x-1)}} = -\infty$$

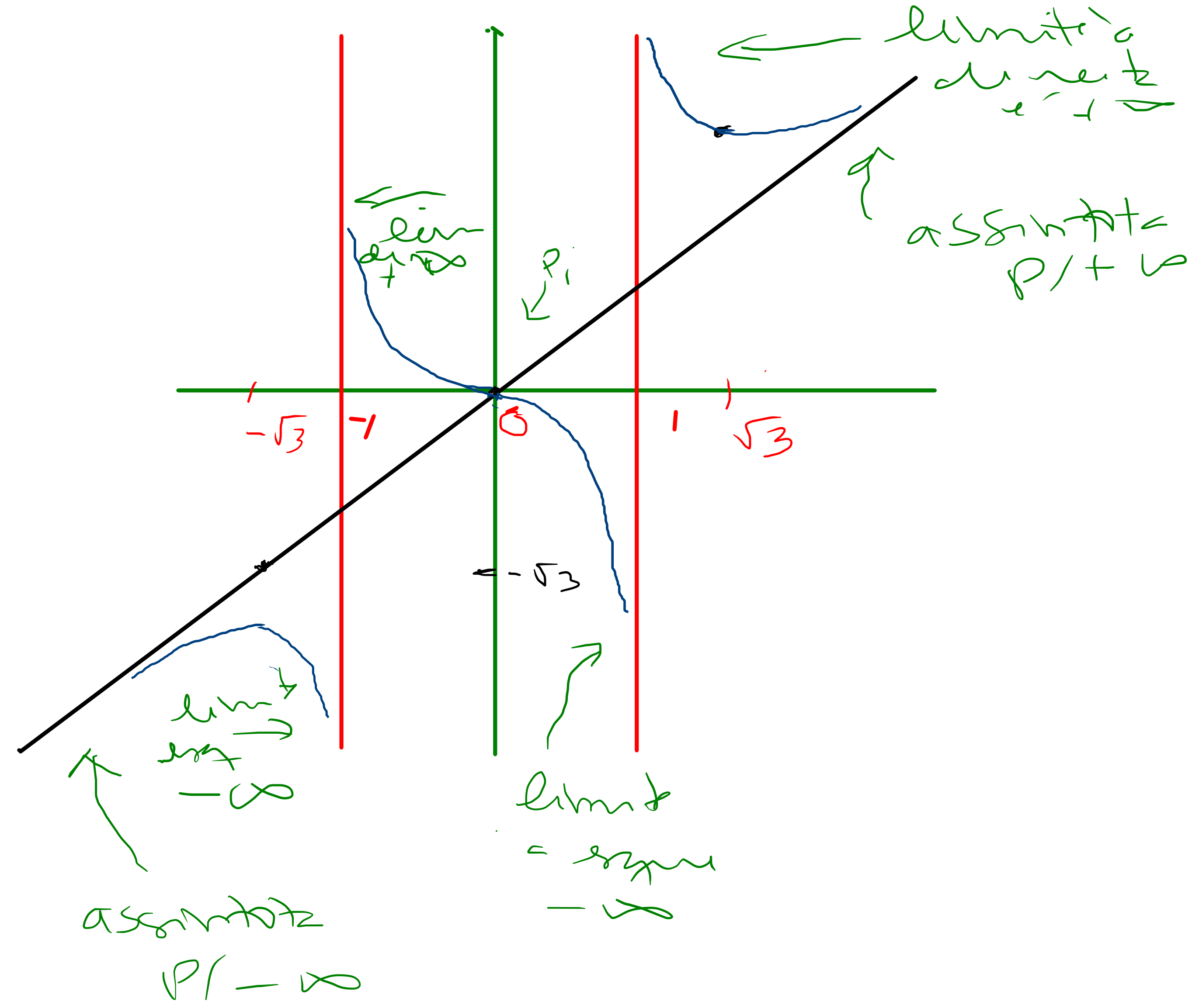
$x < 1$

$x - 1 < 0$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{\overset{\to 1}{x^3}}{\underset{2}{(x+1)}\underset{0^+}{(x-1)}} = +\infty$$

$x > 1$

$x - 1 > 0$



$$f(-\sqrt{3}) = \frac{(-\sqrt{3})^3}{3-1} = -\frac{3\sqrt{3}}{2}$$

$$f(0) = 0$$

$$f(\sqrt{3}) = \frac{(\sqrt{3})^3}{3-1} = \frac{3\sqrt{3}}{2}$$

$$-\sqrt{3} > -\frac{3\sqrt{3}}{2}$$

$$\Leftrightarrow -1 > -\frac{3}{2}$$