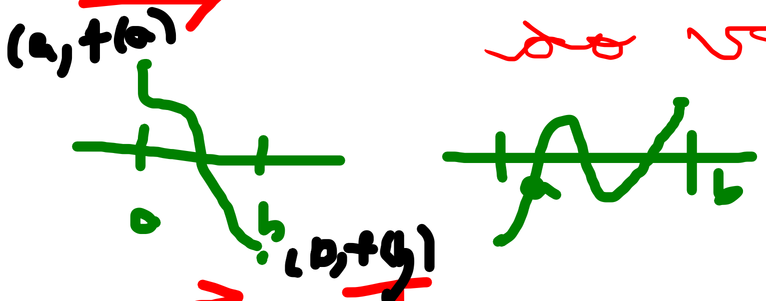
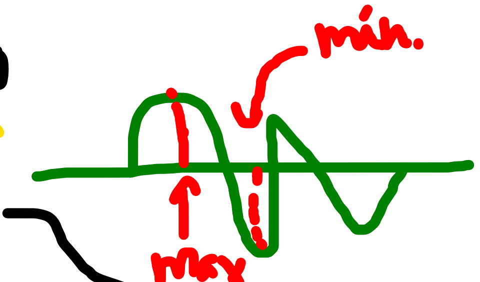
→ TVI - Teorema do valor intermediário



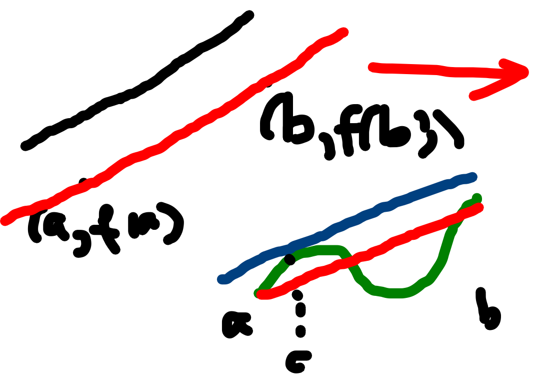
f contínua em $[a,b]$ se $\boxed{f(a)\cdot f(b) < 0}$ então $\exists\, c \in\,]a,b[$ tal que $f(c) = 0$

→ Teorema de Weierstrass

Se f é contínua em $[a,b]$ então f possui ponto de máximo e mínimo em

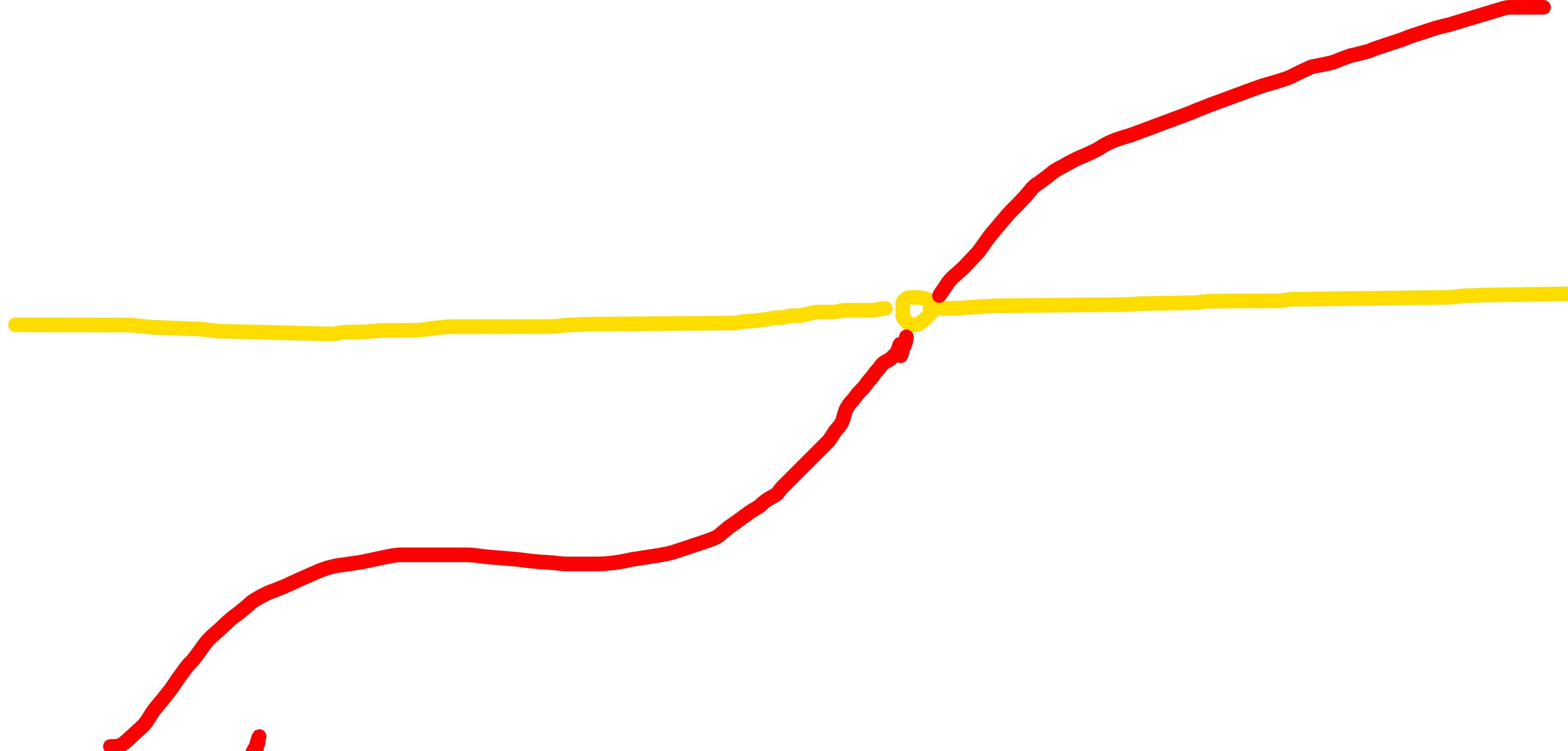


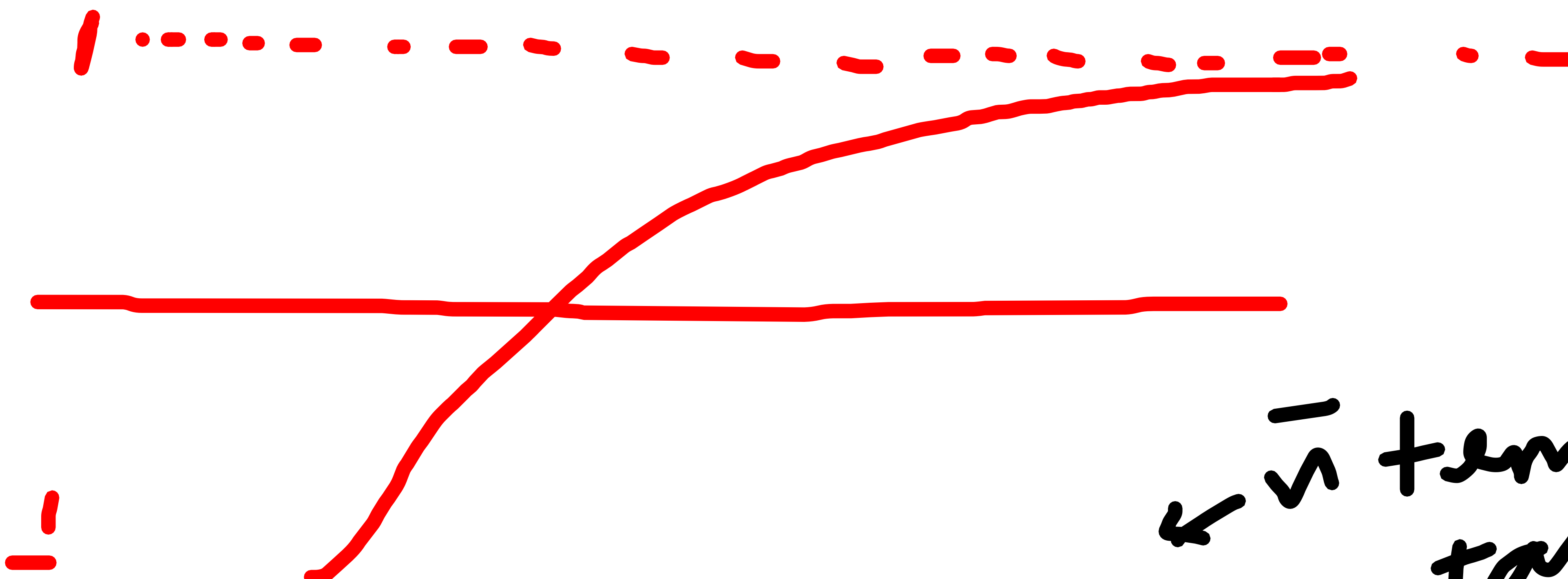
→ TVM = Teorema do valor médio

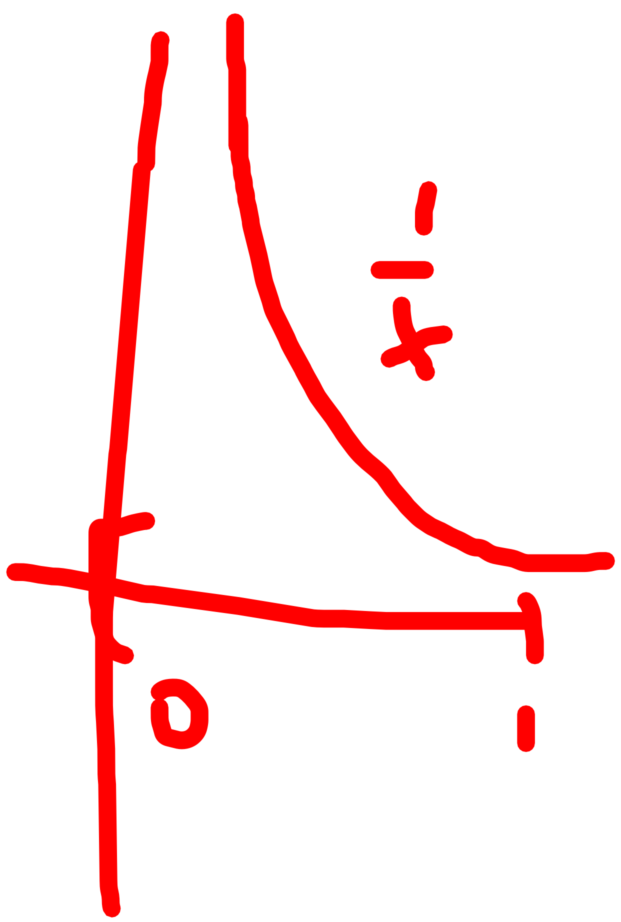


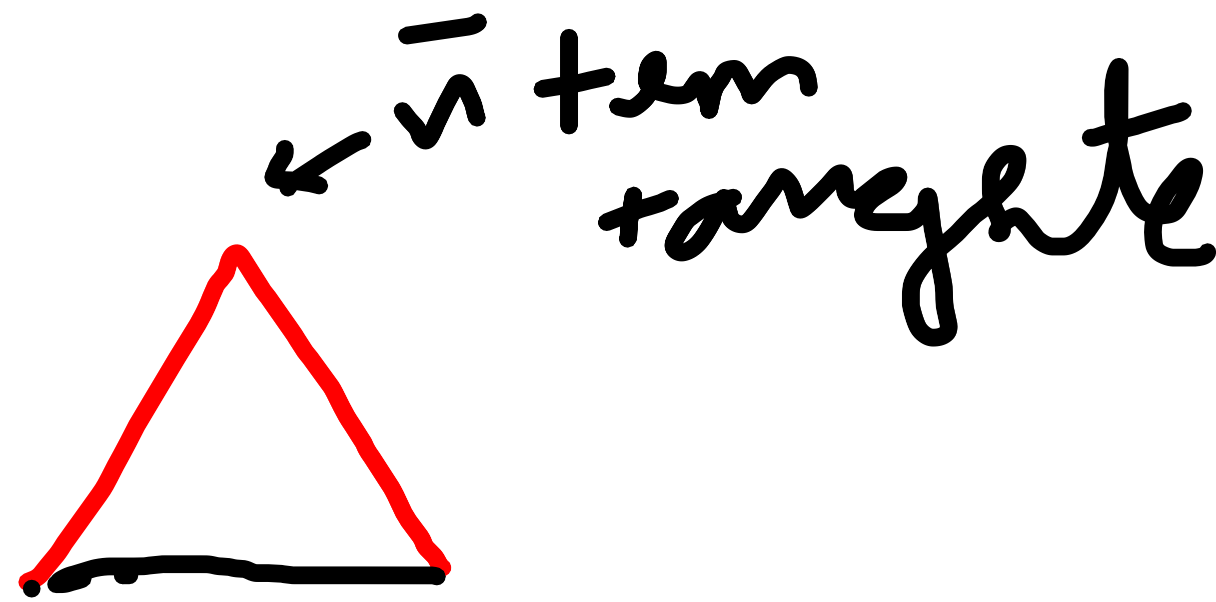
f cont. em $[a,b]$
f derivável em $]a,b[$
$\exists\, c \in\,]a,b[$ $\quad f'(c) = \dfrac{f(b) - f(a)}{b-a}$

→ Regra de L'Hospital

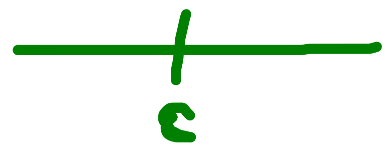



1


1/x
0


ñ tem
tangente

Esboço do TVI



$\exists c$

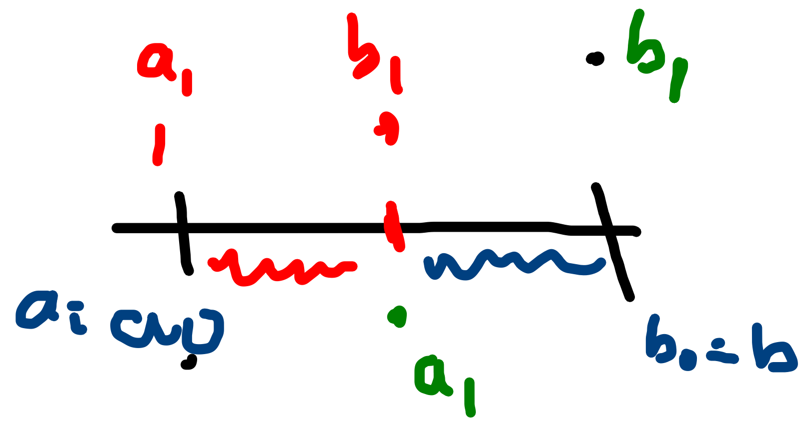
$a_n \le c \le b_n \ \forall n.$

$f(c)=0$

$f(c)$

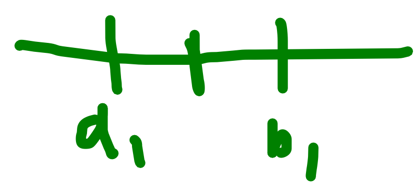
$\exists! c$

a_n b_n

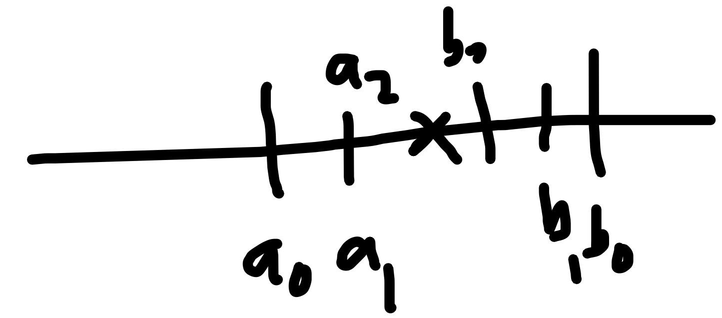


$b_1 - a_1 = \frac{1}{2}(b_0 - a_0)$

$f(a_1).f(b_1) < 0$



Intervalos Encaixantes



$b_n - a_n \to 0$

$$\begin{cases} b_n - a_n = \frac{1}{2^n}(b-a) \\ f(b_n).f(a_n) < 0 \end{cases}$$

sup A existe

$A \cup B$

$\sup A \cup B = \max\{\sup A, \sup B\}$

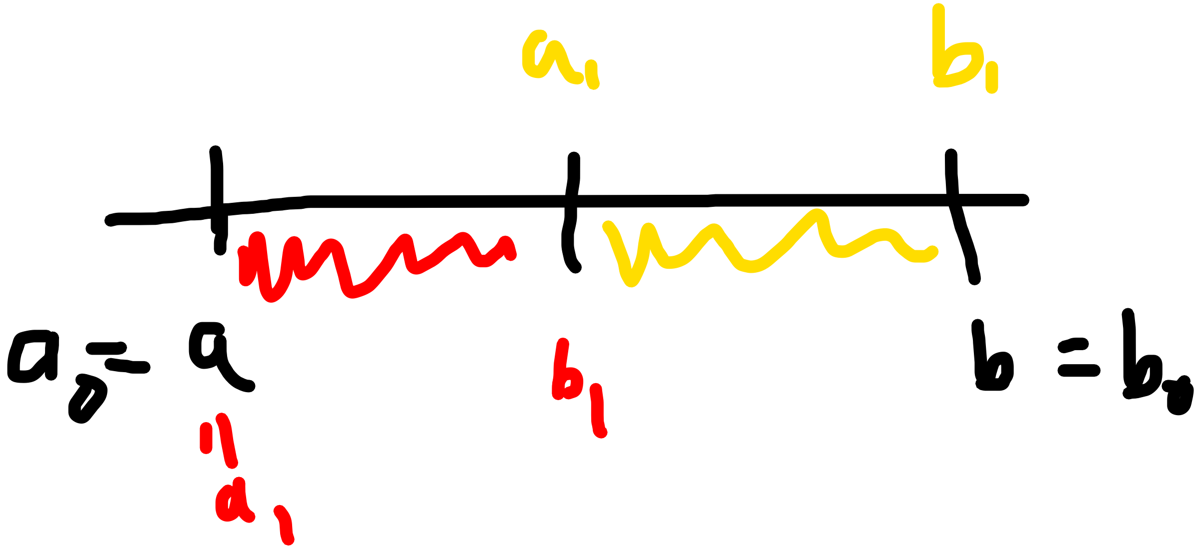
Prop. de $\mathbb{R}$

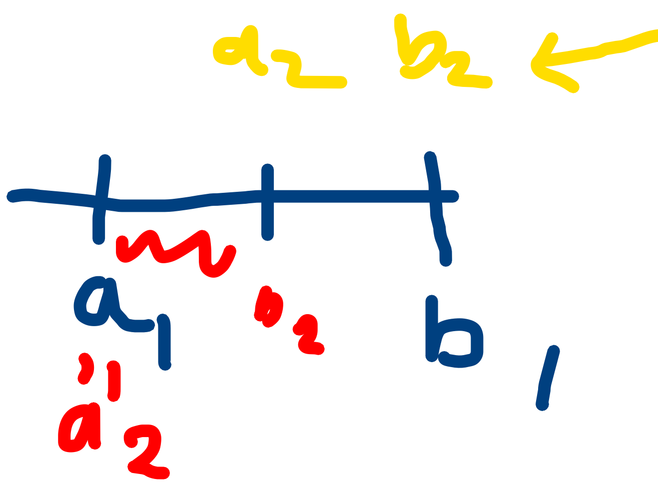
Todo $A \neq \emptyset$ limitado possui supremo.

1) $a \leq \sup A \ \forall a \in A$

2) se $a \leq b \ \forall a \in A$ então $b \geq \sup A$

Teor: Se f é contínua em $[a,b]$ então f é limitada.





$\exists c$.

$a_n \le c \le b_n$

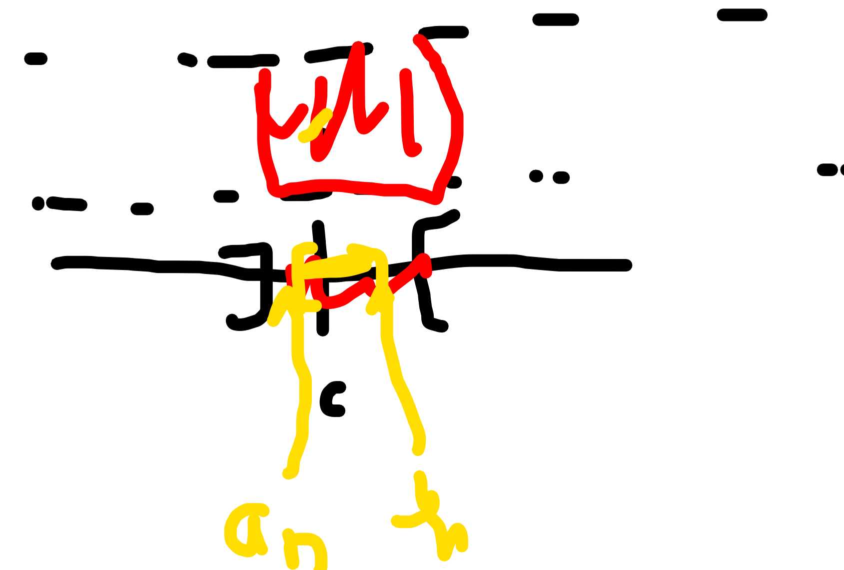
$\varepsilon = 1$

$\exists \delta$

$|x-c| < \delta$

$\Rightarrow |f(x) - f(c)|$

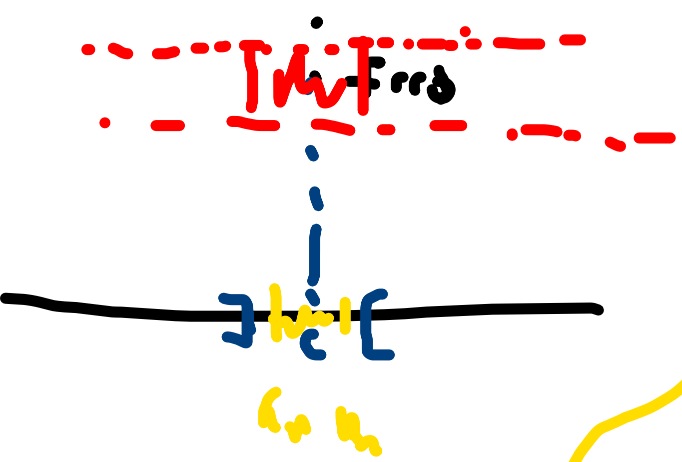
< 1



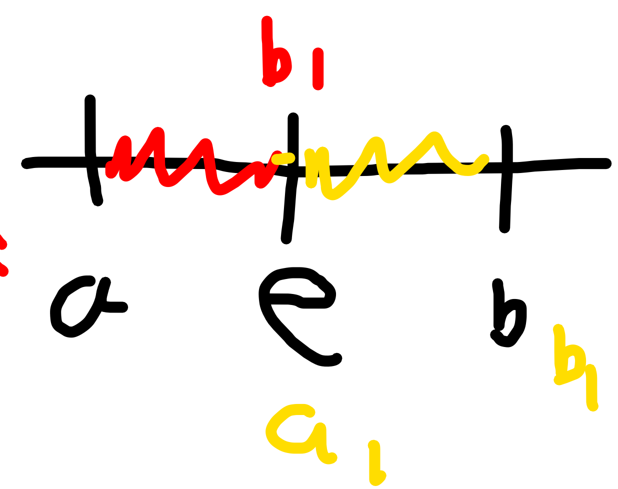
Teor. de Weierstrass

f é limitada

$\sup\{f(x) : x \in [a, b]\}$



$f(c) < \sup\{$



$a_1 =$

$\sup\{f(x) : x \in [a_0, b_0]\}$

$= \sup\{f(x) : x \in [a_0, e]\}$

ou $\sup\{f(x) : x \in [e, b_0]\}$

$\exists c \quad a_n \leq c \leq b_n$

$f(c) \leq \sup\{f(x) : x \in [a,b]\}$

Ex. 36 4b)

$$\lim_{x\to 0} x \cdot \operatorname{sen}\frac{1}{x} = 0$$

limitada

$$\frac{\operatorname{sen}\left(\frac{1}{x}\right)}{\frac{1}{x}}$$

$f(x) \xrightarrow{x\to a} 0$

$|g(x)| \leq M$, $\forall x$ próximo de a

$$\lim_{u\to 0} \frac{\operatorname{sen} u}{u} = 1$$

$$-M|f(x)| \leq |f(x)\cdot g(x)| \leq |f(x)|\cdot M$$

$|f(x)g(x)| \to 0 \Rightarrow f(x)g(x) \to 0$

Ex 3.5.3a)

f definida em $\mathbb{R}$ $\quad p \in \mathbb{R}$

suponha

$$\boxed{\lim_{x \to p} \frac{f(x) - f(p)}{x - p} = L}$$

a) $\displaystyle \lim_{h \to 0} \frac{f(p+h) - f(p)}{h} = L$

$\boxed{x - p = h}$ $\quad x \to p \Rightarrow h = x - p \to 0 \mid x = p + h$

$$L = \lim_{x \to p} \frac{f(x) - f(p)}{x - p} = \lim_{h \to 0} \frac{f(p+h) - f(p)}{(p+h) - p} = \lim_{h \to 0} \frac{f(p+h) - f(p)}{h}$$

b) $\lim_{h\to 0} \frac{f(p+3h)-f(p)}{h} = \lim_{h\to 0} \frac{f(p+3h)-f(p)}{3h} \cdot 3$ $\to L$

$$g(u) = \begin{cases} \frac{f(p+u)-f(p)}{u}, & u \neq 0 \\ L, & u = 0 \end{cases}$$

continua em 0

$u = 3h$

$h \to 0 \quad u \to 0$

$u \to 0 \Rightarrow \frac{f(p+u)-f(p)}{u} = L$

$= \lim_{h\to 0} g(3h) \cdot 3 = 3L$

$\to L$

c) $\lim\limits_{h\to 0} \dfrac{f(p+h)-f(p-h)}{h}$ $\qquad f(p)-f(p)=0$

$= \lim\limits_{h\to 0} \underbrace{\dfrac{f(p+h)-f(p)}{h}}_{L} + \underbrace{\dfrac{f(p)-f(p-h)}{h}}_{L} = 2L$

$u=-h$

$*\ \lim\limits_{h\to 0} -1 \cdot \dfrac{f(p)-f(p-h)}{h} \cdot -1 = \lim\limits_{h\to 0} \dfrac{f(p-h)-f(p)}{-h} = \lim\limits_{u\to 0} \dfrac{f(p+u)-f(p)}{u} = L$

d.) $\lim_{h\to 0} \frac{f(p-h)-f(p)}{h}$

$u = -h$

$= \lim_{u\to 0} \frac{f(p+u)-f(p)}{-u}$

$= \lim_{u\to 0} -1 \cdot \left[\underbrace{\frac{f(p+u)-f(p)}{u}}_{L}\right] = -L$

$a(b+c)$

$= ab + ac$

$$[x^x]' = \left[(e^{\underbrace{\ln x}_{x}})^{x}\right]'$$

$$(e^u)' = e^u \cdot u'$$

$$= \left[e^{\ln x \cdot x}\right]'$$

$$(f \cdot g)'$$

$$= e^{\ln x \cdot x} \cdot (\ln x \cdot x)'$$

$$= f' \cdot g + f g'$$

$$= x^x \cdot \left[\frac{1}{x} \cdot x + \ln x \cdot 1\right] = x^x [1 + \ln x]$$

$\ln a - \ln b$
$= \ln\left(\frac{a}{b}\right)$

$c \ln a$
$= \ln(a^c)$

$\lim_{u\to 0}(1+u)^{\frac{1}{u}} = e$

$$(\ln x)' = \lim_{h\to 0} \frac{\ln(x+h) - \ln x}{h}$$

$$= \lim_{h\to 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h} = \lim_{h\to 0} \frac{1}{h} \ln\left(1 + \frac{h}{x}\right)$$

$$= \lim_{h\to 0} \ln\left(1 + \frac{h}{x}\right)^{\frac{1}{h}} = \lim_{u\to 0} \ln(1+u)^{\frac{1}{u}\cdot\frac{1}{x}}$$

$\frac{1}{h} = \frac{1}{u}\cdot\frac{1}{x}$ $\quad u = \frac{h}{x}$

$$= \lim_{u \to 0} \frac{1}{x} \ln\left((1+u)^{\frac{1}{u}}\right) = \frac{1}{x} \cdot 1 = \frac{1}{x}$$

ln continua

e

$\ln e = 1$

$$\boxed{(e^x)' = e^x}$$

$f(x) = e^x \qquad f'(x) = e^x$

$f^{-1}(x) = \ln x$

$f(f^{-1}(x)) = x \Big\} \ f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$\therefore (\ln x)' = \frac{1}{e^{\ln x}} = \frac{1}{x}$$