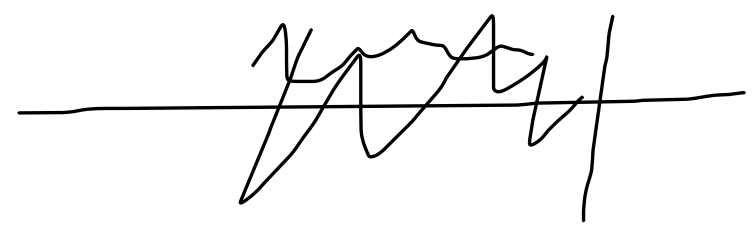
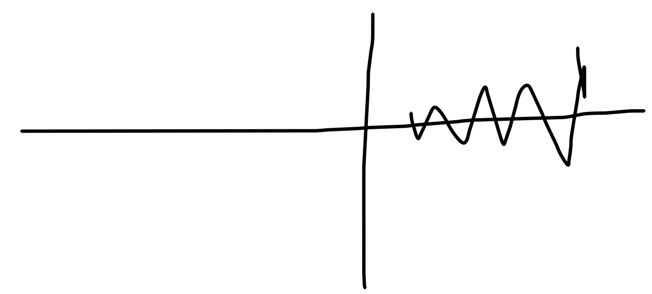
$$\lim_{x \to a} f(x) = L$$

$$\lim_{x \to a^-} f(x)$$



$$\lim_{x \to a^+} f(x)$$



$$\lim_{x \to a} \frac{p(x)}{q(x)} \qquad (x-a)$$

$a \in \mathbb{R}$

$p(a) = 0$

$q(a) = 0$

ou

$q(a) = 0$

$$\lim_{x \to +\infty} \frac{p(x)}{q(x)} \quad \text{ou} \quad \lim_{x \to -\infty} \frac{p(x)}{q(x)}$$

"puxa" o grau mais alto de $p(x)$ e de $q(x)$

$+\infty$, $-\infty$ não são números

Indeterminação → refazer de outro jeito

$$\lim_{x\to 1} \frac{\sqrt{4x^2+2x}-6}{2x^3+x^2-3x} \qquad \sqrt{4x^2+2x}-6 \to \sqrt{6}-6<0 \qquad 2x^3+x^2-3x \to 0$$

1	2	1	-3	0
		2	3	0
	2	3	0	0 resto

$$= \lim_{x\to 1} \frac{\sqrt{4x^2+2x}-6}{(x-1)(2x^2+3x)} \qquad \sqrt{4x^2+2x}-6 \to 6-\sqrt{6}<0 \qquad (x-1)\to 0 \qquad (2x^2+3x)\to 5>0$$

$\bar{n}$ esiste (i limiti laterali ≠)

$$\lim_{x\to 1^{+}/^{-}} \frac{\sqrt{4x^2+2x}-6}{(x-1)(2x^2+3x)} \qquad \sqrt{4x^2+2x}-6 \to (\sqrt{6}-6)<0 \qquad (x-1)\to 0^{+},\ 0^{-}<0 \qquad (2x^2+3x)\to 5>0$$

$$= -\infty \qquad +\infty$$

$$\lim_{x \to 5} \frac{1}{(x-5)^{10}}$$

par

0^{+}

$$\lim_{x \to 5^-} \frac{1}{(x-5)^{11}} = \lim_{x \to 5^-} \frac{1}{(x-5)} \cdot \frac{1}{(x-5)^{10}}$$

> 0

$+ , - , \circ , \div$

$f \circ g$

$f(x) = x^2 + 1$

$g(x) = 3x - 2$

$f \circ g(x) = f(g(x))$

$f(3x-2) = (3x-2)^2 + 1$

$= 9x^2 - 6x - 4 + 1$

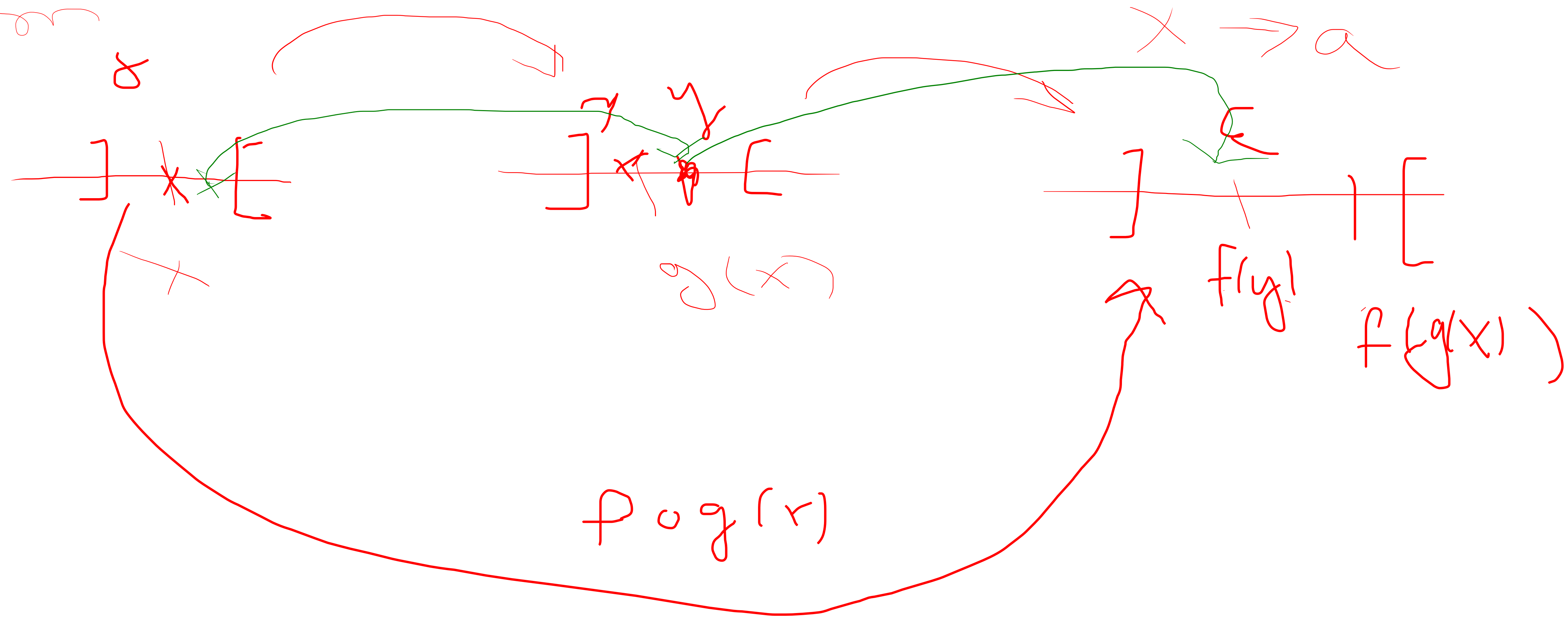
$g \circ f(x) = g(f(x)) = g(x^2+1)$

$= 3(x^2+1) - 2 = 3x^2 + 3 - 2 = 3x^2 + 1$

Tesi: $\lim_{x\to a} f\circ g(x) = f\left(\lim_{x\to a} g(x)\right)$

se f è continua in $\lim_{x\to a} g(x) = L$

Dim.



$\varepsilon > 0 \quad \exists \gamma > 0 \quad |y - L| < \gamma \Rightarrow |f(y) - f(L)| < \varepsilon$

f continua in L

$\exists \delta > 0$

$0 < |x - a| < \delta \Rightarrow |g(x) - L| < \gamma$

Ex $f(x) = x^{20} + 3x^2$

$g(x) = x^{100} + 1$

$$= \lim_{x \to 1} f \circ g(x)$$

$$= f\left(\lim_{x \to 1} g(x)\right) = f(g(1))$$

$$= f(2) = 2^{20} + 12$$

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1 \quad (\text{limite fundamental})$$

$u = 2x \xrightarrow[x \to 0]{} 0$ $\quad \frac{\operatorname{sen} u}{u} \xrightarrow[u \to 0]{} 1$

Ex: $$\lim_{x \to 0} \frac{\operatorname{sen} 2x}{x} = \lim_{x \to 0} \frac{\operatorname{sen} 2x}{2x} \cdot 2$$

$$f(x) = \begin{cases} 1 & \text{se } x = 0 \\ \frac{\operatorname{sen} x}{x} & x \neq 0 \end{cases}$$ é contínua em 0

$$\lim_{x \to 0} f(\overset{\to 0}{2x}) \cdot 2$$

$= f(0) \cdot 2 = 1 \cdot 2 = 2.$

$$\lim_{x \to 0} \frac{\cos x - 1}{x} \cdot \frac{(\cos x + 1)}{(\cos x + 1)}$$

$$= \lim_{x \to 0} \frac{\cos^2 x - 1}{x} \cdot \frac{1}{\cos x + 1}$$

$$= \lim_{x \to 0} \frac{-\sin^2 x}{x} \cdot \frac{1}{\cos x + 1}$$

lim. fund.

$$= \lim_{x \to 0} - \underbrace{\frac{\sin x}{x}}_{1} \cdot \underbrace{\sin x}_{0} \cdot \frac{1}{\underbrace{\cos x + 1}_{2}} = 0$$

$$\lim_{x\to 1} \frac{\operatorname{sen}(2x^2-2)}{x-1}\cdot\frac{2(x+1)}{2(x+1)}$$

$$= \lim_{x\to 1} \underbrace{\frac{\operatorname{sen}(2x^2-2)}{2x^2-2}}_{1}\cdot\underbrace{2(x+1)}_{4}$$

$u = 2x^2-2$

$x\to 1 \Rightarrow u \to 0$

$u \to 0 \Rightarrow \frac{\operatorname{sen} u}{u} \to 1$

$$= 4$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{\operatorname{sen}\left(\frac{\pi}{2} - x\right)}{2x - \pi}$$

$$-2\left(\frac{\pi}{2} - x\right) = -\pi + 2x = 2x - \pi$$

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{\operatorname{sen}\left(\frac{\pi}{2} - x\right)}{-2\left(\frac{\pi}{2} - x\right)} = -\frac{1}{2}$$

1

$u = \frac{\pi}{2} - x \quad x \to \frac{\pi}{2} \Rightarrow u \to 0$

$u \to 0 \Rightarrow \frac{\operatorname{sen} u}{u} \to 1$

lim fund.

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} \cdot \frac{(1+\cos x)}{1+\cos x}$$

$$\frac{\operatorname{sen}^2 x}{x^2} = \frac{(\operatorname{sen} x)^2}{x^2} = \left(\frac{\operatorname{sen} x}{x}\right)^2$$

$$= \lim_{x \to 0} \frac{\operatorname{sen}^2 x}{x^2} \cdot \frac{1}{1+\cos x}$$

lim fund.

$$= \lim_{x \to 0} \left(\frac{\operatorname{sen} x}{x}\right)^2 \cdot \frac{1}{1+\cos x}$$

$\to 1^2 = 1$

$\to \frac{1}{2}$

$$= \frac{1}{2}$$

$\operatorname{sen}(a+b)$

$\cos(a+b)$

$$\frac{\operatorname{sen}(x+h) - \operatorname{sen} x}{h}$$